

INTRO TO AI AND ML

(EE1390)

MATRIX PROJECT

V.REDDYKHAJA , EE17BTECH11044

B.MANOHARREDDY , EE17BTECH11008

14 Feb 2018

PROBLEM:13

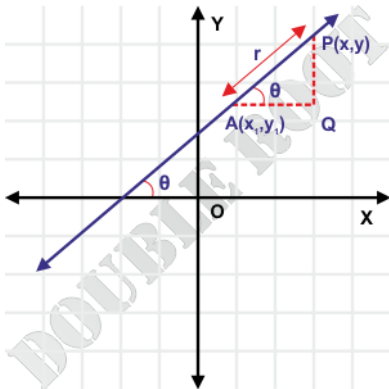
A square of side length 2, lies above the x -axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30° with the positive direction of the x -axis.

Find the sum of the x -coordinates of the vertices of the square?

Solution

Consider a line which has slope $\tan\theta$ and passes through the point $A(x_1, y_1)$.

Let $P(x, y)$ be a point on the line which is at a distance r from the point A .



We have, $\cos\theta = AQ/AP = (x-x_1)/r$ and $\sin\theta = PQ/AP = (y-y_1)/r$

This gives the coordinates of P as $(x_1 + r\cos\theta, y_1 + r\sin\theta)$.

let

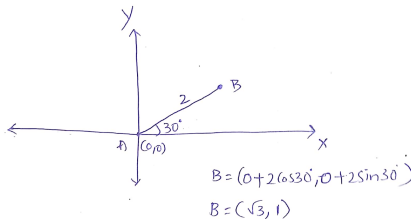
$$p = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_1 & \cos\theta \\ y_1 & \sin\theta \end{bmatrix} \begin{bmatrix} 1 \\ r \end{bmatrix}$$

Given length of side is 2 units and one of the vertex of the square is origin $A(x_1, y_1) = (0, 0)$

let the other vertices of square be B, C, D

Line AB makes an angle of 30° with the positive direction of x-axis in anticlockwise direction



coordinates of the point which is 2 units away from origin and lie above x-axis (i.e Point B) can be written as

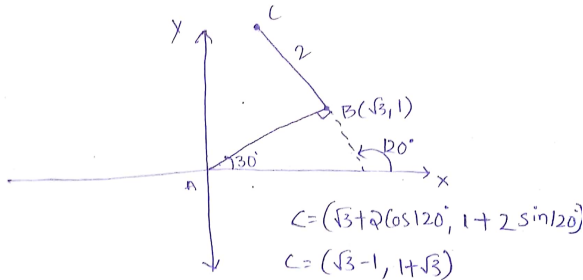
$$B = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 & \cos 30^\circ \\ y_1 & \sin 30^\circ \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & \sqrt{3}/2 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$B = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$$

Similarly line BC makes an angle of 120° (i.e. $(30+90)$) with the positive direction of x-axis in anticlockwise direction



coordinates of the point which is 2 units away from B and lie above x-axis (i.e Point C) can be written as

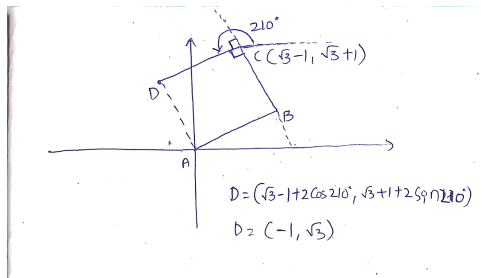
$$C = \begin{bmatrix} x_3 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_2 & \cos 120^\circ \\ y_2 & \sin 120^\circ \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} \sqrt{3} & -1/2 \\ 1 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$C = \begin{bmatrix} \sqrt{3} - 1 \\ 1 + \sqrt{3} \end{bmatrix}$$

line CD makes an angle of 210° (i.e. $(120+90)$) with the positive direction of x-axis in anticlockwise direction



coordinates of the point(i.e Point D) which is 2 units away from C and also 2 units away from A (because it is a square) can be written as

$$D = \begin{bmatrix} x_4 \\ y_4 \end{bmatrix}$$

$$\begin{bmatrix} x_4 \\ y_4 \end{bmatrix} = \begin{bmatrix} x_3 & \cos 210^\circ \\ y_3 & \sin 210^\circ \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x_4 \\ y_4 \end{bmatrix} = \begin{bmatrix} \sqrt{3} - 1 & -\sqrt{3}/2 \\ \sqrt{3} + 1 & -1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$D = \begin{bmatrix} -1 \\ \sqrt{3} \end{bmatrix}$$

Let X be sum of x-coordinates

$$X = x_1 + x_2 + x_3 + x_4$$

$$X = 0 + \sqrt{3} + (\sqrt{3} - 1) + (-1)$$

$$X = 1.464$$