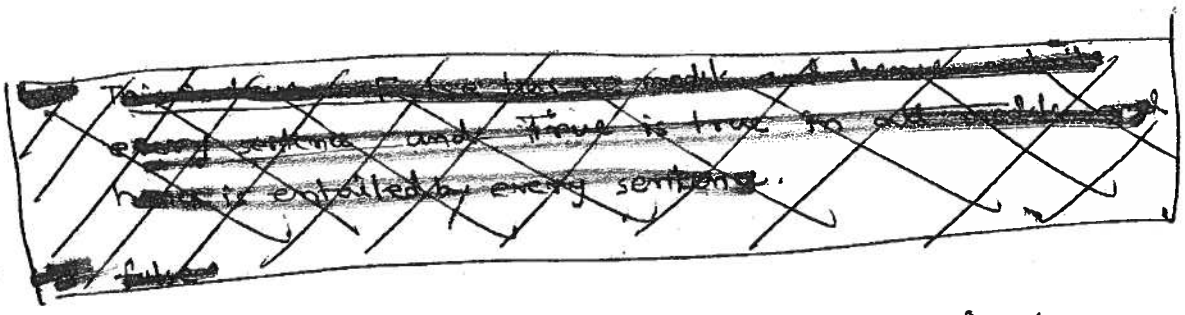
CS-771

HW # 4 Solution

[redacted]

1. (a) $Q \Rightarrow P$
   (b) $Q \Leftrightarrow P$
   (c) $P \vee Q$
   (d) $P \wedge Q \wedge \neg M$
   (e) $T \wedge S$ where T = John is tall, S = Peter has short hair.

2. 

   (a) $(A \wedge B) \models (A \Leftrightarrow B)$ is true. The left hand side has exactly one model in which it is true. A=true and B=true. The right hand side has two models in which it is true (A=true, B=true), (A=false, B=false)

   (b) $(A \Leftrightarrow B) \models (A \vee B)$ is false. because $A \Leftrightarrow B$ has both A and B false, which doesn't satisfy $A \vee B$.

   (c) $(A \Leftrightarrow B) \models \neg A \vee B$ is true. because RHS is $A \Rightarrow B$ which is one of the conjuncts of $A \Leftrightarrow B \equiv (A \Rightarrow B) \wedge (B \Rightarrow A)$

3. (a) valid. $Smoke \Rightarrow Smoke \equiv \neg Smoke \vee Smoke$

   (b) not valid $Smoke \Rightarrow Fire \equiv \neg Smoke \vee Fire$
   for smoke = True and Fire = false it doesn't satisfy

   (c) Not valid.
   $(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F) \equiv (\neg S \vee F) \Rightarrow (S \vee \neg F)$
   $\equiv \neg(\neg S \vee F) \vee (S \vee \neg F) \equiv (S \wedge \neg F) \vee (S \vee \neg F)$
   This is not true for ~~all possible assignments of~~ ~~true and false~~.
   S = false and F = true.

(d) <u>valid</u> Smoke ∨ Fire ∨ ¬Fire No matter what Smoke or Fire is, it is always true.

(e) Big ∨ Dumb ∨ (Big ⇒ Dumb)

≡ Big ∨ Dumb ∨ ¬Big ∨ Dumb

≡ Big ∨ Dumb ∨ ¬Big.

Valid.

14.

6.	S	(3)	AND elimination.
7.	¬W	(3)	AND elimination.
8.	¬S ∨ (¬Q ∧ T)	(2)	Implication elimination.
9.	¬Q ∧ T	(6, 8)	Unit resolution.
10.	¬Q	(9)	And elimination.
11.	¬Q ∨ ¬S	(10)	OR Introduction.
12.	¬(Q ∧ S)	(11)	De Morgan's Law.

5.

5.	¬T ∨ S	(4)	⇒ elimination.
6.	T	(3)	And elimination.
7.	S	(5, 6)	Unit resolution.
8.	S ∨ R	(7)	OR Introduction.