Diocese of Imus Catholic Educational System, Inc.
Office of the Superintendent of the Diocesan Schools
Office of Curriculum and Instruction
L E A R N I N G   A C T I V I T Y   S H E E T

Name: _____ Expert Teacher: _____ Quarter: <u>2</u> Act. #: <u>1</u>

Grade/Year & Section: _____ Subject: <u>Mathematics 10</u>   Date: _____

Please check the box for the type of the activity:

☑ Concept Notes ☐ Illustrations/Examples ☑ Seat work ☐ Written Work *(Pls. Specify WW: _____*

☐ Quiz ☐ Performance Task ☐ Meaning-making Activity ☐ *Others:* _____

Activity Title: <u>Harmonic and Fibonacci Sequence</u>

Learning Target/Competency: *The learner will* illustrate other types of sequences (e.g., harmonic, Fibonacci).

Values/Graduate Attributes: *The learners will become* systematic and orderly.

Reference(s) & Author(s) <u>Grade 10 Mathematics Patterns and Practicalities Gladys C. Nievera Phd.</u>   Page(s) No. <u>53-65</u>

I. CONCEPT/DIGEST

➢ **Harmonic sequence** is a sequence whose reciprocal form is an arithmetic sequence. ➢ If $a_1, a_2, a_3, \ldots a_n$ are terms of an arithmetic sequence, then the sequence of reciprocal of these terms $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \ldots \frac{1}{a_n}$ is called **harmonic sequence**. ➢ The nth term of a harmonic sequence is given by: $a_n = \frac{1}{a_1 + (n-1)d}$	➢ **Fibonacci sequence** was named after Leonardo of Pisa. He was posthumously given the nickname Fibonacci (meaning, the son of Bonacci) and an Italian mathematician who analyzed the Rabbit Problem in his book Liber Abaci, which was published in 1202. ➢ Fibonacci sequence can be obtained by the formula: $a_n = a_{n-1} + a_{n-2}\ for\ n > 2$. ➢ The initial terms are $a_1 = 1\ and\ a_2 = 1$

II. EXAMPLE

1. Given the arithmetic sequence 2, 5, 8, 11…, find the first 7 terms of the corresponding harmonic sequence.

   Solution:

   Since the next three terms of 2, 5, 8, 11 are 14, 17, 20, then the harmonic sequence is:

   $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \frac{1}{14}, \frac{1}{17}, \frac{1}{20}$

2. Find the 10th term of the harmonic sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$.

   Solution: $a_1 = 2$ $d = 2$ $n = 10$

   $a_n = \frac{1}{a_1 + (n-1)d}$ $a_{10} = \frac{1}{2 + (10-1)2}$ $a_8 = \frac{1}{2 + (9)2}$ $a_8 = \frac{1}{2 + 18}$ $a_8 = \frac{1}{20}$

3. Given the Fibonacci sequence: 5, 8, 13, 21, 34, ... find the next 6 terms.

   Solution:

   Since each new term in a Fibonacci sequence can be obtained by adding its two preceding terms, then the next 6 terms are 55, 89, 144, 233, 377, and 610.

III. EXERCISES

  A. **Directions:** Complete the sequence.

   1. $2, \frac{2}{5},$ _____, _____, $\frac{2}{17}, \frac{2}{21}, \frac{2}{25}$

   2. 24, 32, 56, _____, _____

   3. $\frac{2}{5}, \frac{2}{11}, \frac{2}{17}, \frac{2}{23},$ _____, _____

  B. **Directions:** Solve each problem. Show your solution.

   1. What is the first 8 terms in the Fibonacci sequence if the first two terms are 3 and 8?

   2. Find the 15th term of the harmonic sequence $\frac{7}{3}, \frac{7}{6}, \frac{7}{9}, \ldots$