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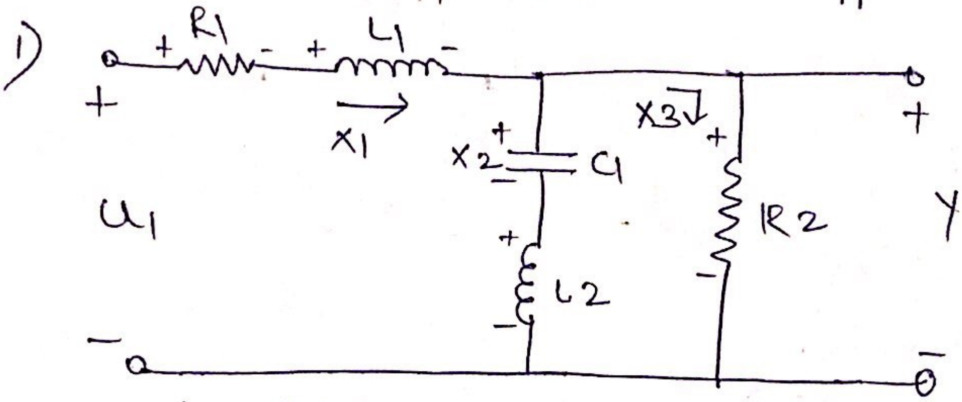
LINEAR SYSTEMS
ASSIGNMENT # 2

1)



Apply KVL,

$$\therefore\ u_1 - R_1 x_1 - L_1 \dot{x}_1 - x_2 - L_2(\dot{x}_1 - \dot{x}_3) = 0 \quad (1)$$

$$+x_2 + R_2 x_3 + L_2(\dot{x}_3 - \dot{x}_1) = 0 \quad - \quad (2)$$

$$\therefore\ L_2(\dot{x}_3 - \dot{x}_1) = -R_2 x_3 - x_2$$

$$\therefore\ L_2(\dot{x}_1 - \dot{x}_3) = R_2 x_3 + x_2 \quad - \quad (3)$$

substitute eqn (3) in eqn (1)

$$\therefore\ u_1 - R_1 x_1 - L_1 \dot{x}_1 - x_2 - R_2 x_3 - x_2 = 0$$

$$\therefore\ L_1 \dot{x}_1 = u_1 - R_1 x_1 - 2x_2 - R_2 x_3$$

$$\therefore\ \boxed{\dot{x}_1 = \frac{u_1}{L_1} - \frac{R_1 x_1}{L_1} - \frac{2x_2}{L_1} - \frac{R_2 x_3}{L_1}}$$

$$L_2 \dot{x}_3 = L_2 \dot{x}_1 - x_2 - R_2 x_3 \ \ldots\ldots \text{(From eqn (2))}$$

$$\therefore\ \dot{x}_3 = \dot{x}_1 - \frac{x_2}{L_2} - \frac{R_2 x_3}{L_2}$$

$$\therefore\ \dot{x}_3 = \frac{u_1}{L_1} - \frac{R_1 x_1}{L_1} - \frac{2x_2}{L_1} - \frac{R_2 x_3}{L_1} - \frac{x_2}{L_2} - \frac{R_2 x_3}{L_2}$$

$$\therefore\ \boxed{\dot{x}_3 = \frac{u_1}{L_1} - \frac{R_1 x_1}{L_1} - x_2\left[\frac{2}{L_1} + \frac{1}{L_2}\right] - x_3\left[\frac{+R_2}{L_1} + \frac{R_2}{L_2}\right]}$$

$$\boxed{\dot{x}_2 = \frac{1}{C}(x_1 - x_3) = \frac{x_1}{C_1} - \frac{x_3}{C_1}}$$

$$\boxed{y = R_2 x_3}$$

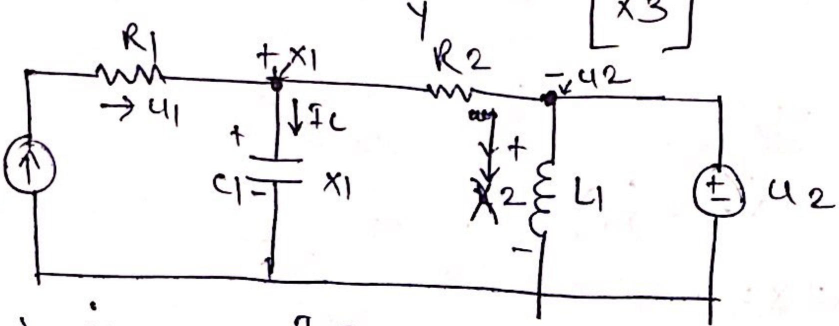
→ state equation matrix,

$$\dot{x} = \begin{bmatrix} \frac{-R_1}{L_1} & -\frac{2}{L_1} & -\frac{R_2}{L_1} \\ \frac{1}{C_1} & 0 & -\frac{1}{C_1} \\ \frac{-R_1}{L_1} & -\left(\frac{2}{L_1}+\frac{1}{L_2}\right) & -\left(\frac{R_2}{L_1}+\frac{R_2}{L_2}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ \frac{1}{L_1} \end{bmatrix} u_1$$

→ output equation matrix →

$$y = \begin{bmatrix} 0 & 0 & R_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

2)



$$\Rightarrow \dot{x}_1 = \frac{I_c}{C_1}$$

$$\dot{x}_1 = \frac{u_1}{C_1} - \frac{y}{R_2 C_1}$$

$$\boxed{y = x_1 - u_2}$$

$$\therefore \dot{x}_1 = \frac{u_1}{C_1} - \frac{(x_1 - u_2)}{R_2 C_1}$$

$$\boxed{\dot{x}_1 = \frac{u_1}{C_1} - \frac{x_1}{R_2 C_1} + \frac{u_2}{R_2 C_1}}$$

$$u_2 = L_1 \dot{x}_2$$

$$\therefore \boxed{\dot{x}_2 = \frac{u_2}{L_1}}$$

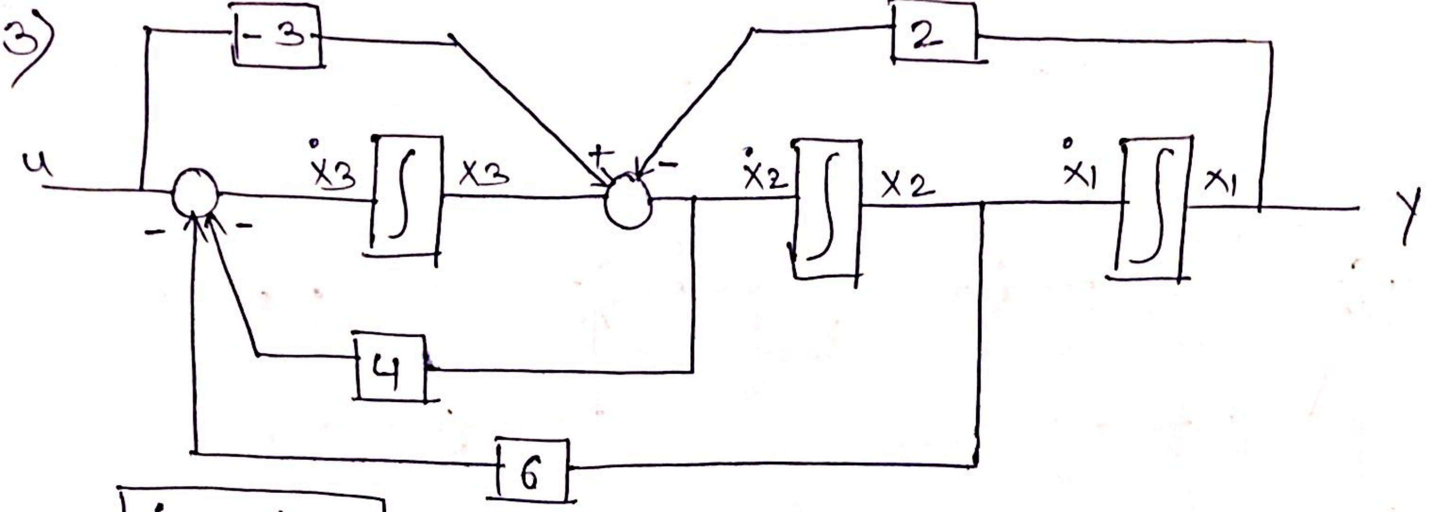
→ state equation matrix →

$$\dot{x} = \begin{bmatrix} -\frac{1}{R_2 C_1} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{C_1} & \frac{1}{R_2 C_1} \\ 0 & 1/L_1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

→ output equation matrix →

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} //$$

3)



$\Rightarrow$ $\boxed{\dot{x}_1 = x_2}$

$$\dot{x}_2 = x_3 + (-3)u - 2x_1$$

$$\boxed{\dot{x}_2 = -2x_1 + x_3 - 3u}$$

$$\dot{x}_3 = u - 4\dot{x}_2 - 6x_2$$

$$= u - 4(-2x_1 + x_3 - 3u) - 6x_2$$

$$= u + 8x_1 - 4x_3 + 12u - 6x_2$$

$$\boxed{\dot{x}_3 = 8x_1 - 6x_2 - 4x_3 + 13u}$$

$$\boxed{y = x_1}$$

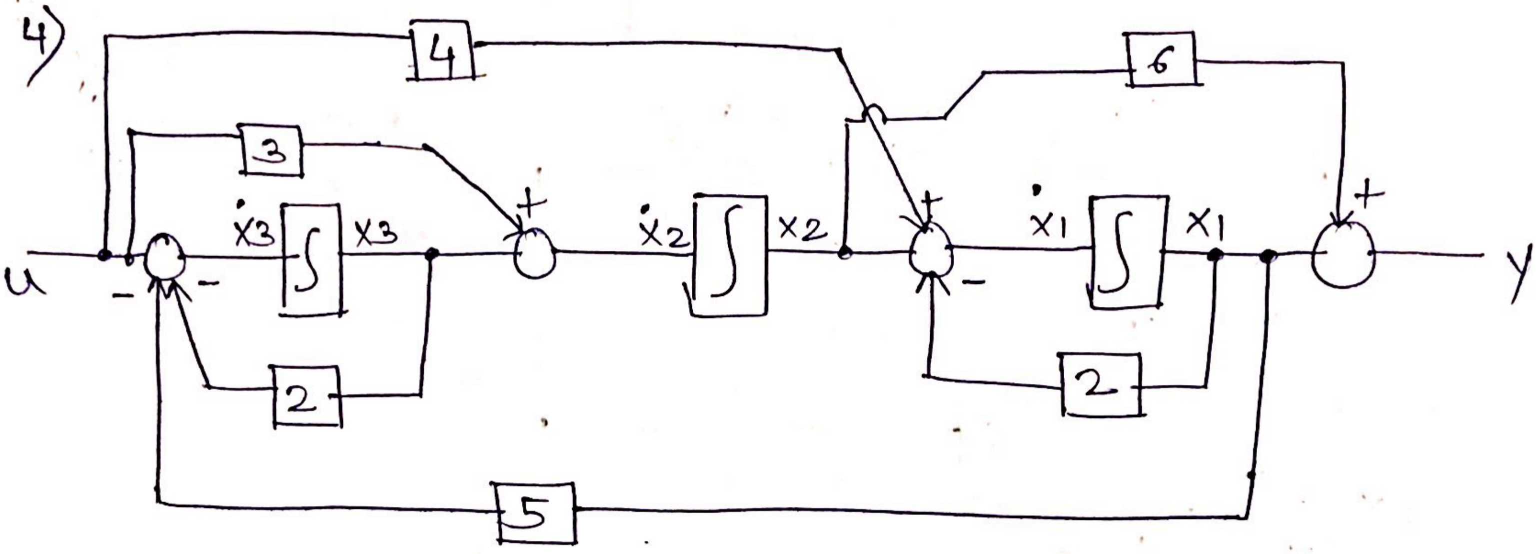
State equation matrix $\Rightarrow$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ -2 & 0 & 1 \\ 8 & -6 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ -3 \\ 13 \end{bmatrix} u$$

output equation matrix $\Rightarrow$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

4)



$\Rightarrow \dot{x}_1 = x_2 + 4u - 2x_1$

$$\boxed{\dot{x}_1 = -2x_1 + x_2 + 4u}$$

$$\boxed{\dot{x}_2 = x_3 + 3u}$$

$\dot{x}_3 = u - 2x_3 - 5x_1$

$$\boxed{\dot{x}_3 = -5x_1 - 2x_3 + u}$$

$$\boxed{y = x_1 + 6x_2}$$

State equation matrix $\Rightarrow$

$$\dot{x} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix} u$$

Output equation matrix $\Rightarrow$

$$y = \begin{bmatrix} 1 & 6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$ //