Linear systems

Exam - 1

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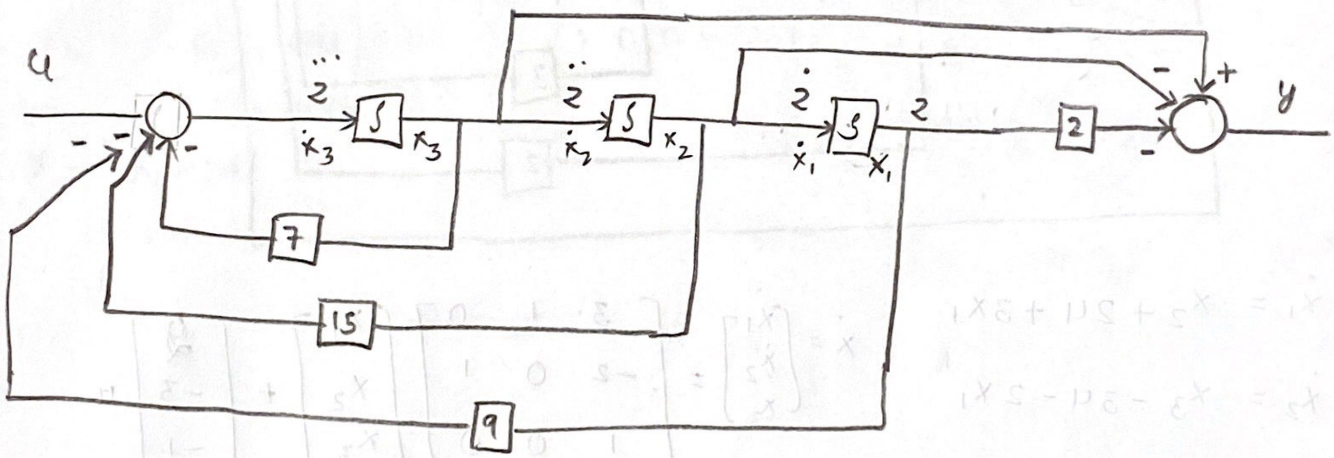
② $\dddot{y} + 7\ddot{y} + 15\dot{y} + 9y = \ddot{u} - \dot{u} - 2u$ (Controllable Canonical Form)

$$\frac{y}{u} = \frac{s^2 - s - 2}{s^3 + 7s^2 + 15s + 9} \Rightarrow \frac{y}{z} \times \frac{z}{u} = \frac{y}{u}$$

$$\Rightarrow \frac{y}{z} = s^2 - s - 2 \qquad \bigg| \qquad \frac{z}{u} = \frac{1}{s^3 + 7s^2 + 15s + 9}$$

$$y = \ddot{z} - \dot{z} - 2z \qquad u = \dddot{z} + 7\ddot{z} + 15\dot{z} + 9z$$

$$\dddot{z} = u - 7\ddot{z} - 15\dot{z} - 9z$$



$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = u - 7x_3 - 15x_2 - 9x_1$

$y = -2x_1 - x_2 + x_3$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -9 & -15 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} -2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0u$$

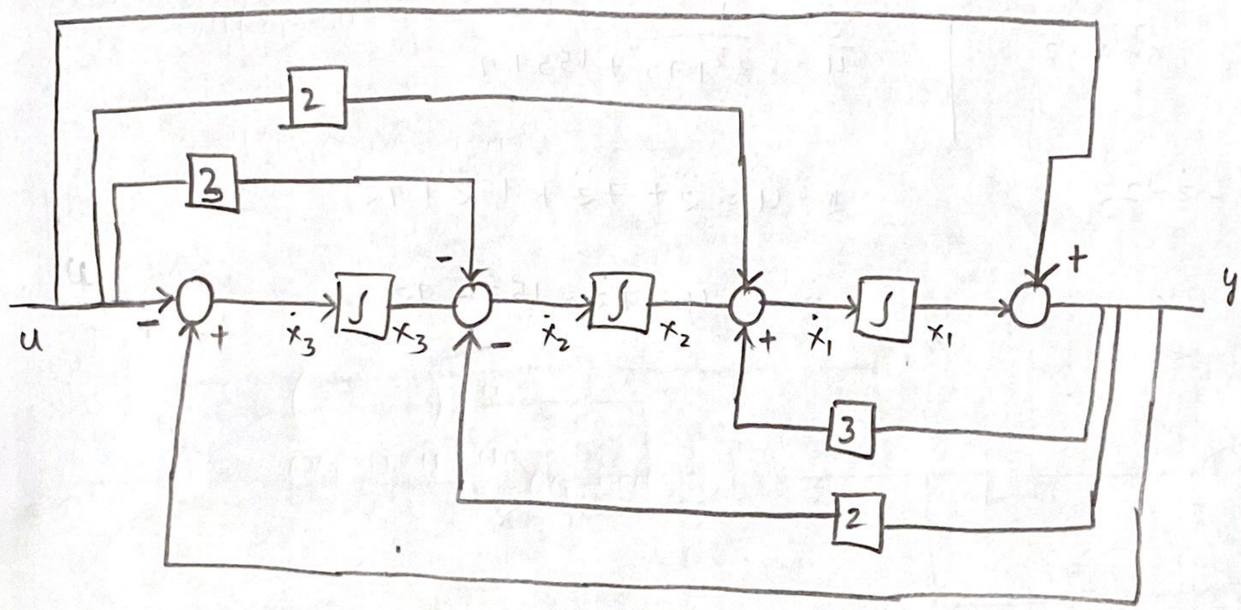
⑤ $\dddot{y} - 3\ddot{y} + 2\dot{y} - y = \dddot{u} + 2\ddot{u} - 3\dot{u} - u$

(observable CF)

Integrating 3 times both sides

$$\dddot{y} = \dddot{u} + 2\ddot{u} - 3\dot{u} - u + 3\ddot{y} - 2\dot{y} + y$$

$$y = u + 2\int u - 3\iint u - \iiint u + 3\int y - 2\iint y + \iiint y$$



$$\dot{x}_1 = x_2 + 2u + 3x_1$$

$$\dot{x}_2 = x_3 - 3u - 2x_1$$

$$\dot{x}_3 = -u + x_1$$

$$\dot{x} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ -2 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} u$$

$$Y = x_1 + u \qquad Y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + 1\,u$$

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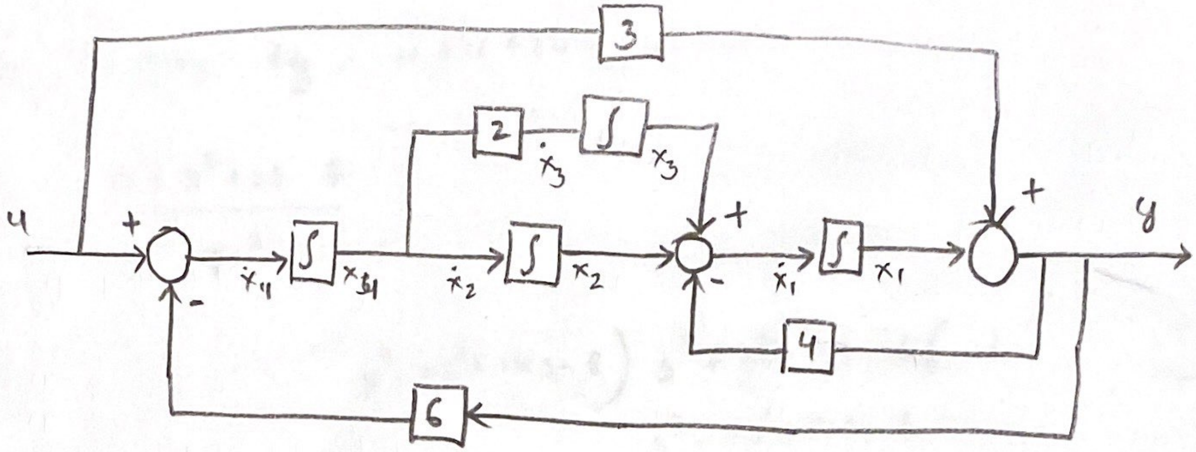


$-4(x_1+3u)$

$$\dot{x}_1 = \cancel{-4x_1} + x_2 + x_3$$

$$= -4x_1 + x_2 + x_3 - 12u$$

$$\dot{x}_2 = x_4$$

$$\dot{x}_3 = 2x_4$$

$$\dot{x}_4 = -6(x_1 + 3u) + u$$

$$= -6x_1 - 18u + u$$

$$= -6x_1 - 17u$$

$$y = x_1 + 3u$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -4 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ -6 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} -12 \\ 0 \\ 0 \\ -17 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + 3u$$

③ $\dddot{y} - 7\ddot{y} + 14\dot{y} - 8y = \dddot{u} + \ddot{u} + 2\dot{u} - 8u$

$$\frac{y}{u} = \frac{s^3 + s^2 + 2s - 8}{s^3 - 7s^2 + 14s - 8}$$

$$s^3 - 7s^2 + 14s - 8 \overline{) s^3 + s^2 + 2s - 8 } (1$$

$$\frac{s^3 - 7s^2 + 14s - 8}{8s^2 - 12s}$$

$$1 + \frac{8s^2 - 12s}{s^3 - 7s^2 + 14s - 8} \Rightarrow 1 + \frac{8s^2 - 12s}{(s-4)(s^2 - 3s + 2)} \Rightarrow 1 + \frac{8s^2 - 12s}{(s-4)(s-2)(s-1)}$$

$$\Rightarrow \frac{8s^2 - 12s}{(s-4)(s-2)(s-1)} = \frac{A}{(s-4)} + \frac{B}{(s-2)} + \frac{C}{(s-1)}$$

$$8s^2 - 12s = A(s-2)(s-1) + B(s-4)(s-1) + C(s-4)(s-2)$$

$$= s^2(A+B+C) + s(-3A - 5B - 6C) + 2A + 4B + 8C$$

$A + B + C = 8$ — ①

$-3A - 5B - 6C = -12$

$3A + 5B + 6C = 12$ — ③

$2A + 4B + 8C = 0$

$A + 2B + 4C = 0$ — ②

① - ② $\quad \cancel{A} + B + C - \cancel{A} - 2B - 4C = 8$

$-B - 3C = 8$

$\boxed{B + 3C = -8}$ — ④

18 $3\times 1 - 3 \Rightarrow \cancel{3A}+3B+3C-\cancel{3A}-5B-6C = 24-12$

$$-2B-3C=12$$

$$\boxed{2B+3C=-12} \quad — \ 5$$

$2\times 4 - 5 \Rightarrow \quad \cancel{2B}+6C-\cancel{2B}-3C = -16+12$

$$3C=-4$$

$$\boxed{C=-4/3}$$

$4 \Rightarrow \quad B+3C=-8$

$$B+\cancel{3}\left(-\frac{4}{\cancel{3}}\right)=-8$$

$$B=-8+4=-4$$

$$\boxed{B=-4}$$

$1 \Rightarrow \quad A+B+C=8$

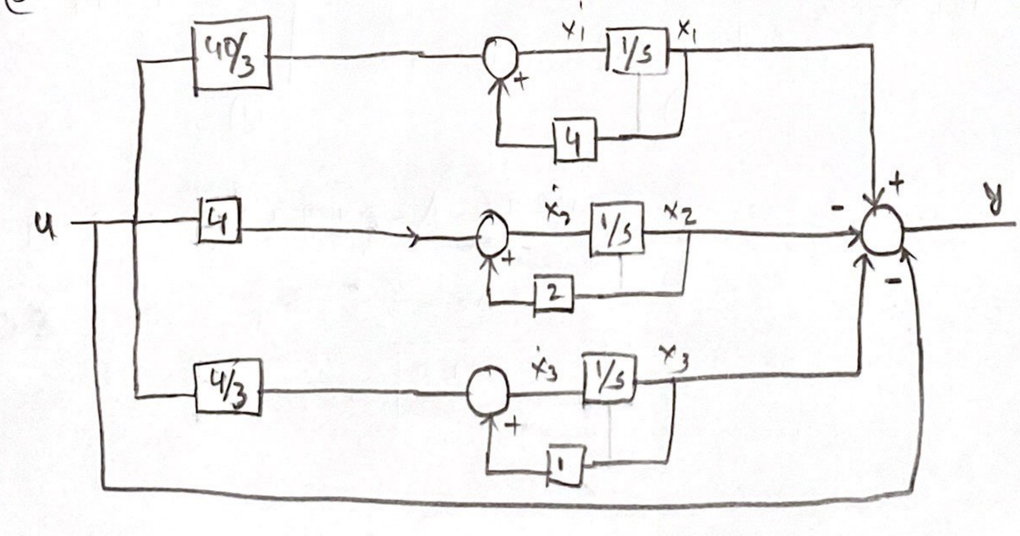
$$A=8-B-C$$

$$A=8+4+\frac{4}{3}$$

$$A=\frac{24+12+4}{3}=40/3$$

$$\boxed{A=40/3}$$

$$\frac{8s^2-12s}{(s-4)(s-2)(s-1)}=\frac{40/3}{(s-4)}+\frac{-4}{(s-2)}+\frac{-4/3}{(s-1)}$$



$$\dot{x}_1 = 4x_1 + \tfrac{40}{3}u$$

$$\dot{x}_2 = 2x_2 + 4u$$

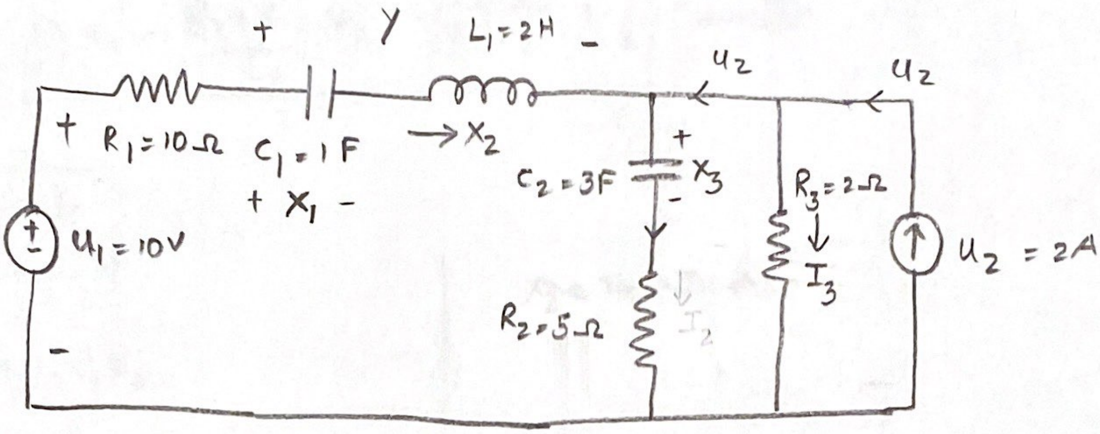
$$\dot{x}_3 = x_3 + \tfrac{4}{3}u$$

$$y = x_1 - x_2 - x_3 + u$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 40/3 \\ 4 \\ 4/3 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + (1)u$$

①



$u_1 = R_1 x_2 + x_1 + L_1 \dot{x}_2 + x_3 + R_2(x_2 + u_2)$ $\boxed{y = x_1 + L_1 \dot{x}_2}$

$L_1 \dot{x}_2 = u_1 - R_1 x_2 - x_1 - x_3 - R_2(x_2 + u_2)$

$$\boxed{\dot{x}_2 = \frac{u_1}{L_1} - \frac{x_1}{L_1} - \frac{R_1}{L_1} x_2 - \frac{x_3}{L_1} - \frac{R_2}{L_1}(x_2 + u_2)}$$

$x_1 = \frac{1}{C_1}\int x_2 \, dt \Rightarrow \boxed{\dot{x}_1 = \frac{x_2}{C_1}}$ $\cancel{\dot{x}_1} =$

$\dot{x}_3 = \cancel{\frac{1}{C_2}\int (x_2 + u_2)\, dt} = \cancel{\frac{x_2 + u_2}{C_2}}$

$$\boxed{\dot{x}_3 = \cancel{\frac{x_2}{C_2} + \frac{u_2}{C_2}}}$$

$\cancel{\dot{x}_2} = \cancel{\frac{10}{2}}$ $\cancel{x_2} + \cancel{u_2} = \cancel{I_2} + \cancel{I_3}$

$$\dot{x}_2 = -\frac{x_1}{L_1} - \frac{R_1}{L_1} x_2 - \frac{R_2 x_2}{L_1} - \frac{R_1}{L_1} x_2 - \frac{x_3}{L_1} + \frac{u_1}{L_1} - \frac{R_2 u_2}{L_1}$$

$$\boxed{\dot{x}_2 = \left(-\frac{1}{L_1}\right)x_1 - \frac{(R_1+R_2)}{L_1}x_2 - \left(\frac{1}{L_1}\right)x_3 + \frac{u_1}{L_1}} \quad \text{— (1)}$$

$$- \frac{R_2 u_2}{L_1}$$

$$\boxed{\dot{x}_1 = \frac{x_2}{C_1}}$$

$$Y = x_1 + L_1 \dot{x}_2$$

$$\boxed{\dot{x}_3 = \frac{x_2 + u_2}{C_2}}$$

$\cancel{x_2 + u_2 = I_3 = I}$.

$$\boxed{\dot{x}_1 = (1)x_2 \quad = x_2}$$

$$\dot{x}_2 = \left(-\frac{1}{2}\right)x_1 - \left(\frac{15}{2}\right)x_2 - \left(\frac{1}{2}\right)x_3 + \frac{1\cancel{0}}{\cancel{2}} - \frac{1\cancel{0}}{\cancel{2}}$$

$$\boxed{\dot{x}_2 = -\frac{1}{2}x_1 - \frac{15}{2}x_2 - \frac{1}{2}x_3}$$

$$\boxed{\dot{x}_3 = \frac{x_2}{3} + \frac{2}{3}}$$

$$Y = x_1 + 2\left(-\frac{1}{2}x_1 - \frac{15}{2}x_2 - \frac{1}{2}x_3\right) + \cancel{0}$$

$$Y = x_1 - \cancel{x_1} - 15x_2 - x_3 + \cancel{0}$$

$$\boxed{Y = -15x_2 - x_3} + \cancel{0}$$

$$Y = \begin{bmatrix} 0 & -15 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0\,u$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1/2 & -15/2 & -1/2 \\ 0 & 1/3 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2/3 \end{bmatrix}u$$