**CS 771 Artificial Intelligence**
**Spring 2019 Homework 6 (120 points)**

**Assigned: Wednesday, April 17, 2019**
**Due: Wednesday, April 24, 2019**

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1. (10 points) Suppose you are a witness to a night time hit-and-run accident involving a taxi in Athens. All taxis in Athens are blue or green. You swear, under oath, that the taxi was blue. Extensive testing shows that, under the dim light conditions, discriminating between blue and green is 75% reliable, meaning the following. The relevant aspect of the world can be described by two random variables: $B$ means the taxi was blue, and $LB$ means the taxi looked blue. The information on the reliability of color identification can be written as $P(LB|B) = 0.75$ and $P(\neg LB|\neg B) = 0.75$. Now answer the following.

   (a) If you do not know prior probability of a taxi being blue, i.e., $P(B)$, is it possible to calculate and answer the most likely color of the taxi, given that it looked blue? Show your work.

   (b) What is the most likely color of the taxi, given that it looked blue if it is revealed to you that 9 out of 10 Athenian taxis are green?

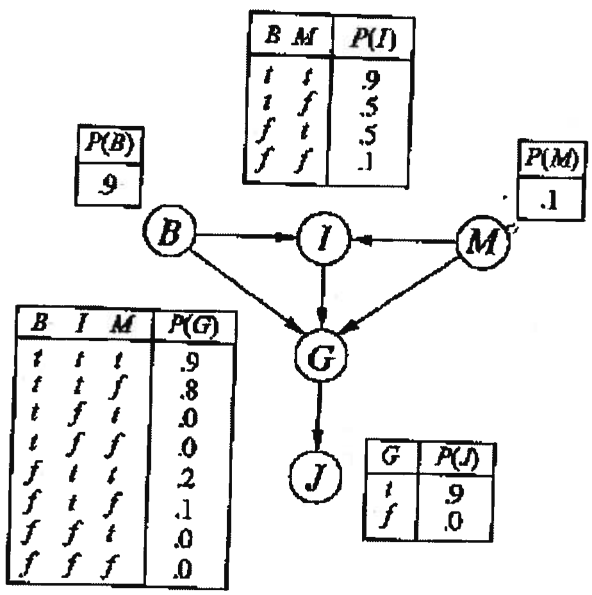
2. (10 points) After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease and that the test is 99% accurate (i.e., the probability of testing positive when you do have the disease is 0.99, also, the probability of testing negative when you don't have the disease). The good news is that this is a rare disease, striking only 1 in 10,000 people of your age. Why is it good news that the disease is rare? What are the chances that you actually have the disease i.e., the probability that you actually have the disease given that you test positive?

3. (10 points) Suppose random variables $X$ and $Y$ are conditionally independent given random variable $Z$, i.e. $P(X, Y|Z) = P(X|Z)P(Y|Z)$. Show that :

   (a) $P(Y|X, Z) = P(Y|Z)$.

   (b) $P(X|Y, Z) = P(X|Z)$.

4. (20 points) Consider the Bayesian network given in Figure 1. Answer the following.




| P(B) |
|---|
| .9 |

| P(M) |
|---|
| .1 |

| B | M | P(I) |
|---|---|---|
| t | t | .9 |
| t | f | .5 |
| f | t | .5 |
| f | f | .1 |

| B | I | M | P(G) |
|---|---|---|---|
| t | t | t | .9 |
| t | t | f | .8 |
| t | f | t | .0 |
| t | f | f | .0 |
| f | t | t | .2 |
| f | t | f | .1 |
| f | f | t | .0 |
| f | f | f | .0 |

| G | P(J) |
|---|---|
| t | .9 |
| f | .0 |

Figure 1: Bayesian Network

(a) Without considering the Bayesian network structure, how many parameters are needed to completely describe the joint probability distribution involving five binary variables $B, I, M, G, J$?

(b) Now if you consider the Bayesian network structure given in Figure 1, how many parameters are needed to completely describe the joint probability distribution involving five binary variables $B, I, M, G, J$?

(c) Express the joint probability distribution involving five binary variables $B, I, M, G, J$ as a product of conditional probability distribution as shown in Figure 1.

(d) Each of the five binary variables above takes either true (t) of false (f) value. Calculate the probability $P(B = t, I = t, M = f, G = t, J = t)$. Note that in figure 1, $P(M)$ means $P(M = t)$ and similar interpretation holds for other variables.

5. (20 points) As part of a comprehensive study of the role of CS 771 on people's happiness we have been collecting important data from WSU students. In an entirely optional hypothetical survey that all students are required to complete, we ask the following highly objective questions:

- Do you party frequently [`Party:` Yes/No]?
- Are you wicked smart [`Smart:` Yes/No]?
- Are you creative [`Creative:` Yes/No]? (Please only answer Yes or No)
- Did you do well on all your homework assignments? [`HW:` Yes/No]
- Do you use a Mac? [`Mac:` Yes/No]
- Did your CS 771 programming assignments succeed? [`Project:` Yes/No]
- Did you succeed in your most important class (CS 771)? [`Success:` Yes/No]
- Are you currently Happy? [`Happy:` Yes/No]

After consulting a behavioral psychologist we obtained the following complete set of conditional relationships:

- `HW` depends only on `Party` and `Smart`.
- `Mac` depends only on `Smart` and `Creative`.
- `Project` depends only on `Smart` and `Creative`.
- `Success` depends only on `HW` and `Project`.
- `Happy` depends only on `Party`, `Mac`, and `Success`.

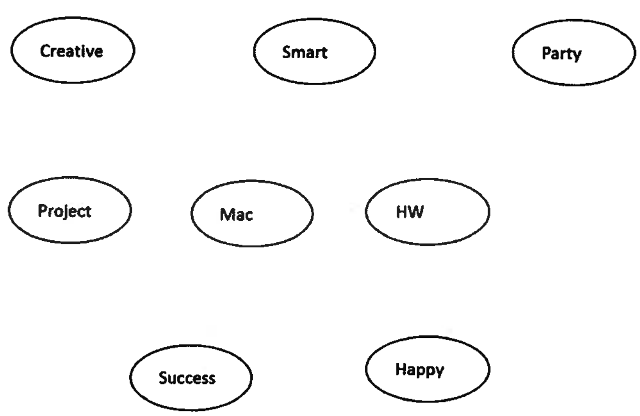



Figure 2: Bayesian Network for happiness in life

Now answer the following.

(a) Draw the Bayesian network given in figure 3.

(b) Write the joint probability as a product of conditional probabilities according to the Bayesian network drawn in part (a).

6. (30 points) For the Bayesian network shown below please indicate if the following cases satisfy conditional independence or not. In Each case, please provide active and inactive triple and justify why the conditional independence assumption hold or does not hold.

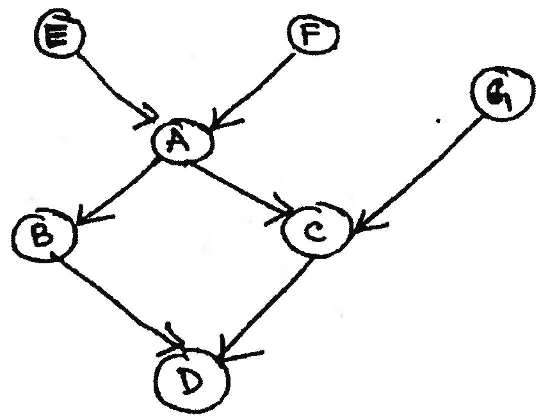



Figure 3: Another Bayesian Network

(a) $E \perp\!\!\!\perp G \mid A$

(b) $E \perp\!\!\!\perp G \mid C$

(c) $E \perp\!\!\!\perp D \mid A$

(d) $E \perp\!\!\!\perp D \mid A,\ B$

(e) $E \perp\!\!\!\perp D \mid B,\ C$

(f) $B \perp\!\!\!\perp G \mid C$

7. (20 points) Consider the Bayesian network shown in the next page involving four variables. Each variable is binary and take positive or negative values as indicated in the CPT table. We are interested in approximation the conditional probability $P(Rain \mid Sprinkler = +s, WetGrass = +w)$, or $P(R \mid +s, +w)$ for short, using likelihood weighting sampling. Suppose the four following observations are sampled that are consistent with the evidence using likelihood weighting sampling.

(a) $+c, +s, +r, +w$

(b) $+c, +s, -r, +w$

(c) $-c, +s, -r, +w$

(d) $-c, +s, +r, +w$

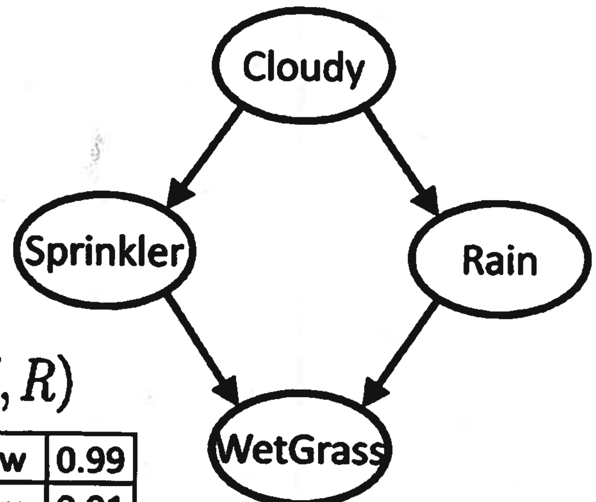
What will be associated wights for each ob these observations? Show your work.

$P(C)$

| | |
|---|---|
| +c | 0.5 |
| -c | 0.5 |

$P(S|C)$

| | | |
|---|---|---|
| +c | +s | 0.1 |
| | -s | 0.9 |
| -c | +s | 0.5 |
| | -s | 0.5 |




$P(R|C)$

| | | |
|---|---|---|
| +c | +r | 0.8 |
| | -r | 0.2 |
| -c | +r | 0.2 |
| | -r | 0.8 |

$P(W|S,R)$

| | | | |
|---|---|---|---|
| +s | +r | +w | 0.99 |
| | | -w | 0.01 |
| | -r | +w | 0.90 |
| | | -w | 0.10 |
| -s | +r | +w | 0.90 |
| | | -w | 0.10 |
| | -r | +w | 0.01 |
| | | -w | 0.99 |