# Approaches to Probability

In this section, we discuss 3 approaches to assigning a probability to an event: **classical, empirical and subjective**

The classical and empirical methods are objective and are based on information and data.

The subjective method is based on a person's belief or estimate of an event's likelihood.

**Objective probability** is subdivided into

    (1) *classical probability* (theoretical)

    (2) *empirical probability (experimental)*

# Classical (Theoretical) Probability

**Classical (theoretical) probability** is based on the assumption that the outcomes of an experiment are _equally likely_

Using the classical viewpoint, the probability of an event happening is computed by dividing the number of favourable outcomes by the number of possible outcomes

$$Probability \ of \ event \ happening = \frac{Number \ of \ favourable \ outcomes}{Total \ number \ of \ possible \ outcomes}$$

# Example – Coin Tossing

Consider an experiment of tossing a coin.

What is the probability of the event "a head appears"?

When flipping a coin...there are
2 possible outcomes...either a
tail or a head...

$$Probability\ of\ a\ head = \frac{Number\ of\ favourable\ outcomes}{Number\ of\ possible\ outcomes} = \frac{1}{2} = 0.5$$

# Empirical (Experimental) Probability

In classical (theoretical) probability approach, the total number of outcomes is known before the experiment...unnecessary to perform the experiment

**Empirical (experimental) probability -** it is based on the number of times an event occurs as a proportion of a known number of trials.

The empirical (experimental) approach to probability is based on what is called the **law of large numbers**. Over a large number of trials, the empirical probability of an event will approach its true probability. The more observations will result in a better estimate of the probability.

$$Empirical\ Probability = \frac{\text{Number of times the event occurs}}{\text{Total number of trials}}$$

# Law of Large Numbers

Suppose we toss a fair coin. If we toss the coin a great number of times, the probability of the outcome of heads will approach 0.5.

The following table reports the results of an experiment of flipping a fair coin 1, 10, 50, 100, 500, 1,000 and 10,000 times and then computing the relative frequency of heads.

As the number of trials increases, the empirical probability approaches 0.5 which is the value based on the classical probability method

The insurance industry is entirely dependent on the empirical probability approach

| Number of Trials | Number of Heads | Relative Frequency of Heads |
|---|---|---|
| 1 | 0 | 0.00 |
| 10 | 3 | 0.30 |
| 50 | 26 | 0.52 |
| 100 | 52 | 0.52 |
| 500 | 236 | 0.472 |
| 1000 | 494 | 0.494 |
| 10000 | 5027 | 0.5027 |

# Example – Coin Tossing

Consider an experiment of tossing a coin.

A coin is tossed 10 times. If head appears 7 times, then what is the probability of heads in this experiment?

If we keep conducting this experiment many more times...the experimental probability value should approach the theoretical value...

$$Probability\ of\ Heads = \frac{\text{Number of times the event occurs}}{\text{Total number of trials}} = \frac{7}{10}$$

# Subjective Probability

If there is little or no experience or information on which to base a probability, then it is estimated <u>subjectively</u>.

An individual evaluates the available opinions and whatever other information is available and then estimates or assigns a probability.

Some examples of subjective probability are

1. Estimating when the next provincial election will be.
2. Estimating the likelihood you will be married before the age of 30.
3. Estimating the chance of the Maple Leafs winning the Stanley Cup.
4. Estimating how long it will take for humans to make contact with another civilization beyond Earth.

# Probability Approaches Summary