06|15|2020

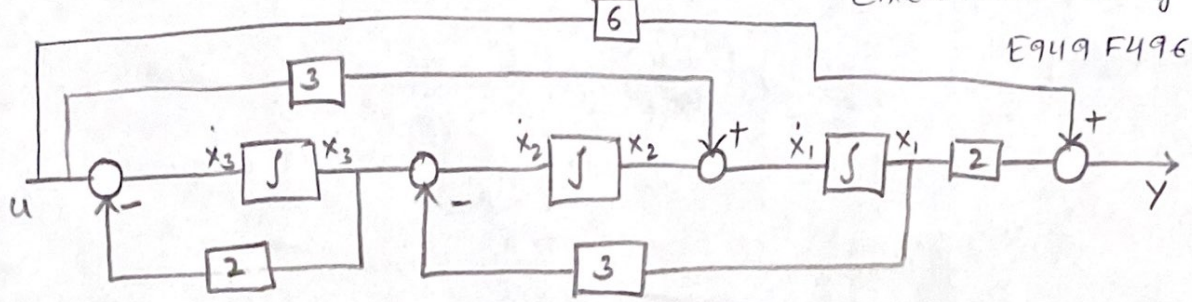
Linear Systems

Homework - 3

Chakradhar Reddy Donuri

E949 F496

(1)




$\dot{x}_1 = x_2 + 3u$

$\dot{x}_2 = x_3 - 3x_1$

$\dot{x}_3 = u - 2x_3$

$y = 2x_1 + 6u$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -3 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 0 & 0 \end{bmatrix} \qquad D = [6$$

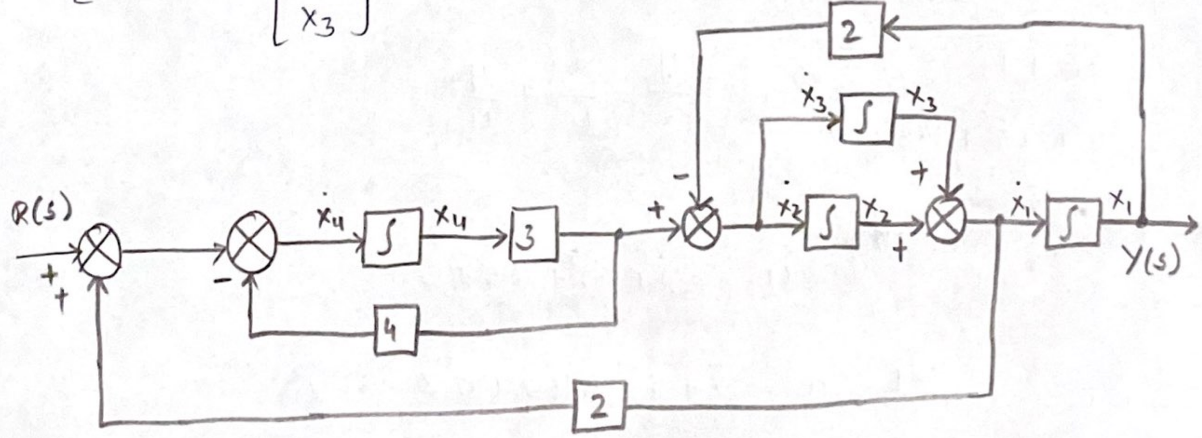
$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -3 & 0 & 1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} u$$

state equation

$$y = \begin{bmatrix} 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 6[u]$$

output equation.

(2)



$$y(s) = x_1$$

$$\dot{x}_1 = x_2 + x_3$$

$$\dot{x}_2 = 3x_4 - 2x_1$$

$$\dot{x}_3 = 3x_4 - 2x_1$$

$$\dot{x}_4 = R(s) + 2(x_2 + x_3) - 12x_4$$

$$\dot{x}_4 = R(s) + 2x_2 + 2x_3 - 12x_4$$

$$\dot{x}_4 = 2x_2 - 12x_4 + R(s) + 2x_3$$

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ -2 & 0 & 0 & 3 \\ -2 & 0 & 0 & 3 \\ 0 & 2 & \underset{2}{\cancel{-10}} & -12 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \quad D = 0$$

State equation:

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ -2 & 0 & 0 & 3 \\ -2 & 0 & 0 & 3 \\ 0 & 2 & 2 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} R(s)$$

Output equation:

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + 0\, R(s)$$

③ $\dddot{y} + 8\ddot{y} - 6\dot{y} + 2y = 4\ddot{u} - 3\dot{u} - 2u$

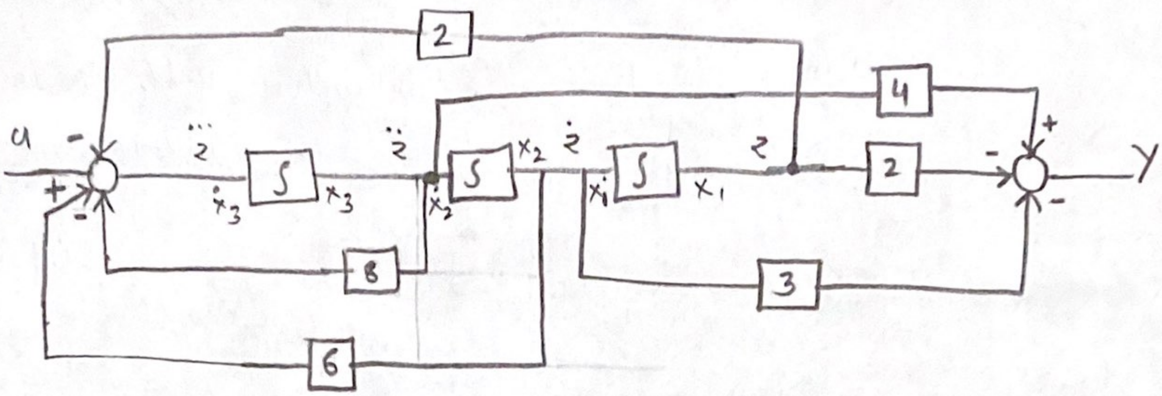
Using 2nd System Concept

$$\frac{y}{u} = \frac{4s^2 - 3s - 2}{s^3 + 8s^2 - 6s + 2} \Rightarrow \frac{y}{z} \cdot \frac{z}{u} = \frac{y}{u}$$

$$\frac{y}{z} = 4s^2 - 3s - 2 \qquad \frac{z}{u} = \frac{1}{s^3 + 8s^2 - 6s + 2}$$

$$Y = 4\ddot{z} - 3\dot{z} - 2z \qquad \dddot{z} + 8\ddot{z} - 6\dot{z} + 2z = u$$

$$\dddot{z} = u - 8\ddot{z} + 6\dot{z} - 2z$$




$\cancel{y = -2z + 4\dot{z}}$

$$Y = -2x_1 + 4x_3 - 3x_2$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = u - 8x_3 + 6x_2 - 2x_1$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 6 & -8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

state equation

$$Y = \begin{bmatrix} -2 & -3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]u$$

output equation.

(4) $\dddot{y} + 7\ddot{y} + 16\dot{y} + 12y = 2\dot{u} + u$

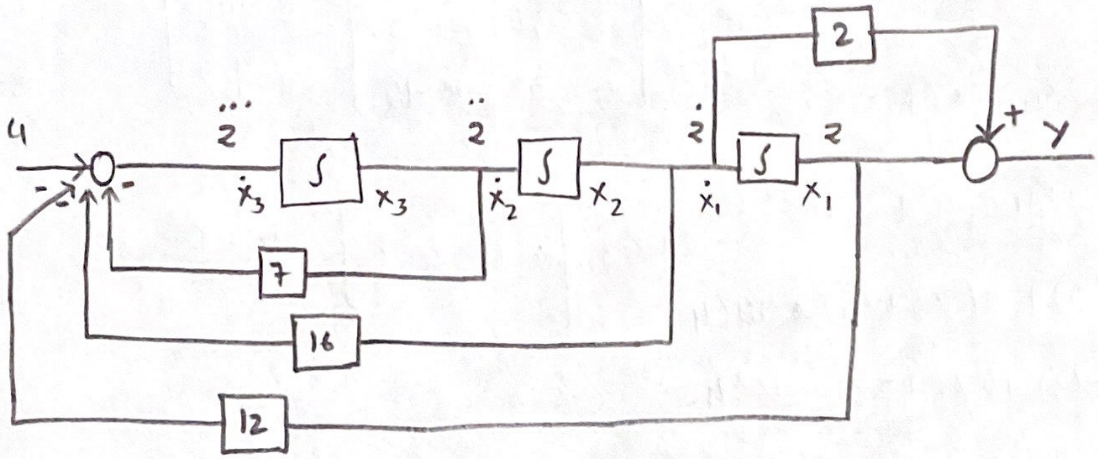
using 2nd system concept

$$\frac{y}{u} = \frac{2s+1}{s^3 + 7s^2 + 16s + 12} \Rightarrow \frac{y}{z} \cdot \frac{z}{u} = \frac{y}{u}$$

$$\frac{y}{z} = 2s + 1 \quad \text{and} \quad \frac{z}{u} = \frac{1}{s^3 + 7s^2 + 16s + 12}$$

$$Y = 2\dot{z} + z \quad \text{and} \quad u = \dddot{z} + 7\ddot{z} + 16\dot{z} + 12z$$

$$\dddot{z} = u - 7\ddot{z} - 16\dot{z} - 12z$$




$$Y = 2x_2 + x_1$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$x_3 = u - 7x_3 - 16x_2 - 12x_1$$

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -12 & -16 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

state equation

$$Y = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0\,u$$