Linear Systems

Homework-5

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1) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -6 & 1 \\ -6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad , \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\dot{X} = AX + Bu \quad , Y = CX + Du$

$H(s) = C[sI - A]^{-1} B + D$

$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -6 & 1 \\ -6 & 0 \end{bmatrix} = \begin{bmatrix} s+6 & -1 \\ 6 & s \end{bmatrix}$

$[sI - A]^{-1} = \dfrac{1}{(s+6)s + 6} \begin{bmatrix} s & 1 \\ -6 & s+6 \end{bmatrix} = \dfrac{1}{s^2 + 6s + 6} \begin{bmatrix} s & 1 \\ -6 & s+6 \end{bmatrix}$

$C[sI - A]^{-1} = \dfrac{1}{s^2+6s+6} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s & 1 \\ -6 & s+6 \end{bmatrix} = \dfrac{1}{s^2+6s+6} \begin{bmatrix} s & 1 \end{bmatrix}$

$C[sI - A]^{-1} B = \dfrac{1}{s^2+6s+6} \begin{bmatrix} s & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \dfrac{1}{s^2+6s+6}$

$\dfrac{y}{u} = \dfrac{1}{s^2+6s+6}$

$ys^2 + 6sy + 6y = u$

$\boxed{\ddot{y} + 6\dot{y} + 6y = u}$

2) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -18 & -27 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$H(s) = C[sI - A]^{-1} B + D$

$$sI - A = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -18 & -27 & -10 \end{bmatrix} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 18 & 27 & s+10 \end{bmatrix}$$

$$[sI-A]^{-1} = \frac{1}{s[s(s+10)+27] + 1(0+18)} \begin{bmatrix} s(s+10)+27 & -18 & -18s \\ s+10 & s(s+10) & -(27s+18) \\ (1) & -(-s) & s^2 \end{bmatrix}^T$$

$$= \frac{1}{s(s^2+10s+27)+18} \begin{bmatrix} s^2+10s+27 & -18 & -18s \\ s+10 & s^2+10s & -(27s+18) \\ 1 & s & s^2 \end{bmatrix}^T$$

$$= \frac{1}{s^3+10s^2+27s+18} \begin{bmatrix} s^2+10s+27 & s+10 & 1 \\ -18 & s^2+10s & s \\ -18s & -(27s+18) & s^2 \end{bmatrix}$$

$$C[sI-A]^{-1} = \frac{1}{s^3+10s^2+27s+18} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} s^2+10s+27 & s+10 & 1 \\ -18 & s^2+10s & s \\ -18s & -(27s+18) & s^2 \end{bmatrix}$$

$$= \frac{1}{s^3+10s^2+27s+18} \begin{bmatrix} s^2+10s+27 & s+10 & 1 \end{bmatrix}$$

$$C[sI-A]^{-1}B = \frac{1}{s^3+10s^2+27s+18} \begin{bmatrix} s^2+10s+27 & s+10 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{s^3+10s^2+27s+18}$$

$$\frac{Y(s)}{U(s)} = \frac{1}{s^3+10s^2+27s+18} \Rightarrow \boxed{\dddot{y} + 10\ddot{y} + 27\dot{y} + 18y = u}$$

3) $A = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 1 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ -5 \end{bmatrix}$, $\cancel{A\ CA} = \left[-8 \right.$

$C \cdot A = \underset{1\times 2}{\begin{bmatrix} 1 & 1 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix}} = \begin{bmatrix} -4 & -5 \end{bmatrix}$

$P_c = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 1 & -5 \end{bmatrix}$ $\det(P_c) = -5 + 4 = -1 \neq 0$

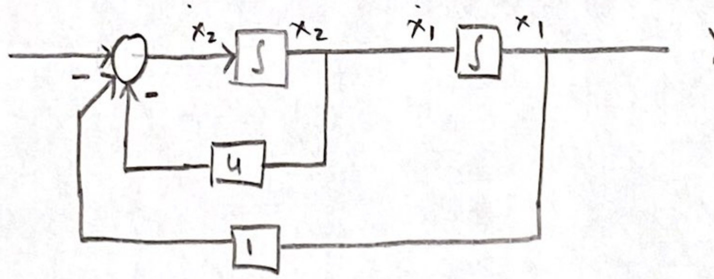
$\therefore$ It is controllable

$P_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -4 & -5 \end{bmatrix}$ $\det(P_o) = -5+4 = -1 \neq 0$

$\therefore$ It is observable

4) $\ddot{y} + 4\dot{y} + y = u$ CCF

$\ddot{y} = u - 4\dot{y} - y$



$y = x_1, \quad \dot{x}_1 = x_2, \quad \dot{x}_2 = -4x_2 - x_1 + u$

state Eq^n

$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

Output Eq^n

$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]\, u$

$$A \cdot B = \begin{bmatrix} 0 & 1 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

$$C \cdot A = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$P_C = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -4 \end{bmatrix} \quad \det(P_C) = -1 \neq 0$$

$\therefore$ It is controllable

$$P_O = \begin{bmatrix} C \\ C \cdot A \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow \det(P_O) = 1 \neq 0$$

$\therefore$ It is observable.