INTRO TO AI AND ML

(EE1390)

MATRIX PROJECT

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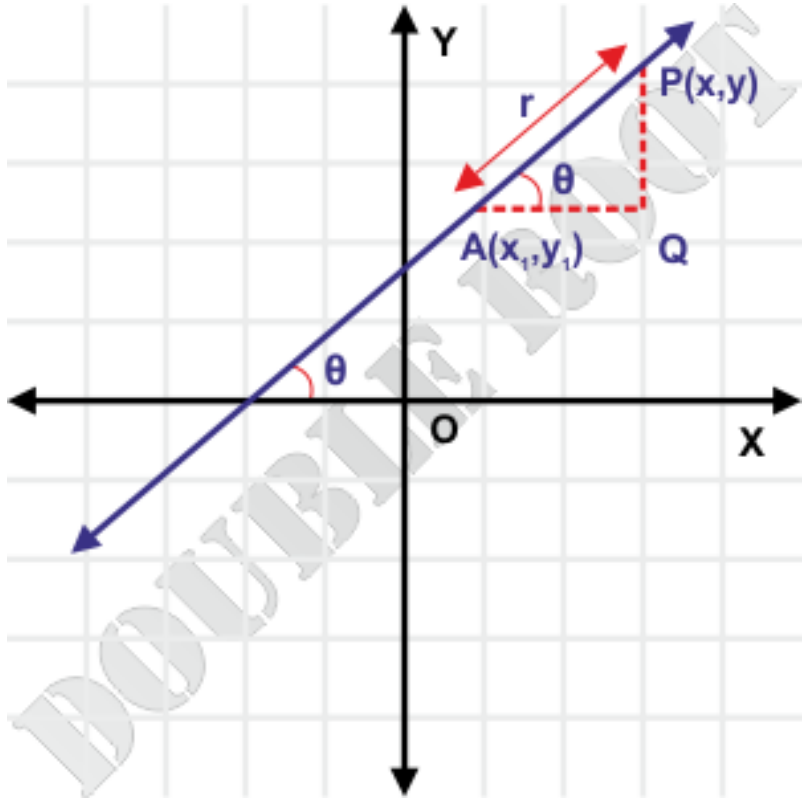
PROBLEM:13

A square of side length 2, lies above the x-axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30° with the positive direction of the x-axis.
Find the sum of the x-coordinates of the vertices of the square?

Solution

Consider a line which has slope $\tan\theta$ and passes through the point $A(x_1, y_1)$.
Let P(x, y) be a point on the line which is at a distance r from the point A.



We have, $\cos\theta = AQ/AP = (x\text{-}x_1)/r$ and $\sin\theta = PQ/AP = (y\text{-}y_1)/r$

This gives the coordinates of P as ($x_1 + rcos\theta$, $y_1 + rsin\theta$).
let

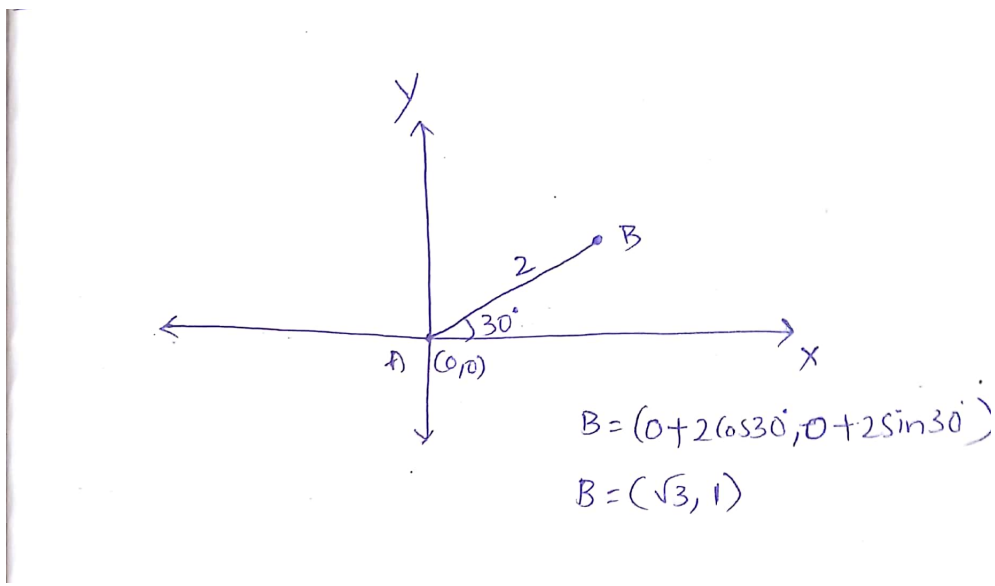
$$p = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x_1 & cos\theta \\ y_1 & sin\theta \end{bmatrix} \begin{bmatrix} 1 \\ r \end{bmatrix}$$

Given length of side is 2 units and one of the vertex of the square is origin $A(x_1, y_1) = (0, 0)$

let the other vertices of square be B,C,D

Line AB makes an angle of 30° with the positive direction of x-axis in anticlockwise direction



coordinates of the point which is 2 units away from origin and lie above x-axis (i.e Point B) can be written as

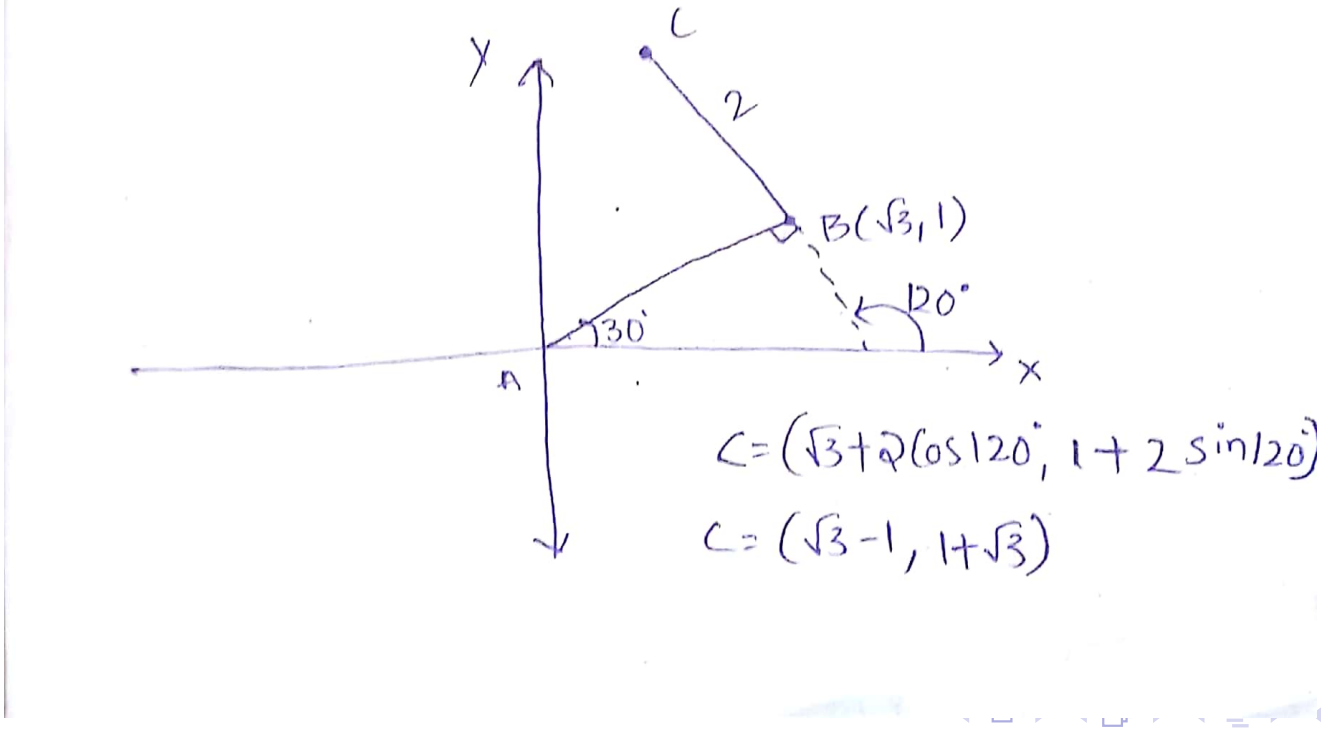
$$B = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 & cos30^\circ \\ y_1 & sin30^\circ \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & \sqrt{3}/2 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$B = \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$$

Similarly line BC makes an angle of 120°(i.e(30+90)) with the positive direction of x-axis in anticlockwise direction



coordinates of the point which is 2 units away from B and lie above x-axis (i.e Point C) can be written as

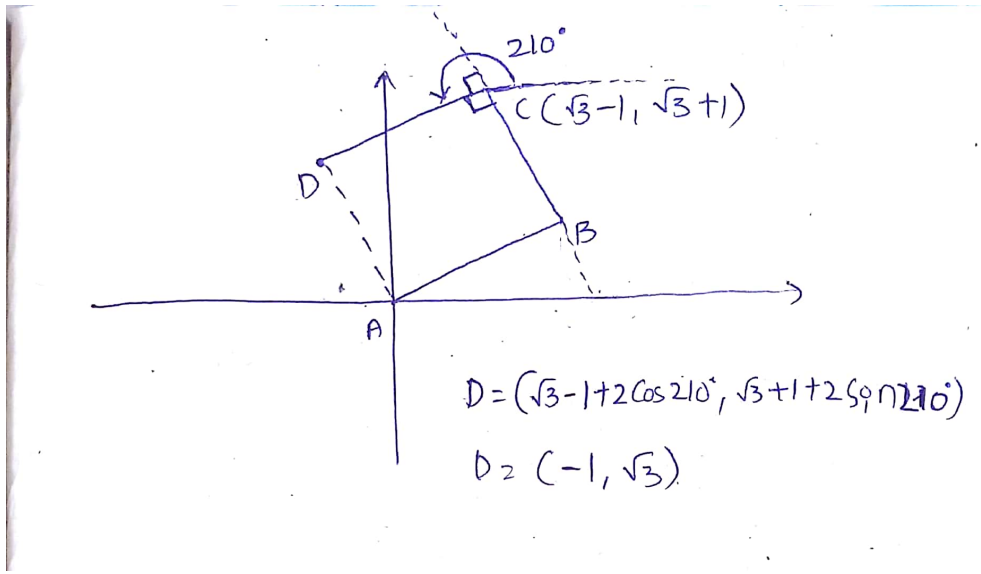
$$C = \begin{bmatrix} x_3 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_2 & cos120^\circ \\ y_2 & sin120^\circ \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = \begin{bmatrix} \sqrt{3} & -1/2 \\ 1 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$C = \begin{bmatrix} \sqrt{3} - 1 \\ 1 + \sqrt{3} \end{bmatrix}$$

line CD makes an angle of 210°(i.e(120+90)) with the positive direction of x-axis in anticlockwise direction



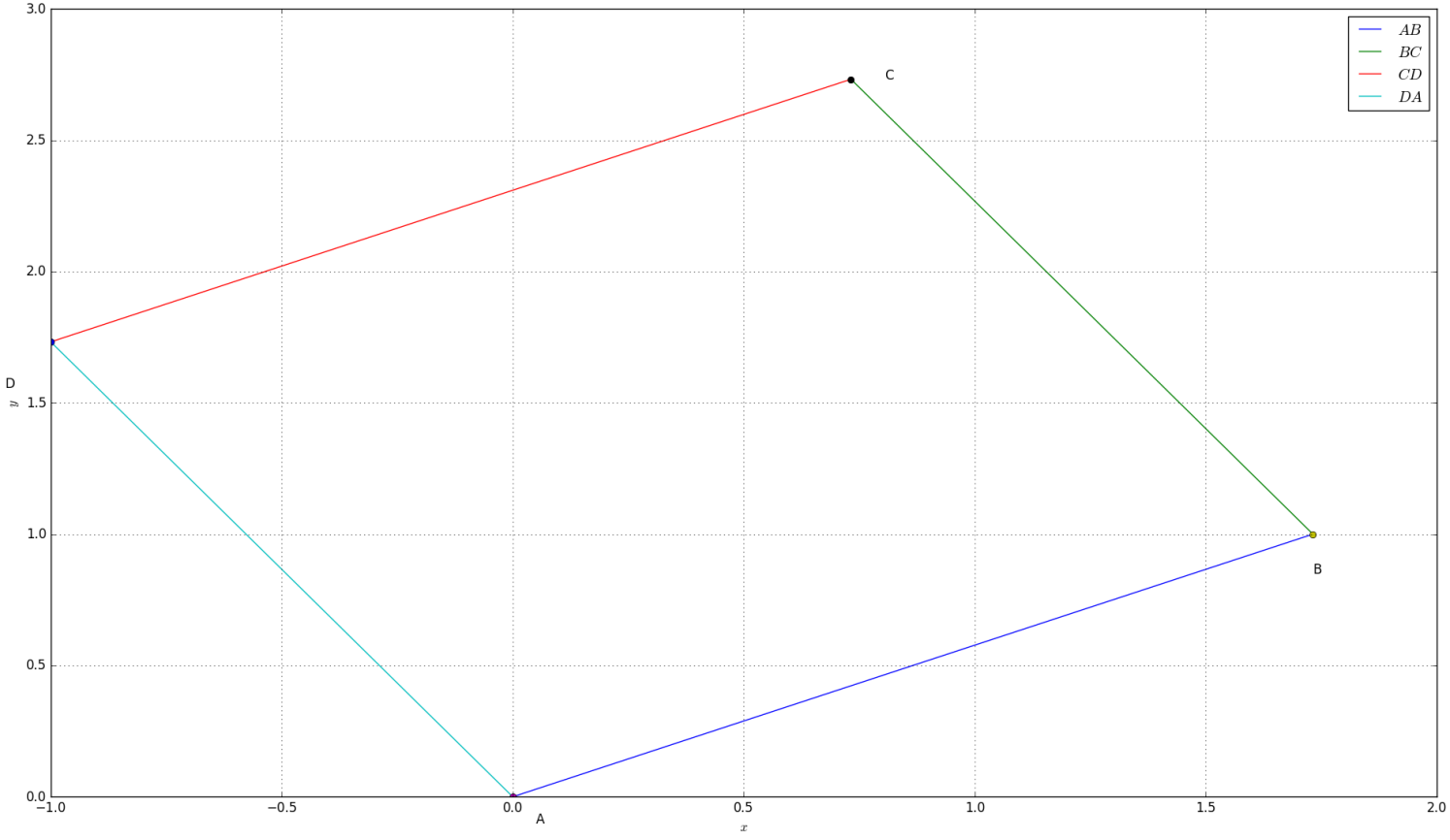
coordinates of the point(i.e Point D) which is 2 units away from C and also 2 units away from A (because it is a square) can be written as

$$D = \begin{bmatrix} x_4 \\ y_4 \end{bmatrix}$$

$$\begin{bmatrix} x_4 \\ y_4 \end{bmatrix} = \begin{bmatrix} x_3 & cos210^\circ \\ y_3 & sin210^\circ \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x_4 \\ y_4 \end{bmatrix} = \begin{bmatrix} \sqrt{3}-1 & -\sqrt{3}/2 \\ \sqrt{3}+1 & -1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$D = \begin{bmatrix} -1 \\ \sqrt{3} \end{bmatrix}$$


AB
BC
CD
DA
C
D
B
A
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
y
x

Let X be sum of x-cordinates

$X = x_1 + x_2 + x_3 + x_4$

$X = 0 + \sqrt{3} + (\sqrt{3} - 1) + (-1)$

$X = 1.464$