# EE-792 TEST-1 Summer-18

**First Name:** Solution

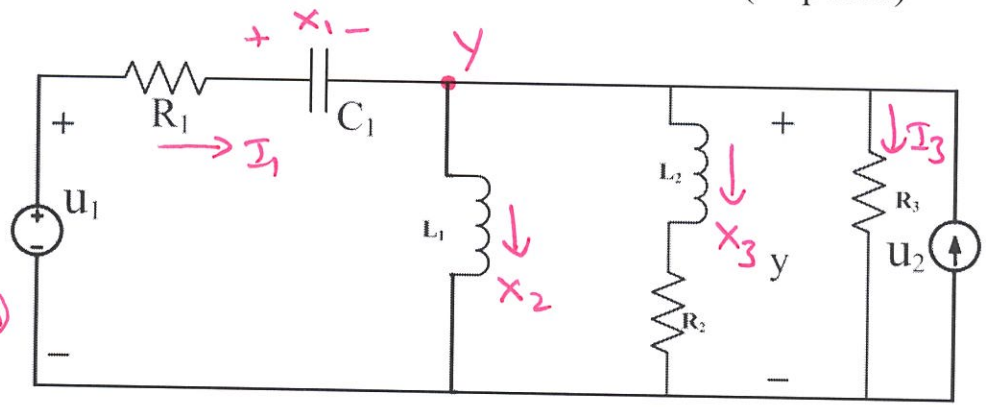
**Last Name:** _____

**WSUID:** _____

- Open text book, and closed notes. One sheet of notes (A4 size, both sides) will be allowed to the exam.
- One sheet of notes must be approved by me.
- Time for this Test is one hour thirty minutes.
- Calculators are allowed for this test (any kind)
- All work in this exam must be your own, sharing of calculators, formula sheet or text book will not be allowed.

(1) Write the state equation and output equations for the following circuit

(25 points)




$\dot{x}_1 = \frac{1}{C} I_1$

$I_1 = \frac{u_1 - x_1 - y}{R_1} \rightarrow (1)$

@ y. apply KCL

$I_1 + u_2 = x_2 + x_3 + I_3 \rightarrow (5)$

$y = L_1 \dot{x}_2 \rightarrow (2)$

$y = L_2 \dot{x}_3 + R_2 x_3 \rightarrow (3)$

$y = -R_3 I_3 \rightarrow (4)$

Sub. eq-1 & eq-4 in eq-5

$$\frac{u_1 - x_1 - y}{R_1} + u_2 = x_2 + x_3 + \frac{y}{R_3}$$

$$\frac{u_1}{R_1} - \frac{x_1}{R_1} + u_2 - x_2 - x_3 = y\left\{\frac{1}{R_3} + \frac{1}{R_1}\right\}$$

$$y = \frac{R_1 R_3}{R_1 + R_3}\left\{\frac{u_1}{R_1} - \frac{x_1}{R_1} + u_2 - x_2 - x_3\right\}$$

$$y = -\frac{R_3}{R_1+R_3} x_1 - \frac{R_1 R_3}{R_1+R_2} x_2 - \frac{R_1 R_3}{R_1+R_2} x_3 + \frac{R_3 R_1}{R_1+R_3} u_2 + \frac{R_3}{R_1+R_3} u_1$$

$$\dot{x}_2 = \frac{1}{L_1} y$$

$$\dot{x}_1 = \frac{1}{CR_1}\{u_1 - x_1 - y\}$$

$$\dot{x}_3 = -\frac{R_2}{L_2} x_3 + \frac{1}{L_2} y$$

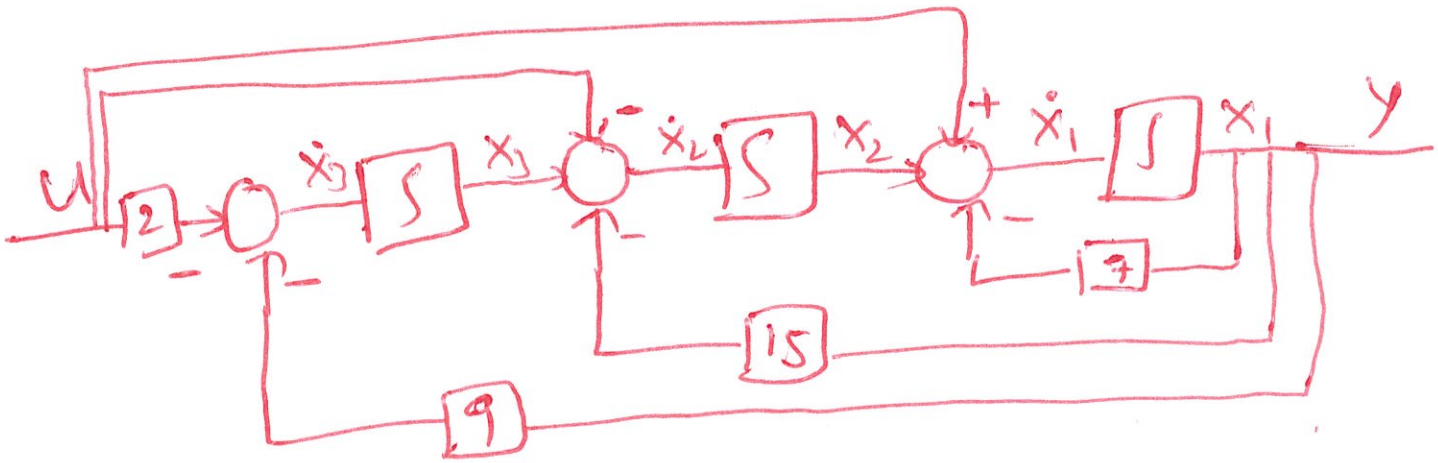
**Please show me your calculations for partial credit**

Substitute y in $\dot{x}_1$, $\dot{x}_2$ & $\dot{x}_3$

(2) $\dddot{y} + 7\ddot{y} + 15\dot{y} + 9y = \ddot{u} - \dot{u} - 2u$, find the state and output equation by using the Observable canonical form. (20 points)

$$\dddot{y} = \ddot{u} - \dot{u} - 2u - 7\ddot{y} - 15\dot{y} - 9y$$

$$y = \int u - \iint u - 2\iiint u - 7\int y - 15\iint y - 9\iiint y$$




$$\dot{x}_1 = -7x_1 + x_2 + u$$

$$\dot{x}_2 = -15x_1 + x_3 - u$$

$$\dot{x}_3 = -9x_1 - 2u$$

$$y = x_1$$

$$A = \begin{bmatrix} -7 & 1 & 0 \\ -15 & 0 & 1 \\ -9 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix} \quad C = [1, 0, 0] \quad D = 0$$

**Please show me your calculations for partial credit**

(3) $\dddot{y} - 3\ddot{y} + 4y = \ddot{u} + 6\ddot{u} + 11\dot{u} + 6u$, find the state and output equation by using the Jordan form. This methamatical modal has a repeated pole at '2'

(20 points)

$$\frac{y}{u} = \frac{s^3+6s^2+11s+6}{s^3-3s^2+4} =$$

$$s^3-3s+4 \;\big)\; s^3+6s^2+11s+6 \;(1$$
$$s^3-3s^2+4$$
$$+9s^2+11s+2$$

$$= 1 + \frac{9s^2+11s+2}{s^3-3s^2+4}$$

$$= 1 + \frac{9s^2+11s+2}{(s-2)^2(s+1)}$$

$$s^2-4s+4 \;\big)\; s^3-3s^2+4 \;(s+1$$
$$s^3-4s^2+4s$$
$$s^2-4s+4$$
$$s^2-4s+4$$
$$0$$

$$= 1 + \frac{A}{s+1} + \frac{B}{(s-2)} + \frac{C}{(s-2)^2}$$

$$\frac{9s^2+11s+2}{(s-2)^2(s+1)} = \frac{A}{s+1} + \frac{B}{s-2} + \frac{C}{(s-2)^2}$$

$$= \frac{A(s-2)^2 + B(s+1)(s-2) + C(s+1)}{(s-2)^2(s+1)}$$

$$\frac{A(s^2-4s+4) + B(s^2-s-2) + C(s+1)}{(s-2)^2(s+1)}$$

$$A = \left.\frac{9s^2+11s+2}{(s-2)^2}\right|_{s=-1}$$

$$= 9-11+2 = 0$$

$$B =$$

$$A+B=9 \qquad A=0 \qquad B=9$$

$$4A-2B+C=2$$

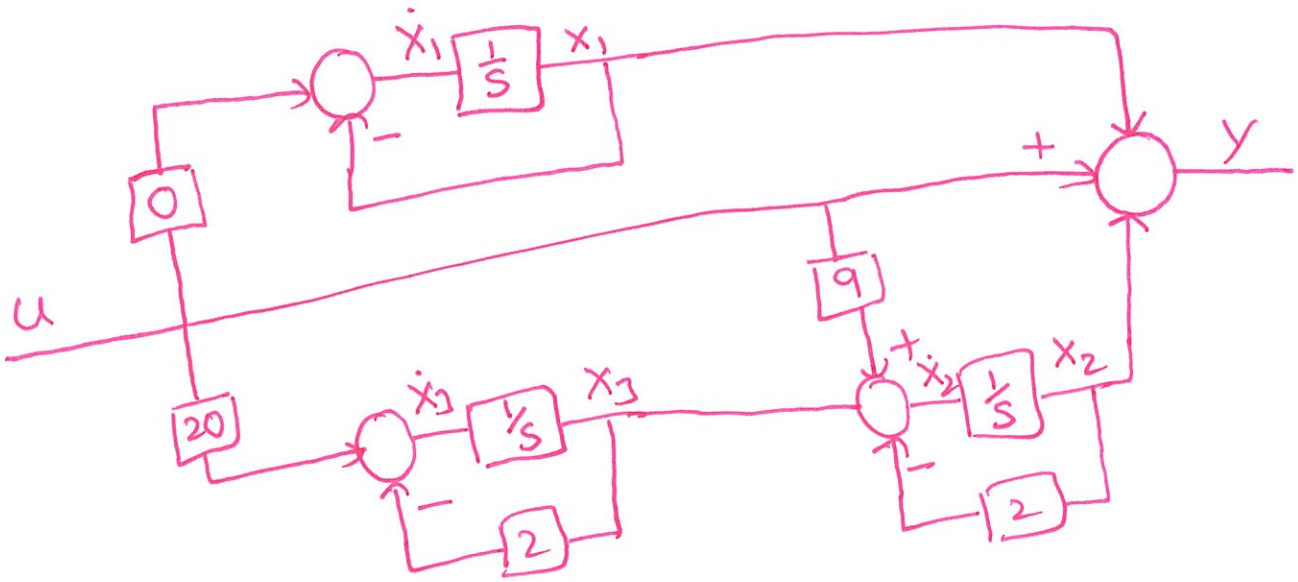
$$0-18+C=2 \qquad C=20$$

**Please show me your calculations for partial credit**

(3) $\dddot{y} - 3\ddot{y} + 4y = \dddot{u} + 6\ddot{u} + 11\dot{u} + 6u$, find the state and output equation by using the Jordan form. This methamatical modal has a repeated pole at '2'

(20 points)

$$\frac{y}{u} = 1 + \frac{0}{s+1} + \frac{9}{s-2} + \frac{20}{(s-2)^2}$$




$$\dot{x}_1 = -x_1 + 0u$$
$$\dot{x}_2 = -2x_2 + x_3 + 9u$$
$$\dot{x}_3 = -2x_3 + 20u$$
$$y = x_1 + x_2 + u$$

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 9 \\ 20 \end{bmatrix} \quad C = \begin{bmatrix} 1, & 1, & 0 \end{bmatrix}$$

$$D = 1$$

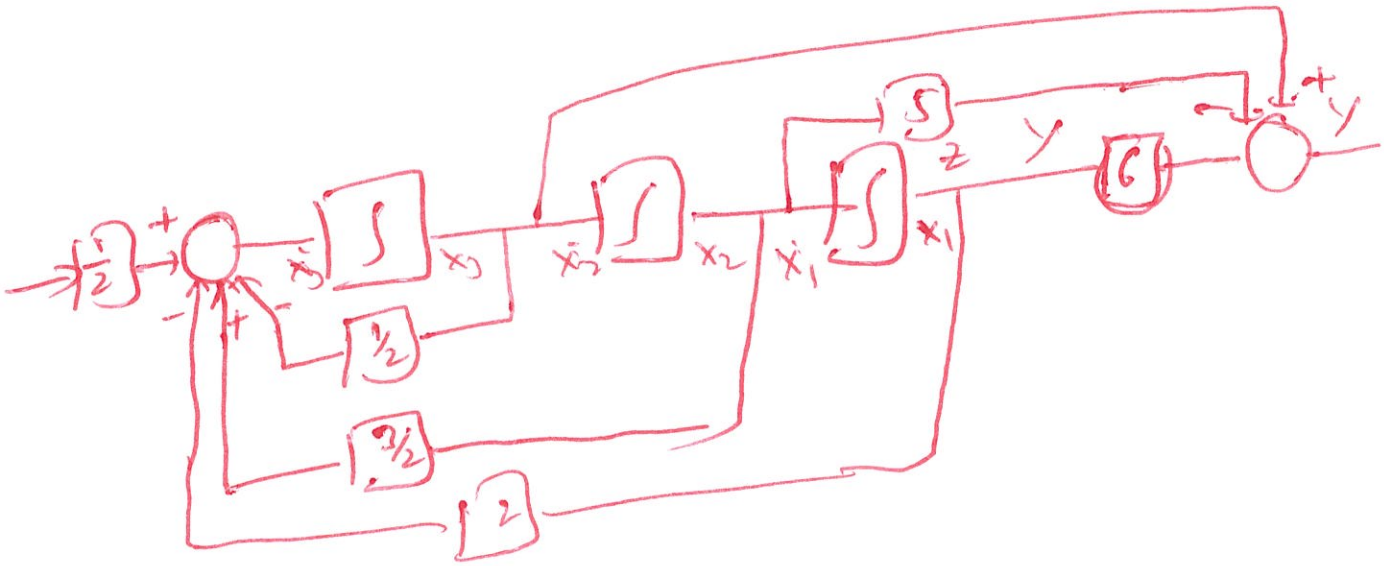
**Please show me your calculations for partial credit**

(4) $2\dddot{y} + \ddot{y} - 3\dot{y} + 4y = \ddot{u} - 5\dot{u} + 6u$, write the state equation and output equation by using the Controllable canonical form (20 points)

$$2\dddot{z} + \ddot{z} - 3\dot{z} + 4z = u$$

$$2\dddot{z} = u - \ddot{z} + 3\dot{z} - 4z$$

$$\dddot{z} = \frac{1}{2}u - \frac{1}{2}\ddot{z} + \frac{3}{2}\dot{z} - 2z$$




$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

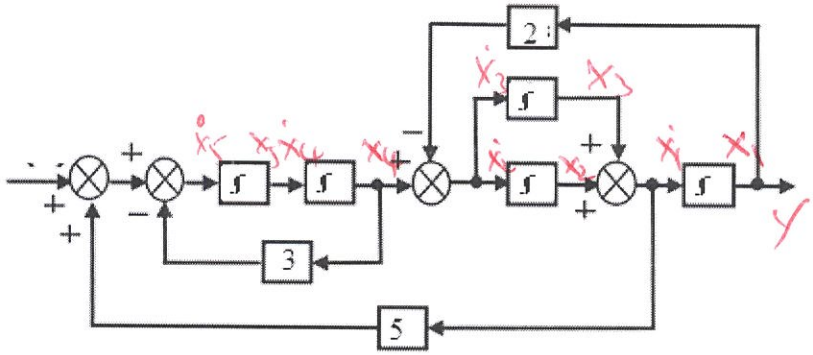
$$\dot{x}_3 = -2x_1 + \frac{3}{2}x_2 - \frac{1}{2}x_3 + \frac{1}{2}u$$

$$y = -5x_2 + x_3 + 6x_1$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & \frac{3}{2} & -\frac{1}{2} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{2} \end{bmatrix} \quad C = [6, -5, 1] \quad D = 0$$

**Please show me your calculations for partial credit**

(5) Find the state and output equations from the black (Simulation) diagram
(15+5 bonus points)




$$\dot{x}_1 = x_2 + x_3$$
$$\dot{x}_2 = -2x_1 + x_4$$
$$\dot{x}_3 = -2x_1 + x_4$$
$$\dot{x}_4 = x_5$$
$$\dot{x}_5 = -3x_4 + 5x_2 + 5x_3 + u$$

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ -2 & 0 & 0 & 1 & 0 \\ -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 5 & 5 & -3 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad C = x_1$$

$$C = [1, 0, 0, 0, 0] \qquad D = 0$$

**Please show me your calculations for partial credit**