

# Value at Risk Estimation Across Asset Classes and Portfolios: From Equity and Fixed Income to Diversified Portfolios

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## **Abstract**

This paper conducts a comprehensive empirical investigation of Value at Risk (VaR) estimation across multiple asset classes. The analysis begins with single-asset risk modeling for equities (S&P 500) and fixed income (10-Year U.S. Treasury Bonds), and extends to a diversified portfolio comprising equities, government bonds, commodities (gold), and foreign exchange (EUR/USD). Three major VaR methodologies are implemented and compared: parametric (variance-covariance), PV01-based fixed income modeling, and Monte Carlo simulation. Each model is applied to real-world financial data, with thorough backtesting and visual diagnostics. The study highlights the strengths, limitations, and practical implications of each approach, providing a robust foundation for advanced risk management strategies.

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# 1 Summary of Analyses

This research is structured around four core empirical studies, each implemented in Python using real financial data and statistical modeling techniques. The studies are designed to evaluate different approaches to Value at Risk (VaR) estimation across asset classes and methodologies:

1. **One-Asset VaR: S&P 500 (Equity)**

A parametric VaR model is applied to the S&P 500 index, focusing on volatility estimation, log return distribution, and model backtesting under normality assumptions.

2. **One-Asset VaR: 10-Year U.S. Treasury Bond (Fixed Income)**

This section models interest rate risk using PV01 sensitivity to estimate potential losses on a Treasury bond position. Rate changes are analyzed, and VaR is validated via P&L exceedances.

3. **Diversified Portfolio VaR: GLD, TLT, SPY, EUR/USD**

A multi-asset portfolio is constructed and analyzed using the variance-covariance method. The effects of diversification, cross-asset correlations, and asset-specific risk contributions are explored.

4. **Monte Carlo VaR on the Diversified Portfolio**

Monte Carlo simulation is used to model portfolio return distributions, incorporating empirical correlations via Cholesky decomposition. The estimated VaR is compared to parametric results.

## 2 Introduction

### 2.1 Importance of Value at Risk (VaR)

Value at Risk (VaR) has become a cornerstone of modern financial risk management. It is widely adopted by financial institutions, asset managers, and regulators to quantify the potential loss of a portfolio over a specified time horizon under normal market conditions. VaR provides a probabilistic estimate of the maximum expected loss at a given confidence level—typically 95% or 99%—and is central to regulatory frameworks such as Basel II and Basel III. By summarizing risk exposure in a single monetary figure, VaR facilitates capital allocation, stress testing, and risk-adjusted performance evaluation.

#### Historical Context

The concept of VaR did not emerge overnight—it evolved in response to major shifts in financial theory, regulatory demand, and market crises:

- **1980s – Rise of Quantitative Risk:** Large financial institutions began integrating statistical models into trading and treasury operations.

- **1994 – JPMorgan Formalizes VaR:** JPMorgan introduced its internal risk model, *RiskMetrics*, popularizing the use of VaR and releasing it publicly to promote transparency and standardization.
- **1996 – Basel Committee Endorsement:** The Basel Committee incorporated VaR into its 1996 Market Risk Amendment, officially recognizing it for determining capital requirements.
- **Post-2008 – Criticism and Evolution:** The Global Financial Crisis exposed VaR’s shortcomings—particularly in capturing tail risk—leading to the adoption of supplementary metrics like Expected Shortfall (ES) in Basel III.

## 2.2 Motivation Behind This Research

Despite its prevalence, VaR estimation is highly sensitive to the chosen methodology, underlying distributional assumptions, and prevailing market dynamics. This research aims to systematically evaluate and compare multiple VaR estimation techniques—namely parametric, PV01-based, and Monte Carlo simulation—across various asset classes, including:

- **Equities:** S&P 500 Index
- **Fixed Income:** 10-Year U.S. Treasury Bonds
- **Multi-Asset Portfolio:** GLD (Gold), TLT (Treasuries), SPY (Equities), EUR/USD (FX)

Through hands-on implementation and backtesting, this study seeks to assess the precision, robustness, and practical limitations of each VaR method, ultimately offering insights into their suitability under different market conditions and risk profiles.

## 3 Literature Review

### 3.1 Overview of VaR Methodologies

Value at Risk (VaR) has been the subject of extensive research in both academic and industry settings. Among the most common estimation methods is the **parametric approach**, such as the variance-covariance method, which assumes that asset returns are normally distributed. This allows for closed-form VaR calculations based on historical volatility and correlation estimates.

Alternatively, the **historical simulation** method is non-parametric and involves re-ordering historical returns to construct an empirical distribution of portfolio losses. This approach avoids explicit distributional assumptions but may be sensitive to the choice of historical window and structural breaks.

The **Monte Carlo simulation** framework introduces the most flexibility. By drawing from theoretical or empirical distributions and applying random sampling techniques, this method can model more complex risk factors, including non-linear exposures and stochastic processes such as Geometric Brownian Motion or jump-diffusion models.

### 3.2 Limitations of the Normality Assumption

Despite its analytical convenience, the normality assumption has been widely criticized in the context of financial return modeling. Empirical studies show that asset returns frequently exhibit **heavy tails**, **asymmetry**, and **volatility clustering**—stylized facts that contradict the Gaussian distribution. Consequently, parametric VaR models may significantly underestimate the likelihood of extreme losses, especially during periods of market stress or structural instability.

### 3.3 Backtesting and Regulatory Context

Model validation through **backtesting** is a regulatory and practical necessity. Institutions are required by frameworks such as Basel II and III to compare predicted VaR estimates against actual daily P&L outcomes. A key metric is the number of **exceptions**—days when realized losses exceed the forecasted VaR.

An effective model should yield an exception rate close to the chosen confidence level (e.g., approximately 5% for a 95% VaR). Statistical tests such as Kupiec’s Proportion of Failures (POF) or Christoffersen’s independence test are often employed to assess both the accuracy and independence of these exceptions.

Regulatory capital requirements are directly influenced by backtesting outcomes, making model reliability not only a methodological concern but a strategic and financial one for institutions.

## 4 Methodology and Results

### 4.1 Parametric VaR Estimation: S&P 500 Index

#### 4.1.1 Methodology: Parametric VaR under Normality

The Value at Risk (VaR) metric estimates the potential loss on a portfolio over a specified time horizon at a given confidence level. Under the parametric method, we assume that asset returns are normally distributed:

$$r_t \sim \mathcal{N}(\mu, \sigma^2) \quad (1)$$

where  $r_t$  is the return at time  $t$ ,  $\mu$  is the mean return, and  $\sigma^2$  is the variance of returns. For short horizons such as daily VaR, the mean  $\mu$  is typically negligible and often assumed to be zero for simplicity.

Given a confidence level  $\alpha$  (e.g., 95%), the one-period VaR for a portfolio of size  $W$  is given by:

$$\text{VaR}_\alpha = -z_\alpha \cdot \sigma \cdot W \quad (2)$$

where:

- $z_\alpha$  is the quantile of the standard normal distribution corresponding to the confidence level (e.g.,  $z_{0.95} \approx 1.645$ ),
- $\sigma$  is the standard deviation (volatility) of returns,
- $W$  is the monetary value of the portfolio.

In this study, we use log returns for volatility estimation:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \quad (3)$$

The empirical volatility  $\hat{\sigma}$  is computed as the sample standard deviation of log returns:

$$\hat{\sigma} = \sqrt{\frac{1}{N-1} \sum_{t=1}^N (r_t - \bar{r})^2} \quad (4)$$

Assuming a \$1,000,000 long position, and using the empirical daily volatility, we compute:

$$\text{VaR}_{95\%} = -z_{0.95} \cdot \hat{\sigma} \cdot 1,000,000 \quad (5)$$

#### 4.1.2 Data Description

We analyze the daily closing prices of the S&P 500 index from January 1, 2019 to December 31, 2024, obtained using the `yfinance` Python package. This dataset captures several market phases including periods of both high and low volatility, making it suitable for risk analysis.

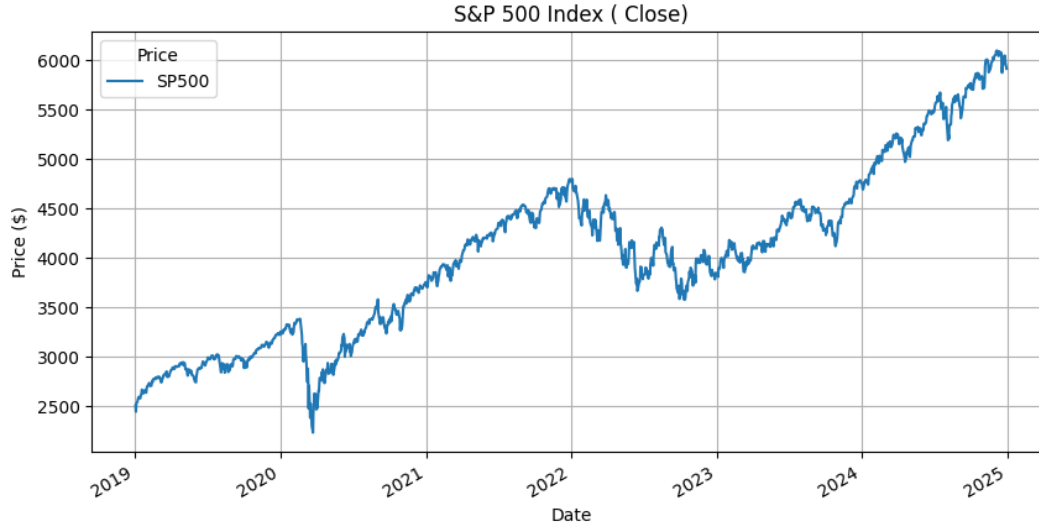


Figure 1: S&P 500 Daily Closing Prices (2019–2024)

#### 4.1.3 Model Assumptions

The parametric Value at Risk (VaR) model employed assumes:

- Log returns are normally distributed with mean zero and constant variance.
- Portfolio consists of a \$1,000,000 long position in the S&P 500 index.
- Daily VaR is calculated at a 95% confidence level.

#### 4.1.4 Return Series and Distribution

Daily log returns were computed and analyzed. As expected, we observe volatility clustering and sharp movements, particularly during crisis periods (e.g., early 2020).



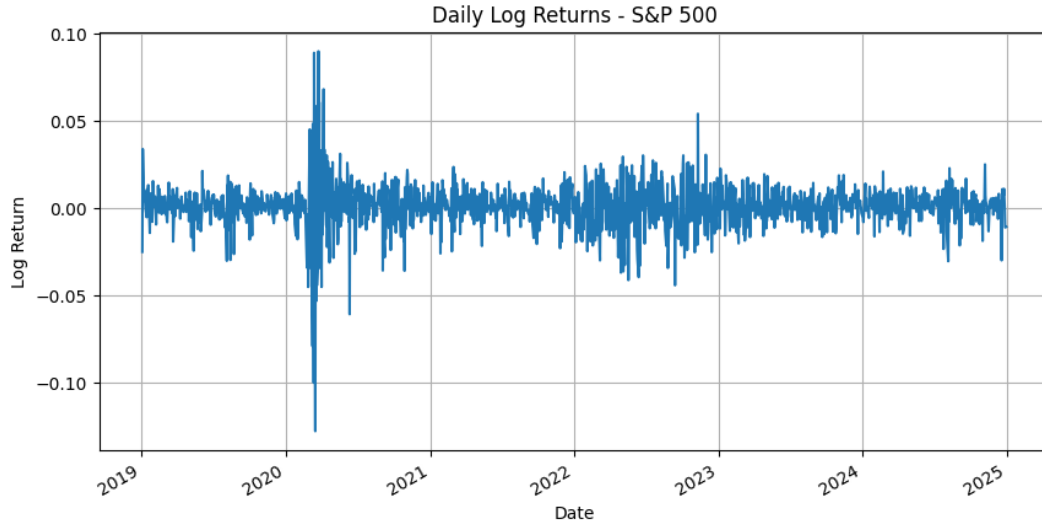


Figure 2: Daily Log Returns of the S&P 500

To assess the normality assumption, the empirical distribution of log returns is compared to a fitted normal distribution.

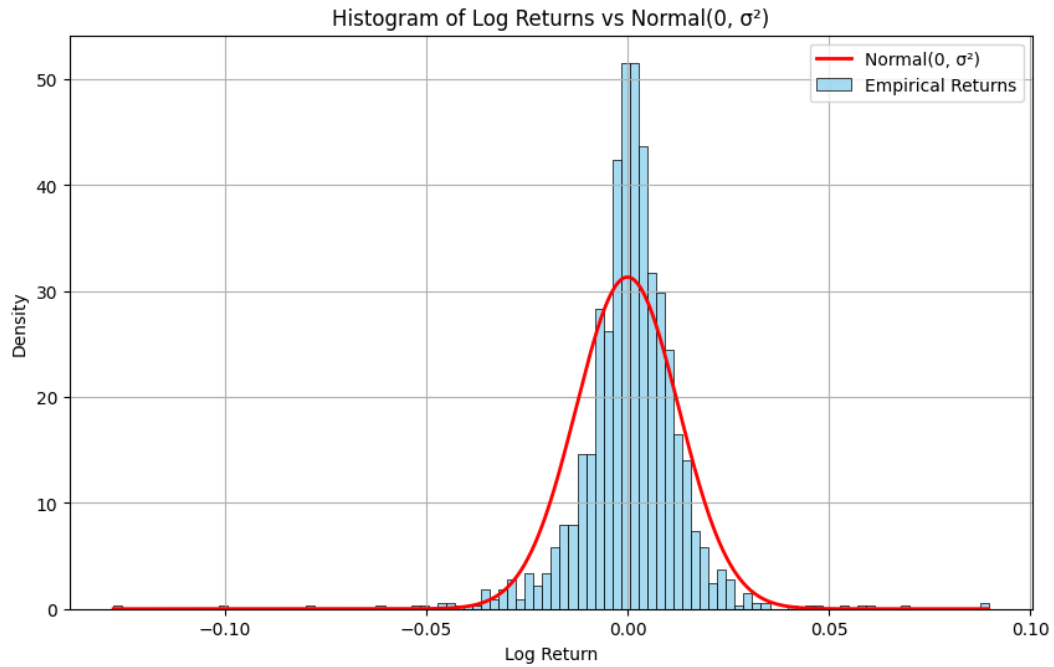


Figure 3: Histogram of Log Returns vs. Normal Distribution

#### 4.1.5 Computation and Results

The estimated daily volatility was found to be:

$$0.012738$$

Using a Z-score of approximately 1.645 for the 95% confidence level, the one-day VaR was calculated as:

**1-Day VaR (95% Confidence): \$20,952.48**

#### 4.1.6 Backtesting and VaR Exceedances

Backtesting was performed by comparing actual daily profit and loss (P&L) to the computed VaR threshold. A breach occurs when the realized loss exceeds the VaR estimate.

- Number of VaR Exceedances: **57**
- Percentage of Days with Loss > VaR: **3.79%**

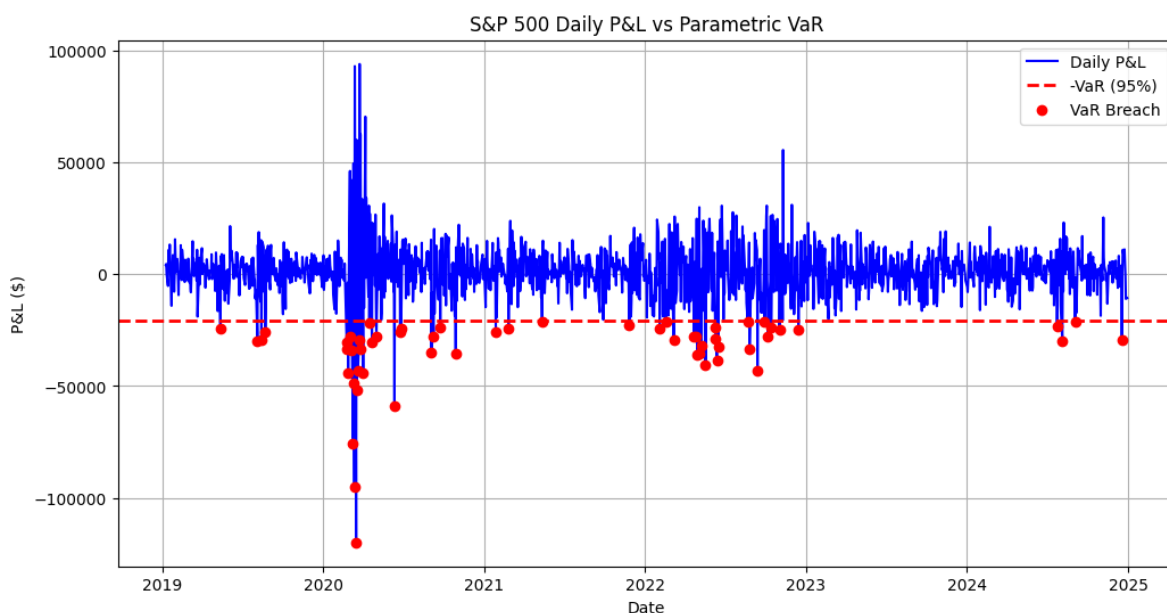


Figure 4: Daily P&L vs. Parametric VaR (95%) with Breaches Highlighted

#### 4.1.7 Interpretation

The number of VaR exceedances observed is **57**, which corresponds to approximately **3.79%** of trading days. This is slightly below the theoretical expectation of **5%** exceedances (i.e., around 63 out of 1260 days), indicating a relatively conservative estimate of risk under the parametric model.

However, the presence of clustered breaches and extreme losses, especially during market stress periods (e.g., early 2020), highlights potential underestimation of tail risk. This behavior suggests that while the model performs acceptably under normal market conditions, it may fail to capture the full severity of losses during turbulent periods due to the normality assumption and constant volatility.

#### 4.1.8 Limitations

- The assumption of normality underestimates tail risk, especially during periods of market stress.
- Volatility is assumed to be constant, which contradicts empirical evidence of volatility clustering.
- No accounting for autocorrelation or structural breaks in the return series.

## 4.2 Parametric VaR Estimation: 10-Year U.S. Treasury Bond

### 4.2.1 Methodology: Fixed-Income VaR via PV01 Sensitivity

Unlike other assets, fixed-income instruments such as Treasury bonds are sensitive to interest rate movements rather than direct price changes. To model interest rate risk, we use the **PV01 (Price Value of a Basis Point)** framework, which measures the change in a bond's price for a 1 basis point (0.01%) change in yield.

The price of a bond is calculated using the present value of its future cash flows:

$$P = \sum_{t=1}^n \frac{C}{(1 + y/f)^t} + \frac{F}{(1 + y/f)^n} \quad (6)$$

where:

- $P$  is the bond price,
- $C$  is the coupon payment,
- $F$  is the face value,
- $y$  is the yield to maturity (YTM),
- $f$  is the compounding frequency (semi-annual),
- $n$  is the total number of periods.

The **PV01** is computed as:

$$\text{PV01} = P(y) - P(y + 0.0001) \quad (7)$$

This represents the price change from a 1bp increase in rates. The daily VaR is then computed as:

$$\text{VaR}_\alpha = -z_\alpha \cdot \sigma_{\Delta y} \cdot \text{PV01} \cdot \text{Position Size} \quad (8)$$

where:

- $z_\alpha$  is the Z-score at confidence level  $\alpha$ ,
- $\sigma_{\Delta y}$  is the standard deviation of daily rate changes (in bps),
- PV01 is computed per dollar and scaled by the total notional,
- The position size is \$1,000,000.

### 4.2.2 Data Description

We used the 10-Year U.S. Treasury Constant Maturity Rate (DGS10), sourced from the Federal Reserve Economic Data (FRED), spanning January 2019 to December 2024.

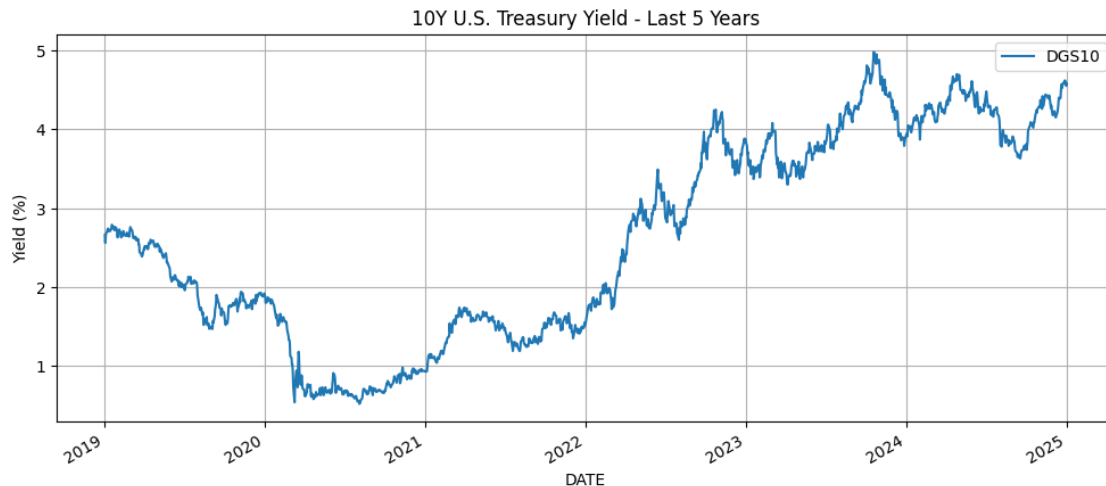


Figure 5: 10Y U.S. Treasury Yield (2019–2024)

### 4.2.3 Rate Changes and Distribution

We approximate daily rate changes in basis points (bps) as the first difference of yields:

$$\Delta y_t = (y_t - y_{t-1}) \times 100 \text{ bps} \quad (9)$$

The resulting series of  $\Delta y_t$  exhibits mean-reverting behavior and fat tails, typical of interest rate dynamics.

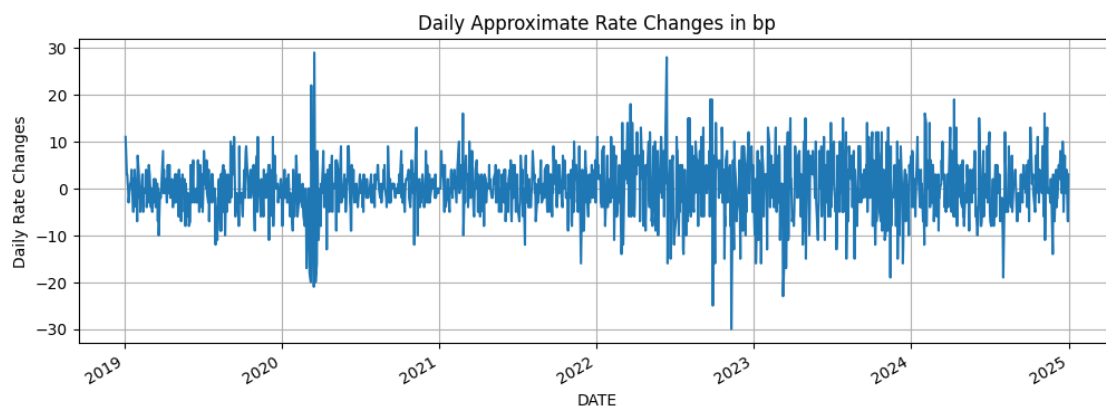


Figure 6: Daily Changes in 10Y Yields (in bps)

The empirical rate change distribution is then compared to a normal distribution.

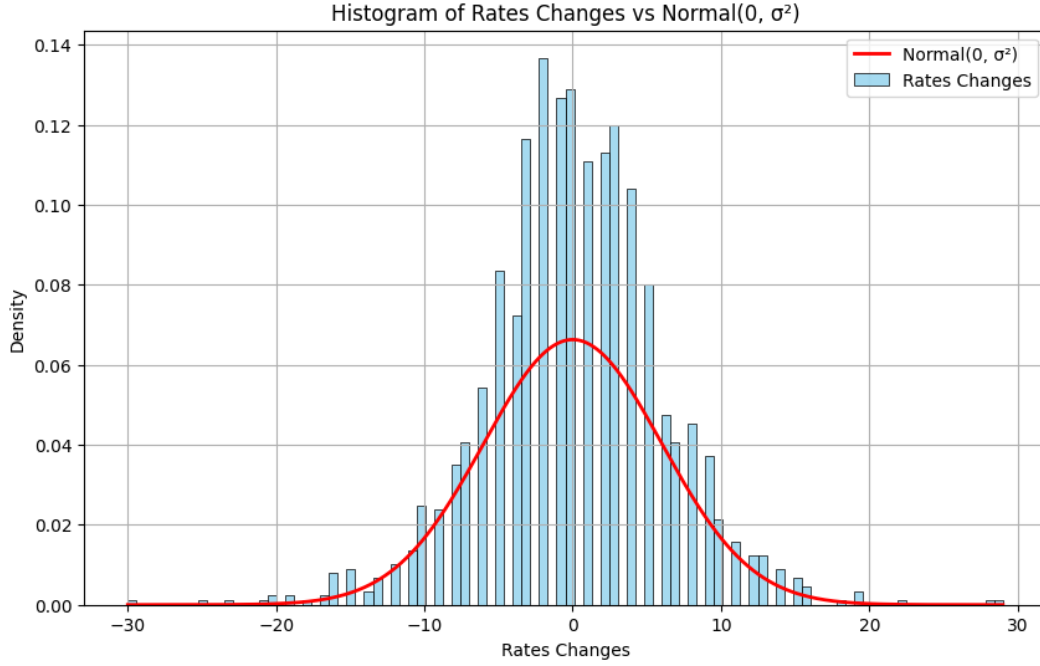


Figure 7: Histogram of Rate Changes vs Normal Distribution

#### 4.2.4 Computation and Results

The computed PV01 of the \$1 million bond position is:

**PV01 (per \$1): 0.000807**  
**Total PV01: \$807.21**

Using a standard deviation of rate changes  $\sigma_{\Delta y} = 6.019$  bps and a Z-score of 1.645 (for 95% confidence), we compute:

**1-Day VaR (95% Confidence): \$7,992.24**

#### 4.2.5 Backtesting and VaR Exceedances

To validate the model, we calculate the daily P&L assuming a linear approximation:

$$\text{PnL}_t = -\text{PV01} \cdot \Delta y_t \cdot \text{Notional} \quad (10)$$

An exceedance occurs when the actual loss exceeds the predicted VaR.

- Number of VaR Exceedances: **80**
- Percentage of Days with Loss > VaR: **5.33%**

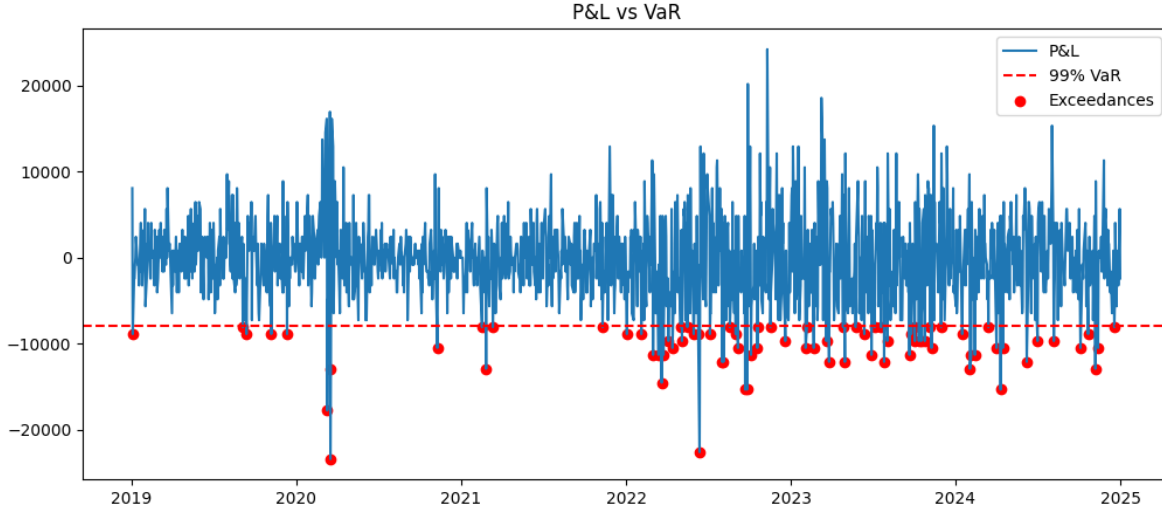


Figure 8: P&L vs 1-Day Parametric VaR (95%) with Exceedances Highlighted

#### 4.2.6 Interpretation

The observed exceedance rate of **5.33%** aligns closely with the theoretical 5% expected under the 95% confidence level. This suggests the parametric VaR model captures interest rate risk with reasonable accuracy. However, visual inspection indicates potential breach clustering during high-volatility periods, which may be attributed to non-normal tails and structural shifts in interest rate regimes.

#### 4.2.7 Limitations

- Linearity assumption from PV01 breaks down for large rate shocks.
- The rate change distribution exhibits fat tails, which the Gaussian model underrepresents.
- No dynamic modeling of yield curve shifts or convexity adjustments.

## 4.3 Parametric VaR Estimation: Diversified Portfolio (GLD, TLT, SPY, EUR/USD)

### 4.3.1 Portfolio Composition

The diversified portfolio analyzed in this section consists of four distinct asset classes, each representing a different segment of the financial markets. The allocations are equal-weighted (25% each), with a total portfolio value of \$1,000,000.

- **GLD** – SPDR Gold Shares ETF: Tracks the price of gold bullion; used as a proxy for commodity exposure.
- **TLT** – iShares 20+ Year Treasury Bond ETF: Tracks long-term U.S. Treasury bonds; represents the fixed income component.
- **SPY** – SPDR S&P 500 ETF Trust: A major equity ETF tracking the S&P 500 index; represents the equity market.
- **EUR/USD** – Euro to U.S. Dollar Exchange Rate: Represents the foreign exchange component, capturing currency risk.

### 4.3.2 Correlation Structure and Asset Behavior

To better understand the portfolio's risk structure, we first examine the correlation matrix and distribution of individual asset returns:

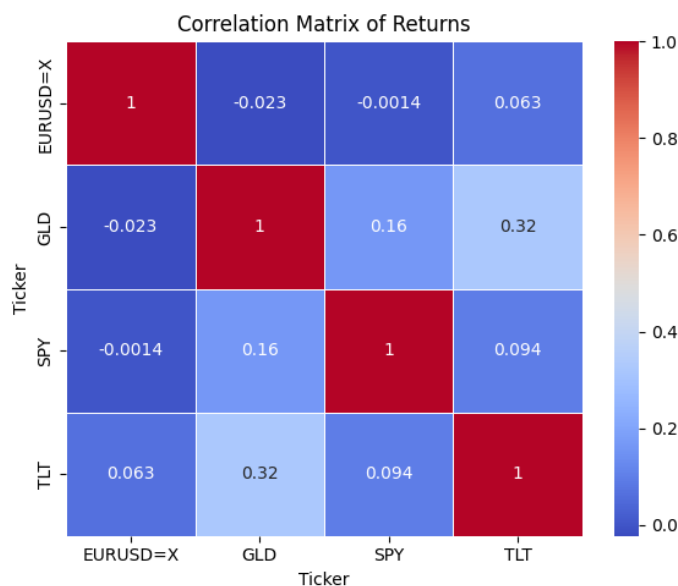


Figure 9: Correlation Matrix of Daily Returns (2022–2024)



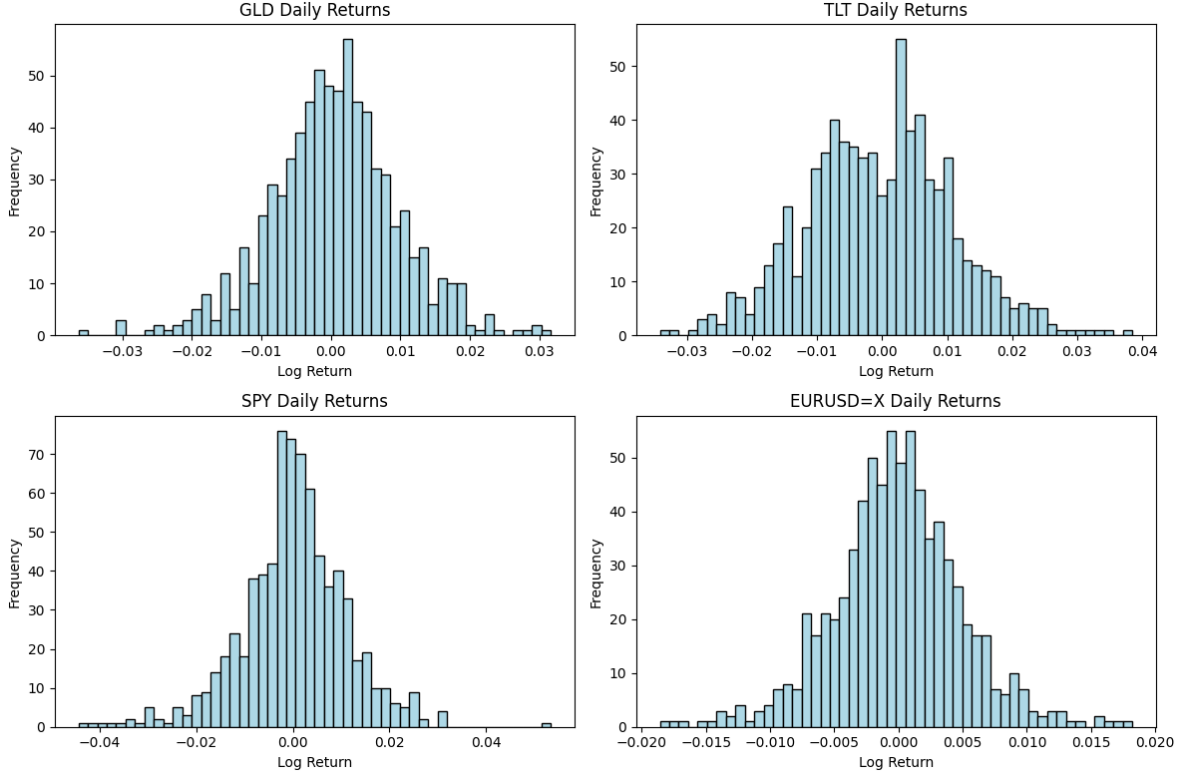


Figure 10: Histograms of Daily Log Returns for Each Asset

These figures reveal low to moderate correlations, indicating potential diversification benefits.

#### 4.3.3 Methodology: Multivariate Risk Aggregation

We estimate the portfolio's parametric VaR using the variance-covariance approach. Let  $\mathbf{w}$  be the portfolio weight vector and  $\mathbf{\Sigma}$  the empirical covariance matrix of log returns. The portfolio's daily variance is:

$$\sigma_p^2 = \mathbf{w}^\top \mathbf{\Sigma} \mathbf{w} \quad (11)$$

The one-day VaR at confidence level  $\alpha$  is then computed as:

$$\text{VaR}_\alpha^{(1d)} = -z_\alpha \cdot \sigma_p \cdot V \quad (12)$$

where:

- $z_\alpha$  is the Z-score at confidence level  $\alpha$  (e.g., 1.645 for 95%),
- $\sigma_p$  is the portfolio's standard deviation,
- $V$  is the portfolio value (\$1,000,000).

#### 4.3.4 Bond Component Handling via PV01

To capture interest rate risk for the TLT bond component, we use the PV01 approach:

$$\text{VaR}_{\text{IR, TLT}} = -z_{\alpha} \cdot \sigma_{\Delta y} \cdot \text{PV01} \cdot \text{Exposure} \quad (13)$$

- PV01 per dollar = 0.000111
- Exposure = \$250,000
- Rate volatility = 109.28 bps

**VaR (TLT): \$4,981.93**

#### 4.3.5 Portfolio Return Evolution

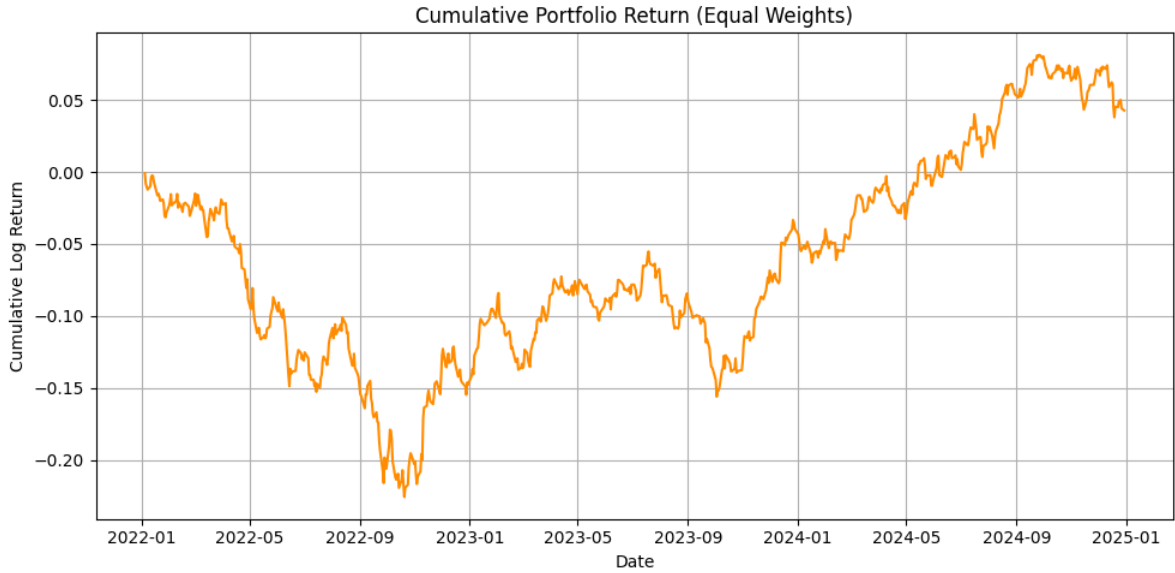


Figure 11: Cumulative Log Return of the Portfolio (Equal Weights)

#### 4.3.6 Computation and Results

The estimated portfolio variance and standard deviation are:

$$\sigma_p^2 = 3.03 \times 10^{-5}, \quad \sigma_p = 0.005507 \quad (14)$$

**1-Day Parametric VaR (95% Confidence): \$9,052.73**

### 4.3.7 Backtesting and Model Validation

To evaluate model effectiveness, we compare the daily P&L with the VaR threshold over the sample period:

- **Number of VaR Exceedances: 38**
- **Percentage of Days with Loss > VaR: 5.06%**

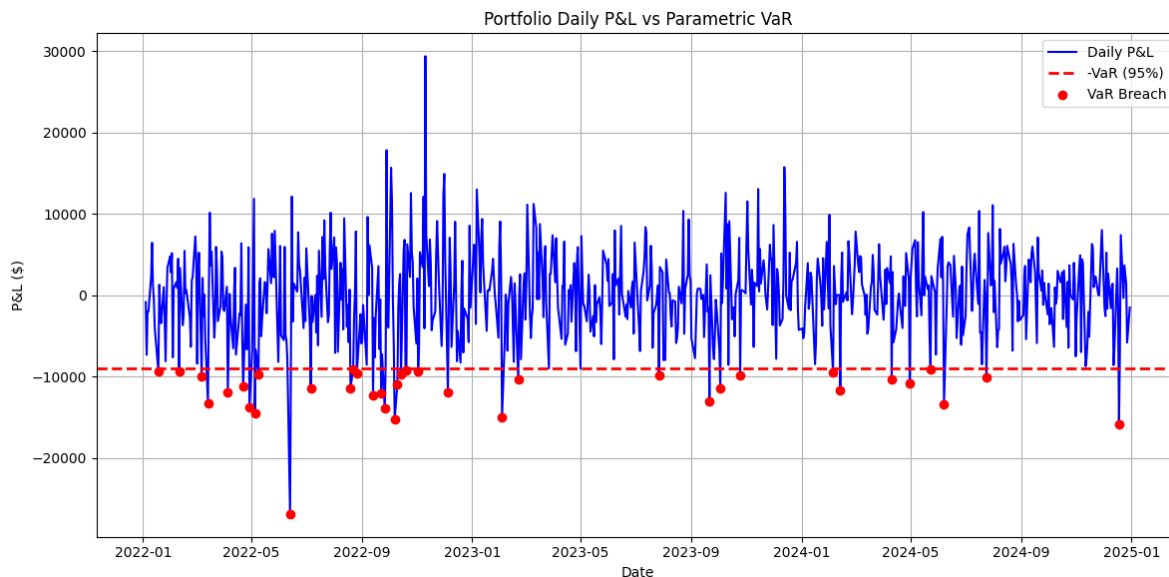


Figure 12: Portfolio P&L vs Parametric VaR (95%) with Breaches Highlighted

### 4.3.8 Individual Asset VaR Estimates

- **GLD: \$3,773.37** (Std Dev = 0.009176)
- **TLT: \$4,981.93** (PV01 Method)
- **SPY: \$4,541.18** (Std Dev = 0.011043)
- **EUR/USD: \$2,071.84** (Std Dev = 0.005038)

If TLT is modeled as a standard asset using historical volatility:

- **TLT (Normal): \$4,612.18** (Std Dev = 0.011216)

### 4.3.9 Interpretation

The observed exceedance rate (5.06%) aligns closely with the theoretical expectation (5%) for a 95% VaR. This suggests that the parametric model performs acceptably in capturing typical risk. However, visual inspection reveals occasional breach clustering, emphasizing the need for more robust tail modeling during volatile periods.

#### 4.3.10 Limitations

- PV01 approximation ignores convexity and non-linear effects in bond pricing.
- The model assumes normality of returns and fixed correlations.
- Potential correlation breakdown and volatility clustering are not captured.
- FX and commodity risk may require more specialized treatment under stressed conditions.

## 4.4 Monte Carlo VaR Estimation: Diversified Portfolio (GLD, TLT, SPY, EUR/USD)

### 4.4.1 Methodology: Simulation-Based Risk Modeling

Monte Carlo simulation provides a robust and flexible framework for estimating Value at Risk (VaR), especially in portfolios that exhibit complex dynamics or nonlinear exposures. Unlike parametric models, which rely on normality and closed-form aggregation, the Monte Carlo approach generates a large number of hypothetical future scenarios by sampling from the empirical return distribution.

The simulation process follows these steps:

1. Generate a matrix of **uniform random variables**:

$$U \sim \mathcal{U}(0, 1) \quad (15)$$

2. Transform uniform variables into **standard normal shocks** using the inverse CDF:

$$Z = \Phi^{-1}(U) \quad (16)$$

3. Use the Cholesky decomposition of the empirical covariance matrix  $\Sigma$  to obtain a lower triangular matrix  $L$ :

$$\Sigma = LL^\top \quad (17)$$

4. Generate **correlated asset returns**:

$$R_{\text{sim}} = ZL^\top \quad (18)$$

5. Compute simulated portfolio returns:

$$r_p^{(i)} = \mathbf{w}^\top R_{\text{sim}}^{(i)} \quad (19)$$

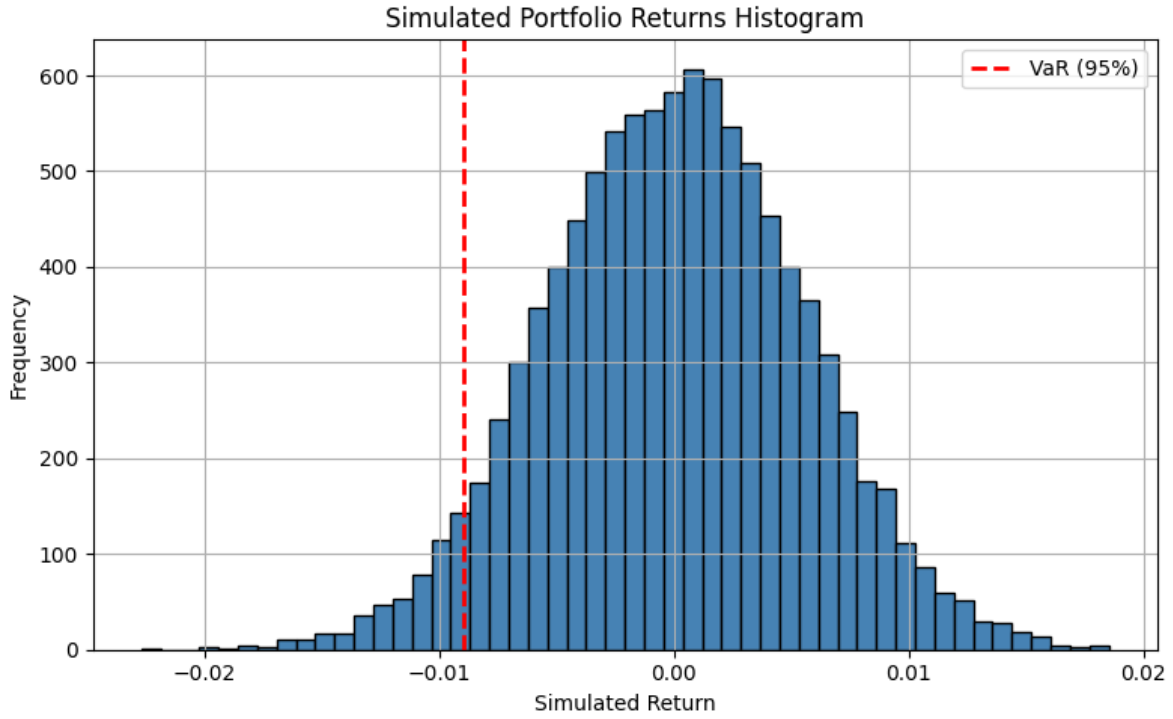


Figure 13: Transformation from Uniform to Normal Distribution (Asset 1)

#### 4.4.2 Simulation Setup and Inputs

- Number of simulations: **10,000**
- Portfolio weights: **[0.25, 0.25, 0.25, 0.25]**
- Data range: **January 2022 – December 2024**
- Assets: GLD, TLT, SPY, EUR/USD

Correlation structure is preserved via Cholesky decomposition:

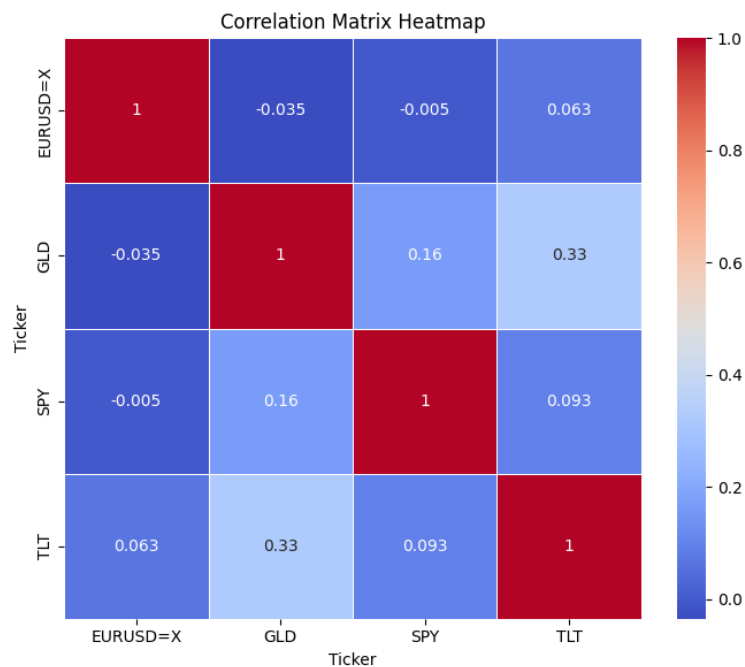


Figure 14: Correlation Matrix of Asset Returns

#### 4.4.3 Empirical Return Analysis and Simulated Distribution

Before simulation, we analyze historical portfolio returns:

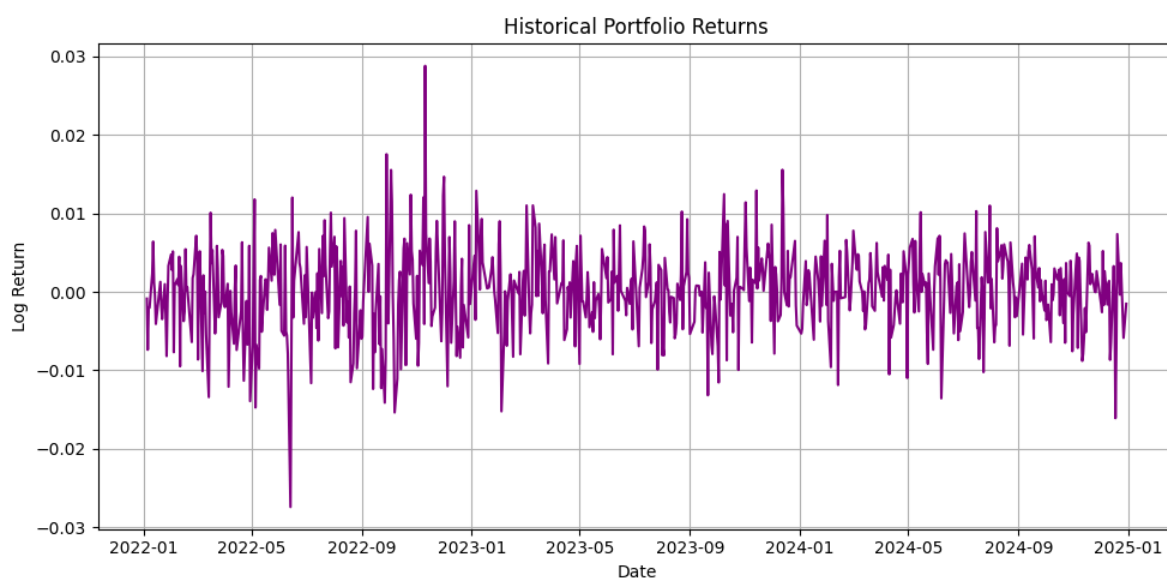


Figure 15: Historical Portfolio Returns (Log Daily)

#### 4.4.4 Results

The Monte Carlo estimate of one-day VaR at 95% confidence level is:

**VaR (95%): 0.8938% of Portfolio Value**  
**VaR in Dollars: \$8,938.00**

#### 4.4.5 Backtesting Results

Backtesting over the same historical window yields:

- **Number of VaR Exceedances: 38**
- **Percentage of Days with Loss > VaR: 5.26%**

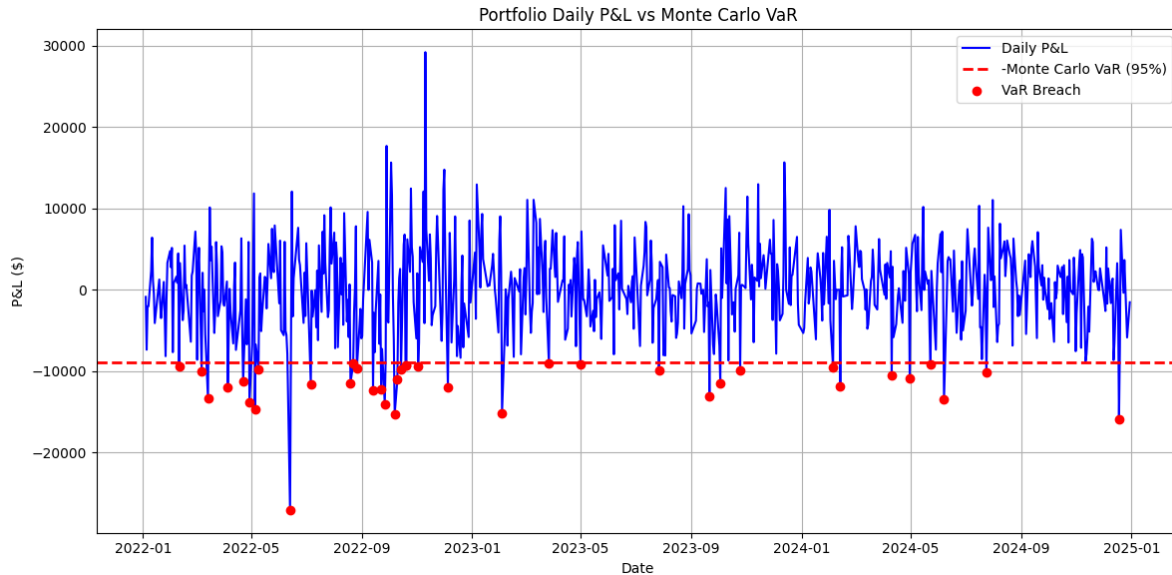


Figure 16: Portfolio Daily P&L vs Monte Carlo VaR (95%) with Breaches Highlighted

#### 4.4.6 Comparison with Parametric VaR

- **Parametric VaR: \$9,052.73 (0.9053%)**
- **Monte Carlo VaR: \$8,938.00 (0.8938%)**

**Interpretation:** While both models yield similar risk estimates, the Monte Carlo VaR is slightly *lower* than the parametric VaR (\$8,938 vs. \$9,052), suggesting a marginally less conservative estimate. This is also reflected in the backtesting results, where the Monte Carlo model resulted in a slightly higher number of exceedances (5.26% vs. 5.06%). This indicates that, despite its flexibility and ability to model joint behavior through simulation, the Monte Carlo VaR underestimated tail risk more frequently in this setup.



#### **4.4.7 Strengths of Monte Carlo Approach**

- Preserves realistic dependency structure across assets.
- Allows flexible modeling of non-linear payoffs and exotic instruments.
- Avoids analytical assumptions on joint distribution shape.

#### **4.4.8 Limitations**

- Computationally intensive for real-time applications or long-term horizons.
- Results are still driven by assumptions of normality in the base case (unless non-normal marginals are used).
- Accuracy depends on simulation granularity and stability of covariance estimates.

## 5 Conclusion

### 5.1 Summary of Findings

This study presented a multi-faceted investigation into Value at Risk (VaR) estimation, applying theoretical and empirical tools across different asset classes and modeling frameworks. Through hands-on Python-based implementations and backtesting using real market data (2019–2024), we explored the risk profiles of:

- **Equities:** via the S&P 500 Index (univariate log returns, Gaussian VaR),
- **Fixed Income:** using 10-Year U.S. Treasury Bonds (rate-based PV01 approximation),
- **Multi-Asset Portfolio:** combining GLD (gold), TLT (bonds), SPY (equity), and EUR/USD (forex).

Three core VaR methodologies were implemented and benchmarked:

- **Parametric VaR:** based on variance-covariance assumptions and normal distribution of returns,
- **PV01-Based Estimation:** accounting for interest rate sensitivity in fixed income securities,
- **Monte Carlo Simulation:** leveraging joint return distributions and Cholesky-decomposed covariance structures.

Key empirical insights include:

- Parametric VaR, while simple and intuitive, performs reasonably well under normal market conditions but may understate tail risk, particularly during volatile periods.
- The PV01-based method enhances bond risk estimation by incorporating rate sensitivity directly, rather than treating bonds like equity.
- Monte Carlo simulation provides more flexibility and realism in capturing joint portfolio behavior, though its benefits are more pronounced when distributions deviate from Gaussian assumptions.
- Backtesting of both portfolio-level VaRs (parametric and Monte Carlo) yielded breach rates slightly above the theoretical 5% threshold, suggesting decent performance under historical conditions, but highlighting mild underestimation of extreme events—particularly in the Monte Carlo case.

## 5.2 Limitations

Despite the breadth of this analysis, several limitations must be acknowledged:

- **Distributional Assumptions:** Both the parametric and Monte Carlo models in this study assume normally distributed returns. This does not fully capture empirical characteristics such as fat tails, skewness, or volatility clustering.
- **Static Covariance:** All estimations relied on historical covariance matrices, implicitly assuming time-invariant risk relationships. This is unlikely to hold in stressed environments.
- **PV01 Approximation:** While effective for a first-order bond sensitivity estimate, PV01 ignores higher-order risks such as convexity and duration drift.
- **No Tail-Specific Modeling:** The analysis did not incorporate extreme value theory (EVT) or empirical distribution fitting to better model rare but impactful events.
- **Linear Aggregation:** All methods assumed linear portfolio behavior, which may be inappropriate for derivatives or strategies involving optionality or leverage.

## 5.3 Final Remarks

Risk management in finance is a delicate balance between model simplicity and robustness. While VaR remains a widely used and powerful tool, its accuracy depends heavily on modeling choices, underlying assumptions, and the quality of input data. As financial markets evolve and risks become more intertwined, a combination of analytical tools, simulation-based techniques, and stress testing frameworks will be essential for navigating uncertainty with greater precision.

# A Appendix: Code and Data Access

All Python code notebooks, datasets, and simulation scripts used in this research are available in the following shared folder:

### Google Drive

The repository includes:

- Google Colab notebooks for each VaR estimation technique (parametric, PV01-based, Monte Carlo),
- Data collection scripts using `yfinance`, `pandas_datareader`, and FRED,
- Visualization modules for plotting risk distributions, correlation matrices, and VaR backtests,

Users are encouraged to explore and adapt the codebase for research, teaching, or applied finance projects.