EE-792 TEST-3 Summer-19

First Name: Solution.

Last Name: ______________________

- Open text book, and closed notes. One sheet of notes (A4 size, both sides) will be allowed to the exam.
- Time for this Test is one hour thirty minutes.
- Calculators are allowed for this test (any kind)
- All work in this exam must be your own, sharing of calculators, formula sheet or text book will not be allowed.

(1) $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}2\\ 1\end{bmatrix}u,\quad y=\begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$ Find the transformation matrix for the given model to convert into observable canonical form. Now by using the transformation matrix find the state transition matrix of the new model. (25 points)

from the solution $T=\begin{bmatrix}3 & 1\\ 2 & 1\end{bmatrix}$ $T^{-1}=\begin{bmatrix}1 & -1\\ -2 & 3\end{bmatrix}$

$$\phi_{(s)}=[sI-A]^{-1}=\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix}-\begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix}=\begin{bmatrix}s-3 & 0\\ 0 & s-2\end{bmatrix}^{-1}$$

$$\phi_{(s)}=\begin{pmatrix}\frac{s-2}{(s-2)(s-3)} & 0\\ 0 & \frac{s-3}{(s-2)(s-3)}\end{pmatrix}=\begin{pmatrix}\frac{1}{s-3} & 0\\ 0 & \frac{1}{s-2}\end{pmatrix}$$

$$\phi_{(t)}=\begin{bmatrix}e^{+3t} & 0\\ 0 & e^{+2t}\end{bmatrix}$$

$$\hat{\phi}_{(t)}=T^{-1}\phi_{(t)}T=\begin{bmatrix}3 & 1\\ 2 & 1\end{bmatrix}\begin{bmatrix}e^{+3t} & 0\\ 0 & e^{+2t}\end{bmatrix}\begin{bmatrix}1 & -1\\ -2 & 3\end{bmatrix}=\begin{bmatrix}3e^{+3t} & e^{+2t}\\ 2e^{+3t} & e^{2t}\end{bmatrix}\begin{bmatrix}1 & -1\\ -2 & 3\end{bmatrix}$$

$$=\begin{bmatrix}3e^{+3t}-2e^{2t} & -3e^{3t}+3e^{2t}\\ 2e^{3t}-2e^{2t} & -2e^{3t}+3e^{2t}\end{bmatrix}$$

$$T^{-1}\phi_{(t)}=\begin{bmatrix}1 & -1\\ -2 & 3\end{bmatrix}\begin{bmatrix}e^{3t} & 0\\ 0 & e^{2t}\end{bmatrix}=\begin{bmatrix}e^{3t} & -e^{2t}\\ -2e^{3t} & 3e^{2t}\end{bmatrix}$$

$$T^{-1}\phi_{t}T=\begin{bmatrix}e^{3t} & -e^{2t}\\ -2e^{3t} & 3e^{2t}\end{bmatrix}\begin{bmatrix}3 & +1\\ +2 & 1\end{bmatrix}$$

$$=\begin{bmatrix}3e^{3t}-2e^{2t} & e^{3t}-e^{2t}\\ -6e^{3t}+6e^{2t} & -2e^{3t}+3e^{2t}\end{bmatrix}$$

Please show me your calculations for partial credit

(2) A model has $A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $C = [1 \quad -1]$ design an observer with a desired poles at $-1 \pm j4$, find the state equation of Observer. (25 points)

$$A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$\mathring{A} = [A - KC] \Rightarrow \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} - \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} - \begin{bmatrix} K_1 & -K_1 \\ K_2 & -K_2 \end{bmatrix}$$

25/25

$$= \begin{bmatrix} -K_1 & (1+K_1) \\ 2-K_2 & (-3+K_2) \end{bmatrix}.$$

$$\det[SI - \mathring{A}] = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -K_1 & (1+K_1) \\ (2-K_2) & (-3+K_2) \end{bmatrix} = \begin{bmatrix} (S+K_1) & -(1+K_1) \\ -(2-K_2) & (S+3+K_2) \end{bmatrix}$$

$$= S^2 + K_1S + 3S + 3K_1 + K_2S + \cancel{K_1K_2} - 2 - 2K_1 + K_2 - \cancel{K_1K_2} = 0$$

$$= S^2 + (K_1 + 3 + K_2)S + K_1 - 2 + K_2 = 0$$

Desired poles at $-1 \pm j4$

$$\Rightarrow (S+1+4j)(S+1-4j) = 0$$

$$\Rightarrow S^2 + S + \cancel{4jS} + S + 1 + \cancel{4j} - \cancel{4jS} - \cancel{4j} + 16.$$

$$\Rightarrow S^2 + 2S + 17 = 0$$

Then we have :—

$$K_1 + 3 + K_2 = 2$$

$$K_1 + K_2 = -1$$

$$2K_1 = 18$$

$$K_1 = 9$$

$$K_1 - 2 + K_2 = 17$$

$$K_1 - K_2 = 19$$

$$K_2 = \cancel{+10}$$

It has Unique Solution

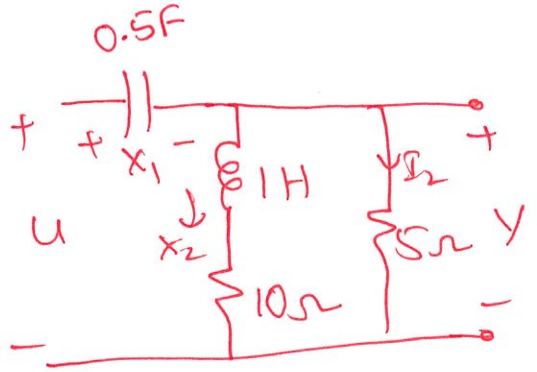
State equation :— $\mathring{\hat{X}} = (A - KC)\hat{X} + Bu + Ky.$

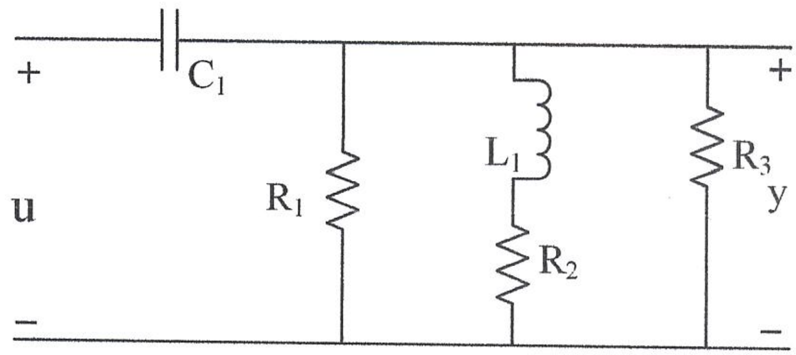
$$\mathring{\hat{x}} = \begin{bmatrix} -9 & 10 \\ -8 & 7 \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 9 \\ +10 \end{bmatrix} y.$$

Please show me your calculations for partial credit

(3) Find the controllability, Observability and stability of the state and output equation obtained from the following electrical circuit, take each resistance value as 10 Ohm, capacitance as 0.5F and inductance as 1H.

(25 points)





$U = X_1 + \dot{X}_2 + 10X_2$

$\boxed{\dot{X}_2 = -X_1 - 10X_2 + U}$

$Y = 5I_2 = \dot{X}_2 + 10X_2 =$

$\boxed{Y = U - X_1}$

$\dot{X}_1 = \frac{1}{0.5}(X_2 + I_2)$

$\boxed{\dot{X}_1 = 2\left(X_2 + \frac{U}{5} - \frac{X_1}{5}\right)}$

$A = \begin{bmatrix} -\frac{2}{5} & 2 \\ -1 & -10 \end{bmatrix} \quad B = \begin{bmatrix} \frac{2}{5} \\ 1 \end{bmatrix}$

$C = [-1, 0] \quad D = 1$

$P_c = [B, AB] = \begin{pmatrix} \frac{2}{5} & \frac{46}{25} \\ 1 & -\frac{52}{5} \end{pmatrix}$

Det $|P_c| \neq 0$ So Controllable

$P_o = \begin{pmatrix} -1 & 0 \\ \frac{2}{5} & -1 \end{pmatrix}$ Det $P_o \neq 0$ So observable

$[SI - A] = \begin{pmatrix} S & 0 \\ 0 & S \end{pmatrix} - \begin{pmatrix} -\frac{2}{5} & 2 \\ -1 & -10 \end{pmatrix} = \begin{pmatrix} S+\frac{2}{5} & -2 \\ 1 & S+10 \end{pmatrix}$

Det $|SI - A| = S^2 + \frac{52}{5}S + 4 - 2 = 0$

$S^2 + \frac{52}{5}S + 2 = 0$

Poles must be stable pole

Please show me your calculations for partial credit

(4) $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u,\quad y = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + 3u$, Transfer the given model into Jordan form by using transformation matrix. (25 points)

$$P_c = \begin{bmatrix}B & AB\end{bmatrix} = \begin{bmatrix}0 & 1\\ 1 & -3\end{bmatrix} \qquad AB = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \begin{bmatrix}1\\ -3\end{bmatrix}$$

$P_c \neq 0.$

$$A - \lambda I = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix} - \begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix} = \begin{bmatrix}-\lambda & 1\\ -2 & -3-\lambda\end{bmatrix}$$

$$|A - \lambda I| = -\lambda(-3-\lambda) + 2$$
$$= \lambda^2 + 3\lambda + 2 = 0$$
$$(\lambda+2)(\lambda+1) = 0$$
$$\lambda = -2, \quad \lambda = -1$$

If $\lambda = -1$

$$(A - \lambda I) = \begin{bmatrix}1 & 1\\ -2 & -2\end{bmatrix}$$

$$\begin{bmatrix}1 & 1\\ -2 & -2\end{bmatrix}\begin{bmatrix}V_{11}\\ V_{21}\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

$V_{11} + V_{21} = 0$

$V_{11} = -V_{21}$

$V_{11} = 1$

$V_{21} = -1$

If $\lambda = -2$

$$\begin{bmatrix}2 & 1\\ -2 & -1\end{bmatrix}\begin{bmatrix}V_{12}\\ V_{22}\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

$2V_{12} + V_{22} = 0$

$V_{12} = 1 \quad V_{22} = -2$

$$T = \begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix} \qquad T^{-1} = \frac{1}{-1}\begin{bmatrix}-2 & -1\\ 1 & 1\end{bmatrix}$$

$$T^{-1} = \begin{bmatrix}2 & 1\\ -1 & -1\end{bmatrix}$$

$$\hat{A} = T^{-1}AT = \begin{bmatrix}2 & 1\\ -1 & -1\end{bmatrix}\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix}$$
$$= \begin{bmatrix}-2 & -1\\ 2 & 2\end{bmatrix}\begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix} = \begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}$$

$$\hat{B} = T^{-1}B = \begin{bmatrix}2 & 1\\ -1 & -1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \begin{bmatrix}1\\ -1\end{bmatrix}$$

$$\hat{C} = CT = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix} = \begin{bmatrix}-1 & -2\end{bmatrix}$$

Please show me your calculations for partial credit