Linear Systems

HW - 1

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① $A = \begin{bmatrix} 2 & 3 & -1 \\ 6 & 1 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -5 \\ -3 & 0 \\ 1 & 2 \end{bmatrix}$ $AB = ?$

$$AB = \underset{2\times3}{\begin{bmatrix} 2 & 3 & -1 \\ 6 & 1 & -2 \end{bmatrix}} \underset{3\times2}{\begin{bmatrix} 4 & -5 \\ -3 & 0 \\ 1 & 2 \end{bmatrix}} = \begin{bmatrix} 8-9-1 & -10+0-2 \\ 24-3-2 & -30+0-4 \end{bmatrix} = \begin{bmatrix} -2 & -12 \\ 19 & -34 \end{bmatrix}_{2\times2}$$

② $A = \begin{bmatrix} 3 & 4 & 2 \\ 5 & 6 & 7 \\ 8 & 7 & 9 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 & 5 \\ 7 & 6 & 2 \\ 1 & 3 & 6 \end{bmatrix}$

$$A+B = \begin{bmatrix} 3 & 4 & 2 \\ 5 & 6 & 7 \\ 8 & 7 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 5 \\ 7 & 6 & 2 \\ 1 & 3 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 7 & 7 \\ 12 & 12 & 9 \\ 9 & 10 & 15 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 3 & 4 & 2 \\ 5 & 6 & 7 \\ 8 & 7 & 9 \end{bmatrix} - \begin{bmatrix} 1 & 3 & 5 \\ 7 & 6 & 2 \\ 1 & 3 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -3 \\ -2 & 0 & 5 \\ 7 & 4 & 3 \end{bmatrix}$$

③ $A = \begin{bmatrix} 3 & 4 & 2 \\ 5 & 6 & 7 \\ 8 & 7 & 9 \end{bmatrix}$ $|A| = ?$

$$|A| = 3\left[6(9) - 7(7)\right] - 4\left[45 - 56\right] + 2\left[35 - 48\right]$$

$$= 3\left[5\right] - 4\left[-11\right] + 2\left[-13\right] = 15 + 44 - 26$$

$$= 33$$

(4) $A = \begin{bmatrix} 3 & 3 \\ 6 & 9 \end{bmatrix}$ $A^{-1} = ?$ we know that $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$|A| = ad - bc = 27 - 18 = 9$

$A^{-1} = \frac{1}{9}\begin{bmatrix} 9 & -3 \\ -6 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1/3 \\ -2/3 & 1/3 \end{bmatrix}$

(5) $A = \begin{bmatrix} 3 & 4 & 2 \\ 5 & 6 & 7 \\ 8 & 7 & 9 \end{bmatrix}$ $A^{-1} = ?$ we know that $A^{-1} = \frac{Adj(A)}{|A|}$

$adj(A) = (\text{cofactor Matrix})^T$

$|A| = 3(54-49) - 4(45-56) + 2(35-48) = 3(5) - 4(-11) + 2(-13)$

$= 33$

Cofactor of $3 = (-1)^{1+1}(54-49) = 5$

Cofactor of $4 = (-1)^{1+2}(45-56) = 11$

Cofactor of $2 = (-1)^{1+3}(35-48) = -13$

Cofactor of $5 = (-1)^{2+1}(36-14) = -22$

Cofactor of $6 = (-1)^{2+2}(27-16) = 11$

Cofactor of $7 = (-1)^{2+3}$ ~~$(27-36)$~~ $= (21-32) = 11$

Cofactor of $8 = (-1)^{3+1}(28-12) = 16$

Cofactor of $7 = (-1)^{3+2}(21-10) = -11$

Cofactor of $9 = (-1)^{3+3}(18-20) = -2$

Cofactor matrix $= \begin{bmatrix} 5 & 11 & -13 \\ -22 & 11 & 11 \\ 16 & -11 & -2 \end{bmatrix}$

$Adj(A) = (\text{Co-f matrix})^T = \begin{bmatrix} 5 & -22 & 16 \\ 11 & 11 & -11 \\ -13 & 11 & -2 \end{bmatrix}$

$A^{-1} = \frac{1}{33}\begin{bmatrix} 5 & -22 & 16 \\ 11 & 11 & -11 \\ -13 & 11 & -2 \end{bmatrix}$

⑥ a) Rank of $\begin{bmatrix} 2 & 3 & 4 & 5 & 6 & -7 \\ 18 & 9 & 0 & 1 & 5 & 4 \\ 6 & 8 & 10 & 12 & 14 & -16 \end{bmatrix}_{3\times 6}$

Rank is 3, ∵ rows & columns are independent to each other

b) $\begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}_{3\times 3}$ = $R_2 = (R_2 - R_1)$, $R_3 = (R_3 - R_1)$

$$= \begin{bmatrix} 2 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

∴ Rank is 1