EE-792 **TEST-2** **FALL-17**

First Name: LAKSHAY

Last Name: ARORA

WSUID: J436R998

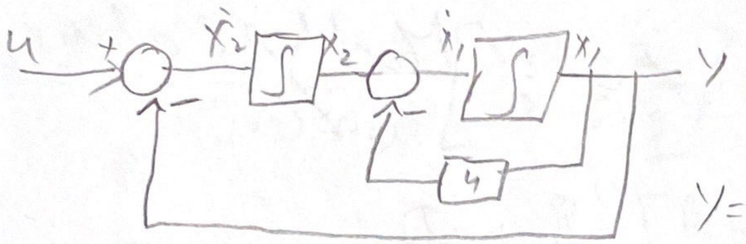
- Open text book, and closed notes. One sheet of notes (A4 size, both sides) will be allowed to the exam.
- Time for this Test is one hour thirty minutes.
- Calculators are allowed for this test (any kind)
- All work in this exam must be your own, sharing of calculators, formula sheet or text book will not be allowed.

$\frac{96}{100}$

(1) A continuous time invariant system is described by $\ddot{y} + 4\dot{y} + y = u$, find the controllability and observability of the model by deriving the model by observable canonical form, also write down the transfer function from the state and output equation of the model.

(25 points)

$$\ddot{y} = \iint u - 4\int y - \iint y$$



$y = x_1$ — O/p eq^n

$\cancel{\Phi} = \dot{x}_1 = -4y + x_2 = -4x_1 + x_2$

$\dot{x}_2 = -y + u = -x_1 + u$

$\frac{25}{25}$

$A = \begin{bmatrix} -4 & 1 \\ -1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $C = [1 \quad 0]$ $D = [0]$,

$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -4 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} s+4 & -1 \\ 1 & s \end{bmatrix}$

$P_c = [B \quad AB]$

$AB = \begin{bmatrix} -4 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$AB = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$P_c = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$|P_c| = -1 \neq 0$

$\therefore$ Model is controllable

Please show me your calculations for partial credit

$$P_o = \begin{bmatrix} C \\ CA \end{bmatrix} \qquad CA = [1 \; 0] \begin{bmatrix} -4 & 1 \\ -1 & 0 \end{bmatrix}$$

$$= [-4 \quad 1]$$

$$= \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix}$$

$|P_o| = -1 \neq 0 \therefore$ Model is observable

$$H(s) = C[sI - A]^{-1}B + D$$

$$[sI - A] = \begin{bmatrix} s+4 & -1 \\ 1 & s \end{bmatrix} \rightarrow [sI - A]^{-1} = \frac{1}{s^2+4s+1} \begin{bmatrix} s & 1 \\ -1 & s+4 \end{bmatrix}$$

$$H(s) = \frac{1}{s^2+4s+1} \underset{1\times2}{[1 \quad 0]} \underset{2\times2}{\begin{bmatrix} s & 1 \\ -1 & s+4 \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} 0 \\ 1 \end{bmatrix}} + 0$$

$$= \frac{1}{s^2+4s+1} \underset{1\times2}{[1 \quad 0]} \underset{2\times1}{\begin{bmatrix} 1 \\ s+4 \end{bmatrix}}$$

$$= \frac{1}{s^2+4s+1} [1]$$

$$H(s) = \frac{1}{s^2+4s+1} \qquad s = -2 \pm \sqrt{3}$$

$s^2+4s+1 \quad \boxed{b^2 - 4ac}$

$16 - 4 = 12$

$\frac{-4 \pm \sqrt{12}}{2}$

$\frac{-4 \pm 2\sqrt{3}}{2}$

$-2 \pm \sqrt{3}$

(2) Find the state response of the following model for zero initial conditions, whose input conditions are $u_t = 3$ $\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \end{bmatrix} u,$ (25 points+ 5 bonus points)

$A = \begin{bmatrix} -3 & 2 \\ 0 & -1 \end{bmatrix}$ $B = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$

$\phi(s) = [sI - A]^{-1}$

$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} s+3 & -2 \\ 0 & s+1 \end{bmatrix}$

$[sI - A]^{-1} = \frac{1}{(s+3)(s+1)} \begin{bmatrix} s+1 & 2 \\ 0 & s+3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+3} & \frac{2}{(s+3)(s+1)} \\ \frac{0}{(s+3)(s+1)} & \frac{1}{s+1} \end{bmatrix}$

$\frac{2}{(s+3)(s+1)} = \frac{A}{s+1} + \frac{B}{s+3} = As + 3A + Bs + B$

$A + B = 0 \Rightarrow A = -B$

$3A + B = 2$

$3A - A = 2$

$2A = 2$

$A = 1 \Rightarrow B = -1$

$= \frac{1}{s+1} - \frac{1}{s+3}$

$\phi(s) = \begin{bmatrix} \frac{1}{s+3} & \frac{1}{s+1} - \frac{1}{s+3} \\ 0 & \frac{1}{s+1} \end{bmatrix}$ $\frac{23}{25}$

$\phi(t) = \begin{bmatrix} e^{-3t} & e^{-t} - e^{-3t} \\ 0 & e^{-t} \end{bmatrix}$

Please show me your calculations for partial credit

Zero initial Condition:

$$X_{(t)} = \phi_t \cancel{X_0} \int_0^t \phi_{(-\tau)} B u \, d\tau$$

$$= \begin{bmatrix} e^{-3t} & e^{-t} - e^{-3t} \\ 0 & e^{-t} \end{bmatrix} \int_0^t \underbrace{\begin{bmatrix} e^{3\tau} & e^{\tau} - e^{3\tau} \\ 0 & e^{\tau} \end{bmatrix}}_{2\times2} \underbrace{\begin{bmatrix} -6 \\ -3 \end{bmatrix}}_{2\times1} d\tau$$

$$= \begin{bmatrix} e^{-3t} & e^{-t} - e^{-3t} \\ 0 & e^{-t} \end{bmatrix} \int_0^t \begin{bmatrix} -6e^{3\tau} - 3e^{\tau} + 3e^{3\tau} \\ -3e^{\tau} \end{bmatrix} d\tau$$

$$= \begin{bmatrix} e^{-3t} & e^{-t} - e^{-3t} \\ 0 & e^{-t} \end{bmatrix} \int_0^t \begin{bmatrix} -3e^{\tau} - 3e^{3\tau} \\ -3e^{\tau} \end{bmatrix} d\tau$$

$$= \begin{bmatrix} e^{-3t} & e^{-t} - e^{-3t} \\ 0 & e^{-t} \end{bmatrix} \begin{bmatrix} -3e^{\tau} - e^{3\tau} \\ -3e^{\tau} \end{bmatrix} \Bigg|_0^t$$

$$= \underbrace{\begin{bmatrix} e^{-3t} & e^{-t} - e^{-3t} \\ 0 & e^{-t} \end{bmatrix}}_{2\times2} \begin{bmatrix} -3e^{t} - e^{3t} - 4 \\ -3e^{t} - 3 \end{bmatrix}$$

$$= \begin{bmatrix} e^{-3t}(-3e^{t} - e^{3t} - 4) + (e^{-t} - e^{-3t})(-3e^{t} - 3) \\ e^{-t}(-3e^{t} - 3) \end{bmatrix}^{2\times1}$$

$$= \begin{bmatrix} \cancel{0} - 3e^{-2t} - 4e^{-3t} - 1 - 3e^{-t} + 3e^{-2t} + 3e^{-3t} - 3 \\ -\cancel{3} - 3 - 3e^{-t} \end{bmatrix} = \begin{bmatrix} -e^{-3t} - 3e^{-t} - 4 \\ -3 - 3e^{-t} \end{bmatrix}$$

(3) $\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\end{bmatrix} = \begin{bmatrix}-1 & 0\\ 2 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u,\ \ y = [1\ \ 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix} + 0u$, (a) find the model is stable or not? (b) Find the model is controlable or not? (c) Find the solution if the desired poles are at -2 and -3 (25 points)

$A = \begin{bmatrix}-1 & 0\\ 2 & -2\end{bmatrix}$ $B = \begin{bmatrix}0\\ 1\end{bmatrix}$ $C = [1\ \ 0]$

$SI - A = \begin{bmatrix}S & 0\\ 0 & S\end{bmatrix} - \begin{bmatrix}-1 & 0\\ 2 & -2\end{bmatrix} = \begin{bmatrix}S+1 & 0\\ -2 & S+2\end{bmatrix}$

$|SI - A| = (S+1)(S+2) - 0 \Rightarrow S = -1, -2$

$= S^2 + 3S + 2$ $\therefore$ Model is stable

$[SI - A]^{-1} = \frac{1}{(S+1)(S+2)}\begin{bmatrix}S+2 & 0\\ 2 & S+1\end{bmatrix}$ $\because$ s values lies on negative axis

25/25

b) $P_c = [B\ \ AB]$

$AB = \begin{bmatrix}-1 & 0\\ 2 & -2\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} = \begin{bmatrix}0\\ -2\end{bmatrix}$

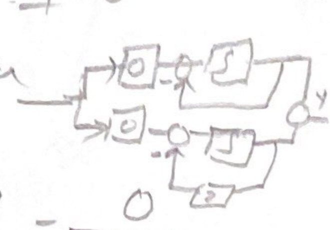
$P_c = \begin{bmatrix}0 & 0\\ 1 & -2\end{bmatrix}$ $|P_c| = 0$

$\therefore$ Model is Uncontrollable

$H(S) = C[SI - A]^{-1}B + D$

$= [1\ \ 0]\frac{1}{S^2+3S+2}\begin{bmatrix}S+2 & 0\\ 2 & S+1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix} + 0$

$= \frac{1}{S^2+3S+2}[1\ \ 0]\begin{bmatrix}0\\ S+1\end{bmatrix}$



Please show me your calculations for partial credit

$= \frac{1}{S^2+3S+2}[0] = \frac{0}{(S+1)(S+2)}$

c) $A_c = A - Bg$

$$= \begin{bmatrix} -1 & 0 \\ 2 & -2 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} [g_1 \ g_2]$$

$$= \begin{bmatrix} -1 & 0 \\ 2 & -2 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ g_1 & g_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 2-g_1 & -2-g_2 \end{bmatrix}$$

$$SI - A_c = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 2-g_1 & -2-g_2 \end{bmatrix}$$

$$= \begin{bmatrix} s+1 & 0 \\ -2+g_1 & s+2+g_2 \end{bmatrix}$$

$$|SI - A_c| = (s+1)(s+2+g_2) - 0$$

$$= s^2 + 2s + sg_2 + s + 2 + g_2$$

$$= s^2 + s(3+g_2) + 2 + g_2$$

Desired Poles -2 & -3

$$(s+2)(s+3)$$

$$s^2 + 3s + 2s + 6 = s^2 + 5s + 6$$

On comparing

$$3 + g_2 = 5 \Rightarrow g_2 = 2$$

$$2 + g_2 = 6$$

$$\Rightarrow g_2 = 4$$

Different value of g_2 exist

$\therefore$ No solution

(4) $\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\end{bmatrix} = \begin{bmatrix}1 & 1\\ 0 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1\\ 0\end{bmatrix}u,\ \ y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + 0u$, (a) find the model is stable or not? (b) Find the model is controlable or not? (c) Choose any two poles such that the model will have a solution and also draw the simulation diagram (25 points)

$A = \begin{bmatrix}1 & 1\\ 0 & -2\end{bmatrix},\ B = \begin{bmatrix}1\\ 0\end{bmatrix}\ C = \begin{bmatrix}1 & 0\end{bmatrix}$

$SI - A = \begin{bmatrix}S & 0\\ 0 & S\end{bmatrix} - \begin{bmatrix}1 & 1\\ 0 & -2\end{bmatrix} = \begin{bmatrix}S-1 & -1\\ 0 & S+2\end{bmatrix}$

$|SI - A| = (S-1)(S+2) = S^2 + 2S - S - 2 = S^2 + S - 2$

$S = 1, -2$

$\therefore$ Model is not stable, since one of the poles lies on the positive axis

b) $P_c = [B \ AB]$ $AB = \begin{bmatrix}1 & 1\\ 0 & -2\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}$

$= \begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$ $\begin{bmatrix}1\\ 0\end{bmatrix}$

23/25

$|P_c| = 0 \ \therefore$ Model is Uncontrollable

If uncontrollable & Unstable poles are there then Model can't be stabilized

$H(S) = C[SI - A]^{-1}B + D$

$= \begin{bmatrix}1 & 0\end{bmatrix}\dfrac{1}{(S-1)(S+2)}\begin{bmatrix}S-1 & -1\\ 0 & S+2\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}$

Please show me your calculations for partial credit

$= \begin{bmatrix}1 & 0\end{bmatrix}_{1\times2}\dfrac{1}{(S-1)(S+2)}\begin{bmatrix}S-1\\ 0\end{bmatrix}_{2\times1}$

$$H(s) = \frac{1}{(s-1)(s+2)} [s-1] = \frac{1}{s+2}$$

$\cancel{H(s)=}$ $\quad$ Pole $= s = -2$

$$A_c = A - Bg = \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} [g_1 \; g_2]$$

$$= \begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix} - \begin{bmatrix} g_1 & g_2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1-g_1 & 1-g_2 \\ 0 & -2 \end{bmatrix}$$

$$sI - A_c = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1-g_1 & 1-g_2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} s+1+g_1 & -1+g_2 \\ 0 & s+2 \end{bmatrix}$$

$$|sI - A_c| = (s+1+g_1)(s+2)$$

$$= s^2 + s + sg_1 + 2s + 2 + 2g_1 \Rightarrow s^2 + s(3+g_1) + 2 + 2g_1$$

c)

$$\frac{A}{s-1} + \frac{B}{s+2} = As + 2A + Bs - B$$

$$A + B = 1$$
$$2A - B = -1$$
$$2A - 1 - A = -1$$
$$A = 0$$
$$B = 1 - A = 1$$

$$= \frac{0}{s+1} + \frac{1}{s+2}$$

$\cancel{3+g_1 = 1}$

$\cancel{g_1 = -2}$

$\cancel{2+2g_1 = -2}$

$\cancel{g_1 = 0}$

$-1 \quad -2$

$\cancel{3+g_1 = -1}$

Choosing Poles $(s+1)(s+2)$

$s^2 + 3s + 2$

$3 + g_1 = 3$

$g_1 = 0$

$2 + 2g_1 = 2$

$g_1 = 0$