CI method
Z-score method } → If the value is lying in a range or not
↓
Interval

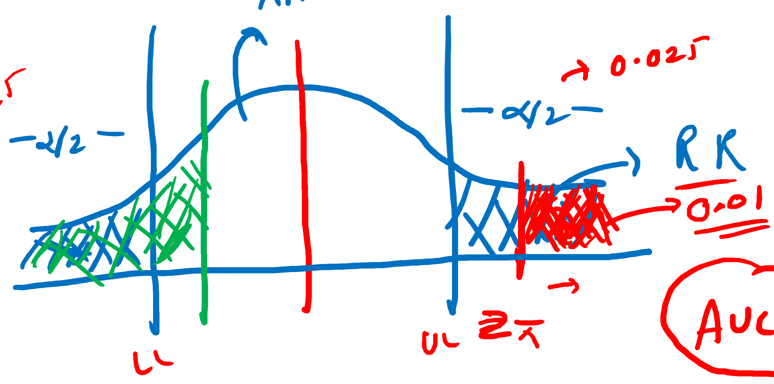
→ P-value method → Compare the AUC (~~Not value~~) for the given sample mean
↳ Rejection Region / significance Area

&

The AUC for the z-score of $\bar{x}$

$P_{calc} < P_\alpha$
⇓ → H_A ✓
$\bar{x}$ is in RR

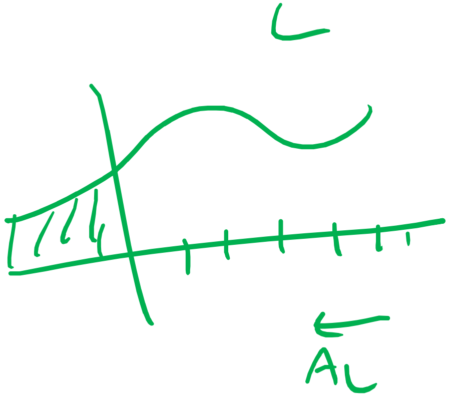
$P_{calc} > P_\alpha$
$\bar{x}$ is in AR
↳ H_0 ✓
H_A ✗

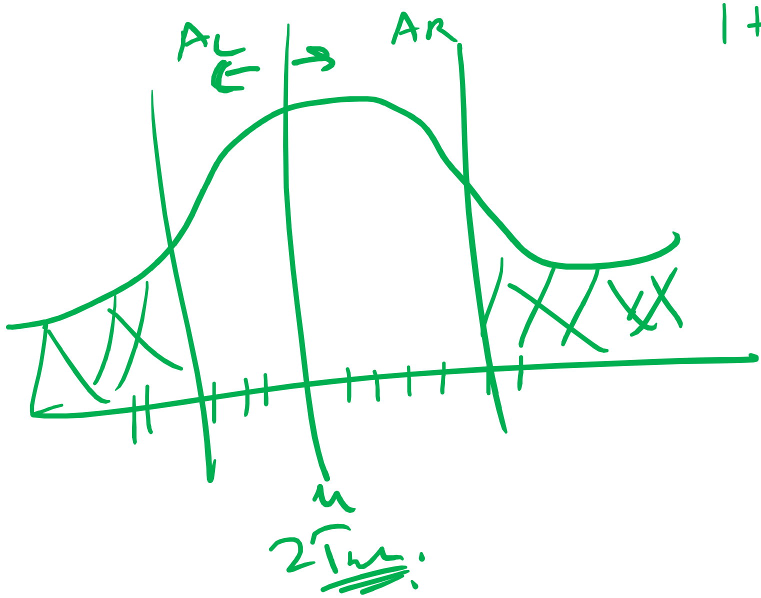


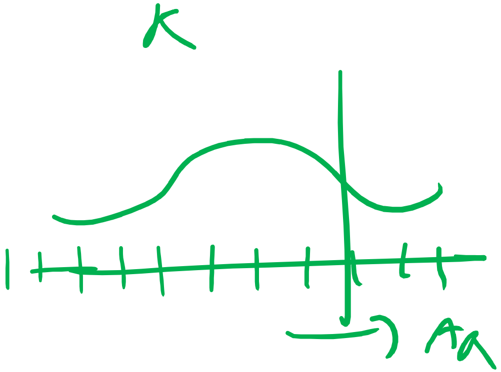
$\alpha = 5\%$ of 1

$AUC_{RR} = 0.05$ ⇒ P_α

$AUC_{z_{\bar{x}}}$ ⇒ # of tails * AUC on the Tail
↳ 0.02 = P_{calc}


L
A_L


A_L
A_R
μ


K
A_R

An inventor has developed a new, energy-efficient lawn mower engine. He claims that the engine will run continuously for more than 5 hours (300 minutes) on a single gallon of regular gasoline. (The leading brand lawnmower engine runs for 300 minutes on 1 gallon of gasoline.)

From his stock of engines, the inventor selects a simple random sample of 50 engines for testing. The engines run for an average of 305 minutes. The true standard deviation σ is known and is equal to 30 minutes, and the run times of the engines are normally distributed.

Test hypothesis that the mean run time is more than 300 minutes. Use a 0.05 level of significance.

$H_0 \Rightarrow \mu \leq 300$

$H_A \Rightarrow \mu > 300$ $\rightarrow$ Right Tail Test

$P_\alpha = 0.05$

$\alpha = 5\%$ $\quad \bar{X} = 305$

$$Z_{\bar{x}} = \frac{305 - 300}{30/\sqrt{50}} = 1.18$$

$P_{calc} =$ # of tails * AUC on the tail

$\Rightarrow 1 * A_{R_{Z\bar{x}}}$

$\Rightarrow 1 - A_{L_{Z\bar{x}}}$

$\Rightarrow 1 - 0.881 \Rightarrow 0.119 = P_{calc}$

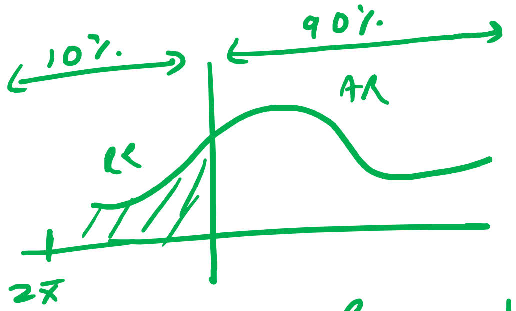
$P_{calc} > P_\alpha \Rightarrow AR$

Failed to Reject H_0, Reject H_A

It is believed that a stock price for a particular company will grow at a rate of $5 per week with a standard deviation of $1. An investor believes the stock won’t grow as quickly. The changes in stock price is recorded for ten weeks and are as follows: $4, $3, $2, $3, $1, $7, $2, $1, $1, $2. Perform a hypothesis test using a 10% level of significance.

$H_0 \Rightarrow \mu \geq 5$

✓ $H_A \Rightarrow \mu < 5$ → Left Tail Test



$\bar{x} = 2.6$

$\sigma = 1$

$\mu = 5$

$N = 10$

$$Z_{\bar{x}} = \frac{2.6 - 5}{1/\sqrt{10}} = -7.59$$

$A_L < -4 = 0$

$P_\alpha = 0.1$

$P_{calc} = 1 * 0 = 0$

$P_{calc} < P_\alpha \Rightarrow RR \Rightarrow$

H_0 is Rejected, H_A is Accepted

Jane has just begun her new job as on the sales force of a very competitive company. In a sample of 16 sales calls it was found that she closed the contract for an average value of 108 dollars with a standard deviation of 12 dollars. Test at 5% significance that the population mean is at least 100 dollars against the alternative that it is less than 100 dollars. Company policy requires that new members of the sales force must exceed an average of $100 per contract during the trial employment period. Can we conclude that Jane has met this requirement at the significance level of 5%?

$\alpha = 5\%$

$P_\alpha = 0.05$

✓ $H_0 \Rightarrow \mu \geq 100$

$H_A \Rightarrow \mu < 100$ → Left Tail Test

$P_{calc} > P_\alpha = AR$

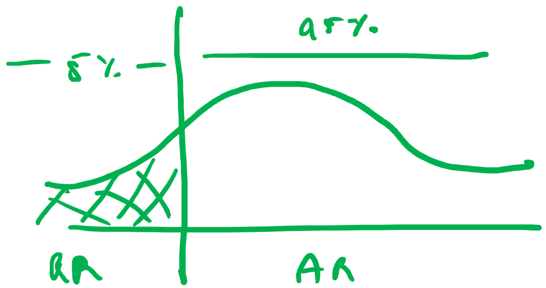
$P_{calc} = A_{L\, z_{\bar{x}}} = 0.99621$

$\bar{X} = 108$

$\sigma = 12$

$\mu = 100$

$N = 16$



$$z_{\bar{x}} = \frac{\bar{X} - \mu}{\sigma / \sqrt{N}}$$

$$\Rightarrow \frac{108 - 100}{12 / \sqrt{16}} = \frac{8 \times 4}{12\,3} = 8/3$$

$$\boxed{z_{\bar{x}} = 2.67}$$

Failed to Reject H_0, Reject H_A

→ pop , sample → One - Sample Z -Test

→ Sample 1 , Sample 2 → Two - Sample Z Test

↓

$H_0 \Rightarrow \bar{x}_1 = \bar{x}_2$
$H_A \rightarrow \bar{x}_1 \neq \bar{x}_2$ } → 2 Tail Test

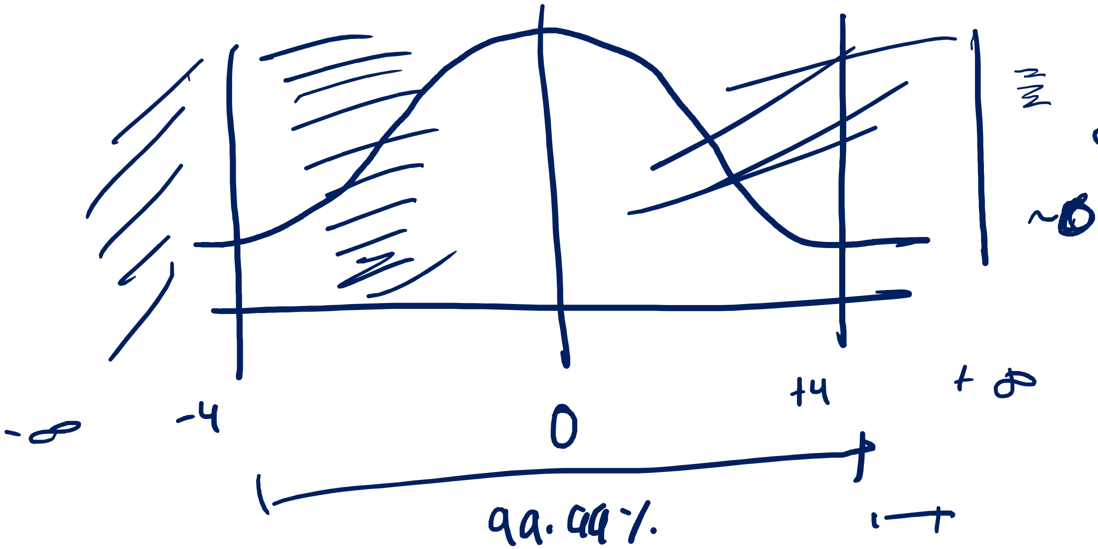
$H_0 = \bar{x}_1 \geq \bar{x}_2$
$H_A = \bar{x}_1 < \bar{x}_2$ } → Left Tail Test

$H_0 \Rightarrow \bar{x}_1 \leq \bar{x}_2$
$H_A = \bar{x}_1 > \bar{x}_2$ } → Right Tail Test

$$Z_{calc} \Rightarrow \frac{\bar{x} - \mu}{\sigma / \sqrt{N}}$$

↳ ↳ $\Sigma \sqrt{\frac{\sigma_i^2}{N_i}}$

$$Z_{calc} \Rightarrow \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}}$$



$\mu = 0$

$\sigma = 1$

$-4 - +4$

$> 99.99\%$

0.01%

~ 1

$A_L > +4 = 1$ $\qquad$ $A_L < -4 = 0$

$A_R > +4 = 0$ $\qquad$ $A_R < -4 = 1$

It is thought that teenagers sleep more than adults on average. A study is done to verify this. A sample of 16 teenagers has a mean of 8.9 hours slept and a standard deviation of 1.2. A sample of 12 adults has a mean of 6.9 hours slept and a standard deviation of 0.6.

$\alpha = 5\%$

$P_\alpha = 0.05$

X_1 teenagers

$\bar{X}_1 = 8.9$

$\sigma_1 = 1.2$

$N_1 = 16$

$H_0 \Rightarrow \bar{X}_1 \geq \bar{X}_2$

$H_A \rightarrow \bar{X}_1 < \bar{X}_2$

Left Tail Test

X_2 Adults

$\bar{X}_2 = 6.9$

$\sigma_2 = 0.6$

$N_2 = 12$

$A_L > +4 = 1$

$P_{calc} = 1$

$P_{calc} > P_\alpha = AR$

failed to reject H_0, Reject H_A

$$Z_{calc} \Rightarrow \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}} \Rightarrow \frac{8.9 - 6.9}{\sqrt{\frac{1.2^2}{16} + \frac{0.6^2}{12}}} = \frac{2}{\sqrt{\frac{1.44}{16} + \frac{0.36}{12}}} \Rightarrow \frac{2}{\sqrt{0.12}}$$

$\Rightarrow 5.77$

It is believed that the average grade on an English essay in a particular school system for females is higher than for males. A random sample of 31 females had a mean score of 82 with a standard deviation of three, and a random sample of 25 males had a mean score of 76 with a standard deviation of four.

10 mins

Female x_1 Male x_2

$$Z_{calc} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}}$$

09:45 PM

$H_0 \Rightarrow \bar{X}_1 \geq \bar{X}_2$

$H_A \Rightarrow \bar{X}_1 < \bar{X}_2$

→ Left Tail Test

$\alpha = 5\%$, $P_\alpha = 0.05$

$P_{calc} \Rightarrow A_{L\, z_{calc}}$

$$Z_{calc} \Rightarrow \frac{82 - 76}{\sqrt{\frac{3^2}{31} + \frac{4^2}{25}}} = 6.22$$

$A_L > +4 \Rightarrow 1$

$P_{calc} = 1$ $P_{calc} > P_\alpha \Rightarrow AR$ Failed to reject H_0, Rejecting H_A