# Rules for Computing Probabilities

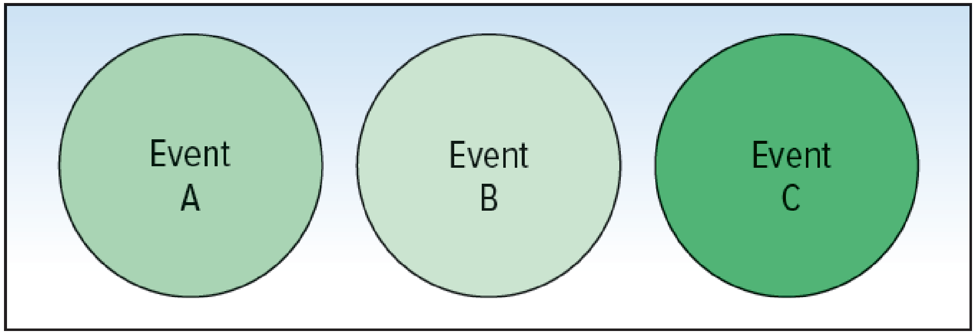
So far we have determined how to compute and assign a probability for an event.

Now we examine how to combine different events by applying rules of addition and multiplication.

Venn diagrams (J. Venn 1834-1923) are useful tools to represent addition or multiplication rules.

First, a rectangular space representing the total of all possible outcomes is drawn.

Next, a circular area (representing an event) is drawn inside the rectangle proportional to the probability of the event.



# Two Important Terms

Recall from statistics (frequencies, distributions...) that when we learned to setup class frequencies, we used the rule that an observation can only be included in <u>one</u> class...(not 2 classes or any overlapping classes)

Such observations or events are **mutually exclusive** if occurrence of one event means that none of the other events can occur at the same time….(e.g. switch on/off….)

If an experiment has a set of events that includes every possible outcome, then the set of events are **collectively exhaustive** if at least one of the events must occur when an experiment is conducted….(die toss experiment)

If the set of events is collectively exhaustive and the events are mutually exclusive, the sum of the probabilities equals 1.

# The Special Rule of Addition

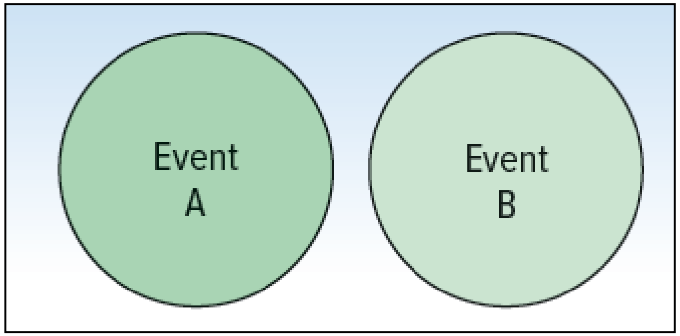
For the Special Rule of Addition, if two events A and B are mutually exclusive, the probability of occurrence of one event or the other event equals the sum of their probabilities.

Recall that mutual exclusive means that when one event occurs, none of the other events can occur at the same time.

In the Venn diagram, events A and B are equally likely to occur and there is no overlapping of events, meaning they are mutually exclusive,

$$P(A \text{ or } B) = P(A) + P(B)$$

$$P(A \text{ or } B \text{ or } C) = P(A) + P(B) + P(C)$$



# Example – Special Rule of Addition

500 people were asked about their preference to which city they would like to reside. The number of preferences for three cities are shown in the table. What is the probability that a preference will be either Halifax or Calgary?

The outcome "Halifax" is the event A. The outcome "Calgary" is the event C. Applying the special rule of addition.

| City | Event | Number of Preferences | Probability of Occurrence |
|---|---|---|---|
| Halifax | A | 208 | 0.42 |
| Yellowknife | B | 137 | 0.27 |
| Calgary | C | 155 | 0.31 |
| | | **500** | **1.00** |

$$P(A \text{ or } C) = P(A) + P(C) = \frac{208}{500} + \frac{155}{500} = 0.42 + 0.31 = 0.73$$

There is a 73% probability that the group of people will choose either Halifax or Calgary for their residence.

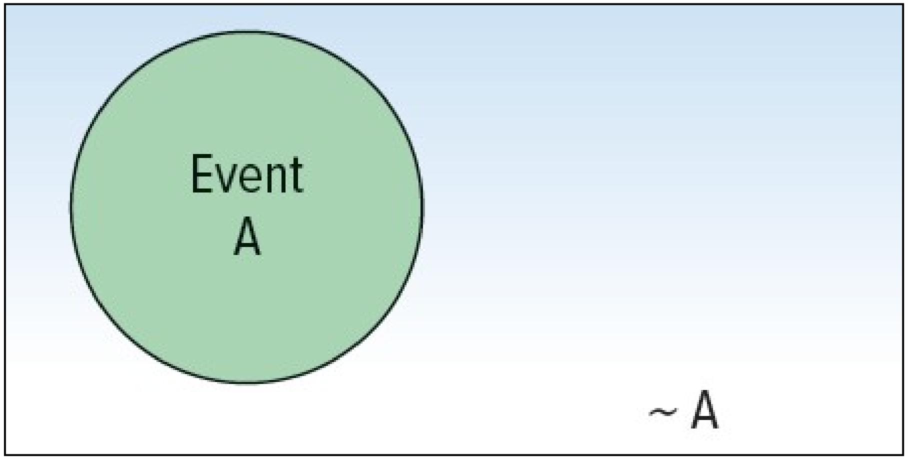
# The Complement Rule

Sometimes it is easier to calculate the probability of an event <u>not</u> happening. The complement rule is used to determine the probability of an event occurring by subtracting the probability of the event <u>not</u> occurring from 1.

$$P(\sim A) = 1 - P(A)$$

**<u>Note:</u>** Events A and ~A are mutually exclusive and collectively exhaustive and hence

$$P(\sim A) + P(A) = 1$$



# Example – City Preference

500 people were asked about their preference to which city they would like to reside. The number of preferences for three cities are shown in the table.

Use the complement rule to show the probability of preference to Yellowknife is 0.27. Show the solution using a Venn diagram.

| **City** | **Event** | **Number of Preferences** | **Probability of Occurrence** |
|---|---|---|---|
| Halifax | A | 208 | 0.42 |
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| Calgary | C | 155 | 0.31 |
| | | **500** | **1.00** |

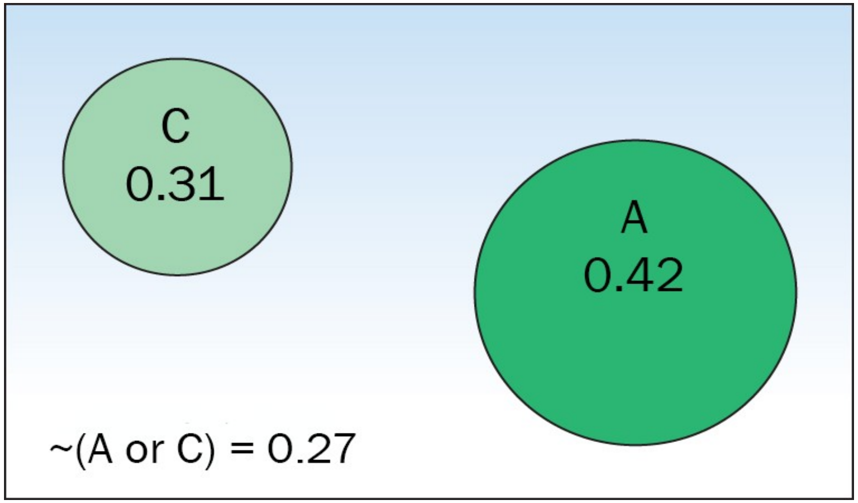
# Solution – City Preference

From the previous example we had determined the probability of preferences for A or C

$$P(A \text{ or } C) = P(A) + P(C) = \frac{208}{500} + \frac{155}{500} = 0.42 + 0.31 = 0.73$$

$$P(\sim(A \text{ or } C)) = 1 - P(A \text{ or } C) = 1 - 0.73 = 0.27$$

The Venn diagram portraying this situation



# Joint Probability

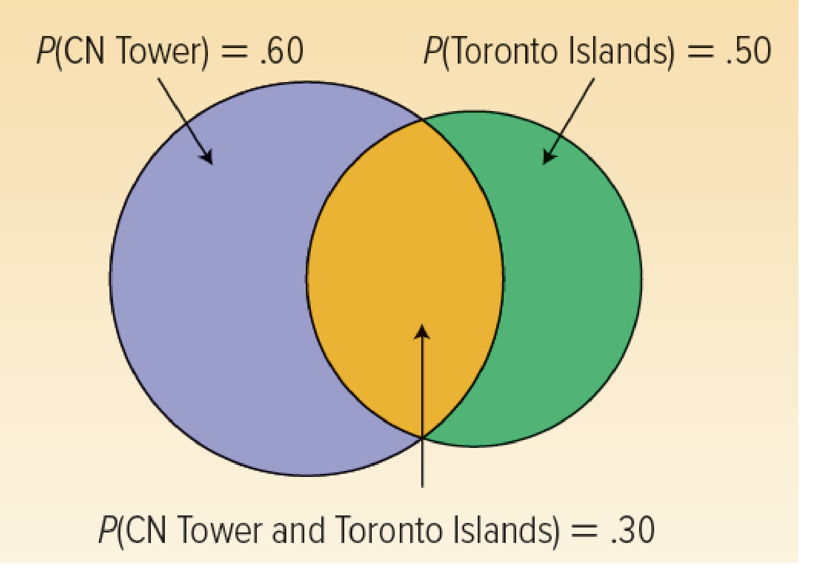
A probability that measures the likelihood of two or more events that will happen concurrently is called a joint probability. Pictured below is the joint probability of A and B, P(A and B).

Tour group of 200 visited various spots in Toronto...120 visited CN Tower, 100 visited Toronto Island...60 visited both places...

What is the probability that a tourist selected at random visited either CN Tower or Toronto Island?

0.6 + 0.5 = 1.10 ???

Possibility that some tourists visited both sites….double counting



# The General Rule of Addition

To account for the double counting or joint probability in some cases, we have to use the General Rule of Addition.

If A and B are two events that are <u>not mutually exclusive</u>, then P(A or B) is given by the following formula

$$P(A \text{ or } B) = P(A \text{ or } B \text{ or } both) = P(A) + P(B) - P(A \text{ and } B)$$

It is important to determine if the events are mutually exclusive.

If the events are mutually exclusive then use the special rule of addition….otherwise we should use the general rule of addition.

# The Special Rule of Multiplication

The Special Rule of Multiplication can be used in the situation wherein 2 events are both happening. The special rule of multiplication requires that the 2 events, A and B, be independent of one another.

Two events are considered independent if the occurrence of one event does not alter the probability of the occurrence of the other event.

$$P(A \text{ and } B) = P(A)P(B)$$

# Example – Marathon

In a marathon race of 25 kilometers, 67% of the runners successfully reach their destination within 4 hours. If 3 runners are selected at random, then what is the total probability that all 3 runners will successfully reach their destination within 4 hours?

The individual probability of the 1st, 2nd and 3rd runners who will successfully reach their destination within 4 hours is 0.67 each.

Assuming they are all independently progressing, no runner will affect another runner in their progress.

$$P(R_1) = P(R_2) = P(R_3) = 0.67$$

$$P(R_1 \text{ and } R_2 \text{ and } R_3) = P(R_1)\,P(R_2)\,P(R_3) = (0.67)(0.67)(0.67) = 0.30$$

# The General Rule of Multiplication

If 2 events are not independent of one another, then they are considered to be dependent on one another.

In the General Rule of Multiplication, the conditional probability is required to compute the joint probability of 2 events that are dependent on one another.

Conditional probability is the probability of a particular event occurring given that another event has occurred.

$$P(A|B)$$

The conditional probability of 2 events, A and B that are dependent is written in the form where the symbol "|" means "given that".

The probability of event A happening given that B has already happened. Probability of event A is conditional on the outcome of event B.

$$P(A \text{ and } B) = P(A)P(B|A)$$

# Example – Umbrellas

A street side seller has 12 umbrellas to sell. Suppose 9 of these are white and the others (3) are blue. He sold 2 umbrellas. What is the total probability that both the umbrellas sold were blue?

The event that the first umbrella sold was blue is $B_1$. The probability is $P(B_1) = 3/12 = 0.25$.

The event that the second umbrella sold was also blue is identified as $B_2$. The conditional probability that the second umbrella sold was blue, given that the first umbrella sold was also blue, is $P(B_2|B_1) = 2/11$.

$$P(B_1 \text{ and } B_2) = P(B_1)\,P(B_2|B_1) = \left(\frac{3}{12}\right)\left(\frac{2}{11}\right) = \frac{6}{132} = 0.045$$