CS 721: Advanced Algorithms & Analysis

Homework 4, Fall 2019, Total 100 points

Assigned on: Tuesday, 10/29/2019
Due on: Wednesday, 11/06/2019

1. (20 points) Suppose T is a binary search tree with m nodes, A is a an array with n elements, and a procedure P employs the Tree insert operation of Page 294 of your textbook repeatedly to insert all elements of A into T, one after another. What is the best case and worst case running time of this operation? Express your answer in Big-Oh or Θ notation in terms of m and n and provide brief justification for your answer.

2. (40 points) In this problem you will consider dynamic tables where only insertions are allowed. Initially the size of the table is 1. The cost of insertion is 1 if the table is not full. When an item is inserted into a full table, it first expands the table and then inserts the item into the new table. The expansion is done by allocating a table of size 3 times larger than the old and copying all elements of the old table into the new table.

   (a) Calculate the amortized cost using accounting method.
   (b) Calculate the amortized cost using potential function method by defining appropriate potential function.

3. (20 points) A pharmacist has W pills and n empty bottles. Let $\{p_1, p_2, \ldots, p_n\}$ denote the number of pills that each bottle can hold. Bottle i has associated cost c_i and the pharmacist wants to minimize the total cost of the bottles used to store all the pills. Give a bottom-up dynamic programming solution for this problem.

4. (20 points) Answer the following.

   (a) Suppose you insert a node with key value 36 in the red-black tree shown on page 310 of your textbook (Figure 13.1). If the inserted node is colored red, will the resulting tree be a red black tree? What if it is colored black?
   (b) Explain why the longest path from a node x in a red black tree to a descendant leaf has length at most twice that of the shortest path from node x to a descendant leaf.
   (c) What is the largest possible number of internal nodes in a red-black tree with black height k? What is the smallest possible number?
   (d) Describe a red-black tree on n keys that realizes the largest possible ratio of red internal nodes to black internal nodes. What is this ratio? What tree has the smallest possible ratio, and what is the ratio?

Submision:

- All texts and diagrams must be electronically produced.

- Your name and page number should appear on each page.

- Entire assignment should be a single PDF file.

- the PDF file should be named in the format HW04_Lastname_Firstname.pdf, for example HW04_Sinha_Kaushik.pdf.

- Submit the pdf file on blackboard.

- **This homework assignment is due at 11:59 pm on the due date.**