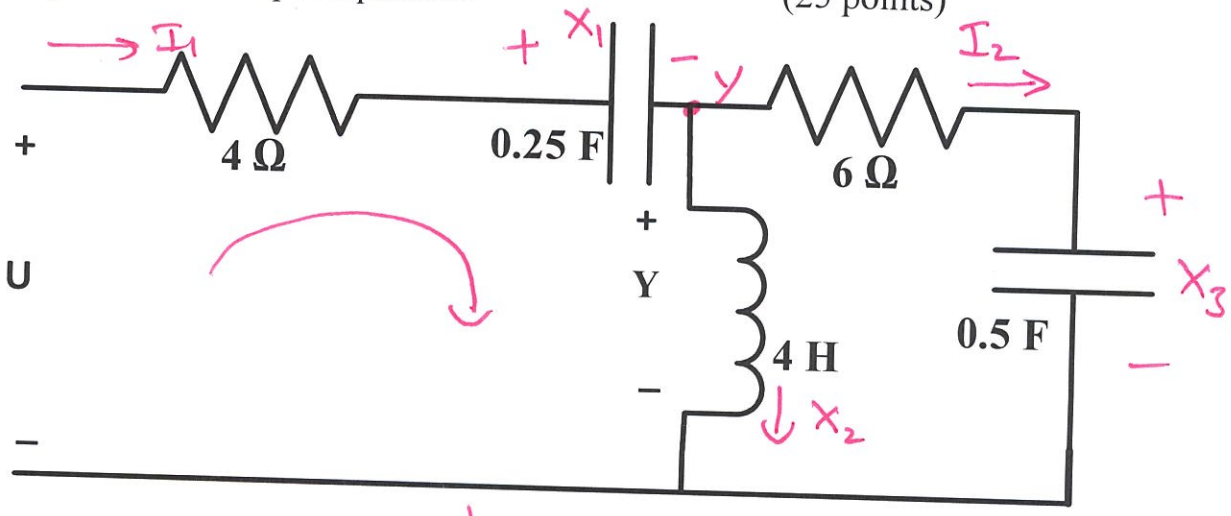
EE-792 TEST-1 Summer-19

First Name: Solution

Last Name: _____

- Open text book, and closed notes. One sheet of notes (A4 size, both sides) will be allowed to the exam.
- Time for this test is one hour thirty minutes.
- Calculators are allowed for this test (any kind)
- All work in this exam must be your own, sharing of calculators, formula sheet or text book will not be allowed.

(1) Write the state equation and output equations (25 points)



loop-1 apply KVL

$$u = 4I_1 + x_1 + 4\dot{x}_2 \rightarrow (1)$$

loop-2 apply KVL

$$4\dot{x}_2 = 6I_2 + x_3 \rightarrow (2)$$

$$\dot{x}_3 = \frac{1}{0.5} I_2 = 2I_2 \rightarrow (3)$$

$$\dot{x}_1 = \frac{1}{0.25} I_1 = 4I_1 \rightarrow (4)$$

$$u - y = 4I_1 + x_1$$

$$y = u - 4I_1 - x_1 \rightarrow (5)$$

Now replace I_1 & I_2 with State Variables.

$$I_1 = x_2 + I_2 \rightarrow (7)$$

$$y = 6I_2 + x_3 \rightarrow 6$$

Now eq.5 & eq.6 Compare

$$6I_2 + x_3 = u - 4I_1 - x_1$$

$$6I_2 = u - 4I_1 - x_1 - x_3 \rightarrow (8)$$

Sub. eq-8 in eq (7)

$$6I_1 = 6I_2 + 6x_2$$

$$6I_2 = 6I_1 - 6x_2$$

$$6I_1 - 6x_2 = u - 4I_1 - x_1 - x_3$$

$$10I_1 = u - x_1 + 6x_2 - x_3$$

Please show me your calculations for partial credit

$$u = \frac{4}{10}\left[u - x_1 + 6x_2 - x_3\right] + x_1 + 4\dot{x}_2$$

$$u - \frac{4}{10}u + \frac{4}{10}x_1 - x_1 - \frac{24}{10}x_2 + \frac{4}{10}x_3 = 4\dot{x}_2$$

$$4\dot{x}_2 = \frac{-6}{10}x_1 - \frac{24}{10}x_2 + \frac{4}{10}x_3 + \frac{6u}{10}$$

$$\boxed{\dot{x}_2 = \frac{-6}{40}x_1 - \frac{24}{40}x_2 + \frac{4}{40}x_3 + \frac{6}{40}u}$$

$$\boxed{\dot{x}_1 = -\frac{4}{10}x_1 + \frac{24}{10}x_2 - \frac{4}{10}x_3 + \frac{4}{10}u}$$

$$\dot{x}_3 = 2I_2 = 2(I_1 - x_2)$$

$$\dot{x}_3 = \frac{2}{10}\left[u - x_1 + 6x_2 - x_3\right] - 2x_2$$

$$= -\frac{2}{10}x_1 + \left[\frac{12}{10} - 2\right]x_2 - \frac{2}{10}x_3 + \frac{2}{10}u$$

$$\boxed{\dot{x}_3 = -\frac{2}{10}x_1 + \frac{8}{10}x_2 + \frac{2}{10}x_3 + \frac{2}{10}u}$$

$$y = u - \frac{4}{10}\left[u - x_1 + 6x_2 - x_3\right] - x_1$$

$$\boxed{y = -\frac{6}{10}x_1 - \frac{24}{10}x_2 + \frac{4}{10}x_3 + \frac{6}{10}u}$$

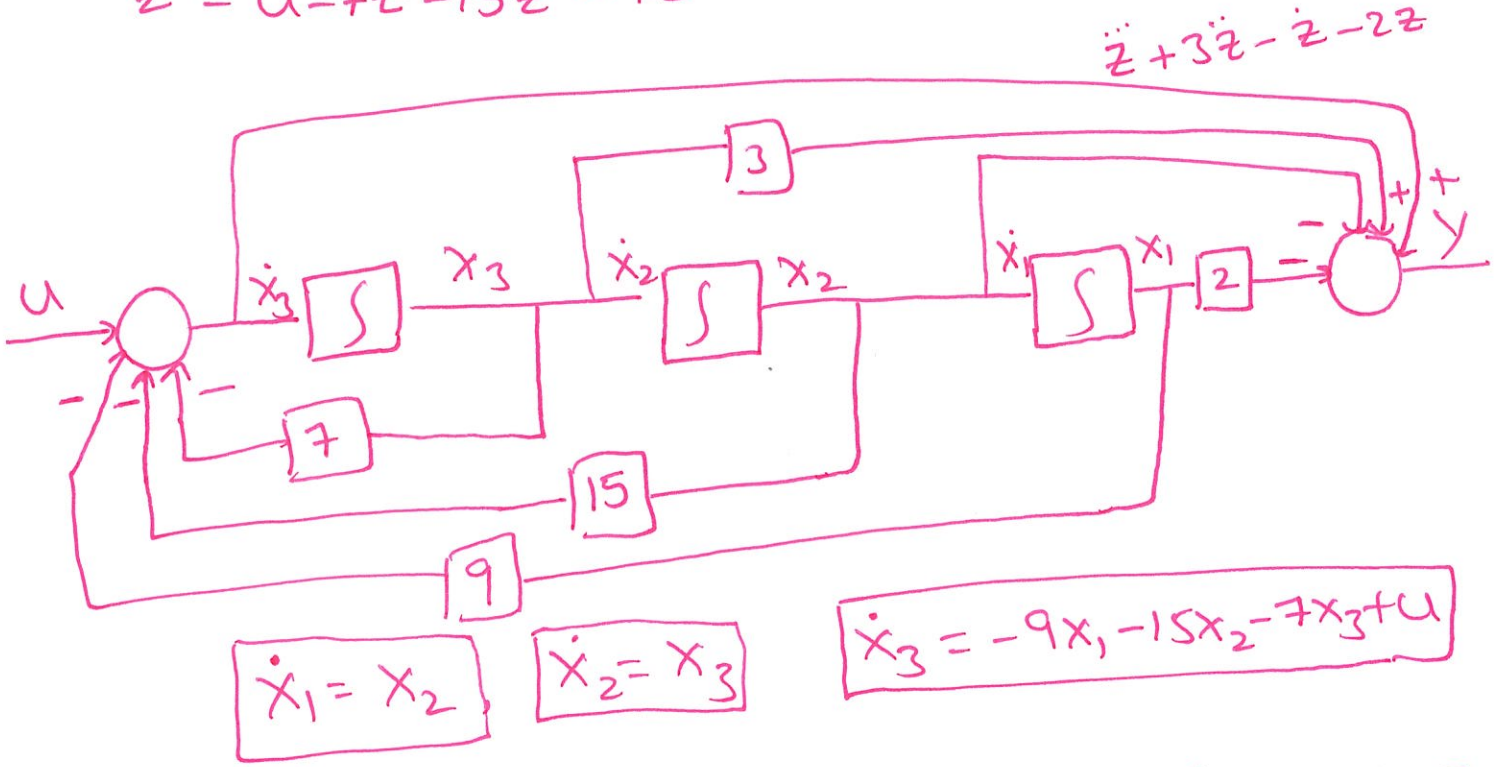
$$A = \begin{pmatrix} \frac{-4}{10} & \frac{24}{10} & \frac{-4}{10} \\ \frac{-6}{40} & \frac{-24}{40} & \frac{4}{40} \\ \frac{-2}{10} & \frac{-8}{10} & \frac{-2}{10} \end{pmatrix} \qquad B = \begin{pmatrix} \frac{4}{10} \\ \frac{6}{40} \\ \frac{2}{10} \end{pmatrix}$$

$$C = \begin{pmatrix} \frac{-6}{10} & \frac{-24}{10} & \frac{4}{10} \end{pmatrix} \qquad D = \frac{6}{10}$$

(2) $\dddot{y} + 7\ddot{y} + 15\dot{y} + 9y = \dddot{u} + 3\ddot{u} - \dot{u} - 2u$, find the state and output equation by using the controllable canonical form. (25 points)

$$\dddot{z} + 7\ddot{z} + 15\dot{z} + 9z = u$$

$$\dddot{z} = u - 7\ddot{z} - 15\dot{z} - 9z$$



$\dot{x}_1 = x_2$ $\quad \dot{x}_2 = x_3$ $\quad \dot{x}_3 = -9x_1 - 15x_2 - 7x_3 + u$

$$y = -2x_1 - x_2 + 3x_3 + \dot{x}_3 = -2x_1 - x_2 + 3x_3 + (-9x_1 - 15x_2 - 7x_3 + u)$$

$$y = -11x_1 - 16x_2 - 4x_3 + u$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -9 & -15 & -7 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} -11 & -16 & -4 \end{bmatrix}$$

$$D = (1)$$

Please show me your calculations for partial credit

(3) $\dddot{y} + 7\ddot{y} + 15\dot{y} + 9y = \dddot{u} + 10\ddot{u} + 20\dot{u} + 8u$, find the state and output equation by using the Jordan form. This methamatical modal has pole at '-1' (25 points)

$$(s^3+7s^2+15s+9)Y = U(s^3+10s^2+20s+8)$$

$$\frac{Y}{U} = \frac{s^3+10s^2+20s+8}{s^3+7s^2+15s+9}$$ Now do the long division to proceed with Laplace.

$$s^3+7s^2+15s+9 \;\overline{)\; s^3+10s^2+20s+8 \;(1}$$

$$s^3+7s^2+15s+9$$

$$3s^2+5s-1$$

$$\frac{Y}{U} = 1 + \frac{3s^2+5s-1}{s^3+7s^2+15s+9}$$

$$= 1 + \frac{3s^2+5s-1}{(s+1)(s^2+6s+9)}$$

$$s+1 \;\overline{)\; s^3+7s^2+15s+9 \;(s^2+6s+9}$$

$$s^3+s^2$$

$$6s^2+15s$$

$$6s^2+6s$$

$$9s+9$$

$$9s+9$$

$$0$$

$$= 1 + \frac{3s^2+5s-1}{(s+1)(s+3)^2}$$

$$\frac{Y}{U} = 1 + \frac{A}{s+1} + \frac{B}{(s+3)^2} + \frac{C}{(s+3)}$$

$$= 1 + \frac{\frac{-3}{4}}{s+1} + \frac{B}{(s+3)^2} + \frac{C}{s+3}$$

$$A = \left.\frac{3s^2+5s-1}{(s+3)^2}\right|_{s=-1}$$

$$A = \frac{3-5-1}{2^2} = \frac{-3}{4}$$

Please show me your calculations for partial credit

$$\frac{3s^2+5s-1}{(s+1)(s+3)^2} = \frac{A}{(s+1)} + \frac{B}{(s+3)^2} + \frac{C}{s+3}$$

$$= \frac{A(s+3)^2 + B(s+1)(s+3) + C(s+1)(s+3)}{(s+1)(s+3)^2}$$

$$\frac{3s^2+5s-1}{(s+1)(s+3)^2} = \frac{A(s^2+6s+9) + B(s+1) + C(s^2+4s+3)}{(s+1)(s+3)^2}$$

$$3 = A + C$$
$$5 = 6A + B + 4C$$
$$-1 = 9A + B + 3C$$

$$A = -\frac{3}{4},$$

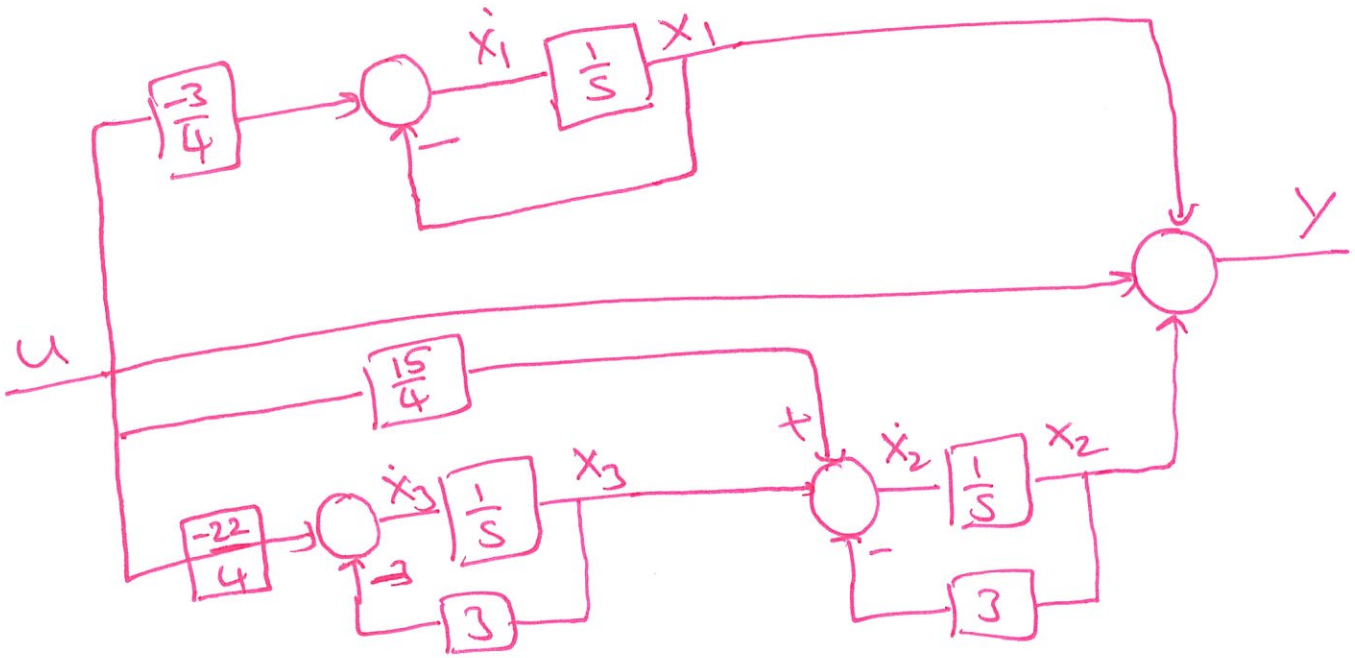
$$C = 3 + \frac{3}{4} = \frac{15}{4}$$

$$5 = 6\left[\frac{-3}{4}\right] + B + 4\left[\frac{15}{4}\right]$$

$$5 = -\frac{18}{4} + B + 15$$

$$B = -10 + \frac{18}{4} = \frac{-22}{4}$$

$$\frac{y}{u} = 1 + \frac{-3/4}{s+1} + \frac{\frac{-22}{4}}{(s+3)^2} + \frac{\frac{15}{4}}{s+3}.$$



$$\dot{x}_1 = -x_1 - \frac{3}{4}u$$

$$\dot{x}_2 = -3x_2 + x_3 + \frac{15}{4}u$$

$$\dot{x}_3 = -3x_3 - \frac{22}{4}u$$

$$y = x_1 + x_2 + u$$

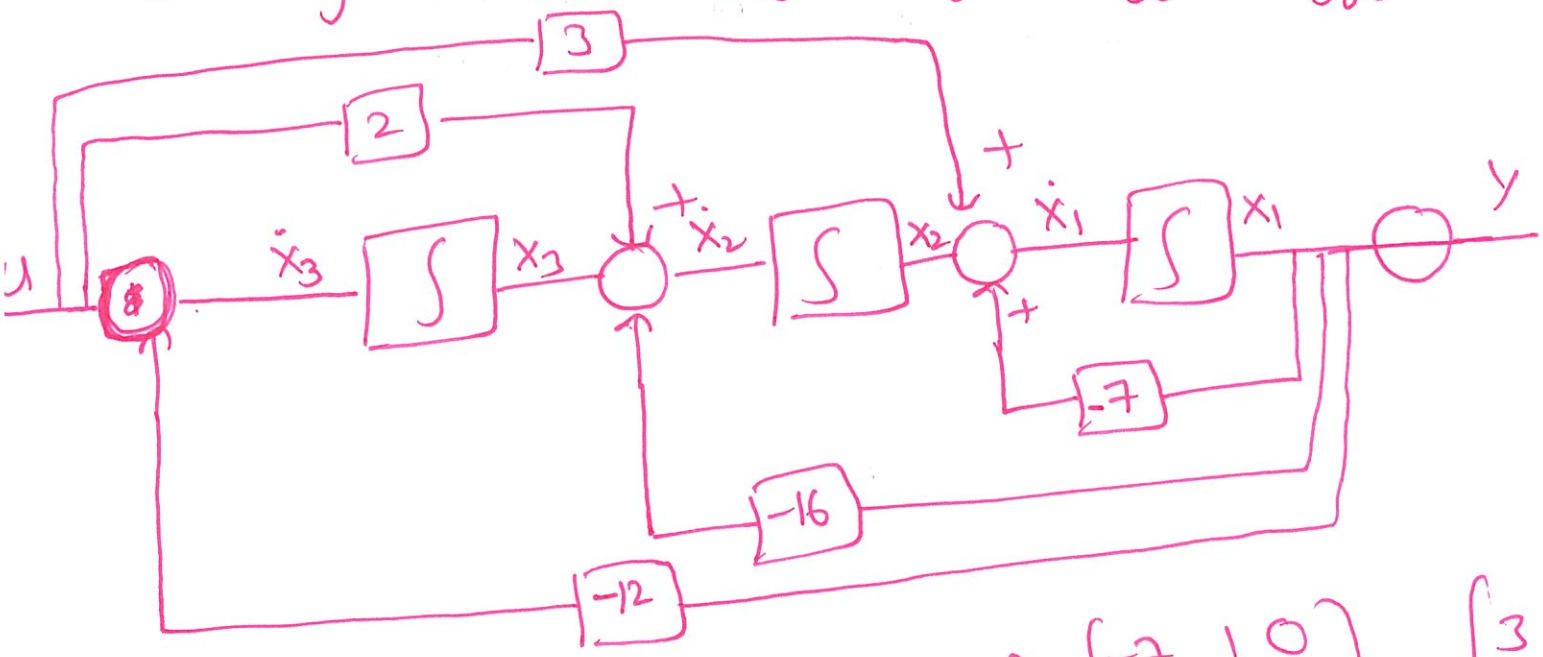
$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix} \qquad B = \begin{pmatrix} -\frac{3}{4} \\ \frac{15}{4} \\ -\frac{22}{4} \end{pmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \qquad D = 1$$

(4) $\dddot{y} + 7\ddot{y} + 16\dot{y} + 12y = 3\ddot{u} + 2\dot{u} + u$, find the state and output equation by using the observer canonical form of the given model. (25 points)

$$\dddot{y} = 3\ddot{u} + 2\dot{u} + u - 7\ddot{y} - 16\dot{y} - 12y$$

$$y = 3\int u + 2\iint u + \iiint u - 7\int y - 16\iint y - 12\iiint y$$



$$Y = X_1$$

$$\dot{X}_1 = -7X_1 + X_2 + 3u$$

$$\dot{X}_2 = -16X_1 + X_3 + 2u$$

$$\dot{X}_3 = -12X_1 + u$$

$$A = \begin{bmatrix} -7 & 1 & 0 \\ -16 & 0 & 1 \\ -12 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \quad D = 0$$

Please show me your calculations for partial credit