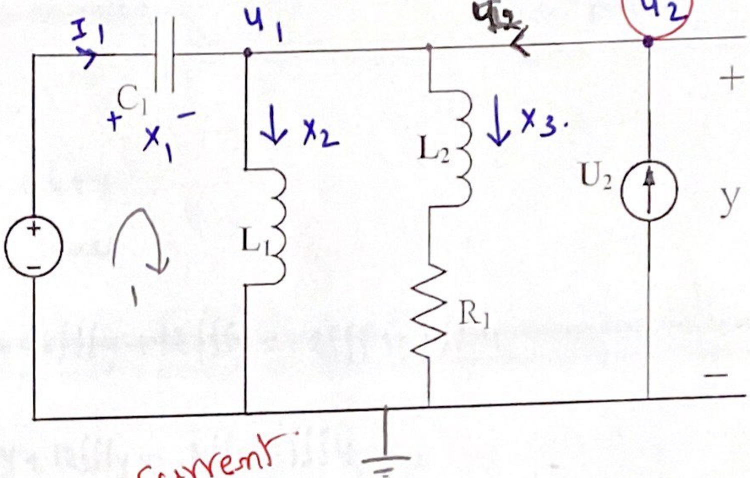
$\frac{75}{100} \times 7$

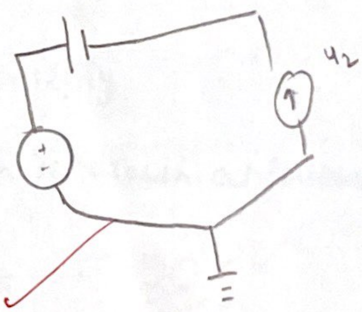
(1) Write the state equation and output equations (25 points)



output function.

$Y = u_2$. u_2 = current

State equations.



we have $x_1 = \frac{1}{C_1}\int I_1\, dt$

$$\boxed{\dot{x}_1 = \frac{1}{C_1} I_1}$$

Loop 1

$$\boxed{u_1 = x_1 + L_1 \dot{x}_2 \Rightarrow} \quad \frac{u_1 - x_1}{L_1} = \dot{x}_2$$

$\frac{5 \times 5}{25}$

$$u_2 = L_2 \dot{x}_3 + R_1 x_3 \Rightarrow \quad \frac{u_2 - R_1 x_3}{L_2} = \dot{x}_3$$

$$\dot{x}_1 = \frac{1}{C_1} u_1 \quad X$$

$$\dot{x}_2 = \frac{u_1}{L_1} - \frac{x_1}{L_1}$$

$$\dot{x}_3 = \frac{u_2}{L_2} - \frac{R_1 x_3}{L_2}$$

$$\dot{x} = \begin{bmatrix} 0 & 0 & 0 \\ -1/L_1 & 0 & 0 \\ 0 & 0 & -\frac{R_1}{L_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1} & 0 \\ 1/L_1 & 0 \\ 0 & 1/L_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Please show me your calculations for partial credit

(2) $\dddot{y} + 7\ddot{y} + 16\dot{y} + 12y = 2\dot{u} + u$, find the state and output equation by using the observer canonical form of the given model. (25 points)

Sol: Given

$$\dddot{y} + 7\ddot{y} + 16\dot{y} + 12y = 2\dot{u} + u$$

Apply integral three times

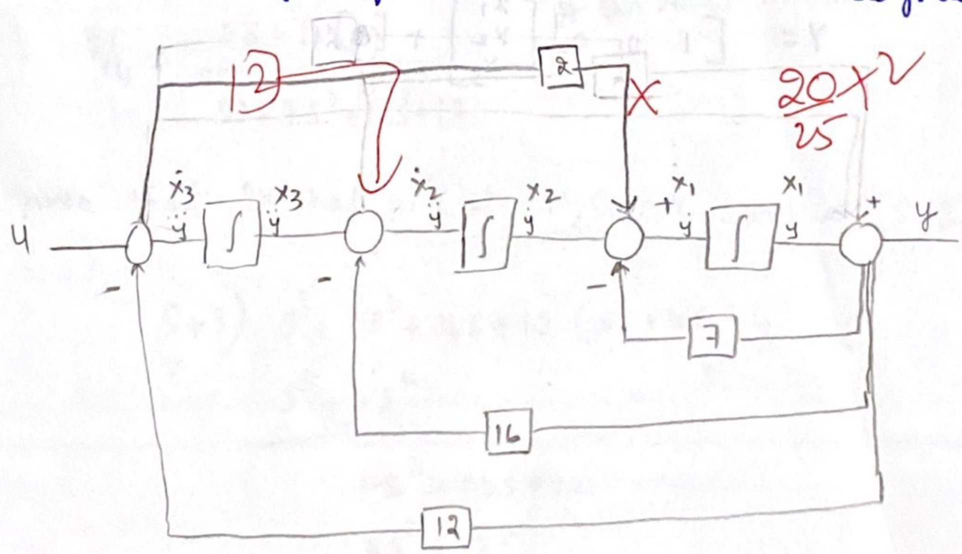
$$\iiint \dddot{y} + 7\iiint \ddot{y} + 16\iiint \dot{y} + 12\iiint y = 2\iiint \dot{u} + \iiint u$$

$$y + 7\int y + 16\iint y + 12\iiint y = 2\iint u + \iiint u$$

Now, we have

$$y = 2\iint u + \iiint u - 7\int y - 16\iint y - 12\iiint y$$

The simulation diagram for the above can be drawn as follows:



from above we get

o/p equation :

$$y = x_1 \quad - ①$$

Please show me your calculations for partial credit

(3) $\dddot{y} + 7\ddot{y} + 16\dot{y} + 12y = 2\dot{u} + u$, find the state and output equation by using the Jordan form. This methamatical modal has pole at '-3' (25 points)

Sol: Given

$$\dddot{y} + 7\ddot{y} + 16\dot{y} + 12y = 2\dot{u} + u \quad -①$$

The above eq ① can be converted into s-domain as follows

$$s^3 y + 7s^2 y + 16sy + 12y = 2su + u.$$

$$y(s^3 + 7s^2 + 16s + 12) = u(2s+1)$$

We have $H_s = y/u$ = Transfer function, writing in the form we get

$$y/u = \frac{2s+1}{s^3+7s^2+16s+12};$$ It can be written in Jordan form as

given that It has pole at -3, So one root is s+3.

$$
\begin{array}{l}
s+3)\; s^3 + 7s^2 + 16s + 12\; (s^2 + 4s + 4 \\
\underline{\;s^3 + 3s^2\;} \\
\quad 4s^2 + 16s + 12 \\
\quad \underline{4s^2 + 12s} \\
\qquad 4s + 12 \\
\qquad \underline{4s + 12} \\
\qquad\;\; (0)
\end{array}
$$

Please show me your calculations for partial credit

$$\begin{array}{c} \\ \\ \end{array}$$

(4) $\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \underbrace{\begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}_{B} u, \quad y = \underbrace{[1 \quad 2 \quad 3]}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, write the transfer function for the above state equation and output equation (25 points)

Transfer function $H_s = C(SI-A)^{-1} \cdot B + D.$

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad C = [1 \ 2 \ 3].$$

$$SI = \begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{bmatrix}$$

$$SI - A = \begin{bmatrix} S & 0 & 0 \\ 0 & S & 0 \\ 0 & 0 & S \end{bmatrix} - \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} S+2 & 0 & 0 \\ 0 & S+1 & 0 \\ 0 & 0 & S+4 \end{bmatrix}$$

25/25

$$(SI-A)^{-1} = \frac{1}{|SI-A|} (\text{Adjoint of } SI-A)$$

$$|SI-A| = (S+2) \begin{vmatrix} S+1 & 0 \\ 0 & S+4 \end{vmatrix} = (S+2)(S+1)(S+4).$$

co-factor Matrix: co-factor of $S+2 = (-1)^{1+1} \begin{vmatrix} S+1 & 0 \\ 0 & S+4 \end{vmatrix} = (S+1)(S+4)$

co-factor of $0 = (-1)^{1+2} \begin{vmatrix} 0 & 0 \\ 0 & S+4 \end{vmatrix} = 0$

co-factor of $0 = (-1)^{1+3} \begin{vmatrix} 0 & S+1 \\ 0 & 0 \end{vmatrix} = 0$

Please show me your calculations for partial credit

(5) What did you observe from the results of Question-2 and 3? (5 bonus points)

Among 2 and 3, we observe as follows

In 2, the input is dependent on only one of the state variable and not dependent on output. where as

In 3, the input is depends on both the state variables and state functions are particularly depends on diagonal elements i.e each state function in the given question dependents on particular state variable.

2/5

4th

$$y/u = \frac{s^2+5s+4+2s^2+12s+16+3s^2+9s+6}{(s^2+3s+2)(s+4)}$$

$$y/u = \cancel{\frac{6s^2+26s+26}{s^3+3s^2+2s+4s^2+12s+8}}$$

$$\Rightarrow y(s^3+3s^2+2s+4s^2+12s+8) = u(6s^2+26s+26)$$

$$\Rightarrow ys^3+7ys^2+14ys+8y = 6us^2+26us+26u$$

$\therefore$ Transfer function is $\dddot{y}+7\ddot{y}+14\dot{y}+8y = 6\ddot{u}+26\dot{u}+26u$

Please show me your calculations for partial credit