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MATH2144.M31.CTTT-Analysis I
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Parametric equations and partial derivetives

1. Parametric equations

Question 1. Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter

1) $x = 1 + 4t - t^2$, $y = 2 - t^3$; $t = 1$
2) $x = t\cos t$, $y = t\sin t$, $t = \pi$
3) $x = 1 + lnt, y = t^2 + 2$ at $(1, 3)$
4) $x = 6\sin t, y = t^2 + t$ at $(0, 0)$

Question 2. Find dy/dy, d^2y/d^2x.

1) $x = t^2 + 1, y = t^2 + t$
2) $x = t^3 + 2, y = t^2 - t$
3) $x = e^t, y = te^{-1}$
4) $x = 2\sin t, y = 3\cos t$, $0 < t < 2\pi$
5) $x = \cos 2t, y = \cos t$, $0 < T < \pi$.

Question 3. Find equations of the tangents to the curve $x = 3t^2 + 1, y = 2t^3 + 1$ that pass through $(4, 3)$

Question 4. Find the points on the curve where the tangent is horizontal or vertical

1) $x = t^3 - 3t$, $y = t^2 - 3$
2) $x = t^3 - 3t, y = t^3 - 3t$
3) $x = \cos\theta, y = \cos 3\theta$
4) $x = e^{\sin\theta}, y = e^{\cos\theta}$.

Question 5. Show that the curve $x = \cos t, y = \sin t$, has two tangents at $(0, 0)$ and find their equations.

Question 6. Find the slope of the tangent line to $x = r\theta - d\sin\theta, y = r - d\cos\theta$ in term of θ.

Question 7. Find the exact length of the curve

1) $x = 1 + 3t^2, y = 4 + 2t^3$, $0 \le t \le 1$
2) $x = e^t + e^{-t}, y = 5 - 2t$, $0 \le t \le 3$
3) $x = t\sin t, y = t\cos t$ $0 \le t \le 1$
4) $x = 3\cos t - 3\cos 3t, y = 3\sin t - \sin 3t, 0 \le t \le \pi$
5) $x = 1 - 2\sin t, y = 1 - 2\cos t, 0 \le t \le 4\pi$
6) $x = e^t\cos t, y = e^t\sin t, 0 \le t \le \pi$.

Question 8. Find the area enclosed by the curve $x = t^2 - 2t, y = \sqrt{t}$ and $y-$axis.

Question 9. Find the area enclosed by the $x-$axis and the curve $x = 1 + e^t, y = t - t^2$.

Question 10. Use the parametric equations of an ellipse, $x = a\cos\theta, y = b\sin\theta, 0 \leq \theta \leq 2\pi$, to find the area that it encloses.

Question 11. Identify the curve by finding a Cartesian equation for the curve

1) $r = 5$
2) $r = 4\cos\theta$,
3) $r^2\cos 2\theta = 1$
4) $\theta = \pi/3$.

Question 12. Find a polar equation for the curve represented by the given Cartesian equation

1) $y = 2$
2) $y = 1 + 3x$
3) y=x
4) $xy = 4$
5) $x^2 + y^2 = 2cx$
6) $4y^2 = x$.

12.1. Function of several variables

Question 13. Let $g(x, y) = \cos(x + 2y)$,

1) evaluate $g(2, -1)$
2) find and sketch the domain of g
3) range of g.

Question 14. Let $g(x, y) = 1 + \sqrt{4 - y^2}$,

1) evaluate $g(3, 1)$
2) find and sketch the domain of g
3) range of g.

Question 15. For each of the following functions describe the domain in words. If possible, draw a sketch of the domain as well.

1) $f(x, y) = \sqrt{10 - x^2 - y^2}$
2) $f(x, y) = \sin(2x + y)$
3) $f(x, y, z) = Ln(16 - 4x^2 - 4y^2 - z^2)$
4) $f(x, y, z) = \sqrt{6 - 2x - 3y - z}$

Question 16. Let $f(x, y) = 2xe^{3y}$, compute

- $f(4, 0)$
- $f(1, Ln2)$.

Question 17. Suppose $f(x, y) = \int_x^y (t^2 - 1)dt$, compute

- $f(-1, 2)$
- $f(0, 2)$.

Question 18. Consider $f(x, y) = x^2 + y^2$, if $x(t) = 1 + t$ and $y(t) = 2 - 3t$, find $f(x(t), y(t))$

Question 19. Sketch the level curves $f(x, y) = k$, for the specified values of k

- $z = 2x - y$; $k = -2, -1, 0, 1, 2$
- $z = y^2 - x^2$; $k = -2, -1, 0, 1, 2$

Question 20. Shown is a contour map of atmospheric pressure in North America on August 12, 2008. On the level curves, the pressure is indicated in millibars (mb).

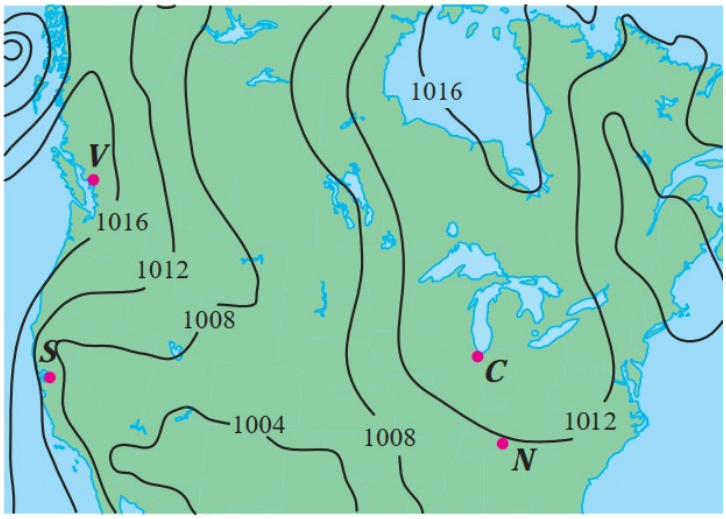

FIGURE 1. Ex8.

- Estimate the pressure at C (Chicago), N (Nashville),
- At which of these locations were the winds strongest?

Question 21. Consider the contour map shown below

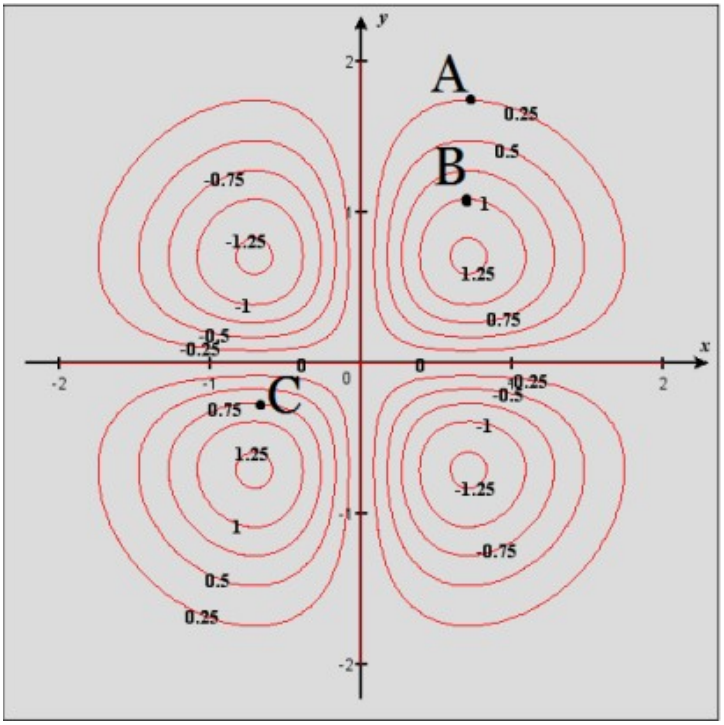

FIGURE 2. Ex9.

1) If a person were walking straight from point A to point B, would s/he be walking uphill or downhill?
2) Is the slope steeper at point B or point C?

3) Starting at C and moving so that x remains constant and y decreases, will the elevation begins to increase or decrease?
4) Starting at B and moving so that y remains constant and x increases, will the elevation begins to increase or decrease?

21.1. Limits and continuity

Question 22. Find the limit, if it exists, or show that the limit does not exist

1) $\lim_{(x,y)\to(1,2)}(x^6y + 2xy$
2) $\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x-y}$
3) $\lim_{(x,y)\to(1,2)}(5x^3 - x^2y^2)$
4) $\lim_{(x,y)\to(1,-1)} e^{-xy}\cos(x+y)$
5) $\lim_{(x,y)\to(2,1)} \frac{4-xy}{x^2+3y^2}$
6) $\lim_{(x,y)\to(1,1)} \frac{xy^2}{x^2+y^4}$
7) $\lim_{(x,y,z)\to(0,0,0)} \frac{xy+yz^2+xz^2}{x^2+y^2+z^4}$
8) $\lim_{(x,y,z)\to(0,0,0)} \frac{yx}{x^2+4y+9z}$

Question 23. Using polar coordinates, if $x = r\cos\theta$ and $y = r\sin\theta$ with $r \geq 0$, find

1) $\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}$
2) $\lim_{(x,y)\to(0,0)}(x^2+y^2)ln(x^2+y^2)$
3) $\lim_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2}-1}{x^2+y^2}$.

Question 24. Does $\lim_{(x,y)\to(0,0)} \dfrac{xy^2}{x^2+y^4}$ exist?

Question 25. Find $\lim_{(x,y)\to(0,0)} \dfrac{3x^2y}{x^2+y^2}$.

Question 26. Determine the set of points at which the function is continuous

1) $f(x,y) = x^2y + 3x^3y^4 - x + 2y$
2) $f(x,y) = \frac{2x-y}{x^2+y^2}$
3) $f(x,y) = \frac{1}{x^2-y}$
4) $f(x,y) = \tan^{-1}\left(\frac{xy^2}{x+y}\right)$
5) $f(x,y,z) = ln(x^2+y^2+z^2-1)$
6) $f(x,y,x) = \sqrt{y^2-x}lnz$
7) $f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{at } (0,0) \end{cases}$
8) $f(x,y) = \begin{cases} \frac{x^2y}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{at } (0,0) \end{cases}$

26.1. Partial derivatives

Question 27.

1) 5-6/pages 936-Jame Stewart
2) 7-8/pages 936-Jame Stewart
3) 9/pages 936-Jame Stewart
4) 10/pages 936-Jame Stewart

Question 28.

1) 11-15-17/pages 936-Jame Stewart
2) 12-22/pages 936-Jame Stewart
3) 31-33
4) 53-54/pages 936-Jame Stewart

Question 29.

1) 53-54-55/pages 936-Jame Stewart
2) 56-57-58/pages 936-Jame Stewart
3) 101/pages 936-Jame Stewart

Question 30.

1) 1-2/pages 946-Jame Stewart
2) 3-4/pages 946-Jame Stewart
3) 5-6/pages 946-Jame Stewart

Question 31.

1) 2/pages 967-Jame Stewart
2) 36/pages 967-Jame Stewart

Question 32.

1) 4-5/pages 967-Jame Stewart
2) 6-7/pages 967-Jame Stewart
3) 8-9/pages 967-Jame Stewart

Question 33.

1) 3/pages 978-Jame Stewart
2) 4/pages 978-Jame Stewart
3) 5-6/pages 978-Jame Stewart
4) 7-8/pages 978-Jame Stewart
5) 9-10/pages 978-Jame Stewart

Question 34.

1) 29-30/pages 978-Jame Stewart
2) 31-32/pages 978-Jame Stewart
3) 33-34/pages 978-Jame Stewart
4) 36-37/pages 978-Jame Stewart

Question 35.

1) 1/pages 987-Jame Stewart
2) 3-4/pages 987-Jame Stewart
3) 5-6/pages 987-Jame Stewart

Question 36. 22/pages 987-Jame Stewart

Question 37. 23/pages 987-Jame Stewart

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