LINEAR SYSTEMS
HOMEWORK # 4

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1) $A = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \end{bmatrix}$

$\Rightarrow$ i) Controllability Test $\Rightarrow$

For controllability test,

$P_c = [B \quad AB]$

Now, $AB = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ -5 \end{bmatrix}$

$\therefore [B \quad AB] = \begin{bmatrix} 1 & -4 \\ 1 & -5 \end{bmatrix}$

determinant of $P_c = -5 + 4 = -1 \neq 0$

$\therefore$ The model is controllable.

ii) Observability Test $\Rightarrow$

$P_o = \begin{bmatrix} C \\ CA \end{bmatrix}$

$CA = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix} [1 \quad 1] = [-4 \quad -5]$

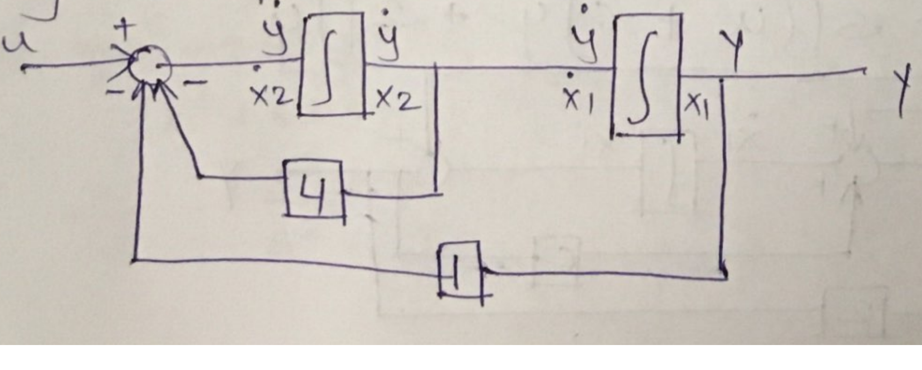
$\therefore P_o = \begin{bmatrix} 1 & 1 \\ -4 & -5 \end{bmatrix}$

$\det|P_o| = -5 + 4 = -1 \neq 0$

$\therefore$ The model is observable.

2) $\ddot{y} + 4\dot{y} + y = u$ (Controllable Canonical Form)

$\ddot{y} = u - 4\dot{y} - y$



$\therefore \dot{x}_1 = x_2$

$\dot{x}_2 = -x_1 - 4x_2 + u$

$y = x_1$

$\therefore A = \begin{bmatrix} 0 & 1 \\ -1 & -4 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

i) Controllability Test $\Rightarrow$

$P_c = [B \quad AB]$

$AB = \begin{bmatrix} 0 & 1 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$

$P_c = \begin{bmatrix} 0 & 1 \\ 1 & -4 \end{bmatrix}$

$\det |P_c| = 0 - 1 = -1 \neq 0$

$\therefore$ Model is controllable.

ii) Observability Test $\Rightarrow$

$P_o = \begin{bmatrix} C \\ CA\cancel{B} \end{bmatrix}$

$CA\cancel{B} = \begin{bmatrix} 0 & 1 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \end{bmatrix}$

$P_o = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

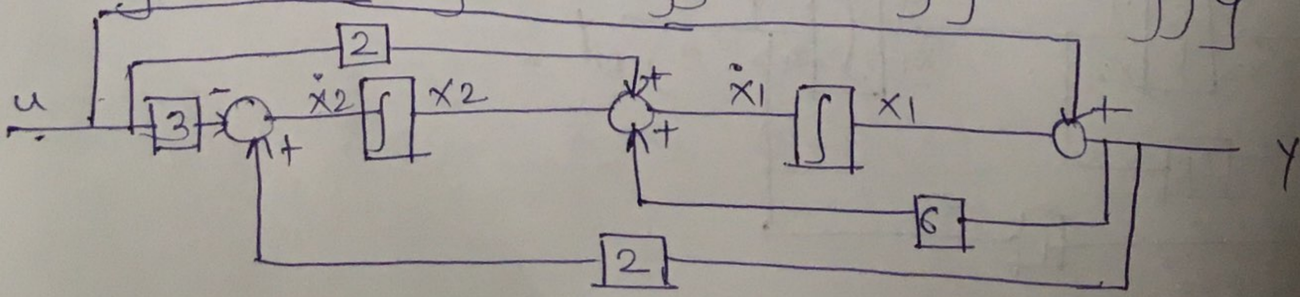
$\det |P_o| = 1 - 0 = 1 \neq 0$

$\therefore$ Model is observable.

3) $\ddot{y} - 6\dot{y} - 2y = \ddot{u} + 2\dot{u} - 3u$

$\Rightarrow \ddot{y} = \ddot{u} + 2\dot{u} - 3u + 6\dot{y} + 2y$

$y = u + 2\int u - 3\iint u + 6\int y + 2\iint y$



$$y = x_1 + u$$

$$\dot{x}_1 = 6y + x_2 + 2u$$

$$\dot{x}_1 = 6x_1 + x_2 + 8u$$

$$\dot{x}_2 = 2y - 3u$$

$$\dot{x}_2 = 2x_1 - u$$

$$\therefore A = \begin{bmatrix} 6 & 1 \\ 2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 8 \\ -1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 1$$

i) Controllability Test $\Rightarrow$

$$P_c = [B \;\; AB]$$

$$AB = \begin{bmatrix} 6 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 8 \\ -1 \end{bmatrix} = \begin{bmatrix} 47 \\ 16 \end{bmatrix}$$

$$P_c = \begin{bmatrix} 8 & 47 \\ -1 & 16 \end{bmatrix}$$

$$\det |P_c| = 128 + 47 = 175 \neq 0$$

$\therefore$ Model is controllable.

ii) Observability Test $\Rightarrow$

$$P_o = \begin{bmatrix} C \\ CA \end{bmatrix}$$

$$CA = \begin{bmatrix} 6 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 1 \end{bmatrix}$$

$$P_o = \begin{bmatrix} 1 & 0 \\ 6 & 1 \end{bmatrix}$$

$$\det |P_o| = 1 - 0 = 1 \neq 0$$

$\therefore$ Model is observable.

4) $\ddot{y} + 6\dot{y} + 8y = \ddot{u} + 2\dot{u} - 3u$ (Jordan Form)

$$\Rightarrow \cancel{y} s^2 y + 6sy + 8y = s^2 u + 2su - 3u$$

$$\therefore y(s^2 + 6s + 8) = u(s^2 + 2s - 3)$$

$$\therefore \frac{y}{u} = \frac{s^2 + 2s - 3}{s^2 + 6s + 8} = \frac{(s-1)(s+3)}{(s+4)(s+2)}$$

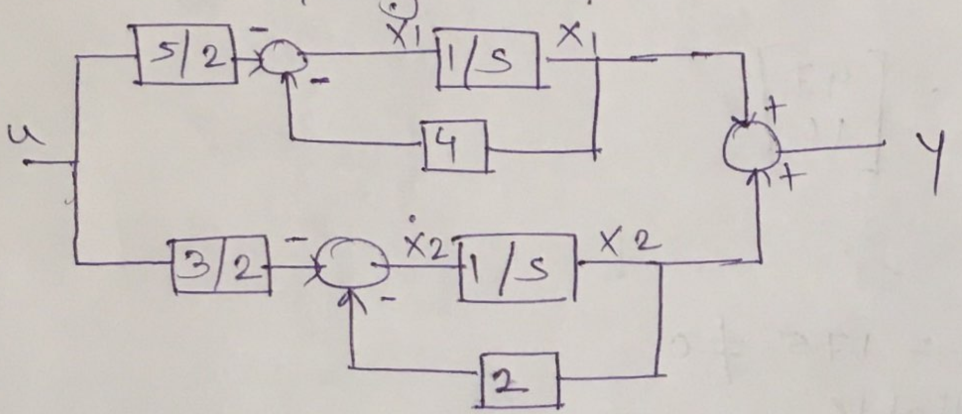
$\therefore H(s) = \frac{A}{s+4} + \frac{B}{s+2}$

$A = \left.\frac{(s-1)(s+3)}{s+2}\right|_{s=-4} \quad \therefore A = -5/2$

$B = \left.\frac{(s-1)(s+3)}{s+4}\right|_{s=-2} \quad \therefore B = -3/2$

$\therefore H(s) = \frac{-5/2}{s+4} - \frac{3/2}{s+2}$

Simulation Diagram $\Rightarrow$



$y = \dot{x}_1 + x_2$

$\dot{x}_1 = -4x_1 - \frac{5}{2}u$

$\dot{x}_2 = -2x_2 - 3/2\,u$

$A = \begin{bmatrix} -4 & 0 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} -5/2 \\ -3/2 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}$

i) Controllability Test $\Rightarrow$

$Q_c = [B \;\; AB]$

$AB = \begin{bmatrix} -4 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -5/2 \\ -3/2 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \end{bmatrix}$

$P_c = \begin{bmatrix} -5/2 & 10 \\ -3/2 & 3 \end{bmatrix}$

$\det(P_c) = \frac{-15}{2} + 15 = \frac{15}{2}$

ii) Observability Test ⇒

$$P_o = \begin{bmatrix} C \\ CA \end{bmatrix}$$

$$CA = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} -4 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -4 & -2 \end{bmatrix}$$

$$P_o = \begin{bmatrix} 1 & 1 \\ -4 & -2 \end{bmatrix}$$

$$\det |P_o| = -2 + 4 = 2$$

∴ Model is observable.