- One of two boxes contains 4 red balls and 2 green balls and the second box contains 4 green and two red balls. By design, the probabilities of selecting box 1 or box 2 at random are 1/3 for box 1 and 2/3 for box 2.
- A box is selected at random and a ball is selected at random from it.
- a) Given that the ball selected is red, what is the probability it was selected from the first box?
- b) Given that the ball selected is red, what is the probability it was selected from the second box?

4 2 — R G — B1

2 4 — R G — B2

$P(B1) = 1/3$ , $P(B2) = 2/3$

$$P(B1/R) \Rightarrow \frac{P(R/B1) * P(B1)}{P(R/B1) * P(B1) + P(R/B2) * P(B2)}$$

$$\frac{4/6 * \frac{1}{3}}{4/6 * \frac{1}{3} + 2/6 * 2/3} = \frac{\cancel{2/3 * 1/3}}{\cancel{2/3 * 1/3} + \cancel{1/3 * 2/3}} = \frac{1}{2} = 0.5$$

$P(B2/R) \Rightarrow$

$\hookrightarrow \frac{1}{2}$

A doctor is called to see a sick child. The doctor has prior information that 90% of sick children in that neighborhood have the flu, while the other 10% are sick with 1 measles. Let F stand for an event of a child being sick with flu and M stand for an event of a child being sick with measles. Assume for simplicity that $F \cup M = \Omega$ i.e., that there no other maladies in that neighborhood. A well-known symptom of measles is a rash (the event of having which we denote R). Assume that the probability of having a rash if one has measles is $P(R \mid M) = 0.95$. However, occasionally children with flu also develop rash, and the probability of having a rash if one has flu is $P(R \mid F) = 0.08$. Upon examining the child, the doctor finds a rash. What is the probability that the child has measles?

10 mins

08:28PM

$P(F) = 0.9$ / $P(M) = 0.1$

$P(R/M) = 0.95$

$P(R/F) = 0.08$

$$P(M/R) = \frac{P(R/M) * P(M)}{P(R/M) * P(M) + P(R/F) * P(F)} \Rightarrow \frac{0.95 * 0.1}{0.95 * 0.1 + 0.08 * 0.9}$$

$$\Rightarrow \frac{0.095}{0.095 + 0.072} = \frac{0.095}{0.167} = \frac{95}{167} \Rightarrow 56.89\%$$

$0.5689$

A factory has two machines A and B. Past record shows that machine A produced 60% of the items of output and machine B produced 40% of the items. Further, 2% of the items produced by machine A were defective and 1% produced by machine B were defective. All the items are put into one stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B?

items — 0.6 — A — Defect: $P(D/A) = 0.02$; Perfect $\rightarrow 0.98$

items — 0.4 — B — Defect: $P(D/B) = 0.01$; Perfect $\rightarrow 0.99$

$P(B/D)$

$$\Rightarrow \frac{P(D/B) * P(B)}{P(D/B) * P(B) + P(D/A) * P(A)}$$

$$\Rightarrow \frac{0.01 * 0.4}{0.01 * 0.4 + 0.02 * 0.6} = \frac{0.004}{0.004 + 0.012} = \frac{0.004}{0.016} = \frac{4}{16} = 0.25$$

A laboratory blood test is 99% effective in detecting a certain disease when it is present. However, the test also yields a false-positive result for 0.5% of the healthy person tested (i.e., if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive?

$P(+ve/D) = 0.99$

$P(+ve/H) = 0.005$

$P(D) = 0.001$

$P(H) = 0.999$

$P(D/+ve) = ?$

$$P(D/+ve) = \frac{P(+ve/D) * P(D)}{P(+ve/D) * P(D) + P(+ve/H) * P(H)}$$

$$\Rightarrow \frac{0.99 * 0.001}{0.99 * 0.001 + 0.005 * 0.999} = \frac{0.00099}{0.00099 + 0.004995}$$

$$\Rightarrow \frac{0.00099}{0.005985} = \frac{990}{5985} = 0.1654$$

$\Rightarrow 16.54\%$