Linear Systems

Homework-4

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① $\dddot{y} - 3\ddot{y} - 6\dot{y} - 9y = 2\ddot{u} - \dot{u} + 5u$

$s^3y - 3s^2y - 6sy - 9y = 2s^2u - su + 5u$

$y(s^3 - 3s^2 - 6s - 9) = u(2s^2 - s + 5)$

$$\frac{y}{u} = \frac{2s^2 - s + 5}{s^3 - 3s^2 - 6s - 9} = \frac{2s^2 - s + 5}{(s + 4.688)(s + 0.1555)(s - 1.8444)}$$

$$\Rightarrow \frac{A}{(s + 4.7)} + \frac{B}{(s + 0.15)} + \frac{C}{(s - 1.8)} = \frac{2s^2 - s + 5}{s^3 - 3s^2 - 6s - 9}$$

$\Rightarrow A(s + 0.15)(s - 1.8) + B(s + 4.7)(s - 1.8) + C(s + 4.7)(s + 0.15) = 2s^2 - s + 5$

$\Rightarrow A(s^2 - 1.65s - 0.27) + B(s^2 + 2.9s - 8.46) + C(s^2 + 4.85s + 0.705) = 2s^2 - s + 5$

$A + B + C = 2$ — ①

$0.27A - 8.46B + 0.705C = 5$ — ②

$-1.65A + 2.9B + 4.85C = -1$ — ③

① & ③ $\Rightarrow$ $1.65(①) + ③ \Rightarrow 3.9B + 5.85C = 2.3$ — ④

$[1.65(A + B + C) = 1.65A + 2.9B + 4.85C = 2(1.65) - 1)$ ↑

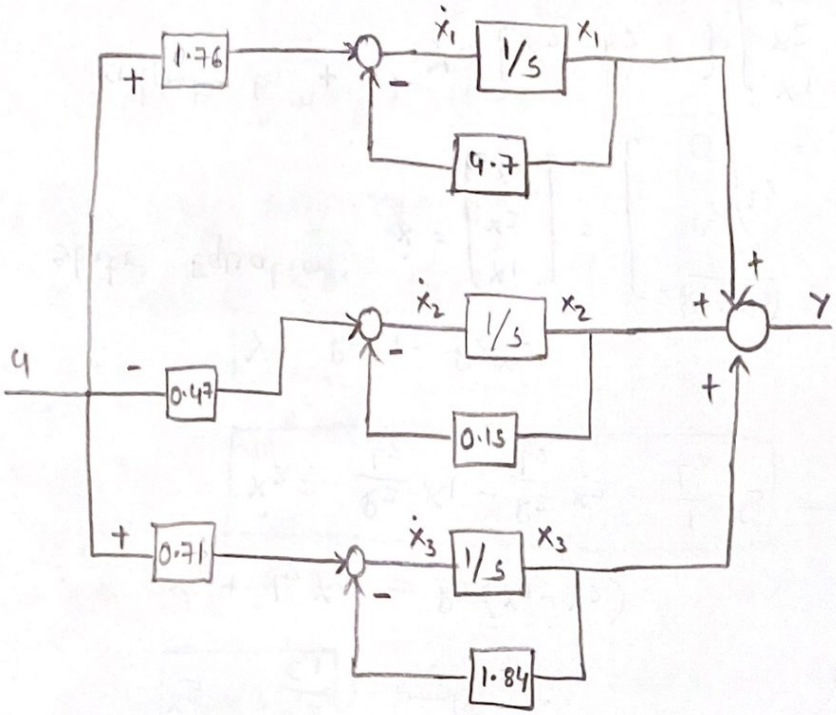
① & ② $\Rightarrow$ $0.27(A + B + C) - 0.27A + 8.46B - 0.705C = 2(0.27) - 5$

$\Rightarrow -8.73B + 0.435C = 4.46$ — ⑤

④ & ⑤ $\Rightarrow -8.73(④) + 3.9(⑤) \Rightarrow 52.767C = 37.473 \Rightarrow \boxed{C = 0.71}$

④ $\Rightarrow \boxed{B = -0.47}$ ① $\Rightarrow \boxed{A = 1.76}$

$$\frac{y}{u} = \frac{1.76}{s + 4.7} - \frac{0.47}{s + 0.15} + \frac{0.71}{s - 1.84}$$



$$Y = x_1 + x_2 + x_3$$

$$\dot{x}_1 = -4.7x_1 + 1.76u$$

$$\dot{x}_2 = -0.15x_2 - 0.47u$$

$$\dot{x}_3 = -1.84x_3 + 0.71u$$

$$A = \begin{bmatrix} -4.7 & 0 & 0 \\ 0 & -0.15 & 0 \\ 0 & 0 & -1.84 \end{bmatrix}$$

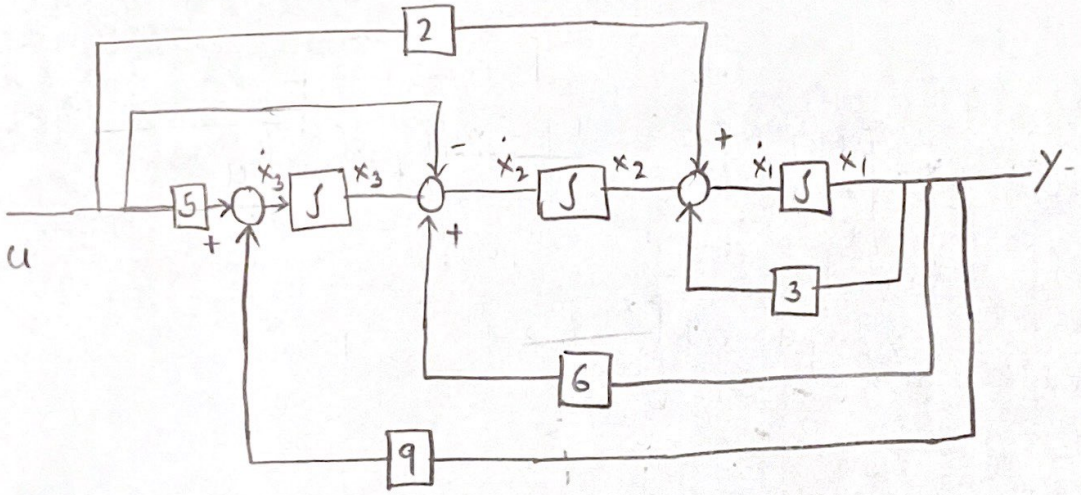
$$B = \begin{bmatrix} 0.71 \\ -0.47 \\ 1.76 \end{bmatrix} \quad B = \begin{bmatrix} 1.76 \\ -0.47 \\ 0.71 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \qquad D = 0$$

2) $\dddot{y} - 3\ddot{y} - 6\dot{y} - 9y = 2\ddot{u} - \dot{u} + 5u$ using Observable canonical form

$\ddot{y} = 2\ddot{u} - \dot{u} + 5u + 3\ddot{y} + 6\dot{y} + 9y$ (Integration both sides 3 times)

$y = 2\int u - \iint u + 5\iiint u + 3\int y + 6\iint y + 9\iiint y$



$y = x_1$ → output Equation

state equations

$\dot{x}_1 = 3x_1 + x_2 + 2u$

$\dot{x}_2 = 6x_1 - u + x_3$

$\dot{x}_3 = 5u + 9x_1$

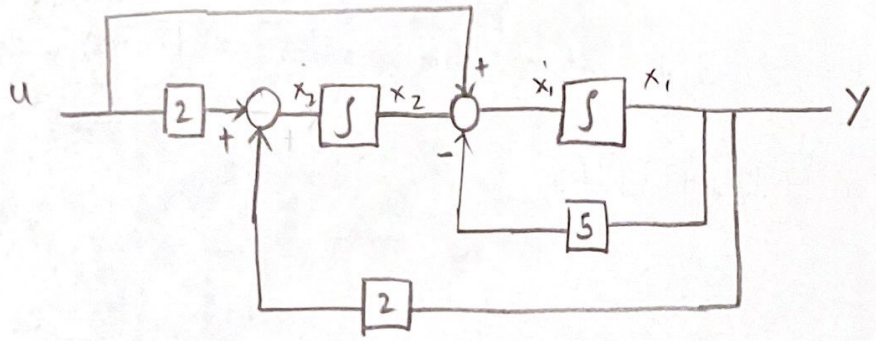
$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 6 & 0 & 1 \\ 9 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0$$

3) $\dddot{y} + 5\ddot{y} - 2\dot{y} = \dot{u} + 2u$ using observable canonical form

$\ddot{y} = \dot{u} + 2u - 5\dot{y} + 2y$ (Integrating twice on both sides)

$y = \int u + 2\iint u - 5\int y + 2\iint y$



Output Equation

$$y = x_1$$

State Equations

$$\dot{x}_1 = -5x_1 + x_2 + u$$

$$\dot{x}_2 = 2x_1 + 2u$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -5 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0$$

4) $\dddot{y} - 3\ddot{y} + 4y = \ddot{u} - u$ using Jordan Form

$$s^3y - 3s^2y + 4y = s^2u - u \Rightarrow y(s^3 - 3s^2 + 4) = u(s^2 - 1)$$

$$\frac{y}{u} = \frac{s^2-1}{s^3-3s^2+4} = \frac{s^2-1}{(s-2)(s^2-s-2)} = \frac{s-1}{(s-2)^2(s+1)}$$

$$\Rightarrow \frac{s-1}{(s-2)^2(s+1)} = \frac{A}{(s-2)} + \frac{B}{(s-2)^2} + \frac{C}{(s+1)}$$

$$s-1 = A(s-2)(s+1) + B(s+1) + C(s-2)^2$$

$$s-1 = A(s^2 - 2s + s - 2) + Bs + B + C(s^2 + 4 - 4s)$$

$$s-1 = As^2 - As - 2A + Bs + B + Cs^2 + 4C - 4Cs$$

$$(s-1) = s^2(A+C) + s(-A+B-4C) - 2A + B + 4C$$

$A+C=0$, $-A+B-4C=1$, $-2A+B+4C=-1$

$A=-C$ — ① $\quad C+B-4C=1$

$B-3C=1$ — ②

$2A-B-4C=1$

$-2C-B-4C=1$

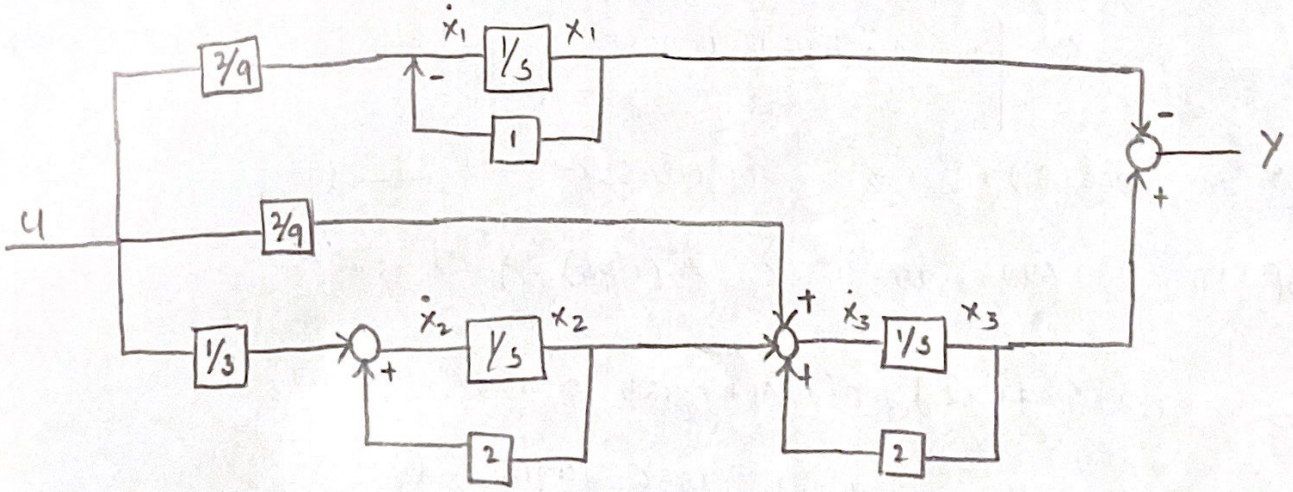
$-B-6C=1$ — ③

② & ③ $\Rightarrow \quad -9C=2 \quad \Rightarrow \quad \boxed{C=-2/9}$

① $\Rightarrow \quad A=-C=2/9 \quad \Rightarrow \quad \boxed{A=2/9}$

② $\Rightarrow \quad B-3\left(\frac{-2}{9_3}\right)=1 \quad \Rightarrow \quad B+\frac{2}{3}=1 \quad \Rightarrow \quad \boxed{B=1/3}$

$$\Rightarrow \frac{2/9}{(s-2)} + \frac{1/3}{(s-2)^2} + \frac{-2/9}{(s+1)}$$



$y = -x_1 + x_3$

$\dot{x}_1 = -x_1 + 2/9\,u$

$\dot{x}_2 = 2x_2 + 1/3\,u$

$\dot{x}_3 = x_2 + 2/9\,u + 2x_3$

$$A=\begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \qquad B=\begin{bmatrix} 2/9 \\ 1/3 \\ 2/9 \end{bmatrix}$$

$C=\begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \qquad D=0$