LINEAR SYSTEMS
HOMEWORK # 3

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1) $\dddot{y} - 3\ddot{y} - 6\dot{y} - 9y = 2\ddot{u} - \dot{u} + 5u$ [controllable canonical Form]

$\Rightarrow \dddot{y} = 2\ddot{u} - \dot{u} + 5u + 3\ddot{y} + 6\dot{y} + 9y$

2nd system concept $\Rightarrow$

$2\ddot{u} - \dot{u} + 5u$ — $\boxed{\text{system - 2}}$ — $2\ddot{z} - \dot{z} + 5z$

$\therefore$ We have,

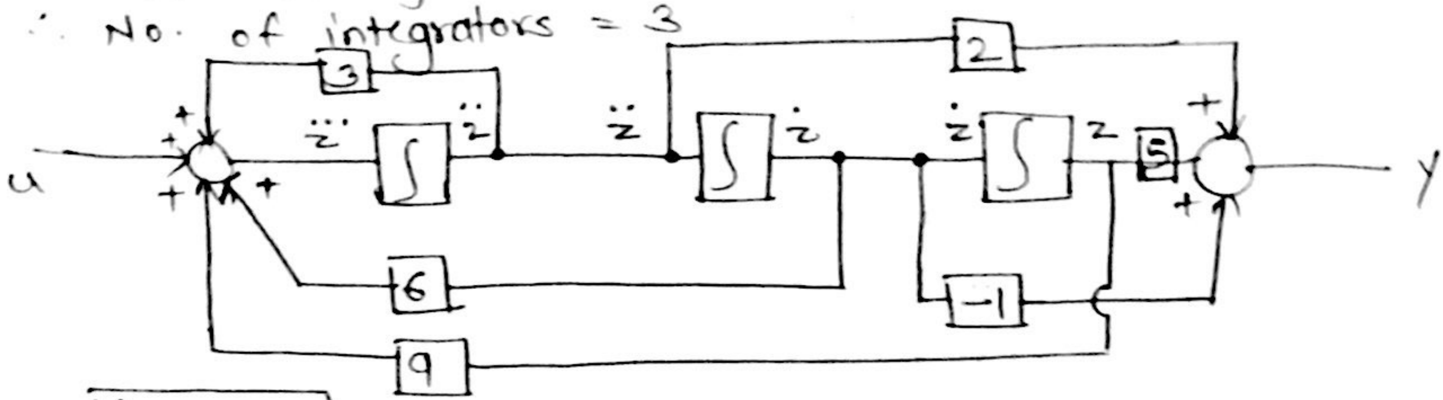
$u = 2\ddot{z} - \dot{z} + 5z$

$\therefore \dddot{z} = u + 3\ddot{z} + 6\dot{z} + 9z$

$y = 2\ddot{z} - \dot{z} + 5z$

Order of system = 3

$\therefore$ No. of integrators = 3



$\therefore$ $\boxed{\dot{x}_1 = x_2}$

$\boxed{\dot{x}_2 = x_3}$

$\dot{x}_3 = u + 6x_2 + 9x_1 + 3x_3$

$\boxed{\dot{x}_3 = 9x_1 + 6x_2 + 3x_3 + u}$

$\boxed{y = 5x_1 + 2x_3 - x_2}$

State Equation Matrix $\rightarrow$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 9 & 6 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

Output Equation Matrix $\rightarrow$

$$y = \begin{bmatrix} 5 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

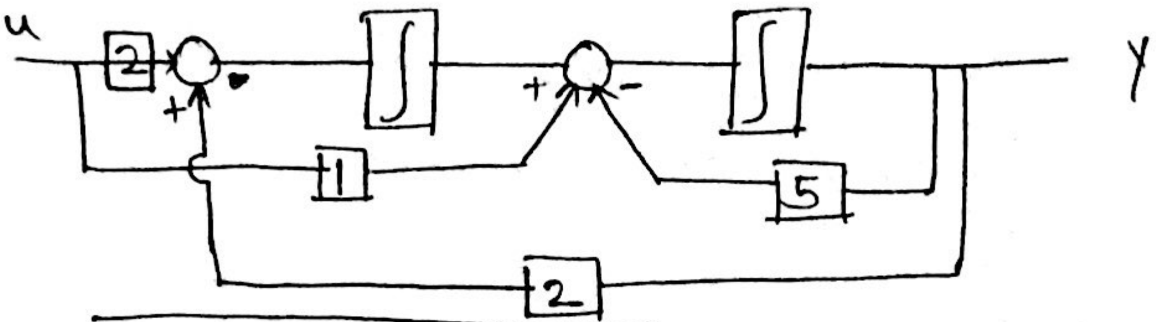
3) $\ddot{y} + 5\dot{y} - 2y = \dot{u} + 2u$ [observable canonical Form).

$\Rightarrow \quad \ddot{y} = \dot{u} + 2u - 5\dot{y} + 2y$

$\therefore \quad y = \int u + 2\iint u - 5\int y + 2\iint y$

Order of system = 2

$\therefore$ # of Integrators = 2.



$\therefore \quad \boxed{\dot{x}_1 = -5x_1 + x_2 + u}$

$\boxed{\dot{x}_2 = 2u + 2x_1}$

$\boxed{y = x_1.}$

State equation matrix $\rightarrow$

$$\dot{x} = \begin{bmatrix} -5 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

output equation matrix $\rightarrow$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

3) $\dddot{y} - 3\ddot{y} + 4y = \dot{u} - u$ [Jordan Form]

$\Rightarrow$ convert into s-domain

$\therefore s^3 y - 3s^2 y + 4y = su - u$

$\therefore (s^3 - 3s^2 + 4)\, y = u(s-1)$

$$\therefore \frac{Y}{u} = \frac{s-1}{s^3 - 3s^2 + 4}$$

$$\frac{Y}{u} = \frac{s-1}{(s+1)(s-2)(s-2)}$$

$$\therefore \frac{Y}{u} = \frac{A}{s+1} + \frac{B}{s-2} + \frac{C}{(s-2)^2}$$

$\therefore s-1 = A(s-2)(s-2) + B(s+1)(s-2) + C(s+1)(s-2)$

Put $s = -1$

$\therefore -2 = 9A$

$\therefore \boxed{A = -2/9}$

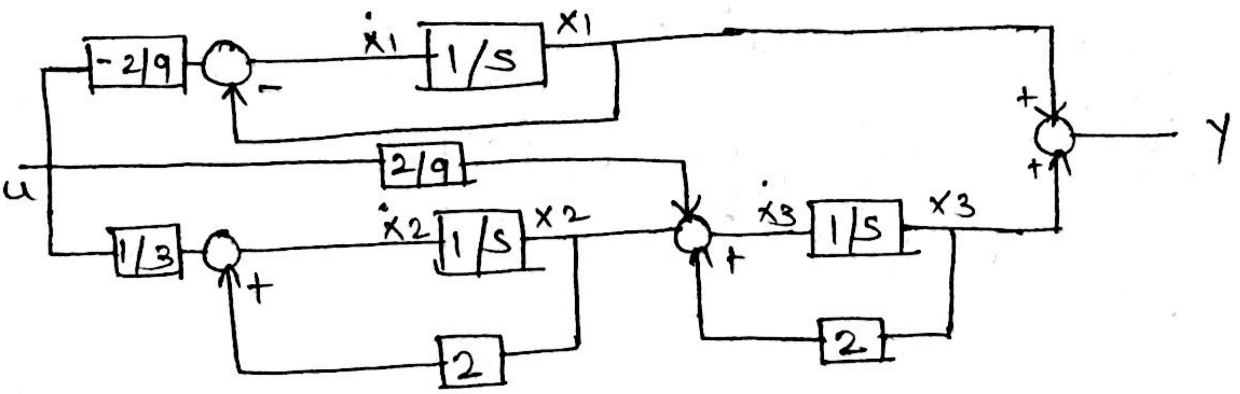
Put $s = 2$

$\therefore 1 = 3C$

$\therefore \boxed{C = 1/3}$

similarly,

$\boxed{B = 2/9}$



$\therefore \dot{x}_1 = -x_1 - (2/9)u$

$\dot{x}_2 = 2x_2 + (1/3)u$

$\dot{x}_3 = x_2 + 2x_3 + (2/9)u$

$y = x_1 + x_3$

State equation matrix →

$$\dot{X} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -2/9 \\ 1/3 \\ 2/9 \end{bmatrix} u$$

Output equation matrix →

$$Y = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

4) $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -6 & 1 \\ -6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$. Find transfer function.

⇒ $A = \begin{bmatrix} -6 & 1 \\ -6 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$

$$H(s) = \frac{Y(s)}{U(s)} = C[sI - A]^{-1} B + D$$

$$\therefore [sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -6 & 1 \\ -6 & 0 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s+6 & -1 \\ 6 & s \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{s^2 + 6s + 6} \begin{bmatrix} s & 1 \\ -6 & s+6 \end{bmatrix}$$

$$C[sI - A]^{-1} = \frac{1}{s^2 + 6s + 6} \begin{bmatrix} s & 1 \\ -6 & s+6 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$= \frac{1}{s^2 + 6s + 6} \begin{bmatrix} s & 1 \end{bmatrix}$$

$$C[sI - A]^{-1} \cdot B = \frac{1}{s^2 + 6s + 6} \begin{bmatrix} s & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2 + 6s + 6} (1)$$

$$\therefore \boxed{H(s) = \frac{1}{s^2 + 6s + 6}} \;//$$

5) $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -18 & -27 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -18 & -27 & -10 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad D = 0$$

$$H(s) = \frac{Y(s)}{U(s)} = C[sI - A]^{-1} B + D.$$

$$[sI - A] = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -18 & -27 & -10 \end{bmatrix}$$

$$[sI - A] = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 18 & 27 & s+10 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{\det|A|} \cdot \text{Adjoint } |A|$$

$$\det|A| = s(s^2+10s+27) - (-1)(0+18) + 0(0+18s)$$

$$= s^3 + 10s^2 + 27s + 18$$

co-factors $\Rightarrow$

co-factor of $a_{11} = (-1)^{1+1}(s^2+10s+27) = s^2+10s+27$

$a_{12} = (-1)^{1+2}(18) = -18$

$a_{13} = (-1)^{1+3}(0 - 18s) = -18s$

$a_{21} = (-1)^{2+1}(-s-10-0) = s+10$

$a_{22} = (-1)^{2+2}(s^2+10s-0) = s^2+10s$

$a_{23} = (-1)^{2+3}(27s+18) = -27s-18$

$a_{31} = (-1)^{3+1}(1+0) = 1$

$a_{32} = (-1)^{3+2}(-s+0) = s$

$a_{33} = (-1)^{3+3}(s^2+0) = s^2$

$\therefore$ Adj $|A|$ = [Co-factor of A]^T

$$\text{Adj } |A| = \begin{bmatrix} s^2+10s+27 & s+10 & 1 \\ -18 & s^2+10s & s \\ -18s & -27-18s & s^2 \end{bmatrix}$$

$$\therefore [sI - A]^{-1} = \frac{1}{s^3+10s^2+27s+18}\begin{bmatrix} s^2+10s+27 & s+10 & 1 \\ -18 & s^2+10s & s \\ -18s & -27s-18 & s^2 \end{bmatrix}$$

$$\therefore C[sI - A^{-1}] = \frac{1}{s^3+10s^2+27s+18}\begin{bmatrix} s^2+10s+27 & s+10 & 1 \\ -18 & s^2+10s & s \\ -18s & -27s-18 & s^2 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$C[sI - A]^{-1} = \frac{1}{s^3+10s^2+27s+18}\begin{bmatrix} s^2+10s+27 & s+10 & 1 \end{bmatrix}$$

$$C[sI - A]^{-1}.B = \frac{1}{s^3+10s^2+27s+18}\begin{bmatrix} s^2+10s+27 & s+10 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = \frac{1}{s^3+10s^2+27s+18} \quad (1)$$

$$\therefore \boxed{H(s) = \frac{1}{s^3+10s^2+27s+18}} \; //$$