PROBLEM I.8: PRIMALITY FUNCTION

ROSIE KEY

1. Examining form 1

The partial product form 1 is given as $S_n = \Pi_{n=1}^{\infty}(1 + \frac{f(n)}{g(n)})$, where both $f(n)$ and $g(n)$ are polynomials. Using the code created, the first and last 15 terms in the partial product series as well as the conditions for convergence and divergence of the partial product can be determined. The following are two different functions entered into the code to demonstrate the conditions of convergence and divergence.

The following is the first example of the partial product examined:

$$S_n = \Pi_{n=1}^{\infty}(1 + \frac{8n+3}{17n^3 + 5n^2 + 73})$$

where $f(n) = 8n+3$ and $g(n) = 17n^3 + 5n^2 + 73$. When inputted into the program, this partial product converges at $S_n = 1.442400$. This differs from the second function inputted,

$$S_n = \Pi_{n=1}^{\infty}(1 + \frac{n^4}{4n^2 + 5}),$$

where $f(n) = n^4$ and $g(n) = 4n^2 + 5$, which goes on to infinity.

Given the examples above, it can be deduced that one condition needed to have a partial product converge is that if the partial product consists of a variable over a variable, the denominator must increase at a faster rate than the numerator. Notice in the first example that up to almost any given point n in the series, the denominator $g(n)$ will be bigger than the numerator $f(n)$ as where the second example is the opposite case. This is because of the following (let k be some integer):

$$lim_(x \to \infty)\frac{k}{x} \approx 0.$$

The equation says that as the denominator becomes an extremely large number, the fraction will get closer to zero. However, there is an exception to this. For fractions with functions raised to the same power or raised to a power close to one another ($\frac{n^2}{n^3}$ for example), the series will diverge if it is reduced down to the following equation:

$$S_n = \Pi_{n=1}^{\infty}(\frac{n+1}{n})$$

2. Examining form 2

The partial product form 2 is given as $S_n = \Pi_{n=1}^{\infty}(1 + b^n)$, where $b > 0$ and is a constant number. The first partial product tested is the following:

$$S_n = \Pi_{n=1}^{\infty}(1 + (\frac{4}{5})^n),$$

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where $b = \frac{4}{5}$. The result when entered into the code is that the series converges at $S_n = 28.461274$. An example of the partial product diverging is the following equation:

$$S_n = \Pi_{n=1}^{\infty}(1 + (\frac{19}{18})^n),$$

where $b = \frac{19}{18}$.

Looking at the examples for form 2, a condition for convergence can be deduced. Much like form 1, the partial product converges or diverges depending on the fact that a fraction with a denominator that increases faster goes to zero and a fraction with a numerator that increases faster goes to infinity. However, in this case, n is neither part of the numerator nor denominator but is instead an exponent. That said, a fraction raised to an exponent can be better thought of as a fraction times itself, so if the numerator is greater than the denominator, the numerator will grow faster and the partial product will go to infinity. Conversely, the partial product will go to zero if the denominator is greater, therefore increasing faster than the numerator when raised to a power. In conclusion, the series converges if $0 < b < 1$, since that is when the numerator is smaller than the denominator.