HW #5 (Solution)

CS 771

(1) (a) $\exists x \; T(x) \wedge F(x)$

(b) $\forall x \; T(x) \Rightarrow C(x)$

(c) $\forall x \; (T(x) \vee C(x)) \Rightarrow \exists y \; (Fr(x,y) \wedge \neg(Fx)$

(d) $\neg \exists x \; (T(x) \wedge C(x) \wedge F(x))$

(e) $\forall x \; C(x) \Rightarrow (\exists y \; Fr(y,x) \wedge C(y))$

(3) (b) is legitimate but not (a). In order to drop an existential quantifier the variable must be set to a constant this not used previously.

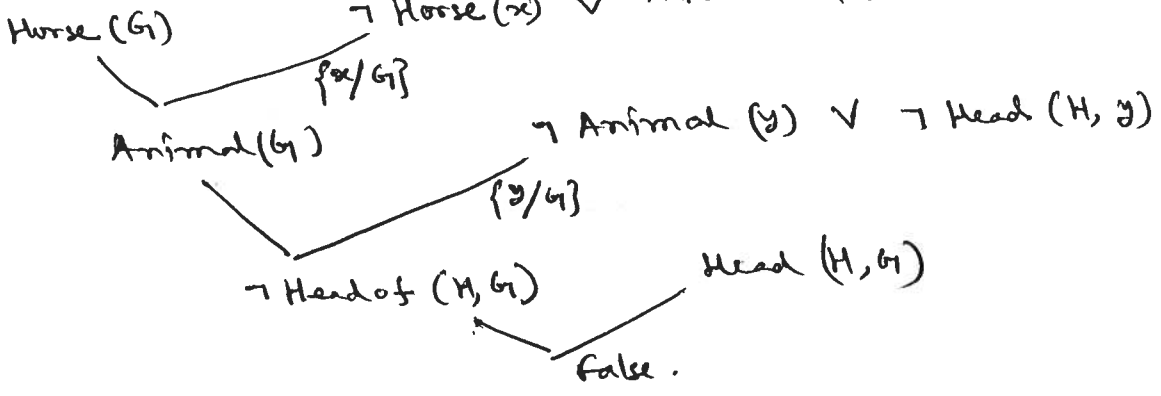
(4) (a) No unifier

(b) $\{x/A, y/B, z/B\}$

(c) No unifier. x can not be bind both y and father(y)

(d) $\{y/John, x/John\}$

(2) (d) Resolution. see part (a), (b), (c) in next page.



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2(a) $\forall x \ \text{Horse}(x) \Rightarrow \text{Animal}(x)$

(b) $\forall x,h \ \text{Horse}(x) \wedge \text{HeadOf}(h,x) \Rightarrow \exists y \ \text{Animal}(y) \wedge \text{HeadOf}(h,y)$

(c) Premise. $\text{Horse}(x) \Rightarrow \text{Animal}(x) \equiv \neg \text{Horse}(x) \vee \text{Animal}(x)$

Conclusion.

$\forall x,h \ \text{Horse}(x) \wedge \text{HeadOf}(h,x) \Rightarrow \exists y \ \text{Animal}(y) \wedge \text{HeadOf}(h,y)$

(negate)

$\neg \left(\forall x,h \ \text{Horse}(x) \wedge \text{HeadOf}(h,x) \Rightarrow \exists y \ \text{Animal}(y) \wedge \text{HeadOf}(h,y) \right)$

(Eliminate Implication)

$\neg \left(\forall x,h \ \neg (\text{Horse}(x) \wedge \text{HeadOf}(h,x)) \vee (\exists y \ \text{Animal}(y) \wedge \text{HeadOf}(h,y)) \right)$

$(\neg \forall x \, p \equiv \exists x \neg p$ }, Take the negation inside)

$\exists x,h \ (\text{Horse}(x) \wedge \text{HeadOf}(h,x)) \wedge \neg (\exists y \ \text{Animal}(y) \wedge \text{HeadOf}(h,y))$

$(\neg \exists x \, p \equiv \forall x \neg p$, Take the negation inside)

$\exists x,h \ (\text{Horse}(x) \wedge \text{HeadOf}(h,x)) \wedge (\forall y \ \neg \text{Animal}(y) \vee \neg \text{HeadOf}(h,y))$

(Drop quantifiers)

$\text{Horse}(G_1) \wedge \text{HeadOf}(H,G_1) \wedge (\neg \text{Animal}(y) \vee \neg \text{HeadOf}(H,y))$

$\equiv$ ~~Horse(x) ∧ Head(H,x)~~ This is CNF format.

So we have,

1. $\neg \text{Horse}(x) \vee \text{Animal}(x)$
2. $\text{Horse}(G_1)$
3. $\text{HeadOf}(H,G_1)$
4. $\neg \text{Animal}(y) \vee \neg \text{HeadOf}(H,y)$