EE-792 TEST-2 Summer-17

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- Open text book, and closed notes. One sheet of notes (A4 size, both sides) will be allowed to the exam.
- Time for this Test is one hour thirty minutes.
- Calculators are allowed for this test (any kind)
- All work in this exam must be your own, sharing of calculators, formula sheet or text book will not be allowed.

$\frac{96}{100}$ $-5+8$

(1) A model has following ABC parameters. $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $C = [\cancel{2 \quad 1}]$ $[1 \quad 2]$ Transfer the given model into observable canonical form.

(25 points)

$|sI-A| = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix} = 0 \therefore s^2+3s+2=0$

$(s+2)(s+1)$

NOW, $\hat{C} = [1 \quad 0]$ $\hat{A} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}$

$P_o = \begin{bmatrix} C \\ CA \end{bmatrix}$

$CA = [1 \quad 2]_{1\times2} \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}_{2\times2}$

$= [-4 \quad -5]$

$= \begin{bmatrix} 1 & 2 \\ -4 & -5 \end{bmatrix}$

$P_o^{-1} = \frac{1}{3} \begin{bmatrix} -5 & -2 \\ 4 & 1 \end{bmatrix}$

$\hat{C}\hat{A} = [1 \quad 0]_{1\times2} \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}_{2\times2}$

$= [-3 \quad 1]$

$\hat{P}_o = \begin{bmatrix} \hat{C} \\ \hat{C}\hat{A} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

$\hat{B} = T^{-1} B$

$T = P_o^{-1} \hat{P}_o$

$= \frac{1}{3} \begin{bmatrix} -5 & -2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}$

$\frac{23}{25}$

$= \frac{1}{9} \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}_{2\times2} \begin{bmatrix} 0 \\ 1 \end{bmatrix}_{2\times1}$

$= \frac{1}{3} \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$

$= \frac{1}{9} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$T^{-1} = \frac{1}{9} \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2/\cancel{9} \quad 1 \\ 1/\cancel{9} \quad 1 \end{bmatrix}$

Please show me your calculations for partial credit

$$\hat{\dot{x}} = \hat{A}x + \hat{B}u$$

$$= \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.2222 \\ 0.1111 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\hat{Y} = \hat{C}x + \hat{D}u \qquad \hat{D} = 0$$

$$\hat{Y} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(2) A model has $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ Find the feedback gain such that the closed loop poles are at $-2 \pm j3$ *and* -1 (25 points)

$\dot{x} = Ax + B\gamma$

$\dot{x} = [A - BF]x + B\gamma$

$BF = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}_{3\times1} \begin{bmatrix} F_1 & F_2 & F_3 \end{bmatrix}_{1\times3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ F_1 & F_2 & F_3 \end{bmatrix}$

$[A - BF] = A_c = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ -F_1 & -F_2 & -3-F_3 \end{bmatrix}$

$\det |sI - A_c| = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ -F_1 & -F_2 & -3-F_3 \end{bmatrix}$

$= \begin{bmatrix} s+1 & 0 & 0 \\ 0 & s+2 & 0 \\ F_1 & F_2 & s+3+F_3 \end{bmatrix}$

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$|sI - A_c| = s+1\,(s^2 + 3s + sF_3 + 2s + 6 + 2F_3)$

$= (s+1)\,(s^2 + 5s + (s+2)F_3 + 6)$

$= s^3 + 5s^2 + s^2F_3 + 2sF_3 + 6s + s^2 + 5s + F_3 s + 2F_3 + 6$

$= s^3 + (6 + F_3)s^2 + s\,[3F_3 + 11] + 2F_3 + 6$

charac polynomial

given → $= (s+1)(s+2+j3)(s+2-j3)$

$= (s+1)\quad s^2 + 2s - \cancel{j3s} + 2s + 4 - \cancel{6j} + \cancel{s3j} + \cancel{6j} - j^2 9$

$= (s+1)\quad (s^2 + 4s + 13)$

Please show me your calculations for partial credit

$= s^3 + 4s^2 + 13s + s^2 + 4s + 13 = \boxed{s^3 + 5s^2 + 17s + 13}$

comparing it with charac polynomial

$6 + F_3 = 5$ $\quad$ $3F_3 + 11 = 17$ $\quad$ $2F_3 + 6 = 13$

$F_3 = -1$ $\quad$ $F_3 = 2$ $\quad$ $F_3 = 7/2$

Feedback gain $[F_1 \ F_2 \ F_3] = [-1 \ 2 \ 7/2]$

No Solution.

Rough work

→ $A(s+1) + B(s+2) = s$

$s=-1$ $\quad B = -1$

$s=-2$ $\quad -A = -2$

$A = 2$

→ $A(s+1) + B(s+2) = s+3$

$s=-1$ $\quad B = 2$

$s=-2$ $\quad -A = 1$

→ $A(s+1) + B(s+2) = 1$

$s=-1$ $\quad B = 1$

$s=-2$ $\quad -A = 1$

→ $A(\overset{-2+1}{s+1}) + B(s+2) = -2$

$s=-1$ $\quad B = -2$

$s=-2$ $\quad -A = -2$

$$\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}$$

$(s+2)(s+1)$

(3) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u,\ \ y = [0 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 3u$, Find the total response of the given model when $X_{1(0)} = 1$, $X_{2(0)} = -1$, & u=5 (25 points)

$\phi(s) = \mathcal{L}^{-1}[sI - A]^{-1}$ $\quad A = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$C = [0 \quad 1]$ $\quad D = 3$

$$[sI - A] = \begin{bmatrix} s+3 & -1 \\ 2 & s \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{s^2+3s+2} \begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s}{(s+2)(s+1)} & \frac{1}{(s+2)(s+1)} \\ \frac{-2}{(s+2)(s+1)} & \frac{s+3}{(s+2)(s+1)} \end{bmatrix}$$

$\frac{24}{25}$

$$\phi(s) = \begin{bmatrix} \frac{2}{s+2} + \frac{-1}{s+1} & \frac{-1}{s+2} + \frac{1}{s+1} \\ \frac{2}{s+2} + \frac{-2}{s+1} & \frac{-1}{s+2} + \frac{2}{s+1} \end{bmatrix}$$

$$\phi(t) = \begin{bmatrix} 2e^{-2t} - e^{-t} & -e^{-2t} + e^{-t} \\ 2e^{-2t} - 2e^{-t} & -e^{-2t} + \cancel{/}2e^{-t} \end{bmatrix}$$

Please show me your calculations for partial credit

$\Rightarrow$ zero i/P cond^{n}

$$X(t) = \phi(t)\, X_0$$

$$= \begin{bmatrix} 2e^{-2t} - e^{-t} & -e^{-2t} + e^{-t} \\ 2e^{-2t} - 2e^{-t} & -e^{-2t} + 2e^{-t} \end{bmatrix}_{2\times2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}_{2\times1}$$

$$= \begin{bmatrix} 2e^{-2t} - e^{-t} + e^{-2t} - e^{-t} \\ 2e^{-2t} - 2e^{-t} + e^{-2t} - 2e^{-t} \end{bmatrix}$$

$$= \begin{bmatrix} 3e^{-2t} - 2e^{-t} \\ 3e^{-2t} - 4e^{-t} \end{bmatrix}$$

$\Rightarrow$ for zero initial cond^{n}

$$X(t) = \phi(t) \int_0^t (\phi_{(-\tau)})\, B \cdot u\, d\tau$$

$$= \begin{bmatrix} 2e^{-2t} - e^{-t} & -e^{-2t} + e^{-t} \\ 2e^{-2t} - 2e^{-t} & -e^{-2t} + 2e^{-t} \end{bmatrix} \int_0^t \begin{bmatrix} 2e^{2\tau} - e^{\tau} & -e^{2\tau} + e^{\tau} \\ 2e^{2\tau} - 2e^{\tau} & -e^{2\tau} + 2e^{\tau} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} 5 \cdot d\tau$$

$$= \begin{bmatrix} 2e^{-2t} - e^{-t} & -e^{-2t} + e^{-t} \\ 2e^{-2t} - 2e^{-t} & -e^{-2t} + 2e^{-t} \end{bmatrix} \int_0^t \begin{bmatrix} 2e^{2\tau} - e^{\tau} \\ 2e^{2\tau} - 2e^{\tau} \end{bmatrix} 5\, d\tau$$

$$= \begin{bmatrix} -2e^{-2t} - e^{-t} & -e^{-2t} + e^{-t} \\ 2e^{-2t} - 2e^{-t} & -e^{-2t} + 2e^{-t} \end{bmatrix} \begin{bmatrix} \frac{2}{2}(e^{2\tau} - 1) - (e^{\tau} - 1) \\ \frac{2}{2}(e^{2\tau} - 1) - 2(e^{\tau} - 1) \end{bmatrix}_0^t \cdot 5$$

$$= \begin{bmatrix} 2e^{-2t} - e^{-t} & -e^{-2t} + e^{-t} \\ 2e^{-2t} - 2e^{-t} & -e^{-2t} + 2e^{-t} \end{bmatrix}_{2\times2} \begin{bmatrix} e^{2t} - e^{t} \\ e^{2t} - 2e^{t} \end{bmatrix}_{2\times1} \cdot 5 =$$

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Continuing Q-3

$$= \begin{bmatrix} 2e^{-2t} - e^{-t} & -e^{-2t} + e^{-t} \\ 2e^{-2t} - 2e^{-t} & -e^{-2t} + 2e^{-t} \end{bmatrix}_{2\times 2} \begin{bmatrix} e^{2t} & -e^{t} \\ e^{2t} & -2e^{t} \end{bmatrix}_{2\times 1} \cdot 5$$

$$= \begin{bmatrix} 2e^{0} - 2e^{-2t}e^{t} - e^{-t}\cdot e^{2t} + e^{0} & -e^{0} + 2e^{-2t}e^{t} + e^{-t}e^{2t} - 2e^{0} \\ 2e^{0} - 2e^{-2t}e^{t} - 2e^{-t}e^{2t} + 2e^{0} & -e^{0} + 2e^{t}\cdot e^{-2t} + 2e^{-t}e^{2t} - 4e^{0} \end{bmatrix}$$

$$= \begin{bmatrix} 3 - 2e^{-t} - e^{t} & -3 + 2e^{-t} + e^{t} \\ 4 - 2e^{-t} - 2e^{t} & -5 + 2e^{-t} + 2e^{t} \end{bmatrix} \cdot 5$$

$$= \begin{bmatrix} 15 - 10e^{-t} - 5e^{t} & -15 + 10e^{-t} + e^{t} \\ 20 - 10e^{-t} - 10e^{t} & -25 + 10e^{-t} + 10e^{t} \end{bmatrix}$$

Total Response = Zero i/p cond^n + Zero initial cond^n

$$= \begin{bmatrix} 3e^{-2t} & -2e^{-t} \\ 3e^{-2t} & -4e^{-t} \end{bmatrix} + \begin{bmatrix} 15 - 10e^{-t} - 5e^{t} & -15 + 10e^{-t} + e^{t} \\ 20 - 10e^{-t} - 10e^{t} & -25 + 10e^{-t} + 10e^{t} \end{bmatrix}$$

$$= \begin{bmatrix} -15 + 10e^{-t} + 5e^{t} + 3e^{-2t} & +15 - 12e^{-t} - e^{t} \\ -20 + 10e^{-t} + 10e^{t} + 3e^{-2t} & 25 - 14e^{-t} - 10e^{t} \end{bmatrix}$$

$(s+2)\ (s+1)$

(4) $\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}-1\\ 1\end{bmatrix}u,\ \ y = [1\ \ 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix} + 0u$, (a) find the system is stable or not? (b) Find the model is controlable or not? (c) if the model is uncontrollable explain why the model is uncontrollable and show me the calculations.

Find the solution if the desired poles are at -2 and -3 **(25 points)**

Find the solution if the desired poles are at -1 and -3 also write your obervation for the above example (5 bonus points)

→ a) $A = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}$ $B = \begin{bmatrix}-1\\ 1\end{bmatrix}$ $C = [1\ \ 0]$ $D = 0$

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$|sI - A| = \begin{bmatrix}s & -1\\ 2 & s+3\end{bmatrix} = 0 \quad \therefore s^2 + 3s + 2 = 0$

$\therefore s = -2 \quad s = -1$

Both poles lies LHS of Y-axis

System is stable

b) $P_c = [B\ \ AB]$ $AB = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}-1\\ 1\end{bmatrix}$

$= \begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$ $= \begin{bmatrix}1\\ -1\end{bmatrix}$

$|P_c| = 1 - 1 = 0$ $|P_c| = 0 \Rightarrow$ model is uncontrollable

This is uncontrollable so the transfer f^n of system has a pole zero cancellation

Yes we can find sol^n

→ $[sI - A]^{-1} = \dfrac{1}{s^2+3s+2}\begin{bmatrix}s+3 & 1\\ -2 & s\end{bmatrix}$

$C[sI-A]^{-1} = \dfrac{1}{(s+2)(s+1)}\begin{bmatrix}s+3 & 1\end{bmatrix}$

Please show me your calculations for partial credit

$C[sI-A]^{-1}B = \dfrac{1}{(s+2)(s+1)}(-s-3+1) = \dfrac{-(s)(s+2)}{(s+2)(s+1)}$

$(S+2)$ is pole zero cancellation

Now, $A_C = A - BF$

$$= \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} -F_1 & -f_2 \\ F_1 & f_2 \end{bmatrix}$$

$$= \begin{bmatrix} F_1 & 1+f_2 \\ -2-F_1 & -3-f_2 \end{bmatrix}$$

$$|SI - A_C| = \begin{bmatrix} S-F_1 & -1-f_2 \\ 2+F_1 & S+3+f_2 \end{bmatrix}$$

$$= (S^2+3S+Sf_2-F_1S-F_1 3-F_1f_2) - (2-2F_2-F_1-F_1F_2)$$

$$= S^2 + S(3+f_2-F_1) - F_1 3 - 2 + 2F_2 + F_1$$

$$= S^2 + S(3+f_2-F_1) + (2f_2-2-2F_1)$$

#
Desired poles $-2, -3$ $(S+2)(S+3) \Rightarrow S^2+5S+6$

$\therefore 3+f_2-f_1 = 5$ $2f_2-2-2f_1 = 6$ $\therefore \infty$ # of solution

$f_2 - F_1 = 2$ $f_2 - F_1 = 4$

So No solution

Desired poles -1 & -3 $(S+1)(S+3) \Rightarrow S^2+4S+3$

$\therefore 3+f_2-f_1 = 4$ $2F_2-2-2f_1 = 3$

$f_2 - f_1 = 1$ $2f_2-2f_1 = 5$

$\therefore F_2 = f_1$

No Solution