Linear Systems

Homework - 6

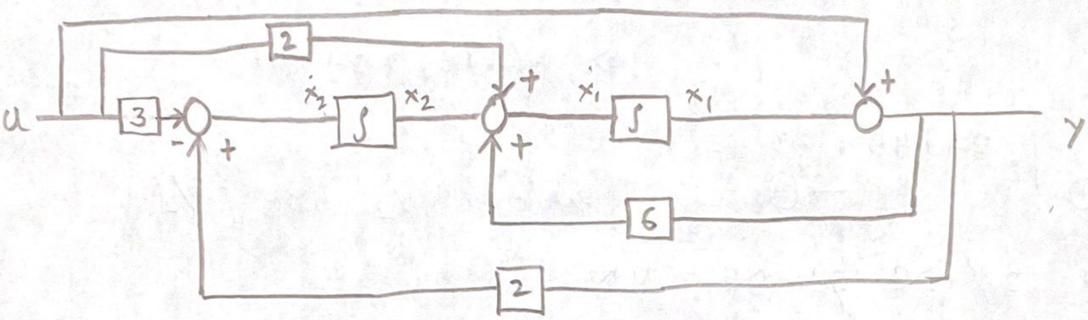
D. chakradhar Reddy

E949F496

① $\ddot{y} - 6\dot{y} - 2y = \ddot{u} + 2\dot{u} - 3u$ (Observable CF)

$\ddot{y} = \ddot{u} + 2\dot{u} - 3u + 6\dot{y} + 2y$ (Integrating twice on both sides)

$y = u + 2\int u - 3\iint u + 6\int y + 2\iint y$



$Y = x_1 + u$

$\dot{x}_1 = \cancel{6x_1 + x_2 + 2u} \quad 6(x_1 + u) + x_2 + 2u \quad = \quad 6x_1 + x_2 + 8u$

$\dot{x}_2 = \cancel{2x_1 + 3u} \quad 2(x_1 + u) - 3u \quad = \quad 2x_1 + 2u - 3u = 2x_1 - u$

$A = \begin{bmatrix} 6 & 1 \\ 2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 8 \\ -1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = 1$

(i) Controllability Test $\Rightarrow P_c = [B \quad AB]$

$AB = \begin{bmatrix} 47 \\ 16 \end{bmatrix} \quad P_c \Rightarrow \begin{bmatrix} 8 & 47 \\ -1 & 16 \end{bmatrix} \quad \det|P_c| = 128 + 47 \neq 0$

$\therefore$ It is controllable

(ii) Observability test $\Rightarrow P_o = \begin{bmatrix} C \\ CA \end{bmatrix}$

$CA = [1 \quad 0] \begin{bmatrix} 6 & 1 \\ 2 & 0 \end{bmatrix} = [6 \quad 1] \Rightarrow P_o = \begin{bmatrix} 1 & 0 \\ 6 & 1 \end{bmatrix} \Rightarrow \det|P_o| = 1 \neq 0$

$\therefore$ It is observable.

② $\ddot{y} + 6\dot{y} + 8y = \ddot{u} + 2\dot{u} - 3u$ (Jordan Form)

$$y(s^2+6s+8) = u(s^2+2s-3)$$

$$\frac{y}{u} = \frac{s^2+2s-3}{s^2+6s+8} = 1 + \frac{-4s-11}{s^2+6s+8}$$

$$\begin{array}{r} s^2+6s+8 \;)\; s^2+2s-3 \;(\; 1 \\ s^2+6s+8 \\ \hline 0 - 4s - 11 \end{array}$$

$$\Rightarrow 1 + \left[\frac{-(4s+11)}{s^2+6s+8}\right] = 1 + \left[\frac{A}{(s+4)} + \frac{B}{s+2}\right]$$

$$-(4s+11) = As + 2A + Bs + 4B$$

$$-(4s+11) = s(A+B) + 2A + 4B$$

$A + B = -4$ — ① $2A + 4B = -11$ — ②

① $*\, 2$ − ② $\Rightarrow 2\cancel{A} + 2B - 2\cancel{A} - 4B = -8 + 11$

$-2B = +3$

$\boxed{B = -3/2}$

① $\Rightarrow A + B = -4 \Rightarrow A = -4 + \frac{3}{2}$

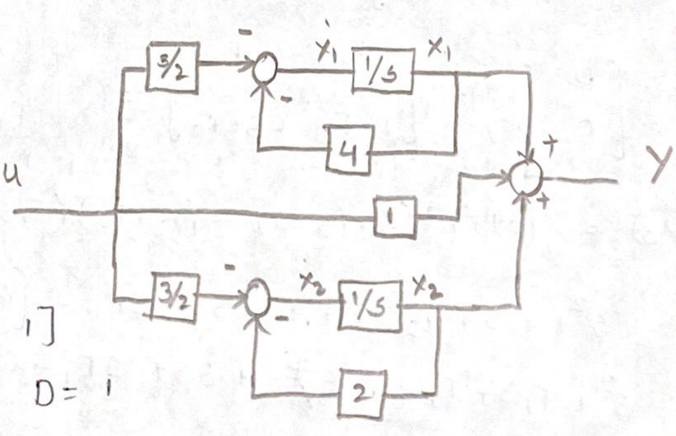
$\boxed{A = -5/2}$

$$\Rightarrow 1 + \frac{-4s-11}{s^2+6s+8} = 1 + \frac{-5/2}{s+4} + \frac{-3/2}{s+2}$$

$Y = x_1 + x_2 + u$

$\dot{x}_1 = -4x_1 - \frac{5}{2}u$

$\dot{x}_2 = -2x_2 - \frac{3}{2}u$



$$A = \begin{bmatrix} -4 & 0 \\ 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} -5/2 \\ -3/2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$D = 1$$

Controllability test

$$P_c = \begin{bmatrix} B & AB \end{bmatrix}, \quad AB = \begin{bmatrix} -4 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -5/2 \\ -3/2 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \end{bmatrix}$$

$$P_c = \begin{bmatrix} -5/2 & 10 \\ -3/2 & 3 \end{bmatrix} \qquad \det|P_c| = -\frac{15}{2} + \frac{30}{2} = 15/2 \neq 0$$

$\therefore$ It is controllable.

(ii) Observability Test

$$P_O = \begin{bmatrix} C \\ CA \end{bmatrix}$$

$$CA = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} -4 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -4 & -2 \end{bmatrix}$$

$$P_O = \begin{bmatrix} 1 & 1 \\ -4 & -2 \end{bmatrix} \quad \det|P_O| = -2+4 = 2 \neq 0$$

$\therefore$ It is Observable.

(3) $A = \begin{bmatrix} 0 & -6 \\ 1 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ closed loop poles at $-1, -4$

$$A_c = A - BF \Rightarrow \begin{bmatrix} 0 & -6 \\ 1 & -5 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} F_1 & F_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -6 \\ 1 & -5 \end{bmatrix} - \begin{bmatrix} F_1 & F_2 \\ F_1 & F_2 \end{bmatrix} \Rightarrow \begin{bmatrix} -F_1 & -6-F_2 \\ 1-F_1 & -5-F_2 \end{bmatrix}$$

$$sI - A_c = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -F_1 & -6-F_2 \\ 1-F_1 & -5-F_2 \end{bmatrix} = \begin{bmatrix} s+F_1 & 6+F_2 \\ F_1-1 & s+F_2+5 \end{bmatrix}$$

$$\det(sI - A_c) = (s+F_1)(s+F_2+5) - (6+F_2)(F_1-1)$$

$$= s^2 + F_1 s + F_2 s + F_1F_2 + 5s + 5F_1 - \left[6F_1 - 6 + F_1F_2 - F_2\right]$$

$$= s^2 + F_1 s + F_2 s + \cancel{F_1F_2} + 5s + 5F_1 - 6F_1 + 6 - \cancel{F_1F_2} + F_2$$

$$= s^2 + s(F_1 + F_2 + 5) - F_1 + F_2 + 6 \quad \text{—(1)}$$

Desired poles at $(s+1)(s+4) = s^2 + 5s + 4$ (Desired characteristic polynomial) —(2)

(1) & (2) $\Rightarrow$ $F_1 + F_2 + 5 = 5$ —(3)

$$-F_1 + F_2 + 6 = 4$$

$$-F_1 + F_2 = -2$$

$$F_1 - F_2 = 2 \quad \text{—(4)}$$

③ + ④ $\Rightarrow$ $F_1 + \cancel{F_2} + F_1 - \cancel{F_2} = 2$

$$\boxed{F_1 = 1}$$

④ $\Rightarrow$ $F_1 - F_2 = 2$

$$\boxed{F_2 = -1}$$

$u = -Fx + r \Rightarrow \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + r$

We can design feedback with desired poles -1 and -4 but unique solution exists

④ $A = \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ closed loop poles at $-3 \pm 4j$

$A_C = A - BF \Rightarrow \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} F_1 & F_2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ F_1 & F_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -F_1 & -3-F_2 \end{bmatrix}$

$sI - A_C = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -F_1 & -3-F_2 \end{bmatrix} = \begin{bmatrix} s-2 & -1 \\ F_1 & s+3+F_2 \end{bmatrix}$

$\det|sI - A_C| \Rightarrow (s-2)(s+3+F_2) - (-F_1)$

$\Rightarrow s^2 + 3s + F_2 s - 2s - 6 - 2F_2 + F_1$

$\Rightarrow s^2 + s(1 + F_2) - 2F_2 - 6 + F_1$ — ①

Desired poles $\Rightarrow (s + 3 + 4j)(s + 3 - 4j) \Rightarrow$ ~~s+3~~ | $j^2 = 1$

$\Rightarrow s^2 + 3s - \cancel{j4s} + 3s + 9 - \cancel{j12} + \cancel{j4s} + \cancel{j12} - j^2 16$ | $j = \sqrt{-1}$

$\Rightarrow s^2 + 6s + 9 + 16 \Rightarrow s^2 + 6s + 25$ — ②

Compare ① & ② $\Rightarrow 1 + F_2 = 6$ | $-2F_2 + F_1 - 6 = 25$ — ④

$$\boxed{F_2 = 5} \quad \text{— ③}$$

④ $\Rightarrow \quad F_1 = 25 + 16 = 41$

$$\boxed{F_1 = 41}$$

$u =$ [illegible]