

# Describing Data:

## Frequency Tables, Frequency Distributions, and Graphic Presentation

### Learning Objectives

When you have completed this chapter, you will be able to:

- L01** Make a frequency table for a set of data.
- L02** Organize data into a bar chart.
- L03** Present a set of data in a pie chart.
- L04** Create a frequency distribution for a data set.
- L05** Understand a relative frequency distribution.
- L06** Present data from a frequency distribution in a histogram or frequency polygon.
- L07** Construct and interpret a cumulative frequency distribution.



Merrill Lynch recently completed a study of online investment portfolios for a sample of clients. For the 70 participants in the study, organize these data into a frequency distribution. (See Exercise 43 and L04.)

## 2.1 Introduction

The highly competitive automobile retailing industry in the United States has changed dramatically in recent years. These changes spurred events such as the:



- bankruptcies of General Motors and Chrysler in 2009.
- elimination of well-known brands such as Pontiac and Saturn.
- closing of over 1,500 local dealerships.
- collapse of consumer credit availability.
- consolidation dealership groups.

Traditionally, a local family owned and operated the community dealership, which might have included one or two manufacturers or brands, like Pontiac and GMC Trucks or Chrysler and the popular Jeep line. Recently, however, skillfully managed and well-financed companies have been acquiring local dealerships in large regions of the country. As these groups acquire the local dealerships, they often bring standardized selling practices, common software and hardware technology platforms, and management reporting techniques. The goal of these new organizations is to provide an improved buying experience for the consumer, while increasing profitability. Megadealerships often employ over 10,000 people, generate several billion dollars in annual sales, own more than 50 franchises, and are traded on the New York Stock Exchange or NASDAQ. Today, the largest megadealership is AutoNation (ticker symbol AN). Others include Penske Auto Group (PAG and second largest), Asbury Automotive Group (ABG), and Hendrick Auto Group (which is privately held).

The Applewood Auto Group is an ownership group that includes four dealerships. The group sells a wide range of vehicles, including the inexpensive but popular Korean brands Kia and Hyundai, BMW and Volvo sedans and luxury SUVs, and a full line of Ford and Chevrolet cars and trucks.

Ms. Kathryn Ball is a member of the senior management team at Applewood Auto Group, which has its corporate offices adjacent to Hilltop Motors. She is responsible for tracking and analyzing vehicle sales and the profitability of those vehicles. Kathryn would like to summarize the profit earned on the vehicles sold with tables, charts, and graphs that she would review monthly. She wants to know the profit per vehicle sold, as well as the lowest and highest amount of profit. She is also interested in describing the demographics of the buyers. What are their ages? How many vehicles have they previously purchased from one of the Applewood dealerships? What type of vehicle did they purchase?

The Applewood Auto Group operates four dealerships:

	A	B	C	D	E
1	Age	Profit	Location	Vehicle-Type	Previous
2	21	\$1,387	Tionesta	Sedan	0
3	23	\$1,754	Sheffield	SUV	1
4	24	\$1,817	Sheffield	Hybrid	1
5	25	\$1,040	Sheffield	Compact	0
6	26	\$1,273	Kane	Sedan	1
7	27	\$1,529	Sheffield	Sedan	1
8	27	\$3,082	Kane	Truck	0
9	28	\$1,951	Kane	SUV	1
10	28	\$2,692	Tionesta	Compact	0
11	29	\$1,206	Sheffield	Sedan	0
12	29	\$1,342	Kane	Sedan	2
13	30	\$443	Kane	Sedan	3
14	30	\$754	Olean	Sedan	2
15	30	\$1,621	Sheffield	Truck	1

- **Tionesta Ford Lincoln Mercury** sells the Ford, Lincoln, and Mercury cars and trucks.
- **Olean Automotive Inc.** has the Nissan franchise as well as the General Motors brands of Chevrolet, Cadillac, and GMC Trucks.
- **Sheffield Motors Inc.** sells Buick, GMC trucks, Hyundai, and Kia.
- **Hilltop Motors** offers the Chrysler, Dodge, and Jeep line as well as BMW and Volvo.

Every month, Ms. Ball collects data from each of the four dealerships and enters it

into an Excel spreadsheet. Last month the Applewood Auto Group sold 180 vehicles at the four dealerships. A copy of the first few observations appears at the bottom of the previous page. The variables collected include:

- **Profit**—the amount earned by the dealership on the sale of each vehicle.
- **Age**—the age of the buyer at the time of the purchase.
- **Location**—the dealership where the vehicle was purchased.
- **Vehicle type**—SUV, sedan, compact, hybrid, or truck.
- **Previous**—the number of vehicles previously purchased at any of the four Applewood dealerships by the consumer.

The entire data set is available at the McGraw-Hill website and in Appendix A.5 at the end of the text.

## 2.2 Constructing a Frequency Table

Recall from Chapter 1 that techniques used to describe a set of data are called descriptive statistics. To put it another way, descriptive statistics organize data to show the general pattern of the data and where values tend to concentrate and to expose extreme or unusual data values. The first procedure we discuss is a frequency table.

**FREQUENCY TABLE** A grouping of qualitative data into mutually exclusive classes showing the number of observations in each class.

**L01** Make a frequency table for a set of data.

In Chapter 1, we distinguished between qualitative and quantitative variables. To review, a qualitative variable is nonnumeric, that is, it can only be classified into distinct categories. There is no particular order to these categories. Examples of qualitative data include political affiliation (Republican, Democrat, Independent), state of birth (Alabama, . . . , Wyoming), and method of payment for a purchase at Barnes & Noble (cash, check, debit, or credit). On the other hand, quantitative variables are numerical in nature. Examples of quantitative data relating to college students include the price of their textbooks, their age, and the hours they spend studying each week of the semester.

In the Applewood Auto Group data set, there are five variables for each vehicle sale: age of the buyer, amount of profit, dealer that made the sale, type of vehicle sold, and number of previous purchases by the buyer. The dealer and the type of vehicle are *qualitative* variables. The amount of profit, the age of the buyer, and the number of previous purchases are *quantitative* variables.

Suppose Ms. Ball wanted to summarize last month's sales by location. To summarize this qualitative data, we classify the vehicles sold last month according to their location: Tionesta, Olean, Sheffield, or Hilltop. We use location to develop a frequency table with four mutually exclusive (distinctive) classes. This means that a particular vehicle cannot belong to more than one class. Each vehicle is uniquely classified into one of the four mutually exclusive locations. This frequency table is shown in Table 2-1. The number of observations, representing the sales at each location, is called the class frequency. So the class frequency for vehicles sold at the Kane location is 52.



## Relative Class Frequencies

You can convert class frequencies to relative class frequencies to show the fraction of the total number of observations in each class. A relative frequency captures the

**TABLE 2-1** Frequency Table for Vehicles Sold Last Month at Applewood Auto Group by Location

Location	Number of Cars
Kane	52
Olean	40
Sheffield	45
Tionesta	43
Total	180

relationship between a class total and the total number of observations. In the vehicle sales example, we may want to know the percentage of total cars sold at each of the four locations. To convert a frequency distribution to a relative frequency distribution, each of the class frequencies is divided by the total number of observations. For example, the fraction of vehicles sold last month at the Kane location is 0.289, found by 52 divided by 180. The relative frequency for each location is shown in Table 2-2.

**TABLE 2-2** Relative Frequency Table of Vehicles Sold by Type Last Month at Applewood Auto Group

Location	Number of Cars	Relative Frequency
Kane	52	.289
Olean	40	.222
Sheffield	45	.250
Tionesta	43	.239
Total	180	1.000

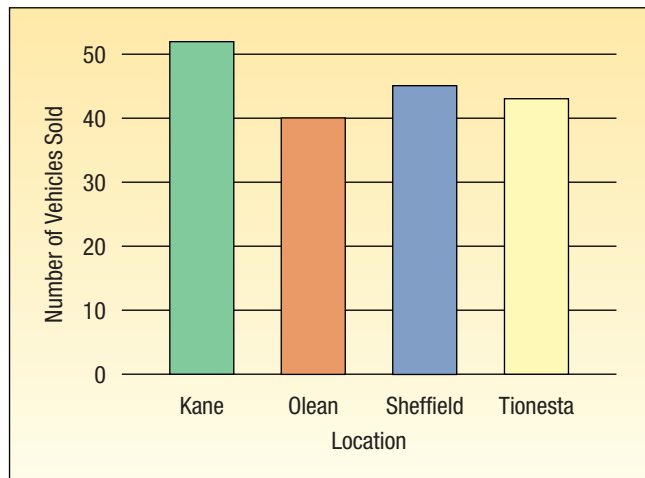
## Graphic Presentation of Qualitative Data

**L02** Organize data into a bar chart.

The most common graphic form to present a qualitative variable is a **bar chart**. In most cases, the horizontal axis shows the variable of interest. The vertical axis shows the frequency or fraction of each of the possible outcomes. A distinguishing feature of a bar chart is there is distance or a gap between the bars. That is, because the variable of interest is qualitative, the bars are not adjacent to each other. Thus, a bar chart graphically describes a frequency table using a series of uniformly wide rectangles, where the height of each rectangle is the class frequency.

**BAR CHART** A graph that shows qualitative classes on the horizontal axis and the class frequencies on the vertical axis. The class frequencies are proportional to the heights of the bars.

We use the Applewood Auto Group data as an example (Chart 2-1). The variable of interest is the location where the vehicle was sold and the number of vehicles sold at each location is the class frequency. We label the horizontal axis with the four locations and scale the vertical axis with the number sold. The height of the bars, or rectangles, corresponds to the number of vehicles at each location. There were 52 vehicles sold last month at the Kane location, so the height of the Kane bar is 52; the height of the bar for the Olean location is 40. The variable



**CHART 2-1** Number of Vehicles Sold by Location

location is of nominal scale, so the order of the locations on the horizontal axis does not matter. Listing this variable alphabetically or by some type of geographical arrangement might also be appropriate.

Another useful type of chart for depicting qualitative information is a **pie chart**.

**PIE CHART** A chart that shows the proportion or percentage that each class represents of the total number of frequencies.

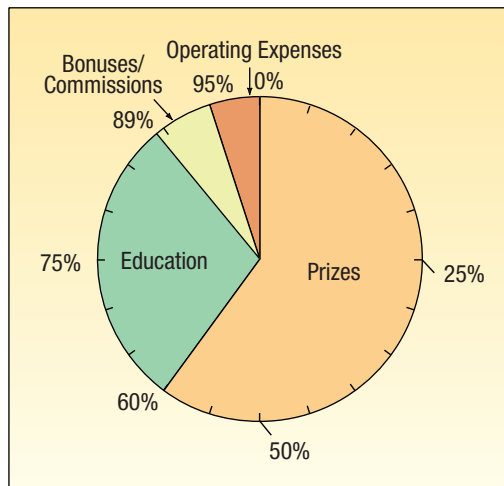
We explain the details of constructing a pie chart using the information in Table 2-3, which shows a breakdown of the expenses for the Ohio State Lottery in 2009.

**TABLE 2-3** Ohio State Lottery Expenses in 2009

Use of Sales	Amount (\$ million)	Percentage of Sales
Prizes	1,460.0	60
Education	702.3	29
Bonuses	150.0	6
Expenses	124.3	5
Total	2,436.6	100

**L03** Present a set of data using a pie chart.

The first step to develop a pie chart is to record the percentages 0, 5, 10, 15, and so on evenly around the circumference of a circle (see Chart 2-2). To plot the 60 percent share awarded for prizes, draw a line from the center of the circle to 0 and another line from the center of the circle to 60 percent. The area in this “slice” represents the lottery proceeds that were awarded in prizes. Next, add the 60 percent of expenses awarded in prizes to the 29 percent payments to education; the result is 89 percent. Draw a line from the center of the circle to 89 percent, so the area between 60 percent and 89 percent depicts the payments made to education.



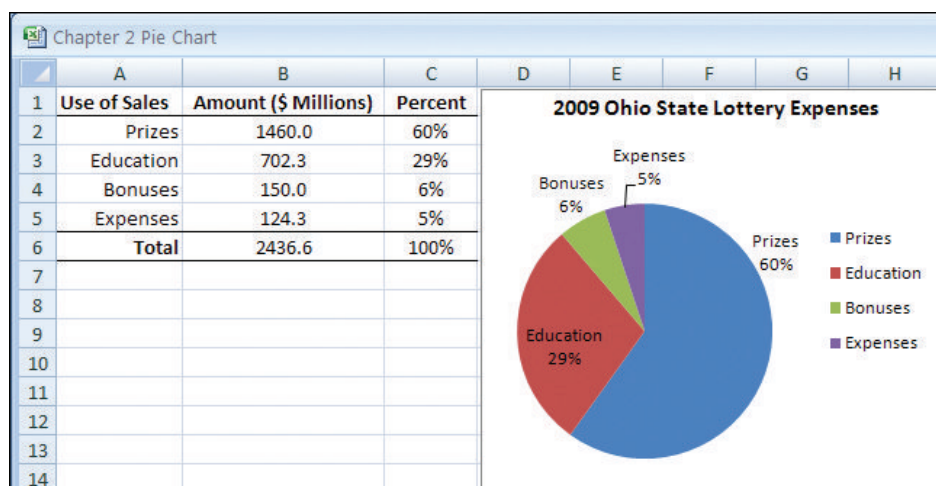
**CHART 2-2** Pie Chart of Ohio Lottery Expenses in 2009

Continuing, add the 6 percent for bonuses, which gives us a total of 95 percent. Draw a line from the center of the circle to 95, so the “slice” between 89 percent and 95 percent represents the payment of bonuses. The remainder, 5 percent, is for operating expenses.

Because each slice of the pie represents the relative share of each component, we can easily compare them:

- The largest expense of the Ohio Lottery is for prizes.
- About 30 percent of the proceeds is transferred to education.
- Operating expenses account for only 5 percent of the proceeds.

We can use software to quickly create a visually appealing and informative pie chart. The following chart, using the information in Table 2–3, depicts the uses of Ohio Lottery expenses in 2009.



Pie charts and bar charts serve much the same function. What are the criteria for selecting one over the other? In most cases, pie charts are the most informative



when the goal is to compare the relative difference in the percentage of observations for each of the nominal scale variables. Bar charts are preferred when the goal is to compare the number of observations in each category.

### Example

SkiLodges.com is test marketing its new website and is interested in how easy its Web page design is to navigate. It randomly selected 200 regular Internet users and asked them to perform a search task on the Web page. Each person was asked to rate the relative ease of navigation as poor, good, excellent, or awesome. The results are shown in the following table:

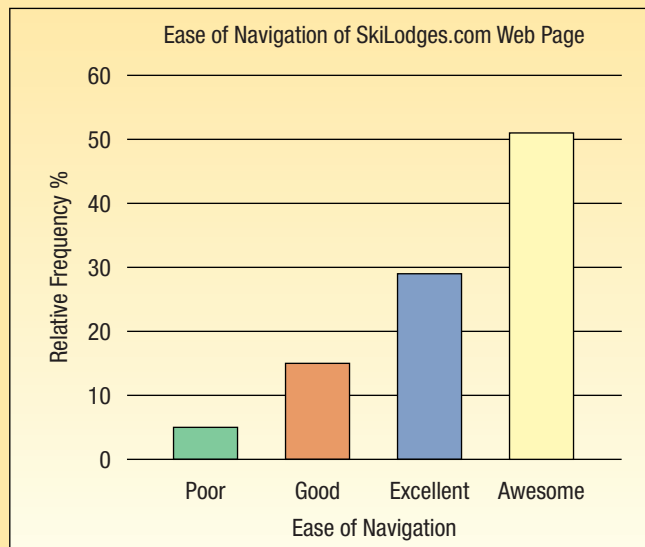
Awesome	102
Excellent	58
Good	30
Poor	10

1. What type of measurement scale is used for ease of navigation?
2. Draw a bar chart for the survey results.
3. Draw a pie chart for the survey results.

### Solution

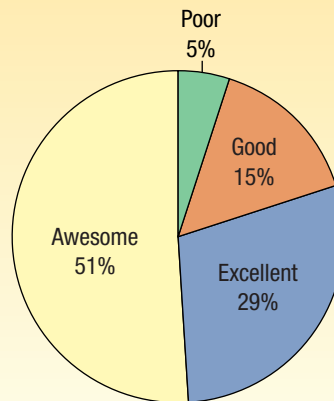
The data are measured on an ordinal scale. That is, the scale is ranked in relative ease when moving from “poor” to “awesome.” Also, the interval between each rating is unknown so it is impossible, for example, to conclude that a rating of good is twice the value of a poor rating.

We can use a bar chart to graph the data. The vertical scale shows the relative frequency and the horizontal shows the values of the ease of navigation scale.



A pie chart can also be used to graph this data. The pie chart emphasizes that more than half of the respondents rate the relative ease of using the website awesome.

Ease of Navigation of SkiLodges.com Web Page

**Self-Review 2-1**

The answers are at the end of the chapter.



DeCenzo Specialty Food and Beverage Company has been serving a cola drink with an additional flavoring, Cola-Plus, that is very popular among its customers. The company is interested in customer preferences for Cola-Plus versus Coca-Cola, Pepsi, and a lemon-lime beverage. They ask 100 randomly sampled customers to take a taste test and select the beverage they preferred most. The results are shown in the following table:

Beverage	Number
Cola-Plus	40
Coca-Cola	25
Pepsi	20
Lemon-Lime	15
Total	100

- Is the data qualitative or quantitative? Why?
- What is the table called? What does it show?
- Develop a bar chart to depict the information.
- Develop a pie chart using the relative frequencies.

## Exercises

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The answers to the odd-numbered exercises are at the end of the book.

- A pie chart shows the relative market share of cola products. The “slice” for Pepsi-Cola has a central angle of 90 degrees. What is its market share?
- In a marketing study, 100 consumers were asked to select the best digital music player from the iPod, the iRiver, and the Magic Star MP3. To summarize the consumer responses with a frequency table, how many classes would the frequency table have?
- A total of 1,000 residents in Minnesota were asked which season they preferred. The results were 100 liked winter best, 300 liked spring, 400 liked summer, and 200 liked fall. If the data were summarized in a frequency table, how many classes would be used? What would be the relative frequencies for each class?
- Two thousand frequent midwestern business travelers are asked which midwestern city they prefer: Indianapolis, Saint Louis, Chicago, or Milwaukee. The results were 100 liked Indianapolis best, 450 liked Saint Louis, 1,300 liked Chicago, and the remainder preferred Milwaukee. Develop a frequency table and a relative frequency table to summarize this information.



5. Wellstone Inc. produces and markets replacement covers for cell phones in a variety of colors. The company would like to allocate its production plans to five different colors: bright white, metallic black, magnetic lime, tangerine orange, and fusion red. The company set up a kiosk in the Mall of America for several hours and asked randomly selected people which cover color was their favorite. The results follow:

Bright white	130
Metallic black	104
Magnetic lime	325
Tangerine orange	455
Fusion red	286

- What is the table called?
  - Draw a bar chart for the table.
  - Draw a pie chart.
  - If Wellstone Inc. plans to produce 1 million cell phone covers, how many of each color should it produce?
6. A small business consultant is investigating the performance of several companies. The fourth quarter sales for last year (in thousands of dollars) for the selected companies were:

Corporation	Fourth-Quarter Sales (\$ thousands)
Hoden Building Products	\$ 1,645.2
J & R Printing Inc.	4,757.0
Long Bay Concrete Construction	8,913.0
Mancell Electric and Plumbing	627.1
Maxwell Heating and Air Conditioning	24,612.0
Mizelle Roofing & Sheet Metals	191.9

The consultant wants to include a chart in his report comparing the sales of the six companies. Use a bar chart to compare the fourth-quarter sales of these corporations and write a brief report summarizing the bar chart.

## 2.3 Constructing Frequency Distributions: Quantitative Data

**L04** Create a frequency distribution for a data set.

In Chapter 1 and earlier in this chapter, we distinguished between qualitative and quantitative data. In the previous section, using the Applewood Automotive Group data, we summarized a qualitative variable, location of the sale, using a frequency table, a relative frequency table, and a bar chart.

The Applewood Auto Group data also includes several quantitative variables: the age of the buyer, the profit earned on the sale of the vehicle, and the number of previous purchases. Suppose Ms. Ball wants to summarize last month's sales by profit earned. We can describe profit using a **frequency distribution**.

**FREQUENCY DISTRIBUTION** A grouping of data into mutually exclusive classes showing the number of observations in each class.

How do we develop a frequency distribution? The first step is to tally the data into a table that shows the classes and the number of observations in each class. The steps in constructing a frequency distribution are best described by an example. Remember, our goal is to construct tables, charts, and graphs that will quickly reveal the concentration, extreme values, and shape of the data.

## Example

We return to the situation where Ms. Kathryn Ball of the Applewood Auto Group wants to develop tables, charts, and graphs to show the typical profit for each sale. Table 2–4 reports the profit on each of the 180 vehicles sold last month at the four Applewood locations. What is the *typical* profit on each sale? What is the *largest* profit on any sale? What is the *lowest* profit on any sale? Around what value did the profits tend to cluster?

**TABLE 2–4** Profit on Vehicles Sold Last Month by the Applewood Auto Group

\$1,387	\$2,148	\$2,201	\$ 963	\$ 820	\$2,230	\$3,043	\$2,584	\$2,370
1,754	2,207	996	1,298	1,266	2,341	1,059	2,666	2,637
1,817	2,252	2,813	1,410	1,741	3,292	1,674	2,991	1,426
1,040	1,428	323	1,553	1,772	1,108	1,807	934	2,944
1,273	1,889	352	1,648	1,932	1,295	2,056	2,063	2,147
1,529	1,166	482	2,071	2,350	1,344	2,236	2,083	1,973
3,082	1,320	1,144	2,116	2,422	1,906	2,928	2,856	2,502
1,951	2,265	1,485	1,500	2,446	1,952	1,269	2,989	783
2,692	1,323	1,509	1,549	369	2,070	1,717	910	1,538
1,206	1,761	1,638	2,348	978	2,454	1,797	1,536	2,339
1,342	1,919	1,961	2,498	1,238	1,606	1,955	1,957	2,700
443	2,357	2,127	294	1,818	1,680	2,199	2,240	2,222
754	2,866	2,430	1,115	1,824	1,827	2,482	2,695	2,597
1,621	732	1,704	1,124	1,907	1,915	2,701	1,325	2,742
870	1,464	1,876	1,532	1,938	2,084	3,210	2,250	1,837
1,174	1,626	2,010	1,688	1,940	2,639	377	2,279	2,842
1,412	1,761	2,165	1,822	2,197	842	1,220	2,626	2,434
1,809	1,915	2,231	1,897	2,646	1,963	1,401	1,501	1,640
2,415	2,119	2,389	2,445	1,461	2,059	2,175	1,752	1,821
1,546	1,766	335	2,886	1,731	2,338	1,118	2,058	2,487

Lowest

Highest

## Solution

Table 2–4 shows the profits from the 180 sales. We refer to this unorganized information as **raw data** or **ungrouped data**. With a little searching, we can find the smallest profit (\$294) and the highest profit (\$3,292), but that is about all. It is difficult to determine a typical amount of profit. It is also difficult to visualize where the profits tend to cluster. The raw data are more easily interpreted if organized into a frequency distribution.

**Step 1: Decide on the number of classes.** The goal is to use just enough groupings or **classes** to reveal the shape of the set of observations. Some judgment is needed here. Too many classes or too few classes might not reveal the basic shape of the data set. In the vehicle profit example, three classes would not give much insight into the pattern of the data (see Table 2–5).

**TABLE 2–5** An Example of Too Few Classes

Vehicle Profit (\$)	Number of Vehicles
\$ 200 up to \$1,400	42
1,400 up to 2,600	115
2,600 up to 3,800	23
Total	180

A useful recipe to determine the number of classes ( $k$ ) is the “2 to the  $k$  rule.” This guide suggests you select the smallest number ( $k$ ) for the number of classes such that  $2^k$  (in words, 2 raised to the power of  $k$ ) is greater than the number of observations ( $n$ ). In the Applewood Auto Group

The steps for organizing data into a frequency distribution.



### Statistics in Action

In 1788, James Madison, John Jay, and Alexander Hamilton anonymously published a series of essays entitled *The Federalist*. These Federalist papers were an attempt to convince the people of New York that they should ratify the Constitution. In the course of history, the authorship of most of these papers became known, but 12 remained contested. Through the use of statistical analysis, and particularly the study of the frequency of the use of various words, we can now conclude that James Madison is the likely author of the 12 papers. In fact, the statistical evidence that Madison is the author is overwhelming.

example, there were 180 vehicles sold. So  $n = 180$ . If we try  $k = 7$ , which means we would use 7 classes,  $2^7 = 128$ , which is less than 180. Hence, 7 is too few classes. If we let  $k = 8$ , then  $2^8 = 256$ , which is greater than 180. So the recommended number of classes is 8.

**Step 2: Determine the class interval or class width.** Generally the **class interval** or **class width** is the same for all classes. The classes all taken together must cover at least the distance from the lowest value in the data up to the highest value. Expressing these words in a formula:

$$i \geq \frac{H - L}{k}$$

where  $i$  is the class interval,  $H$  is the highest observed value,  $L$  is the lowest observed value, and  $k$  is the number of classes.

For the Applewood Auto Group, the lowest value is \$294 and the highest value is \$3,292. If we need 8 classes, the interval should be:

$$i \geq \frac{H - L}{k} = \frac{\$3,292 - \$294}{8} = \$374.75$$

In practice, this interval size is usually rounded up to some convenient number, such as a multiple of 10 or 100. The value of \$400 is a reasonable choice.

In frequency distributions, equal class intervals are preferred. However, unequal class intervals may be necessary in certain situations to avoid a large number of empty, or almost empty, classes. Such is the case in Table 2-6. The Internal Revenue Service used unequal-sized class intervals to report the adjusted gross income on individual tax returns. Had they used an equal-sized interval of, say, \$1,000, more than 1,000 classes would have been required to describe all the incomes. A frequency distribution with 1,000 classes would be difficult to interpret. In this case, the distribution is easier to understand in spite of the unequal classes. Note also that the number of income tax returns or “frequencies” is reported in thousands in this particular table. This also makes the information easier to understand.

**TABLE 2-6** Adjusted Gross Income for Individuals Filing Income Tax Returns

Adjusted Gross Income		Number of Returns (in thousands)
No adjusted gross income		178.2
\$ 1 up to	\$ 5,000	1,204.6
5,000 up to	10,000	2,595.5
10,000 up to	15,000	3,142.0
15,000 up to	20,000	3,191.7
20,000 up to	25,000	2,501.4
25,000 up to	30,000	1,901.6
30,000 up to	40,000	2,502.3
40,000 up to	50,000	1,426.8
50,000 up to	75,000	1,476.3
75,000 up to	100,000	338.8
100,000 up to	200,000	223.3
200,000 up to	500,000	55.2
500,000 up to	1,000,000	12.0
1,000,000 up to	2,000,000	5.1
2,000,000 up to	10,000,000	3.4
10,000,000 or more		0.6

**Step 3: Set the individual class limits.** State clear class limits so you can put each observation into only one category. This means you must avoid overlapping or unclear class limits. For example, classes such as “\$1,300–\$1,400” and “\$1,400–\$1,500” should not be used, because it is not clear whether the value \$1,400 is in the first or second class. Classes stated as “\$1,300–\$1,400” and “\$1,500–\$1,600” are frequently used, but may also be confusing without the additional common convention of rounding all data at or above \$1,450 up to the second class and data below \$1,450 down to the first class. In this text, we will generally use the format \$1,300 up to \$1,400 and \$1,400 up to \$1,500 and so on. With this format it is clear that \$1,399 goes into the first class and \$1,400 in the second.

Because we round the class interval up to get a convenient class size, we cover a larger than necessary range. For example, using 8 classes with a width of \$400 in the Applewood Auto Group example results in a range of  $8(\$400) = \$3,200$ . The actual range is \$2,998, found by  $(\$3,292 - \$294)$ . Comparing that value to \$3,200, we have an excess of \$202. Because we need to cover only the distance  $(H - L)$ , it is natural to put approximately equal amounts of the excess in each of the two tails. Of course, we should also select convenient class limits. A guideline is to make the lower limit of the first class a multiple of the class interval. Sometimes this is not possible, but the lower limit should at least be rounded. So here are the classes we could use for this data.

Classes
\$ 200 up to \$ 600
600 up to 1,000
1,000 up to 1,400
1,400 up to 1,800
1,800 up to 2,200
2,200 up to 2,600
2,600 up to 3,000
3,000 up to 3,400

**Step 4: Tally the vehicle profit into the classes.** To begin, the profit from the sale of the first vehicle in Table 2–4 is \$1,387. It is tallied in the \$1,000 up to \$1,400 class. The second profit in the first row of Table 2–4 is \$2,148. It is tallied in the \$1,800 up to \$2,200 class. The other profits are tallied in a similar manner. When all the profits are tallied, the table would appear as:

Profit	Frequency
\$ 200 up to \$ 600	
600 up to 1,000	
1,000 up to 1,400	
1,400 up to 1,800	
1,800 up to 2,200	
2,200 up to 2,600	
2,600 up to 3,000	
3,000 up to 3,400	
Total	

**Step 5: Count the number of items in each class.** The number of observations in each class is called the **class frequency**. In the \$200 up to \$600 class there are 8 observations, and in the \$600 up to \$1,000 class there are 11 observations. Therefore, the class frequency in the first class is 8 and the class frequency in the second class is 11. There are a total of

180 observations or frequencies in the entire set of data. So the sum of all the frequencies should be equal to 180.

**TABLE 2-7** Frequency Distribution of Profit for Vehicles Sold Last Month at Applewood Auto Group

Profit	Frequency
\$ 200 up to \$ 600	8
600 up to 1,000	11
1,000 up to 1,400	23
1,400 up to 1,800	38
1,800 up to 2,200	45
2,200 up to 2,600	32
2,600 up to 3,000	19
3,000 up to 3,400	4
Total	180

Now that we have organized the data into a frequency distribution, we can summarize the pattern in the profits of the vehicles for the Applewood Auto Group. Observe the following:

1. The profit from a vehicle ranged between \$200 and \$3,400.
2. The profits are concentrated between \$1,000 and \$3,000. The profit on 157 vehicles or 87 percent was within this range.
3. The largest concentration, or highest frequency, is in the \$1,800 up to \$2,200 class. There are 45 observations. The middle of this class is \$2,000. So we say that the typical profit on selling a vehicle is \$2,000.

By presenting this information to Ms. Ball, we give her a clear picture of the distribution of the vehicle profits for last month.

We admit that arranging the information on profits into a frequency distribution does result in the loss of some detailed information. That is, by organizing the data into a frequency distribution, we cannot pinpoint the exact profit on any vehicle, such as \$1,387, \$2,148, or \$2,201. Further, we cannot tell that the actual lowest amount of profit for any vehicle sold is \$294 or that the most profit was \$3,292. However, the lower limit of the first class and the upper limit of the largest class convey essentially the same meaning. Likely, Ms. Ball will make the same judgment if she knows the lowest price is about \$200 that she will if she knows the exact profit is \$292. The advantages of condensing the data into a more understandable and organized form more than offset this disadvantage.

### Self-Review 2-2



The commissions earned for the first quarter of last year by the 11 members of the sales staff at Master Chemical Company are:

\$1,650 \$1,475 \$1,510 \$1,670 \$1,595 \$1,760 \$1,540 \$1,495 \$1,590 \$1,625 \$1,510

- (a) What are the values such as \$1,650 and \$1,475 called?
- (b) Using \$1,400 up to \$1,500 as the first class, \$1,500 up to \$1,600 as the second class, and so forth, organize the quarterly commissions into a frequency distribution.
- (c) What are the numbers in the right column of your frequency distribution called?
- (d) Describe the distribution of quarterly commissions, based on the frequency distribution. What is the largest concentration of commissions earned? What is the smallest, and the largest? What is the typical amount earned?

We will use two other terms frequently: **class midpoint** and **class interval**. The midpoint is halfway between the lower limits of two consecutive classes. It is computed by adding the lower limits of consecutive classes and dividing the result by 2.

Referring to Table 2–7, the lower class limit of the first class is \$200 and the next class limit is \$600. The class midpoint is \$400, found by  $(\$600 + \$200)/2$ . The midpoint of \$400 best represents, or is typical of, the profits of the vehicles in that class.

To determine the class interval, subtract the lower limit of the class from the lower limit of the next class. The class interval of the Applewood data is \$400, which we find by subtracting the lower limit of the first class, \$200, from the lower limit of the next class; that is, \$600. ( $\$600 - \$200 = \$400$ .) You can also determine the class interval by finding the difference between consecutive midpoints. The midpoint of the first class is \$400 and the midpoint of the second class is \$800. The difference is \$400.

## 2.4 A Software Example

As we mentioned in Chapter 1, there are many software packages that perform statistical calculations. Throughout this text we will show the output from Microsoft Excel; from MegaStat, which is an add-in to Microsoft Excel; and from Minitab. The commands necessary to generate the outputs are given in the **Software Commands** section at the end of each chapter. By following these commands, you will be able to duplicate the output.

The following is a frequency distribution, produced by MegaStat, showing the prices of the 180 vehicles sold last month at the Applewood Auto Group. The form of the output is somewhat different than the frequency distribution of Table 2–7, but the overall conclusions are the same.

Frequency Distribution-Quantitative

Profit			Midpoint	Width	Frequency	Percent	Cumulative	
Lower	Upper						Frequency	Percent
200	< 600		400	400	8	4.4	8	4.4
600	< 1,000		800	400	11	6.1	19	10.6
1,000	< 1,400		1,200	400	23	12.8	42	23.3
1,400	< 1,800		1,600	400	38	21.1	80	44.4
1,800	< 2,200		2,000	400	45	25.0	125	69.4
2,200	< 2,600		2,400	400	32	17.8	157	87.2
2,600	< 3,000		2,800	400	19	10.6	176	97.8
3,000	< 3,400		3,200	400	4	2.2	180	100.0
					180	100.0		

### Self-Review 2–3



Barry Bonds of the San Francisco Giants established a new single-season Major League Baseball home run record by hitting 73 home runs during the 2001 season. The longest of these home runs traveled 488 feet and the shortest 320 feet. You need to construct a frequency distribution of these home run lengths.

- How many classes would you use?
- What class interval would you suggest?
- What actual classes would you suggest?

## 2.5 Relative Frequency Distribution

**L05** Understand a relative frequency distribution.

A relative frequency converts the frequency to a percentage.

It may be desirable, as we did earlier with qualitative data, to convert class frequencies to relative class frequencies to show the proportion of the total number of observations in each class. In our vehicle profits, we may want to know what percentage of the vehicle profits are in the \$1,000 up to \$1,400 class. In another study, we may want to know what percentage of the employees used 5 up to 10 personal leave days last year. To convert a frequency distribution to a *relative* frequency distribution, each of the class frequencies is divided by the total number of observations. From the distribution of vehicle profits, Table 2–7, the relative frequency for the \$1,000 up to \$1,400 class is 0.128, found by dividing 23 by 180. That is, profit



on 12.8 percent of the vehicles sold is between \$1,000 and \$1,400. The relative frequencies for the remaining classes are shown in Table 2–8.

**TABLE 2–8** Relative Frequency Distribution of Profit for Vehicles Sold Last Month at Applewood Auto Group

Profit	Frequency	Relative Frequency	Found by
\$ 200 up to \$ 600	8	.044	8/180
600 up to 1,000	11	.061	11/180
1,000 up to 1,400	23	.128	23/180
1,400 up to 1,800	38	.211	38/180
1,800 up to 2,200	45	.250	45/180
2,200 up to 2,600	32	.178	32/180
2,600 up to 3,000	19	.106	19/180
3,000 up to 3,400	4	.022	4/180
Total	180	1.000	

### Self-Review 2–4




Refer to Table 2–8, which shows the relative frequency distribution for the profit earned on vehicles sold last month at the Applewood Auto Group.

- How many vehicles are in the \$1,800 up to \$2,200 class?
- What proportion of the vehicles sold for a profit of between \$1,800 up to \$2,200?
- What proportion of the vehicles sold for a profit of \$2,200 or more?


## Exercises

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- A set of data consists of 38 observations. How many classes would you recommend for the frequency distribution?
- A set of data consists of 45 observations between \$0 and \$29. What size would you recommend for the class interval?
- A set of data consists of 230 observations between \$235 and \$567. What class interval would you recommend?
- A set of data contains 53 observations. The lowest value is 42 and the largest is 129. The data are to be organized into a frequency distribution.
  - How many classes would you suggest?
  - What would you suggest as the lower limit of the first class?
- Wachesaw Manufacturing Inc. produced the following number of units in the last 16 days.  This icon indicates that the data is available at the text website: [www.mhhe.com/lind15e](http://www.mhhe.com/lind15e). You will be able to download the data directly into Excel or Minitab from this site.

27	27	27	28	27	25	25	28
26	28	26	28	31	30	26	26

The information is to be organized into a frequency distribution.

- How many classes would you recommend?
  - What class interval would you suggest?
  - What lower limit would you recommend for the first class?
  - Organize the information into a frequency distribution and determine the relative frequency distribution.
  - Comment on the shape of the distribution.
- The Quick Change Oil Company has a number of outlets in the metropolitan Seattle area. The daily number of oil changes at the Oak Street outlet in the past 20 days are: 

65	98	55	62	79	59	51	90	72	56
70	62	66	80	94	79	63	73	71	85

The data are to be organized into a frequency distribution.

- How many classes would you recommend?
- What class interval would you suggest?

- c. What lower limit would you recommend for the first class?
  - d. Organize the number of oil changes into a frequency distribution.
  - e. Comment on the shape of the frequency distribution. Also determine the relative frequency distribution.
13. The manager of the BiLo Supermarket in Mt. Pleasant, Rhode Island, gathered the following information on the number of times a customer visits the store during a month. The responses of 51 customers were:

5	3	3	1	4	4	5	6	4	2	6	6	6	7	1
1	14	1	2	4	4	4	5	6	3	5	3	4	5	6
8	4	7	6	5	9	11	3	12	4	7	6	5	15	1
1	10	8	9	2	12									

- a. Starting with 0 as the lower limit of the first class and using a class interval of 3, organize the data into a frequency distribution.
  - b. Describe the distribution. Where do the data tend to cluster?
  - c. Convert the distribution to a relative frequency distribution.
14. The food services division of Cedar River Amusement Park Inc. is studying the amount families who visit the amusement park spend per day on food and drink. A sample of 40 families who visited the park yesterday revealed they spent the following amounts:

\$77	\$18	\$63	\$84	\$38	\$54	\$50	\$59	\$54	\$56	\$36	\$26	\$50	\$34	\$44
41	58	58	53	51	62	43	52	53	63	62	62	65	61	52
60	60	45	66	83	71	63	58	61	71					

- a. Organize the data into a frequency distribution, using seven classes and 15 as the lower limit of the first class. What class interval did you select?
- b. Where do the data tend to cluster?
- c. Describe the distribution.
- d. Determine the relative frequency distribution.

## 2.6 Graphic Presentation of a Frequency Distribution

**L06** Present data from a frequency distribution in a histogram or a frequency polygon.

Sales managers, stock analysts, hospital administrators, and other busy executives often need a quick picture of the distributions of sales, stock prices, or hospital costs. These distributions can often be depicted by the use of charts and graphs. Three charts that will help portray a frequency distribution graphically are the histogram, the frequency polygon, and the cumulative frequency polygon.

### Histogram

A **histogram** for a frequency distribution based on quantitative data is similar to the bar chart showing the distribution of qualitative data. The classes are marked on the horizontal axis and the class frequencies on the vertical axis. The class frequencies are represented by the heights of the bars. However, there is one important difference based on the nature of the data. Quantitative data are usually measured using scales that are continuous, not discrete. Therefore, the horizontal axis represents all possible values, and the bars are drawn adjacent to each other to show the continuous nature of the data.

**HISTOGRAM** A graph in which the classes are marked on the horizontal axis and the class frequencies on the vertical axis. The class frequencies are represented by the heights of the bars, and the bars are drawn adjacent to each other.

**Example**

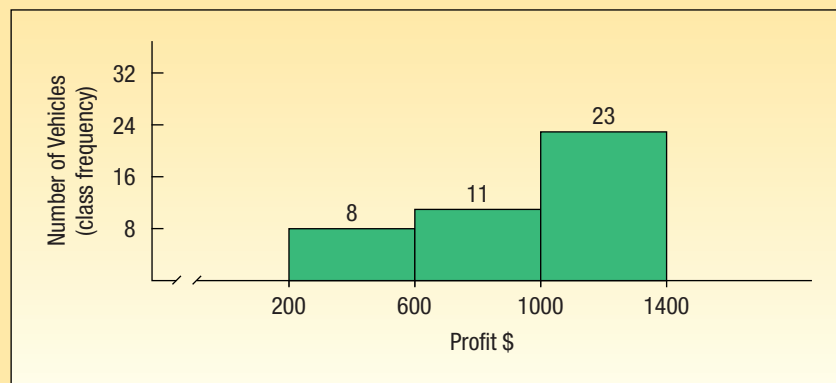
Below is the frequency distribution of the profits on vehicle sales last month at the Applewood Auto Group.

Profit	Frequency
\$ 200 up to \$ 600	8
600 up to 1,000	11
1,000 up to 1,400	23
1,400 up to 1,800	38
1,800 up to 2,200	45
2,200 up to 2,600	32
2,600 up to 3,000	19
3,000 up to 3,400	4
Total	180

Construct a histogram. What observations can you reach based on the information presented in the histogram?

**Solution**

The class frequencies are scaled along the vertical axis (Y-axis) and either the class limits or the class midpoints along the horizontal axis. To illustrate the construction of the histogram, the first three classes are shown in Chart 2–3.



**CHART 2–3** Construction of a Histogram

From Chart 2–3 we note the profit on eight vehicles was \$200 up to \$600. Therefore, the height of the column for that class is 8. There are 11 vehicles sales where the profit was \$600 up to \$1,000. So, logically, the height of that column is 11. The height of the bar represents the number of observations in the class.

This procedure is continued for all classes. The complete histogram is shown in Chart 2–4. Note that there is no space between the bars. This is a feature of the histogram. Why is this so? Because the variable plotted on the horizontal axis is quantitative and the ratio scale of measurement. In a bar chart, the scale of measurement is nominal and the vertical bars are separated. These are important distinctions between the histogram and the bar chart.

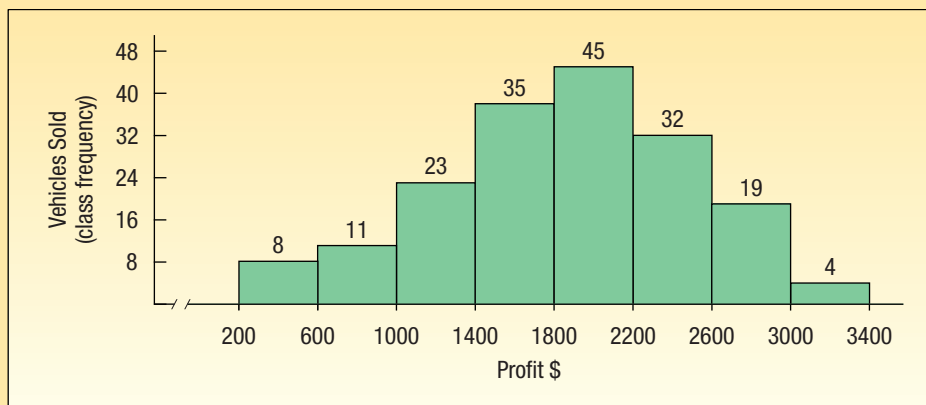
From Chart 2–4 we can make the following statement:

1. The profit from a vehicle ranged from about \$200 up to about \$3,400.
2. The profits are concentrated between \$1,000 and \$3,000. The profit on 157 vehicles, or 87 percent, was within this range.
3. The largest concentration, or highest frequency, is in the \$1,800 up to \$2,200 class. The middle of this class is \$2,000. So we say that the typical profit on selling a vehicle is \$2,000.



### Statistics in Action

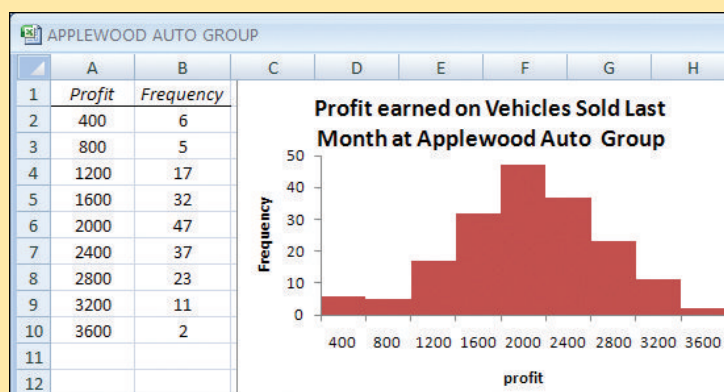
Florence Nightingale is known as the founder of the nursing profession. However, she also saved many lives by using statistical analysis. When she encountered an unsanitary condition or an undersupplied hospital, she improved the conditions and then used statistical data to document the improvement. Thus, she was able to convince others of the need for medical reform, particularly in the area of sanitation. She developed original graphs to demonstrate that, during the Crimean War, more soldiers died from unsanitary conditions than were killed in combat.



**CHART 2-4** Histogram of the Profit on 180 Vehicles Sold at the Applewood Auto Group

Thus, the histogram provides an easily interpreted visual representation of a frequency distribution. We should also point out that we would have made the same observations and the shape of the histogram would have been the same had we used a relative frequency distribution instead of the actual frequencies. That is, if we had used the relative frequencies of Table 2-8, we would have had a histogram of the same shape as Chart 2-4. The only difference is that the vertical axis would have been reported in percentage of vehicles instead of the number of vehicles.

We use the Microsoft Excel system to produce the histogram for the Applewood Auto Group vehicle sales data. Note that class midpoints are used as the labels for the classes. The software commands to create this output are given in the **Software Commands** section at the end of the chapter.



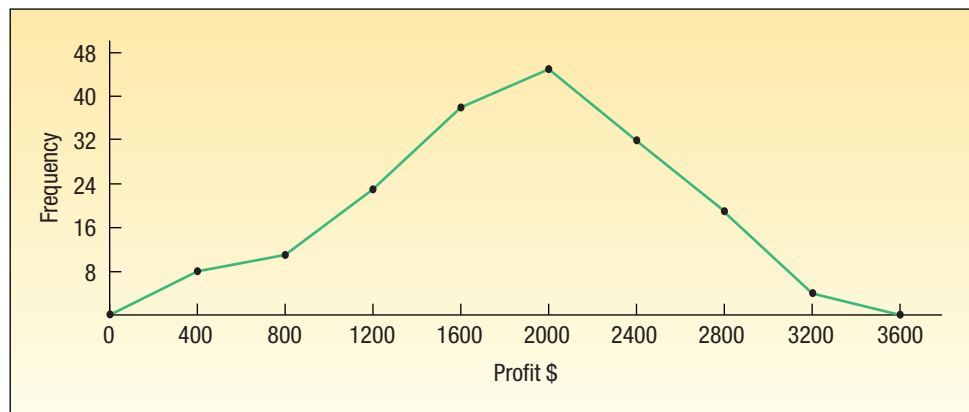
## Frequency Polygon

A **frequency polygon** also shows the shape of a distribution and is similar to a histogram. It consists of line segments connecting the points formed by the intersections of the class midpoints and the class frequencies. The construction of a frequency polygon is illustrated in Chart 2-5 (on page 39). We use the profits from the cars sold last month at the Applewood Auto Group. The midpoint of each class is scaled on the X-axis and the class frequencies on the Y-axis. Recall that the class midpoint is the value at the center of a class and represents the typical values in

that class. The class frequency is the number of observations in a particular class. The profit earned on the vehicles sold last month by the Applewood Auto Group is repeated below.

Profit	Midpoint	Frequency
\$ 200 up to \$ 600	\$ 400	8
600 up to 1,000	800	11
1,000 up to 1,400	1,200	23
1,400 up to 1,800	1,600	38
1,800 up to 2,200	2,000	45
2,200 up to 2,600	2,400	32
2,600 up to 3,000	2,800	19
3,000 up to 3,400	3,200	4
Total		180

As noted previously, the \$200 up to \$600 class is represented by the midpoint \$400. To construct a frequency polygon, move horizontally on the graph to the midpoint, \$400, and then vertically to 8, the class frequency, and place a dot. The *X* and the *Y* values of this point are called the *coordinates*. The coordinates of the next point are  $X = 800$  and  $Y = 11$ . The process is continued for all classes. Then the points are connected in order. That is, the point representing the lowest class is joined to the one representing the second class and so on. Note in Chart 2–5 that, to complete the frequency polygon, midpoints of \$0 and \$3,600 are added to the *X*-axis to “anchor” the polygon at zero frequencies. These two values, \$0 and \$3,600, were derived by subtracting the class interval of \$400 from the lowest midpoint (\$400) and by adding \$400 to the highest midpoint (\$3,200) in the frequency distribution.



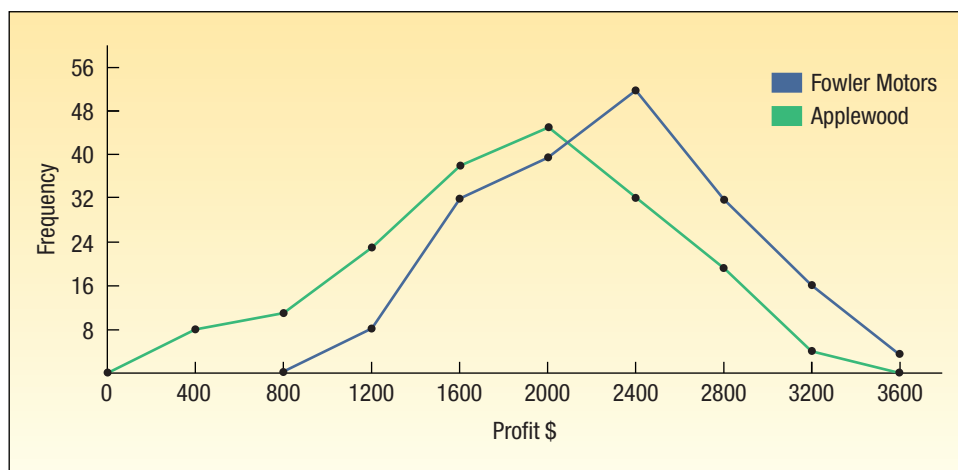
**CHART 2–5** Frequency Polygon of Profit on 180 Vehicles Sold at Applewood Auto Group

Both the histogram and the frequency polygon allow us to get a quick picture of the main characteristics of the data (highs, lows, points of concentration, etc.). Although the two representations are similar in purpose, the histogram has the advantage of depicting each class as a rectangle, with the height of the rectangular bar representing the number in each class. The frequency polygon, in turn, has an advantage over the histogram. It allows us to compare directly two or more

frequency distributions. Suppose Ms. Ball wants to compare the profit per vehicle sold at Applewood Auto Group with a similar auto group, Fowler Auto in Grayling, Michigan. To do this, two frequency polygons are constructed, one on top of the other, as in Chart 2-6. Two things are clear from the chart:

- The typical vehicle profit is larger at Fowler Motors—about \$2,000 for Applewood and about \$2,400 for Fowler.
- There is less dispersion in the profits at Fowler Motors than at Applewood. The lower limit of the first class for Applewood is \$0 and the upper limit is \$3,600. For Fowler Motors, the lower limit is \$800 and the upper limit is the same: \$3,600.

The total number of cars sold at the two dealerships is about the same, so a direct comparison is possible. If the difference in the total number of cars sold is large, then converting the frequencies to relative frequencies and then plotting the two distributions would allow a clearer comparison.



**CHART 2-6** Distribution of Profit at Applewood Auto Group and Fowler Motors

### Self-Review 2-5



The annual imports of a selected group of electronic suppliers are shown in the following frequency distribution.

Imports (\$ millions)	Number of Suppliers	Imports (\$ millions)	Number of Suppliers
2 up to 5	6	11 up to 14	10
5 up to 8	13	14 up to 17	1
8 up to 11	20		

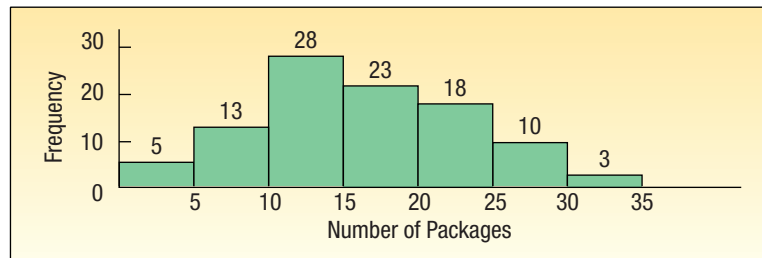
- Portray the imports as a histogram.
- Portray the imports as a relative frequency polygon.
- Summarize the important facets of the distribution (such as classes with the highest and lowest frequencies).



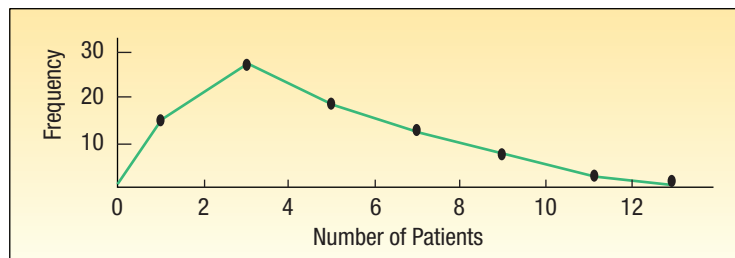
## Exercises



15. Molly's Candle Shop has several retail stores in the coastal areas of North and South Carolina. Many of Molly's customers ask her to ship their purchases. The following chart shows the number of packages shipped per day for the last 100 days.



- What is this chart called?
  - What is the total number of frequencies?
  - What is the class interval?
  - What is the class frequency for the 10 up to 15 class?
  - What is the relative frequency of the 10 up to 15 class?
  - What is the midpoint of the 10 up to 15 class?
  - On how many days were there 25 or more packages shipped?
16. The following chart shows the number of patients admitted daily to Memorial Hospital through the emergency room.



- What is the midpoint of the 2 up to 4 class?
  - How many days were 2 up to 4 patients admitted?
  - Approximately how many days were studied?
  - What is the class interval?
  - What is this chart called?
17. The following frequency distribution reports the number of frequent flier miles, reported in thousands, for employees of Brumley Statistical Consulting Inc. during the most recent quarter.

Frequent Flier Miles (000)	Number of Employees
0 up to 3	5
3 up to 6	12
6 up to 9	23
9 up to 12	8
12 up to 15	<u>2</u>
Total	50

- How many employees were studied?
- What is the midpoint of the first class?
- Construct a histogram.

- d. A frequency polygon is to be drawn. What are the coordinates of the plot for the first class?
  - e. Construct a frequency polygon.
  - f. Interpret the frequent flier miles accumulated using the two charts.
18. Ecommerce.com, a large Internet retailer, is studying the lead time (elapsed time between when an order is placed and when it is filled) for a sample of recent orders. The lead times are reported in days.

Lead Time (days)	Frequency
0 up to 5	6
5 up to 10	7
10 up to 15	12
15 up to 20	8
20 up to 25	7
Total	40

- a. How many orders were studied?
- b. What is the midpoint of the first class?
- c. What are the coordinates of the first class for a frequency polygon?
- d. Draw a histogram.
- e. Draw a frequency polygon.
- f. Interpret the lead times using the two charts.

## Cumulative Frequency Distributions

**L07** Construct and interpret a cumulative frequency distribution.

Consider once again the distribution of the profits on vehicles sold by the Applewood Auto Group. Suppose we were interested in the number of vehicles that sold for a profit of less than \$1,400 or the profit earned on the lowest selling 40 percent of the vehicles. These values can be approximated by developing a **cumulative frequency distribution** and portraying it graphically in a **cumulative frequency polygon**.

### Example

The frequency distribution of the profits earned at Applewood Auto Group is repeated from Table 2-7.

Profit	Frequency
\$ 200 up to \$ 600	8
600 up to 1,000	11
1,000 up to 1,400	23
1,400 up to 1,800	38
1,800 up to 2,200	45
2,200 up to 2,600	32
2,600 up to 3,000	19
3,000 up to 3,400	4
Total	180

Construct a cumulative frequency polygon. Seventy-five percent of the vehicles sold earned a profit of less than what amount? Sixty of the vehicles earned a profit of less than what amount?

### Solution

As the names imply, a cumulative frequency distribution and a cumulative frequency polygon require *cumulative frequencies*. To construct a cumulative frequency distribution, refer to the preceding table and note that there were eight vehicles in which

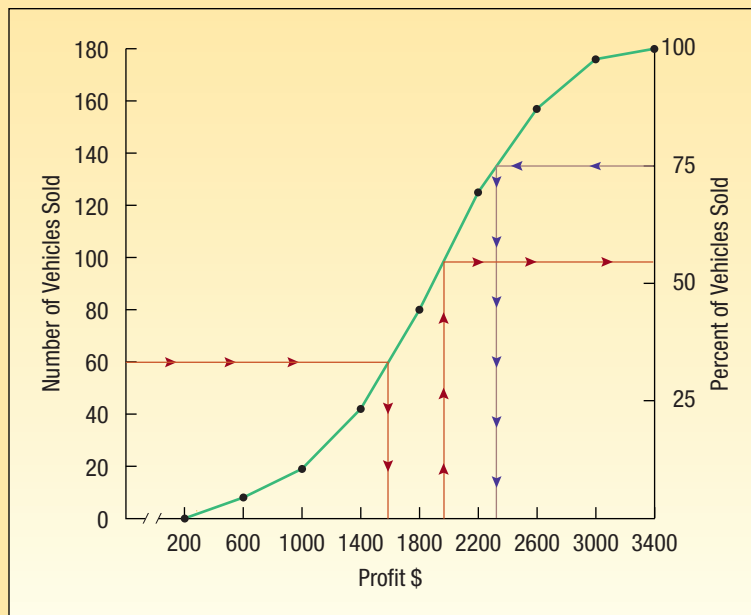
the profit earned was less than \$600. Those 8 vehicles, plus the 11 in the next higher class, for a total of 19, earned a profit of less than \$1,000. The cumulative frequency for the next higher class is 42, found by  $8 + 11 + 23$ . This process is continued for all the classes. All the vehicles earned a profit of less than \$3,400. (See Table 2–9.)

**TABLE 2–9** Cumulative Frequency Distribution for Profit on Vehicles Sold Last Month at Applewood Auto Group

Profit	Frequency	Cumulative Frequency	Found by
\$ 200 up to \$ 600	8	8	8
600 up to 1,000	11	19	$8 + 11$
1,000 up to 1,400	23	42	$8 + 11 + 23$
1,400 up to 1,800	38	80	$8 + 11 + 23 + 30$
1,800 up to 2,200	45	125	$8 + 11 + 23 + 30 + 45$
2,200 up to 2,600	32	157	$8 + 11 + 23 + 30 + 45 + 32$
2,600 up to 3,000	19	176	$8 + 11 + 23 + 30 + 45 + 32 + 19$
3,000 up to 3,400	4	180	$8 + 11 + 23 + 30 + 45 + 32 + 19 + 4$
Total	180		

To plot a cumulative frequency distribution, scale the upper limit of each class along the X-axis and the corresponding cumulative frequencies along the Y-axis. To provide additional information, you can label the vertical axis on the left in units and the vertical axis on the right in percent. In the Applewood Auto Group, the vertical axis on the left is labeled from 0 to 180 and on the right from 0 to 100 percent. The value of 50 percent corresponds to 90 vehicles.

To begin, the first plot is at  $X = 200$  and  $Y = 0$ . None of the vehicles sold for a profit of less than \$200. The profit on 8 vehicles was less than \$600, so the next plot is at  $X = 600$  and  $Y = 8$ . Continuing, the next plot is  $X = 1,000$  and  $Y = 19$ . There were 19 vehicles that sold for a profit of less than \$1,000. The rest of the points are plotted and then the dots connected to form the chart below.



**CHART 2–7** Cumulative Frequency Polygon for Profit on Vehicles Sold Last Month at Applewood Auto Group

To find the amount of profit earned on 75 percent of the cars sold, draw a horizontal line from the 75 percent mark on the right-hand vertical axis over to the polygon, then drop down to the X-axis and read the amount of profit. The value on the X-axis is about \$2,300, so we estimate that 75 percent of the vehicles sold earned a profit for the Applewood group of \$2,230.

To find the profit earned on 60 vehicles, we locate the value of 60 on the left-hand vertical axis. Next, we draw a horizontal line from the value of 60 to the polygon and then drop down to the X-axis and read the profit. It is about \$1,590, so we estimate that 60 of the vehicles sold for a profit of less than \$1,590. We can also make estimates of the percentage of vehicles that sold for less than a particular amount. To explain, suppose we want to estimate the percentage of vehicles that sold for a profit of less than \$1,600. We begin by locating the value of \$1,600 on the X-axis, move vertically to the polygon, and then horizontally to the vertical axis on the right. The value is about 56 percent, so we conclude that 56 percent of the vehicles sold for a profit of less than \$1,600.

### Self-Review 2–6



A sample of the hourly wages of 15 employees at Home Depot in Brunswick, Georgia, was organized into the following table.

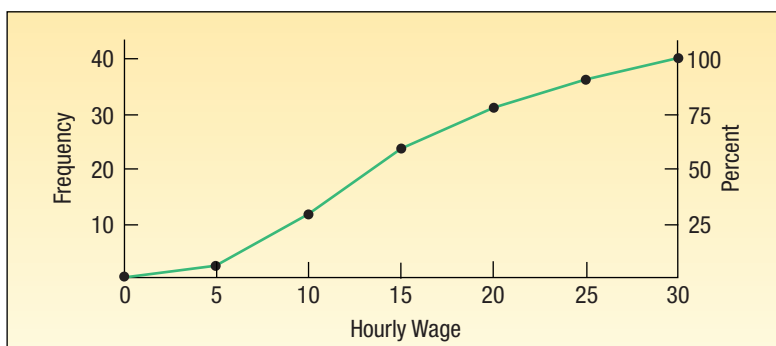
Hourly Wages	Number of Employees
\$ 8 up to \$10	3
10 up to 12	7
12 up to 14	4
14 up to 16	1

- What is the table called?
- Develop a cumulative frequency distribution and portray the distribution in a cumulative frequency polygon.
- On the basis of the cumulative frequency polygon, how many employees earn \$11 an hour or less? Half of the employees earn an hourly wage of how much or more? Four employees earn how much or less?

## Exercises

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19. The following chart shows the hourly wages of a sample of certified welders in the Atlanta, Georgia area.



- How many welders were studied?
- What is the class interval?

- c. About how many welders earn less than \$10.00 per hour?  
 d. About 75 percent of the welders make less than what amount?  
 e. Ten of the welders studied made less than what amount?  
 f. What percent of the welders make less than \$20.00 per hour?
20. The following chart shows the selling price (\$000) of houses sold in the Billings, Montana area.



- a. How many homes were studied?  
 b. What is the class interval?  
 c. One hundred homes sold for less than what amount?  
 d. About 75 percent of the homes sold for less than what amount?  
 e. Estimate the number of homes in the \$150,000 up to \$200,000 class.  
 f. About how many homes sold for less than \$225,000?
21. The frequency distribution representing the number of frequent flier miles accumulated by employees at Brumley Statistical Consulting Company is repeated from Exercise 17.

Frequent Flier Miles (000)	Frequency
0 up to 3	5
3 up to 6	12
6 up to 9	23
9 up to 12	8
12 up to 15	2
Total	50

- a. How many employees accumulated less than 3,000 miles?  
 b. Convert the frequency distribution to a cumulative frequency distribution.  
 c. Portray the cumulative distribution in the form of a cumulative frequency polygon.  
 d. Based on the cumulative frequency polygon, about 75 percent of the employees accumulated how many miles or less?
22. The frequency distribution of order lead time at Ecommerce.com from Exercise 18 is repeated below.

Lead Time (days)	Frequency
0 up to 5	6
5 up to 10	7
10 up to 15	12
15 up to 20	8
20 up to 25	7
Total	40

- a. How many orders were filled in less than 10 days? In less than 15 days?  
 b. Convert the frequency distribution to a cumulative frequency distribution.  
 c. Develop a cumulative frequency polygon.  
 d. About 60 percent of the orders were filled in less than how many days?

## Chapter Summary

- I. A frequency table is a grouping of qualitative data into mutually exclusive classes showing the number of observations in each class.
- II. A relative frequency table shows the fraction of the number of frequencies in each class.
- III. A bar chart is a graphic representation of a frequency table.
- IV. A pie chart shows the proportion each distinct class represents of the total number of frequencies.
- V. A frequency distribution is a grouping of data into mutually exclusive classes showing the number of observations in each class.
  - A. The steps in constructing a frequency distribution are:
    1. Decide on the number of classes.
    2. Determine the class interval.
    3. Set the individual class limits.
    4. Tally the raw data into classes.
    5. Count the number of tallies in each class.
  - B. The class frequency is the number of observations in each class.
  - C. The class interval is the difference between the limits of two consecutive classes.
  - D. The class midpoint is halfway between the limits of consecutive classes.
- VI. A relative frequency distribution shows the percent of observations in each class.
- VII. There are three methods for graphically portraying a frequency distribution.
  - A. A histogram portrays the number of frequencies in each class in the form of a rectangle.
  - B. A frequency polygon consists of line segments connecting the points formed by the intersection of the class midpoint and the class frequency.
  - C. A cumulative frequency distribution shows the number or percent of observations below given values.




## Chapter Exercises

23. Describe the similarities and differences of qualitative and quantitative variables. Be sure to include:
  - a. What level of measurement is required for each variable type?
  - b. Can both types be used to describe both samples and populations?
24. Describe the similarities and differences of a frequency table and a frequency distribution. Be sure to include which requires qualitative data and which requires quantitative data.
25. Alexandra Damonte will be building a new resort in Myrtle Beach, South Carolina. She must decide how to design the resort based on the type of activities that the resort will offer to its customers. A recent poll of 300 potential customers showed the following results about customers' preferences for planned resort activities:

Like planned activities	63
Do not like planned activities	135
Not sure	78
No answer	24


- a. What is the table called?
  - b. Draw a bar chart to portray the survey results.
  - c. Draw a pie chart for the survey results.
  - d. If you are preparing to present the results to Ms. Damonte as part of a report, which graph would you prefer to show? Why?
26. Speedy Swift is a package delivery service that serves the greater Atlanta, Georgia metropolitan area. To maintain customer loyalty, one of Speedy Swift's performance objectives is on-time delivery. To monitor its performance, each delivery is measured on the following scale: early (package delivered before the promised time), on-time (package delivered within 5 minutes of the promised time), late (package delivered more than 5 minutes past the promised time), lost (package never delivered). Speedy Swift's objective




is to deliver 99 percent of all packages either early or on-time. Another objective is to never lose a package. 

Speedy collected the following data for last month's performance:

On-time	On-time	Early	Late	On-time	On-time	On-time	On-time	Late	On-time
Early	On-time	On-time	Early	On-time	On-time	On-time	On-time	On-time	On-time
Early	On-time	Early	On-time	On-time	On-time	Early	On-time	On-time	On-time
Early	On-time	On-time	Late	Early	Early	On-time	On-time	On-time	Early
On-time	Late	Late	On-time	On-time	On-time	On-time	On-time	On-time	On-time
On-time	Late	Early	On-time	Early	On-time	Lost	On-time	On-time	On-time
Early	Early	On-time	On-time	Late	Early	Lost	On-time	On-time	On-time
On-time	On-time	Early	On-time	Early	On-time	Early	On-time	Late	On-time
On-time	Early	On-time	On-time	On-time	Late	On-time	Early	On-time	On-time
On-time	On-time	On-time	On-time	On-time	Early	Early	On-time	On-time	On-time


- What scale is used to measure delivery performance? What kind of variable is delivery performance?
  - Construct a frequency table for delivery performance for last month.
  - Construct a relative frequency table for delivery performance last month.
  - Construct a bar chart of the frequency table for delivery performance for last month.
  - Construct a pie chart of on-time delivery performance for last month.
  - Analyze the data summaries and write an evaluation of last month's delivery performance as it relates to Speedy Swift's performance objectives. Write a general recommendation for further analysis.
- A data set consists of 83 observations. How many classes would you recommend for a frequency distribution?
  - A data set consists of 145 observations that range from 56 to 490. What size class interval would you recommend?
  - The following is the number of minutes to commute from home to work for a group of automobile executives. 

28	25	48	37	41	19	32	26	16	23	23	29	36
31	26	21	32	25	31	43	35	42	38	33	28	

- How many classes would you recommend?
  - What class interval would you suggest?
  - What would you recommend as the lower limit of the first class?
  - Organize the data into a frequency distribution.
  - Comment on the shape of the frequency distribution.
- The following data give the weekly amounts spent on groceries for a sample of households. 


\$271	\$363	\$159	\$ 76	\$227	\$337	\$295	\$319	\$250
279	205	279	266	199	177	162	232	303
192	181	321	309	246	278	50	41	335
116	100	151	240	474	297	170	188	320
429	294	570	342	279	235	434	123	325

- How many classes would you recommend?
- What class interval would you suggest?
- What would you recommend as the lower limit of the first class?
- Organize the data into a frequency distribution.


31. A social scientist is studying the use of iPods by college students. A sample of 45 students revealed they played the following number of songs yesterday. 

4	6	8	7	9	6	3	7	7	6	7	1	4	7	7
4	6	4	10	2	4	6	3	4	6	8	4	3	3	6
8	8	4	6	4	6	5	5	9	6	8	8	6	5	10


Organize the above information into a frequency distribution.

- How many classes would you suggest?
  - What is the most suitable class interval?
  - What is the lower limit of the initial class?
  - Create the frequency distribution.
  - Describe the profile of the distribution.
32. David Wise handles his own investment portfolio, and has done so for many years. Listed below is the holding time (recorded to the nearest whole year) between purchase and sale for his collection of stocks. 

8	8	6	11	11	9	8	5	11	4	8	5	14	7	12	8	6	11	9	7
9	15	8	8	12	5	9	8	5	9	10	11	3	9	8	6				

- How many classes would you propose?
  - What class interval would you suggest?
  - What quantity would you use for the lower limit of the initial class?
  - Using your responses to parts (a), (b), and (c), create a frequency distribution.
  - Identify the appearance of the frequency distribution.
33. You are exploring the music in your iTunes library. The total play counts over the past year for the songs on your “smart playlist” are shown below. Make a frequency distribution of the counts and describe its shape. It is often claimed that a small fraction of a person’s songs will account for most of their total plays. Does this seem to be the case here? 

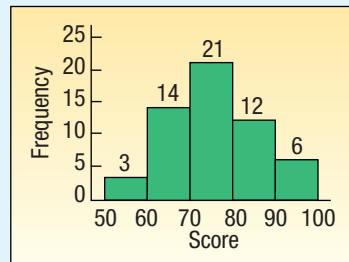
128	56	54	91	190	23	160	298	445	50
578	494	37	677	18	74	70	868	108	71
466	23	84	38	26	814	17			

34. The *Journal of Finance* made its content available on the Internet starting in July of 2005. The table below shows the number of times a monthly version was downloaded and the number of articles that were viewed during each month. Suppose you wish to make a frequency distribution of the number of downloads. 

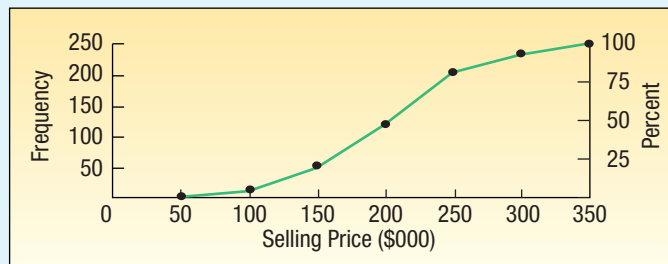
312	2,753	2,595	6,057	7,624	6,624	6,362	6,575	7,760	7,085	7,272
5,967	5,256	6,160	6,238	6,709	7,193	5,631	6,490	6,682	7,829	7,091
6,871	6,230	7,253	5,507	5,676	6,974	6,915	4,999	5,689	6,143	7,086

- How many classes would you propose?
- What class interval would you suggest?
- What quantity would you use for the lower limit of the initial class?
- Using your responses to parts (a), (b), and (c), create a frequency distribution.
- Identify the appearance of the frequency distribution.

35. The following histogram shows the scores on the first exam for a statistics class.



- How many students took the exam?
  - What is the class interval?
  - What is the class midpoint for the first class?
  - How many students earned a score of less than 70?
36. The following chart summarizes the selling price of homes sold last month in the Sarasota, Florida, area.



- What is the chart called?
  - How many homes were sold during the last month?
  - What is the class interval?
  - About 75 percent of the houses sold for less than what amount?
  - One hundred seventy-five of the homes sold for less than what amount?
37. A chain of sport shops catering to beginning skiers, headquartered in Aspen, Colorado, plans to conduct a study of how much a beginning skier spends on his or her initial purchase of equipment and supplies. Based on these figures, it wants to explore the possibility of offering combinations, such as a pair of boots and a pair of skis, to induce customers to buy more. A sample of cash register receipts revealed these initial purchases:

\$140	\$ 82	\$265	\$168	\$ 90	\$114	\$172	\$230	\$142
86	125	235	212	171	149	156	162	118
139	149	132	105	162	126	216	195	127
161	135	172	220	229	129	87	128	126
175	127	149	126	121	118	172	126	

- Arrive at a suggested class interval. Use six classes, and let the lower limit of the first class be \$70.
- What would be a better class interval?
- Organize the data into a frequency distribution using a lower limit of \$80.
- Interpret your findings.


38. Following is the number of shareholders for a selected group of large companies (in thousands):

Company	Number of Shareholders (thousands)	Company	Number of Shareholders (thousands)
Southwest Airlines	144	Standard Oil (Indiana)	173
General Public Utilities	177	Home Depot	195
Occidental Petroleum	266	Detroit Edison	220
Middle South Utilities	133	Eastman Kodak	251
Chrysler	209	Dow Chemical	137
Standard Oil of California	264	Pennsylvania Power	150
Bethlehem Steel	160	American Electric Power	262
Long Island Lighting	143	Ohio Edison	158
RCA	246	Transamerica Corporation	162
Greyhound Corporation	151	Columbia Gas System	165
Pacific Gas & Electric	239	International Telephone & Telegraph	223
Niagara Mohawk Power	204	Union Electric	158
E. I. du Pont de Nemours	204	Virginia Electric and Power	162
Westinghouse Electric	195	Public Service Electric & Gas	225
Union Carbide	176	Consumers Power	161
BankAmerica	175		
Northeast Utilities	200		


The shareholder numbers are to be organized into a frequency distribution and several graphs drawn to portray the distribution.

- Using seven classes and a lower limit of 130, construct a frequency distribution.
  - Portray the distribution as a frequency polygon.
  - Portray the distribution in a cumulative frequency polygon.
  - According to the polygon, three out of four (75 percent) of the companies have how many shareholders or less?
  - Write a brief analysis of the number of shareholders based on the frequency distribution and graphs.
39. A recent survey showed that the typical American car owner spends \$2,950 per year on operating expenses. Below is a breakdown of the various expenditure items. Draw an appropriate chart to portray the data and summarize your findings in a brief report.


Expenditure Item	Amount
Fuel	\$ 603
Interest on car loan	279
Repairs	930
Insurance and license	646
Depreciation	492
Total	\$2,950

40. Midland National Bank selected a sample of 40 student checking accounts. Below are their end-of-the-month balances. 

\$404	\$ 74	\$234	\$149	\$279	\$215	\$123	\$ 55	\$ 43	\$321
87	234	68	489	57	185	141	758	72	863
703	125	350	440	37	252	27	521	302	127
968	712	503	489	327	608	358	425	303	203

- a. Tally the data into a frequency distribution using \$100 as a class interval and \$0 as the starting point.
  - b. Draw a cumulative frequency polygon.
  - c. The bank considers any student with an ending balance of \$400 or more a “preferred customer.” Estimate the percentage of preferred customers.
  - d. The bank is also considering a service charge to the lowest 10 percent of the ending balances. What would you recommend as the cutoff point between those who have to pay a service charge and those who do not?
41. Residents of the state of South Carolina earned a total of \$69.5 billion in adjusted gross income. Seventy-three percent of the total was in wages and salaries; 11 percent in dividends, interest, and capital gains; 8 percent in IRAs and taxable pensions; 3 percent in business income pensions; 2 percent in Social Security, and the remaining 3 percent from other sources. Develop a pie chart depicting the breakdown of adjusted gross income. Write a paragraph summarizing the information.
42. A recent study of home technologies reported the number of hours of personal computer usage per week for a sample of 60 persons. Excluded from the study were people who worked out of their home and used the computer as a part of their work. 

9.3	5.3	6.3	8.8	6.5	0.6	5.2	6.6	9.3	4.3
6.3	2.1	2.7	0.4	3.7	3.3	1.1	2.7	6.7	6.5
4.3	9.7	7.7	5.2	1.7	8.5	4.2	5.5	5.1	5.6
5.4	4.8	2.1	10.1	1.3	5.6	2.4	2.4	4.7	1.7
2.0	6.7	1.1	6.7	2.2	2.6	9.8	6.4	4.9	5.2
4.5	9.3	7.9	4.6	4.3	4.5	9.2	8.5	6.0	8.1

- a. Organize the data into a frequency distribution. How many classes would you suggest? What value would you suggest for a class interval?
  - b. Draw a histogram. Interpret your result.
43. Merrill Lynch recently completed a study regarding the size of online investment portfolios (stocks, bonds, mutual funds, and certificates of deposit) for a sample of clients in the 40- to 50-year-old age group. Listed following is the value of all the investments in thousands of dollars for the 70 participants in the study. 

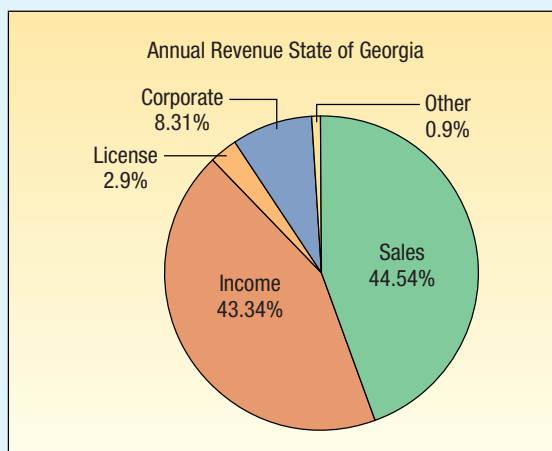
\$669.9	\$ 7.5	\$ 77.2	\$ 7.5	\$125.7	\$516.9	\$ 219.9	\$645.2
301.9	235.4	716.4	145.3	26.6	187.2	315.5	89.2
136.4	616.9	440.6	408.2	34.4	296.1	185.4	526.3
380.7	3.3	363.2	51.9	52.2	107.5	82.9	63.0
228.6	308.7	126.7	430.3	82.0	227.0	321.1	403.4
39.5	124.3	118.1	23.9	352.8	156.7	276.3	23.5
31.3	301.2	35.7	154.9	174.3	100.6	236.7	171.9
221.1	43.4	212.3	243.3	315.4	5.9	1,002.2	171.7
295.7	437.0	87.8	302.1	268.1	899.5		

- a. Organize the data into a frequency distribution. How many classes would you suggest? What value would you suggest for a class interval?
  - b. Draw a histogram. Interpret your result.
44. A total of 5.9 percent of the prime time viewing audience watched shows on ABC, 7.6 percent watched shows on CBS, 5.5 percent on Fox, 6.0 percent on NBC, 2.0 percent on Warner Brothers, and 2.2 percent on UPN. A total of 70.8 percent of the audience watched shows on other cable networks, such as CNN and ESPN. You can find the latest information on TV viewing from the following website: <http://tv.zap2it.com/news/ratings>. Develop a pie chart or a bar chart to depict this information. Write a paragraph summarizing your findings.

45. Refer to the following chart, which appeared recently in the Snapshot section of *USA Today*.



- What is the name given to this type of chart?
  - If you studied 500 weddings, how many would you expect to take place in a house of worship?
  - Would it be reasonable to conclude that about 80 percent of weddings take place in either a house of worship or outdoors? Cite evidence.
46. The following chart depicts the annual revenues, by type of tax, for the state of Georgia. The chart was developed using Kids Zone, a NCES, project. Their website is: [nces.ed.gov/nceskids/createagraph/](http://nces.ed.gov/nceskids/createagraph/).




- What percentage of the state revenue is accounted for by sales tax and individual income tax?
  - Which category will generate more revenue, corporate taxes or license fees?
  - The total annual revenue for the state of Georgia is \$6.3 billion. Estimate the amount of revenue in billions of dollars for sales taxes and for individual taxes.
47. In 2006, Canada exported \$303.4 billion worth of products to the United States. The five largest were:


Product	Amount
Petroleum products	\$63.7 billion
Passenger cars	36.6
Car parts and accessories	15.6
Aluminum	7.7
Lumber	6.6

- Use a software package to develop a bar chart.
- What percentage of Canada's *total* exports to the United States is represented by the two categories "Petroleum products" and "Passenger cars"?
- Of the top five exported products, what percentage of the total do "Petroleum products" and "Passenger cars" represent?



48. Farming has changed from the early 1900s. In the early 20th century, machinery gradually replaced animal power. For example, in 1910 U.S. farms used 24.2 million horses and mules and only about 1,000 tractors. By 1960, 4.6 million tractors were used and only 3.2 million horses and mules. In 1920, there were over 6 million farms in the United States. Today there are fewer than 2 million. Listed below is the number of farms, in thousands, for each of the 50 states. Write a paragraph summarizing your findings. 

47	1	8	46	76	26	4	3	39	45
4	21	80	63	100	65	91	29	7	15
7	52	87	39	106	25	55	2	3	8
14	38	59	33	76	71	37	51	1	24
35	86	185	13	7	43	36	20	79	9

49. One of the most popular candies in the United States is M&M's, which are produced by the Mars Company. In the beginning M&M's were all brown; more recently they were produced in red, green, blue, orange, brown, and yellow. You can read about the history of the product, find ideas for baking, purchase the candies in the colors of your school or favorite team, and learn the percent of each color in the standard bags at [www.m-ms.com](http://www.m-ms.com). Recently, the purchase of a 14-ounce bag of M&M's Plain had 444 candies with the following breakdown by color: 130 brown, 98 yellow, 96 red, 35 orange, 52 blue, and 33 green. Develop a chart depicting this information and write a paragraph summarizing the results.
50. The number of families who used the Minneapolis YWCA day care service was recorded during a 30-day period. The results are as follows: 

31	49	19	62	24	45	23	51	55	60
40	35	54	26	57	37	43	65	18	41
50	56	4	54	39	52	35	51	63	42

- Construct a cumulative frequency distribution.
- Sketch a graph of the cumulative frequency polygon.
- How many days saw fewer than 30 families utilize the day care center?
- How busy were the highest 80 percent of the days?

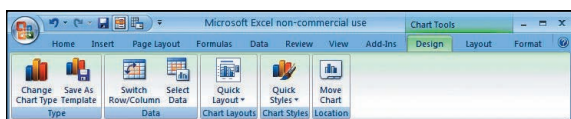
## Data Set Exercises

51. Refer to the Real Estate data at the end of the book, which reports information on homes sold in the Goodyear, Arizona, area during the last year. Select an appropriate class interval and organize the selling prices into a frequency distribution. Write a brief report summarizing your finding. Be sure to answer the following questions in your report.
- Around what values do the data tend to cluster?
  - What is the largest selling price? What is the smallest selling price?
  - Draw a cumulative frequency distribution. How many homes sold for less than \$200,000? Estimate the percent of the homes that sold for more than \$220,000. What percent of the homes sold for less than \$125,000?
  - Refer to the variable regarding the townships. Draw a bar chart showing the number of homes sold in each township. Are there any differences or is the number of homes sold about the same in each township?
52. Refer to the Baseball 2009 data, which reports information on the 30 Major League Baseball teams for the 2009 season. Select an appropriate class interval and organize the team salaries into a frequency distribution.
- What is a typical team salary? What is the range of salaries?
  - Comment on the shape of the distribution. Does it appear that any of the team salaries are out of line with the others?
  - Draw a cumulative frequency distribution. Forty percent of the teams are paying less than what amount in total team salary? About how many teams have total salaries of less than \$80,000,000?

53. Refer to the Buena School District bus data. Select the variable referring to the number of miles traveled last month, and then organize these data into a frequency distribution.
- What is a typical amount of miles traveled? What is the range?
  - Comment on the shape of the distribution. Are there any outliers in terms of miles driven?
  - Draw a cumulative frequency distribution. Forty percent of the buses were driven fewer than how many miles? How many buses were driven less than 850 miles?
  - Refer to the variables regarding the bus type and the number of seats in each bus. Draw a pie chart of each variable and comment on your findings.

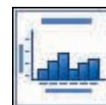
## Software Commands

- The Excel commands for the pie chart on page 26 are:
  - Set cell *A1* as the active cell and type the words *Use of Sales*. In cells *A2* through *A5* type *Prizes*, *Education*, *Bonuses*, and *Expense*.
  - Set *B1* as the active cell and type *Amount (\$ Millions)* and in cells *B2* through *B5* enter the data. When finished entering data in *B5*, hit the **Enter** button.
  - From the top row of tabs, select **Insert**. In the Chart tool bar, select **Pie**. Select the top-left **2-D** pie chart. A blank chart will appear.



- In Excel's top row, a **Chart Tools** tab will appear. Select the **Design** option. Select the **Select Data** option from the tool bar. A **Select Data** window will appear. For the **Chart Data Range**, using the mouse, select all cells from *A1* to *B5*. Click **OK**.
  - Click on the pie chart. Right-click for the options menu. Select **Add Data Labels**. Click on the pie chart again. Right-click for the options menu. Select **Format Data Labels**, then uncheck any boxes currently checked in the dialog box. Next select **Category**, **Percent**, and **Leader Lines**. Then click **Close**.
  - Double-click on the chart title and rename **Ohio State Lottery Expenses**.
- The MegaStat commands for the frequency distribution on page 34 are:
    - Open Excel and from the CD provided, select **Data Sets**, and select the Excel format; go to Chapter 2, and select Applewood data. Click on **Mega-Stat**, **Frequency Distribution**, and select **Quantitative**.
    - In the dialog box, input the range from *A1:A181*, select **Equal width intervals**, use 400 as the interval width, 2000 as the lower boundary of the first interval, select **Histogram**, and then click **OK**.

- The Excel commands for the histogram on page 38 are:
  - In cell *A1* indicate that the column of data is the profit and in *B1* that it is the frequency. In cells *A2* to *A9*, insert the midpoints of the profits. In *B2* to *B9*, record the class frequencies. When finished entering data in cell *B9*, hit the **Enter** key.
  - With your mouse, highlight cells *B2* through *B9*.
  - From the tabs, select **Insert**. From the Charts, select **Column**, then **2-D column** and pick the first chart type. A graph will appear.
  - When the graph area is active, a **Chart Tools** tab appears at the top of the screen. Select the **Design** tab, and then select **Data**. Under **Horizontal (Category) Axis Labels**, click **Edit**, then use the mouse to select cells *A2* through *A9* and click **OK** twice. The horizontal axis now shows the class midpoints.
  - With the **Chart Tools** displayed at the top, select the **Design** tab. Select **Chart Layout**. Select the layout:

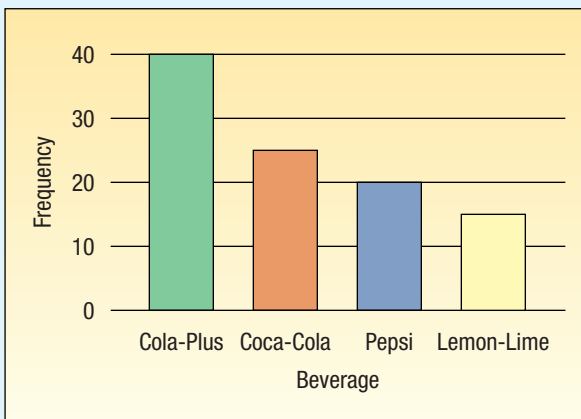


- With **Chart Tools** displayed at the top, select the **Layout** tab. Double-click on the **Chart Title** and type in an appropriate title. Next, under the same **Layout** tab, select **Axis Titles**. Using **Primary Vertical Axis Title**, name the vertical axis *Frequency* and delete the words *vertical axis*. Using the **Primary Horizontal Axis Title**, name it *Profit* \$. Next select **Legend** and select **None**.
- Double-click one of the columns in the graph. Then from the Tabs across the top select **Layout**. On the left-hand side of the tool bar, click the words **Format Selection**. A dialog box will appear. Under **Series Option**, change the **Gap Width** to 0% by moving the arrow all the way to the left, and then click the **Close** button at the bottom of the dialog box.

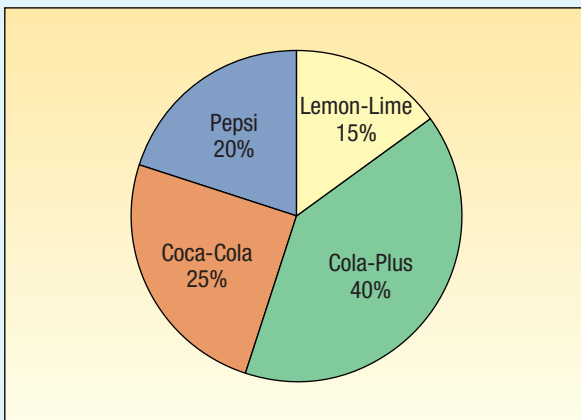


## Chapter 2 Answers to Self-Review

- 2-1**
- Qualitative data, because the customers' response to the taste test is the name of a beverage.
  - Frequency table. It shows the number of people who prefer each beverage.
  -



d.



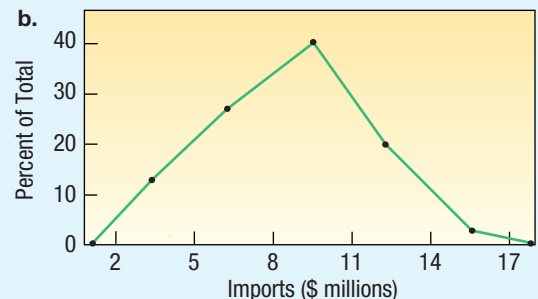
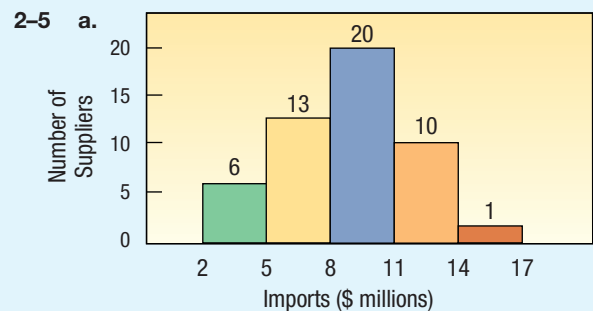
- 2-2**
- The raw data or ungrouped data.
  -

Commission	Number of Salespeople
\$1,400 up to \$1,500	2
1,500 up to 1,600	5
1,600 up to 1,700	3
1,700 up to 1,800	1
Total	11

- Class frequencies.
- The largest concentration of commissions is \$1,500 up to \$1,600. The smallest commission is about \$1,400 and the largest is about \$1,800. The typical amount earned is \$15,500.

- 2-3**
- $2^6 = 64 < 73 < 128 = 2^7$ . So seven classes are recommended.
  - The interval width should be at least  $(488 - 320)/7 = 24$ . Class intervals of 25 or 30 feet are both reasonable.
  - If we use a class interval of 25 feet and begin with a lower limit of 300 feet, eight classes would be necessary. A class interval of 30 feet beginning with 300 feet is also reasonable. This alternative requires only seven classes.

- 2-4**
- 45
  - .250
  - .306, found by  $.178 + .106 + .022$



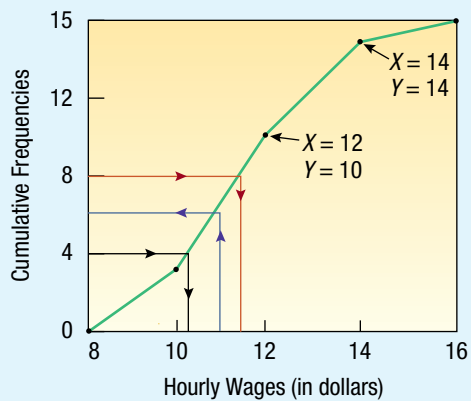
The plots are: (3.5, 12), (6.5, 26), (9.5, 40), (12.5, 20), and (15.5, 2).

- The smallest annual volume of imports by a supplier is about \$2 million, the largest about \$17 million. The highest frequency is between \$8 million and \$11 million.

2-6 a. A frequency distribution.

b.

Hourly Wages	Cumulative Number
Less than \$8	0
Less than \$10	3
Less than \$12	10
Less than \$14	14
Less than \$16	15



- c. About seven employees earn \$11.00 or less.  
 About half the employees earn \$11.25 or more.  
 About four employees earn \$10.25 or less.