EE-792 TEST-3 Summer-18

First Name: Solution

Last Name: ____________________

- Open text book, and closed notes. One sheet of notes (A4 size, both sides) will be allowed to the exam.
- Time for this Test is one hour thirty minutes.
- Calculators are allowed for this test (any kind)
- All work in this exam must be your own, sharing of calculators, formula sheet or text book will not be allowed.

(1) $\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\end{bmatrix} = \begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}2\\ 1\end{bmatrix}u, \quad y = \begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$ Transfer the given model into observable canonical form by using transformation matrix. (25 points)

$$[SI-A] = \begin{bmatrix}S & 0\\ 0 & S\end{bmatrix} - \begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix} = \begin{bmatrix}S-3 & 0\\ 0 & S-2\end{bmatrix}$$

$$\text{Det}|SI-A| = (S-3)(S-2) = S^2-5S+6=0$$

$$\hat{A} = \begin{bmatrix}5 & 1\\ -6 & 0\end{bmatrix} \quad \hat{C} = [1, 0]$$

$$CA = [1 \ -1]\begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix} = [3 \ -2]$$

$$P_0 = \begin{bmatrix}C\\ CA\end{bmatrix} = \begin{bmatrix}1 & -1\\ 3 & -2\end{bmatrix}$$

$$\hat{C}\hat{A} = [1, 0]\begin{bmatrix}5 & 1\\ -6 & 0\end{bmatrix} = [5, 0]$$

$$\hat{P}_0 = \begin{bmatrix}1 & 0\\ 5 & 0\end{bmatrix}$$

$$P_0^{-1} = \frac{1}{-2+3}\begin{bmatrix}-2 & 1\\ -3 & 1\end{bmatrix}$$

$$P_0^{-1} = \begin{bmatrix}-2 & 1\\ -3 & 1\end{bmatrix}$$

$$T = P_0^{-1}\hat{P}_0 = \begin{bmatrix}-2 & 1\\ -3 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}$$

$$= \begin{bmatrix}3 & 1\\ 2 & 1\end{bmatrix}$$

$$T^{-1} = \frac{1}{3-2}\begin{bmatrix}1 & -1\\ -2 & 3\end{bmatrix} = \begin{bmatrix}1 & -1\\ -2 & 3\end{bmatrix}$$

$$\hat{B} = T^{-1}B = \begin{bmatrix}1 & -1\\ -2 & 3\end{bmatrix}\begin{bmatrix}2\\ 1\end{bmatrix}$$

$$\hat{B} = \begin{bmatrix}1\\ -1\end{bmatrix}$$

$$\hat{C} = [1, 0]$$

$$\hat{A} = \begin{bmatrix}5 & 1\\ -6 & 0\end{bmatrix}$$

Please show me your calculations for partial credit

(2) A model has $A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $C = [1 \quad -1]$ design an observer with a desired poles at $-1 \pm j4$ (25 points)

$$A_o = A - KC = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} - \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} [1 \quad -1] = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} - \begin{bmatrix} k_1 & -k_1 \\ k_2 & -k_2 \end{bmatrix}$$

$$A_o = \begin{bmatrix} -k_1 & 1+k_1 \\ 2-k_2 & k_2-3 \end{bmatrix}$$

$$[sI - A_o] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -k_1 & 1+k_1 \\ 2-k_2 & k_2-3 \end{bmatrix} = \begin{bmatrix} s+k_1 & -1-k_1 \\ -2+k_2 & s+3-k_2 \end{bmatrix}$$

$$\text{Det } |sI - A_o| = \left[(s+k_1)(s+3-k_2) - -(k_1+1)(k_2-2) \right]$$

$$= s^2 + 3k_1 - \cancel{k_1 k_2} + 3s - k_2 s + k_1 s + (\cancel{k_1 k_2} + k_2 - 2k_1 - 2)$$

$$= s^2 + (3 - k_2 + k_1) s + k_1 + k_2 - 2 = 0$$

desired pole

$$(s+1+j4)(s+1-j4) = 0$$

$$s^2 + s - \cancel{j4s} + s + 1 - \cancel{j4} + \cancel{j4s} + \cancel{j4} - j^2 16 = 0$$

$$s^2 + 2s + 1 + 16 = 0$$

$$s^2 + 2s + 17 = 0$$

$$3 - k_2 + k_1 = 2$$

$$1 = k_2 - k_1$$

$$k_1 + k_2 - 2 = 17$$

$$k_1 + k_2 = 19$$

$$-k_1 + k_2 = 1$$

$(+)$

$$2k_2 = 20$$

$$k_2 = 10$$

$$k_1 = 10 - 1 = 9$$

$$k_1 = 9 \qquad k_2 = 10$$

Please show me your calculations for partial credit

Unique Solution

(3) $\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\end{bmatrix}=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}2\\ 1\end{bmatrix}u,\ \ y=\begin{bmatrix}1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+3u,$ convert the given model into jordan form, also find the controllability and observability of the given model before and after trasferring the model

(25 points+ 5 bonus points)

$P_c = [B,\ AB] = \begin{bmatrix}2 & 1\\ 1 & -7\end{bmatrix}$

$AB = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}2\\ 1\end{bmatrix} = \begin{bmatrix}1\\ -7\end{bmatrix}$

$\text{Det}|P_c| = -14-1 = -15$

it is Controllable

$P_o = \begin{bmatrix}C\\ CA\end{bmatrix} = \begin{bmatrix}1 & 1\\ -2 & -2\end{bmatrix}$

$CA = (1\ \ 1)\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix} = [-2\ \ -2]$

it is not observable

$[A-\lambda I] = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix} - \begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix} = \begin{bmatrix}-\lambda & 1\\ -2 & -3-\lambda\end{bmatrix}$

$\lambda(3+\lambda)+2=0$

$\lambda=-1,\ -2$

for $\lambda=-1$

$\begin{bmatrix}1 & 1\\ -2 & -2\end{bmatrix}\begin{bmatrix}V_{11}\\ V_{12}\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$

$V_{11}+V_{12}=0 \quad V_{11}=-V_{12}$

$V_{11}=1 \quad V_{12}=-1$

$\hat{D}=3$

$\lambda=-2$

$\begin{bmatrix}2 & 1\\ -2 & -1\end{bmatrix}$ $\quad 2V_{21}+1V_{22}=0$

$V_{22}=-2V_{21}$

$V_{21}=1 \quad V_{22}=-2$

$T=\begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix}$

$T^{-1} = \frac{1}{-1}\begin{bmatrix}-2 & -1\\ 1 & 1\end{bmatrix} = \begin{bmatrix}2 & 1\\ -1 & -1\end{bmatrix}$

$\hat{A}=T^{-1}AT = \begin{bmatrix}2 & 1\\ -1 & -1\end{bmatrix}\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix}$

$= \begin{bmatrix}-2 & -1\\ 2 & 2\end{bmatrix}\begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix} = \begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}$

$\hat{B} = T^{-1}B = \begin{bmatrix}2 & 1\\ -1 & -1\end{bmatrix}\begin{bmatrix}2\\ 1\end{bmatrix} = \begin{bmatrix}5\\ -3\end{bmatrix}$

$\hat{C} = CT = (1\ \ 1)\begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix} = [0\ \ -1]$

$\hat{P}_c = [\hat{B}\ \ \hat{A}\hat{B}] = \begin{bmatrix}5 & -5\\ -3 & 6\end{bmatrix}$ $\quad |P_c| \neq 0$

Controllable

$\hat{P}_o = \begin{bmatrix}\hat{C}\\ \hat{C}\hat{A}\end{bmatrix} = \begin{bmatrix}0 & -1\\ 0 & 2\end{bmatrix}$ $\quad \hat{P}_o = 0$

not observable.

Please show me your calculations for partial credit

(4) $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u, \quad y = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + 3u$, Transfer the given model into Jordan form by using transformation matrix. (25 points)

$A = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}$

$A - \lambda I$ from 3rd problem

$T = \begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix} \quad T^{-1} = \begin{bmatrix}2 & 1\\ -1 & -1\end{bmatrix}$

$\hat{A} = \begin{bmatrix}-1 & 0\\ 0 & -2\end{bmatrix}$

$\hat{B} = T^{-1}B = \begin{bmatrix}2 & 1\\ -1 & -1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \begin{bmatrix}1\\ -1\end{bmatrix}$

$\hat{C} = CT = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix} = \begin{bmatrix}-1, & -2\end{bmatrix}$

Please show me your calculations for partial credit