CS-771 Artificial Intelligence

Final project Homework

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1) a) we know that

$$P(B|LB) = \frac{P(LB|B)\,P(B)}{P(LB)} = \alpha\, P(LB|B)\,P(B)$$

$$= \alpha\,(0.75)\,P(B) \quad \text{— ①}$$

$$P(\neg B|LB) = \frac{P(LB|\neg B)\,P(\neg B)}{P(LB)} = \alpha\, P(LB|\neg B)\,P(\neg B)$$

$$= \alpha\,[1 - P(\neg LB|\neg B)]\,[1 - P(B)]$$

$$= \alpha\,(1 - 0.75)(1 - P(B))$$

$$= \alpha\,(0.25)(1 - P(B)) \quad \text{— ②}$$

from ① and ②

$$P(B|LB) + P(\neg B|LB) = \alpha\, 0.75\, P(B) + \alpha\,(0.25)(1 - P(B)) = 1$$

$$\Rightarrow \alpha = 1 / 0.75\,P(B) + 0.25(1 - P(B))$$

$$\therefore P(B|LB) = \frac{0.75\,P(B)}{0.75\,P(B) + 0.25\,[1 - P(B)]} = 3P(B) / 3P(B) + [1 - P(B)]$$

$$P(\neg B|LB) = \frac{0.25\ (1-P(B))}{0.75\ P(B) + 0.25(1-P(B))} = \frac{1-P(B)}{3P(B) + [1-P(B)]}$$

Unless we know $P(B)$, we can't conclude if

$$P(B|LB) > P(\neg B|LB) \quad (or) \quad P(B|LB) \leq P(\neg B|LB)$$

1) b) $P(B) = 1/10 = 0.1$

$$\therefore P(B|LB) = \frac{3(0.1)}{3(0.1)+(1-0.1)} = 0.25$$

$$P(\neg B|LB) = \frac{1-0.1}{3(0.1)+(1-0.1)} = 0.75$$

$\because P(\neg B|LB) > P(B|LB)$, most likely color of taxi is green.

2) Test positive : TP , Have disease : HD

$P(TP|HD) = 0.99$, $P(\neg TP|\neg HD) = 0.99$

$P(HD) = 1/10000 = 0.0001$

We need to find $P(HD|TP)$ from Baye's rule

$$P(HD/TP) = \frac{P(TP|HD)\ P(HD)}{P(TP)} \quad — ①$$

Now, $P(TP) = P(TP|HD)\,P(HD) + P(TP|\neg HD)\,P(\neg HD)$

$$= 0.99(0.0001) + (1-P(\neg TP|\neg HD))\,(1-P(HD))$$

$= 0.99(0.0001) + (1-0.99)(1-0.0001)$

$= 0.99(0.0001) + (0.01)(0.9999)$

Substituting in ①

$$P(HD|TP) = \frac{0.99\ (0.0001)}{(0.99)0.0001 + (0.01)0.9999} = 0.0098$$

$P(HD)$ is small $\therefore$ the disease is really rare, inspite of testing positive probability of actually having the disease is really small.

3) Given that, ~~$P(X)$~~ $P(X, Y|Z) = P(X|Z)\,P(Y|Z)$

a) $$P(Y|X,Z) = \frac{P(X,Y|Z)}{P(X|Z)} = \frac{P(X|Z)\,P(Y|Z)}{P(X|Z)} = P(Y|Z)$$

b) $$P(X|Y,Z) = \frac{P(X,Y|Z)}{P(Y|Z)} = \frac{P(X|Z)\,P(Y|Z)}{P(Y|Z)} = P(X|Z)$$

4) a) We have '5' binary variable

so, $2^5 - 1 = 31$ variables are needed

b) $1 + 4 + 1 + 8 + 2 = 16$

c) $P(B, I, M, J, G) = P(B)\,P(M)\,P(I|B,M)\,P(G|B,I,M)$
$P(J|G)$

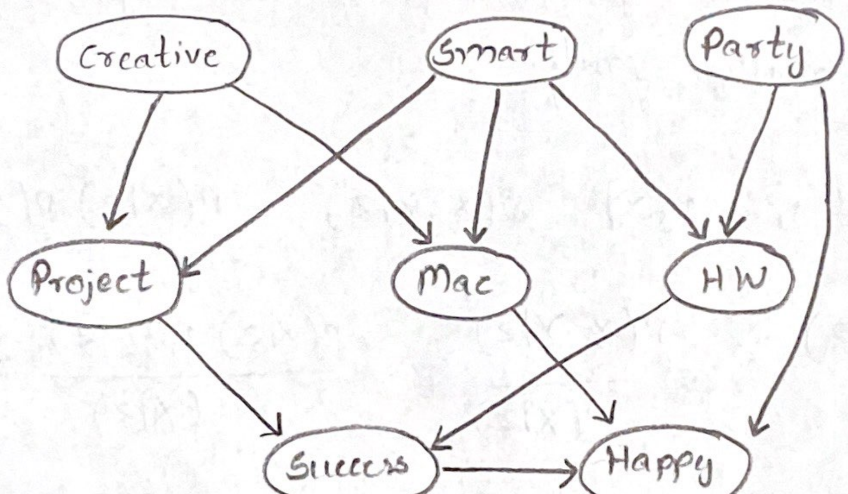
d) $P(B=t, I=t, M=f, G=t, J=t)$

$$= P(B=t)\,P(M=f)\,P(I=t \mid B=t, M=f)\,P(G=t \mid B=t, I=t, M=f)\,P(J=t \mid G=t)$$

$$= 0.9 \times (1-0.1) \times 0.5 \times 0.8 \times 0.9$$

$$= 0.2916$$

5) a)



b) $P(\text{creative}, \text{smart}, \text{party}, \text{project}, \text{Mac}, \text{HW}, \text{success}, \text{Happy})$

$$= P(\text{Creative})\,P(\text{Smart})\,P(\text{Party})\,P(\text{Project} \mid \text{creative}, \text{Smart})$$

$$\times P(\text{Mac} \mid \text{Creative}, \text{Smart})\,P(\text{HW} \mid \text{Smart}, \text{Party})$$

$$\times P(\text{Success} \mid \text{Project}, \text{HW})\,P(\text{Happy} \mid \text{Success}, \text{Mac}, \text{Party})$$