EE-792 **TEST-2** **Summer-19**

First Name: Solution

Last Name: ______________________

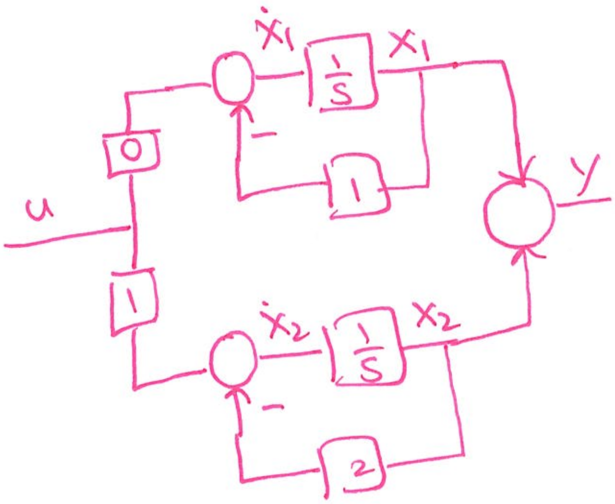
- Open text book, and closed notes. One sheet of notes (A4 size, both sides) will be allowed to the exam.
- Time for this Test is one hour thirty minutes.
- Calculators are allowed for this test (any kind)
- All work in this exam must be your own, sharing of calculators, formula sheet or text book will not be allowed.

(1) A continues time invariant system is described by $\ddot{y} + 3\dot{y} + 2y = \dot{u} + u$ find the controllability and observability of the model by deriving the model in Jordan form. (20 points)

$$(s^2+3s+2)y = (s+1)(u)$$

$$\frac{y}{u} = \frac{(s+1)}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} = \frac{0}{s+1} + \frac{1}{s+2}$$

$$\left.\frac{s+1}{s+1}\right|_{s=-2} \qquad \frac{-1}{-1} = 1 \qquad \left.\frac{s+1}{s+2}\right|$$



$$\dot{x}_1 = -\cancel{2}x_1 + 0u$$

$$\dot{x}_2 = -2x_2 + u$$

$$y = x_1 + x_2$$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$P_c = [B, AB] = \begin{bmatrix} 0 & 0 \\ 1 & -2 \end{bmatrix}$$ Det $|P_c| = 0$ so not controllable.

$$P_o = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$$ Det $|P_o| \neq 0$ so observable

Please show me your calculations for partial credit

(2) A model has $A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ Find the feedback gain such that the closed loop poles are at $-2 \pm j3$ (20 points)

$A_c = A - BF = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} F_1 & F_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -F_1 & -2-F_2 \end{bmatrix}$

$SI - A_c = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ -F_1 & -2-F_2 \end{bmatrix} = \begin{bmatrix} S+1 & 0 \\ F_1 & S+2+F_2 \end{bmatrix}$

Det $|SI - A_c| =$

$(S+1)(s+2+F_2) = 0 \rightarrow$ charactristic Polynomial equation

$s^2 + 2s + sF_2 + s + 2 + F_2 = 0$

$s^2 + (2+F_2+1)s + (2+F_2) = 0.$

$F_2 = 1$ $F_2 = 11$

NO Solution

closed loop Poles

$(s+2+j3)(s+2-j3) = 0$

$(s+2)^2 - (j3)^2 = 0$

$s^2 + 4s + \cancel{13} = 0$

$2 + F_2 + 1 = 4$

$F_2 = 1$

$2 + F_2 = 13$

$F_2 =$

$= 11$

Please show me your calculations for partial credit

(3) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u, \quad y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, Find the zero initial condition response of the given model when u=5 (20 points)

$$[sI-A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} s+3 & -1 \\ 2 & s \end{bmatrix}$$

$$[sI-A]^{-1} = \frac{1}{s^2+3s+2} \begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix} = \frac{1}{(s+1)(s+2)} \begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}$$

$$\varphi(s) = \begin{bmatrix} \frac{s}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s+3}{(s+1)(s+2)} \end{bmatrix} = \begin{bmatrix} \frac{A}{s+1} + \frac{B}{s+2} & \frac{c}{s+1} + \frac{d}{s+2} \\ \frac{e}{s+1} + \frac{f}{s+2} & \frac{g}{s+1} + \frac{h}{s+2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{s+1} + \frac{2}{s+2} & \frac{1}{s+1} + \frac{-1}{s+2} \\ \frac{-2}{s+1} + \frac{2}{s+2} & \frac{2}{s+1} + \frac{-1}{s+2} \end{bmatrix} \qquad \varphi(t) = \begin{bmatrix} -e^{-t} + 2e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & 2e^{-t} - e^{-2t} \end{bmatrix}$$

$$x(t) = \varphi(t) \int_0^t \varphi(-\tau) B u_\tau \, d\tau = \varphi(t) \int_0^t \begin{pmatrix} (-e^{\tau} + 2e^{+2\tau})5 \\ (-2e^{\tau} + 2e^{2\tau})5 \end{pmatrix} d\tau$$

$$x(t) = \varphi(t) 5 \begin{bmatrix} -e^{t} + e^{2t} \\ -2e^{t} + e^{2t} + 1 \end{bmatrix} = 5 \begin{bmatrix} -e^{-t} + 2e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & 2e^{-t} - e^{-2t} \end{bmatrix} \begin{bmatrix} -e^{t} + e^{2t} \\ -2e^{t} + e^{2t} + 1 \end{bmatrix}$$

Please show me your calculations for partial credit

$= 5 \Big($

$$x(t) = \begin{bmatrix} -e^{-t}+2e^{-2t} & e^{-t}-e^{-2t} \\ -2e^{-t}+2e^{-2t} & 2e^{-t}-e^{-2t} \end{bmatrix} \begin{bmatrix} -5e^{t}+5e^{2t} \\ -10e^{t}+5e^{2t}+5 \end{bmatrix}$$

$$= \begin{bmatrix} 5e^{-t}e^{t}-5e^{-t}e^{2t}-10e^{-2t}e^{t}+10e^{-2t}e^{2t}-10e^{t}e^{-t}+5e^{2t}e^{-t}+5e^{-t}+10e^{t}e^{-2t}-5e^{2t}e^{-2t}-5e^{-2t} \\ 10e^{-t}e^{t}-10e^{2t}e^{-t}-10e^{-2t}e^{t}+10e^{-2t}e^{2t}-20e^{-t}e^{t}+10e^{-t}e^{2t}+10e^{-t}+10e^{t}e^{-2t}-5e^{2t}e^{-2t}+5e^{-2t} \end{bmatrix}$$

$$= \begin{bmatrix} 5e^{-t}-5e^{-2t} \\ 10e^{-t}-5e^{-2t}-5 \end{bmatrix}$$

(4) $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}-1\\ 1\end{bmatrix}u,\ y = [1\ \ 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix} + 0u$, (a) find the system is stable or not? (b) Find the model is controlable or not? (c) if the model is uncontrollable explain why the model is uncontrollable and show me the calculations. Find the solution if the desired poles are at -2 and -3 **(25 points)**

$$[SI-A] = \begin{bmatrix}S & 0\\ 0 & S\end{bmatrix} - \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix} = \begin{bmatrix}S & -1\\ 2 & S+3\end{bmatrix}$$

$$\text{Det } |SI-A| = S^2+3S+2=0 \qquad (S+2)(S+1)=0$$

$S=-1$ $\quad S=-2$ $\quad$ System is stable.

$P_c = [B, AB] = \begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$ $\quad |P_c| = 0$ $\therefore$ The system is uncontrollable

$$H(s) = C[SI-A]^{-1}B = \frac{1}{(S^2+3S+2)}[1, 0]\begin{bmatrix}S+3 & 1\\ -2 & S\end{bmatrix}\begin{bmatrix}-1\\ 1\end{bmatrix}$$

$$= \frac{1}{(S+2)(S+1)}[S+3, 1]\begin{bmatrix}-1\\ 1\end{bmatrix} = \frac{-S-3+1}{(S+2)(S+1)} = \frac{-(S+2)}{(S+2)(S+1)}$$

→ The model is uncontrollable since there is a pole zero cancellation

$$A_c = A - BF = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix} - \begin{bmatrix}-1\\ 1\end{bmatrix}[F_1\ F_2] = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix} - \begin{bmatrix}-F_1 & -F_2\\ F_1 & F_2\end{bmatrix}$$

$$= \begin{bmatrix}F_1 & 1+F_2\\ -2-F_1 & -3-F_2\end{bmatrix} \qquad |SI-A_c| = \begin{bmatrix}S & 0\\ 0 & S\end{bmatrix} - \begin{bmatrix}F_1 & 1+F_2\\ -2-F_1 & -3-F_2\end{bmatrix}$$

Please show me your calculations for partial credit

$$= \begin{bmatrix}S-F_1 & -1-F_2\\ 2+F_1 & S+3+F_2\end{bmatrix}$$

$$\text{Det}|sI - A_c| = (s - f_1)(s + 3 + f_2) + (+1 + f_2)(2 + f_1)$$

$$= s^2 + 3s + sf_2 - f_1 s - 3f_1 - f_1 f_2 + (2 + f_1 + 2f_2 + f_2 f_1)$$

$$= s^2 + (3 + f_2 - f_1)s - 3f_1 + 2 + f_1 + 2f_2$$

$$= s^2 + (3 + f_2 - f_1)s + (2f_2 - 2f_1 + 2) = 0$$

$$(s+2)(s+3) = 0$$

$$s^2 + 5s + 6$$

$3 + f_2 - f_1 = 5$ $\qquad$ $2f_2 - 2f_1 + 2 = 6$

$\underline{f_2 - f_1 = 2}$ $\qquad$ $\underline{f_2 - f_1 = 2}$

$\Rightarrow$ many <u>solutions</u>

(5) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u,\ y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, Write the transfer function by using the above state and output equations. (15 points+2 bonus points)

$$[sI - A] = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} = \begin{bmatrix} s+1 & 0 & 0 \\ 0 & s+2 & 0 \\ 0 & 0 & s+3 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s+1)(s+2)(s+3)} \begin{bmatrix} (s+2)(s+3) & 0 & 0 \\ 0 & (s+1)(s+3) & 0 \\ 0 & 0 & (s+1)(s+2) \end{bmatrix}$$

$$C[sI - A]^{-1} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} (s+2)(s+3) & 0 & 0 \\ 0 & (s+1)(s+3) & 0 \\ 0 & 0 & (s+1)(s+2) \end{bmatrix} \frac{1}{(s+1)(s+2)(s+3)}$$

$$C[sI - A]^{-1}B = \begin{bmatrix} (s+2)(s+3) & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \frac{1}{(s+1)(s+2)(s+3)}$$

$$\boxed{H(s) = \frac{(s+2)(s+3)}{(s+1)(s+2)(s+3)} = \frac{y}{u}}$$

Please show me your calculations for partial credit