



UNIVERSITÉ DU  
LUXEMBOURG

# Natural Language Processing

**RNNs and Transformer**

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# Lecture Plan

1. Recurrent Neural Networks

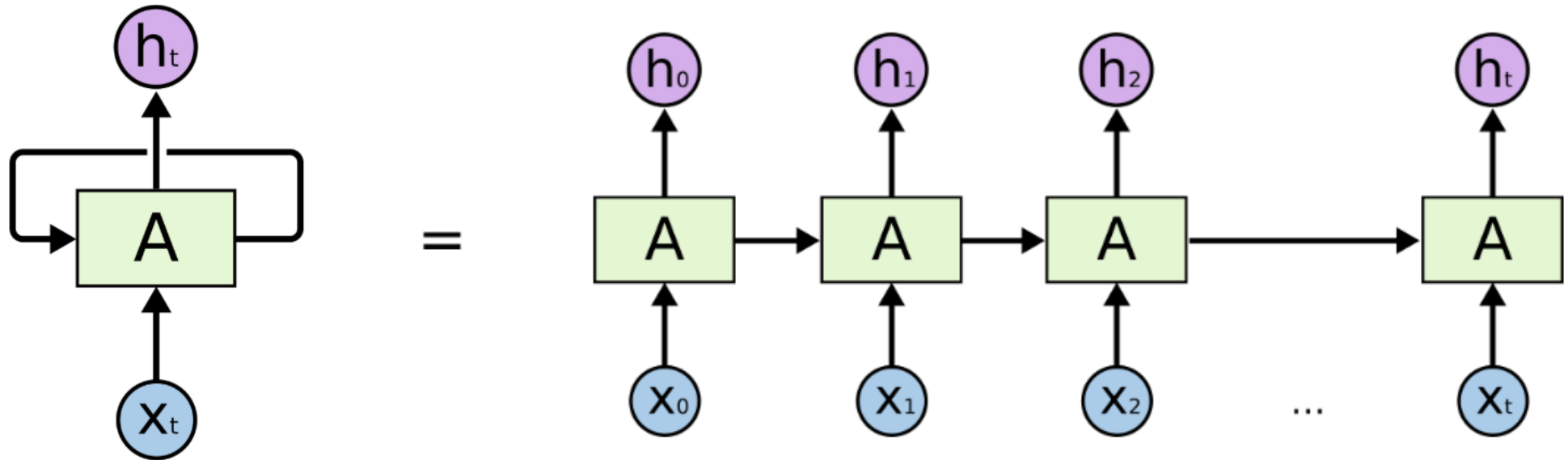
2. Transformers

# *Recurrent Neural Networks*

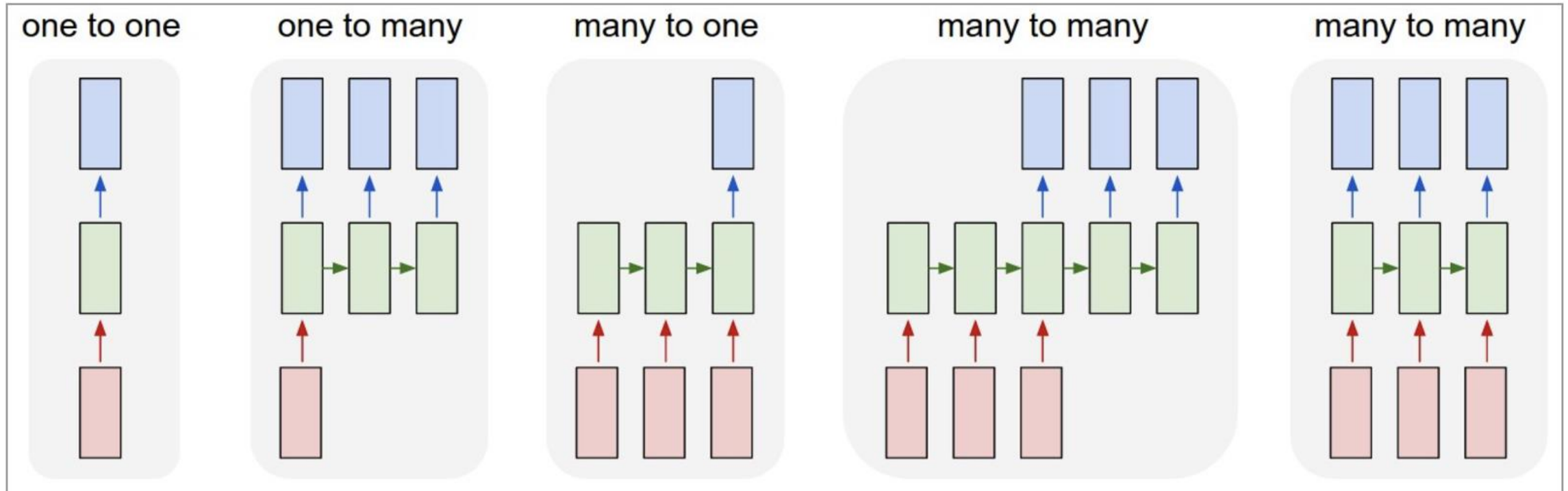
# Why do we need RNNs

- How to represent these two sentences with word embeddings?  
Adding vectors? Averaging?
  - the dog eats the cat.
  - the cat eats the dog.

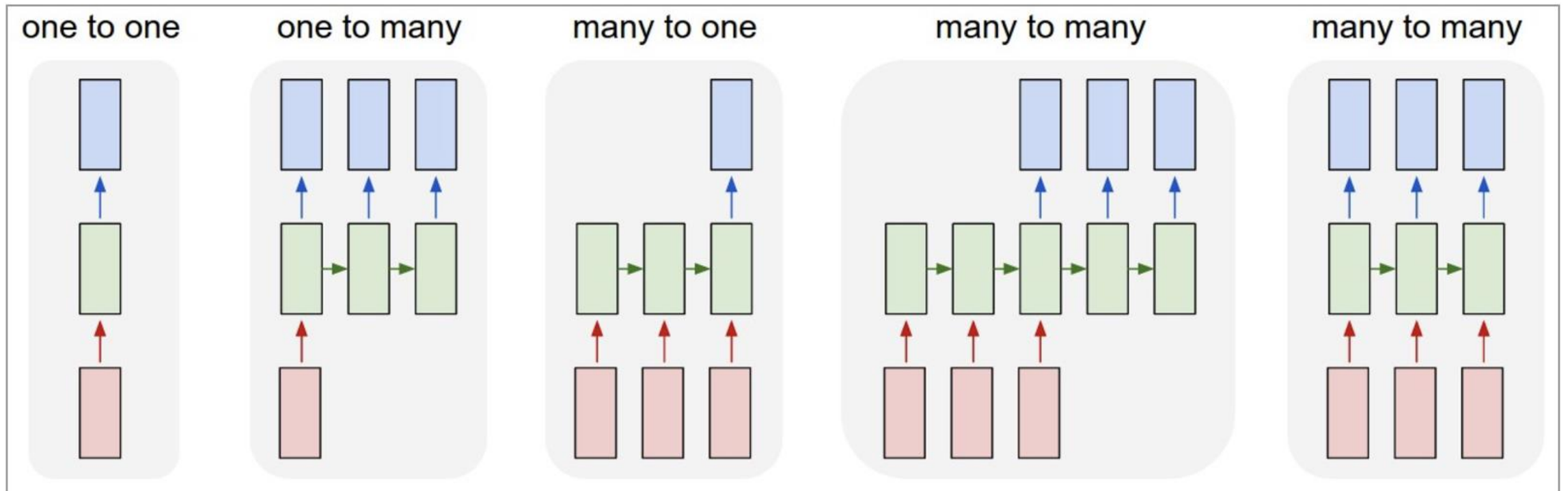
# RNN design



# Sequence Learning



# Sequence Learning



Vanilla NN

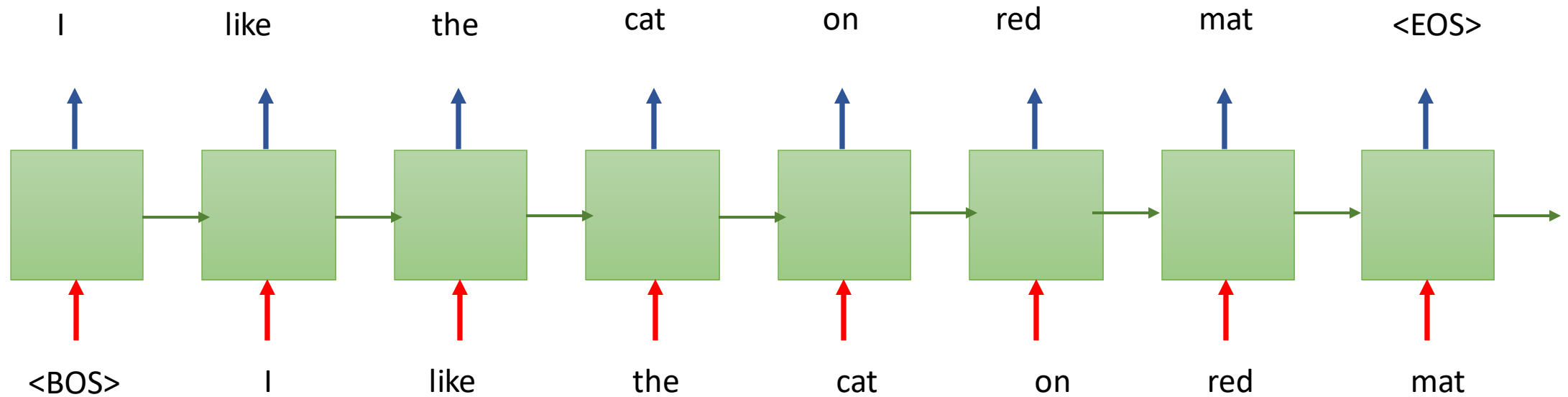
Image Captioning

Classification  
(Sentiment analysis)

Sequence to Sequence  
Machine Translation  
Chatbot

Language Modelling  
Video captioning  
POS Tagging  
NER

# Neural Language Modelling





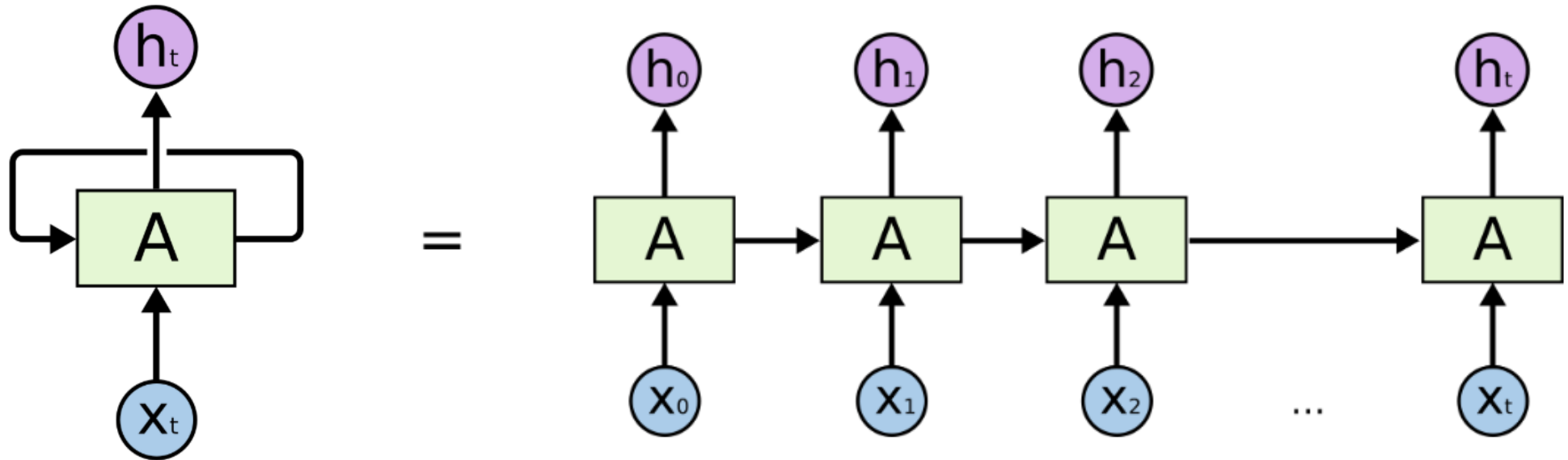
# Introduction to RNNs

- **Language & Temporal Phenomenon:** Language unfolds over time; requires sequential modeling.
- **RNNs:** Designed to process sequences, capturing dependencies across time without fixed context windows.
- **Applications:** Language modeling, sequence labeling, text classification.

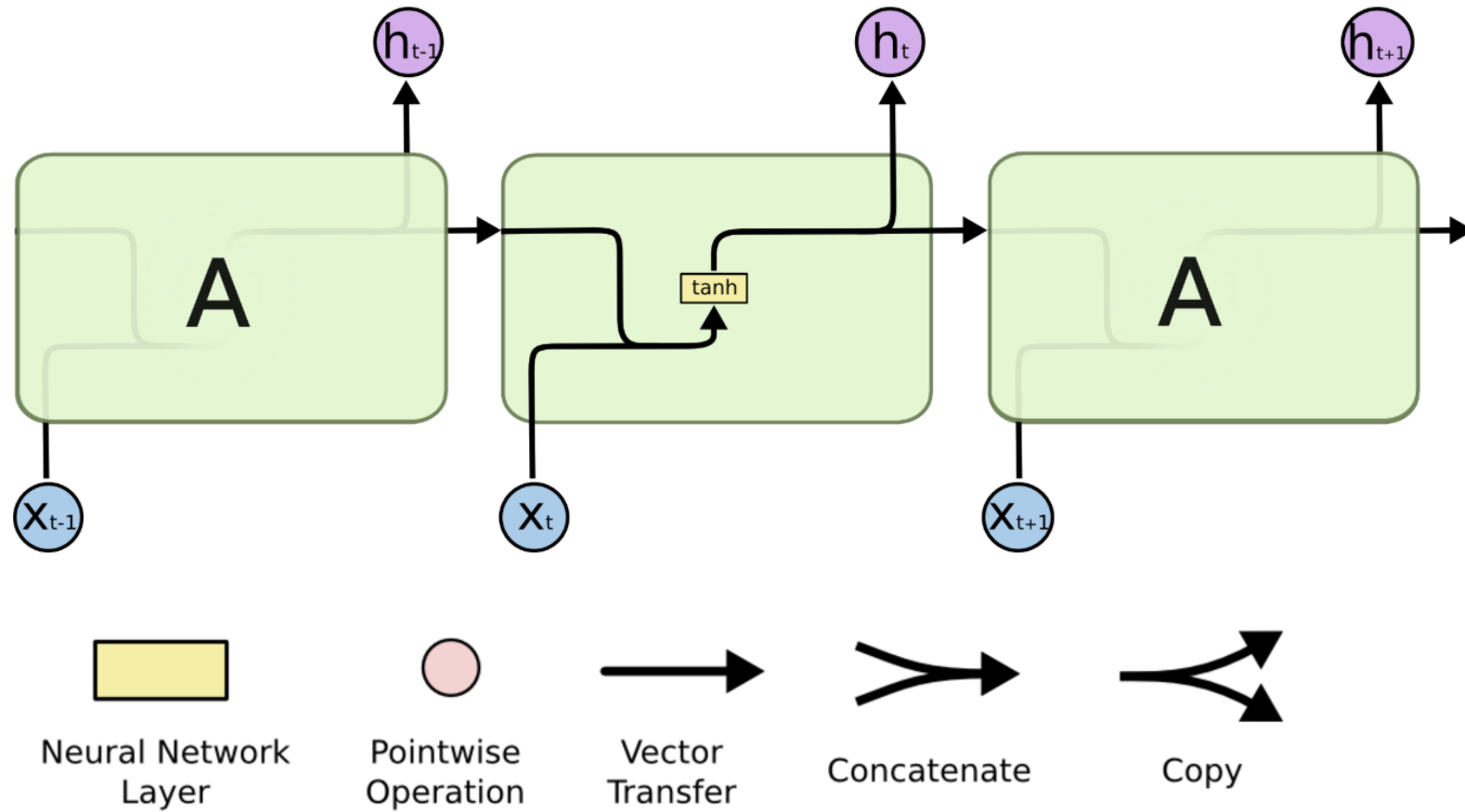
# Introduction to RNNs

- Core Concept: Recurrent connections enable "memory" of previous states.
- Unrolling: Process input sequence step-by-step; weights shared across time.

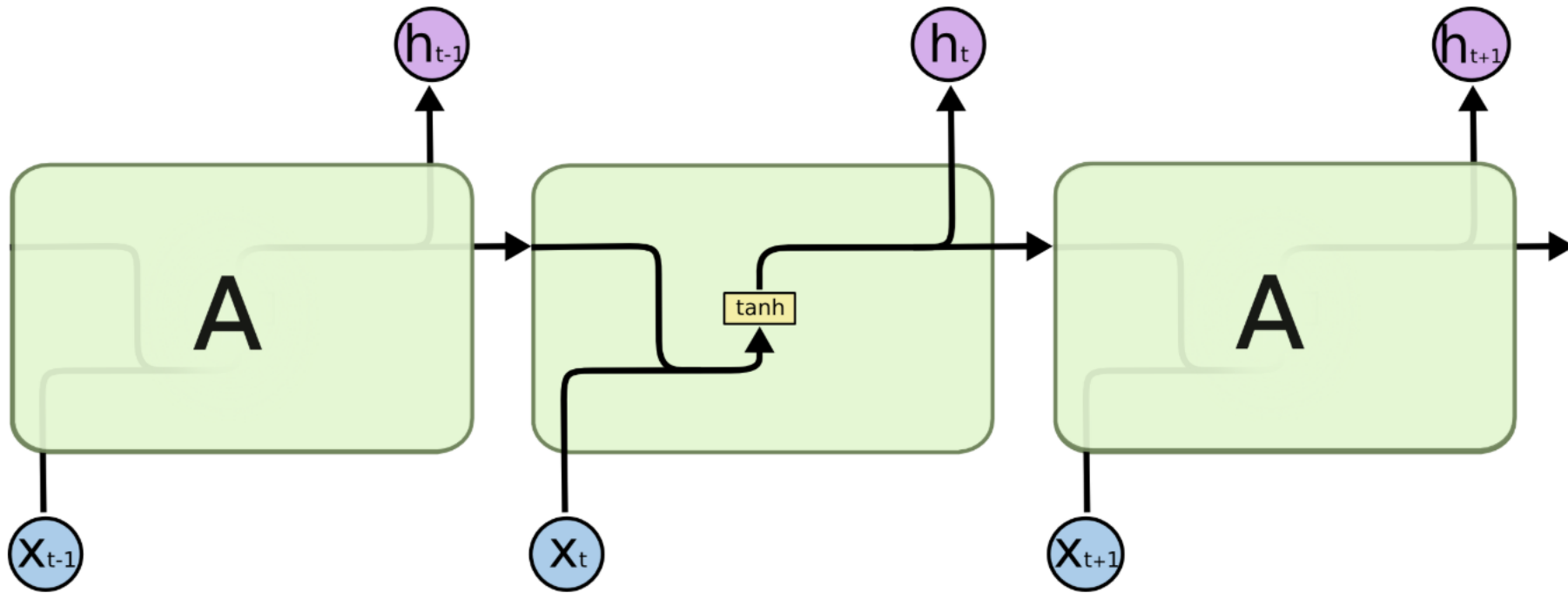
# Recurrent Neural Network



# Simple RNN



# Simple RNN



$$h_t = \tanh(W_h h_{t-1} + W_x x_t + b)$$

# Training RNNs

- Backpropagation Through Time (BPTT): Two-phase training:
  - **Forward pass**: Compute activations and loss.
  - **Backward pass**: Compute gradients through the unrolled network.

# Problems with RNN

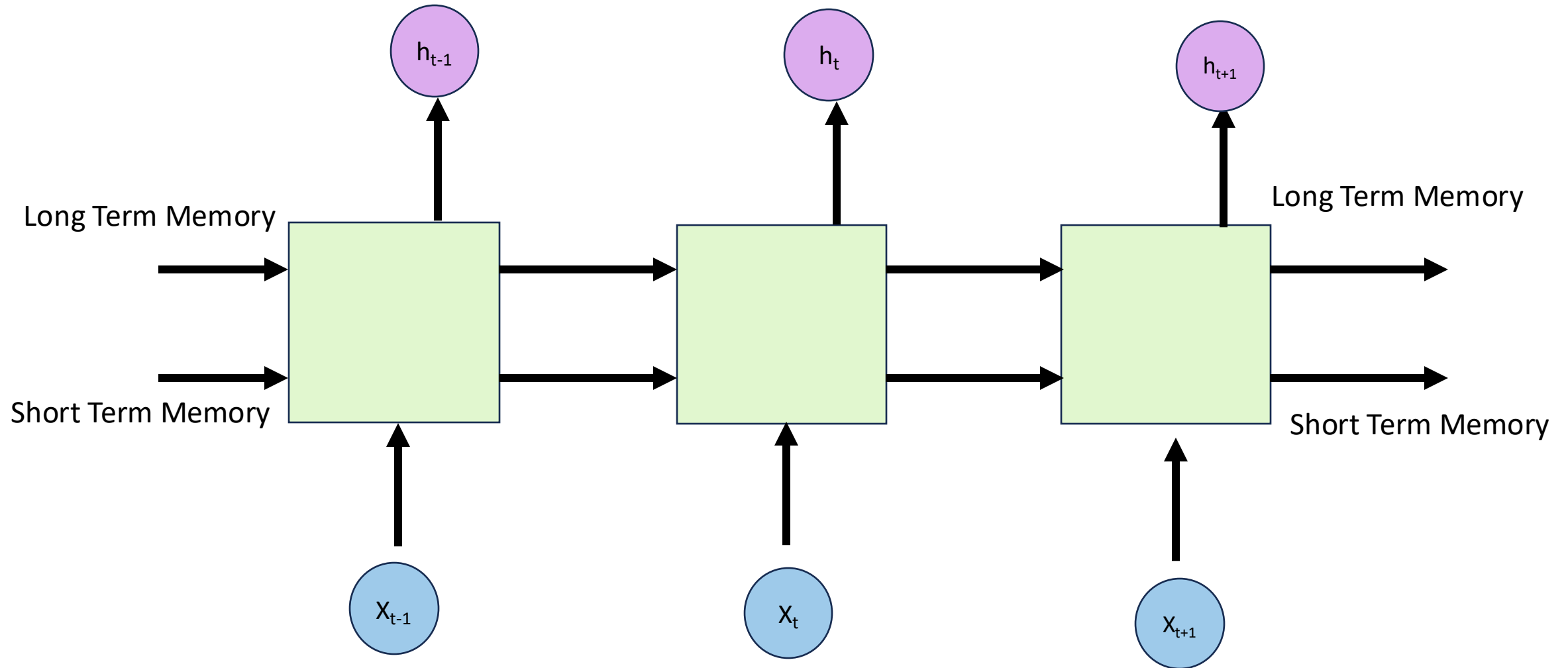
- **Vanishing Gradient Problem**

- gradients can become very small (vanish) as they are propagated backward through time

- **RNNs struggle to maintain information over long sequences:**

- They are inherently biased toward recent inputs

# LSTM (Long Short Term Memory)





# LSTM (Long Short Term Memory)

- LSTMs include a "memory cell" that can maintain information over long periods.
- By using memory cells and gates, LSTMs allow gradients to flow through time *more smoothly*, avoiding both vanishing and exploding gradient problems.

# LSTM (Long Short Term Memory)

- LSTMs fix this by introducing:
  - A **cell state**  $c_t$  that can carry information *almost* unchanged over many steps.
  - **Gates** that control what to keep, what to forget, and what to output.

# an Example

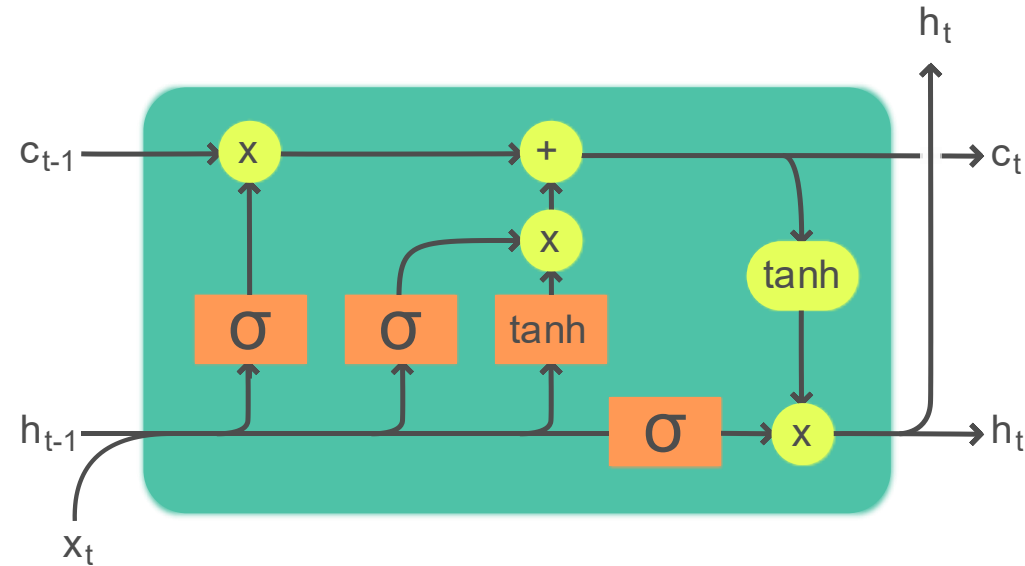
- "In **2010**, she moved to Paris. She lived there for **five years**."



Imagine network is at this timestep

- And we want to predict:  
in **2015** she returned

# LSTM architecture



Legend:

Layer



Componentwise



Copy

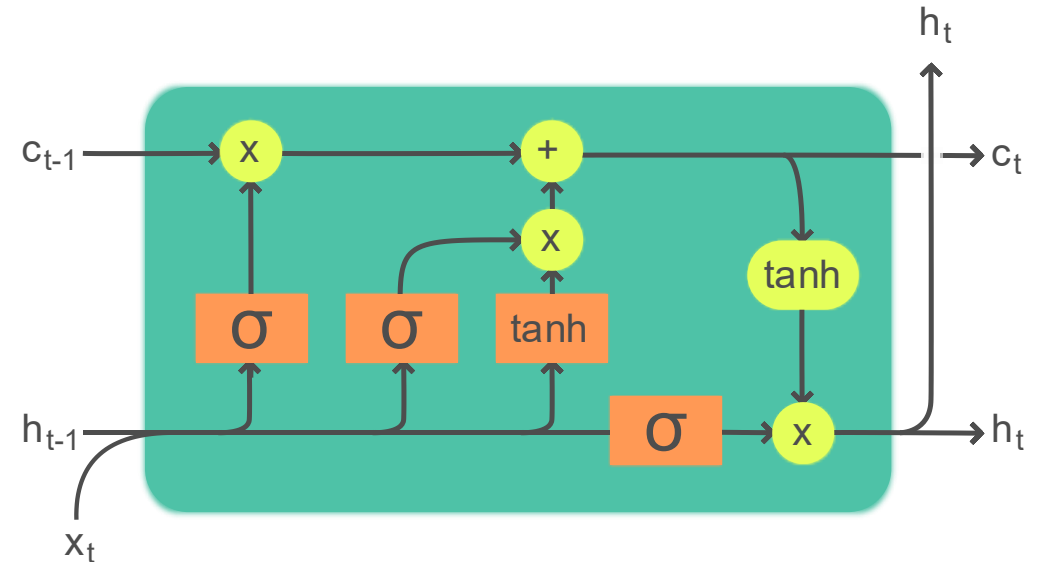


Concatenate

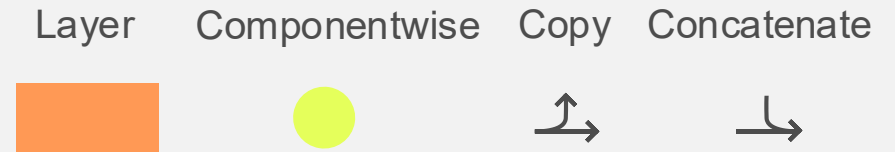


# LSTM architecture

- **Forget gate:** what part of old memory to erase.
- **Input gate:** what new information to write.
- **Output gate:** what part of the memory to expose as the hidden state.



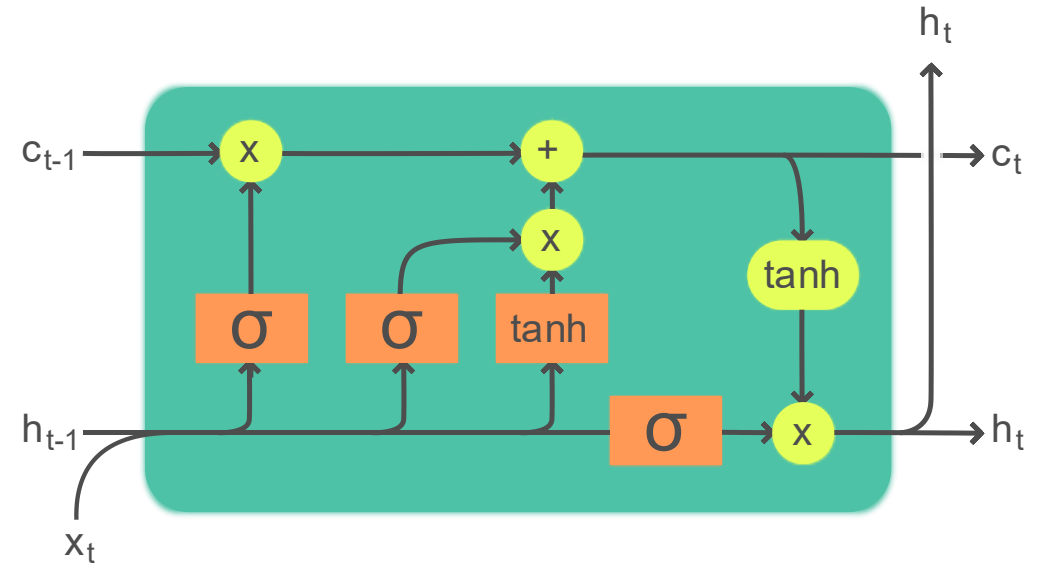
Legend:



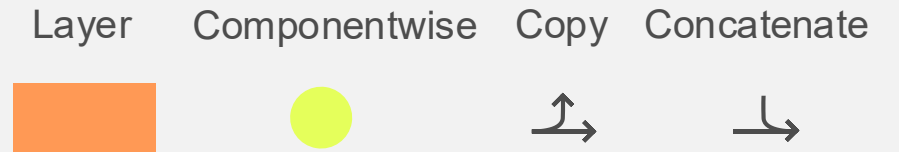
# LSTM architecture

- **Concatenate** input and previous hidden state:

$$z_t = [h_{t-1}; x_t]$$



Legend:



# LSTM architecture

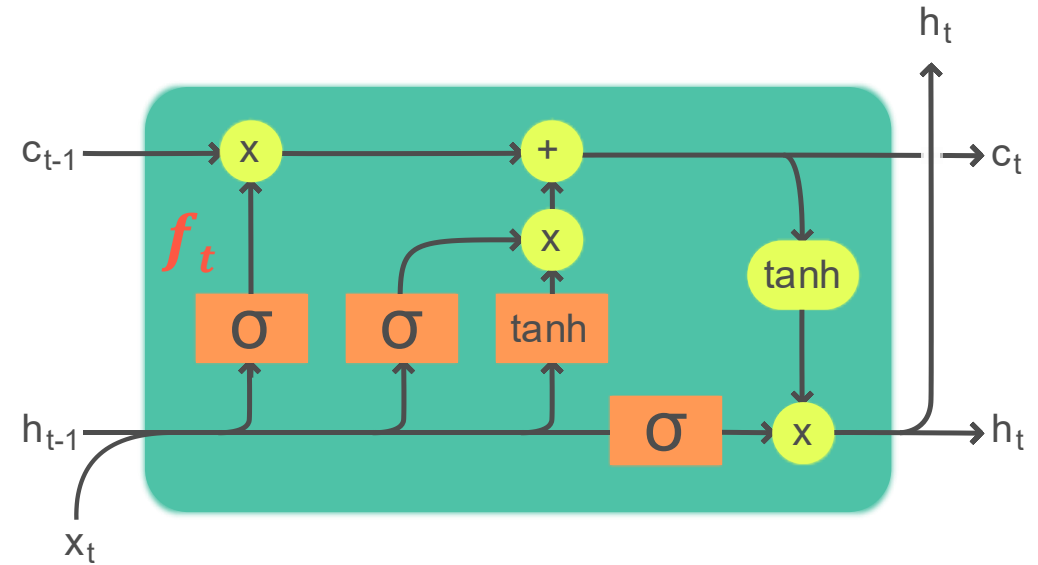
- **Concatenate** input and previous hidden state:

$$z_t = [h_{t-1}; x_t]$$

- **Gates** (each element is in  $(0, 1)$  via sigmoid  $\sigma$ ):

- Forget gate:

$$f_t = \sigma(W_f z_t + b_f)$$



Legend:

Layer



Componentwise



Copy



Concatenate



# LSTM architecture

- **Concatenate** input and previous hidden state:

$$z_t = [h_{t-1}; x_t]$$

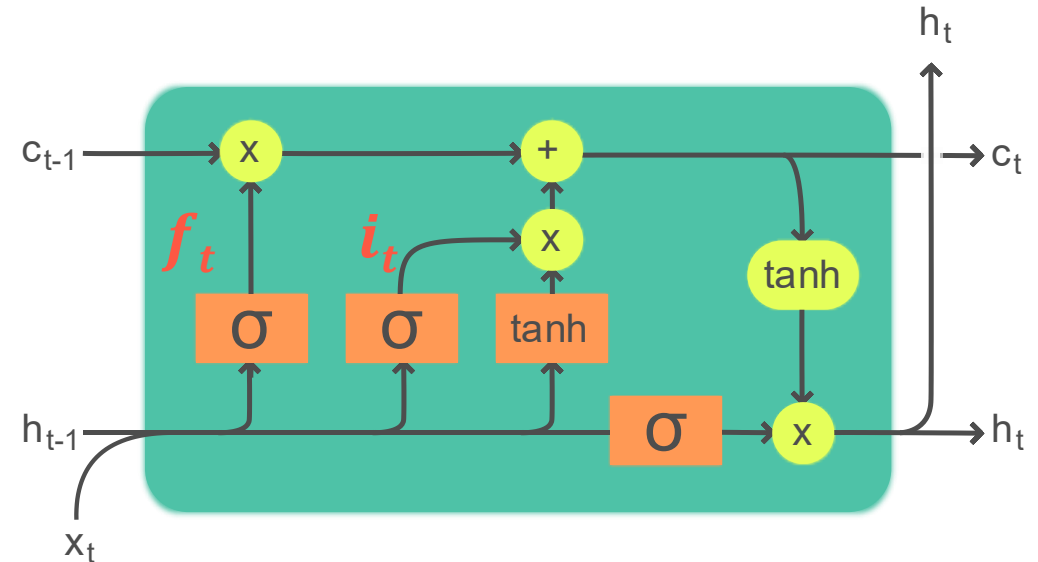
- **Gates** (each element is in  $(0, 1)$  via sigmoid  $\sigma$ ):

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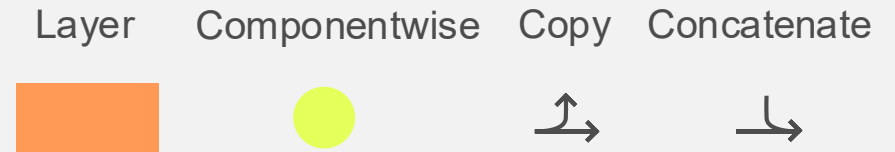
$$f_t = \sigma(W_f z_t + b_f)$$

- Input gate:

$$i_t = \sigma(W_i z_t + b_i)$$



Legend:





# LSTM architecture

- **Concatenate** input and previous hidden state:

$$z_t = [h_{t-1}; x_t]$$

- **Gates** (each element is in  $(0, 1)$  via sigmoid  $\sigma$ ):

- Forget gate:

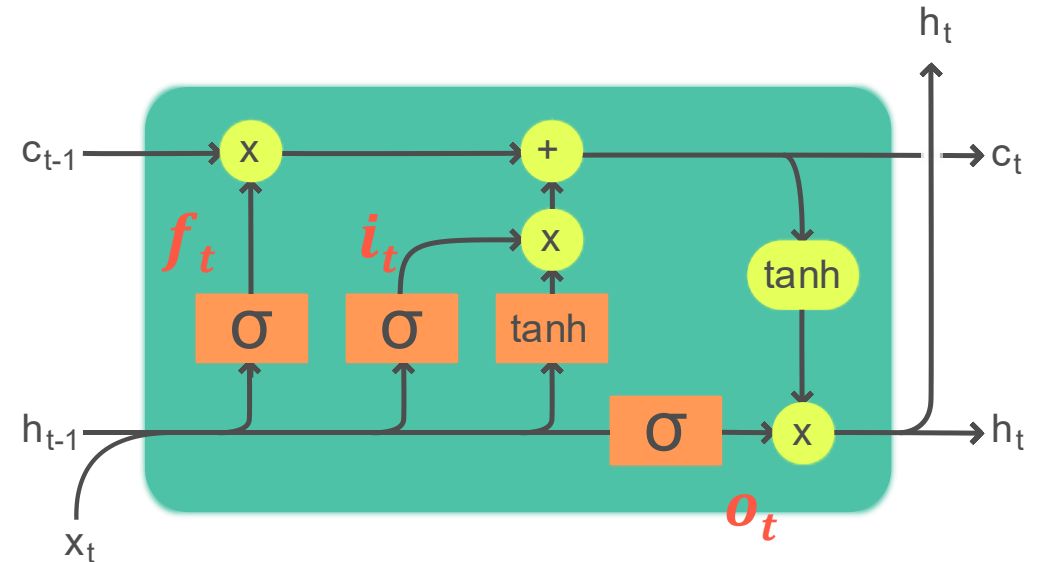
$$f_t = \sigma(W_f z_t + b_f)$$

- Input gate:

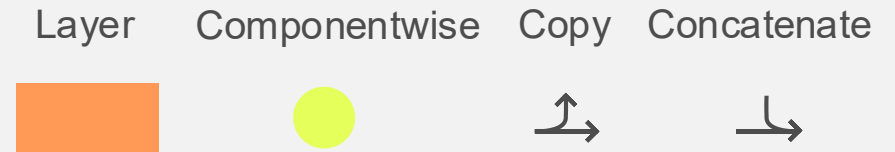
$$i_t = \sigma(W_i z_t + b_i)$$

- Output gate:

$$o_t = \sigma(W_o z_t + b_o)$$



Legend:



# LSTM architecture

- **Concatenate** input and previous hidden state:

$$z_t = [h_{t-1}; x_t]$$

- **Gates** (each element is in  $(0, 1)$  via sigmoid  $\sigma$ ):

- Forget gate:

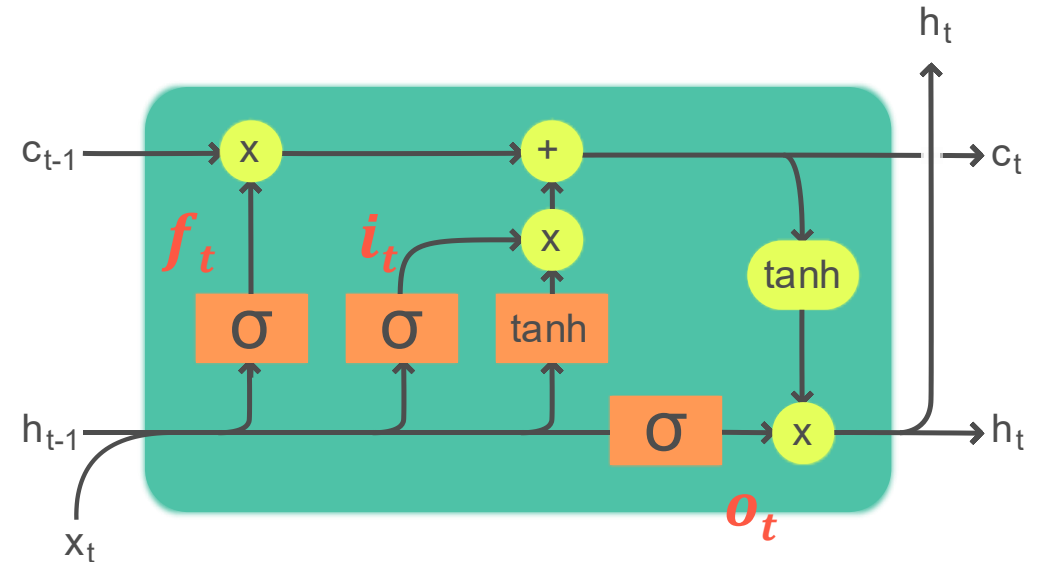
$$f_t = \sigma(W_f z_t + b_f)$$

- Input gate:

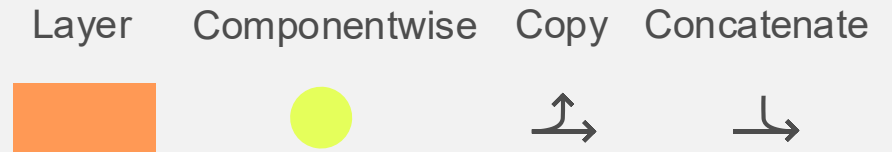
$$i_t = \sigma(W_i z_t + b_i)$$

- Output gate:

$$o_t = \sigma(W_o z_t + b_o)$$



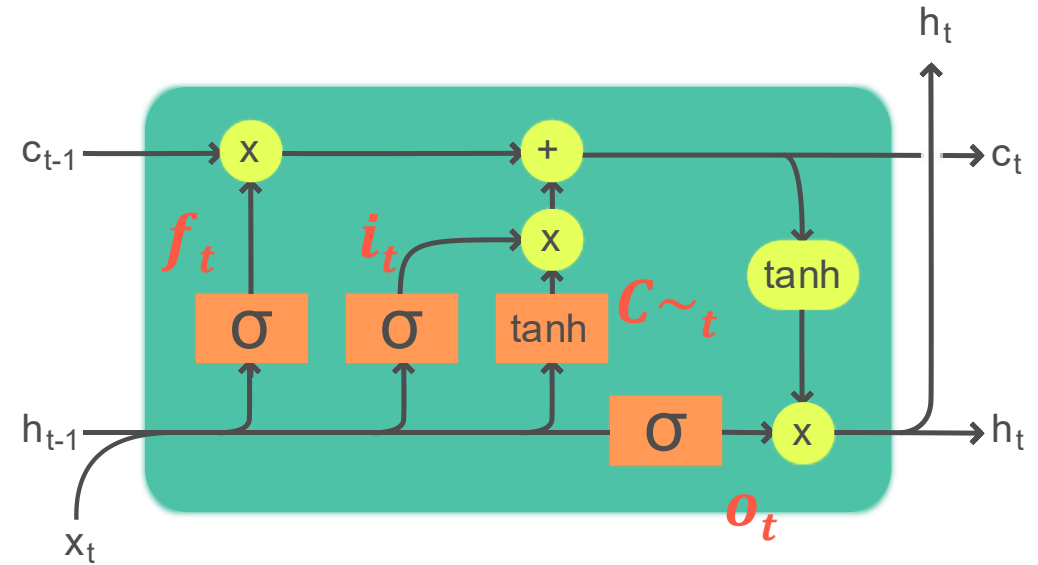
Legend:



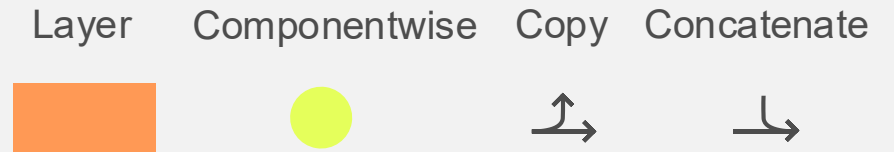
# LSTM architecture

- **Candidate content** to add to memory  
tanh squashes to  $(-1, 1)$ :

- $c_{\sim_t} = \tanh(W_c z_t + b_c)$

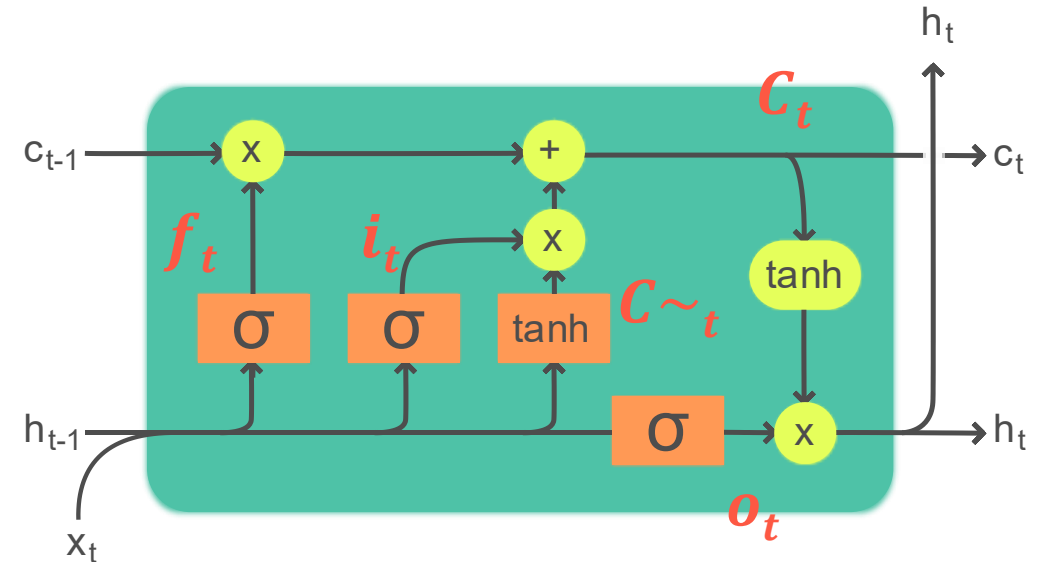


Legend:

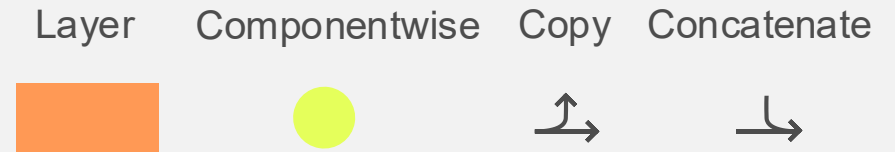


# LSTM architecture

- **Candidate content** to add to memory  
tanh squashes to  $(-1, 1)$ :
  - $c_{\sim t} = \tanh(W_c z_t + b_c)$
- **Update cell state:**
  - $c_t = f_t \odot c_{t-1} + i_t \odot \tilde{c}_t$
- If a component of  $f_t$  is close to 1, that part of the old memory is kept.
- If a component of  $i_t$  is close to 1, we write the new candidate into memory.



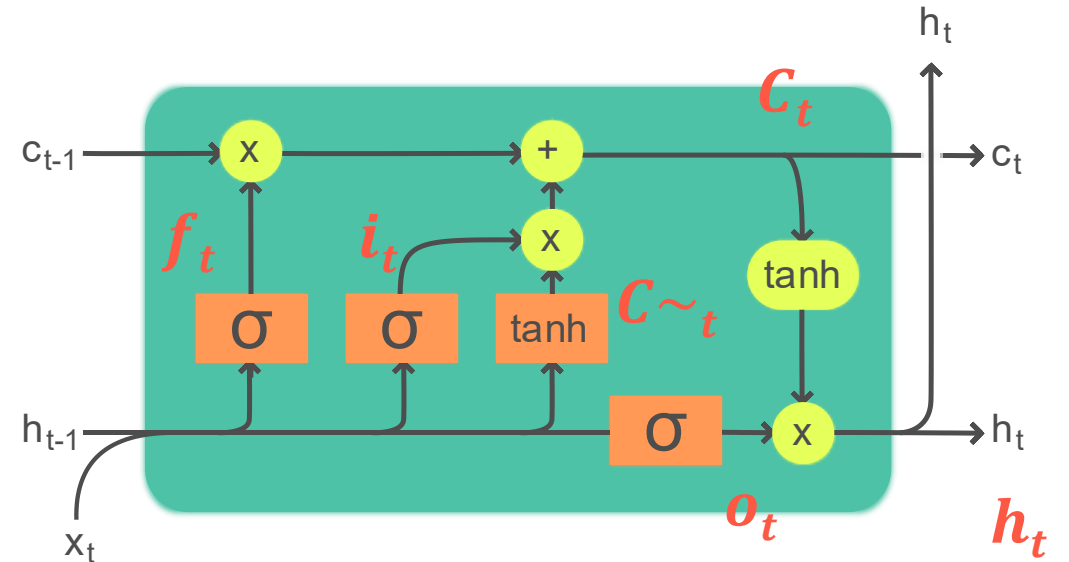
Legend:



# LSTM architecture

- **Update hidden state:**

$$h_t = o_t \odot \tanh(c_t)$$



Legend:

Layer



Componentwise



Copy



Concatenate

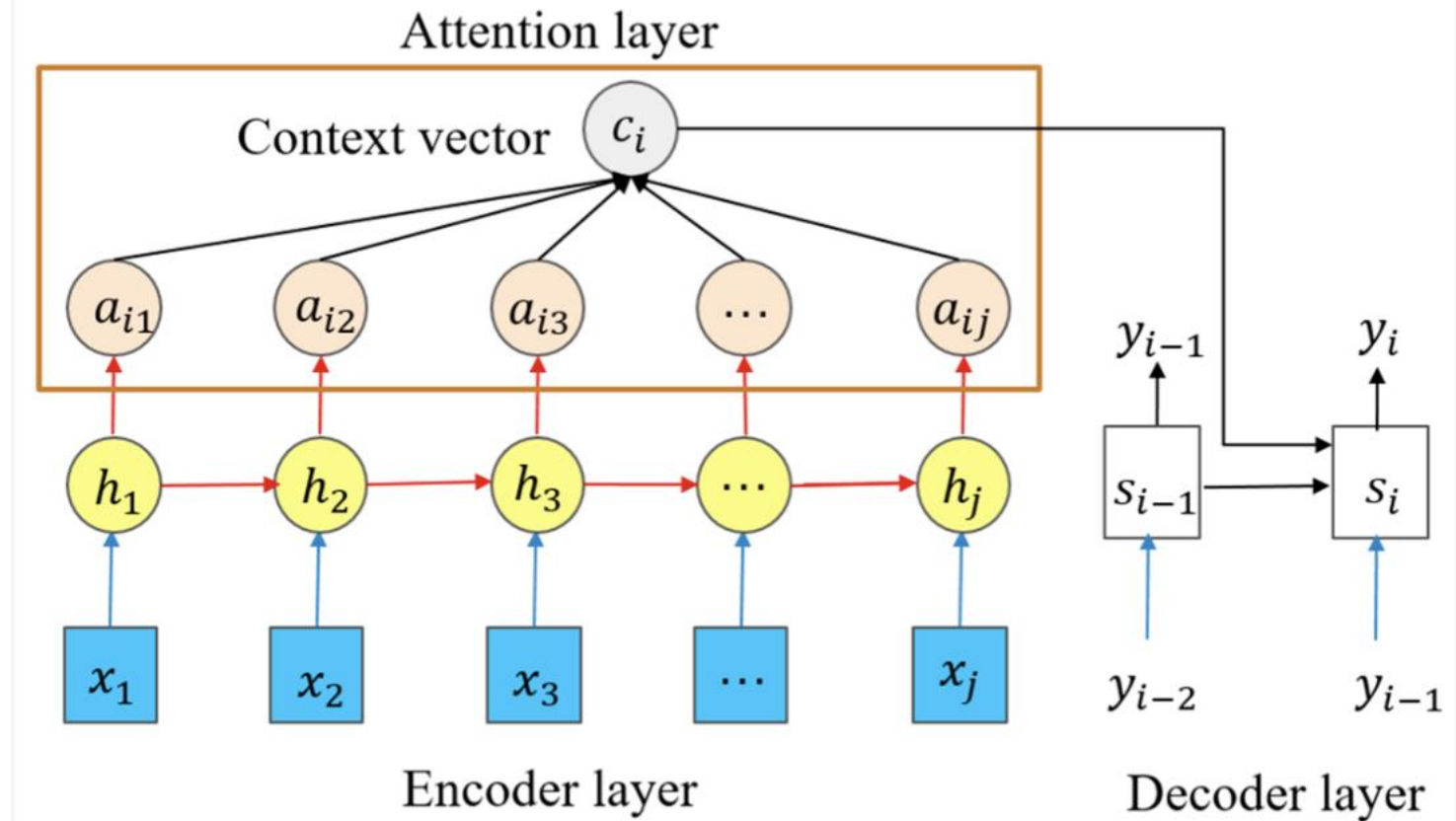


# LSTM Variants

- Gated Recurrent Unit
  - SEE FOR YOURSELF!

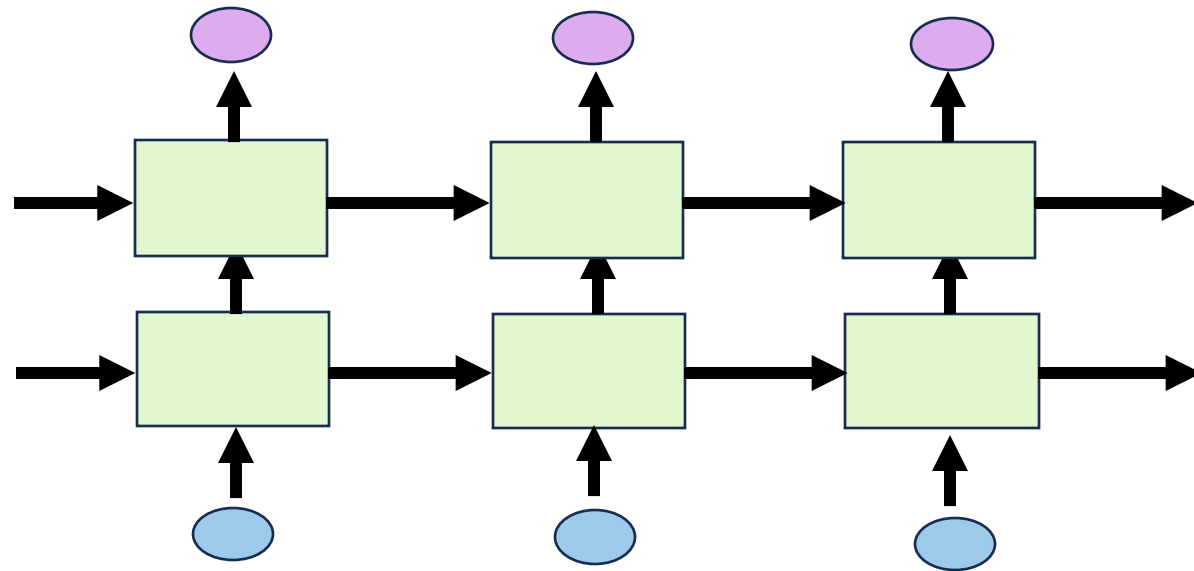
# LSTM with attention

- LSTM attention mechanism
- Attention = extra mechanism that lets the decoder look back at all encoder hidden states instead of only a single fixed vector.



# Variants of RNNs

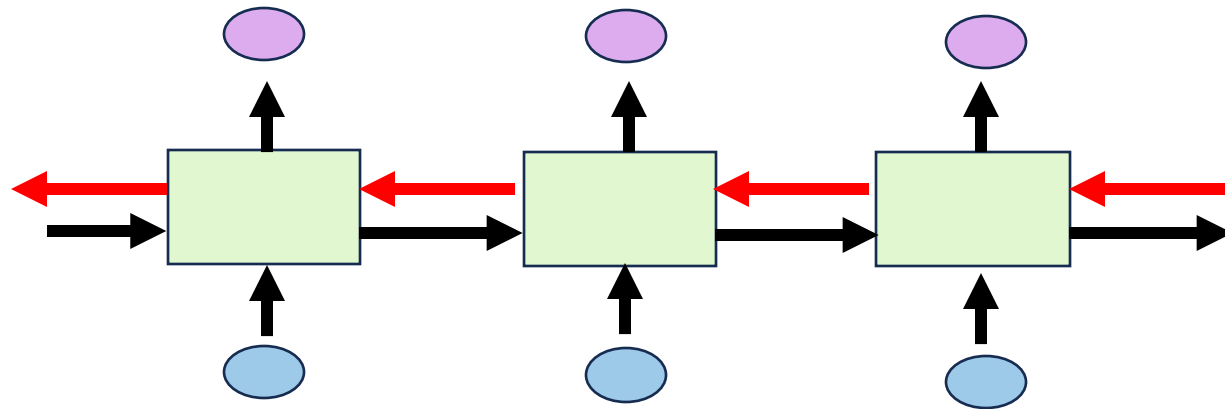
- Stacked RNNs:
  - Multiple RNN layers; output of one feeds into the next.





# Variants of RNNs

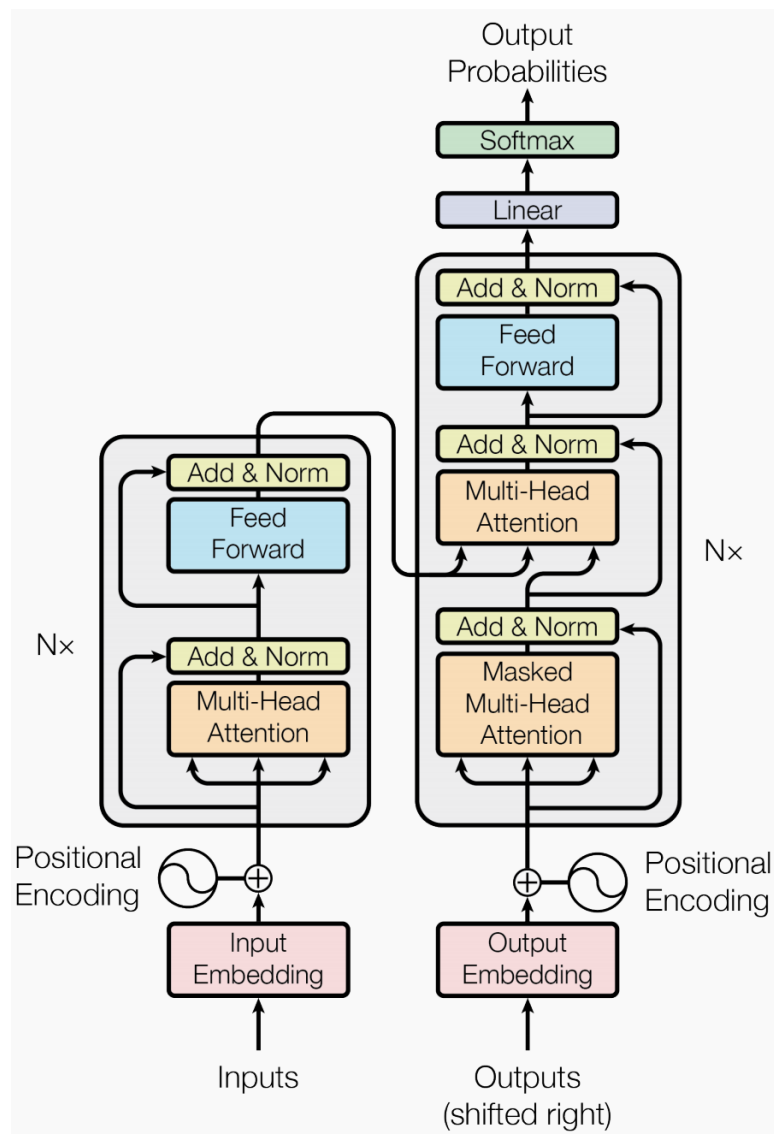
- Bidirectional RNNs:
  - Combines forward and backward RNNs for richer context.
  - Applications: Sequence labelling and classification.



# *Transformers*

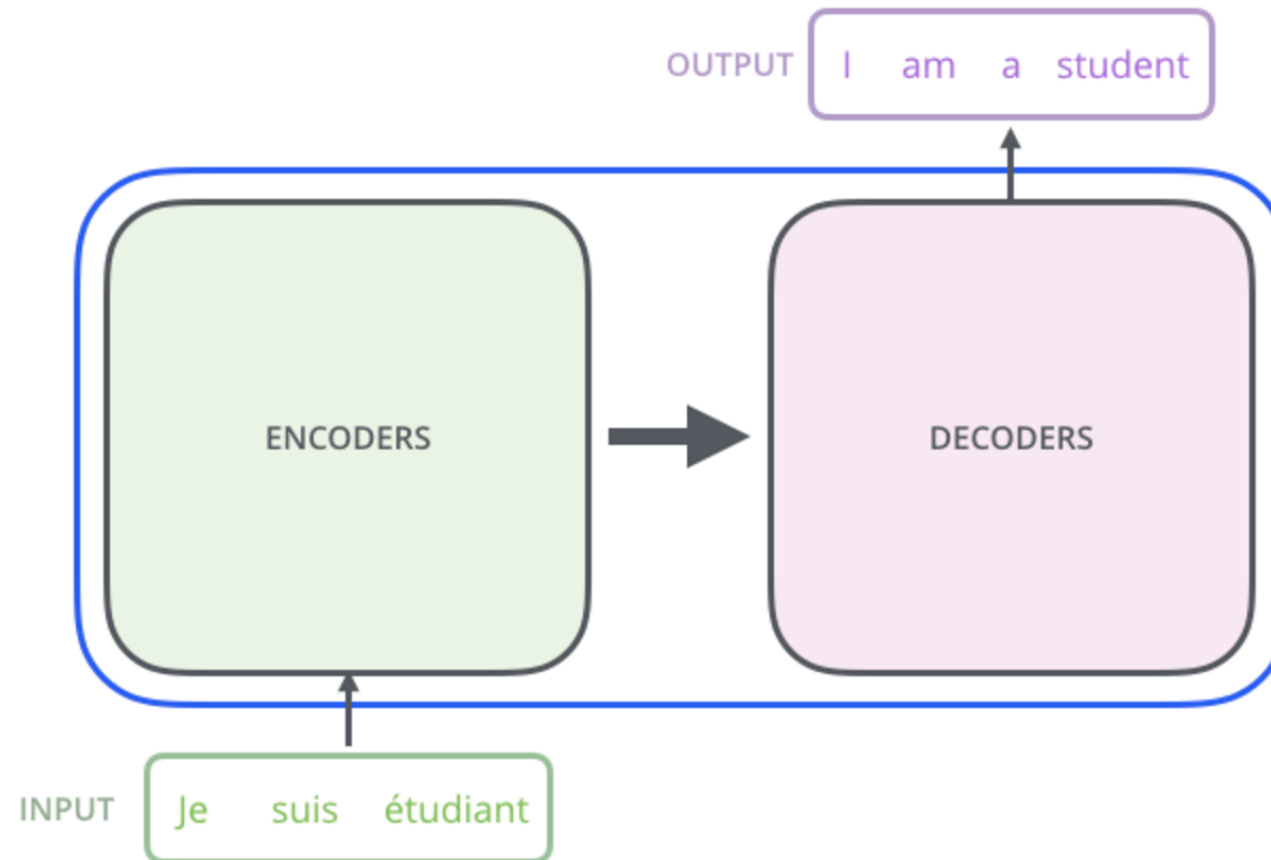
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# Transformers



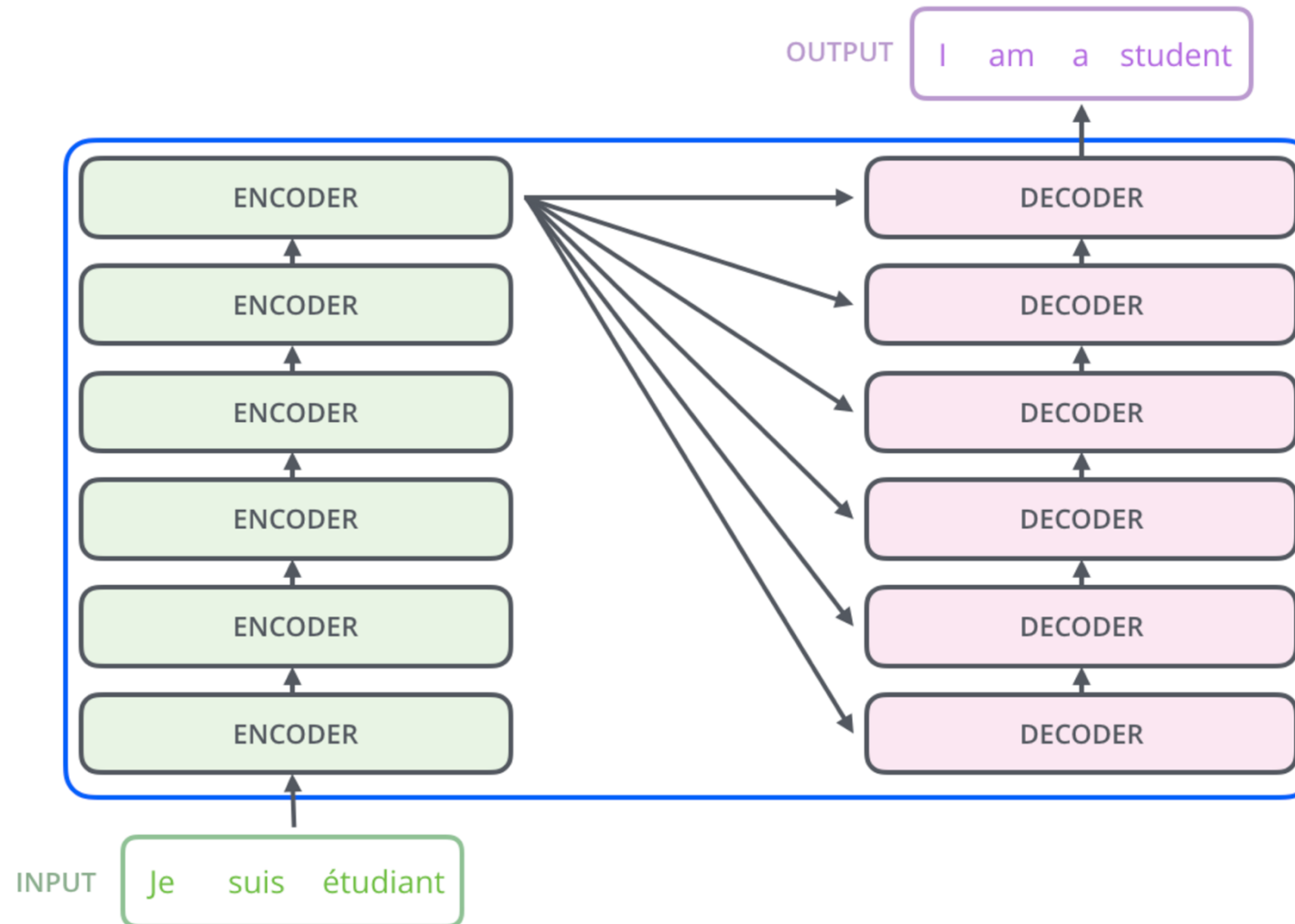
# Introduction to Transformers

- Stacks of **encoders** and **decoders**



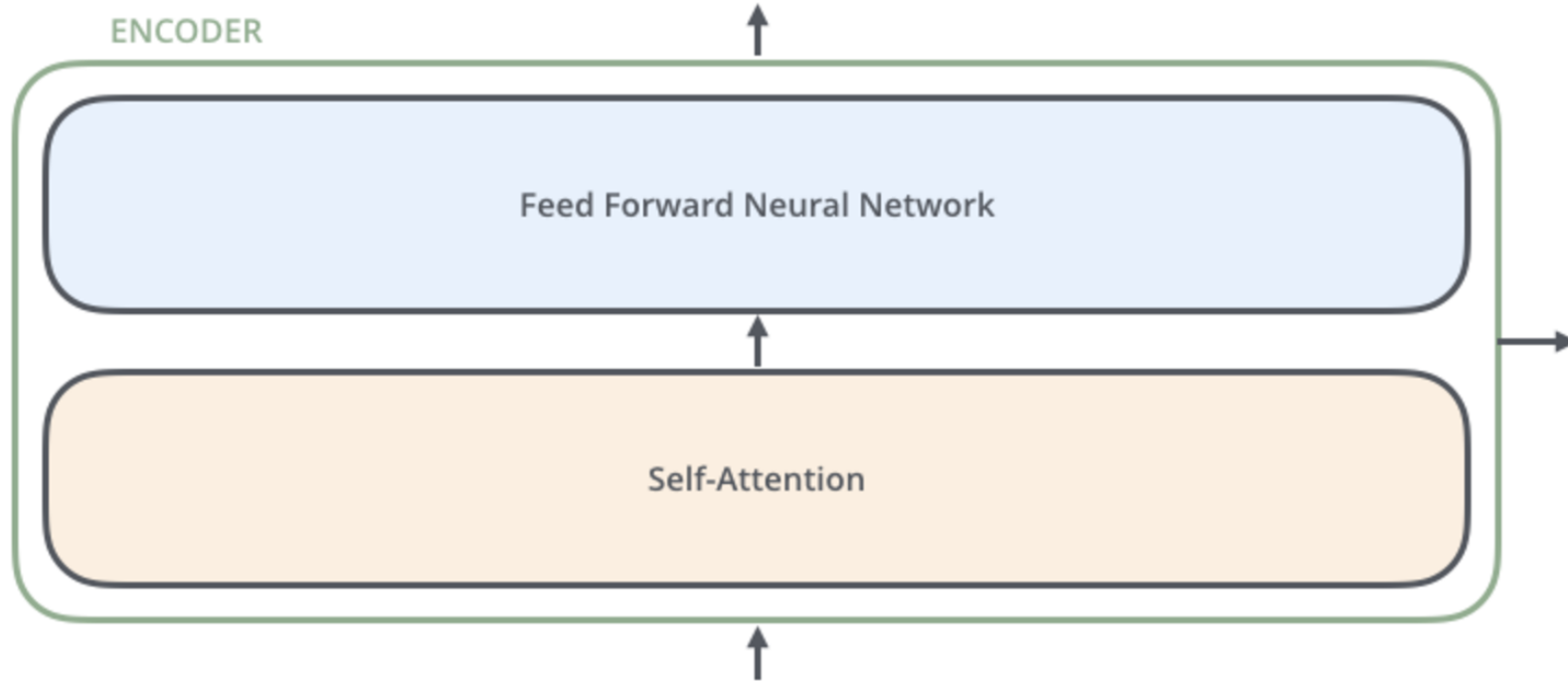
# Introduction to Transformers

- Stacks of **encoders** and **decoders**



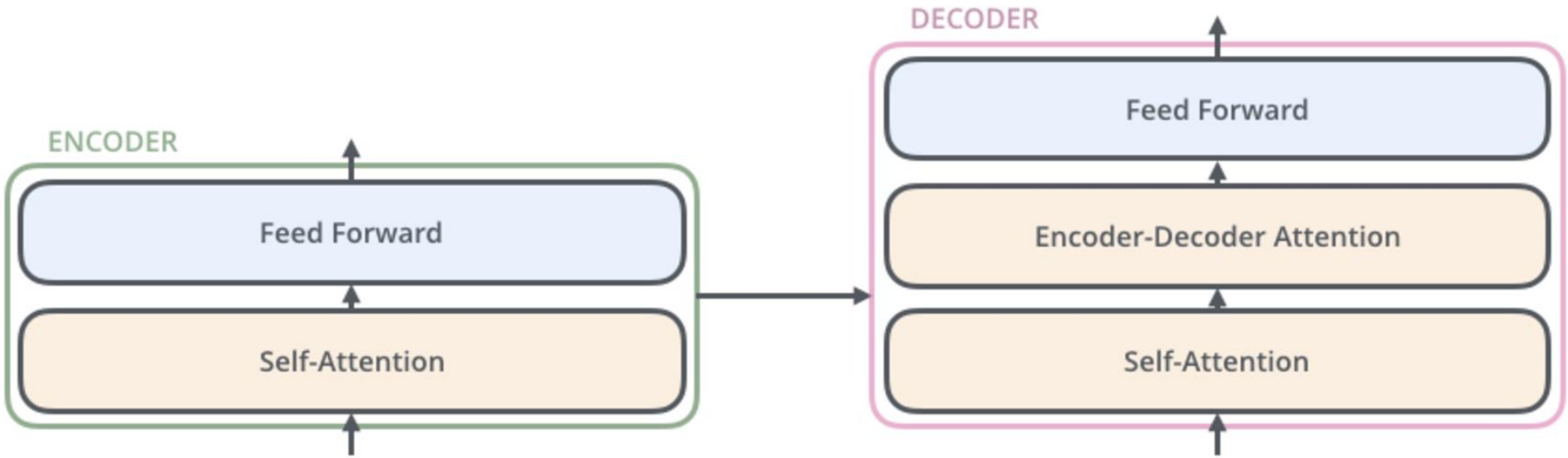
# Encoder

- Stacks of **encoders**

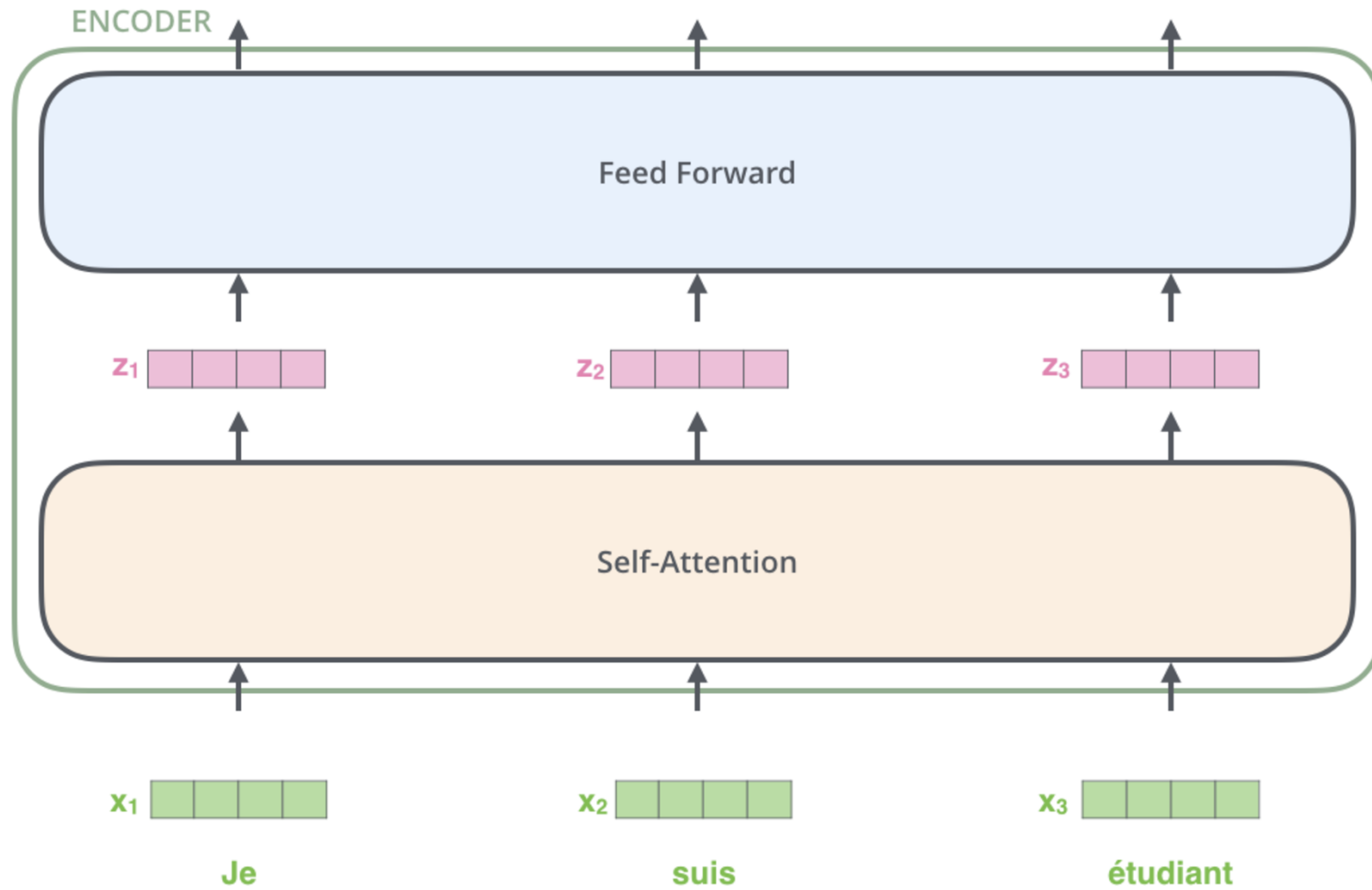


# Encoder - Decoder

- Stacks of **encoders** and **decoders**

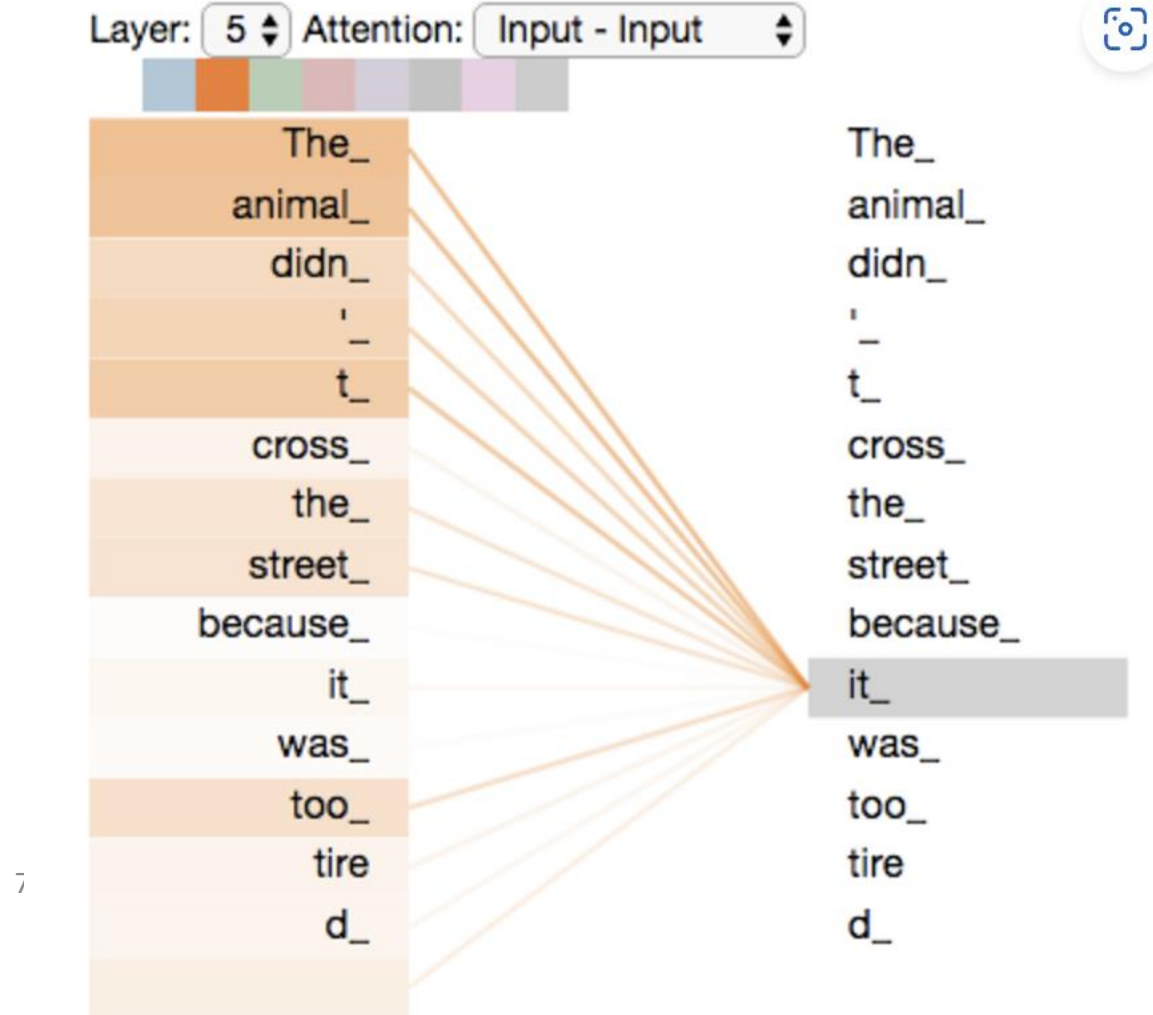


# Attention mechanism





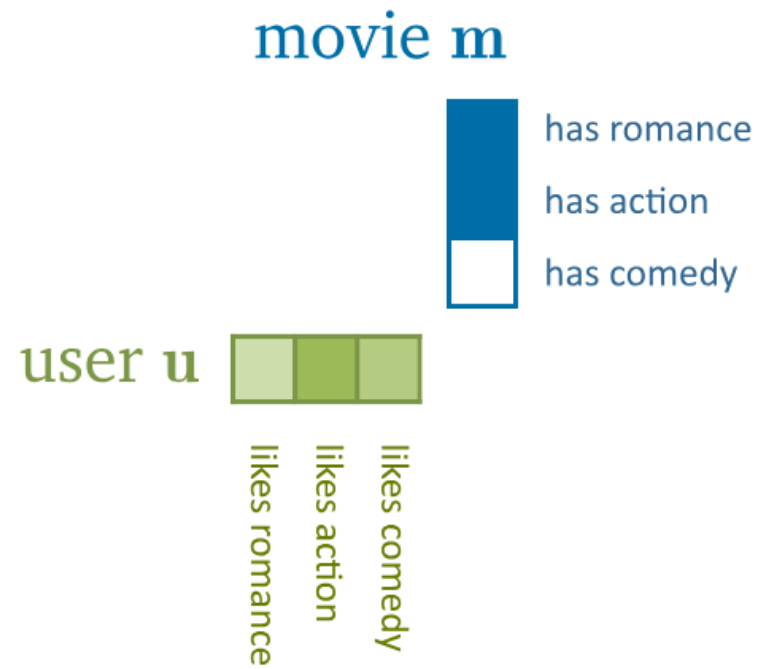
# Self-Attention



# Key Mechanism: Attention

- **Self-Attention:** Computes contextualized token embeddings by attending to other tokens.
- **Multi-Head Attention:**
  - Multiple parallel attention heads for capturing diverse linguistic features

# Dot product



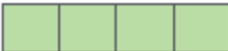
# Self-Attention

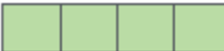
Input

Thinking


Machines

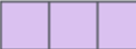
Embedding

$x_1$  

$x_2$  

Queries

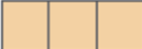
$q_1$  

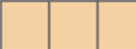
$q_2$  



$W^Q$

Keys

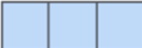
$k_1$  

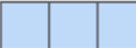
$k_2$  



$W^K$

Values

$v_1$  

$v_2$  



$W^V$

# Self-attention

- Start with input token embeddings as a matrix  $X \in \mathbb{R}^{T \times d_{model}}$ :
- $T$  :number of tokens in the sequence
- $d_{model}$  :embedding / hidden size
- The transformer makes three new matrices:
$$Q = XW_Q, \quad K = XW_K, \quad V = XW_V$$
- where  $W_Q, W_K, W_V$  are learned weight matrices.

# Matrix Calculation of Self-Attention

$$\text{softmax}\left(\frac{\begin{matrix} \text{Q} \\ \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array} \end{matrix} \times \begin{matrix} \text{K}^T \\ \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \end{matrix}}{\sqrt{d_k}}\right) \begin{matrix} \text{V} \\ \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array} \end{matrix}$$

=

$\text{Z}$

$\begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array}$

# Matrix Calculation of Self-Attention

$$\text{Attention}(Q, K, V) = \text{Softmax}\left(\frac{QK^{\top}}{\sqrt{d_k}}\right)V$$

# Self-attention

- **1. Query matrix  $Q$**

Shape:  $T \times d_k$

Row  $Q_i$  = “What does token  $i$  want to look for in other tokens?”

- **Intuition:**

Each token asks a question: *“Which other tokens are relevant to me, given my role in this position?”*

That question is encoded as the query vector  $Q_i$ .

- You use these queries when you compute attention scores:
- $\text{score}_{i,j} = Q_i \cdot K_j$
- $Q_i$  is the “search pattern” of token  $i$ .



# Self-attention

- **2. Key matrix  $K$**

- Shape:  $T \times d_k$
- Row  $K_j$  “=What information does token  $j$  offer?”

- **Intuition:**

Each token advertises what it is about.

The key vector  $K_j$  describes features that others might want to attend to: subject, object, position, etc., depending on what the model has learned.

- In the score  $Q_i \cdot K_j$ , you are matching:

- “What token  $i$  is looking for” (query)  
with  
“What token  $j$  offers” (key).

- High dot product = strong match = high attention weight from  $i$  to  $j$ .

# Self-attention

- **3.Value matrix  $V$**

- Shape:  $T \times d_v$
- Row  $V_j$  “=What information does token  $j$  pass along if you decide to attend to it?”

- **Intuition:**

Once the model decides that token  $j$  is relevant to token  $i$  (via query–key similarity), it still needs to know *what* to take from  $j$ . That payload is  $V_j$ .

- The final output for token  $i$  is a weighted average of values:

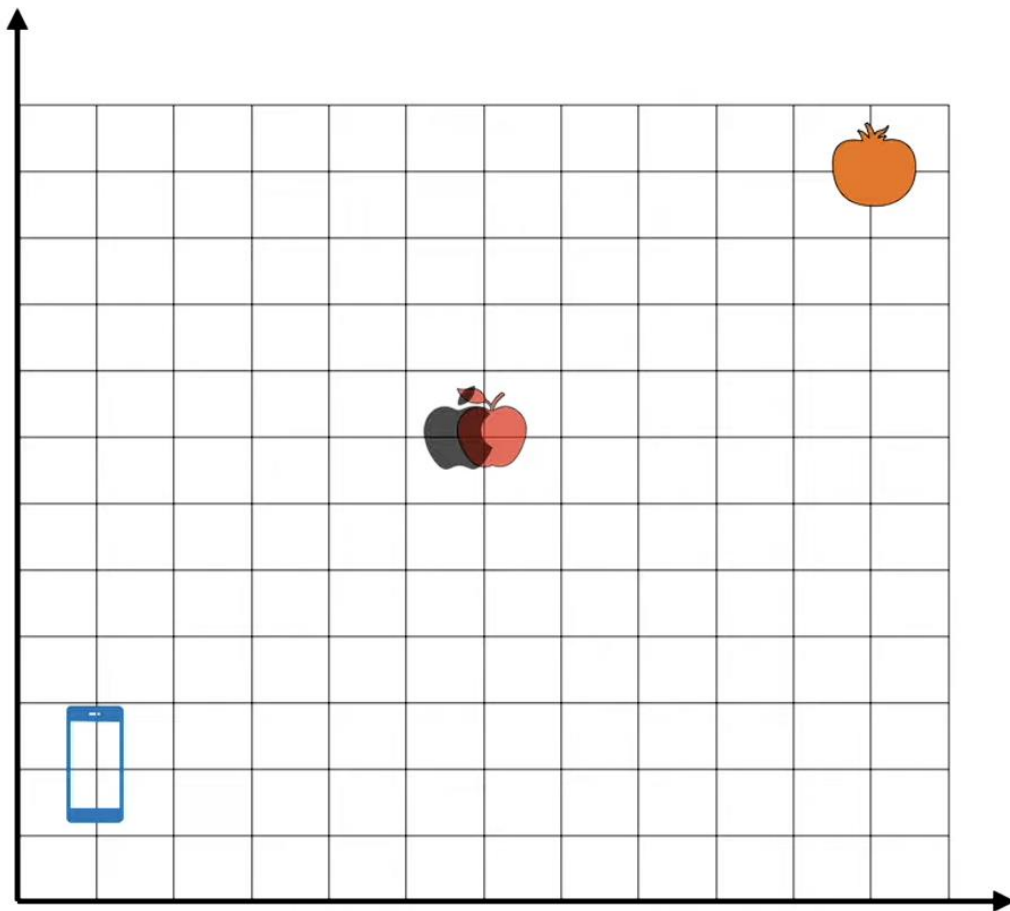
$$\text{output}_i = \sum_{j=1}^T \alpha_{i,j} V_j,$$

- where  $\alpha_{i,j}$  are the attention weights (softmax of scores).

# Self-attention recap

- Queries: what each token is asking for.
- Keys: what each token offers.
- Values: what information each token contributes when others attend to it.

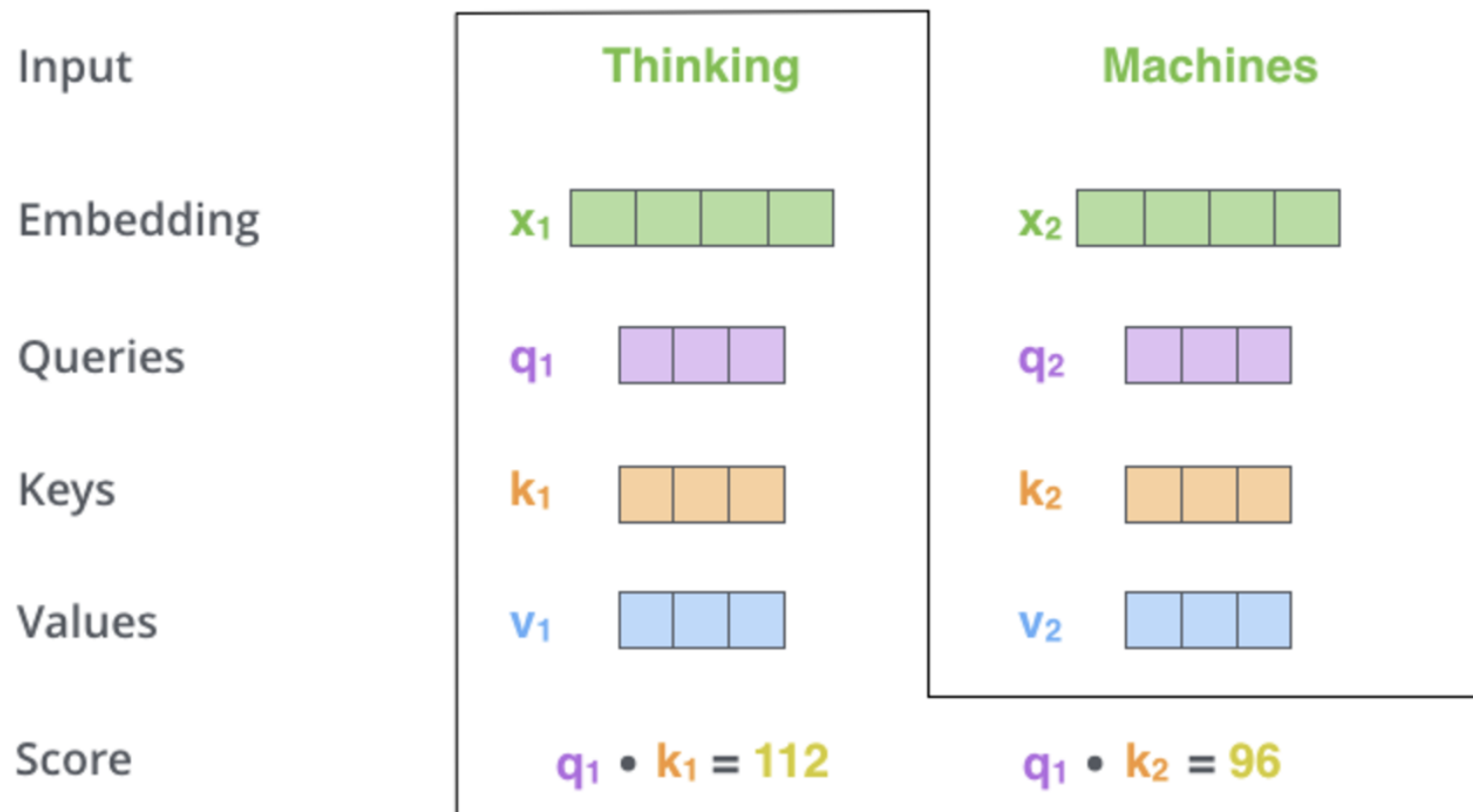
# Attention



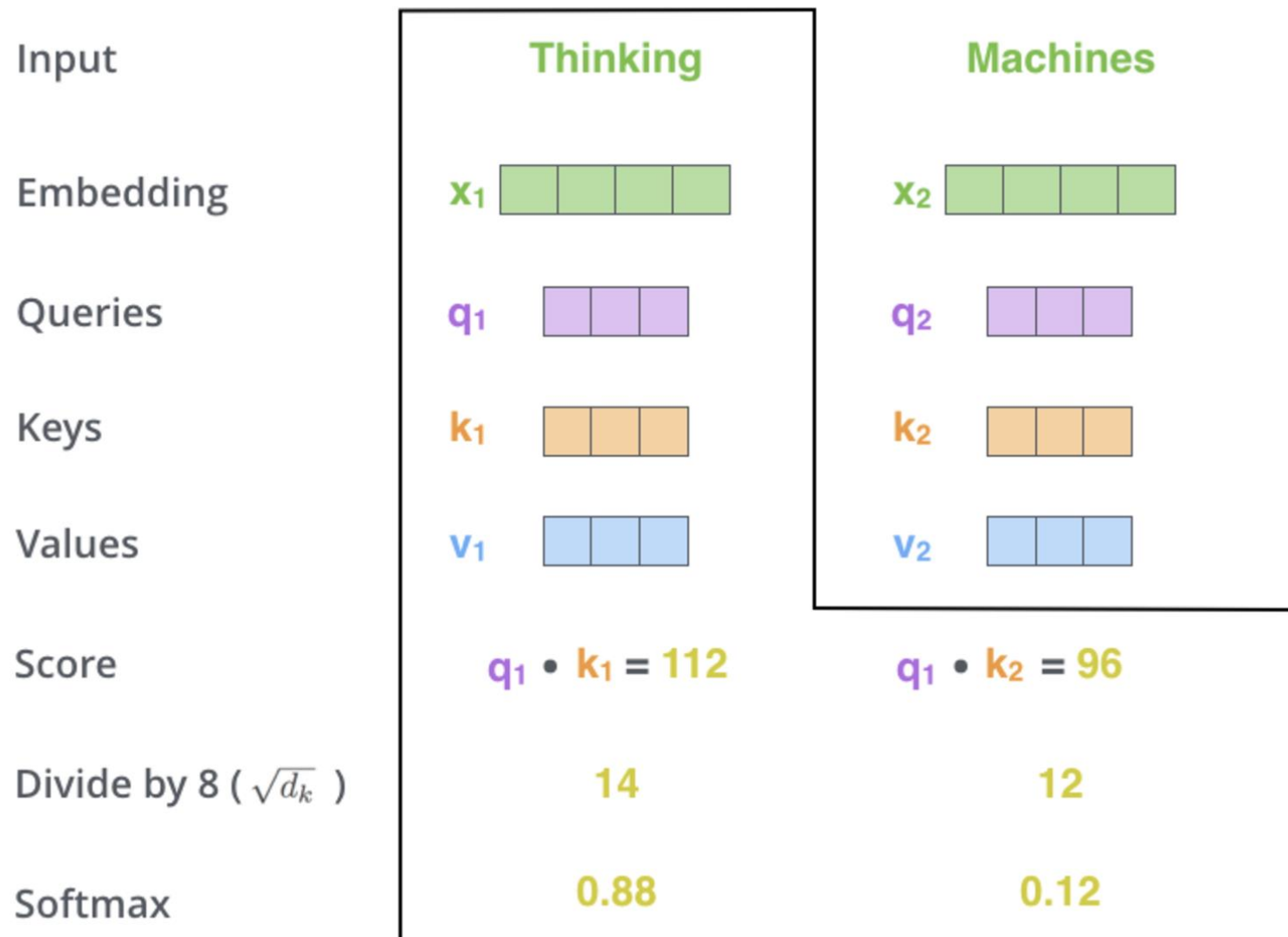
please buy an **apple** and an **orange**

**apple** unveiled the new phone

# Self-Attention



# Self-Attention



# Self-Attention

- **Efficiency:**
  - Computation parallelized across sequence tokens.
  - Masking applied for causal language models.

Input

Embedding

Queries

Keys

Values

Score

Divide by 8 (  $\sqrt{d_k}$  )

Softmax

Softmax

X

Value

Sum

Thinking

$x_1$

$q_1$

$k_1$

$v_1$

$$q_1 \cdot k_1 = 112$$

14

0.88

$v_1$

$z_1$

Machines

$x_2$

$q_2$

$k_2$

$v_2$

$$q_1 \cdot k_2 = 96$$

12

0.12

$v_2$

$z_2$

# Matrix Calculation of Self-Attention

$$\mathbf{X} \times \mathbf{W}^Q = \mathbf{Q}$$


The diagram illustrates the calculation of the Query matrix  $\mathbf{Q}$ . It shows a green matrix  $\mathbf{X}$  (2 rows by 4 columns) multiplied by a purple matrix  $\mathbf{W}^Q$  (4 rows by 3 columns) to produce a purple matrix  $\mathbf{Q}$  (2 rows by 3 columns).

$$\mathbf{X} \times \mathbf{W}^K = \mathbf{K}$$


The diagram illustrates the calculation of the Key matrix  $\mathbf{K}$ . It shows a green matrix  $\mathbf{X}$  (2 rows by 4 columns) multiplied by an orange matrix  $\mathbf{W}^K$  (4 rows by 3 columns) to produce an orange matrix  $\mathbf{K}$  (2 rows by 3 columns).

$$\mathbf{X} \times \mathbf{W}^V = \mathbf{V}$$


The diagram illustrates the calculation of the Value matrix  $\mathbf{V}$ . It shows a green matrix  $\mathbf{X}$  (2 rows by 4 columns) multiplied by a blue matrix  $\mathbf{W}^V$  (4 rows by 3 columns) to produce a blue matrix  $\mathbf{V}$  (2 rows by 3 columns).



# Matrix Calculation of Self-Attention

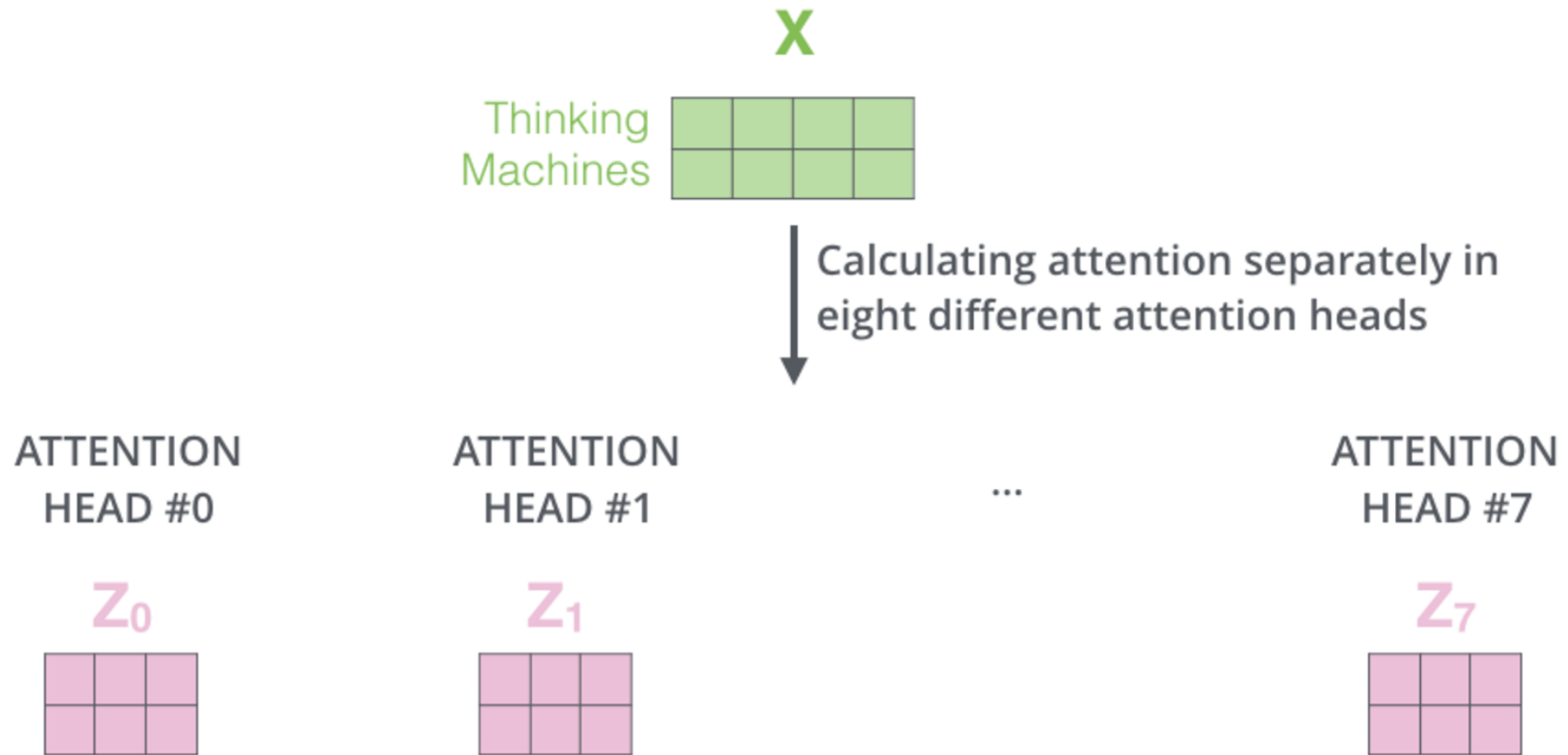
$$\text{softmax}\left(\frac{\begin{matrix} \text{Q} \\ \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array} \end{matrix} \times \begin{matrix} \text{K}^T \\ \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \end{matrix}}{\sqrt{d_k}}\right) \begin{matrix} \text{V} \\ \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array} \end{matrix}$$

=

$\text{Z}$

$\begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \square & \square & \square \\ \hline \end{array}$

# Multiple Attention Head

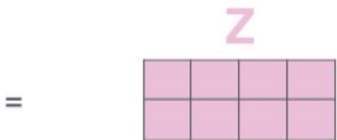


# Multiple Attention Head

1) Concatenate all the attention heads

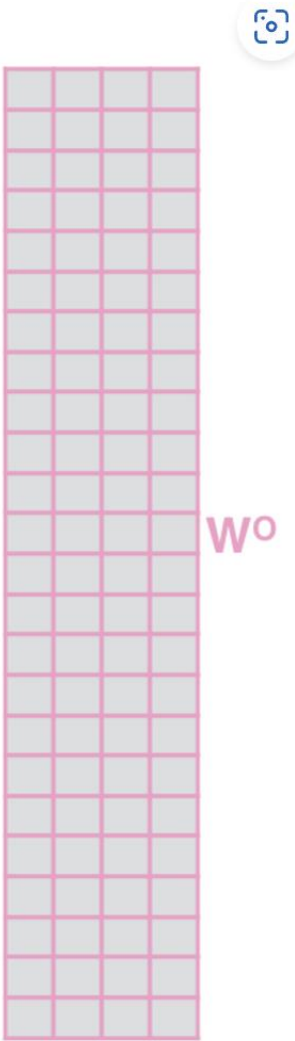


3) The result would be the  $Z$  matrix that captures information from all the attention heads. We can send this forward to the FFNN



2) Multiply with a weight matrix  $W^O$  that was trained jointly with the model

$\times$

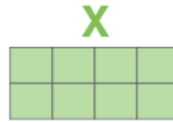


# Multiple Attention Head- Matrix operation

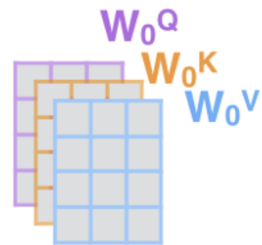
1) This is our input sentence\*

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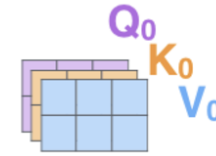
2) We embed each word\*



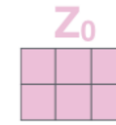
3) Split into 8 heads. We multiply  $X$  or  $R$  with weight matrices



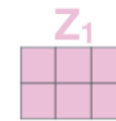
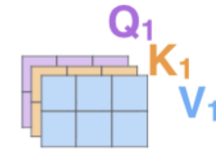
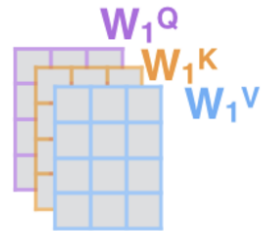
4) Calculate attention using the resulting  $Q/K/V$  matrices



5) Concatenate the resulting  $Z$  matrices, then multiply with weight matrix  $W^O$  to produce the output of the layer



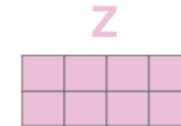
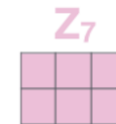
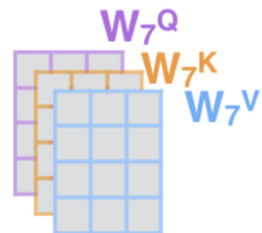
\* In all encoders other than #0, we don't need embedding. We start directly with the output of the encoder right below this one



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# Example

- Assume we already have 2-dimensional token embeddings:

Token 1: “I”

Token 2: “like”

Token 3: “pizza”

- Write them as a matrix  $X \in \mathbb{R}^{3 \times 2}$ :

$$X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

- So:

3 tokens (rows)

embedding size  $d_{model} = 2$

# Example

- The transformer learns three weight matrices:

$$W_Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, W_K = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, W_V = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

# Example

- For self-attention in a transformer:
- $Q = XW_Q, K = XW_K, V = XW_V$
- Given our choices,  $W_Q$  and  $W_K$  are identity, so:

$$Q = X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, K = X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

- Now compute values  $V$ :

$$V = XW_V = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + 0 \cdot 1 & 1 \cdot 1 + 0 \cdot 0 \\ 0 \cdot 1 + 1 \cdot 1 & 0 \cdot 1 + 1 \cdot 0 \\ 1 \cdot 1 + 1 \cdot 1 & 1 \cdot 1 + 1 \cdot 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}$$

# Example

- So each token has:

$$Q_1 = [1, 0] , K_1 = [1, 0] , V_1 = [1, 1]$$

$$Q_2 = [0, 1] , K_2 = [0, 1] , V_2 = [1, 0]$$

$$Q_3 = [1, 1] , K_3 = [1, 1] , V_3 = [2, 1]$$



# Example

## Compute attention scores $QK^\top$

- Attention scores between all query–key pairs:

$$\text{scores} = \frac{QK^\top}{\sqrt{d_k}}, d_k = 2$$

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, K = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} = \text{So } QK^\top = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

- Now scale by  $1/\sqrt{2} \approx 0.7071$ :

$$\text{scores} \approx \begin{bmatrix} 0.7071 & 0 & 0.7071 \\ 0 & 0.7071 & 0.7071 \\ 0.7071 & 0.7071 & 1.4142 \end{bmatrix}$$

# Example

- Interpretation:
  - Row 1 are the *raw* attention scores for token 1 “I” toward tokens {1, 2, 3}.
  - Row 2 for token 2 “like”.
  - Row 3 for token 3 “pizza”.
- $\text{scores} = \text{Softmax}\left(\frac{QK^T}{\sqrt{d_k}}, d_k\right) \Rightarrow A \approx \begin{bmatrix} 0.40 & 0.20 & 0.40 \\ 0.20 & 0.40 & 0.40 \\ 0.25 & 0.25 & 0.50 \end{bmatrix}$
- Each row sums to 1.

# Example

- *AV*

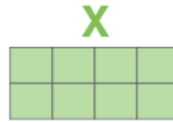
$$\begin{bmatrix} 0.40 & 0.20 & 0.40 \\ 0.20 & 0.40 & 0.40 \\ 0.25 & 0.25 & 0.50 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 2 & 1 \end{bmatrix} \approx \begin{bmatrix} 1.40 & 0.80 \\ 1.40 & 0.60 \\ 1.50 & 0.75 \end{bmatrix}$$

# Multiple Attention Head- Matrix operation

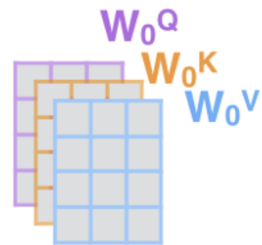
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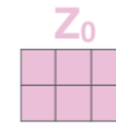
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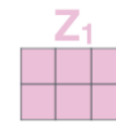
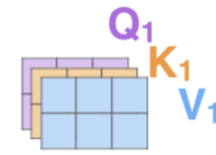
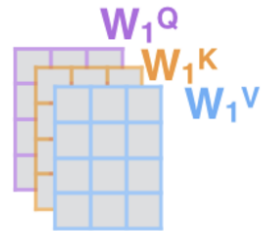
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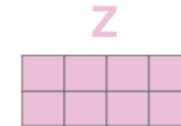
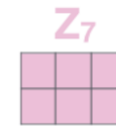
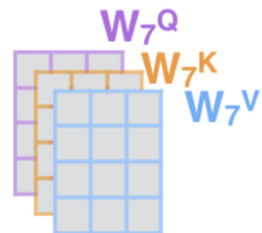
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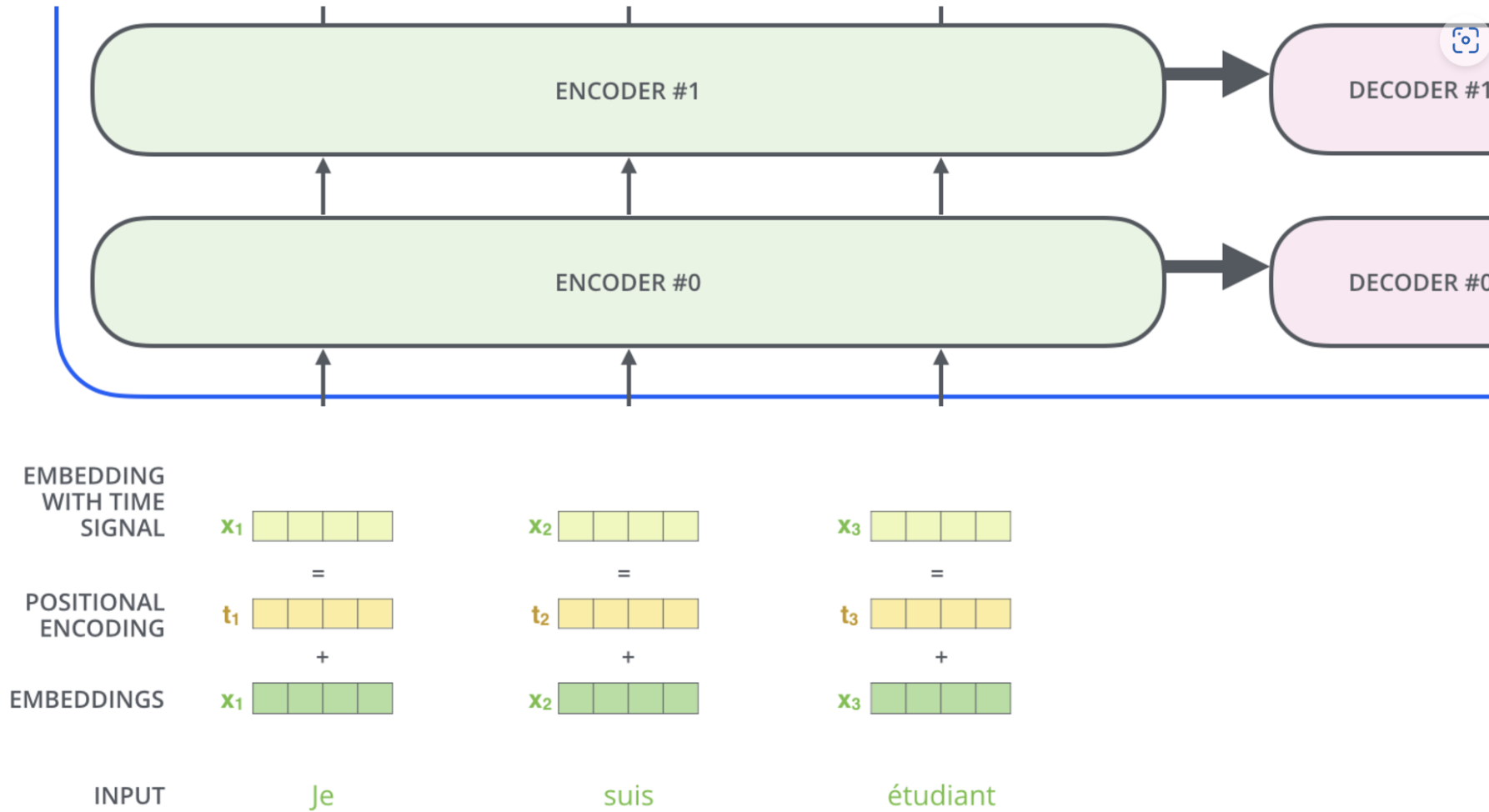
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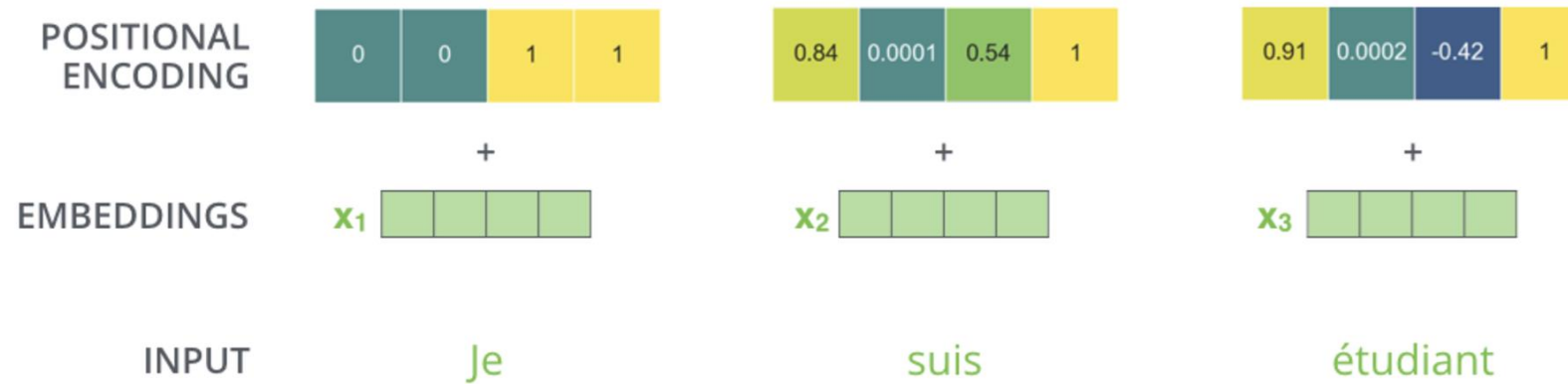


# Representing The Order

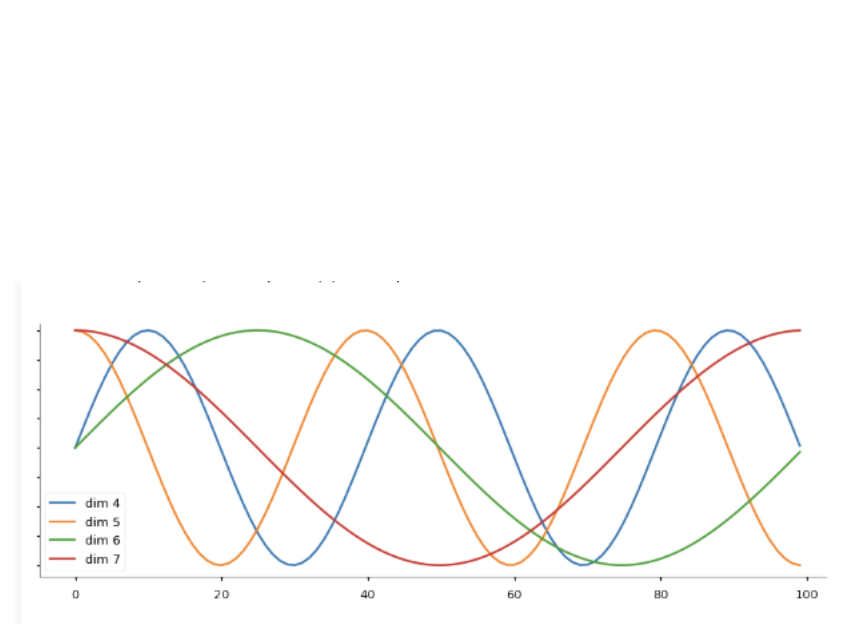
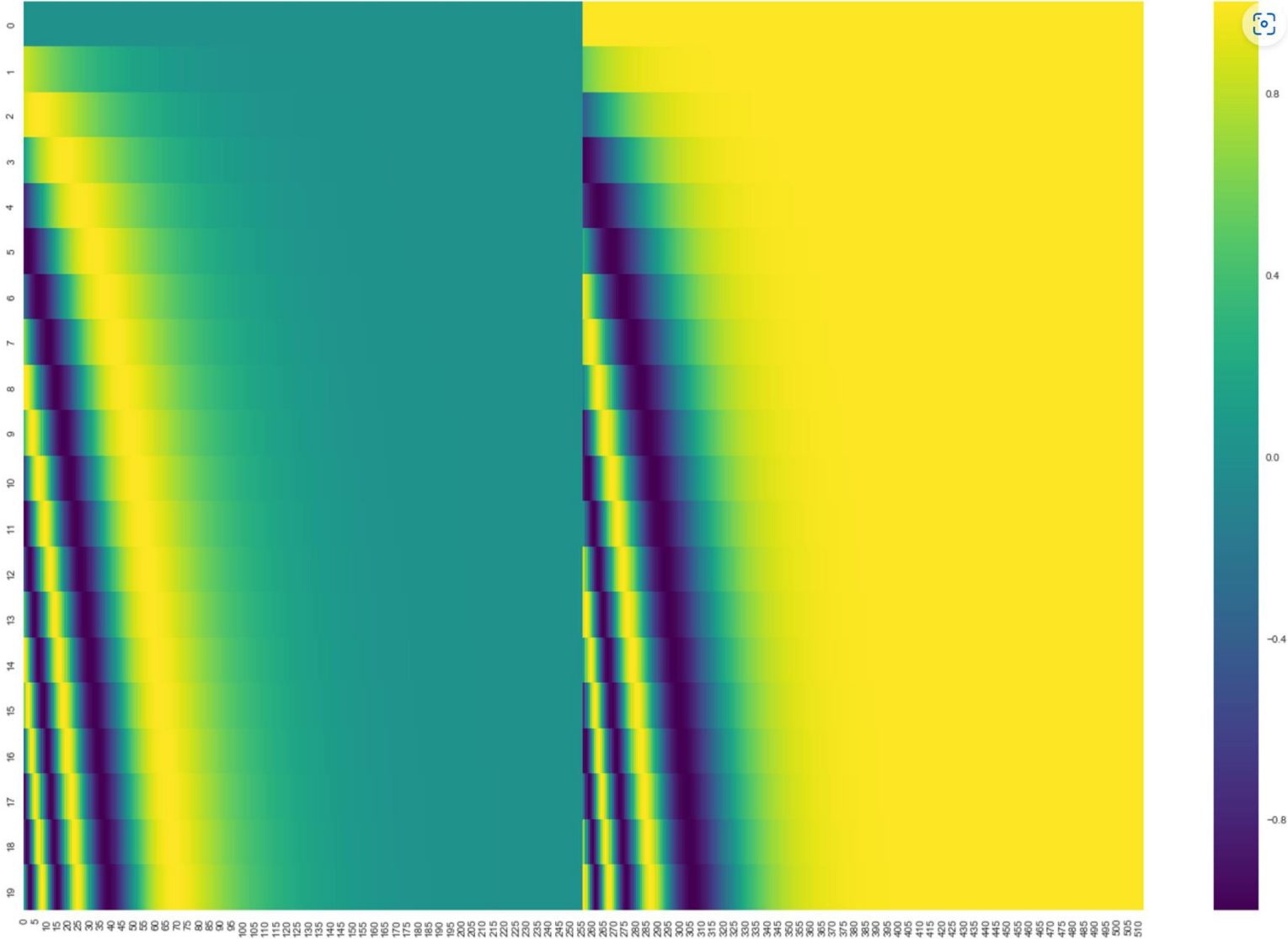


# Representing the Order

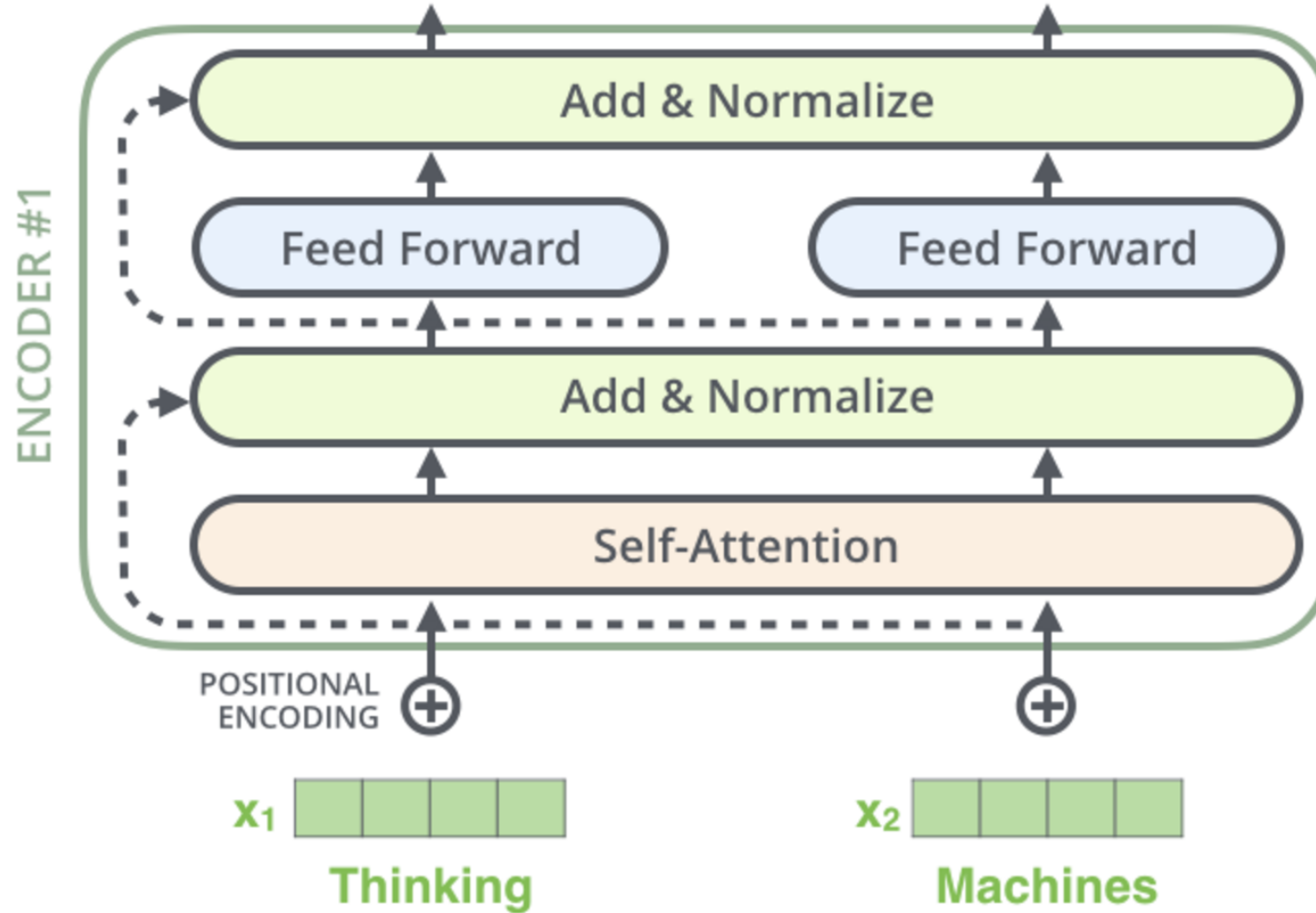
- We also assign each position in our vocabulary an embedding vector



# Representing the Order

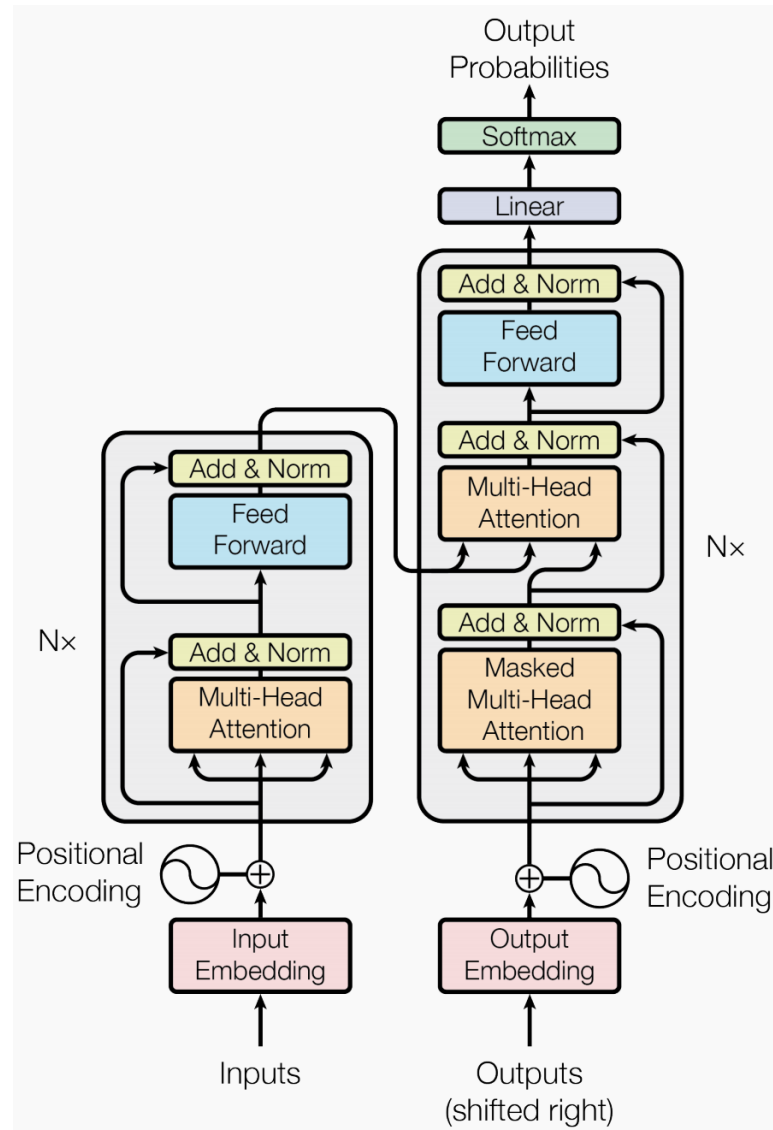


# The Residuals





# Transformers



# RNN - Transformers

- [The Illustrated Transformer – Jay Alammar – Visualizing machine learning one concept at a time.](#)
- [Understanding LSTM Networks -- colah's blog](#)