Z score → Z table → P-value (AUC) → A_c

Z_α

Df., # tails, α → T-Table → T_α → t-test, CI range

Manually → comparing the value

P-value

Programming → coding

compare the area

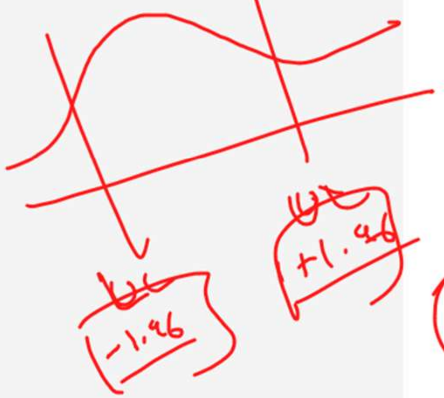
Solving by your Hands
n=1
n=2
n=3
Python = function
Zα
tα

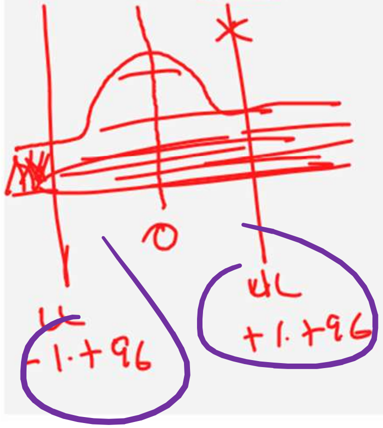
t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300

$(1-\alpha)$
1.17 value
1000
49
50
Table

n = 12
α = 0.05
Left tail Test
d.f. = n − 1
= 12 − 1 = 11
1.796
μ = 0
σ = 1





t-table.pdf

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Tools sjsu.edu / t-table Sign in

CI → UL & LL → Table

T-Table

n = 12

α = 0.05

dof table

df = 11

n-1

α

tail

d.f

± 1.796

t table

-1.796 - +∞

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
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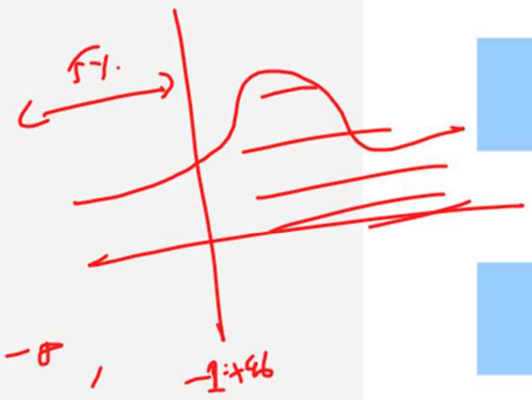
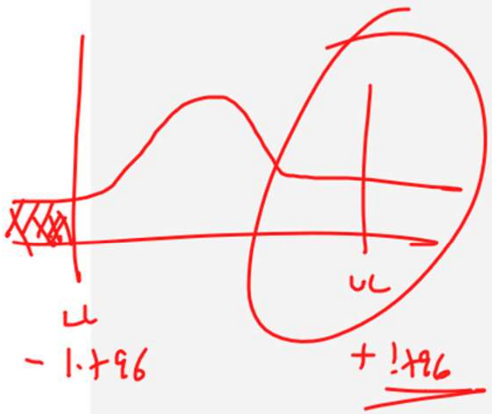
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300





Previously, an organization reported that teenagers spent 4.5 hours per week, on average (μ), on the phone. [The organization thinks that, currently, the mean is ~~higher~~.] Fifteen randomly chosen teenagers were asked how many hours per week they spend on the phone. The sample mean was 4.75 hours with a sample standard deviation of 2.0. Conduct a hypothesis test for 90% confidence level.

$H_0 \Rightarrow \mu \leq 4.5$

$H_A \Rightarrow \mu > 4.5$

→ Right tail Test

T-test

$\boxed{Z\text{-Test}}$

$\boxed{1T, 10\%}$

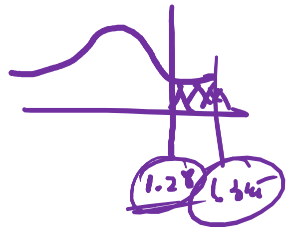
$z_\alpha = 1.28$

$\mu = 4.5$

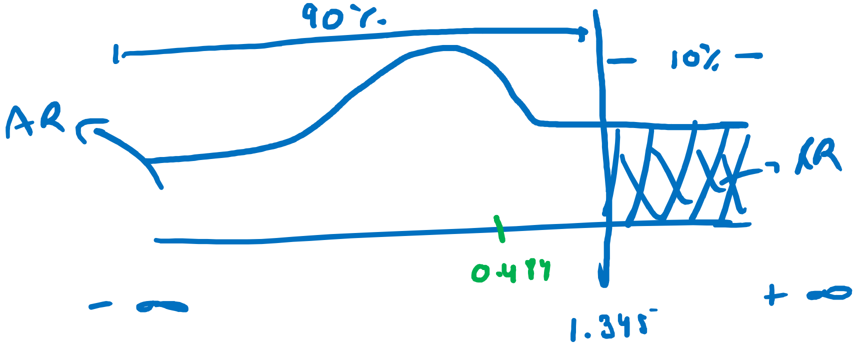
$\boxed{\bar{x} = 4.75}$

$n = 15$

$\sigma = 2$



$$t_{calc} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$



CL = 90%

$\alpha = 10\%$

$\alpha = 0.10$

$d.f = n - 1 = 15 - 1 = 14$

One-Tail

$t_\alpha = 1.345$

$$t_{calc} = \frac{4.75 - 4.5}{2/\sqrt{15}}$$

$$= \frac{0.25 * \sqrt{15}}{2}$$

$$= 0.125 * \sqrt{15} = 0.484$$

→ AR →

Failed to reject H_0, Rejected H_A

A manufacturer of running shoes knows that the average lifetime for a particular model of shoes is 15 months. Someone in the research and development division of the shoe company claims to have developed a longer lasting product. This new product was worn by 30 individuals and lasted on average for 17 months. The variability of the original shoe is estimated based on the standard deviation of the new group which is 5.5 months. Is the designer's claim of a better shoe supported by the trial results? level of significance of p = 0.10.

T - Test

$\mu = 15$

$\bar{X} = 17$

$\boxed{t_{calc} = \frac{\bar{X} - \mu}{\sigma / \sqrt{N}}}$

$\sigma = 5.5$

$N = 30$

$d.f = 29$

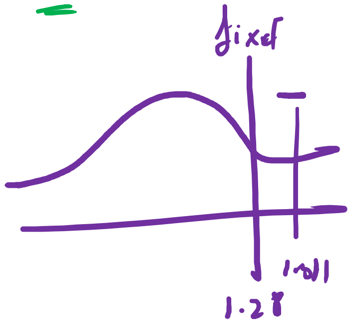
$\alpha = 0.10$

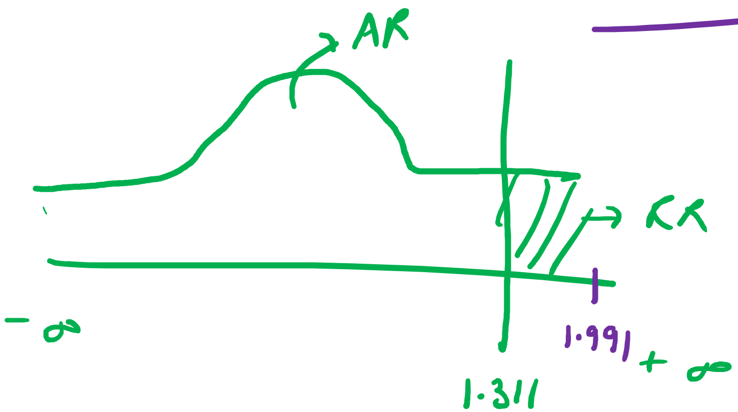
One-tail Test

$t_\alpha = 1.311$

$H_0 = \mu \le 15 \rightarrow$ Right Tail Test

✓ $H_A = \mu > 15$





$t_{calc} = \frac{17 - 15}{5.5 / \sqrt{30}}$

$t_{calc} \Rightarrow \frac{2 * \sqrt{30}}{5.5}$

$\boxed{t_{calc} \rightarrow 1.991}$ → RR → Rejecting H_0, Accept H_A

Average heart rate for Americans is 72 beats/minute. A group of 25 individuals participated in an aerobics fitness program to lower their heart rate. After six months the group was evaluated to identify is the program had significantly slowed their heart. The mean heart rate for the group was 69 beats/minute with a standard deviation of 6.5. Was the aerobics program effective in lowering heart rate?

$72 \rightarrow \mu$, $69 \rightarrow \bar{x}$

T-Test

$\mu = 72$

$\bar{x} = 69$

$\sigma = 6.5$

$N = 25$

$d.f = 25 - 1 = 24$

$\alpha = 0.05$

one-Tail Test

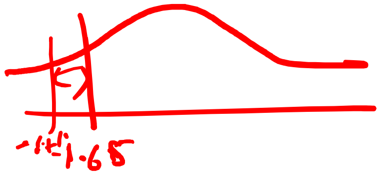
$t_\alpha = \pm 1.711$

$H_0 \Rightarrow \mu \geq 72$

$\checkmark H_A \Rightarrow \mu < 72$

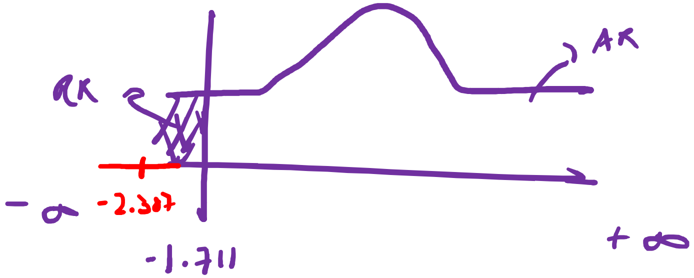
$\rightarrow$ Left Tail Test

z - Test



$$t_{calc} = \frac{\bar{x} - \mu}{\sigma/\sqrt{N}}$$

$$\Rightarrow \frac{69 - 72}{6.5/\sqrt{25}}$$



$\Rightarrow -2.307 \rightarrow$ RR $\rightarrow$ Reject H_0, Accept H_A

2 - Sample T-Test

$$t_{calc} \Rightarrow \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$d.f. \Rightarrow d.f._1 + d.f._2 = (n_1 - 1) + (n_2 - 1) \Rightarrow \boxed{n_1 + n_2 - 2 = d.f.}$$

Mio, a restaurant owner, wants to test if her two managers perform at the same level. To do that, she collects data about the number of customer complaints at two random samples of shifts (one for each manager). Here is a summary of the results:

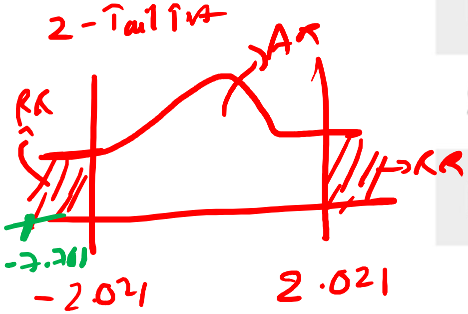
$H_0 \Rightarrow \bar{x}_1 = \bar{x}_2$

$H_A \Rightarrow \bar{x}_1 \neq \bar{x}_2$

2-Tail Test

$t_\alpha = \pm 2.021$

$t_{calc} \Rightarrow \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$



	Manager A (x_1)	Manager B (x_2)
Mean	$\bar{x}_1 =$ 4 complaints	$\bar{x}_2 =$ 5 complaints
Standard deviation	σ_1 0.3 complaints	σ_2 0.5 complaints
Number of shifts	$n_1 =$ 19	$n_2 =$ 21

$d.f. = n_1 + n_2 - 2$

T-Test $\alpha = 0.05$

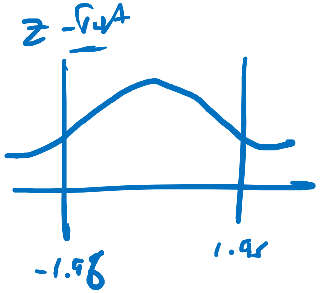
$d.f. = 19 + 21 - 2 = 38$

~ 40

To learn, dist unknown → T or Bernoulli

Mio wants to use these results to carry out a two-sample t test to determine if the mean numbers of complaints are significantly different for the two managers.

Z-Test



$$t_{calc} = \frac{4 - 5}{\sqrt{\dfrac{0.3^2}{19} + \dfrac{0.5^2}{21}}} = \frac{-1}{\sqrt{\dfrac{0.09}{19} + \dfrac{0.25}{21}}} = \frac{-1}{\sqrt{0.0047 + 0.0119}} = \frac{-1}{\sqrt{0.0166}}$$

$$\Rightarrow \frac{-1}{0.129} = -7.761$$

↳ RR

Reject H_0, Accept H_A

A research study was conducted to examine the differences between older and younger adults on perceived life satisfaction. A pilot study was conducted to examine this hypothesis. Ten older adults (over the age of 70) and ten younger adults (between 20 and 30) were giving a life satisfaction test (known to have high reliability and validity). Scores on the measure range from 0 to 60 with high scores indicative of high life satisfaction; low scores indicative of low life satisfaction. The data are presented below. Compute the appropriate t-test.

$H_0 - \bar{x}_1 \geq \bar{x}_2$

$H_A - \bar{x}_1 < \bar{x}_2$

left tail

$\alpha = 0.05$

$t_\alpha = \pm 2.101$

$d.f. = 10 + 10 - 2 = 18$

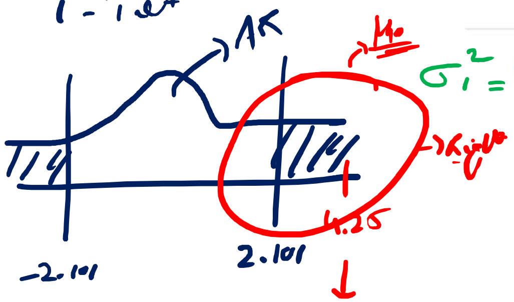
$H_0 = \bar{x}_1 = \bar{x}_2$

$H_A = \bar{x}_1 \neq \bar{x}_2$

2-Tail Test

T-Test

Older Adults	Younger Adults
$\bar{x}_1 =$ Mean = 44.5	$\bar{x}_2 =$ Mean = 28.1
$\sigma_1 =$ S = 8.682677518 $n_1 = 10$	$\sigma_2 =$ S = 8.543353492 $n_2 = 10$
$\sigma_1^2 =$ S2 = 75.388888888	$\sigma_2^2 =$ S2 = 72.988888888



$$t_{calc} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \Rightarrow \frac{44.5 - 28.1}{\sqrt{\frac{75.39}{10} + \frac{72.99}{10}}} = \frac{16.4}{\sqrt{14.838}}$$

$$\Rightarrow \frac{16.4}{3.85} = \boxed{4.25}$$

Reject H_0, Accept H_A

→ Z-Test
→ T-Test } One / Two Sample → Analysis of Mean

for > 2 samples we use ANOVA (Analysis of Variance)

↳ Last Topic of Stats

Very much Imp → Stats → along Data Speak ↳ Python Inference of Hypothesis

Imp → ML

Data Science

90% → Data Preprocess

10% → ML

EDA ↳ Exploring Data Analysis

Th - ML

Pr - ML

Library

Matrix