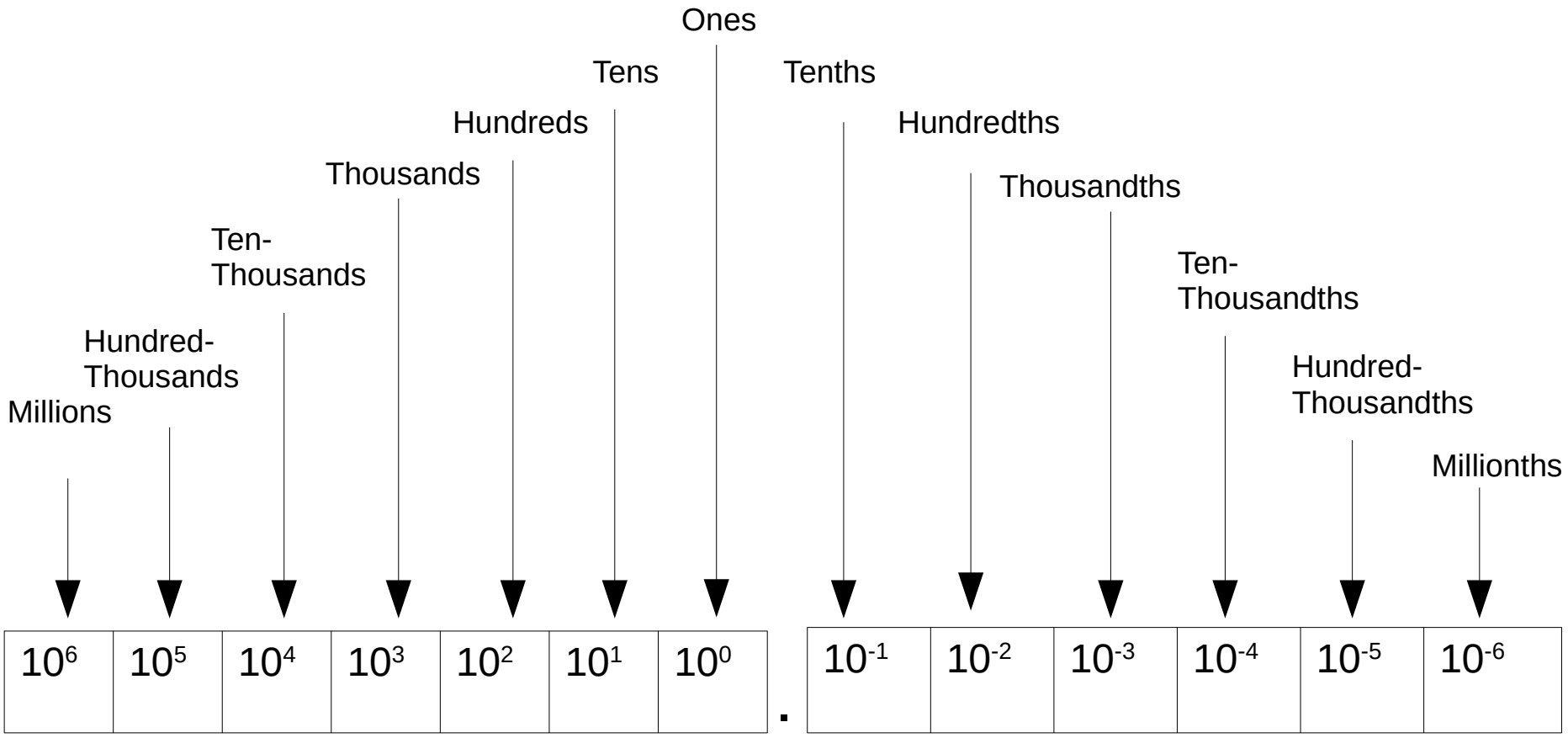
Decimal Number System – Base 10

In the decimal number system, the number 7123.48 can be written as 7000 + 100 + 20 + 3 + 0.4 + 0.08

$7 \times 10^3 + 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0 + 4 \times 10^{-1} + 8 \times 10^{-2}$



Binary Number System – Base 2

In the binary number system, there are only 2 numbers (0,1) available.

The exponent denotes the value of each position, very similar to what we had with the base 10 number system.

The only difference is that the base number is now 2 instead of 10.

MSB	7SB	6SB	5SB	4SB	3SB	2SB	LSB		Fractional Base 2	
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}
128	64	32	16	8	4	2	1		0.5	0.25

Binary digits are called bits. The digit in the first position is the least significant bit (LSB)..and the digit in the last position is the most significant bit (MSB)

Binary to Decimal Conversion and Vice Versa

Some examples of conversion between the number systems

11010_2

1010.11_2

34_{10} Use long division method, divide by 2 each time

203_{10}

Octal Number System – Base 8

In the octal base system, there are 8 digits (0,1,2,3,4,5,6,7) available and the value of each position has a base of 8

$1025_8 = 1 \times 8^3 + 0 \times 8^2 + 2 \times 8^1 + 5 \times 8^0 = 533_{10}$

MSB	5SB	4SB	3SB	2SB	LSB		Fractional Base 8	
8^5	8^4	8^3	8^2	8^1	8^0	.	8^{-1}	8^{-2}
32768	4096	512	64	8	1		0.125	0.015625

Octal to Decimal Conversion and Vice Versa

Some examples of conversion between the number systems

432_8

671.3_8

78_{10} Use long division method, divide by 8 each time

1463_{10}

Hexadecimal Number System – Base 16

In the hexadecimal base system, there are 16 digits (0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F) available and the value of each position has a base of 16

$$567_{16} = 5 \times 16^2 + 6 \times 16^1 + 7 \times 16^0 = 1383_{10}$$

MSB	3SB	2SB	LSB		Fractional Base 16	
16^3	16^2	16^1	16^0	.	16^{-1}	16^{-2}
4096	256	16	1		0.0625	0.00390625

Dec	Hex
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

Hexadecimal to Decimal Conversion and Vice Versa

Some examples of conversion between the number systems

$12\,AB.6_{16}$

$2\,A\,4\,F_{16}$

672_{10} Use long division method, divide by 16 each time

12760_{10}

Binary to Octal Conversion and Vice Versa

All octal digits, (0,1,2,3,4,5,6,7) can be represented by 3 bits

Use 3 bit grouping method for conversion...start at decimal point

1010111100001_2

11101001.101011_2

417_8

362.741_8

Bin	Oct
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

Hexadecimal to Binary and Vice Versa

All hexadecimal digits (0,1,2,3,4,5,6,7,8,9,A,B,C,D, E,F) can be represented by 4 digits

Use 4 bit grouping method for conversion...start at decimal point

$$1110110011011_2$$

$$1110011010110 1.11011011_2$$

$$C7F2_{16}$$

$$B259.C4_{16}$$

Bin	Hex
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

Fractional Decimal Conversion to Fractional Binary

To convert 34.67_{10} to binary, earlier we used long division to convert the integer part to binary...which resulted in...
$34_{10} = 100010_2$

To convert the fractional part to binary, we multiply the fraction part by 2 as such….

Now we can add the fractional part to the integer part

$34.67_{10} = 100010.1010101110_2$

We can stop the process after 10 digits

The fractional part of the number will terminate if it is a multiple of $1/2^a$ (a = 0,1,2,3…)...otherwise it can have a repeating pattern or be irrational

0.67 x 2 =	1.34	1
0.34 x 2 =	0.68	0
0.68 x 2 =	1.36	1
0.36 x 2 =	0.72	0
0.72 x 2 =	1.44	1
0.44 x 2 =	0.88	0
0.88 x 2 =	1.76	1
0.76 x 2 =	1.52	1
0.52 x 2 =	1.04	1
0.04 x 2 =	0.08	0
0.08 x 2 =	0.16	0
0.16 x 2 =	0.32	0
0.32 x 2 =	0.64	0
0.64 x 2 =	1.28	1
0.28 x 2 =	0.56	0
0.56 x 2 =	1.12	1
0.12 x 2 =	0.24	0
0.24 x 2 =	0.48	0
0.48 x 2 =	0.96	0
0.96 x 2 =	1.92	1

Fractional Decimal Conversion to Fractional Octal

To convert 78.24_{10} to octal, earlier we used long division to convert the integer part to octal...which resulted in...
$78_{10} = 116_8$

To convert the fractional part to octal, we multiply the fraction part by 8 as such….

Now we can add the fractional part to the integer part

$78.24_{10} = 116.17270_8$

We can stop the process after 5 digits

The fractional part of the number will terminate if it is a multiple of $1/8^a$ (a = 0,1,2,3…)...otherwise it can have a repeating pattern or be irrational

0.24 x 8 =	1.92	1
0.92 x 8 =	7.36	7
0.36 x 8 =	2.88	2
0.88 x 8 =	7.04	7
0.04 x 8 =	0.32	0
0.32 x 8 =	2.56	2
0.56 x 8 =	4.48	4
0.48 x 8 =	3.84	3
0.84 x 8 =	6.72	6

Fractional Decimal Conversion to Fractional Hexadecimal

To convert 672.39_{10} to hexadecimal, earlier we used long division to convert the integer part to hexadecimal...which resulted in...
$672_{10} = 2A0_{16}$

To convert the fractional part to hexadecimal, we multiply the fraction part by 16 as such….

0.39 x 16 =	6.24	6
0.24 x 16 =	3.84	3
0.84 x 16 =	13.44	13 = D
0.44 x 16 =	7.04	7
0.04 x 16 =	0.64	0
0.64 x 16 =	10.24	10 = A
0.24 x 16 =	3.84	3
0.84 x 16 =	13.44	13 = D
0.44 x 16 =	7.04	7

Now we can add the fractional part to the integer part

$672.39_{10} = 2A0.63D70_{16}$

We can stop the process after 5 digits

The fractional part of the number will terminate if it is a multiple of $1/16^a$ (a = 0,1,2,3…)...otherwise it can have a repeating pattern or be irrational

American Standard Code for Information Interchange (ASCII)

Decimal	Hexadecimal	Binary	Octal	Char
0	0	0	0	*[NULL]*
1	1	1	1	*[START OF HEADING]*
2	2	10	2	*[START OF TEXT]*
3	3	11	3	*[END OF TEXT]*
4	4	100	4	*[END OF TRANSMISSION]*
5	5	101	5	*[ENQUIRY]*
6	6	110	6	*[ACKNOWLEDGE]*
7	7	111	7	*[BELL]*
8	8	1000	10	*[BACKSPACE]*
9	9	1001	11	*[HORIZONTAL TAB]*
10	A	1010	12	*[LINE FEED]*
11	B	1011	13	*[VERTICAL TAB]*
12	C	1100	14	*[FORM FEED]*
13	D	1101	15	*[CARRIAGE RETURN]*
14	E	1110	16	*[SHIFT OUT]*
15	F	1111	17	*[SHIFT IN]*
16	10	10000	20	*[DATA LINK ESCAPE]*
17	11	10001	21	*[DEVICE CONTROL 1]*
18	12	10010	22	*[DEVICE CONTROL 2]*
19	13	10011	23	*[DEVICE CONTROL 3]*
20	14	10100	24	*[DEVICE CONTROL 4]*
21	15	10101	25	*[NEGATIVE ACKNOWLEDGE]*
22	16	10110	26	*[SYNCHRONOUS IDLE]*
23	17	10111	27	*[END OF TRANS. BLOCK]*
24	18	11000	30	*[CANCEL]*
25	19	11001	31	*[END OF MEDIUM]*
26	1A	11010	32	*[SUBSTITUTE]*
27	1B	11011	33	*[ESCAPE]*
28	1C	11100	34	*[FILE SEPARATOR]*
29	1D	11101	35	*[GROUP SEPARATOR]*
30	1E	11110	36	*[RECORD SEPARATOR]*
31	1F	11111	37	*[UNIT SEPARATOR]*
32	20	100000	40	*[SPACE]*
33	21	100001	41	!
34	22	100010	42	"
35	23	100011	43	#
36	24	100100	44	$
37	25	100101	45	%
38	26	100110	46	&
39	27	100111	47	'
40	28	101000	50	(
41	29	101001	51	)
42	2A	101010	52	*
43	2B	101011	53	+
44	2C	101100	54	,
45	2D	101101	55	-
46	2E	101110	56	.
47	2F	101111	57	/
48	30	110000	60	**0**
49	31	110001	61	**1**
50	32	110010	62	**2**
51	33	110011	63	**3**
52	34	110100	64	**4**
53	35	110101	65	**5**
54	36	110110	66	**6**
55	37	110111	67	**7**
56	38	111000	70	**8**
57	39	111001	71	**9**
58	3A	111010	72	:
59	3B	111011	73	;
60	3C	111100	74	<
61	3D	111101	75	=
62	3E	111110	76	>
63	3F	111111	77	?
64	40	1000000	100	@
65	41	1000001	101	**A**
66	42	1000010	102	**B**
67	43	1000011	103	**C**
68	44	1000100	104	**D**
69	45	1000101	105	**E**
70	46	1000110	106	**F**
71	47	1000111	107	**G**
72	48	1001000	110	**H**
73	49	1001001	111	**I**
74	4A	1001010	112	**J**
75	4B	1001011	113	**K**
76	4C	1001100	114	**L**
77	4D	1001101	115	**M**
78	4E	1001110	116	**N**
79	4F	1001111	117	**O**
80	50	1010000	120	**P**
81	51	1010001	121	**Q**
82	52	1010010	122	**R**
83	53	1010011	123	**S**
84	54	1010100	124	**T**
85	55	1010101	125	**U**
86	56	1010110	126	**V**
87	57	1010111	127	**W**
88	58	1011000	130	**X**
89	59	1011001	131	**Y**
90	5A	1011010	132	**Z**
91	5B	1011011	133	**[**
92	5C	1011100	134	\
93	5D	1011101	135	**]**
94	5E	1011110	136	^
95	5F	1011111	137	_
96	60	1100000	140	`
97	61	1100001	141	**a**
98	62	1100010	142	**b**
99	63	1100011	143	**c**
100	64	1100100	144	**d**
101	65	1100101	145	**e**
102	66	1100110	146	**f**
103	67	1100111	147	**g**
104	68	1101000	150	**h**
105	69	1101001	151	**i**
106	6A	1101010	152	**j**
107	6B	1101011	153	**k**
108	6C	1101100	154	**l**
109	6D	1101101	155	**m**
110	6E	1101110	156	**n**
111	6F	1101111	157	**o**
112	70	1110000	160	**p**
113	71	1110001	161	**q**
114	72	1110010	162	**r**
115	73	1110011	163	**s**
116	74	1110100	164	**t**
117	75	1110101	165	**u**
118	76	1110110	166	**v**
119	77	1110111	167	**w**
120	78	1111000	170	**x**
121	79	1111001	171	**y**
122	7A	1111010	172	**z**
123	7B	1111011	173	**{**
124	7C	1111100	174	|
125	7D	1111101	175	**}**
126	7E	1111110	176	~
127	7F	1111111	177	*[DEL]*

https://commons.wikimedia.org/wiki/File:ASCII-Table.svg

Summary of Base Conversions – Base n to Base 10

To convert from base 2 to base 10...multiply each base 2 digit by the proper place value 2^a...(a = 0, 1, 2, 3...) then sum up all the values

MSB	7SB	6SB	5SB	4SB	3SB	2SB	LSB		Fractional Base 2	
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0	.	2^{-1}	2^{-2}

To convert from base 8 to base 10...multiply each base 8 digit by the proper place value 8^a...(a = 0, 1, 2, 3...) then sum up all the values

MSB	5SB	4SB	3SB	2SB	LSB		Fractional Base 8	
8^5	8^4	8^3	8^2	8^1	8^0	.	8^{-1}	8^{-2}

To convert from base 16 to base 10...multiply each base 16 digit by the proper place value 16^a...(a = 0, 1, 2, 3...) then sum up all the values

MSB	3SB	2SB	LSB		Fractional Base 16	
16^3	16^2	16^1	16^0	.	16^{-1}	16^{-2}

Summary of Base Conversions – Base 10 to Base n

To convert from base 10 to base 2...use long division method, where you divide the base 10 number by 2 each time, use the answer from the first division to divide again by 2...keep track of all the remainders which will eventually become the base 2 number

To convert from base 10 to base 8...use long division method, where you divide the base 10 number by 8 each time, use the answer from the first division to divide again by 8...keep track of all the remainders which will eventually become the base 8 number

To convert from base 10 to base 16...use long division method, where you divide the base 10 number by 16 each time, use the answer from the first division to divide again by 16...keep track of all the remainders which will eventually become the base 16 number

Review Questions

Review question set 4