likelihood

chances of occurrence

Mathematical Tool

Basic + Adv

PROBABILITY AND DISTRIBUTION

Distribution of Data

N.D., B.D.

PROBABILITY → [0, 1]

0.78 × 100 78%

0.635

p * 100 => %

Impossible Event → 7 on a dice, 12 in cards → 0

- Probability implies 'likelihood' or 'chance'. When an event is certain to happen then the probability of occurrence of that event is 1 and when it is certain that the event cannot happen then the probability of that event is 0.

Sure Event → Sun rising → 1

- Hence the value of probability ranges from 0 to 1.

Classical Definition of Probability

- As the name suggests the classical approach to defining probability is the oldest approach. It states that if there are n exhaustive, mutually exclusive and equally likely cases out of which m cases are favourable to the happening of event A,
- Then the probabilities of event A is defined as given by the following probability function:

1, 2, 3, 4, 5, 6

roll a dice → Experiment

$$P(A) = \frac{\text{Number of favorable outcomes to A}}{\text{Total number of outcomes}}$$

Event

Experiment

$P(\text{Prime \#}) = \frac{3}{6} = 1/2$

$P(\text{composite \#}) = \frac{2}{6} = 1/3$

Event → Even # of any

Prime

odd

composite

multiple of 2

→ Exhaustive
↳ covering Everything → Every possible outcome (cases)
→ for an Experiment

→ Mutually Exclusive → Any 2 outcomes cannot occur together

→ Equally likely
↳ same chances to occur

Event | Experiment → outcome

Use case

The thing/set of things we are evaluating.

Tossing a coin
→ Head Appearing
1/2 ⇒

2
H, H
T, T
H, T
T, H → 0 H&T 0 H&T

→ 1, 2
→ 1, 1
1, 3
⋮

4, 2, 3 ... K

1 (2,2)

→ (1,1) | 1, 2 | (1,3) ....

$n=2$

1
toss a coin → Exp
→ H, T
Class of student
↳ 30
Exp → choose 1 student

$n=30$

S1
S2
S3 ... S30
S31

rolling a dice
→ 1, 2, 3, 4, 5, 6
$n=6$

PROBABILITY

Example

Problem Statement:

- A coin is tossed. What is the probability of getting a head?

$$P(A) = \frac{\text{Number of favorable outcomes to A}}{\text{Total number of outcomes}}$$

Solution:

- Number of outcomes favourable to head (m) = 1
- Total number of outcomes (n) = 2 (i.e. head or tail)

PROBABILITY - BASIC CONCEPTS

fair coin → Random Experiment

Tampered coin → n → Baised Experiment

- **Random Experiment**

An experiment is said to be a random experiment, if it's out-come can't be predicted with certainty.

Example

If a coin is tossed, we can't say, whether head or tail will appear. So it is a random experiment.

list → duplicate

set → unique Elem

- **Sample Space**

The set of all possible out-comes of an experiment is called the sample space. It is denoted by 'S' and its number of elements are n(s).

Example

In throwing a dice, the number that appears at top is any one of 1,2,3,4,5,6. So here:

S ={1,2,3,4,5,6} and n(s) = 6

Similarly in the case of a coin, S={Head,Tail} or {H,T} and n(s)=2.

PROBABILITY - BASIC CONCEPTS

Event

Every subset of a sample space is an event. It is denoted by 'E'.

Example

In throwing a dice S={1,2,3,4,5,6}, the appearance of an even number will be the event E={2,4,6}.

Clearly E is a sub set of S.

Sample Space

1
2
3
4
5
6

Exhaustive Events

(1, {1,2} {2,3,4} {2,3,5}
(2, {1,3} 2,4,6) {1,4,5,2}
(3, {1,4} {1,2,3,
(4 (1,5) 1,4,6,
(5 (1,6) 1,2,3,4
(6 (1,1(

PROBABILITY - BASIC CONCEPTS

- **Equally likely events**

Events are said to be equally likely, if the probability of occurrence of the events are same.

Example

When a dice is thrown, all the six faces {1,2,3,4,5,6} are equally likely to come up.

- **Exhaustive events**

When every possible out come of an experiment is considered.

Example

A dice is thrown, cases 1,2,3,4,5,6 form an exhaustive set of events.

PROBABILITY - BASIC CONCEPTS

- **Mutually exclusive or Disjoint event**

If two or more events can't occur simultaneously,

that is no two of them can occur together.

Example

When a coin is tossed, the event of occurrence of a head and the event of occurrence of a tail are mutually exclusive events.

- **Independent or Mutually independent events**

Two or more events are said to be independent if occurrence or non-occurrence of any of them does not affect the probability of occurrence or non-occurrence of the other event.

Example

When a coin is tossed twice, the event of occurrence of head in the first throw and the event of occurrence of head in the second throw are independent events.

Difference between mutually exclusive and mutually independent events

Mutually exclusiveness is used when the events are taken from the same experiment, where as independence is used when the events are taken from different experiments.

One Experiment

Exp

E_1 E_2 E_3 E_4

Mutually Exclusive

Mutually Inclusive

K Q

K Q 4

52

rolling 1 dice

$E_1 \Rightarrow$ Odd #

$E_2 \Rightarrow$ Even #

K

2 Experiments

Exp 2

E_1 E_2 E_3 E_4

Exp 2

E_1 E_2 E_3 E_4

A

rolling 1 dice

E_1

B

rolling 1 dice

E_2

Independent Event

1st card

K

2nd card

Q

E1

→ dependent Event

ADDITIVE THEOREM OF PROBABILITY

Events from Same Experiment

- For Non Mutually Exclusive Events

→ mutually Inclusive → CAN occur together

Statement: If A and B are not mutually exclusive events, the probability of the occurrence of either A or B or both is equal to the probability that event A occurs, plus the probability that event B occurs minus the probability of occurrence of the events common to both A and B. In other words the probability of occurrence of at least one of them is given by

2 both (P(A∩B))

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

$$P(A \cup B) = P(A) + P(B) - P(AB)$$

P(A∩B) → P(A) * P(B)

To make the count 1

Sample Space

Counted Twice

(A+B)

Overlap

- **For Mutually Exclusive Events**

Statement: If A and B are two mutually exclusive events, then the probability of occurrence of either A or B is the sum of the individual probabilities of A and B. Symbolically

$$P(A\ or\ B) = P(A) + P(B)$$

$$P(A \cup B) = P(A) + P(B)$$

(A+B)

E ⇒ hit a target $P(S_2) = \frac{2}{5}$

ADDITIVE THEOREM OF PROBABILITY - EXAMPLES

$P(A \text{ or } B) \Rightarrow P(A) + P(B) - P(A \cap B)$

- For Non Mutually Exclusive Events $P(S_1) = \frac{3}{7}$

1. A shooter is known to hit a target 3 out of 7 shots; whereas another shooter is known to hit the target 2 out of 5 shots. Find the probability of the target being hit at all when both of them try.

2. In a math class of 30 students, 17 are boys and 13 are girls. On a unit test, 4 boys and 5 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A student?

$$P(S_1 \text{ or } S_2) = P(S_1) + P(S_2) - P(S_1 \cap S_2) \rightarrow P(S_1) * P(S_2)$$

$$\Rightarrow \frac{3}{7} + \frac{2}{5} - \frac{3}{7} * \frac{2}{5} \Rightarrow \frac{15+14-6}{35} = \frac{23}{35}$$

$E_1 \Rightarrow$ girl

$E_2 \Rightarrow$ A grade

$$\Rightarrow P(g \text{ or } A) = P(g) + P(A) - P(g \cap A) \rightarrow \text{Samples} \ldots \frac{5}{30}$$

$$= \frac{13}{30} + \frac{9}{30} - \frac{5}{30} = \frac{17}{30}$$

SOLUTIONS - 1

- Probability of first shooter hitting the target P (A) = 3/7
- Probability of second shooter hitting the target P (B) = 2/5
- Event A and B are not mutually exclusive as both the shooters may hit target. Hence the additive rule applicable is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{7} + \frac{2}{5} - (\frac{3}{7} \times \frac{2}{5})$$

$$= \frac{29}{35} - \frac{6}{35}$$

$$= \frac{23}{35}$$

ADDITIVE THEOREM OF PROBABILITY - EXAMPLES

$P(K \text{ or } Q) = P(K) + P(Q)$

$\Rightarrow \frac{\cancel{4}^{1}}{\cancel{52}_{13}} + \frac{\cancel{4}^{1}}{\cancel{52}_{13}} = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$

$P(2 \text{ or } 5) \Rightarrow P(2) + P(5) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$

- For Mutually Exclusive Events

1. A card is drawn from a pack of 52, what is the probability that it is a king or a queen?
2. A single 6-sided die is rolled. What is the probability of rolling a 2 or a 5?

SOLUTIONS - 2

- Let Event (A) = Draw of a card of king
- Event (B) Draw of a card of queen
- P (card draw is king or queen) = P (card is king) + P (card is queen)

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{4}{52} + \frac{4}{52}$$

$$= \frac{1}{13} + \frac{1}{13}$$

$$= \frac{2}{13}$$