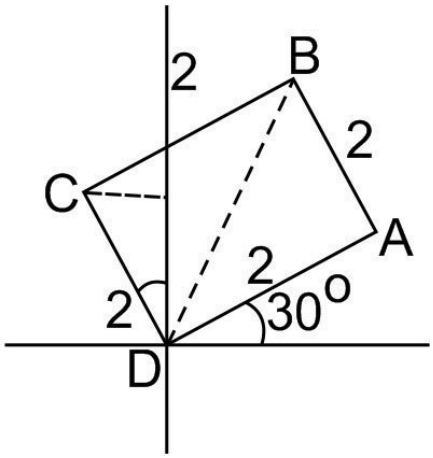
Matix solving

February 14, 2019

1 Question

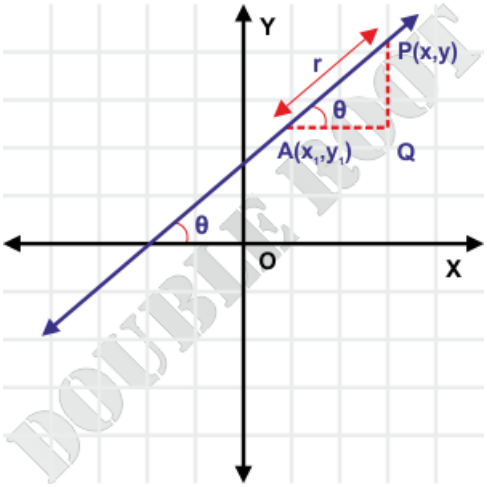
A square, of each side 2, lies above the x-axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30with the positive direction of the x-axis, then find the sum of the x-coordinates of the vertices of the square.

2 solution



Consider a line which has slope tanθ and passes through the point A(x1, y1).

Let P(x, y) be a point on the line which is at a distance r from the point A.



We have, $\cos\theta$ = AQ/AP = (x-x1)/r and $\sin\theta$ = PQ/AP= (y-y1)/r

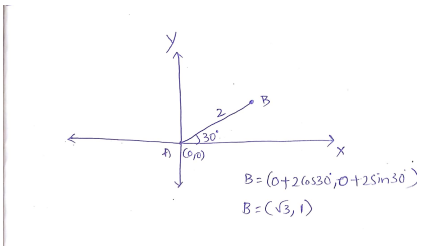
This gives the coordinates of P as (x1 + r$\cos\theta$, y1 + r$\sin\theta$).

And this is the parametric form of the equation of a straight line: x = x1 + r$\cos\theta$, y = y1 + r$\sin\theta$.

This can also be written in a fancy way as xx1/$\cos\theta$=yy1/$\sin\theta$=r

Given length of side is 2 units and one of the vertex of the square is origin A(x1,y1)=(0,0)

Let other vertices be B,C,D

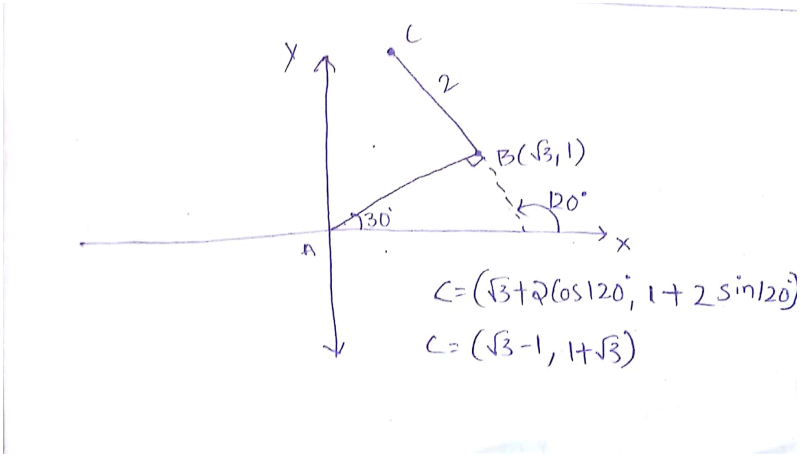


Line AB makes an angle of 30degrees with the positive direction of x-axis in anticlockwise direction then coordinates of the point which is 2 units away from origin and lie above x-axis (i.e Point B) can be written as

B(x2,y2)=(x1+2cos30° , x2+2sin30°)

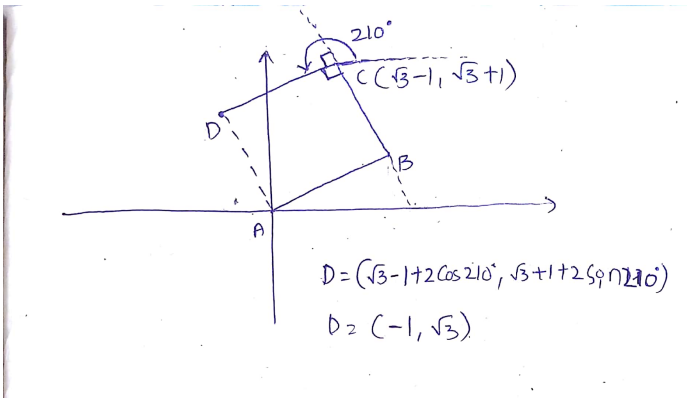
B(x2,y2)=(0+2cos30° , 0+2sin30°)

B(x2,y2)=$(\sqrt{3}, 1)$



Similarly line BC makes an angle of 120degrees(30+90) with the positive direction of x-axis in anticlockwise direction then coordinates of the point which is 2 units away from B and lie above x-axis (i.e Point C) can be written as

C(x3,y3)=(x2+2cos(120)° , y2+2sin(120)°)
C(x3,y3)=($\sqrt{3}$ + 2cos(120)° , 1+2sin(120)°)
C(x3,y3)=($\sqrt{3}-1, 1+\sqrt{3}$)



Similarly line CD makes an angle of 210degrees(120+90) with the positive direction of x-axis in anticlockwise direction then coordinates of the point(i.e Point D) which is 2 units away from C and also 2 units away from A (because it is a square) can be written as

D(x4,y4)=(x3+2cos(210)° , y3+2sin(210)°)
D(x4,y4)=($\sqrt{3} - 1 + 2cos(210)°$,$\sqrt{3} + 1 + 2sin(210)°$)
D(x4,y4)=(-1,$\sqrt{3}$)
Let X be sum of x-cordinates
X=x1+x2+x3+x4
X=0+ $\sqrt{3} + (\sqrt{3} - 1) + (-1)$
X=1.464