$\frac{89}{100}$ V=IR

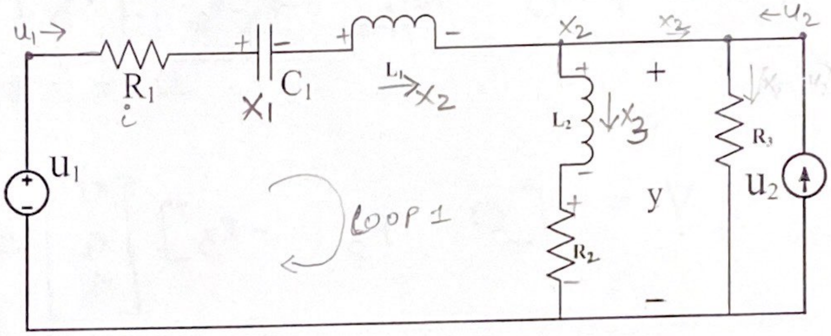
IA →

$x_1 + x_2$

$x_2 = x_3 +$

(1) Write the state equation and output equations for the following circuit

(25 points)



$u_1 = R_1 i + x_1 + L_1 \dot{x}_2 + L_2 \dot{x}_3 + R_2 x_3$

$x_1 = \frac{1}{C_1} \int i\,dt$

$u_1 = R_1 x_2 + x_1 + L_1 \dot{x}_2 + L_2 \dot{x}_3 + R_2 x_3$

$\dot{x}_1 = \frac{x_2}{C_1}$ ✓ —— ①

$u_2 + y = L_2 \dot{x}_3 + R_2 x_3$ ③

$\rightarrow \quad L_2 \dot{x}_3 + R_2 (x_3) = R_3 (x_2 - x_3 + u_2) = u_2$ ②

$y = R_3 (x_2 - x_3 + u_2)$

whole loop $\quad R_1 x_2 + x_1 + L_1 \dot{x}_2 + R_3 (x_2 - x_3) = u_1$ —— ④

$\dot{x}_2 = \frac{u_1}{L_1} - \frac{R_1}{L_1} x_2 - \frac{x_1}{L_1} - \frac{R_3}{L_1} (x_2 - x_3)$

from ②

$\dot{x}_3 = \frac{R_3 (x_2 - x_3)}{L_2} - \frac{R_2 x_3}{L_2} = \frac{x_2 R_3}{L_2} - \frac{x_3 R_3}{L_2} - \frac{R_2 x_3}{L_2}$

$\frac{18}{25}$

$\dot{x}_2 = \frac{u_1}{L_1} - x_1 \left(\frac{1}{L_1} \right) + R_3 \frac{x_3}{L_1} + \frac{x_2}{L} (-R_1 - R_3)$

$y = R_3 (x_2 - x_3) = R_3 x_2 - R_3 x_3$

Please show me your calculations for partial credit

(2) $\dddot{y} + 7\ddot{y} + 15\dot{y} + 9y = \ddot{u} - \dot{u} - 2u$, find the state and output equation by using the controlable canonical form. (20 points)

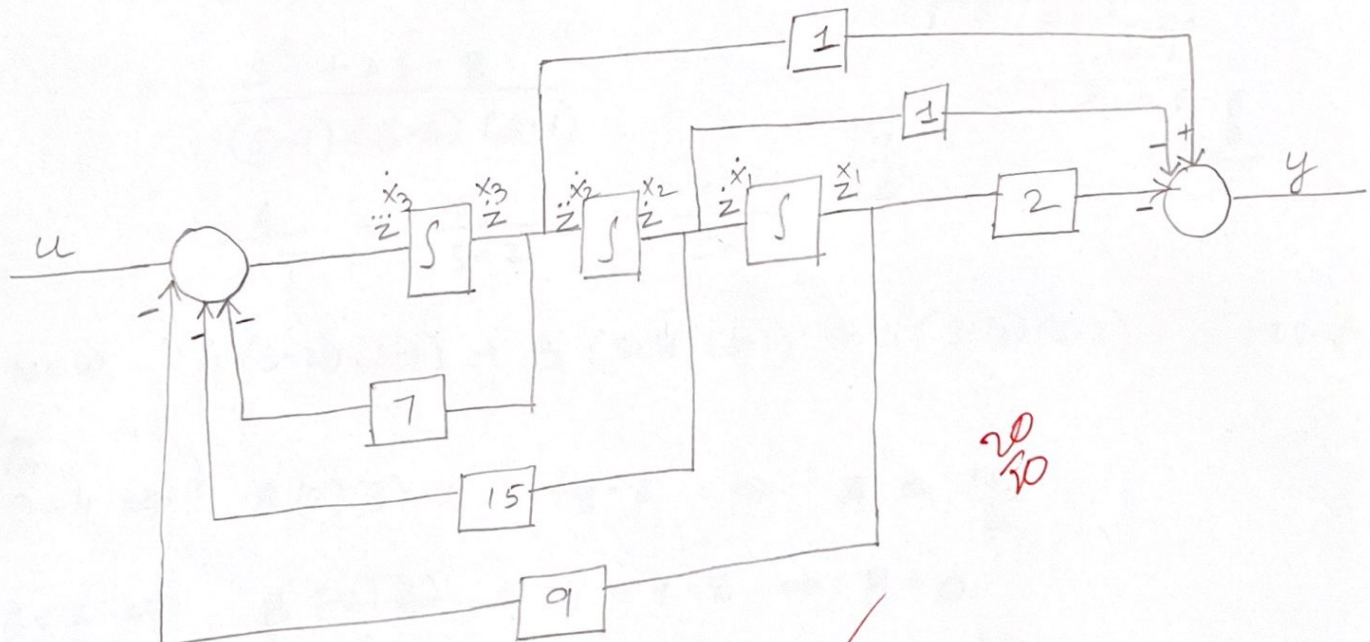
$$\dddot{z} + 7\ddot{z} + 15\dot{z} + 9z = u$$

&

$$y = \ddot{z} - \dot{z} - 2z$$

$$\rightarrow \dddot{z} = u - 7\ddot{z} - 15\dot{z} - 9z$$

$$z = \iiint u - 7\int z - 15\iint z - 9\iiint z$$



20/20

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = u - 7x_3 - 15x_2 - 9x_1$$

$$y = -2x_1 - \dot{x}_1 + \dot{x}_2$$

$$= -2x_1 + (-x_2) + x_3$$

Please show me your calculations for partial credit

(3) $\dddot{y} - 7\ddot{y} + 14\dot{y} - 8y = \ddot{u} + 2\dot{u} - 8u$, find the state and output equation by using the Jordan form. This methamatical modal has pole at '~~+~~4' (20 points)

$$H(s) = \frac{y}{u} = \frac{\ddot{u} + 2\dot{u} - 8u}{\dddot{y} - 7\ddot{y} + 14\dot{y} - 8y}$$

$$= \frac{s^2 + 2s - 8}{s^3 - 7s^2 + 14s - 8}$$

$$\begin{array}{r|l} & s^2 - 3s + 2 \\ \hline s-4 & s^3 - 7s^2 + 14s - 8 \\ & s^3 - 4s^2 \\ \hline & 0 \quad -3s^2 + 14s \\ & -3s^2 + 12s \\ \hline & 0 \quad 2s - 8 \\ & 2s - 8 \\ \hline & 0 \end{array}$$

$$H(s) = \frac{s^2 + 2s - 8}{(s-4)(s^2 - 3s + 2)}$$

$$= \frac{s^2 + 2s - 8}{(s-4)(s-2)(s-1)}$$

$$= \frac{A}{s-4} + \frac{B}{s-2} + \frac{C}{s-1}$$

$\frac{17}{20}$

Now, $A(s-2)(s-1) + B(s-4)(s-1) + C(s-4)(s-2) = s^2 + 2s - 8$

for

$s = 4 \Rightarrow \quad A(2)(3) = 16 + 8 - 8 \quad \Rightarrow \quad A = \frac{16}{6}$

$s = 2 \Rightarrow \quad B(-2)(1) = 4 + 4 - 8 \quad \Rightarrow \quad B = 0$

$s = 1 \Rightarrow \quad C(-3)(-1) = 1 + 2 - 8 \quad \Rightarrow \quad C = -5/3$

$$H(s) = \frac{\frac{16}{6}}{s-4} + \frac{-5/3}{s-1}$$

Please show me your calculations for partial credit

(4) $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \overset{A}{\begin{bmatrix}3 & 1\\ 0 & 2\end{bmatrix}}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \overset{B}{\begin{bmatrix}2\\ 1\end{bmatrix}}u,\quad y = \overset{C}{[-1 \quad 1]}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$, $D=0$, write the transfer function for the above state equation and output equation (20 points)

$$\frac{Y}{U} = C\,[SI-A]^{-1}\,B + D$$

$$SI-A = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} S-3 & -1 \\ 0 & S-2 \end{bmatrix}$$

$\frac{18}{20}$

$$[SI-A]^{-1} = \frac{1}{(S-3)(S-2)+0}\begin{bmatrix} S-2 & 1 \\ 0 & S-3 \end{bmatrix}$$

$$C\,[SI-A]^{-1} = \frac{1}{S^2-5S+6}\,[-1 \quad 1]_{1\times 2}\begin{bmatrix} S-2 & 1 \\ 0 & S-3 \end{bmatrix}_{2\times 2}$$

$$= \frac{1}{S^2-5S+6}\,[-S+2+0 \qquad -1+S-3]$$

$$= \frac{1}{S^2-5S+6}\,[S-2 \qquad S-4]$$

$$C\,[SI-A]^{-1}B = \frac{1}{S^2-5S+6}\,[S-2 \qquad S-4]_{1\times 2}\begin{bmatrix} 2 \\ 1 \end{bmatrix}_{2\times 1}$$

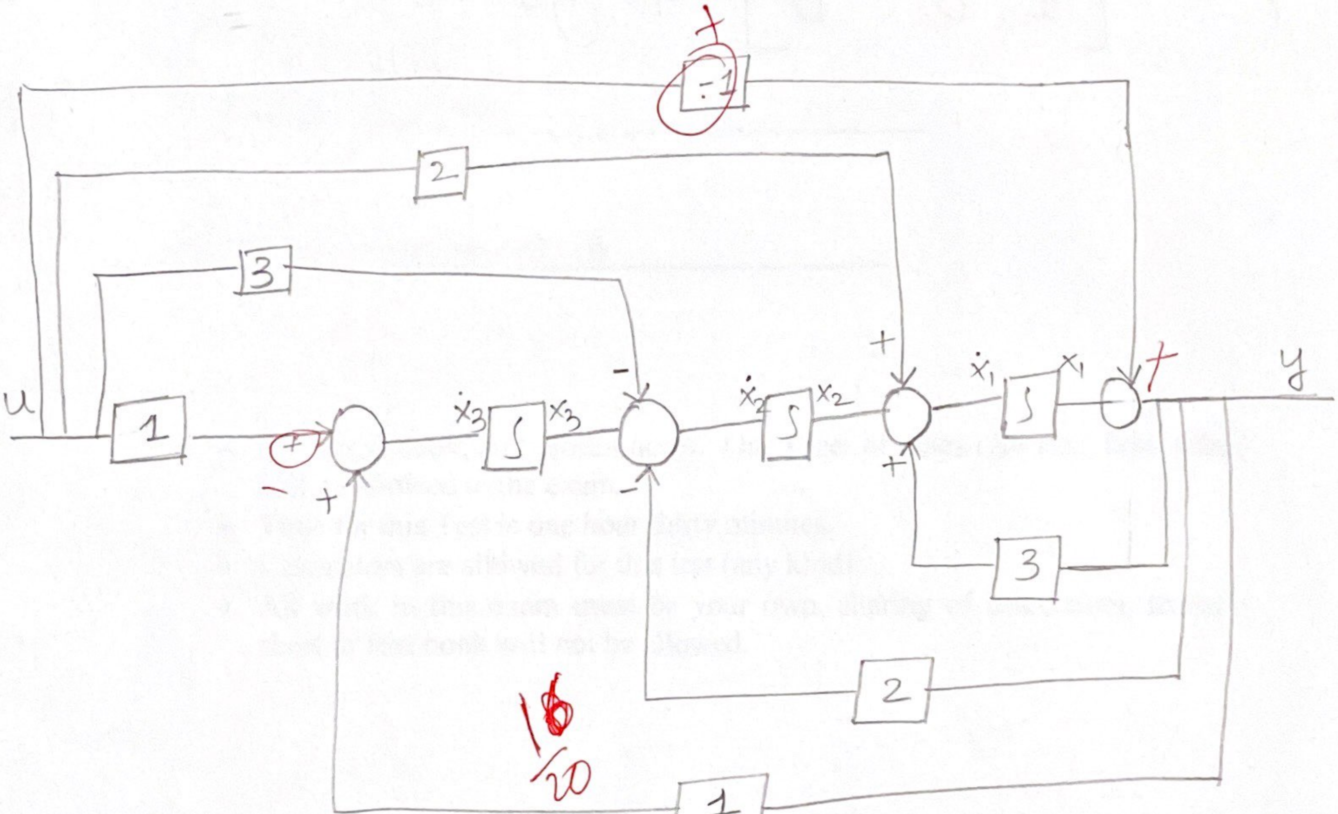
Please show me your calculations for partial credit

(5) $\dddot{y} - 3\ddot{y} + 2\dot{y} - y = \dddot{u} + 2\ddot{u} - 3\dot{u} - u$, find the state and output equation by using observable canonical form (15+5 bonus points)

$$\dddot{y} = \dddot{u} + 2\ddot{u} - 3\dot{u} - u + 3\ddot{y} - 2\dot{y} + y$$

integrating

$$y = u + 2\int u - 3\iint u - \iiint u + 3\int y - 2\iint y + \iiint y$$



18/20

$\dot{x}_1 = x_2 + 2u + 3x_1$

$\dot{x}_2 = x_3 - 3u - 2x_2$

$\dot{x}_3 = u + x_3$

$Y = x_1 - u$

$$\dot{X} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 3 & 1 & 0 \\ -2 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 5 \\ -5 \\ 0 \end{pmatrix}$$

$Y = (1\ 0\ 0)$

Please show me your calculations for partial credit