

These problems are used in subsequent chapters of the book to illustrate different algorithm design techniques and methods of algorithm analysis.

## Sorting

The *sorting problem* is to rearrange the items of a given list in nondecreasing order. Of course, for this problem to be meaningful, the nature of the list items must allow such an ordering. (Mathematicians would say that there must exist a relation of total ordering.) As a practical matter, we usually need to sort lists of numbers, characters from an alphabet, character strings, and, most important, records similar to those maintained by schools about their students, libraries about their holdings, and companies about their employees. In the case of records, we need to choose a piece of information to guide sorting. For example, we can choose to sort student records in alphabetical order of names or by student number or by student grade-point average. Such a specially chosen piece of information is called a *key*. Computer scientists often talk about sorting a list of keys even when the list's items are not records but, say, just integers.

Why would we want a sorted list? To begin with, a sorted list can be a required output of a task such as ranking Internet search results or ranking students by their GPA scores. Further, sorting makes many questions about the list easier to answer. The most important of them is searching: it is why dictionaries, telephone books, class lists, and so on are sorted. You will see other examples of the usefulness of list presorting in Section 6.1. In a similar vein, sorting is used as an auxiliary step in several important algorithms in other areas, e.g., geometric algorithms and data compression. The greedy approach—an important algorithm design technique discussed later in the book—requires a sorted input.

By now, computer scientists have discovered dozens of different sorting algorithms. In fact, inventing a new sorting algorithm has been likened to designing the proverbial mousetrap. And I am happy to report that the hunt for a better sorting mousetrap continues. This perseverance is admirable in view of the following facts. On the one hand, there are a few good sorting algorithms that sort an arbitrary array of size  $n$  using about  $n \log_2 n$  comparisons. On the other hand, no algorithm that sorts by key comparisons (as opposed to, say, comparing small pieces of keys) can do substantially better than that.

There is a reason for this embarrassment of algorithmic riches in the land of sorting. Although some algorithms are indeed better than others, there is no algorithm that would be the best solution in all situations. Some of the algorithms are simple but relatively slow, while others are faster but more complex; some work better on randomly ordered inputs, while others do better on almost-sorted lists; some are suitable only for lists residing in the fast memory, while others can be adapted for sorting large files stored on a disk; and so on.

Two properties of sorting algorithms deserve special mention. A sorting algorithm is called *stable* if it preserves the relative order of any two equal elements in its input. In other words, if an input list contains two equal elements in positions  $i$  and  $j$  where  $i < j$ , then in the sorted list they have to be in positions  $i'$  and  $j'$ ,

respectively, such that  $i' < j'$ . This property can be desirable if, for example, we have a list of students sorted alphabetically and we want to sort it according to student GPA: a stable algorithm will yield a list in which students with the same GPA will still be sorted alphabetically. Generally speaking, algorithms that can exchange keys located far apart are not stable, but they usually work faster; you will see how this general comment applies to important sorting algorithms later in the book.

The second notable feature of a sorting algorithm is the amount of extra memory the algorithm requires. An algorithm is said to be *in-place* if it does not require extra memory, except, possibly, for a few memory units. There are important sorting algorithms that are in-place and those that are not.

## Searching

The *searching problem* deals with finding a given value, called a *search key*, in a given set (or a multiset, which permits several elements to have the same value). There are plenty of searching algorithms to choose from. They range from the straightforward sequential search to a spectacularly efficient but limited binary search and algorithms based on representing the underlying set in a different form more conducive to searching. The latter algorithms are of particular importance for real-world applications because they are indispensable for storing and retrieving information from large databases.

For searching, too, there is no single algorithm that fits all situations best. Some algorithms work faster than others but require more memory; some are very fast but applicable only to sorted arrays; and so on. Unlike with sorting algorithms, there is no stability problem, but different issues arise. Specifically, in applications where the underlying data may change frequently relative to the number of searches, searching has to be considered in conjunction with two other operations: an addition to and deletion from the data set of an item. In such situations, data structures and algorithms should be chosen to strike a balance among the requirements of each operation. Also, organizing very large data sets for efficient searching poses special challenges with important implications for real-world applications.

## String Processing

In recent decades, the rapid proliferation of applications dealing with nonnumerical data has intensified the interest of researchers and computing practitioners in string-handling algorithms. A *string* is a sequence of characters from an alphabet. Strings of particular interest are text strings, which comprise letters, numbers, and special characters; bit strings, which comprise zeros and ones; and gene sequences, which can be modeled by strings of characters from the four-character alphabet {A, C, G, T}. It should be pointed out, however, that string-processing algorithms have been important for computer science for a long time in conjunction with computer languages and compiling issues.