5

A Survey of Probability Concepts

Learning Objectives

When you have completed this chapter, you will be able to:

LO1 Explain the terms *experiment, event,* and *outcome.*

LO2 Identify and apply the appropriate approach to assigning probabilities.

LO3 Calculate probabilities using the rules of addition.

LO4 Define the term *joint probability.*

LO5 Calculate probabilities using the rules of multiplication.

LO6 Define the term *conditional probability.*

LO7 Compute probabilities using a contingency table.

LO8 Calculate probabilities using Bayes' theorem.

LO9 Determine the number of outcomes using the appropriate principle of counting.

It was found that 60 percent of the tourists to China visited the Forbidden City, the Temple of Heaven, the Great Wall, and other historical sites in or near Beijing. Forty percent visited Xi'an and its magnificent terracotta soldiers, horses, and chariots, which lay buried for over 2,000 years. Thirty percent of the tourists went to both Beijing and Xi'an. What is the probability that a tourist visited at least one of these places? (See Exercise 76 and LO4.)

5.1 Introduction

The emphasis in Chapters 2, 3, and 4 is on descriptive statistics. In Chapter 2, we organize the profits on 180 vehicles sold by the Applewood Auto Group into a frequency distribution. This frequency distribution shows the smallest and the largest profits and where the largest concentration of data occurs. In Chapter 3, we use numerical measures of location and dispersion to locate a typical profit on vehicle sales and to examine the variation in the profit of a sale. We describe the variation in the profits with such measures of dispersion as the range and the standard deviation. In Chapter 4, we develop charts and graphs, such as a scatter diagram, to further describe the data graphically.

Descriptive statistics is concerned with summarizing data collected from past events. We now turn to the second facet of statistics, namely, *computing the chance that something will occur in the future*. This facet of statistics is called **statistical inference** or **inferential statistics.**

Seldom does a decision maker have complete information to make a decision. For example:

- Toys and Things, a toy and puzzle manufacturer, recently developed a new game based on sports trivia. It wants to know whether sports buffs will purchase the game. "Slam Dunk" and "Home Run" are two of the names under consideration. One way to minimize the risk of making an incorrect decision is to hire a market research firm to select a sample of 2,000 consumers from the population and ask each respondent for a reaction to the new game and its proposed titles. Using the sample results, the company can estimate the proportion of the population that will purchase the game.
- The quality assurance department of a Bethlehem Steel mill must assure management that the quarter-inch wire being produced has an acceptable tensile strength. Obviously, not all the wire produced can be tested for tensile strength because testing requires the wire to be stretched until it breaks—thus destroying it. So a random sample of 10 pieces is selected and tested. Based on the test results, all the wire produced is deemed to be either acceptable or unacceptable.
- Other questions involving uncertainty are: Should the daytime drama *Days of Our Lives* be discontinued immediately? Will a newly developed mint-flavored cereal be profitable if marketed? Will Charles Linden be elected to county auditor in Batavia County?

Statistical inference deals with conclusions about a population based on a sample taken from that population. (The populations for the preceding illustrations are: all consumers who like sports trivia games, all the quarter-inch steel wire produced, all television viewers who watch soaps, all who purchase breakfast cereal, and so on.)

Because there is uncertainty in decision making, it is important that all the known risks involved be scientifically evaluated. Helpful in this evaluation is *probability theory,* which has often been referred to as the science of uncertainty. The use of probability theory allows the decision maker with only limited information to analyze the risks and minimize the gamble inherent, for example, in marketing a new product or accepting an incoming shipment possibly containing defective parts.

Because probability concepts are so important in the field of statistical inference (to be discussed starting with Chapter 8), this chapter introduces the basic language of probability, including such terms as *experiment, event, subjective probability,* and *addition* and *multiplication rules*.

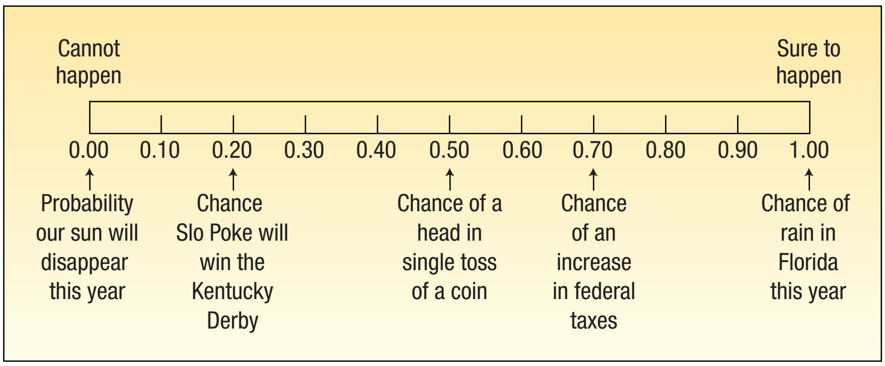
5.2 What Is a Probability?

No doubt you are familiar with terms such as *probability, chance,* and *likelihood*. They are often used interchangeably. The weather forecaster announces that there is a 70 percent chance of rain for Super Bowl Sunday. Based on a survey of consumers who tested a newly developed pickle with a banana taste, the probability is .03 that, if marketed, it will be a financial success. (This means that the chance of the banana-flavor pickle being accepted by the public is rather remote.) What is a probability? In general, it is a number that describes the chance that something will happen.

PROBABILITY A value between zero and one, inclusive, describing the relative possibility (chance or likelihood) an event will occur.

A probability is frequently expressed as a decimal, such as .70, .27, or .50. However, it may be given as a fraction such as 7/10, 27/100, or 1/2. It can assume any number from 0 to 1, inclusive. If a company has only five sales regions, and each region's name or number is written on a slip of paper and the slips put in a hat, the probability of selecting one of the five regions is 1. The probability of selecting from the hat a slip of paper that reads "Pittsburgh Steelers" is 0. Thus, the probability of 1 represents something that is certain to happen, and the probability of 0 represents something that cannot happen.

The closer a probability is to 0, the more improbable it is the event will happen. The closer the probability is to 1, the more sure we are it will happen. The relationship is shown in the following diagram along with a few of our personal beliefs. You might, however, select a different probability for Slo Poke's chances to win the Kentucky Derby or for an increase in federal taxes.



Three key words are used in the study of probability: **experiment, outcome,** and **event.** These terms are used in our everyday language, but in statistics they have specific meanings.

EXPERIMENT A process that leads to the occurrence of one and only one of several possible observations.

LO1 Explain the terms *experiment, event,* and *outcome.*

This definition is more general than the one used in the physical sciences, where we picture someone manipulating test tubes or microscopes. In reference to probability, an experiment has two or more possible results, and it is uncertain which will occur.

OUTCOME A particular result of an experiment.

For example, the tossing of a coin is an experiment. You may observe the toss of the coin, but you are unsure whether it will come up "heads" or "tails." Similarly, asking 500 college students whether they would purchase a new Dell computer system at a particular price is an experiment. If the coin is tossed, one particular outcome is a "head." The alternative outcome is a "tail." In the computer purchasing experiment, one possible outcome is that 273 students indicate they would purchase the computer. Another outcome is that 317 students would purchase the computer. Still another outcome is that 423 students indicate that they would purchase it. When one or more of the experiment's outcomes are observed, we call this an event.

EVENT A collection of one or more outcomes of an experiment.

Examples to clarify the definitions of the terms *experiment, outcome,* and *event* are presented in the following figure.

In the die-rolling experiment, there are six possible outcomes, but there are many possible events. When counting the number of members of the board of directors for Fortune 500 companies over 60 years of age, the number of possible outcomes can be anywhere from zero to the total number of members. There are an even larger number of possible events in this experiment.

		
Experiment	Roll a die	Count the number of members of the board of directors for Fortune 500 companies who are over 60 years of age
All possible outcomes	Observe a 1 Observe a 2 Observe a 3 Observe a 4 Observe a 5 Observe a 6	None are over 60 One is over 60 Two are over 60 ... 29 are over 60 ... ... 48 are over 60 ...
Some possible events	Observe an even number Observe a number greater than 4 Observe a number 3 or less	More than 13 are over 60 Fewer than 20 are over 60

Self-Review 5–1

Video Games Inc. recently developed a new video game. Its playability is to be tested by 80 veteran game players.

(a) What is the experiment?
(b) What is one possible outcome?
(c) Suppose 65 players tried the new game and said they liked it. Is 65 a probability?
(d) The probability that the new game will be a success is computed to be -1.0. Comment.
(e) Specify one possible event.

5.3 Approaches to Assigning Probabilities

Two approaches to assigning probabilities to an event will be discussed, namely, the *objective* and the *subjective* viewpoints. **Objective probability** is subdivided into (1) *classical probability* and (2) *empirical probability*.

Classical Probability

LO2 Identify and apply the appropriate approach to assigning probabilities.

Classical probability is based on the assumption that the outcomes of an experiment are *equally likely*. Using the classical viewpoint, the probability of an event happening is computed by dividing the number of favorable outcomes by the number of possible outcomes:

CLASSICAL PROBABILITY
$$\text{Probability of an event} = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}} \qquad [5\text{–}1]$$

Example

Consider an experiment of rolling a six-sided die. What is the probability of the event "an even number of spots appear face up"?

Solution

The possible outcomes are:

a one-spot		a four-spot	
a two-spot		a five-spot	
a three-spot		a six-spot	

There are three "favorable" outcomes (a two, a four, and a six) in the collection of six equally likely possible outcomes. Therefore:

$$\text{Probability of an even number} = \frac{3}{6} \begin{array}{l} \leftarrow \\ \leftarrow \end{array} \boxed{\frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}}$$
$$= .5$$

The mutually exclusive concept appeared earlier in our study of frequency distributions in Chapter 2. Recall that we create classes so that a particular value is included in only one of the classes and there is no overlap between classes. Thus, only one of several events can occur at a particular time.

MUTUALLY EXCLUSIVE The occurrence of one event means that none of the other events can occur at the same time.

The variable "gender" presents mutually exclusive outcomes, male and female. An employee selected at random is either male or female but cannot be both. A manufactured part is acceptable or unacceptable. The part cannot be both acceptable and unacceptable at the same time. In a sample of manufactured parts, the event of selecting an unacceptable part and the event of selecting an acceptable part are mutually exclusive.

If an experiment has a set of events that includes every possible outcome, such as the events "an even number" and "an odd number" in the die-tossing experiment, then the set of events is **collectively exhaustive.** For the die-tossing experiment, every outcome will be either even or odd. So the set is collectively exhaustive.

COLLECTIVELY EXHAUSTIVE At least one of the events must occur when an experiment is conducted.

If the set of events is collectively exhaustive and the events are mutually exclusive, the sum of the probabilities is 1. Historically, the classical approach to probability was developed and applied in the 17th and 18th centuries to games of chance, such as cards and dice. It is unnecessary to do an experiment to determine the probability of an event occurring using the classical approach because the total number of outcomes is known before the experiment. The flip of a coin has two possible outcomes; the roll of a die has six possible outcomes. We can logically arrive at the probability of getting a tail on the toss of one coin or three heads on the toss of three coins.

The classical approach to probability can also be applied to lotteries. In South Carolina, one of the games of the Education Lottery is "Pick 3." A person buys a lottery ticket and selects three numbers between 0 and 9. Once per week, the three numbers are randomly selected from a machine that tumbles three containers each with balls numbered 0 through 9. One way to win is to match the numbers and the order of the numbers. Given that 1,000 possible outcomes exist (000 through 999), the probability of winning with any three-digit number is 0.001, or 1 in 1,000.

Empirical Probability

Empirical or **relative frequency** is the second type of objective probability. It is based on the number of times an event occurs as a proportion of a known number of trials.

EMPIRICAL PROBABILITY The probability of an event happening is the fraction of the time similar events happened in the past.

In terms of a formula:

$$\text{Empirical probability} = \frac{\text{Number of times the event occurs}}{\text{Total number of observations}}$$

The empirical approach to probability is based on what is called the law of large numbers. The key to establishing probabilities empirically is that more observations will provide a more accurate estimate of the probability.

LAW OF LARGE NUMBERS Over a large number of trials, the empirical probability of an event will approach its true probability.

To explain the law of large numbers, suppose we toss a fair coin. The result of each toss is either a head or a tail. With just one toss of the coin the empirical probability for heads is either zero or one. If we toss the coin a great number of times, the probability of the outcome of heads will approach .5. The following table reports the results of an experiment of flipping a fair coin 1, 10, 50, 100, 500, 1,000, and 10,000 times and then computing the relative frequency of heads. Note as we increase the number of trials the empirical probability of a head appearing approaches .5, which is its value based on the classical approach to probability.

Number of Trials	Number of Heads	Relative Frequency of Heads
1	0	.00
10	3	.30
50	26	.52
100	52	.52
500	236	.472
1,000	494	.494
10,000	5,027	.5027

What have we demonstrated? Based on the classical definition of probability, the likelihood of obtaining a head in a single toss of a fair coin is .5. Based on the empirical or relative frequency approach to probability, the probability of the event happening approaches the same value based on the classical definition of probability.

This reasoning allows us to use the empirical or relative frequency approach to finding a probability. Here are some examples.

- Last semester, 80 students registered for Business Statistics 101 at Scandia University. Twelve students earned an A. Based on this information and the empirical approach to assigning a probability, we estimate the likelihood a student will earn an A is .15.
- Kobe Bryant of the Los Angeles Lakers made 403 out of 491 free throw attempts during the 2009–10 NBA season. Based on the empirical rule of probability, the likelihood of him making his next free throw attempt is .821.

Life insurance companies rely on past data to determine the acceptability of an applicant as well as the premium to be charged. Mortality tables list the likelihood a person of a particular age will die within the upcoming year. For example, the likelihood a 20-year-old female will die within the next year is .00105.

The empirical concept is illustrated with the following example.

Example

On February 1, 2003, the Space Shuttle Columbia exploded. This was the second disaster in 113 space missions for NASA. On the basis of this information, what is the probability that a future mission is successfully completed?

Solution

To simplify, letters or numbers may be used. P stands for probability, and in this case $P(A)$ stands for the probability a future mission is successfully completed.

$$\text{Probability of a successful flight} = \frac{\text{Number of successful flights}}{\text{Total number of flights}}$$

$$P(A) = \frac{111}{113} = .98$$

We can use this as an estimate of probability. In other words, based on past experience, the probability is .98 that a future space shuttle mission will be safely completed.

Subjective Probability

If there is little or no experience or information on which to base a probability, it may be arrived at subjectively. Essentially, this means an individual evaluates the available opinions and information and then estimates or assigns the probability. This probability is aptly called a **subjective probability.**

SUBJECTIVE CONCEPT OF PROBABILITY The likelihood (probability) of a particular event happening that is assigned by an individual based on whatever information is available.

Illustrations of subjective probability are:

1. Estimating the likelihood the New England Patriots will play in the Super Bowl next year.
2. Estimating the likelihood you will be married before the age of 30.
3. Estimating the likelihood the U.S. budget deficit will be reduced by half in the next 10 years.

The types of probability are summarized in Chart 5–1. A probability statement always assigns a likelihood to an event that has not yet occurred. There is, of course, a considerable latitude in the degree of uncertainty that surrounds this probability, based primarily on the knowledge possessed by the individual concerning the underlying process. The individual possesses a great deal of knowledge about the toss of a die and can state that the probability that a one-spot will appear face up on the toss of a true die is one-sixth. But we know very little concerning the acceptance in the marketplace of a new and untested product. For example, even though a market research director tests a newly developed product in 40 retail stores and states that there is a 70 percent chance that the product will have sales of more than 1 million units, she has limited knowledge of how consumers will react when it is marketed nationally. In both cases (the case of the person rolling a die and the testing of a new product), the individual is assigning a probability value to an event of interest, and a difference exists only in the predictor's confidence in the precision of the estimate. However, regardless of the viewpoint, the same laws of probability (presented in the following sections) will be applied.

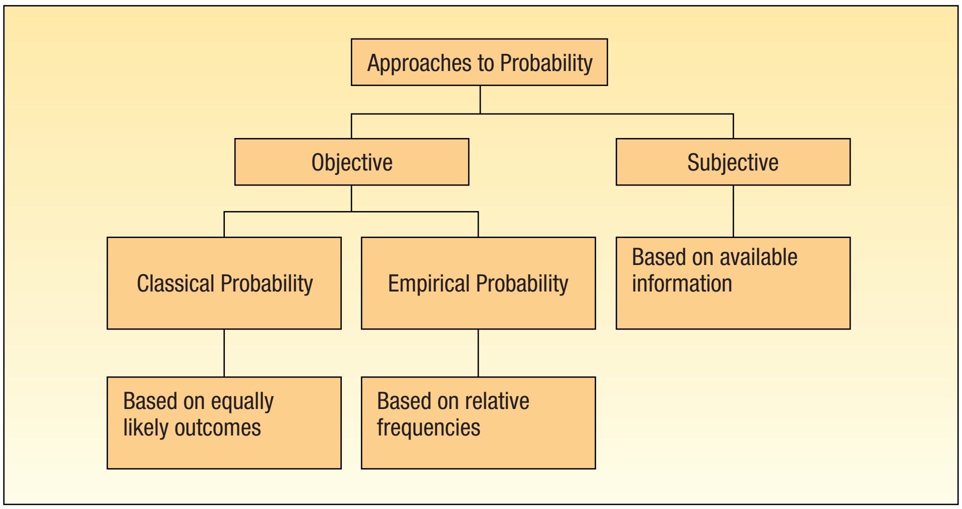


CHART 5–1 Summary of Approaches to Probability

Self-Review 5–2

1. One card will be randomly selected from a standard 52-card deck. What is the probability the card will be a queen? Which approach to probability did you use to answer this question?
2. The Center for Child Care reports on 539 children and the marital status of their parents. There are 333 married, 182 divorced, and 24 widowed parents. What is the probability a particular child chosen at random will have a parent who is divorced? Which approach did you use?
3. What is the probability that the Dow Jones Industrial Average will exceed 12,000 during the next 12 months? Which approach to probability did you use to answer this question?

Exercises

connect

1. Some people are in favor of reducing federal taxes to increase consumer spending and others are against it. Two persons are selected and their opinions are recorded. Assuming no one is undecided, list the possible outcomes.
2. A quality control inspector selects a part to be tested. The part is then declared acceptable, repairable, or scrapped. Then another part is tested. List the possible outcomes of this experiment regarding two parts.
3. A survey of 34 students at the Wall College of Business showed the following majors:

Accounting	10
Finance	5
Economics	3
Management	6
Marketing	10

   Suppose you select a student and observe his or her major.
   a. What is the probability he or she is a management major?
   b. Which concept of probability did you use to make this estimate?
4. A large company that must hire a new president prepares a final list of five candidates, all of whom are equally qualified. Two of these candidates are members of a minority group. To avoid bias in the selection of the candidate, the company decides to select the president by lottery.
   a. What is the probability one of the minority candidates is hired?
   b. Which concept of probability did you use to make this estimate?
5. In each of the following cases, indicate whether classical, empirical, or subjective probability is used.
   a. A baseball player gets a hit in 30 out of 100 times at bat. The probability is .3 that he gets a hit in his next at bat.
   b. A seven-member committee of students is formed to study environmental issues. What is the likelihood that any one of the seven is chosen as the spokesperson?
   c. You purchase one of 5 million tickets sold for Lotto Canada. What is the likelihood you will win the $1 million jackpot?
   d. The probability of an earthquake in northern California in the next 10 years above 5.0 on the Richter Scale is .80.
6. A firm will promote two employees out of a group of six men and three women.
   a. List the chances of this experiment if there is particular concern about gender equity.
   b. Which concept of probability would you use to estimate these probabilities?
7. A sample of 40 oil industry executives was selected to test a questionnaire. One question about environmental issues required a yes or no answer.
   a. What is the experiment?
   b. List one possible event.
   c. Ten of the 40 executives responded yes. Based on these sample responses, what is the probability that an oil industry executive will respond yes?
   d. What concept of probability does this illustrate?
   e. Are each of the possible outcomes equally likely and mutually exclusive?
8. A sample of 2,000 licensed drivers revealed the following number of speeding violations.

Number of Violations	Number of Drivers
0	1,910
1	46
2	18
3	12
4	9
5 or more	5
Total	2,000

   a. What is the experiment?
   b. List one possible event.

c. What is the probability that a particular driver had exactly two speeding violations?
d. What concept of probability does this illustrate?

9. Bank of America customers select their own three-digit personal identification number (PIN) for use at ATMs.
   a. Think of this as an experiment and list four possible outcomes.
   b. What is the probability Mr. Jones and Mrs. Smith select the same PIN?
   c. Which concept of probability did you use to answer (b)?
10. An investor buys 100 shares of AT&T stock and records its price change daily.
    a. List several possible events for this experiment.
    b. Estimate the probability for each event you described in (a).
    c. Which concept of probability did you use in (b)?

5.4 Some Rules for Computing Probabilities

Now that we have defined probability and described the different approaches to probability, we turn our attention to computing the probability of two or more events by applying rules of addition and multiplication.

Rules of Addition

There are two rules of addition, the special rule of addition and the general rule of addition. We begin with the special rule of addition.

LO3 Calculate probabilities using the rules of addition.

Special Rule of Addition To apply the **special rule of addition,** the events must be *mutually exclusive*. Recall that mutually exclusive means that when one event occurs, none of the other events can occur at the same time. An illustration of mutually exclusive events in the die-tossing experiment is the events "a number 4 or larger" and "a number 2 or smaller." If the outcome is in the first group {4, 5, and 6}, then it cannot also be in the second group {1 and 2}. Another illustration is a product coming off the assembly line cannot be defective and satisfactory at the same time.

If two events A and B are mutually exclusive, the special rule of addition states that the probability of one *or* the other event's occurring equals the sum of their probabilities. This rule is expressed in the following formula:

SPECIAL RULE OF ADDITION $$P(A \text{ or } B) = P(A) + P(B) \qquad \textbf{[5–2]}$$

For three mutually exclusive events designated A, B, and C, the rule is written:

$$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)$$

An example will help to show the details.

Example

A machine fills plastic bags with a mixture of beans, broccoli, and other vegetables. Most of the bags contain the correct weight, but because of the variation in the size of the beans and other vegetables, a package might be underweight or overweight. A check of 4,000 packages filled in the past month revealed:

Weight	Event	Number of Packages	Probability of Occurrence	
Underweight	A	100	.025	← $\frac{100}{4,000}$
Satisfactory	B	3,600	.900	
Overweight	C	300	.075	
		4,000	1.000	

What is the probability that a particular package will be either underweight or overweight?

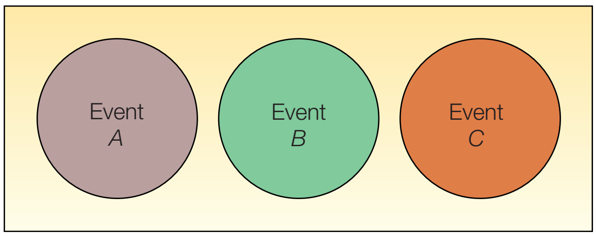
Solution

The outcome "underweight" is the event A. The outcome "overweight" is the event C. Applying the special rule of addition:

$$P(A \text{ or } C) = P(A) + P(C) = .025 + .075 = .10$$

Note that the events are mutually exclusive, meaning that a package of mixed vegetables cannot be underweight, satisfactory, and overweight at the same time. They are also collectively exhaustive; that is, a selected package must be either underweight, satisfactory, or overweight.

English logician J. Venn (1834–1923) developed a diagram to portray graphically the outcome of an experiment. The *mutually exclusive* concept and various other rules for combining probabilities can be illustrated using this device. To construct a Venn diagram, a space is first enclosed representing the total of all possible outcomes. This space is usually in the form of a rectangle. An event is then represented by a circular area which is drawn inside the rectangle proportional to the probability of the event. The following Venn diagram represents the *mutually exclusive* concept. There is no overlapping of events, meaning that the events are mutually exclusive. In the following diagram, assume the events A, B, and C are about equally likely.



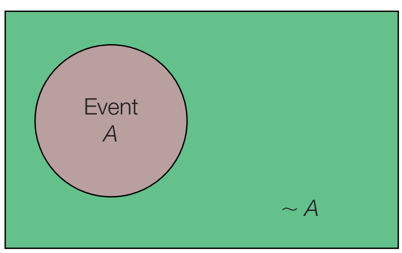
Complement Rule The probability that a bag of mixed vegetables selected is underweight, $P(A)$, plus the probability that it is not an underweight bag, written $P(\sim A)$ and read "not A," must logically equal 1. This is written:

$$P(A) + P(\sim A) = 1$$

This can be revised to read:

COMPLEMENT RULE $$P(A) = 1 - P(\sim A) \quad [5\text{–}3]$$

This is the **complement rule.** It is used to determine the probability of an event occurring by subtracting the probability of the event not occurring from 1. This rule is useful because sometimes it is easier to calculate the probability of an event happening by determining the probability of it not happening and subtracting the result from 1. Notice that the events A and $\sim A$ are mutually exclusive and collectively exhaustive. Therefore, the probabilities of A and $\sim A$ sum to 1. A Venn diagram illustrating the complement rule is shown as:

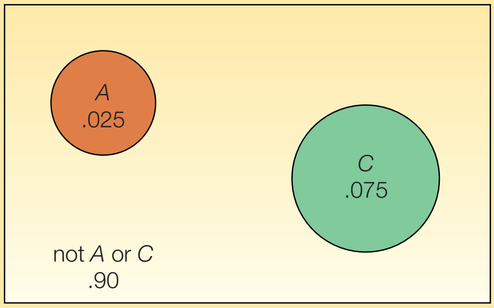


Example

Recall the probability a bag of mixed vegetables is underweight is .025 and the probability of an overweight bag is .075. Use the complement rule to show the probability of a satisfactory bag is .900. Show the solution using a Venn diagram.

Solution

The probability the bag is unsatisfactory equals the probability the bag is overweight plus the probability it is underweight. That is, $P(A \text{ or } C) = P(A) + P(C) = .025 + .075 = .100$. The bag is satisfactory if it is not underweight or overweight, so $P(B) = 1 - [P(A) + P(C)] = 1 - [.025 + .075] = 0.900$. The Venn diagram portraying this situation is:



Self-Review 5–3

A sample of employees of Worldwide Enterprises is to be surveyed about a new health care plan. The employees are classified as follows:

Classification	Event	Number of Employees
Supervisors	*A*	120
Maintenance	*B*	50
Production	*C*	1,460
Management	*D*	302
Secretarial	*E*	68

(a) What is the probability that the first person selected is:
  (i) either in maintenance or a secretary?
  (ii) not in management?
(b) Draw a Venn diagram illustrating your answers to part (a).
(c) Are the events in part (a)(i) complementary or mutually exclusive or both?

The General Rule of Addition The outcomes of an experiment may not be mutually exclusive. Suppose, for illustration, that the Florida Tourist Commission selected a sample of 200 tourists who visited the state during the year. The survey revealed that 120 tourists went to Disney World and 100 went to Busch Gardens near Tampa. What is the probability that a person selected visited either Disney World or Busch Gardens? If the special rule of addition is used, the probability of selecting a tourist who went to Disney World is .60, found by 120/200. Similarly, the probability of a tourist going to Busch Gardens is .50. The sum of these probabilities is 1.10. We know, however, that this probability cannot be greater than 1. The explanation is that many tourists visited both attractions and are being counted twice! A check of the survey responses revealed that 60 out of 200 sampled did, in fact, visit both attractions.

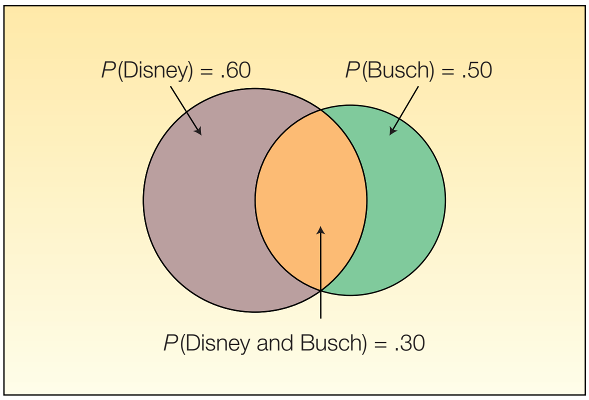
To answer our question, "What is the probability a selected person visited either Disney World or Busch Gardens?" (1) add the probability that a tourist visited Disney

World and the probability he or she visited Busch Gardens, and (2) subtract the probability of visiting both. Thus:

$$P(\text{Disney or Busch}) = P(\text{Disney}) + P(\text{Busch}) - P(\text{both Disney and Busch})$$
$$= .60 + .50 - .30 = .80$$

When two events both occur, the probability is called a **joint probability.** The probability that a tourist visits both attractions (.30) is an example of a joint probability.

The following Venn diagram shows two events that are not mutually exclusive. The two events overlap to illustrate the joint event that some people have visited both attractions.



Statistics in Action

If you wish to get some attention at the next gathering you attend, announce that you believe that at least two people present were born on the same date—that is, the same day of the year but not necessarily the same year. If there are 30 people in the room, the probability of a duplicate is .706. If there are 60 people in the room, the probability is .994 that at least two people share the same birthday. With as few as 23 people the chances are even, that is .50, that at least two people share the same birthday. Hint: To compute this, find the probability everyone was born on a different day and use the complement rule. Try this in your class.

LO4 Define the term *joint probability.*

JOINT PROBABILITY A probability that measures the likelihood two or more events will happen concurrently.

This rule for two events designated *A* and *B* is written:

GENERAL RULE OF ADDITION $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ **[5–4]**

For the expression $P(A \text{ or } B)$, the word *or* suggests that A may occur or B may occur. This also includes the possibility that A and B may occur. This use of *or* is sometimes called an **inclusive.** You could also write $P(A \text{ or } B \text{ or both})$ to emphasize that the union of the events includes the intersection of A and B.

If we compare the general and special rules of addition, the important difference is determining if the events are mutually exclusive. If the events *are* mutually exclusive, then the joint probability $P(A \text{ and } B)$ is 0 and we could use the special rule of addition. Otherwise, we must account for the joint probability and use the general rule of addition.

Example

What is the probability that a card chosen at random from a standard deck of cards will be either a king or a heart?

Solution

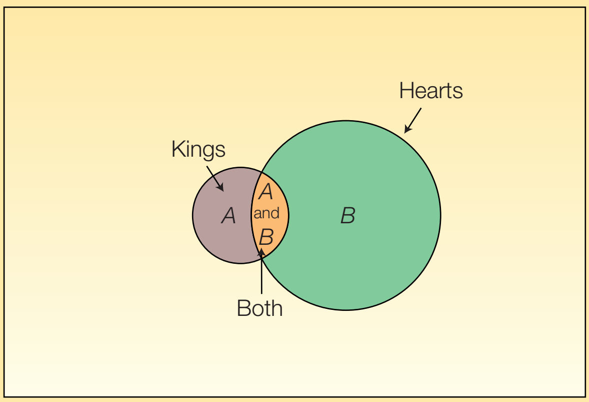
We may be inclined to add the probability of a king and the probability of a heart. But this creates a problem. If we do that, the king of hearts is counted with the kings and also with the hearts. So, if we simply add the probability of a king (there are 4 in a deck of 52 cards) to the probability of a heart (there are 13 in a deck of 52 cards) and report that 17 out of 52 cards meet the requirement, we have counted the king of hearts twice. We need to subtract 1 card from the 17 so the king of hearts is counted only once. Thus, there are 16 cards that are either hearts or kings. So the probability is $16/52 = .3077$.

Card	Probability	Explanation
King	$P(A) = 4/52$	4 kings in a deck of 52 cards
Heart	$P(B) = 13/52$	13 hearts in a deck of 52 cards
King of Hearts	$P(A \text{ and } B) = 1/52$	1 king of hearts in a deck of 52 cards

From formula (5–4):

$$\begin{aligned} P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B) \\ &= 4/52 + 13/52 - 1/52 \\ &= 16/52, \text{ or } .3077 \end{aligned}$$

A Venn diagram portrays these outcomes, which are not mutually exclusive.



Self-Review 5–4

Routine physical examinations are conducted annually as part of a health service program for General Concrete Inc. employees. It was discovered that 8 percent of the employees need corrective shoes, 15 percent need major dental work, and 3 percent need both corrective shoes and major dental work.

(a) What is the probability that an employee selected at random will need either corrective shoes or major dental work?

(b) Show this situation in the form of a Venn diagram.

Exercises

connect™

11. The events A and B are mutually exclusive. Suppose $P(A) = .30$ and $P(B) = .20$. What is the probability of either A or B occurring? What is the probability that neither A nor B will happen?

12. The events X and Y are mutually exclusive. Suppose $P(X) = .05$ and $P(Y) = .02$. What is the probability of either X or Y occurring? What is the probability that neither X nor Y will happen?

13. A study of 200 advertising firms revealed their income after taxes:

Income after Taxes	Number of Firms
Under \$1 million	102
\$1 million to \$20 million	61
\$20 million or more	37

a. What is the probability an advertising firm selected at random has under \$1 million in income after taxes?

b. What is the probability an advertising firm selected at random has either an income between \$1 million and \$20 million, or an income of \$20 million or more? What rule of probability was applied?

14. The chair of the board of directors says, "There is a 50 percent chance this company will earn a profit, a 30 percent chance it will break even, and a 20 percent chance it will lose money next quarter."

a. Use an addition rule to find the probability the company will not lose money next quarter.

b. Use the complement rule to find the probability it will not lose money next quarter.

15. Suppose the probability you will get an A in this class is .25 and the probability you will get a B is .50. What is the probability your grade will be above a C?

16. Two coins are tossed. If A is the event "two heads" and B is the event "two tails," are A and B mutually exclusive? Are they complements?

17. The probabilities of the events A and B are .20 and .30, respectively. The probability that both A and B occur is .15. What is the probability of either A or B occurring?

18. Let $P(X) = .55$ and $P(Y) = .35$. Assume the probability that they both occur is .20. What is the probability of either X or Y occurring?

19. Suppose the two events A and B are mutually exclusive. What is the probability of their joint occurrence?

20. A student is taking two courses, history and math. The probability the student will pass the history course is .60, and the probability of passing the math course is .70. The probability of passing both is .50. What is the probability of passing at least one?

21. A survey of grocery stores in the Southeast revealed 40 percent had a pharmacy, 50 percent had a floral shop, and 70 percent had a deli. Suppose 10 percent of the stores have all three departments, 30 percent have both a pharmacy and a deli, 25 percent have both a floral shop and deli, and 20 percent have both a pharmacy and floral shop.

a. What is the probability of selecting a store at random and finding it has both a pharmacy and a floral shop?

b. What is the probability of selecting a store at random and finding it has both a pharmacy and a deli?

c. Are the events "select a store with a deli" and "select a store with a pharmacy" mutually exclusive?
d. What is the name given to the event of "selecting a store with a pharmacy, a floral shop, and a deli?"
e. What is the probability of selecting a store that does *not* have all three departments?

22. A study by the National Park Service revealed that 50 percent of vacationers going to the Rocky Mountain region visit Yellowstone Park, 40 percent visit the Tetons, and 35 percent visit both.
a. What is the probability a vacationer will visit at least one of these attractions?
b. What is the probability .35 called?
c. Are the events mutually exclusive? Explain.

Rules of Multiplication

When we used the rules of addition in the previous section, we found the likelihood of combining two events. In this section, we find the likelihood that two events both happen. For example, a marketing firm may want to estimate the likelihood that a person is 21 years old or older *and* buys a Hummer. Venn diagrams illustrate this as the intersection of two events. To find the likelihood of two events happening we use the rules of multiplication. There are two rules of multiplication, the special rule and the general rule.

Special Rule of Multiplication The special rule of multiplication requires that two events A and B are independent. Two events are independent if the occurrence of one event does not alter the probability of the occurrence of the other event.

INDEPENDENCE The occurrence of one event has no effect on the probability of the occurrence of another event.

One way to think about independence is to assume that events A and B occur at different times. For example, when event B occurs after event A occurs, does A have any effect on the likelihood that event B occurs? If the answer is no, then A and B are independent events. To illustrate independence, suppose two coins are tossed. The outcome of a coin toss (head or tail) is unaffected by the outcome of any other prior coin toss (head or tail).

For two independent events A and B, the probability that A and B will both occur is found by multiplying the two probabilities. This is the **special rule of multiplication** and is written symbolically as:

LO5 Calculate probabilities using the rules of multiplication.

SPECIAL RULE OF MULTIPLICATION $P(A \text{ and } B) = P(A)P(B)$ **[5–5]**

For three independent events, A, B, and C, the special rule of multiplication used to determine the probability that all three events will occur is:

$$P(A \text{ and } B \text{ and } C) = P(A)P(B)P(C)$$

Example

A survey by the American Automobile Association (AAA) revealed 60 percent of its members made airline reservations last year. Two members are selected at random. What is the probability both made airline reservations last year?

Solution

The probability the first member made an airline reservation last year is .60, written $P(R_1) = .60$, where R_1 refers to the fact that the first member made a reservation.

The probability that the second member selected made a reservation is also .60, so $P(R_2) = .60$. Since the number of AAA members is very large, you may assume that R_1 and R_2 are independent. Consequently, using formula (5–5), the probability they both make a reservation is .36, found by:

$$P(R_1 \text{ and } R_2) = P(R_1)P(R_2) = (.60)(.60) = .36$$

All possible outcomes can be shown as follows. R means a reservation is made, and NR means no reservation was made.

With the probabilities and the complement rule, we can compute the joint probability of each outcome. For example, the probability that neither member makes a reservation is .16. Further, the probability of the first or the second member (special addition rule) making a reservation is .48 (.24 + .24). You can also observe that the outcomes are mutually exclusive and collectively exhaustive. Therefore, the probabilities sum to 1.00.

Outcomes	Joint Probability	
R_1 R_2	(.60)(.60) =	.36
R_1 NR_2	(.60)(.40) =	.24
NR_1 R_2	(.40)(.60) =	.24
NR_1 NR_2	(.40)(.40) =	.16
Total		1.00

Self-Review 5–5

From experience, Teton Tire knows the probability is .95 that a particular XB-70 tire will last 60,000 miles before it becomes bald or fails. An adjustment is made on any tire that does not last 60,000 miles. You purchase four XB-70s. What is the probability all four tires will last at least 60,000 miles?

General Rule of Multiplication If two events are not independent, they are referred to as **dependent.** To illustrate dependency, suppose there are 10 cans of soda in a cooler, 7 are regular and 3 are diet. A can is selected from the cooler. The probability of selecting a can of diet soda is 3/10, and the probability of selecting a can of regular soda is 7/10. Then a second can is selected from the cooler, without returning the first. The probability the second is diet depends on whether the first one selected was diet or not. The probability that the second is diet is:

2/9, if the first can is diet. (Only two cans of diet soda remain in the cooler.)
3/9, if the first can selected is regular. (All three diet sodas are still in the cooler.)

The fraction 2/9 (or 3/9) is aptly called a **conditional probability** because its value is conditional on (dependent on) whether a diet or regular soda was the first selection from the cooler.

LO6 Define the term *conditional probability.*

CONDITIONAL PROBABILITY The probability of a particular event occurring, given that another event has occurred.

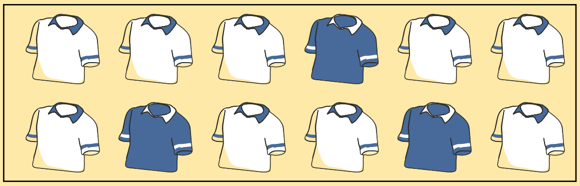
We use the general rule of multiplication to find the joint probability of two events when the events are not independent. For example, when event *B* occurs after event *A* occurs, and *A* has an effect on the likelihood that event *B* occurs, then *A* and *B* are not independent.

The general rule of multiplication states that for two events, *A* and *B*, the joint probability that both events will happen is found by multiplying the probability that event *A* will happen by the conditional probability of event *B* occurring given that *A* has occurred. Symbolically, the joint probability, $P(A \text{ and } B)$, is found by:

GENERAL RULE OF MULTIPLICATION $P(A \text{ and } B) = P(A)P(B|A)$ **[5–6]**

Example

A golfer has 12 golf shirts in his closet. Suppose 9 of these shirts are white and the others blue. He gets dressed in the dark, so he just grabs a shirt and puts it on. He plays golf two days in a row and does not do laundry. What is the likelihood both shirts selected are white?



Solution

The event that the first shirt selected is white is W_1. The probability is $P(W_1) = 9/12$ because 9 of the 12 shirts are white. The event that the second shirt selected is also white is identified as W_2. The conditional probability that the second shirt selected is white, given that the first shirt selected is also white, is $P(W_2|W_1) = 8/11$. Why is this so? Because after the first shirt is selected there are only 11 shirts remaining in the closet and 8 of these are white. To determine the probability of 2 white shirts being selected, we use formula (5–6).

$$P(W_1 \text{ and } W_2) = P(W_1)P(W_2|W_1) = \left(\frac{9}{12}\right)\left(\frac{8}{11}\right) = .55$$

So the likelihood of selecting two shirts and finding them both to be white is .55.

Incidentally, it is assumed that this experiment was conducted *without replacement*. That is, the first shirt was not laundered and put back in the closet before the second was selected. So the outcome of the second event is conditional or dependent on the outcome of the first event.

We can extend the general rule of multiplication to more than two events. For three events *A*, *B*, and *C*, the formula is:

$$P(A \text{ and } B \text{ and } C) = P(A)P(B|A)P(C|A \text{ and } B)$$

In the case of the golf shirt example, the probability of selecting three white shirts without replacement is:

$$P(W_1 \text{ and } W_2 \text{ and } W_3) = P(W_1)P(W_2|W_1)P(W_3|W_1 \text{ and } W_2) = \left(\frac{9}{12}\right)\left(\frac{8}{11}\right)\left(\frac{7}{10}\right) = .38$$

So the likelihood of selecting three shirts without replacement and all being white is .38.

Self-Review 5–6

The board of directors of Tarbell Industries consists of eight men and four women. A four-member search committee is to be chosen at random to conduct a nationwide search for a new company president.

(a) What is the probability all four members of the search committee will be women?
(b) What is the probability all four members will be men?
(c) Does the sum of the probabilities for the events described in parts (a) and (b) equal 1? Explain.

Statistics in Action

In 2000 George W. Bush won the U.S. presidency by the slimmest of margins. Many election stories resulted, some involving voting irregularities, others raising interesting election questions. In a local Michigan election, there was a tie between two candidates for an elected position. To break the tie, the candidates drew a slip of paper from a box that contained two slips of paper, one marked "Winner" and the other unmarked. To determine which candidate drew first, election officials flipped a coin. The winner of the coin flip also drew the winning slip of paper. But was the coin flip really necessary? No, because the two events are independent. Winning the coin flip did not alter the probability of either candidate drawing the winning slip of paper.

5.5 Contingency Tables

Often we tally the results of a survey in a two-way table and use the results of this tally to determine various probabilities. We described this idea beginning on page 126 in Chapter 4. To review, we refer to a two-way table as a contingency table.

CONTINGENCY TABLE A table used to classify sample observations according to two or more identifiable characteristics.

A contingency table is a cross-tabulation that simultaneously summarizes two variables of interest and their relationship. The level of measurement can be nominal. Below are several examples.

- A survey of 150 adults classified each as to gender and the number of movies attended last month. Each respondent is classified according to two criteria—the number of movies attended and gender.

	Gender		
Movies Attended	**Men**	**Women**	**Total**
0	20	40	60
1	40	30	70
2 or more	10	10	20
Total	70	80	150

- The American Coffee Producers Association reports the following information on age and the amount of coffee consumed in a month.

	Coffee Consumption			
Age (Years)	**Low**	**Moderate**	**High**	**Total**
Under 30	36	32	24	92
30 up to 40	18	30	27	75
40 up to 50	10	24	20	54
50 and over	26	24	29	79
Total	90	110	100	300

According to this table, each of the 300 respondents is classified according to two criteria: (1) age and (2) the amount of coffee consumed.

LO7 Compute probabilities using a contingency table.

The following example shows how the rules of addition and multiplication are used when we employ contingency tables.

Example

A sample of executives were surveyed about loyalty to their company. One of the questions was, "If you were given an offer by another company equal to or slightly better than your present position, would you remain with the company or take the other position?" The responses of the 200 executives in the survey were cross-classified with their length of service with the company. (See Table 5–1.)

TABLE 5–1 Loyalty of Executives and Length of Service with Company

	Length of Service				
Loyalty	**Less than 1 Year, B_1**	**1–5 Years, B_2**	**6–10 Years, B_3**	**More than 10 Years, B_4**	**Total**
Would remain, A_1	10	30	5	75	120
Would not remain, A_2	25	15	10	30	80
	35	45	15	105	200

What is the probability of randomly selecting an executive who is loyal to the company (would remain) and who has more than 10 years of service?

Solution

Note that two events occur at the same time—the executive would remain with the company, and he or she has more than 10 years of service.

1. Event A_1 happens if a randomly selected executive will remain with the company despite an equal or slightly better offer from another company. To find the probability that event A_1 will happen, refer to Table 5–1. Note there are 120 executives out of the 200 in the survey who would remain with the company, so $P(A_1) = 120/200$, or .60.
2. Event B_4 happens if a randomly selected executive has more than 10 years of service with the company. Thus, $P(B_4|A_1)$ is the conditional probability that an executive with more than 10 years of service would remain with the company despite an equal or slightly better offer from another company. Referring to the contingency table, Table 5–1, 75 of the 120 executives who would remain have more than 10 years of service, so $P(B_4|A_1) = 75/120$.

Solving for the probability that an executive randomly selected will be one who would remain with the company and who has more than 10 years of service with the company, using the general rule of multiplication in formula (5–6), gives:

$$P(A_1 \text{ and } B_4) = P(A_1)P(B_4|A_1) = \left(\frac{120}{200}\right)\left(\frac{75}{120}\right) = \frac{9{,}000}{24{,}000} = .375$$

To find the probability of selecting an executive who would remain with the company *or* has less than 1 year of experience, we use the general rule of addition, formula (5–4).

1. Event A_1 refers to executives that would remain with the company. So $P(A_1) = 120/200 = .60$.
2. Event B_1 refers to executives that have been with the company less than 1 year. The probability of B_1 is $P(B_1) = 35/200 = .175$.
3. The events A_1 and B_1 are not mutually exclusive. That is, an executive can both be willing to remain with the company and have less than 1 year of experience.

We write this probability, which is called the joint probability, as $P(A_1$ and $B_1)$. There are 10 executives who would both stay with the company and have less than 1 year of service, so $P(A_1$ and $B_1) = 10/200 = .05$. These 10 people are in both groups, those who would remain with the company and those with less than 1 year with the company. They are actually being counted twice, so we need to subtract out this value.

4. We insert these values in formula (5–4) and the result is as follows.

$$P(A_1 \text{ or } B_1) = P(A_1) + P(B_1) - P(A_1 \text{ and } B_1)$$
$$= .60 + .175 - .05 = .725$$

So the likelihood that a selected executive would either remain with the company or has been with the company less than 1 year is .725.

Self-Review 5–7

Refer to Table 5–1 on page 163 to find the following probabilities.

(a) What is the probability of selecting an executive with more than 10 years of service?
(b) What is the probability of selecting an executive who would not remain with the company, given that he or she has more than 10 years of service?
(c) What is the probability of selecting an executive with more than 10 years of service or one who would not remain with the company?

5.6 Tree Diagrams

The **tree diagram** is a graph that is helpful in organizing calculations that involve several stages. Each segment in the tree is one stage of the problem. The branches of a tree diagram are weighted by probabilities. We will use the data in Table 5–1 to show the construction of a tree diagram.

1. To construct a tree diagram, we begin by drawing a heavy dot on the left to represent the root of the tree (see Chart 5–2).
2. For this problem, two main branches go out from the root, the upper one representing "would remain" and the lower one "would not remain." Their probabilities are written on the branches, namely, 120/200 and 80/200. These probabilities could also be denoted $P(A_1)$ and $P(A_2)$.
3. Four branches "grow" out of each of the two main branches. These branches represent the length of service—less than 1 year, 1–5 years, 6–10 years, and more than 10 years. The conditional probabilities for the upper branch of the tree, 10/120, 30/120, 5/120, and so on are written on the appropriate branches. These are $P(B_1|A_1)$, $P(B_2|A_1)$, $P(B_3|A_1)$, and $P(B_4|A_1)$, where B_1 refers to less than 1 year of service, B_2 1 to 5 years, B_3 6 to 10 years, and B_4 more than 10 years. Next, write the conditional probabilities for the lower branch.
4. Finally, joint probabilities, that the events A_1 and B_i or the events A_2 and B_i will occur together, are shown on the right side. For example, the joint probability of randomly selecting an executive who would remain with the company and who has less than 1 year of service, from formula (5–6), is:

$$P(A_1 \text{ and } B_1) = P(A_1)P(B_1|A_1) = \left(\frac{120}{200}\right)\left(\frac{10}{120}\right) = .05$$

Because the joint probabilities represent all possible outcomes (would remain, 6–10 years service; would not remain, more than 10 years of service; etc.), they must sum to 1.00 (see Chart 5–2).

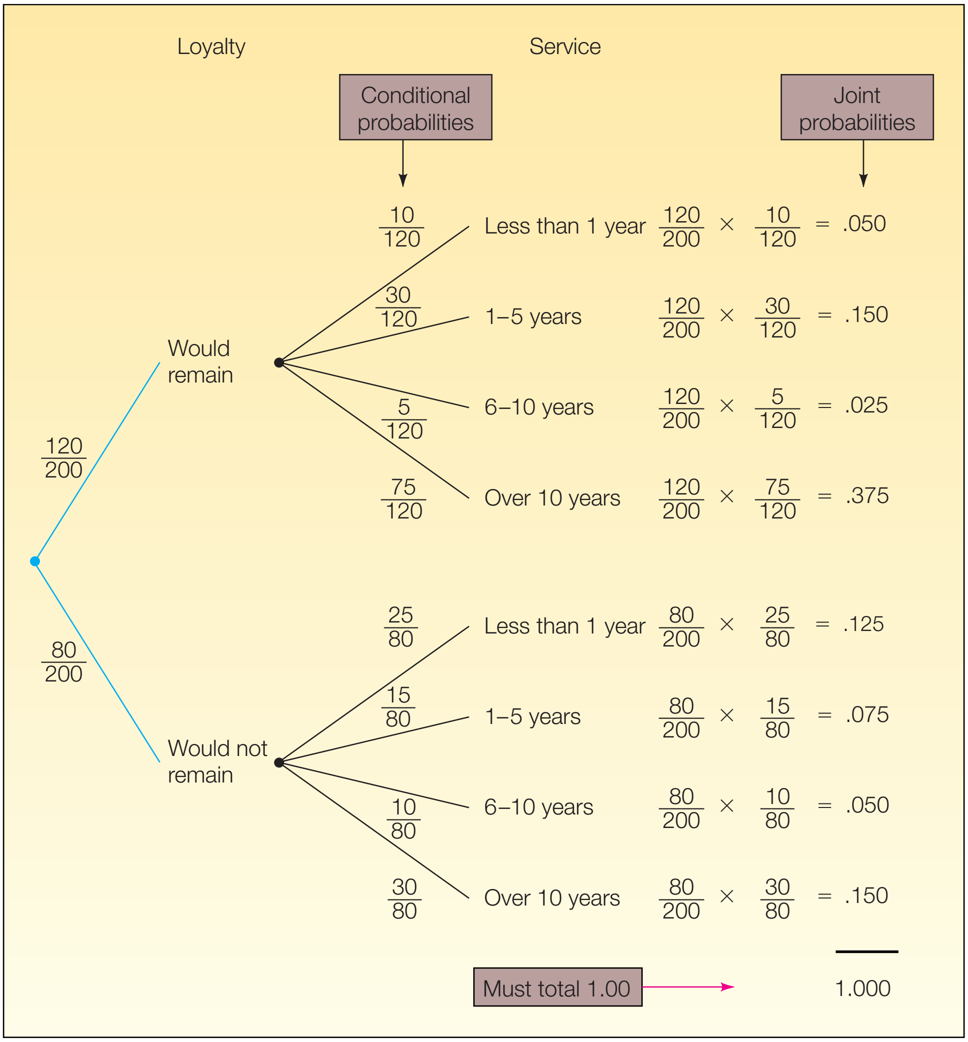


CHART 5–2 Tree Diagram Showing Loyalty and Length of Service

Self-Review 5–8

Consumers were surveyed on the relative number of visits to a Sears store (often, occasional, and never) and if the store was located in an enclosed mall (yes and no). When variables are measured nominally, such as these data, the results are usually summarized in a contingency table.

	Enclosed Mall		
Visits	**Yes**	**No**	**Total**
Often	60	20	80
Occasional	25	35	60
Never	5	50	55
	90	105	195

(a) Are the number of visits and enclosed mall variables independent? Why? Interpret your conclusion.
(b) Draw a tree diagram and determine the joint probabilities.

Exercises

23. Suppose $P(A) = .40$ and $P(B|A) = .30$. What is the joint probability of A and B?
24. Suppose $P(X_1) = .75$ and $P(Y_2|X_1) = .40$. What is the joint probability of X_1 and Y_2?
25. A local bank reports that 80 percent of its customers maintain a checking account, 60 percent have a savings account, and 50 percent have both. If a customer is chosen at random, what is the probability the customer has either a checking or a savings account? What is the probability the customer does not have either a checking or a savings account?
26. All Seasons Plumbing has two service trucks that frequently need repair. If the probability the first truck is available is .75, the probability the second truck is available is .50, and the probability that both trucks are available is .30, what is the probability neither truck is available?
27. Refer to the following table.

	First Event			
Second Event	A_1	A_2	A_3	Total
B_1	2	1	3	6
B_2	1	2	1	4
Total	3	3	4	10

   a. Determine $P(A_1)$.
   b. Determine $P(B_1|A_2)$.
   c. Determine $P(B_2 \text{ and } A_3)$.
28. Three defective electric toothbrushes were accidentally shipped to a drugstore by Cleanbrush Products along with 17 nondefective ones.
   a. What is the probability the first two electric toothbrushes sold will be returned to the drugstore because they are defective?
   b. What is the probability the first two electric toothbrushes sold will not be defective?
29. Each salesperson at Puchett, Sheets, and Hogan Insurance Agency is rated either below average, average, or above average with respect to sales ability. Each salesperson is also rated with respect to his or her potential for advancement—either fair, good, or excellent. These traits for the 500 salespeople were cross-classified into the following table.

	Potential for Advancement		
Sales Ability	Fair	Good	Excellent
Below average	16	12	22
Average	45	60	45
Above average	93	72	135

   a. What is this table called?
   b. What is the probability a salesperson selected at random will have above average sales ability and excellent potential for advancement?
   c. Construct a tree diagram showing all the probabilities, conditional probabilities, and joint probabilities.
30. An investor owns three common stocks. Each stock, independent of the others, has equally likely chances of (1) increasing in value, (2) decreasing in value, or (3) remaining the same value. List the possible outcomes of this experiment. Estimate the probability at least two of the stocks increase in value.
31. The board of directors of a small company consists of five people. Three of those are "strong leaders." If they buy an idea, the entire board will agree. The other "weak" members have no influence. Three salespeople are scheduled, one after the other, to make sales presentations to a board member of the salesperson's choice. The salespeople are convincing but do not know who the "strong leaders" are. However, they will know who the previous salespeople spoke to. The first salesperson to find a strong leader will win the account. Do the three salespeople have the same chance of winning the account? If not, find their respective probabilities of winning.

32. If you ask three strangers about their birthdays, what is the probability: (a) All were born on Wednesday? (b) All were born on different days of the week? (c) None were born on Saturday?

5.7 Bayes' Theorem

LO8 Calculate probabilities using Bayes' theorem.

In the 18th century, Reverend Thomas Bayes, an English Presbyterian minister, pondered this question: Does God really exist? Being interested in mathematics, he attempted to develop a formula to arrive at the probability God does exist based on evidence available to him on earth. Later Pierre-Simon Laplace refined Bayes' work and gave it the name "Bayes' theorem." In a workable form, **Bayes' theorem** is:

BAYES' THEOREM

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2)} \qquad \textbf{[5–7]}$$

Assume in formula 5–7 that the events A_1 and A_2 are mutually exclusive and collectively exhaustive, and A_i refers to either event A_1 or A_2. Hence A_1 and A_2 are in this case complements. The meaning of the symbols used is illustrated by the following example.

Suppose 5 percent of the population of Umen, a fictional Third World country, have a disease that is peculiar to that country. We will let A_1 refer to the event "has the disease" and A_2 refer to the event "does not have the disease." Thus, we know that if we select a person from Umen at random, the probability the individual chosen has the disease is .05, or $P(A_1) = .05$. This probability, $P(A_1) = P(\text{has the disease}) = .05$, is called the **prior probability.** It is given this name because the probability is assigned before any empirical data are obtained.

PRIOR PROBABILITY The initial probability based on the present level of information.

The prior probability a person is not afflicted with the disease is therefore .95, or $P(A_2) = .95$, found by $1 - .05$.

There is a diagnostic technique to detect the disease, but it is not very accurate. Let B denote the event "test shows the disease is present." Assume that historical evidence shows that if a person actually has the disease, the probability that the test will indicate the presence of the disease is .90. Using the conditional probability definitions developed earlier in this chapter, this statement is written as:

$$P(B|A_1) = .90$$

Assume the probability is .15 that for a person who actually does not have the disease the test will indicate the disease is present.

$$P(B|A_2) = .15$$

Let's randomly select a person from Umen and perform the test. The test results indicate the disease is present. What is the probability the person actually has the disease? In symbolic form, we want to know $P(A_1|B)$, which is interpreted as: $P(\text{has the disease} \mid \text{the test results are positive})$. The probability $P(A_1|B)$ is called a **posterior probability.**

POSTERIOR PROBABILITY A revised probability based on additional information.

With the help of Bayes' theorem, formula (5–7), we can determine the posterior probability.

Statistics in Action

A recent study by the National Collegiate Athletic Association (NCAA) reported that of 150,000 senior boys playing on their high school basketball team, 64 would make a professional team. To put it another way, the odds of a high school senior basketball player making a professional team are 1 in 2,344. From the same study:

1. The odds of a high school senior basketball player playing some college basketball are about 1 in 40.
2. The odds of a high school senior playing college basketball as a senior in college are about 1 in 60.
3. If you play basketball as a senior in college, the odds of making a professional team are about 1 in 37.5.

$$P(A_1|B) = \frac{P(A_1)P(B|A_1)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2)}$$

$$= \frac{(.05)(.90)}{(.05)(.90) + (.95)(.15)} = \frac{.0450}{.1875} = .24$$

So the probability that a person has the disease, given that he or she tested positive, is .24. How is the result interpreted? If a person is selected at random from the population, the probability that he or she has the disease is .05. If the person is tested and the test result is positive, the probability that the person actually has the disease is increased about fivefold, from .05 to .24.

In the preceding problem, we had only two mutually exclusive and collectively exhaustive events, A_1 and A_2. If there are n such events, $A_1, A_2, \cdots, A_n$, Bayes' theorem, formula (5–7), becomes

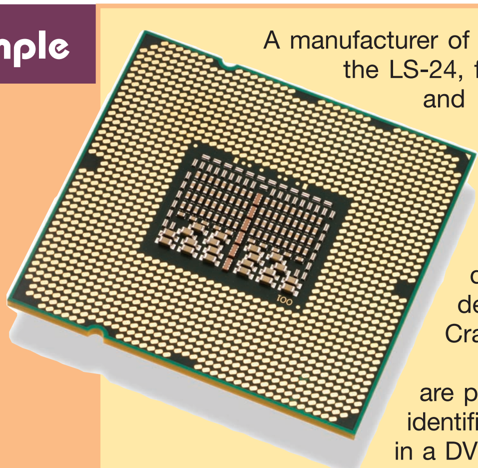
$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \cdots + P(A_n)P(B|A_n)}$$

With the preceding notation, the calculations for the Umen problem are summarized in the following table.

Event, A_i	Prior Probability, $P(A_i)$	Conditional Probability, $P(B\|A_i)$	Joint Probability, $P(A_i$ and $B)$	Posterior Probability, $P(A_i\|B)$
Disease, A_1	.05	.90	.0450	.0450/.1875 = .24
No disease, A_2	.95	.15	.1425	.1425/.1875 = .76
			$P(B)$ = .1875	1.00

Another illustration of Bayes' theorem follows.

Example



A manufacturer of DVD players purchases a particular microchip, called the LS-24, from three suppliers: Hall Electronics, Schuller Sales, and Crawford Components. Thirty percent of the LS-24 chips are purchased from Hall Electronics, 20 percent from Schuller Sales, and the remaining 50 percent from Crawford Components. The manufacturer has extensive histories on the three suppliers and knows that 3 percent of the LS-24 chips from Hall Electronics are defective, 5 percent of chips from Schuller Sales are defective, and 4 percent of the chips purchased from Crawford Components are defective.

When the LS-24 chips arrive at the manufacturer, they are placed directly in a bin and not inspected or otherwise identified by supplier. A worker selects a chip for installation in a DVD player and finds it defective. What is the probability that it was manufactured by Schuller Sales?

Solution

As a first step, let's summarize some of the information given in the problem statement.

- There are three mutually exclusive and collectively exhaustive events, that is, three suppliers.

A_1 The LS-24 was purchased from Hall Electronics.
A_2 The LS-24 was purchased from Schuller Sales.
A_3 The LS-24 was purchased from Crawford Components.

- The prior probabilities are:

  $P(A_1) = .30$ The probability the LS-24 was manufactured by Hall Electronics.
  $P(A_2) = .20$ The probability the LS-24 was manufactured by Schuller Sales.
  $P(A_3) = .50$ The probability the LS-24 was manufactured by Crawford Components.

- The additional information can be either:

  B_1 The LS-24 appears defective, or
  B_2 The LS-24 appears not to be defective.

- The following conditional probabilities are given.

  $P(B_1|A_1) = .03$ The probability that an LS-24 chip produced by Hall Electronics is defective.
  $P(B_1|A_2) = .05$ The probability that an LS-24 chip produced by Schuller Sales is defective.
  $P(B_1|A_3) = .04$ The probability that an LS-24 chip produced by Crawford Components is defective.

- A chip is selected from the bin. Because the chips are not identified by supplier, we are not certain which supplier manufactured the chip. We want to determine the probability that the defective chip was purchased from Schuller Sales. The probability is written $P(A_2|B_1)$.

Look at Schuller's quality record. It is the worst of the three suppliers. Now that we have found a defective LS-24 chip, we suspect that $P(A_2|B_1)$ is greater than $P(A_2)$. That is, we expect the revised probability to be greater than .20. But how much greater? Bayes' theorem can give us the answer. As a first step, consider the tree diagram in Chart 5–3.

The events are dependent, so the prior probability in the first branch is multiplied by the conditional probability in the second branch to obtain the joint probability. The joint probability is reported in the last column of Chart 5–3. To construct the tree diagram of Chart 5–3, we used a time sequence that moved from the supplier to the determination of whether the chip was acceptable or unacceptable.

What we need to do is reverse the time process. That is, instead of moving from left to right in Chart 5–3, we need to move from right to left. We have a defective chip, and we want to determine the likelihood that it was purchased from Schuller Sales. How is that accomplished? We first look at the joint probabilities as relative frequencies out of 1,000 cases. For example, the likelihood of a defective LS-24 chip that was produced by Hall Electronics is .009. So of 1,000 cases, we would expect to find 9 defective chips produced by Hall Electronics. We observe that in 39 of 1,000 cases the LS-24 chip selected for assembly will be defective, found by 9 + 10 + 20. Of these 39 defective chips, 10 were produced by Schuller Sales. Thus, the probability that the defective LS-24 chip was purchased from Schuller Sales is 10/39 = .2564. We have now determined the revised probability of $P(A_2|B_1)$. Before we found the defective chip, the likelihood that it was purchased from Schuller Sales was .20. This likelihood has been increased to .2564.

This information is summarized in the following table.

Event, A_i	Prior Probability, $P(A_i)$	Conditional Probability, $P(B_1\|A_i)$	Joint Probability, $P(A_i \text{ and } B_1)$	Posterior Probability, $P(A_i\|B_1)$
Hall	.30	.03	.009	.009/.039 = .2308
Schuller	.20	.05	.010	.010/.039 = .2564
Crawford	.50	.04	.020	.020/.039 = .5128
			$P(B_1) = .039$	1.0000

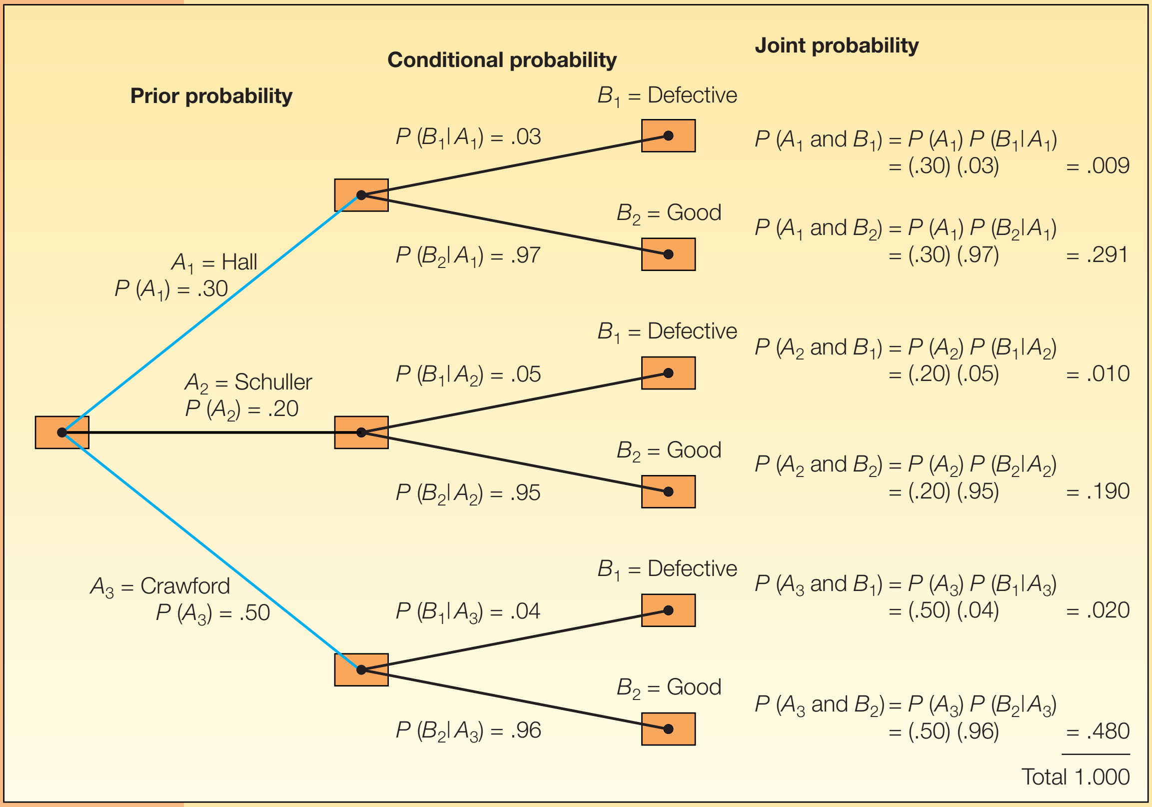


CHART 5–3 Tree Diagram of DVD Manufacturing Problem

The probability the defective LS-24 chip came from Schuller Sales can be formally found by using Bayes' theorem. We compute $P(A_2|B_1)$, where A_2 refers to Schuller Sales and B_1 to the fact that the selected LS-24 chip was defective.

$$P(A_2|B_1) = \frac{P(A_2)P(B_1|A_2)}{P(A_1)P(B_1|A_1) + P(A_2)P(B_1|A_2) + P(A_3)(B_1|A_3)}$$

$$= \frac{(.20)(.05)}{(.30)(.03) + (.20)(.05) + (.50)(.04)} = \frac{.010}{.039} = .2564$$

This is the same result obtained from Chart 5–3 and from the conditional probability table.

Self-Review 5–9

Refer to the preceding example and solution.
(a) Design a formula to find the probability the part selected came from Crawford Components, given that it was a good chip.
(b) Compute the probability using Bayes' theorem.

Exercises

connect

33. $P(A_1) = .60$, $P(A_2) = .40$, $P(B_1|A_1) = .05$, and $P(B_1|A_2) = .10$. Use Bayes' theorem to determine $P(A_1|B_1)$.

34. $P(A_1) = .20$, $P(A_2) = .40$, $P(A_3) = .40$, $P(B_1|A_1) = .25$, $P(B_1|A_2) = .05$, and $P(B_1|A_3) = .10$. Use Bayes' theorem to determine $P(A_3|B_1)$.

35. The Ludlow Wildcats baseball team, a minor league team in the Cleveland Indians organization, plays 70 percent of their games at night and 30 percent during the day. The team wins 50 percent of their night games and 90 percent of their day games. According to today's newspaper, they won yesterday. What is the probability the game was played at night?

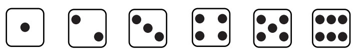
36. Dr. Stallter has been teaching basic statistics for many years. She knows that 80 percent of the students will complete the assigned problems. She has also determined that among those who do their assignments, 90 percent will pass the course. Among those students who do not do their homework, 60 percent will pass. Mike Fishbaugh took statistics last semester from Dr. Stallter and received a passing grade. What is the probability that he completed the assignments?

37. The credit department of Lion's Department Store in Anaheim, California, reported that 30 percent of their sales are cash or check, 30 percent are paid with a credit card and 40 percent with a debit card. Twenty percent of the cash or check purchases, 90 percent of the credit card purchases, and 60 percent of the debit card purchases are for more than \$50. Ms. Tina Stevens just purchased a new dress that cost \$120. What is the probability that she paid cash or check?

38. One-fourth of the residents of the Burning Ridge Estates leave their garage doors open when they are away from home. The local chief of police estimates that 5 percent of the garages with open doors will have something stolen, but only 1 percent of those closed will have something stolen. If a garage is robbed, what is the probability the doors were left open?

5.8 Principles of Counting

If the number of possible outcomes in an experiment is small, it is relatively easy to count them. There are six possible outcomes, for example, resulting from the roll of a die, namely:



LO9 Determine the number of outcomes using the appropriate principle of counting.

If, however, there are a large number of possible outcomes, such as the number of heads and tails for an experiment with 10 tosses, it would be tedious to count all the possibilities. They could have all heads, one head and nine tails, two heads and eight tails, and so on. To facilitate counting, we discuss three formulas: the **multiplication formula** (not to be confused with the multiplication *rule* described earlier in the chapter), the **permutation formula,** and the **combination formula.**

The Multiplication Formula

We begin with the multiplication formula.

MULTIPLICATION FORMULA If there are m ways of doing one thing and n ways of doing another thing, there are $m \times n$ ways of doing both.

In terms of a formula:

MULTIPLICATION FORMULA Total number of arrangements $= (m)(n)$ **[5–8]**

This can be extended to more than two events. For three events m, n, and o:

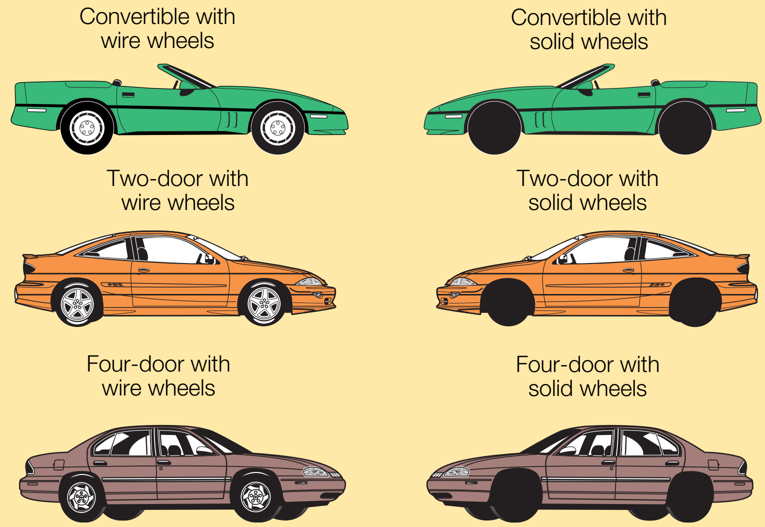
$$\text{Total number of arrangements} = (m)(n)(o)$$

Example

An automobile dealer wants to advertise that for \$29,999 you can buy a convertible, a two-door sedan, or a four-door model with your choice of either wire wheel covers or solid wheel covers. How many different arrangements of models and wheel covers can the dealer offer?

Solution

Of course the dealer could determine the total number of arrangements by picturing and counting them. There are six.



We can employ the multiplication formula as a check (where *m* is the number of models and *n* the wheel cover type). From formula (5–8):

$$\text{Total possible arrangements} = (m)(n) = (3)(2) = 6$$

It was not difficult to count all the possible model and wheel cover combinations in this example. Suppose, however, that the dealer decided to offer eight models and six types of wheel covers. It would be tedious to picture and count all the possible alternatives. Instead, the multiplication formula can be used. In this case, there are $(m)(n) = (8)(6) = 48$ possible arrangements.

Note in the preceding applications of the multiplication formula that there were *two or more groupings from which you made selections*. The automobile dealer, for example, offered a choice of models and a choice of wheel covers. If a home builder offered you four different exterior styles of a home to choose from and three interior floor plans, the multiplication formula would be used to find how many different arrangements were possible. There are 12 possibilities.

Self-Review 5–10

1. The Women's Shopping Network on cable TV offers sweaters and slacks for women. The sweaters and slacks are offered in coordinating colors. If sweaters are available in five colors and the slacks are available in four colors, how many different outfits can be advertised?
2. Pioneer manufactures three models of stereo receivers, two MP3 docking stations, four speakers, and three CD carousels. When the four types of components are sold together, they form a "system." How many different systems can the electronics firm offer?

The Permutation Formula

As noted, the multiplication formula is applied to find the number of possible arrangements for two or more groups. The **permutation formula** is applied to find the possible number of arrangements when there is only *one* group of objects. Illustrations of this type of problem are:

- Three electronic parts are to be assembled into a plug-in unit for a television set. The parts can be assembled in any order. How many different ways can the three parts be assembled?
- A machine operator must make four safety checks before starting his machine. It does not matter in which order the checks are made. In how many different ways can the operator make the checks?

One order for the first illustration might be: the transistor first, the LEDs second, and the synthesizer third. This arrangement is called a **permutation.**

PERMUTATION Any arrangement of r objects selected from a single group of n possible objects.

Note that the arrangements $a\ b\ c$ and $b\ a\ c$ are different permutations. The formula to count the total number of different permutations is:

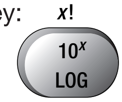
PERMUTATION FORMULA $$_nP_r = \frac{n!}{(n-r)!} \quad \textbf{[5–9]}$$

where:

n is the total number of objects.
r is the number of objects selected.

Before we solve the two problems illustrated, note that permutations and combinations (to be discussed shortly) use a notation called *n factorial*. It is written $n!$ and means the product of $n(n-1)(n-2)(n-3)\cdots(1)$. For instance, $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.

Many of your calculators have a button with $x!$ that will perform this calculation for you. It will save you a great deal of time. For example the Texas Instrument TI-36X calculator has the following key: **$x!$**



It is the "third function" so check your users' manual or the Internet for instructions.

The factorial notation can also be canceled when the same number appears in both the numerator and the denominator, as shown below.

$$\frac{6!3!}{4!} = \frac{6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}(3 \cdot 2 \cdot 1)}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 180$$

By definition, zero factorial, written $0!$, is 1. That is, $0! = 1$.

Example

Referring to the group of three electronic parts that are to be assembled in any order, in how many different ways can they be assembled?

Solution

There are three electronic parts to be assembled, so $n = 3$. Because all three are to be inserted in the plug-in unit, $r = 3$. Solving using formula (5–9) gives:

$$_nP_r = \frac{n!}{(n-r)!} = \frac{3!}{(3-3)!} = \frac{3!}{0!} = \frac{3!}{1} = 6$$

We can check the number of permutations arrived at by using the permutation formula. We determine how many "spaces" have to be filled and the possibilities for each "space." In the problem involving three electronic parts, there are three locations in the plug-in unit for the three parts. There are three possibilities for the first place, two for the second (one has been used up), and one for the third, as follows:

$$(3)(2)(1) = 6 \text{ permutations}$$

The six ways in which the three electronic parts, lettered A, B, C, can be arranged are:

ABC	*BAC*	*CAB*	*ACB*	*BCA*	*CBA*

In the previous example, we selected and arranged all the objects, that is $n = r$. In many cases, only some objects are selected and arranged from the n possible objects. We explain the details of this application in the following example.

Example

Betts Machine Shop Inc. has eight screw machines but only three spaces available in the production area for the machines. In how many different ways can the eight machines be arranged in the three spaces available?

Solution

There are eight possibilities for the first available space in the production area, seven for the second space (one has been used up), and six for the third space. Thus:

$$(8)(7)(6) = 336,$$

that is, there are a total of 336 different possible arrangements. This could also be found by using formula (5–9). If $n = 8$ machines, and $r = 3$ spaces available, the formula leads to

$$_nP_r = \frac{n!}{(n-r)!} = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{(8)(7)(6)\cancel{5!}}{\cancel{5!}} = 336$$

The Combination Formula

If the order of the selected objects is *not* important, any selection is called a **combination.** The formula to count the number of r object combinations from a set of n objects is:

COMBINATION FORMULA $$_nC_r = \frac{n!}{r!(n-r)!}$$ **[5–10]**

For example, if executives Able, Baker, and Chauncy are to be chosen as a committee to negotiate a merger, there is only one possible combination of these three; the committee of Able, Baker, and Chauncy is the same as the committee of Baker, Chauncy, and Able. Using the combination formula:

$$_nC_r = \frac{n!}{r!(n-r)!} = \frac{3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1(1)} = 1$$

Example

The marketing department has been given the assignment of designing color codes for the 42 different lines of compact disks sold by Goody Records. Three colors are to be used on each CD, but a combination of three colors used for one CD cannot be rearranged and used to identify a different CD. This means that if green, yellow, and violet were used to identify one line, then yellow, green, and violet (or any other combination of these three colors) cannot be used to identify another line. Would seven colors taken three at a time be adequate to color-code the 42 lines?

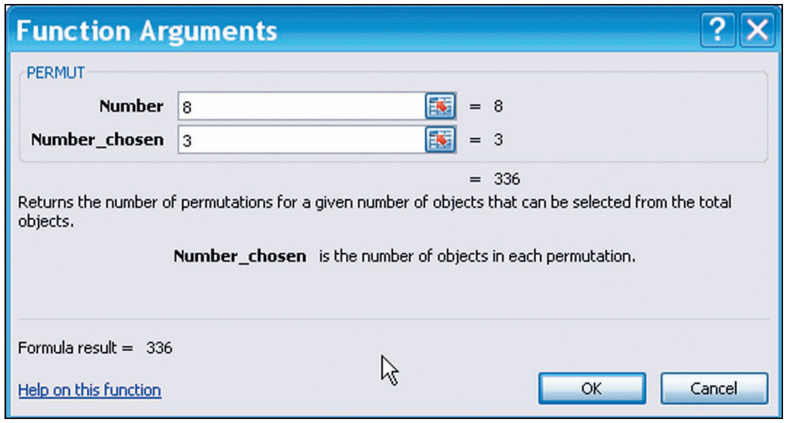
Solution

According to formula (5–10), there are 35 combinations, found by

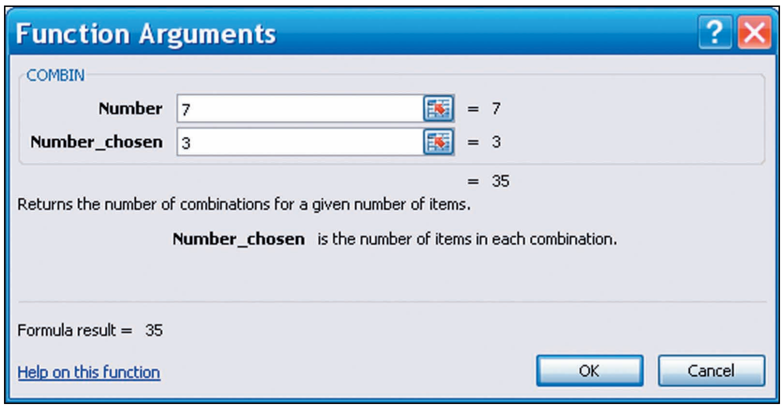
$$_7C_3 = \frac{n!}{r!(n-r)!} = \frac{7!}{3!(7-3)!} = \frac{7!}{3!4!} = 35$$

The seven colors taken three at a time (i.e., three colors to a line) would not be adequate to color-code the 42 different lines because they would provide only 35 combinations. Eight colors taken three at a time would give 56 different combinations. This would be more than adequate to color-code the 42 different lines.

When the number of permutations or combinations is large, the calculations are tedious. Computer software and handheld calculators have "functions" to compute these numbers. The Excel output for the location of the eight screw machines in the production area of Betts Machine Shop Inc. is shown below. There are a total of 336 arrangements.



Below is the output for the color codes at Goody Records. Three colors are chosen from among seven possible. The number of combinations possible is 35.



Self-Review 5–11

1. A musician wants to write a score based on only five chords: B-flat, C, D, E, and G. However, only three chords out of the five will be used in succession, such as C, B-flat, and E. Repetitions, such as B-flat, B-flat, and E, will not be permitted.
   (a) How many permutations of the five chords, taken three at a time, are possible?
   (b) Using formula (5–9), how many permutations are possible?
2. The 10 numbers 0 through 9 are to be used in code groups of four to identify an item of clothing. Code 1083 might identify a blue blouse, size medium; the code group 2031 might identify a pair of pants, size 18; and so on. Repetitions of numbers are not permitted. That is, the same number cannot be used twice (or more) in a total sequence. For example, 2256, 2562, or 5559 would not be permitted. How many different code groups can be designed?
3. In the above example involving Goody Records, we said that eight colors taken three at a time would give 56 different combinations.
   (a) Use formula (5–10) to show this is true.
   (b) As an alternative plan for color-coding the 42 different lines, it has been suggested that only two colors be placed on a disk. Would 10 colors be adequate to color-code

the 42 different lines? (Again, a combination of two colors could be used only once—that is, if pink and blue were coded for one line, blue and pink could not be used to identify a different line.)

4. In a lottery game, three numbers are randomly selected from a tumbler of balls numbered 1 through 50.
   (a) How many permutations are possible?
   (b) How many combinations are possible?

Exercises

39. Solve the following:
- **a.** 40!/35!
- **b.** ${}_7P_4$
- **c.** ${}_5C_2$

40. Solve the following:
- **a.** 20!/17!
- **b.** ${}_9P_3$
- **c.** ${}_7C_2$

41. A pollster randomly selected 4 of 10 available people. How many different groups of 4 are possible?

42. A telephone number consists of seven digits, the first three representing the exchange. How many different telephone numbers are possible within the 537 exchange?

43. An overnight express company must include five cities on its route. How many different routes are possible, assuming that it does not matter in which order the cities are included in the routing?

44. A representative of the Environmental Protection Agency (EPA) wants to select samples from 10 landfills. The director has 15 landfills from which she can collect samples. How many different samples are possible?

45. A national pollster has developed 15 questions designed to rate the performance of the president of the United States. The pollster will select 10 of these questions. How many different arrangements are there for the order of the 10 selected questions?

46. A company is creating three new divisions and seven managers are eligible to be appointed head of a division. How many different ways could the three new heads be appointed? Hint: Assume the division assignment makes a difference.

Chapter Summary

I. A probability is a value between 0 and 1 inclusive that represents the likelihood a particular event will happen.
- **A.** An experiment is the observation of some activity or the act of taking some measurement.
- **B.** An outcome is a particular result of an experiment.
- **C.** An event is the collection of one or more outcomes of an experiment.

II. There are three definitions of probability.
- **A.** The classical definition applies when there are *n* equally likely outcomes to an experiment.
- **B.** The empirical definition occurs when the number of times an event happens is divided by the number of observations.
- **C.** A subjective probability is based on whatever information is available.

III. Two events are mutually exclusive if by virtue of one event happening the other cannot happen.

IV. Events are independent if the occurrence of one event does not affect the occurrence of another event.

V. The rules of addition refer to the union of events.

Statistics in Action

Government statistics show there are about 1.7 automobile-caused fatalities for every 100,000,000 vehicle-miles. If you drive 1 mile to the store to buy your lottery ticket and then return home, you have driven 2 miles. Thus the probability that you will join this statistical group on your next 2 mile round trip is 2 × 1.7/100,000,000 = 0.000000034. This can also be stated as "One in 29,411,765." Thus if you drive to the store to buy your Powerball ticket, your chance of being killed (or killing someone else) is more than 4 times greater than the chance that you will win the Powerball Jackpot, one chance in 120,526,770. http://www.durangobill.com/Powerball Odds.html

A. The special rule of addition is used when events are mutually exclusive.

$$P(A \text{ or } B) = P(A) + P(B) \qquad \textbf{[5–2]}$$

B. The general rule of addition is used when the events are not mutually exclusive.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \qquad \textbf{[5–4]}$$

C. The complement rule is used to determine the probability of an event happening by subtracting the probability of the event not happening from 1.

$$P(A) = 1 - P(\sim A) \qquad \textbf{[5–3]}$$

VI. The rules of multiplication refer to the product of events.

A. The special rule of multiplication refers to events that are independent.

$$P(A \text{ and } B) = P(A)P(B) \qquad \textbf{[5–5]}$$

B. The general rule of multiplication refers to events that are not independent.

$$P(A \text{ and } B) = P(A)P(B|A) \qquad \textbf{[5–6]}$$

C. A joint probability is the likelihood that two or more events will happen at the same time.

D. A conditional probability is the likelihood that an event will happen, given that another event has already happened.

E. Bayes' theorem is a method of revising a probability, given that additional information is obtained. For two mutually exclusive and collectively exhaustive events:

$$P(A_1|B) = \frac{P(A_1)P(B|A_1)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2)} \qquad \textbf{[5–7]}$$

VII. There are three counting rules that are useful in determining the number of outcomes in an experiment.

A. The multiplication rule states that if there are m ways one event can happen and n ways another event can happen, then there are mn ways the two events can happen.

$$\text{Number of arrangements} = (m)(n) \qquad \textbf{[5–8]}$$

B. A permutation is an arrangement in which the order of the objects selected from a specific pool of objects is important.

$$_nP_r = \frac{n!}{(n-r)!} \qquad \textbf{[5–9]}$$

C. A combination is an arrangement where the order of the objects selected from a specific pool of objects is not important.

$$_nC_r = \frac{n!}{r!(n-r)!} \qquad \textbf{[5–10]}$$

Pronunciation Key

SYMBOL	MEANING	PRONUNCIATION
$P(A)$	Probability of A	P of A
$P(\sim A)$	Probability of not A	P of not A
$P(A$ and $B)$	Probability of A and B	P of A and B
$P(A$ or $B)$	Probability of A or B	P of A or B
$P(A\|B)$	Probability of A given B has happened	P of A given B
$_nP_r$	Permutation of n items selected r at a time	*Pnr*
$_nC_r$	Combination of n items selected r at a time	*Cnr*

Chapter Exercises

47. The marketing research department at Pepsico plans to survey teenagers about a newly developed soft drink. Each will be asked to compare it with his or her favorite soft drink.
  a. What is the experiment?
  b. What is one possible event?

48. The number of times a particular event occurred in the past is divided by the number of occurrences. What is this approach to probability called?

49. The probability that the cause and the cure for all cancers will be discovered before the year 2020 is .20. What viewpoint of probability does this statement illustrate?

50. Berdine's Chicken Factory has several stores in the Hilton Head, South Carolina, area. When interviewing applicants for server positions, the owner would like to include information on the amount of tip a server can expect to earn per check (or bill). A study of 500 recent checks indicated the server earned the following amounts in tips per 8-hour shift.

Amount of Tip	Number
$0 up to $ 20	200
20 up to 50	100
50 up to 100	75
100 up to 200	75
200 or more	50
Total	500

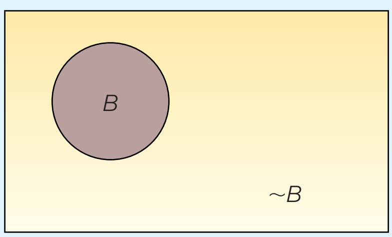
  a. What is the probability of a tip of $200 or more?
  b. Are the categories "$0 up to $20," "$20 up to $50," and so on considered mutually exclusive?
  c. If the probabilities associated with each outcome were totaled, what would that total be?
  d. What is the probability of a tip of up to $50?
  e. What is the probability of a tip of less than $200?

51. Winning all three "Triple Crown" races is considered the greatest feat of a pedigree racehorse. After a successful Kentucky Derby, Big Brown is a 1 to 2 favorite to win the Preakness Stakes.
  a. If he is a 1 to 2 favorite to win the Belmont Stakes as well, what is his probability of winning the Triple Crown?
  b. What do his chances for the Preakness Stakes have to be in order for him to be "even money" to earn the Triple Crown?

52. The first card selected from a standard 52-card deck is a king.
  a. If it is returned to the deck, what is the probability that a king will be drawn on the second selection?
  b. If the king is not replaced, what is the probability that a king will be drawn on the second selection?
  c. What is the probability that a king will be selected on the first draw from the deck and another king on the second draw (assuming that the first king was not replaced)?

53. Armco, a manufacturer of traffic light systems, found that under accelerated-life tests, 95 percent of the newly developed systems lasted 3 years before failing to change signals properly.
  a. If a city purchased four of these systems, what is the probability all four systems would operate properly for at least 3 years?
  b. Which rule of probability does this illustrate?
  c. Using letters to represent the four systems, write an equation to show how you arrived at the answer to part (a).

54. Refer to the following picture.



a. What is the picture called?
b. What rule of probability is illustrated?
c. B represents the event of choosing a family that receives welfare payments. What does $P(B) + P(\sim B)$ equal?

55. In a management trainee program at Claremont Enterprises, 80 percent of the trainees are female and 20 percent male. Ninety percent of the females attended college, and 78 percent of the males attended college.
    a. A management trainee is selected at random. What is the probability that the person selected is a female who did not attend college?
    b. Are gender and attending college independent? Why?
    c. Construct a tree diagram showing all the probabilities, conditional probabilities, and joint probabilities.
    d. Do the joint probabilities total 1.00? Why?
56. Assume the likelihood that any flight on Delta Airlines arrives within 15 minutes of the scheduled time is .90. We select four flights from yesterday for study.
    a. What is the likelihood all four of the selected flights arrived within 15 minutes of the scheduled time?
    b. What is the likelihood that none of the selected flights arrived within 15 minutes of the scheduled time?
    c. What is the likelihood at least one of the selected flights did not arrive within 15 minutes of the scheduled time?
57. There are 100 employees at Kiddie Carts International. Fifty-seven of the employees are production workers, 40 are supervisors, 2 are secretaries, and the remaining employee is the president. Suppose an employee is selected:
    a. What is the probability the selected employee is a production worker?
    b. What is the probability the selected employee is either a production worker or a supervisor?
    c. Refer to part (b). Are these events mutually exclusive?
    d. What is the probability the selected employee is neither a production worker nor a supervisor?
58. Joe Mauer of the Minnesota Twins had the highest batting average in the 2009 Major League Baseball season. His average was .365. So assume the probability of getting a hit is .365 for each time he batted. In a particular game, assume he batted three times.
    a. This is an example of what type of probability?
    b. What is the probability of getting three hits in a particular game?
    c. What is the probability of not getting any hits in a game?
    d. What is the probability of getting at least one hit?
59. Four sports teams remain in a single-elimination playoff competition. If one team is favored in its semi-final match by odds of 2 to 1 and another squad is favored in its contest by odds of 3 to 1. What is the probability that:
    a. Both favored teams win their games?
    b. Neither favored team wins its game?
    c. At least one of the favored teams wins its game?
60. There are three clues labeled "daily double" on the game show *Jeopardy.* If three equally matched contenders play, what is the probability that:
    a. A single contestant finds all three "daily doubles"?
    b. The returning champion gets all three of the "daily doubles"?
    c. Each of the players selects precisely one of the "daily doubles"?
61. Brooks Insurance Inc. wishes to offer life insurance to men age 60 via the Internet. Mortality tables indicate the likelihood of a 60-year-old man surviving another year is .98. If the policy is offered to five men age 60:
    a. What is the probability all five men survive the year?
    b. What is the probability at least one does not survive?
62. Forty percent of the homes constructed in the Quail Creek area include a security system. Three homes are selected at random:
    a. What is the probability all three of the selected homes have a security system?
    b. What is the probability none of the three selected homes have a security system?
    c. What is the probability at least one of the selected homes has a security system?
    d. Did you assume the events to be dependent or independent?
63. Refer to Exercise 62, but assume there are 10 homes in the Quail Creek area and 4 of them have a security system. Three homes are selected at random:
    a. What is the probability all three of the selected homes have a security system?
    b. What is the probability none of the three selected homes have a security system?
    c. What is the probability at least one of the selected homes has a security system?
    d. Did you assume the events to be dependent or independent?

64. There are 20 families living in the Willbrook Farms Development. Of these families, 10 prepared their own federal income taxes for last year, 7 had their taxes prepared by a local professional, and the remaining 3 by H&R Block.
   a. What is the probability of selecting a family that prepared their own taxes?
   b. What is the probability of selecting two families, both of which prepared their own taxes?
   c. What is the probability of selecting three families, all of which prepared their own taxes?
   d. What is the probability of selecting two families, neither of which had their taxes prepared by H&R Block?

65. The board of directors of Saner Automatic Door Company consists of 12 members, 3 of whom are women. A new policy and procedures manual is to be written for the company. A committee of 3 is randomly selected from the board to do the writing.
   a. What is the probability that all members of the committee are men?
   b. What is the probability that at least 1 member of the committee is a woman?

66. A recent survey reported in *BusinessWeek* dealt with the salaries of CEOs at large corporations and whether company shareholders made money or lost money.

	CEO Paid More Than $1 Million	CEO Paid Less Than $1 Million	Total
Shareholders made money	2	11	13
Shareholders lost money	4	3	7
Total	6	14	20

If a company is randomly selected from the list of 20 studied, what is the probability:
   a. The CEO made more than $1 million?
   b. The CEO made more than $1 million or the shareholders lost money?
   c. The CEO made more than $1 million given the shareholders lost money?
   d. Of selecting 2 CEOs and finding they both made more than $1 million?

67. Althoff and Roll, an investment firm in Augusta, Georgia, advertises extensively in the *Augusta Morning Gazette,* the newspaper serving the region. The *Gazette* marketing staff estimates that 60 percent of Althoff and Roll's potential market read the newspaper. It is further estimated that 85 percent of those who read the *Gazette* remember the Althoff and Roll advertisement.
   a. What percent of the investment firm's potential market sees and remembers the advertisement?
   b. What percent of the investment firm's potential market sees, but does not remember the advertisement?

68. An Internet company located in Southern California has season tickets to the Los Angeles Lakers basketball games. The company president always invites one of the four vice presidents to attend games with him, and claims he selects the person to attend at random. One of the four vice presidents has not been invited to attend any of the last five Lakers home games. What is the likelihood this could be due to chance?

69. A computer-supply retailer purchased a batch of 1,000 CD-R disks and attempted to format them for a particular application. There were 857 perfect CDs, 112 CDs were usable but had bad sectors, and the remainder could not be used at all.
   a. What is the probability a randomly chosen CD is not perfect?
   b. If the disk is not perfect, what is the probability it cannot be used at all?

70. An investor purchased 100 shares of Fifth Third Bank stock and 100 shares of Santee Electric Cooperative stock. The probability the bank stock will appreciate over a year is .70. The probability the electric utility will increase over the same period is .60.
   a. What is the probability both stocks appreciate during the period?
   b. What is the probability the bank stock appreciates but the utility does not?
   c. What is the probability at least one of the stocks appreciates?

71. Flashner Marketing Research Inc. specializes in providing assessments of the prospects for women's apparel shops in shopping malls. Al Flashner, president, reports that he assesses the prospects as good, fair, or poor. Records from previous assessments show that 60 percent of the time the prospects were rated as good, 30 percent of the time fair, and 10 percent of the time poor. Of those rated good, 80 percent made a profit the first year; of those rated fair, 60 percent made a profit the first year; and of those rated poor, 20 percent made a profit the first year. Connie's Apparel was one of Flashner's clients. Connie's Apparel made a profit last year. What is the probability that it was given an original rating of poor?

72. Two boxes of men's Old Navy shirts were received from the factory. Box 1 contained 25 mesh polo shirts and 15 Super-T shirts. Box 2 contained 30 mesh polo shirts and 10

Super-T shirts. One of the boxes was selected at random, and a shirt was chosen at random from that box to be inspected. The shirt was a mesh polo shirt. Given this information, what is the probability that the mesh polo shirt came from box 1?

73. With each purchase of a large pizza at Tony's Pizza, the customer receives a coupon that can be scratched to see if a prize will be awarded. The odds of winning a free soft drink are 1 in 10, and the odds of winning a free large pizza are 1 in 50. You plan to eat lunch tomorrow at Tony's. What is the probability:
   a. That you will win either a large pizza or a soft drink?
   b. That you will not win a prize?
   c. That you will not win a prize on three consecutive visits to Tony's?
   d. That you will win at least one prize on one of your next three visits to Tony's?

74. For the daily lottery game in Illinois, participants select three numbers between 0 and 9. A number cannot be selected more than once, so a winning ticket could be, say, 307 but not 337. Purchasing one ticket allows you to select one set of numbers. The winning numbers are announced on TV each night.
   a. How many different outcomes (three-digit numbers) are possible?
   b. If you purchase a ticket for the game tonight, what is the likelihood you will win?
   c. Suppose you purchase three tickets for tonight's drawing and select a different number for each ticket. What is the probability that you will not win with any of the tickets?

75. Several years ago, Wendy's Hamburgers advertised that there are 256 different ways to order your hamburger. You may choose to have, or omit, any combination of the following on your hamburger: mustard, ketchup, onion, pickle, tomato, relish, mayonnaise, and lettuce. Is the advertisement correct? Show how you arrive at your answer.

76. It was found that 60 percent of the tourists to China visited the Forbidden City, the Temple of Heaven, the Great Wall, and other historical sites in or near Beijing. Forty percent visited Xi'an with its magnificent terracotta soldiers, horses, and chariots, which lay buried for over 2,000 years. Thirty percent of the tourists went to both Beijing and Xi'an. What is the probability that a tourist visited at least one of these places?

77. A new chewing gum has been developed that is helpful to those who want to stop smoking. If 60 percent of those people chewing the gum are successful in stopping smoking, what is the probability that in a group of four smokers using the gum at least one quits smoking?

78. Reynolds Construction Company has agreed not to erect all "look-alike" homes in a new subdivision. Five exterior designs are offered to potential home buyers. The builder has standardized three interior plans that can be incorporated in any of the five exteriors. How many different ways can the exterior and interior plans be offered to potential home buyers?

79. A new sports car model has defective brakes 15 percent of the time and a defective steering mechanism 5 percent of the time. Let's assume (and hope) that these problems occur independently. If one or the other of these problems is present, the car is called a "lemon." If both of these problems are present, the car is a "hazard." Your instructor purchased one of these cars yesterday. What is the probability it is:
   a. A lemon?
   b. A hazard?

80. The state of Maryland has license plates with three numbers followed by three letters. How many different license plates are possible?

81. There are four people being considered for the position of chief executive officer of Dalton Enterprises. Three of the applicants are over 60 years of age. Two are female, of which only one is over 60.
   a. What is the probability that a candidate is over 60 and female?
   b. Given that the candidate is male, what is the probability he is less than 60?
   c. Given that the person is over 60, what is the probability the person is female?

82. Tim Bleckie is the owner of Bleckie Investment and Real Estate Company. The company recently purchased four tracts of land in Holly Farms Estates and six tracts in Newburg Woods. The tracts are all equally desirable and sell for about the same amount.
   a. What is the probability that the next two tracts sold will be in Newburg Woods?
   b. What is the probability that of the next four sold at least one will be in Holly Farms?
   c. Are these events independent or dependent?

83. A computer password consists of four characters. The characters can be one of the 26 letters of the alphabet. Each character may be used more than once. How many different passwords are possible?

84. A case of 24 cans contains 1 can that is contaminated. Three cans are to be chosen randomly for testing.
   a. How many different combinations of 3 cans could be selected?
   b. What is the probability that the contaminated can is selected for testing?

85. A puzzle in the newspaper presents a matching problem. The names of 10 U.S. presidents are listed in one column, and their vice presidents are listed in random order in the second column. The puzzle asks the reader to match each president with his vice president. If you make the matches randomly, how many matches are possible? What is the probability all 10 of your matches are correct?

86. Two components, *A* and *B*, operate in series. Being in series means that for the system to operate, both components *A* and *B* must work. Assume the two components are independent. What is the probability the system works under these conditions? The probability *A* works is .90 and the probability *B* functions is also .90.

87. Horwege Electronics Inc. purchases TV picture tubes from four different suppliers. Tyson Wholesale supplies 20 percent of the tubes, Fuji Importers 30 percent, Kirkpatricks 25 percent, and Parts Inc. 25 percent. Tyson Wholesale tends to have the best quality, as only 3 percent of its tubes arrive defective. Fuji Importers' tubes are 4 percent defective, Kirkpatricks' 7 percent, and Parts Inc. are 6.5 percent defective.

a. What is the overall percent defective?

b. A defective picture tube was discovered in the latest shipment. What is the probability that it came from Tyson Wholesale?

88. ABC Auto Insurance classifies drivers as good, medium, or poor risks. Drivers who apply to them for insurance fall into these three groups in the proportions 30 percent, 50 percent, and 20 percent, respectively. The probability a "good" driver will have an accident is .01, the probability a "medium" risk driver will have an accident is .03, and the probability a "poor" driver will have an accident is .10. The company sells Mr. Brophy an insurance policy and he has an accident. What is the probability Mr. Brophy is:

a. A "good" driver?

b. A "medium" risk driver?

c. A "poor" driver?

89. You take a trip by air that involves three independent flights. If there is an 80 percent chance each specific leg of the trip is done on time, what is the probability all three flights arrive on time?

90. The probability a HP network server is down is .05. If you have three independent servers, what is the probability that at least one of them is operational?

91. Twenty-two percent of all liquid crystal displays (LCDs) are manufactured by Samsung. What is the probability that in a collection of three independent LCD purchases, at least one is a Samsung?

Data Set Exercises

92. Refer to the Real Estate data, which reports information on homes sold in the Goodyear, Arizona, area during the last year.

a. Sort the data into a table that shows the number of homes that have a pool versus the number that don't have a pool in each of the five townships. If a home is selected at random, compute the following probabilities.

1. The home is in Township 1 or has a pool.
2. Given that it is in Township 3, that it has a pool.
3. The home has a pool and is in Township 3.

b. Sort the data into a table that shows the number of homes that have a garage attached versus those that don't in each of the five townships. If a home is selected at random, compute the following probabilities:

1. The home has a garage attached.
2. The home does not have a garage attached, given that it is in Township 5.
3. The home has a garage attached and is in Township 3.
4. The home does not have a garage attached or is in Township 2.

93. Refer to the Baseball 2009 data, which reports information on the 30 Major League Baseball teams for the 2009 season. Set up three variables:

- Divide the teams into two groups, those that had a winning season and those that did not. That is, create a variable to count the teams that won 81 games or more, and those that won 80 or less.
- Create a new variable for attendance, using three categories: attendance less than 2.0 million, attendance of 2.0 million up to 3.0 million, and attendance of 3.0 million or more.
- Create a variable that shows the teams that play in a stadium less than 15 years old versus one that is 15 years old or more.

Answer the following questions.

a. Create a table that shows the number of teams with a winning season versus those with a losing season by the three categories of attendance. If a team is selected at random, compute the following probabilities:
  1. The team had a winning season.
  2. The team had a winning season or attendance of more than 3.0 million.
  3. The team had a winning season given attendance was more than 3.0 million.
  4. The team has a winning season and attracted fewer than 2.0 million fans.

b. Create a table that shows the number of teams with a winning season versus those that play in new or old stadiums. If a team is selected at random, compute the following probabilities:
  1. Selecting a team with a winning season.
  2. The likelihood of selecting a team with a winning record and playing in a new stadium.
  3. The team had a winning record or played in a new stadium.

94. Refer to the data on the school buses in the Buena School District. Set up a variable that divides the age of the buses into three groups: new (less than 5 year old), medium (5 but less than 10 years), and old (10 or more years). The median maintenance cost is $456. Based on this value, create a variable for those less than the median (low maintenance) and those more than the median (high maintenance). Finally, develop a table to show the relationship between maintenance cost and age of the bus.

a. What percentage of the buses are new?
b. What percentage of the new buses have low maintenance?
c. What percentage of the old buses have high maintenance?
d. Does maintenance cost seem to be related to the age of the bus? Hint: Compare the maintenance cost of the old buses with the cost of the new buses? Would you conclude maintenance cost is independent of the age?

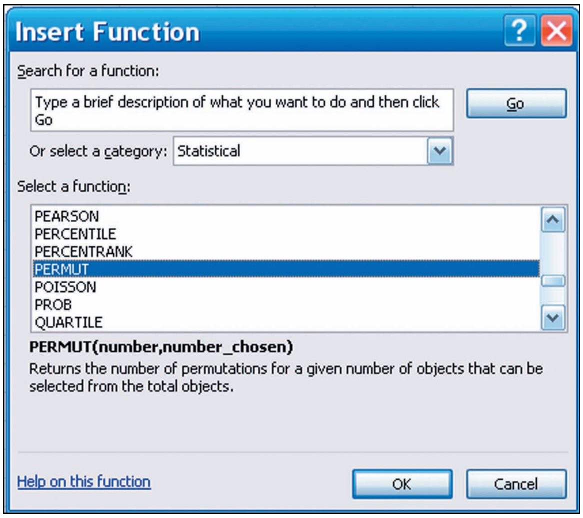
Software Commands

1. The Excel Commands to determine the number of permutations shown on page 175 are:

a. Click on the **Formulas** tab in the top menu, then, on the far left, select **Insert Function** *fx*.

b. In the **Insert Function** box, select **Statistical** as the category, then scroll down to **PERMUT** in the **Select a function list.** Click **OK.**

c. In the **PERM** box after **Number**, enter *8* and in the **Number_chosen** box enter *3*. The correct answer of *336* appears twice in the box.

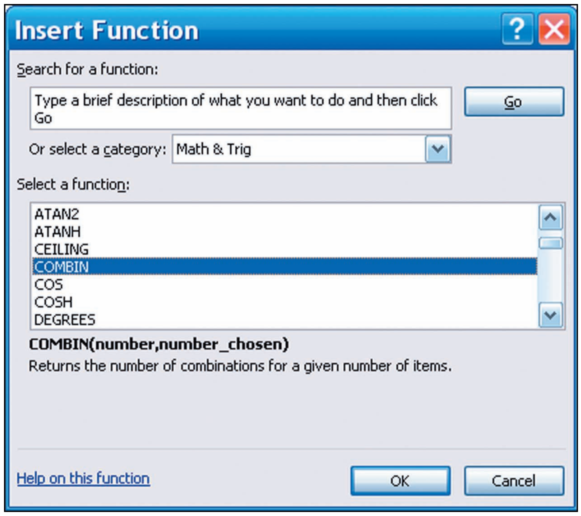


2. The Excel Commands to determine the number of combinations shown on page 175 are:

a. Click on the **Formulas** tab in the top menu, then, on the far left, select **Insert Function** *fx*.

b. In the **Insert Function** box, select **Math & Trig** as the category, then scroll down to **COMBIN** in the **Select a function list**. Click **OK.**

c. In the **COMBIN** box after **Number**, enter *7* and in the **Number_chosen** box enter *3*. The correct answer of *35* appears twice in the box.



Chapter 5 Answers to Self-Review

5–1 **a.** Count the number who think the new game is playable.

b. Seventy-three players found the game playable. Many other answers are possible.

c. No. Probability cannot be greater than 1. The probability that the game, if put on the market, will be successful is 65/80, or .8125.

d. Cannot be less than 0. Perhaps a mistake in arithmetic.

e. More than half of the players testing the game liked it. (Of course, other answers are possible.)

5–2 **1.** $\dfrac{\text{4 queens in deck}}{\text{52 cards total}} = \dfrac{4}{52} = .0769$

Classical.

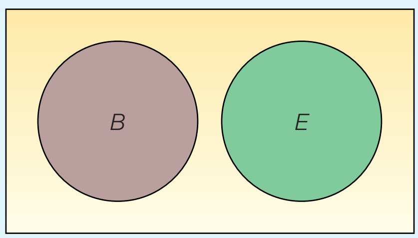
2. $\dfrac{182}{539} = .338$ Empirical.

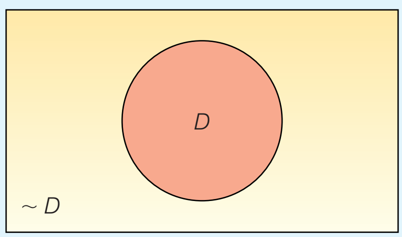
3. The author's view when writing the text of the chance that the DJIA will climb to 12,000 is .25. You may be more optimistic or less optimistic. Subjective.

5–3 **a.** **i.** $\dfrac{(50 + 68)}{2{,}000} = .059$

ii. $1 - \dfrac{302}{2{,}000} = .849$

b.



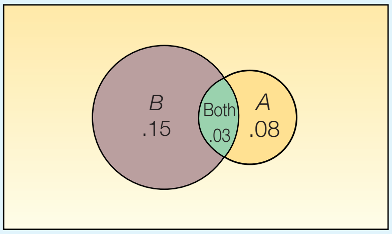


c. They are not complementary, but are mutually exclusive.

5–4 **a.** Need for corrective shoes is event A. Need for major dental work is event B.

$$
\begin{aligned}
P(A \text{ or } B) &= P(A) + P(B) - P(A \text{ and } B)\\
&= .08 + .15 - .03\\
&= .20
\end{aligned}
$$

b. One possibility is:



5–5 $(.95)(.95)(.95)(.95) = .8145$

5–6 **a.** .002, found by:

$$\left(\frac{4}{12}\right)\left(\frac{3}{11}\right)\left(\frac{2}{10}\right)\left(\frac{1}{9}\right) = \frac{24}{11{,}880} = .002$$

b. .14, found by:

$$\left(\frac{8}{12}\right)\left(\frac{7}{11}\right)\left(\frac{6}{10}\right)\left(\frac{5}{9}\right) = \frac{1{,}680}{11{,}880} = .1414$$

c. No, because there are other possibilities, such as three women and one man.

5–7 **a.** $P(B_4) = \dfrac{105}{200} = .525$

b. $P(A_2 \mid B_4) = \dfrac{30}{105} = .286$

c. $P(A_2 \text{ or } B_4) = \dfrac{80}{200} + \dfrac{105}{200} - \dfrac{30}{200} = \dfrac{155}{200} = .775$

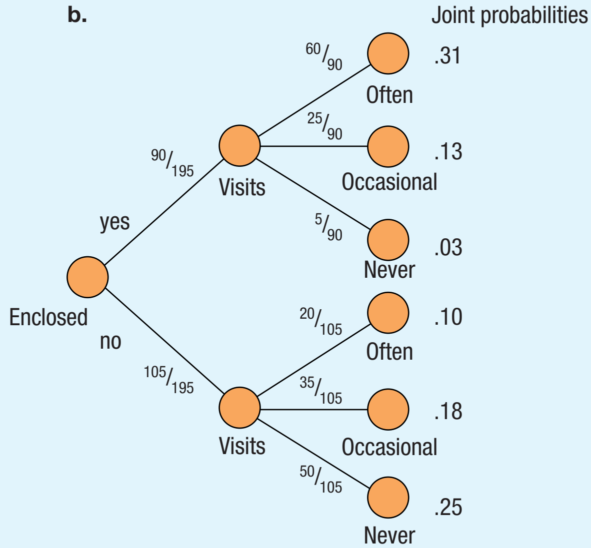
5–8 **a.** Independence requires that $P(A \mid B) = P(A)$. One possibility is:

$P(\text{visit often} \mid \text{yes enclosed mall}) = P(\text{visit often})$

Does $60/90 = 80/195$? No, the two variables are *not* independent.

Therefore, any joint probability in the table must be computed by using the general rule of multiplication.

b.



5–9 **a.** $P(A_3 \mid B_2) = \dfrac{P(A_3)P(B_2 \mid A_3)}{P(A_1)P(B_2 \mid A_1) + P(A_2)P(B_2 \mid A_2) + P(A_3)P(B_2 \mid A_3)}$

b. $= \dfrac{(.50)(.96)}{(.30)(.97) + (.20)(.95) + (.50)(.96)}$

$= \dfrac{.480}{.961} = .499$

5–10 **1.** $(5)(4) = 20$

2. $(3)(2)(4)(3) = 72$

5–11 **1.** **a.** 60, found by (5)(4)(3).

b. 60, found by:

$$\frac{5!}{(5-3)!} = \frac{5 \cdot 4 \cdot 3 \cdot \cancel{2 \cdot 1}}{\cancel{2 \cdot 1}}$$

2. 5,040, found by:

$$\frac{10!}{(10-4)!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}$$

3. **a.** 56 is correct, found by:

$${}_8C_3 = \frac{n!}{r!(n-r)!} = \frac{8!}{3!(8-3)!} = 56$$

b. Yes. There are 45 combinations, found by:

$${}_{10}C_2 = \frac{n!}{r!(n-r)!} = \frac{10!}{2!(10-2)!} = 45$$

4. **a.** ${}_{50}P_3 = \dfrac{50!}{(50-3)!} = 117{,}600$

b. ${}_{50}C_3 = \dfrac{50!}{3!\,(50-3)!} = 19{,}600$