

likelihood  
Chances of Occurrence  
Mathematical Tool

# PROBABILITY AND DISTRIBUTION

Basic + Adv  
Distribution of Data  
N.D., B.O.

# PROBABILITY $\rightarrow [0, 1]$

$p \times 100 \Rightarrow \%$

Impossible Event  $\rightarrow$  7 on a dice, 12 in cards  $\rightarrow 0$

$0.78 \times 100 = 78\%$   
 $0.635$

Sure Event  $\rightarrow$  Sun rising  $\rightarrow 1$

- Probability implies 'likelihood' or 'chance'. When an event is certain to happen then the probability of occurrence of that event is 1 and when it is certain that the event cannot happen then the probability of that event is 0.
- Hence the value of probability ranges from 0 to 1.

## Classical Definition of Probability

- As the name suggests the classical approach to defining probability is the oldest approach. It states that if there are  $n$  exhaustive, mutually exclusive and equally likely cases out of which  $m$  cases are favourable to the happening of event A,
- Then the probabilities of event A is defined as given by the following probability function:

$$P(\text{Prime \#}) = \frac{3}{6} = 1/2$$

$$P(\text{Composite \#}) = \frac{2}{6} = 1/3$$

$$P(A) = \frac{\text{Number of favorable outcomes to A}}{\text{Total number of outcomes}}$$

Event

Experiment

roll a dice

Experiment

Even  $\rightarrow$  Even # of faces  
Prime  
Odd  
Composite  
Multiple of 2

→ Exhaustive  
 ↳ covering Everything → Every possible outcome (cases)

→ For an Experiment

→ Mutually Exclusive → Any 2 outcomes cannot occur together

1, 2, 3 ... K

$n=2$

1  
 toss a coin → Exp  
 → H, T Out

Classes of student  
 ↳ 30

Exp → choose 1 student

$n=30$

S<sub>1</sub>  
 S<sub>2</sub>  
 S<sub>3</sub> ... S<sub>30</sub>

rolling a dice

→ 1, 2, 3, 4, 5, 6

$n=6$

→ (1,1) 1,2 (1,3) ...

→ Equally likely  
 ↳ same chance to

Occur

Event

Experiment

Use case

The thing/set of things we are evaluating.

→ Outcome

Tossing a coin

→ Head Appearing

1/2 ⇒

2

H, H

T, T

(H, T)

(T, H)

→ HT HT

→ 1, 2  
 → 1, 1

1

1

(1,2)

...

# PROBABILITY

Example

**Problem Statement:**

- A coin is tossed. What is the probability of getting a head?

**Solution:**

- Number of outcomes favourable to head ( $m$ ) = 1
- Total number of outcomes ( $n$ ) = 2 (i.e. head or tail)

$$P(A) = \frac{\text{Number of favorable outcomes to A}}{\text{Total number of outcomes}}$$



# PROBABILITY - BASIC CONCEPTS

Fair coin  $\rightarrow$  Random Experiment  
Tampered coin  $\rightarrow$   $\eta$   $\rightarrow$  Baised Experiment

- **Random Experiment**

An experiment is said to be a random experiment, if its out-come can't be predicted with certainty.

Example

If a coin is tossed, we can't say, whether head or tail will appear. So it is a random experiment.

- **Sample Space**

The set of all possible out-comes of an experiment is called the sample space. It is denoted by 'S' and its number of elements are  $n(s)$ .

Example

In throwing a dice, the number that appears at top is any one of 1,2,3,4,5,6. So here:

$S = \{1, 2, 3, 4, 5, 6\}$  and  $n(s) = 6$

Similarly in the case of a coin,  $S = \{\text{Head}, \text{Tail}\}$  or  $\{H, T\}$  and  $n(s) = 2$ .

dist  $\rightarrow$  duplicate  
set  $\rightarrow$  unique element

# PROBABILITY - BASIC CONCEPTS

## Event

Every subset of a sample space is an event. It is denoted by 'E'.

Example

In throwing a dice  $S=\{1,2,3,4,5,6\}$ , the appearance of an even number will be the event  $E=\{2,4,6\}$ .

Clearly E is a sub set of S.

Sample Space

- 1  
- 2  
- 3  
- 4  
- 5  
- 6

Exhaustive Events

(1)  $\{1,2\}$   $\{2,3,4\}$   $\{2,3,5\}$   
(2)  $\{1,3\}$   $\{2,4,6\}$   $\{1,4,5,2\}$   
(3)  $\{1,4\}$   $\{1,2,3\}$   
(4)  $\{1,5\}$   $\{1,4,6\}$   
(5)  $\{1,6\}$   
(6)  $\{1,1\}$   $\{1,2,3,4\}$

# PROBABILITY - BASIC CONCEPTS

- **Equally likely events** ✨

Events are said to be equally likely, if the probability of occurrence of the events are same.

Example

When a dice is thrown, all the six faces  $\{1,2,3,4,5,6\}$  are equally likely to come up.

- **Exhaustive events** ✨

When every possible out come of an experiment is considered.

Example

A dice is thrown, cases  $1,2,3,4,5,6$  form an exhaustive set of events.

# PROBABILITY - BASIC CONCEPTS

- **Mutually exclusive or Disjoint event**

If two or more events can't occur simultaneously, that is no two of them can occur together.

## Example

When a coin is tossed, the event of occurrence of a head and the event of occurrence of a tail are mutually exclusive events.

- **Independent or Mutually independent events**

Two or more events are said to be independent if occurrence or non-occurrence of any of them does not affect the probability of occurrence or non-occurrence of the other event.

## Example

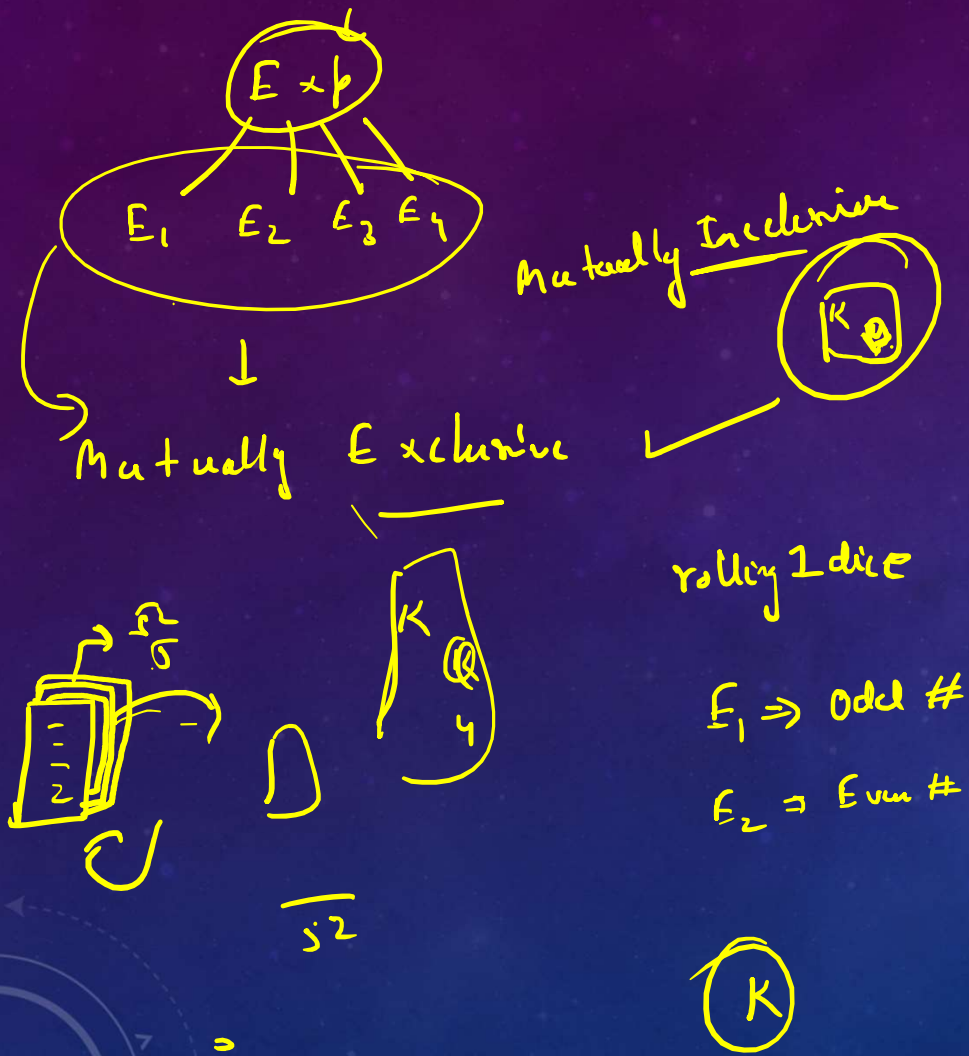
When a coin is tossed twice, the event of occurrence of head in the first throw and the event of occurrence of head in the second throw are independent events.

## ***Difference between mutually exclusive and mutually independent events***

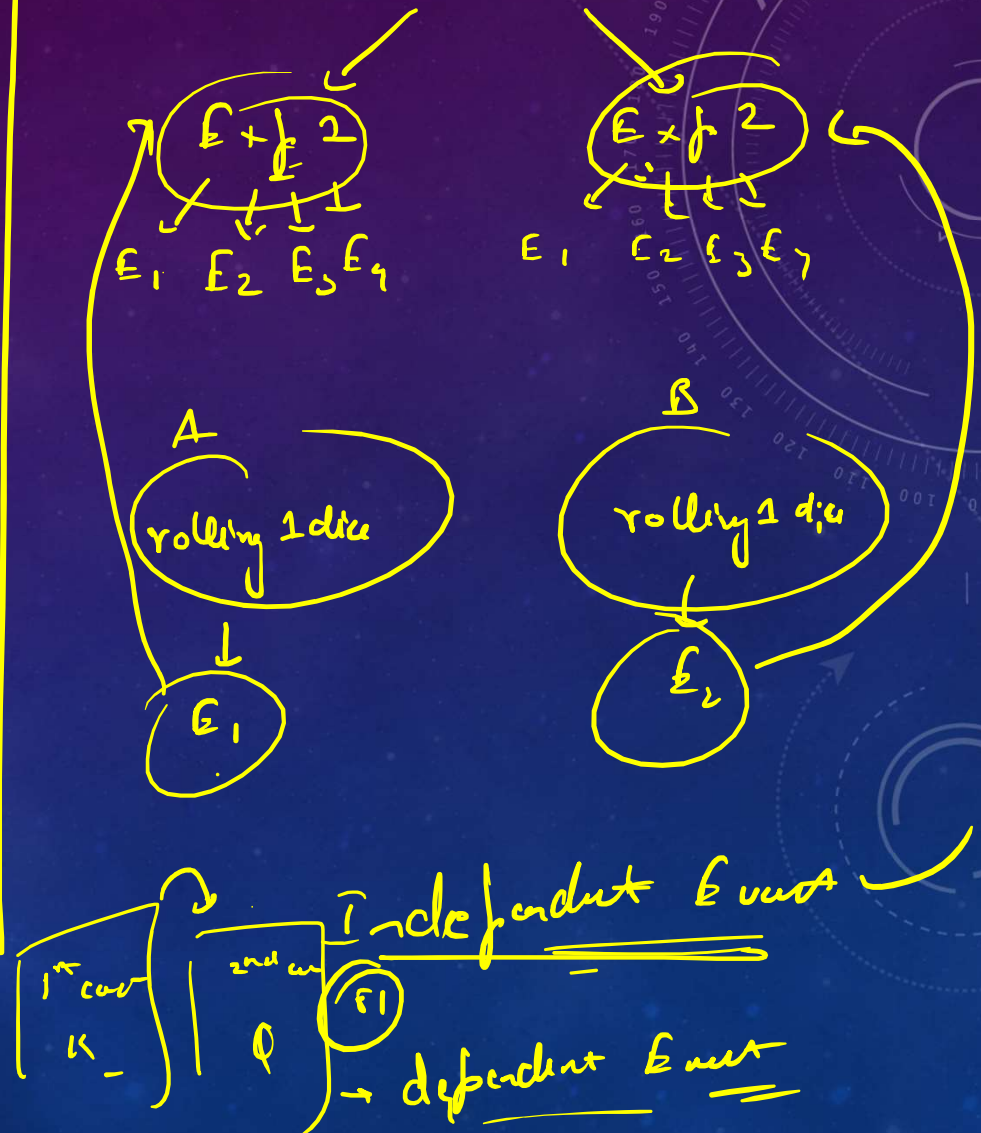
*Mutually exclusiveness is used when the events are taken from the same experiment, where as independence is used when the events are taken from different experiments.*



## One Experiment



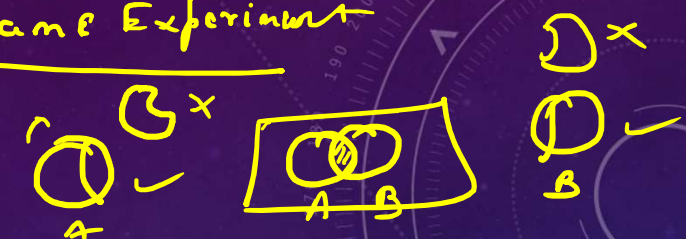
## 2 Experiments



Events from Same Experiment

# ADDITIVE THEOREM OF PROBABILITY

- For Non Mutually Exclusive Events ~ mutually Inclusive → CAN occur together



**Statement:** If A and B are not mutually exclusive events, the probability of the occurrence of either A or B or both is equal to the probability that event A occurs, plus the probability that event B occurs minus the probability of occurrence of the events common to both A and B. In other words the probability of occurrence of at least one of them is given by

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \cup B) = P(A) + P(B) - P(AB)$$

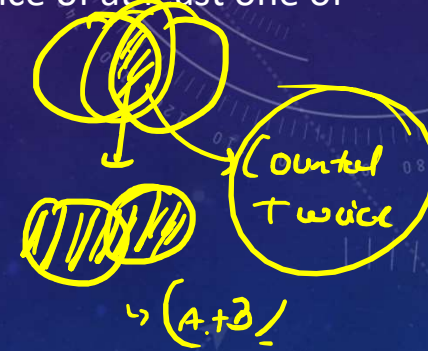
2 parts (A and B)

P(A and B)

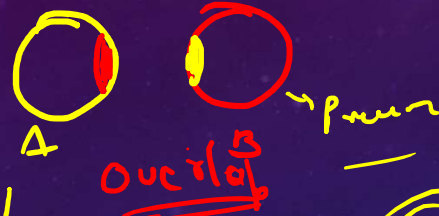
P(A) + P(B)

To make the count 1

Sample space



K



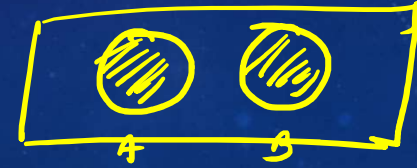
2

- For Mutually Exclusive Events

**Statement:** If A and B are two mutually exclusive events, then the probability of occurrence of either A or B is the sum of the individual probabilities of A and B. Symbolically

$$P(A \text{ or } B) = P(A) + P(B)$$

$$P(A \cup B) = P(A) + P(B)$$



$E \Rightarrow$  hit a target

$$P(S_2) = \frac{2}{5}$$

# ADDITIVE THEOREM OF PROBABILITY - EXAMPLES

- For Non Mutually Exclusive Events

$$P(S_1) = \frac{3}{7}$$

$$P(A \text{ or } B) \Rightarrow P(A) + P(B) - P(A \cap B)$$

1. A shooter is known to hit a target 3 out of 7 shots; whereas another shooter is known to hit the target 2 out of 5 shots. Find the probability of the target being hit at all when both of them try.
2. In a math class of 30 students, 17 are boys and 13 are girls. On a unit test, 4 boys and 5 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A student?

$$P(S_1 \text{ or } S_2) = P(S_1) + P(S_2) - P(S_1 \cap S_2) \rightarrow P(S_1) * P(S_2)$$

$$\Rightarrow \frac{3}{7} + \frac{2}{5} - \frac{3}{7} * \frac{2}{5} \Rightarrow \frac{15+14-6}{35} = \frac{23}{35}$$



$E_1 \Rightarrow$  girl  
 $E_2 \Rightarrow$  A grade

$\Rightarrow$

$$P(g \text{ or } A) = P(g) + P(A) - P(g \cap A)$$

$$= \frac{13}{30} + \frac{9}{30} - \frac{5}{30} = \frac{17}{30}$$

Sample size

$$\frac{5}{30}$$



# SOLUTIONS - 1

- Probability of first shooter hitting the target  $P(A) = 3/7$
- Probability of second shooter hitting the target  $P(B) = 2/5$
- Event A and B are not mutually exclusive as both the shooters may hit target. Hence the additive rule applicable is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{7} + \frac{2}{5} - \left(\frac{3}{7} \times \frac{2}{5}\right)$$

$$= \frac{29}{35} - \frac{6}{35}$$

$$= \frac{23}{35}$$



# ADDITIVE THEOREM OF PROBABILITY - EXAMPLES

$$P(K \text{ or } Q) = P(K) + P(Q)$$
$$\Rightarrow \frac{4}{52} + \frac{4}{52} = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$$
$$P(2 \text{ or } 5) = P(2) + P(5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

- For Mutually Exclusive Events

1. A card is drawn from a pack of 52, what is the probability that it is a king or a queen?
2. A single 6-sided die is rolled. What is the probability of rolling a 2 or a 5?

# SOLUTIONS - 2

- Let Event (A) = Draw of a card of king
- Event (B) Draw of a card of queen
- $P(\text{card draw is king or queen}) = P(\text{card is king}) + P(\text{card is queen})$

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{4}{52} + \frac{4}{52}$$

$$= \frac{1}{13} + \frac{1}{13}$$

$$= \frac{2}{13}$$