Principles of Counting

If the number of possible outcomes in an experiment is small, then it is easy to count them by hand.

However, if the number of possible outcomes in an experiment is large, then it will become difficult to count them by hand...use a counting formula instead.

There are three principles for counting the number of outcomes

- The multiplication formula
- The permutation formula
- The combination formula

The Multiplication Formula

If there are m ways of doing one thing and n ways of doing another thing, then there are (m x n) ways of doing both.

The multiplication formula is useful in finding the number of possible outcomes from 2 or more groups

$$Total\ Number\ of\ Arrangements = (m)(n)$$

The Permutation Formula

The permutation formula is useful in finding the possible number of unique arrangements when there is only one group of objects.

Any arrangement of r objects selected from a single group of n possible objects when order is considered.

Note: The arrangements "abc" and "bac" are different permutations.

The formula to count the total number of different permutations is

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

n = total number of objects

r = number of objects selected

Example – Seats in a Class

Suppose there are 5 students (A,B,C,D,E) and only 3 seats available in a small class. How many unique arrangements of students in the 3 seats can be made?

If n = 5 students and r = 3 seats available, then the permutation formula can be used to make the possible unique arrangements.

$$_nP_r = \frac{n!}{(n-r)!}$$

$$_5P_3 = \frac{5!}{(5-3)!} = \frac{5\times4\times3\times2\times1}{2\times1} = \frac{120}{2} = 60$$

Therefore, there are 60 unique ways of students A,B,C,D,E sitting in 3 seats.

The Combination Formula

If the order of the selected objects is <u>not</u> important then, the possible <u>(non-unique) arrangements</u> can be computed using the combination formula.

The combination formula removes the redundant arrangements and as a result is always smaller than the permutation arrangements.

$$ {}_{n}C_{r} = \frac{n!}{(n-r)!\,r!} $$

“Combination lock should really be called a permutation lock”

Example – Seats in a Class

Suppose there are 5 students (A,B,C,D,E) and only 3 seats available in a small class. How many non-unique arrangements of students in the 3 seats can be made?

If n = 5 students and r = 3 seats available, then the combination formula can be used to make the possible non-unique arrangements.

$$_nC_r = \frac{n!}{(n-r)!r!}$$

$$_5C_3 = \frac{5!}{(5-3)!3!} = \frac{120}{(2)(6)} = 10$$

Therefore, there are 10 non-unique ways of students A,B,C,D,E sitting in 3 seats….removing the 50 redundant arrangements.