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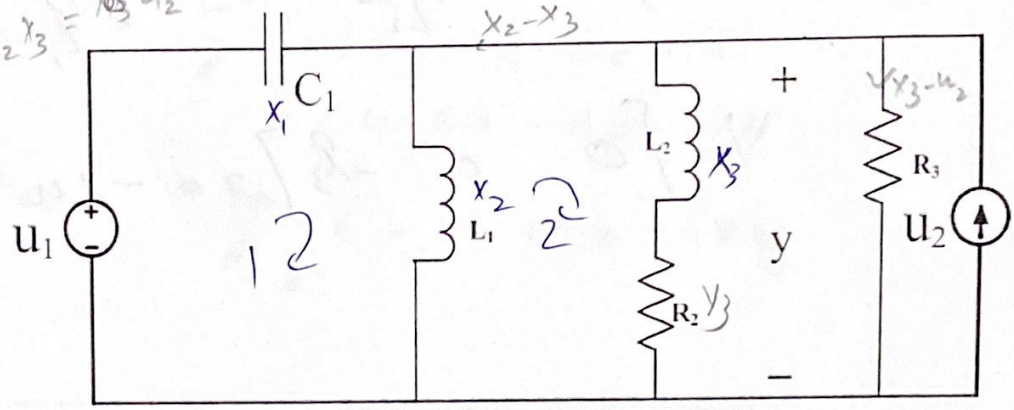
(1) Write the state equation and output equations for the following circuit (25 points)



$y = L_2 \dot{x}_3 + R_2 x_3 = R_3 u_2$

$u_1 = x_1 + L_1 \dot{x}_2$

$\dot{x}_2 = \frac{u_1}{L_1} - \frac{x_1}{L_1}$

$\dot{x}_1 = \frac{1}{C_1} \int x_2 - x_3$

$\dot{x}_1 = \frac{1}{C} [x_2 - x_3]$

$u_1 = x_1 + L_1 \dot{x}_2 \qquad y = L_2 \dot{x}_3 + R_2 x_3 = (x_3 - u_2) R_3$

$\dot{x}_2 =$

$\boxed{u_1 = x_1 + L_1 \dot{x}_2}$

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$\dot{x}_2 = \frac{u_1}{L_1} - \frac{x_1}{L_1}$

$\dot{x}_1 = \frac{1}{C_1} \int x_2 - x_3 + u_2$

$\dot{x}_1 = \frac{1}{C_1} [x_2 - x_3 + u_2]$

$u_1 - y = x_1$

$\boxed{u_1 - x_1 = y}$

Loop 2

$L_2 \dot{x}_2 = L_2 \dot{x}_3 + x_3 R_2 = (x_3 - u_2) R_3$

$L_2 \dot{x}_2 = L_2 \dot{x}_3 + x_3 R_2$

$L_2 \dot{x}_3 = L_2 \dot{x}_2 - x_3 R_2$

$L_2 \dot{x}_3 = L_2 \left[\frac{u_1}{L_1} - \frac{x_1}{L_1} \right] - x_3 R_2 \Rightarrow$

$\dot{x}_3 = x_3 R_3 - u_2 R_3$

$\boxed{\dot{x}_3 = \frac{u_1}{L_1} - \frac{x_1}{L_1} - \frac{x_3 R_2}{L_2}}$

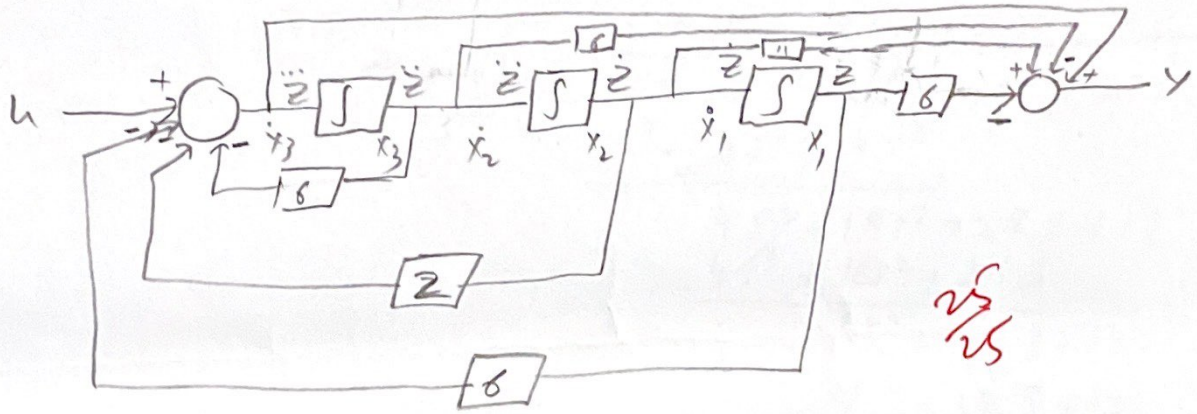
Please show me your calculations for partial credit

(2) $\dddot{y} + 6\ddot{y} + 2\dot{y} + 6y = \dddot{u} - 6\ddot{u} + 11\dot{u} - 6u$, find the state and output equation by using the controlable canonical form. (25 points)

System 1 = $\dddot{z} + 6\ddot{z} + 2\dot{z} + 6z = u$

$\dddot{z} = u - 6\ddot{z} - 2\dot{z} - 6z$

System 2 $y = \dddot{z} - 6\ddot{z} + 11\dot{z} - 6z$



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$Y = -6x_1 + 11x_2 - 6x_3 - 6x_3 - 2x_2 - 6x_1 + u$

$= -12x_1 + 8x_2 - 12x_3 + u$

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = -6x_3 - 2x_2 - 6x_1 + u$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -2 & -6 \end{bmatrix}}_{[A]} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{[B]} u$$

Please show me your calculations for partial credit

(3) $\dddot{y} + 8\ddot{y} + 23\dot{y} + 28y = \dot{u} + u$ Find the state and output equations of the given 4th order model by using Jordan form, and the model has repeated pole at −2. (25 points)

$$(s^4 + 8s^3 + 23s^2 + 28s + 12)\,Y = (s+1)\,U$$

$$\frac{Y}{U} = \frac{s+1}{(s^4 + 8s^3 + 23s^2 + 28s + 12)}$$

$$(s+2)^2 = s^2 + 4s + 4$$

$$\begin{array}{r|l} s^2 + 4s + 4 & s^4 + 8s^3 + 23s^2 + 28s + 12 \\ \hline & s^2 + 4s + 3 \\ & -s^4 - 4s^3 - 4s^2 \\ \hline & 4s^3 + 19s^2 + 28s + 12 \\ & -4s^3 - 16s^2 - 16s \\ \hline & 3s^2 + 12s + 12 \\ & -3s^2 - 12s - 12 \\ \hline & 0 \end{array}$$

$$s^4 + 8s^3 + 23s^2 + 28s + 12 = (s^2 + 4s + 4)(s^2 + 4s + 3)$$
$$= (s+2)^2 (s+1)(s+3)$$

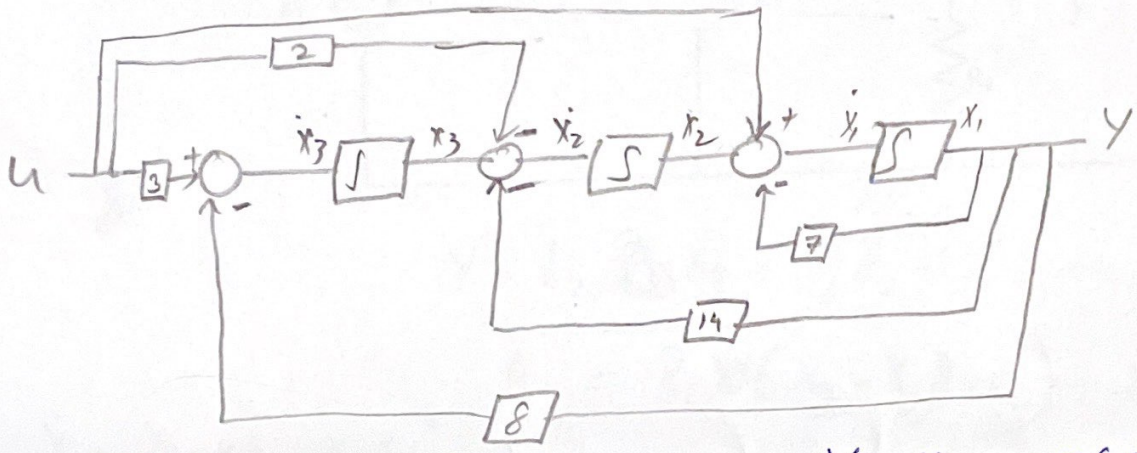
$$\frac{Y}{U} = \frac{s+1}{(s+2)^2(s+1)(s+3)} = \frac{A}{(s+1)} + \frac{B}{(s+3)} + \frac{C}{(s+2)} + \frac{D}{(s+2)^2}$$

Please show me your calculations for partial credit

(4) $\dddot{y} + 7\ddot{y} + 14\dot{y} + 8y = \ddot{u} - 2\dot{u} + 3u$, find the state and output equation by using the observable canonical form. (25 points)

Integrating 3 times

$$\ddot{y} = \int u - 2\iint u + 3\iiint u - 7\int y - 14\iint y - 8\iiint y$$



$Y = X_1$ o/p eq^n

$\dot{X}_1 = -7X_1 + u + X_2 = -7X_1 + u + X_2$

$\dot{X}_2 = -14y - 2u + X_3 = -14X_1 + X_3 - 2u$

$\dot{X}_3 = -8y + 3u = -8X_1 + 3u$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -7 & 1 & 0 \\ -14 & 0 & 1 \\ -8 & 0 & 0 \end{bmatrix}}_{[A]} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}}_{[B]} u$$

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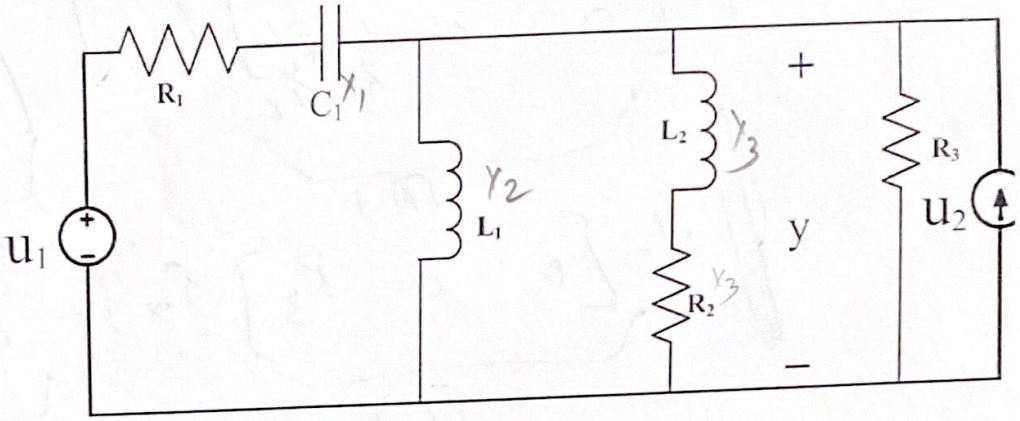
$$Y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{[C]} x + \underbrace{0}_{[D]} u$$

Please show me your calculations for partial credit

(5) Write the output equation for the following circuit. (5 bonus points)



$$y = L_2 \dot{x}_3 + x_3 R_2 = (x_3 - u_2) R_3$$

$$u_1 = R_1 (x_2 - x_3 + u_2) + x_1 + L_1 \dot{x}_2$$

$$\dot{x}_3 = \frac{x_3 - u_2}{\frac{R_3 L_2}{R_3}} - \frac{x_3 R_2}{L_2}$$

$$= x_3 \left[\frac{1}{R_3 L_2} - \frac{R_2}{L_2} \right] - \frac{u_2}{R_3}$$

$$x_1 = \frac{1}{C_1} \int [x_2 - x_3 + u_2]$$

$$\dot{x}_1 = \frac{1}{C_1} [x_2 - x_3 + u_2]$$

$$L_1 \dot{x}_2 = (x_3 - u_2) R_3$$

$$\dot{x}_2 = \frac{x_3 R_3}{L_1} - \frac{u_2 R_3}{L_1}$$

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Please show me your calculations for partial credit