Linear Systems

Homework-2 Chakradhar Reddy Donuri

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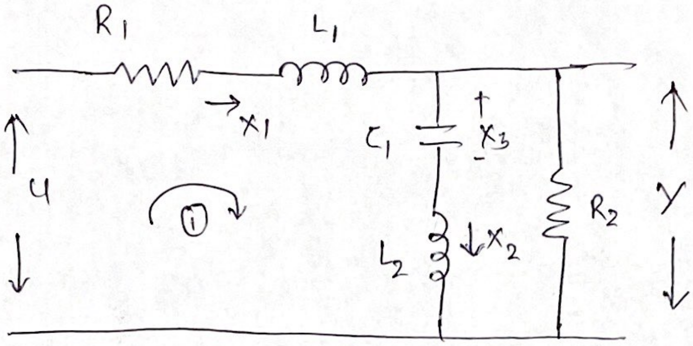
(1)



Apply KVL in (1) $\quad u = R_1 x_1 + L_1 \dot{x}_1 + x_3 + L_2 \dot{x}_2$

but $x_3 + L_2 \dot{x}_2 = R_2(x_1 - x_2)$

$\Rightarrow u = R_1 x_1 + L_1 \dot{x}_1 + R_2(x_1 - x_2)$

$L_1 \dot{x}_1 = u - R_1 x_1 - R_2 x_1 + R_2 x_2$

$$\boxed{\dot{x}_1 = -\frac{(R_1 + R_2)}{L_1} x_1 + \frac{R_2}{L_1} x_2 + \frac{u}{L_1}} \quad \text{— (1)}$$

$$\boxed{\dot{x}_3 = \frac{x_2}{C_1}} \quad \text{— (2)}$$

$x_3 + L_2 \dot{x}_2 = R_2(x_1 - x_2)$

$$\boxed{\dot{x}_2 = \frac{R_2}{L_2} x_1 - \frac{R_2}{L_2} x_2 - \frac{1}{L_2} x_3} \quad \text{— (3)}$$

$$\boxed{y = R_2 x_1 - R_2 x_2}$$

state Equation: $$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{(R_1+R_2)}{L_1} & R_2/L_1 & 0 \\ R_2/L_2 & -R_2/L_2 & -1/L_2 \\ 0 & 1/C_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1/L_1 \\ 0 \\ 0 \end{bmatrix} u$$

output Eq^n: $$y = \begin{bmatrix} R_2 & -R_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

and $x_2 = \frac{1}{C_1}\int x_1 - x_3 \, dt$

$$\boxed{\dot{x}_2 = \frac{1}{C_1}x_1 - \frac{1}{C_1}x_3}$$

$$\boxed{Y = R_2 x_3}$$

∴ State Equation matrix

$$\dot{x} = \begin{bmatrix} -R_1/L_1 & -2/L_1 & -R_2/L_1 \\ 1/C_1 & 0 & -1/C_1 \\ -R_1/L_1 & -\frac{2}{L_1} - \frac{1}{L_2} & -\frac{R_2}{L_1} - \frac{R_2}{L_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1/L_1 \\ 0 \\ 1/L_1 \end{bmatrix} u$$

output equation matrix

$$Y = \begin{bmatrix} 0 & 0 & R_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

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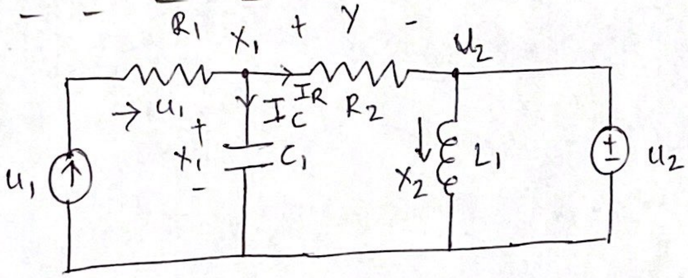
(2)



$\dot{x}_1 = \frac{I_C}{C_1}$ and $u_1 = I_C + I_R$

$u_1 = I_C + \frac{Y}{R_2}$ but $\boxed{Y = x_1 - u_2}$

$\therefore u_1 = C_1\dot{x}_1 + \frac{x_1 - u_2}{R_2}$

$C_1\dot{x}_1 = u_1 - \left[\frac{x_1 - u_2}{R_2}\right]$

$\dot{x}_1 = \frac{u_1}{C_1} - \frac{x_1}{C_1 R_2} + \frac{u_2}{C_1 R_2}$ —— (1)

$u_2 = L_1 \dot{x}_2$

$$\boxed{\dot{x}_2 = \frac{u_2}{L_1}} \quad — ②$$

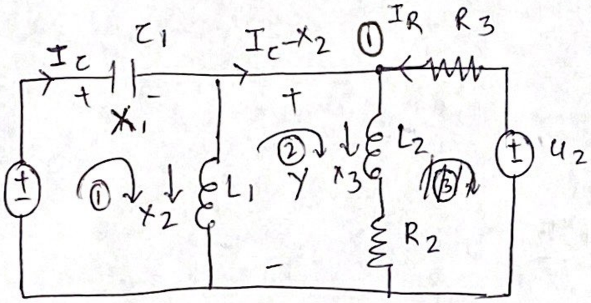
state equation matrix

$$\dot{x} = \begin{bmatrix} -1/C_1R_2 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1/C_1 & 1/C_1R_2 \\ 0 & 1/L_1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

output equation matrix

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

③



loop ① $\Rightarrow$ $u = x_1 + L_1 \dot{x}_2$

$L_1 \dot{x}_2 = u_1 - x_1$

$$\boxed{\dot{x}_2 = -\frac{x_1}{L_1} + \frac{u_1}{L_1}} \quad —①$$

$$\boxed{y = u_1 - x_1}$$

loop ② $L_2 \dot{x}_3 + R_2 x_3 - L_1 \dot{x}_2 = 0$

$L_2 \dot{x}_3 = L_1 \dot{x}_2 - R_2 x_3$

$L_2 \dot{x}_3 = L_1 \left[\frac{-x_1}{L_1} + \frac{u_1}{L_1} \right] - R_2 x_3$

$L_2 \dot{x}_3 = -x_1 + u - R_2 x_3$

$$\boxed{\dot{x}_3 = -\frac{x_1}{L_2} - \frac{R_2}{L_2} x_3 + \frac{u_1}{L_2}} \quad —②$$

~~loop~~ ③ $x_1 = \frac{1}{C_1} \int I_c \, dt$

$$\boxed{\dot{x}_1 = \frac{1}{C_1} I_c}$$

but I_c = at node ①

$I_c - x_2 + I_R = x_3$

we know that

$$Y = u_1 - x_1 \quad \text{and} \quad Y - u_2 = R_3 I_R$$

$$\longrightarrow \quad Y = R_3 I_R + u_2$$

$$u_1 - x_1 = R_3 I_R + u_2$$

$$R_3 I_R = u_1 - x_1 - u_2$$

$$\boxed{I_R = \frac{u_1 - x_1 - u_2}{R_3}}$$

Now $I_c - x_2 + I_R = x_3$ where $I_c = C_1 \dot{x}_1$

$$\therefore \quad C_1 \dot{x}_1 - x_2 + \frac{u_1 - x_1 - u_2}{R_3} = x_3$$

$$C_1 \dot{x}_1 = -\frac{x_1}{R_3} + x_2 + x_3 - \frac{u_1}{R_3} + \frac{u_2}{R_3}$$

$$\boxed{\dot{x}_1 = -\frac{x_1}{C_1 R_3} + \frac{1}{C_1} x_2 + \frac{1}{C_1} x_3 - \frac{u_1}{C_1 R_3} + \frac{u_2}{C_1 R_3}}$$

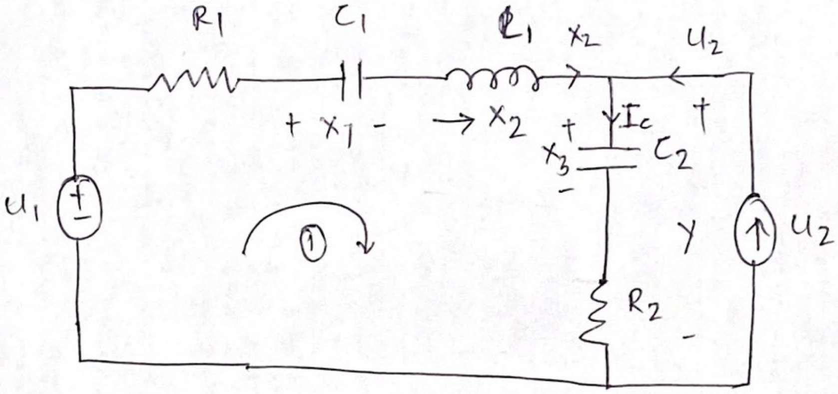
state equation :-

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1/C_1 R_3 & 1/C_1 & 1/C_1 \\ -1/L_1 & 0 & 0 \\ -1/L_2 & 0 & -R_2/L_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -1/C_1 R_3 & 1/C_1 R_3 \\ 1/L_1 & 0 \\ 1/L_2 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Output Equation :-

$$Y = \begin{bmatrix} -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

(4)



$I_c = x_2 + u_2$

KVL at loop ① $\Rightarrow$ $u_1 = R_1 x_2 + x_1 + L_1 \dot{x}_2 + x_3 + R_2(I_c)$

$$L_1 \dot{x}_2 = -R_1 x_2 - x_1 - x_3 - R_2(x_2 + u_2) + u_1$$

$$\dot{x}_2 = -\frac{x_1}{L_1} - \frac{(R_1 + R_2)}{L_1} x_2 - \frac{1}{L_1} x_3 + \frac{u_1}{L_1} - \frac{R_2 u_2}{L_1} \quad \text{— ①}$$

$$\dot{x}_1 = \frac{1}{C_1} x_2 \quad \text{— ②}$$

$$x_3 = \frac{1}{C_2} \int (x_2 + u_2)\, dt$$

$$\dot{x}_3 = \frac{1}{C_2} x_2 + \frac{u_2}{C_2} \quad \text{— ③}$$

$I_c = x_2 + u_2$

$y = x_3 + R_2 [I_c]$

$$\boxed{y = x_3 + R_2 x_2 + R_2 u_2}$$

state Equation matrix

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1/C_1 & 0 \\ -1/L_1 & -\frac{(R_1+R_2)}{L_1} & -1/L_1 \\ 0 & 1/C_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1/L_1 & -R_2/L_1 \\ 0 & 1/C_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

output Equation matrix

$$y = \begin{bmatrix} 0 & R_2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & R_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$