

42. Describe the order of growth of each of the following functions using  $O$  notation.

- a.  $N^2 + 3N$
- b.  $3N^2 + N$
- c.  $N^5 + 100N^3 + 245$
- d.  $3N \log_2 N + N^2$
- e.  $1 + N + N^2 + N^3 + N^4$
- f.  $(N * (N - 1)) / 2$

a.)  $N^2 + 3N$

$$N^2 + 3N = O(N^2)$$

$N^2$  is the highest order term.

$3N$  is the lower order term that is ignored.

The constant factor 3 is ignored.

b.)  $3N^2 + N$

$$= O(3N^2)$$

$$= O(N^2)$$

The highest order term is  $3N^2$ .

The lower order term  $N$  is ignored.

The constant factor 3 is ignored.

c.)  $N^5 + 100N^3 + 245$

$$= O(N^5 + 100N^3)$$

$$= O(N^5 + N^3)$$

$$= O(N^5)$$

The highest order term is  $N^5$ .

The lower order term  $100N^3$  and 245 are ignored.

The constant factor 100 is ignored.

d.)  $3N \log_2 N + N^2$

$$= O(N \log_2 N + N^2)$$

$$= O(N^2)$$

$N^2$  is the highest order term.

The constant factor 3 is ignored.

$N \log_2$  is the lower order term to be ignored.

e.)  $1 + N + N^2 + N^3 + N^4$

$$= O(N + N^2 + N^3 + N^4)$$

$$= O(N^2 + N^3 + N^4)$$

$$= O(N^3 + N^4)$$

$$= O(N^4)$$

The highest order term is  $N^4$ .

The lower order terms to ignore are 1,  $N$ ,  $N^2$ , and  $N^3$ .

The constant factor 1 is ignored.

f.)  $(N * (N - 1)) / 2$

$$= O\left(\frac{N^2 - N}{2}\right)$$

$$= O\left(\frac{N^2}{2}\right)$$

$$= O(N^2)$$

The highest order term is  $N^2$ .

$1/2$  is the constant factor to ignore as well as  $N$  which is the lower order term.

43. Describe the order of growth of each of the following code sections, using  $O$  notation:

```
a. count = 0;
   for (i = 1; i <= N; i++)
       count++;

b. count = 0;
   for (i = 1; i <= N; i++)
       for (j = 1; j <= N; j++)
           count++;

c. value = N;
   count = 0;
   while (value > 1)
   {
       value = value / 2;
       count++;
   }

d. count = 0;
   value = N;
   value = N * (N - 1);
   count = count + value;

e. count = 0;
   for (i = 1; i <= N; i++)
       count++;
   for (i = N; i >= 0; i--)
       count++;

f. count = 0;
   for (i = 1; i <= N; i++)
       for (j = 1; j <= 5; j++)
           count++;
```

a.) The complexity is  $O(N)$ .  
The code goes from 0 to count  $N$ . The  $i$  value goes from 1 to  $N$  incrementing count by 1.

b.) This is a nested loop running from 1 to  $N$ . For both the outer and inner loop the is  $O(N)$ .

c.) This is a time complexity of  $O(\log N)$ . With the value going by 'value/2' the loop runs from value =  $N$ .

d.) Since there is no loop, the single statement complexity is  $O(1)$ .

e.) The first two loops are  $O(N)$ . The complexity will be added since the 2 loops aren't nested.  
' $O(N+N) = O(2N)$ ' is the code complexity, while ignoring the constant, 2. The statement is  $O(N)$ .

f.) There are two nested loops, the outer loop is  $O(N)$  while the inner loop is  $O(5)$  since it runs 5 times. Ignore the constant 5. The complexity is  $O(5 \cdot N) = O(N)$ .