LINEAR SYSTEMS
HOMEWORK #5

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1) $A = \begin{bmatrix} 0 & -6 \\ 1 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Poles are at -1 & -4

$\Rightarrow \quad \dot{x} = Ax + Bu$

$A_c = A - BF = \begin{bmatrix} 0 & -6 \\ 1 & -5 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} F_1 & F_2 \end{bmatrix}$

$= \begin{bmatrix} 0 & -6 \\ 1 & -5 \end{bmatrix} - \begin{bmatrix} F_1 & F_2 \\ F_1 & F_2 \end{bmatrix} = \begin{bmatrix} -F_1 & -6-F_2 \\ 1-F_1 & -5-F_2 \end{bmatrix}$

$A_c(s) = sI - A_c = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -F_1 & -6-F_2 \\ 1-F_1 & -5-F_2 \end{bmatrix}$

$\Delta_c(s) = \begin{bmatrix} s+F_1 & 6+F_2 \\ -1+F_1 & s+5+F_2 \end{bmatrix}$

$\det(A_c(s)) = (s+F_1)(s+5+F_2) - (-1+F_1)(6+F_2)$

$= s^2 + F_1 s + 5s + 5F_1 + F_2 s + F_2 F_1 + 6 + F_2 - F_1 F_2$

$= s^2 + s(F_1+F_2+5) + \cancel{5}F_1 + F_2 + 6 \quad - (1)$

Desired poles $\Rightarrow$

$\Delta d(s) = (s+1)(s+4) = s^2 + 5s + 4 \quad - (2)$

Compare (1) & (2)

$\therefore \quad +F_1 + F_2 + 5 = 5$

$F_1 + F_2 = 0$

$-F_1 + F_2 + 6 = 4$

$-\cancel{F_1} + F_2 = -2$

$\cancel{F_1} + F_2 = 0$

$2F_2 = -2$

$\therefore \quad \boxed{F_2 = -1}$

$\therefore \quad \boxed{F_1 = 1}$

2) $A = \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

closed loop poles are at $-3 \pm j4$

$\Rightarrow$ $A_c = A - BF = \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} F_1 & F_2 \end{bmatrix}$

$$= \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ F_1 & F_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -F_1 & -3-F_2 \end{bmatrix}$$

$$SI - A_c = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -F_1 & -3-F_2 \end{bmatrix} = \begin{bmatrix} s-2 & -1 \\ F_1 & s+3+F_2 \end{bmatrix}$$

$$\det|A_c(s)| = (s-2)(s+3+F_2) + F_1$$

$$= s^2 - 2s + 3s - 6 + F_2 s - 2F_2 + F_1$$

$$= s^2 + s(1+F_2) - 2F_2 - 6 + F_1 \quad - (1)$$

Desired poles $\Rightarrow$

$$(s+3+j4)(s+3-j4)$$

$$= (s-3)^2 - (j4)^2$$

$$= s^2 + 6s + 9 + 16$$

$$= s^2 + 6s + 25 \quad - (2)$$

$\therefore$ $F_2 + 1 = 6$

$F_2 = 5$

$F_1 = 25 + 16 = 41$

$\therefore$ $\boxed{F_2 = 5}$

$\boxed{F_1 = 41}$

3) $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 1 \end{bmatrix}$

$H(s) = C[sI-A]^{-1} \cdot B + D$

$$\Rightarrow sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} s-2 & 0 \\ 0 & s-1 \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} s-1 & 0 \\ 0 & s-2 \end{bmatrix} \cdot \frac{1}{s^2 - 3s + 2}$$

$$C[sI-A]^{-1} = \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} s-1 & 0 \\ 0 & s-2 \end{bmatrix} \frac{1}{s^2 - 3s + 2}$$

$$= \begin{bmatrix} -s+1 & s-2 \end{bmatrix} \frac{1}{s^2 - 3s + 2}$$

$$\therefore C[sI-A]^{-1} \cdot B = \frac{1}{s^2 - 3s + 2} \begin{bmatrix} -s+1 & s-2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$H(s) = \frac{-1}{s^2 - 3s + 2}$$

$$\therefore H(s) = \frac{A}{s-1} + \frac{B}{s-2}$$

$$\therefore A = \left. \frac{-1}{s-2} \right|_{s=1} = 1$$

$$B = \left. \frac{-1}{s-1} \right|_{s=2} = -1$$

$$\therefore H(s) = \frac{1}{s-1} - \frac{1}{s-2}$$

$$A_C = A - BF = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} F_1 & F_2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-F_1 & -F_2 \\ -F_1 & 1-F_2 \end{bmatrix}$$

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$$sI - A_C = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2-F_1 & -F_2 \\ -F_1 & 1-F_2 \end{bmatrix}$$

$$= \begin{bmatrix} s-2+F_1 & F_2 \\ F_1 & s-1+F_2 \end{bmatrix}$$

$$\det(sI - A_C) = (s-2+F_1)(s-1+F_2) - F_1F_2$$

$$= s^2 - 2s + F_1 s - s + 2 - F_1 + sF_2 - 2F_2 + F_1F_2 - F_1F_2$$

$$= s^2 + s(-3+F_1+F_2) + 2 - 2F_2 - F_1 \quad - (1)$$

Desired poles $\Rightarrow$

$$(s+1)(s+2) = s^2 + 3s + 2 \quad - (2)$$

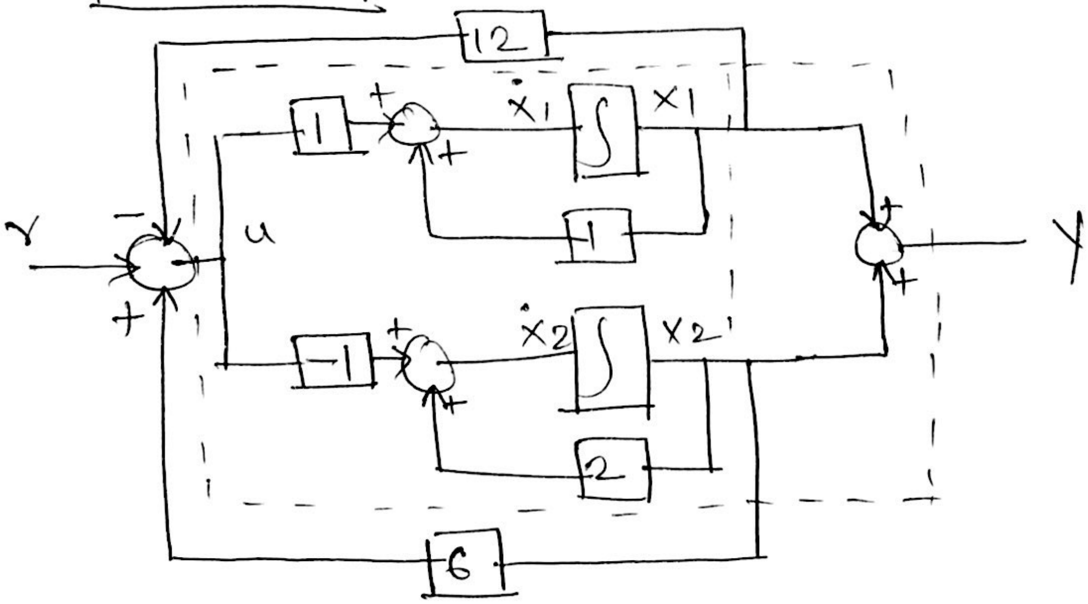
$$\therefore \quad F_1 + F_2 = 6$$

$$-2F_2 - \cancel{F_1} = 0$$

$$F_2 + \cancel{F_1} = 6$$

$$\boxed{F_2 = -6}$$

$$\therefore \boxed{F_1 = 12}$$



$$u = -Fx + r = \begin{bmatrix} -F_1 & -F_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + r$$

$$u = \begin{bmatrix} -12 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + r$$

4) $A = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

$$\Rightarrow \phi(t) = e^{At} = L^{-1}[SI - A]^{-1}$$

$$SI - A = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} -4 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} s+4 & 0 & 0 \\ 0 & s+3 & 0 \\ 0 & 0 & s+2 \end{bmatrix}$$

$$SI - A^{-1} = \frac{1}{(s+4)(s+3)(s+2)} \begin{bmatrix} (s+3)(s+2) & 0 & 0 \\ 0 & (s+4)(s+2) & 0 \\ 0 & 0 & (s+3)(s+4) \end{bmatrix}$$

$$SI - A^{-1} = \begin{bmatrix} \frac{1}{s+4} & 0 & 0 \\ 0 & \frac{1}{s+3} & 0 \\ 0 & 0 & \frac{1}{s+2} \end{bmatrix}$$

$$\therefore L^{-1}[SI - A]^{-1} = \begin{bmatrix} e^{-4t} & 0 & 0 \\ 0 & e^{-3t} & 0 \\ 0 & 0 & e^{-2t} \end{bmatrix} = \phi(t)$$

$$\phi(t) = \begin{bmatrix} e^{-4t} & 0 & 0 \\ 0 & e^{-3t} & 0 \\ 0 & 0 & e^{-2t} \end{bmatrix}$$ is the state transition matrix.