

Problem N.1: Collatz Conjecture

Rosie Key

March 2019

1 Conjecture 1

Let the initial term for the Collatz sequence be such that $a_0 = 2^n$ for some integer $n > 0$. This means that a_0 can be written as

$$a_0 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \dots$$

This means that a_0 is not only an even number, but a double of an even number. Recall that if a term in the Collatz sequence is even, the term that will come after it will be that term halved. Half an even number such that its a product of 2 and another even number is an even number. Therefore, if the initial term of the Collatz sequence $a_0 = 2^n$ for some integer $n > 0$, the entire sequence will consist of even numbers with the exception of 1.

2 Conjecture 2

If the initial term of the Collatz sequence $a_0 = 2^n$ for some integer $n > 0$, the number of terms x the sequence will take before it gets to 1 can be written as the following equation:

$$x = \log_2(a_0) + 1$$

3 Conjecture 3

If the initial term of the Collatz sequence $a_0 = 4^n$ for some integer $n > 0$, the number of terms it will take to get to 1 will be odd.