Introduction

- Up until now we have studied descriptive statistics where we organized data into tables, graphs and charts
- We compute measurements of location (mean, median, mode) as well as the spread of the data (range, standard deviation, variance)
- These measurements described something that had <u>already happened</u>
- We also studied inferential statistics where we made inferences about a population based on the sample
- These measurements described something that <u>could happen</u>

Discrete Probability Distribution

A discrete probability distribution shows a listing of all the outcomes of an experiment and the probability associated with each outcome

The discrete probability distribution is similar to relative frequency distribution but in this case it is describing a likely future event.

Below are the major characteristics of a discrete probability distribution

1. The probability of a particular outcome is between 0 and 1 inclusive

2. The outcomes are mutually exclusive events

3. The list is exhaustive. So, the sum of the probabilities of the various events is equal to 1

Mean of a Discrete Probability Distribution

The mean (μ) is a typical value used to represent the central location within a discrete probability distribution.

It is also the long-run average value of the random variable.

The mean of a discrete probability distribution is also referred to as its expected value

It is a weighted average where the possible values of a random variable are weighted by their corresponding probabilities of occurrence.

P(x) is the probability of a particular random variable (x)

$$\mu = \sum [xP(x)]$$

Variance and Standard Deviation of a Discrete Probability Distribution

The variance (σ^2) describes the amount of spread in a distribution. The variance of a random variable can be computed using the formula

The computational steps are

1. Subtract the mean from each value, and square this difference
2. Multiply each squared difference by its probability
3. Sum the resulting products to arrive at the variance

$$\sigma^2 = \sum \left[(x-\mu)^2 P(x) \right]$$

The standard deviation (σ) is found by taking the positive square root of the variance.

$$\sigma = \sqrt{\sigma^2}$$

Example – Real Estate Sales

A realtor sells houses for a real estate firm. The realtor has compiled the following distribution of the number of houses they have sold over the past year.

1. What type of distribution is this?
2. How many homes does the realtor expect to sell next month?
3. What is the variance and standard deviation of the distribution?

Number of houses sold in any given month, *x*	**Probability, *P*(*x*)**
0	0.05
1	0.15
2	0.35
3	0.25
4	0.15
5	0.05
	Total 1.0

Solution – Real Estate Sales

This is a discrete probability distribution for the random variable “number of houses sold in any given month” because the realtor expects to sell only within a certain range of houses….0, 1, 2, 3 or 4.

The outcomes are mutually exclusive – he cannot sell a total of 3 and 4 houses in the same month. It can only be one or the other.

The sum of all possible outcomes equals 1.0.

Hence, these circumstances qualify as a discrete probability distribution.

Solution – Real Estate Sales

The mean number of houses sold is computed by multiplying the number of houses sold by the probability of selling that number. Then we sum up all the values.

The realtor expects to sell 2.45 houses in the next month...provided there are no major disturbances in the world economy...

$$\mu = \sum [xP(x)]$$

μ = 2.45 houses sold

Number of houses sold in any given month, *x*	**Probability, *P*(*x*)**	***xP*(*x*)**
0	0.05	0.00
1	0.15	0.15
2	0.35	0.70
3	0.25	0.75
4	0.15	0.60
5	0.05	0.25
	Total 1.0	**2.45**

Solution – Real Estate Sales

Again, a table is useful for organizing the calculations for the variance and standard deviation. Recall that μ = 2.45 houses sold.

Number of houses sold, *x*	**Probability, *P*(*x*)**	***(x-μ)***	***(x-μ)²***	***(x-μ)²P*(*x*)**
0	0.05	0 – 2.45 = -2.45	$(-2.45)^2 = 6.0025$	(6.0025)(0.05) = 0.300125
1	0.15	1 – 2.45 = -1.45	$(-1.45)^2 = 2.1025$	(2.1025)(0.15) = 0.315375
2	0.35	2 – 2.45 = -0.45	$(-0.45)^2 = 0.2025$	(0.2025)(0.35) = 0.070875
3	0.25	3 – 2.45 = 0.55	$(0.55)^2 = 0.3025$	(0.3025)(0.25) = 0.075625
4	0.15	4 – 2.45 = 1.55	$(1.55)^2 = 2.4025$	(2.4025)(0.15) = 0.360375
5	0.05	5 – 2.45 = 2.55	$(2.55)^2 = 6.5025$	(6.5025)(0.05) = 0.325125
				1.4475

$$\sigma^2 = \sum \left[(x-\mu)^2 P(x) \right]$$

$$\sigma = \sqrt{\sigma^2}$$

$$\sigma = \sqrt{1.4475} = 1.2031$$

σ^2 = 1.4475 (houses sold)^2

σ = 1.20 houses sold

The realtor expects to sell 2.45 ± 1.20 houses in the next month

Review Questions

Review question set 12