→ >1 Experiment

MULTIPLICATIVE THEOREM OF PROBABILITY

Exp 1 → Exp 2
one After other

- **For Independent Events**

Statement: The theorem states that the probability of the simultaneous occurrence of two events that are independent is given by the product of their individual probabilities.

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$P(AB) = P(A) \times P(B)$$

Exp 1 → Exp 2

Exp 1
Exp 2 } - Simultaneous

toss a coin

Exp → tossing 2 coins

1st 2nd

- **For Dependent Events (Conditional Probability)**

If we recall dependent event(), the earlier stated multiplicative theorem is not applicable for dependent events. For dependent event, we have an another theorem called the conditional probability which is given as:
*The probability of **event B given event A** equals the probability of **event A and event B** divided by the probability of **event A***

$P(A \cap B) \rightarrow P(A) \times P(B/A)$

Exp → Toss a coin

1 Chance → 2 chance, 1 Extra

2 chance → 2 chance

$$P(B/A) = \frac{P(AB)}{P(A)} \text{ or } \frac{P(A \cap B)}{P(A)}$$

MULTIPLICATIVE THEOREM/CONDITIONAL PROBABILITY - EXAMPLES

Independent Event: You have a cowboy hat, a top hat, and an Indonesian hat called a songkok. You also have four shirts: white, black, green, and pink. If you choose one hat and one shirt at random, what is the probability that you choose the songkok and the black shirt?

$\boxed{3}$ $\boxed{4}$

$P(A \cap B) = P(A) * P(B)$

$P(S \cap B) = P(S) * P(B)$

$\Rightarrow \frac{1}{3} * \frac{1}{4} = \frac{1}{12}$

MULTIPLICATIVE THEOREM/CONDITIONAL PROBABILITY - EXAMPLES

Independent Event: You have a cowboy hat, a top hat, and an Indonesian hat called a songkok. You also have four shirts: white, black, green, and pink. If you choose one hat and one shirt at random, what is the probability that you choose the songkok and the black shirt?

The two events are independent events; the choice of hat has no effect on the choice of shirt.
There are three different hats, so the probability of choosing the songkok is 1/3 .
There are four different shirts, so the probability of choosing the black shirt is 1/4.

So, by the Multiplication Rule:

P(songkok and black shirt)=(1/3)·(1/4)=1/12

MULTIPLICATIVE THEOREM/CONDITIONAL PROBABILITY - EXAMPLES

$$P(A \cap B) = P(A) * P(B/A)$$

Bag

Dependent Event: An urn contains 20 red and 10 blue balls. Two balls are drawn from a bag one after the other without replacement. What is the probability that both the balls drawn are red?

$$n(S) = 30$$

$$P(R \cap R) = P(R) * P(R/R)$$

$$\frac{20}{30} * \frac{19}{29}$$

$$\rightarrow \frac{38}{87}$$

Exp → Drawing a Ball

B_1 → B_2

30   ~~30~~ 29

MULTIPLICATIVE THEOREM/CONDITIONAL PROBABILITY - EXAMPLES

Dependent Event: An urn contains 20 red and 10 blue balls. Two balls are drawn from a bag one after the other without replacement. What is the probability that both the balls drawn are red?

Solution: Let A and B denote the events that first and second ball drawn are red balls. We have to find P(A∩B) or P(AB).

P(A) = P(red balls in first draw) = 20/30

Now, only 19 red balls and 10 blue balls are left in the bag. Probability of drawing a red ball in second draw too is an example of conditional probability where drawing of second ball depends on the drawing of first ball.

Hence Conditional probability of B on A will be,

P(B|A) = 19/29

By multiplication rule of probability,

P(A∩B) = P(A) × P(B|A)

$$P(A \cap B) = \frac{20}{30} \times \frac{19}{29} = \frac{38}{87}$$

For detailed discussion on multiplication rule of probability, download Byju's-the learning app.

$$P(A \cap B) = P(A) * P(B/A)$$

$$P(B \cap A) = P(B) * P(A/B)$$

$A \cap B$

$B \cap A$

$A \cap B = B \cap A$

$$P(A) * P(B/A) = P(B) * P(A|B)$$

$$\boxed{P(A|B) = \frac{P(B/A) * P(A)}{P(B)}}$$

→ Bayes Theorem

BAYES THEOREM

Covid Pandemic → RT-PCR: +ve, -ve

Actual: -ve, +ve

- Named after Thomas Bayes
- Bayes' Theorem describes the probability of an event, based on prior knowledge of conditions that might be related to the event.
- Note: This *conditional probability is known as a hypothesis*. This hypothesis is calculated through previous evidence or knowledge. This conditional probability is the probability of the occurrence of an event, given that some other event has already happened.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Hypothesis

Evi

	RT-PCR	Actual
1	+ve	+ve
2	+ve	-ve
3	-ve	+ve
4	-ve	-ve

Example:

Suppose the weather of the day is cloudy. Now, you need to know whether it would rain today, given the cloudiness of the day. Therefore, you are supposed to calculate the probability of rainfall, given the evidence of cloudiness.

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

A|B

B|A

Salary

Salary

60 | 63

Excellent

Money

ATM CASH

A/C

ATM

A

B

P (ATM / A/C)

P (ATM / A/C)

$\boxed{P(A/C \mid ATM)}$

Fclup?

84 / 268

BAYES THEOREM – FROM WHERE IT CAME?

We know from Conditional Probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

Rearranging Equation 1:

$P(A \cap B)$ = $P(A|B)$ * $P(B)$

Similarly: $P(B \cap A)$ = $P(B|A)$ * $P(A)$

Since $P(A \cap B)$ = $P(B \cap A)$

Hence, P (B|A).P (A) = P (A|B).P (B)

Finally: $\longrightarrow$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

BAYES THEOREM – GENERALIZED FORM?

when P(B) is not known / cannot be calculated

$$p(H \mid E) = \frac{p(E \mid H)\ p(H)}{p(E)}$$

$$\Pr(H|E) = \frac{\Pr(E|H)\Pr(H)}{\Pr(E|H)\Pr(H) + \Pr(E|\text{not } H)\Pr(\text{not } H)}$$

	A	A'
(B)	$A \cap B$	$A' \cap B$
B'	$A \cap B'$	$A' \cap B'$

A'

$P(B) = P(A \cap B) + P(A' \cap B)$

$\Rightarrow P(B/A) * P(A) + P(B/A') * P(A')$

$$P(A/B) \Rightarrow \frac{P(B/A) * P(A)}{P(B/A) * P(A) + P(B/A') * P(A')}$$

BAYES THEOREM - EXAMPLES

Cancer Test

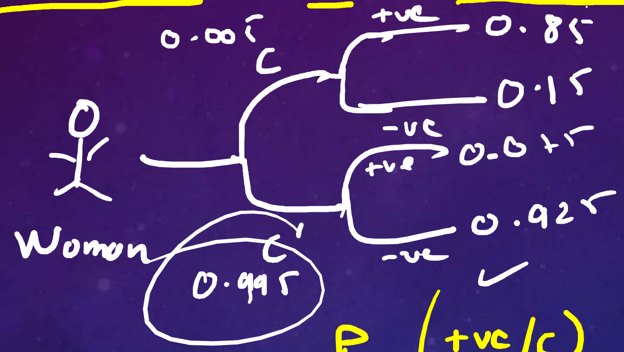
- Epidemiologists claim that the probability of breast cancer among Caucasian women in their mid -50s is 0.005. An established test identified people who had breast cancer and those that were healthy. A new mammography test in clinical trials has a probability of 0.85 for detecting cancer correctly. In women without breast cancer, it has a chance of 0.925 for a negative result. If a 55-year-old Caucasian woman tests positive for breast cancer, what is the probability that she, in fact, has breast cancer?

$-ve / C'$

$P(\text{Cancer}) = 0.005$

$P(+ve \mid C) = 0.85$

$P(-ve \mid c') = 0.925$



$$P(C/+ve) \Rightarrow \frac{P(+ve/C) * P(C)}{P(+ve/C) * P(C) + P(+ve/C') * P(C')}$$

$$\Rightarrow \frac{0.85 * 0.005}{0.85 * 0.005 + 0.075 * 0.995}$$

BAYES THEOREM - EXAMPLES

- Epidemiologists claim that the probability of breast cancer among Caucasian women in their mid -50s is 0.005. An established test identified people who had breast cancer and those that were healthy. A new mammography test in clinical trials has a probability of 0.85 for detecting cancer correctly. In women without breast cancer, it has a chance of 0.925 for a negative result. If a 55-year-old Caucasian woman tests positive for breast cancer, what is the probability that she, in fact, has breast cancer?

Solution:

- P(Cancer) = 0.005
- P(Test Positive | Cancer) = 0.85
- P(Test -ve|No cancer) = 0.925
- P(Cancer|Test +ve)= P(Cancer) * P(Test Positive | Cancer) / P(Test Positive)

	Probability of having Cancer or not	Test being Positive	Test being Negative
Cancer	0.005	0.005*0.85 = 0.00425	0.005*0.15 = 0.00075
No Cancer	0.995	0.995*0.075 = 0.074625	0.995*0.925 = 0.920375
Total	1.00	0.078875	0.921125

BAYES THEOREM - EXAMPLES

(SPAM) (NOT SPAM)

85% $\frac{85 \times 1}{100}$ 0.85

0.1% $\frac{0.1}{100} = 0.001$

Symantec works by having users train the system. It looks for patterns in the words in emails marked as spam by the user. For example, it may have learned that the word "free" appears in 20% of the emails marked as spam. Assuming 0.1% of non-spam mail includes the word "free" and 50% of all emails received by the user is spam, find the probability that a mail is a spam if the word "free" appears in it.

$$P(A|B) = \frac{P(B/A) * P(A)}{P(B/A) * P(A) + P(B/A') * P(A')}$$

Solution:

$P(F/S) = 0.20$

$P(F/S') = 0.001$

$P(S) = 0.50$ | $P(S') = 0.50$

$$P(S/F) = \frac{P(F/S) * P(S)}{P(F/S) * P(S) + P(F/S') * P(S')}$$

$$\Rightarrow \frac{0.2 * 0.5}{0.2 * 0.5 + 0.001 * 0.5} = \frac{0.1}{0.1 + 0.0005} = \frac{0.1}{0.1005} = \frac{1000}{1005}$$

$\boxed{\Rightarrow 0.995}$