PREVIEW 7

Basic Algebra

Learning Objectives	Sample Problems		For Help Go to Page
When you finish this unit you will be able to:			
1. Evaluate formulas and literal expressions.	If $x = 2, y = 3, a = 5, b = 6$, find the value of		332
	(a) $2x + y$	________	
	(b) $\dfrac{1 + x^2 + 2a}{y}$	________	
	(c) $A = x^2y$	$A =$ ________	
	(d) $T = \dfrac{2(a + b + 1)}{3x}$	$T =$ ________	
	(e) $P = abx^2$	$P =$ ________	
2. Perform the basic algebraic operations.	(a) $3ax^2 + 4ax^2 - ax^2$	$=$ ________	340
	(b) $5x - 3y - 8x + 2y$	$=$ ________	
	(c) $5x - (x + 2)$	$=$ ________	
	(d) $3(x^2 + 5x) - 4(2x - 3)$	$=$ ________	
3. Solve linear equations in one unknown and solve formulas.	(a) $3x - 4 = 11$	$x =$ ________	357
	(b) $2x = 18$	$x =$ ________	352
	(c) $2x + 7 = 43 - x$	$x =$ ________	369
	(d) Solve the following formula for N: $S = \dfrac{N}{2} + 26$	$N =$ ________	374
	(e) Solve the following formula for A: $M = \dfrac{(A - B)L}{8}$	$A =$ ________	

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4. Translate simple English phrases and sentences into algebraic expressions and equations.	Write each phrase as an algebraic expression or equation.		
	(a) Four times the area.	_______________	381
	(b) Current squared times resistance.	_______________	
	(c) Efficiency is equal to 100 times the output divided by the input.	_______________	
	(d) Resistance is equal to 12 times the length divided by the diameter squared.	_______________	
	(e) An electrician collected \$1265.23 for a job that included \$840 labour, which was not taxed, and the rest for parts, which were taxed, at 6%. Determine how much of the total was tax.	_______________	386
	(f) A plumber's helper earns \$18/h plus \$27/h overtime. If the helper works 40 regular hours during a week, how much overtime was worked to earn \$1000?	_______________	
5. Multiply and divide simple algebraic expressions.	(a) $2y \cdot 3y$	_______________	391
	(b) $(6x^4y^2)(-2xy^2)$	_______________	
	(c) $3x(y - 2x)$	_______________	
	(d) $\dfrac{10x^7}{-2x^2}$	_______________	393
	(e) $\dfrac{6a^2b^4}{18a^5b^2}$	_______________	
	(f) $\dfrac{12m^4 - 9m^3 + 15m^2}{3m^2}$	_______________	
6. Use scientific notation.	Write in scientific notation.		
	(a) 0.000184	_______________	396
	(b) 213,000	_______________	
	Calculate.		
	(c) $(3.2 \times 10^{-6}) \times (4.5 \times 10^2)$	_______________	399
	(d) $(1.56 \times 10^{-4}) \div (2.4 \times 10^3)$	_______________	

(Answers to these preview problems are given in the back of the book.)

If you are certain that you can work *all* these problems correctly, turn to page 405 for the set of practice problems. If you cannot work one or more of the preview problems, turn to the page indicated to the right of the problem. Those who wish to master this material should turn to Section 7-1 and begin work there.

7 Basic Algebra

Learning Objectives

1. Evaluate formulas and literal expressions.
2. Perform the basic algebraic operations.
3. Solve linear equations in one unknown and solve formulas.
4. Translate simple English phrases and sentences into algebraic expressions and equations.
5. Multiply and divide simple algebraic expressions.
6. Use scientific notation.

In this chapter you will study algebra, but not the very formal algebra that deals with theorems, proofs, sets, and abstract problems. Instead, we shall study practical or applied algebra as actually used by technical and trades workers. Let's begin with a look at the language of algebra and algebraic formulas.

7-1 ALGEBRAIC LANGUAGE AND FORMULAS

The most obvious difference between algebra and arithmetic is that in algebra letters are used to replace or to represent numbers. A mathematical statement in which letters are used to represent numbers is called a *literal* expression. Algebra is the arithmetic of literal expressions—a kind of symbolic arithmetic.

Any letters will do, but in practical algebra we use the normal lower- and uppercase letters of the English alphabet. It is helpful in practical problems to choose the letters to be used on the basis of their memory value: t for time, D for diameter, C for cost, A for area, and so on. The letter used reminds you of its meaning.

Multiplication

Most of the usual arithmetic symbols have the same meaning in algebra that they have in arithmetic. For example, the addition (+) and subtraction (−) signs are used in exactly the same way. However, the multiplication sign (×) of arithmetic looks like the letter x and to avoid confusion we have other ways to show multiplication in algebra. The product of two algebraic quantities a and b, "a times b," may be written using

A raised dot	$a \cdot b$				
Brackets or parentheses	$a(b)$	or	$(a)b$	or	$(a)(b)$
Nothing at all	ab				

Obviously, this last way of showing multiplication won't do in arithmetic; we cannot write "two times four" as "24"—it looks like twenty-four. But it is a quick and easy way to write a multiplication in algebra.

NOTE ▶ Placing two quantities side by side to show multiplication is not new and it is not only an algebra gimmick; we use it every time we write 20 cents or 4 feet.

20 cents $= 20 \times 1$ cent
4 feet $= 4 \times 1$ foot ◀

YOUR TURN

Write the following multiplications using no multiplication symbols.

(a) 8 times a = ________ (b) m times p = ________

(c) 2 times s times t = ________ (d) 3 times x times x = ________

ANSWERS

(a) $8a$ (b) mp (c) $2st$ (d) $3x^2$

Did you notice in problem (d) that powers are written just as in arithmetic:

$x \cdot x = x^2$

$x \cdot x \cdot x = x^3$

$x \cdot x \cdot x \cdot x = x^4$ and so on.

Brackets

Brackets are used in arithmetic to show that some complicated quantity is to be treated as a unit. For example,

$2 \cdot (13 + 14 - 6)$

means that the number 2 multiplies *all* of the quantity in the brackets.

In exactly the same way in algebra, parentheses ( ), brackets [ ], or braces { } are used to show that whatever is enclosed in them should be treated as a single quantity. An expression such as

$(3x^2 - 4ax + 2by^2)^2$

should be thought of as (something)^2. The expression

$(2x + 3a - 4) - (x^2 - 2a)$

should be thought of as (first quantity) – (second quantity). Brackets are the punctuation marks of algebra. Like the period, comma, or semicolon in regular sentences, they tell you how to read an expression and get its correct meaning.

Division

In arithmetic we would write "48 divided by 2" as

$2\overline{)48}$ or $48 \div 2$ or $\frac{48}{2}$

But the first two ways of writing division are used very seldom in algebra. Division is usually written as a fraction.

"x is divided by y" is written $\dfrac{x}{y}$

"$(2n + 1)$ divided by $(n - 1)$" is written $\dfrac{(2n + 1)}{(n - 1)}$ or $\dfrac{2n + 1}{n - 1}$

YOUR TURN

Write the following using algebraic notation.

(a) 8 times $(2a + b) =$ ________ (b) $(a + b)$ times $(a - b) =$ ________

(c) x divided by $y^2 =$ ________ (d) $(x + 2)$ divided by $(2x - 1) =$ ________

ANSWERS

(a) $8(2a + b)$ (b) $(a + b)(a - b)$

(c) $\dfrac{x}{y^2}$ (d) $\dfrac{x + 2}{2x - 1}$

Algebraic Expressions

The word "expression" is used very often in algebra. An *expression* is a general name for any collection of numbers and letters connected by arithmetic signs. For example,

$x + y \qquad 2x^2 + 4 \qquad 3(x^2 - 2ab)$

$\dfrac{D}{T} \qquad \sqrt{x^2 + y^2} \qquad$ and $\qquad (b - 1)^2$

are all algebraic expressions.

If the algebraic expression has been formed by multiplying quantities, each multiplier is called a *factor* of the expression.

Expression	**Factors**
ab	a and b
$2x(x + 1)$	$2x$ and $(x + 1)$
$(R - 1)(2R + 1)$	$(R - 1)$ and $(2R + 1)$

The algebraic expression can also be a sum or difference of simpler quantities or *terms*.

Expression	**Terms**	
$x + 4y$	x and $4y$ ←	The first term is x The second term is $4y$
$2x^2 + xy$	$2x^2$ and xy	
$A - R$	A and R	

YOUR TURN

Now let's check to see if you understand the difference between terms and factors. In each algebraic expression below, tell whether the portion quoted is a term or a factor.

(a) $2x^2 - 3xy$ y is a ______________

(b) $7x - 4$ $7x$ is a ______________

(c) $4x(a + 2b)$ $4x$ is a ______________

(d) $-2y(3y - 5)$ $(3y - 5)$ is a ______________

(e) $3x^2y + 8y^2 - 9$ $3x^2y$ is a ______________

ANSWERS

(a) factor (b) term (c) factor (d) factor (e) term

Evaluating Formulas

One of the most useful algebraic skills for any technical or practical work involves finding the value of an algebraic expression when the letters are given numerical values. A *formula* is a rule for calculating the numerical value of one quantity from the values of the other quantities. The formula or rule is usually written in mathematical form because algebra gives a brief, convenient to use, and easy to remember form for the rule. Here are a few examples of rules and formulas used in the trades.

1. **Rule:** The voltage across a simple resistor is equal to the product of the current through the resistor and the value of its resistance.

   Formula: $V = iR$

2. **Rule:** The cost of setting type is equal to the total ems set in the job multiplied by the hourly wage divided by the rate at which the type is set, in ems per hour.

   Formula: $C = \dfrac{TH}{E}$

3. **Rule:** The number of standard inch-measure bricks needed to build a wall is about 21 times the volume of the wall.

   Formula: $N = 21\,LWH$

Evaluating a formula or algebra expression means to find its value by substituting numbers for the letters in the expression.

EXAMPLE

In retail stores the following formula is used:

$M = R - C$ where M is the markup on an item, R is the retail selling price, and C is the original cost

Find M if $R = \$25$ and $C = \$21$.

$M =$ ________

$M = \$25 - \$21 = \$4$

Easy? Of course. Simply substitute the numbers for the correct letters and then do the arithmetic. A formula is a recipe for a calculation.

YOUR TURN

Automotive engineers use the following formula to calculate the horsepower rating of an engine.

$H = \dfrac{D^2N}{2.5}$ where D is the diameter of a cylinder in inches, and N is the number of cylinders

Find H when $D = 3\frac{1}{2}$ in. and $N = 6$.

SOLUTION

$$H = \frac{(3.5)^2(6)}{2.5}$$

3.5 $\boxed{x^2}$ $\boxed{\times}$ **6** $\boxed{\div}$ **2.5** $\boxed{=}$ $\rightarrow$ *29.4*

$H = 29.4$ hp or roughly 29 horsepower.

To be certain you do it correctly, follow this two-step process:

Step 1 Place the numbers being substituted in brackets, and then substitute them in the formula.

Step 2 Do the arithmetic carefully *after* the numbers are substituted.

EXAMPLE

(a) Find $A = x + 2y$ for $x = 5$ m, y $= 3$ m.

Step 1	$A = (5 \text{ m}) + 2(3 \text{ m})$	Put 5 m and 3 m in brackets.
Step 2	$A = 5 \text{ m} + 6 \text{ m}$	Do the arithmetic.
	$A = 11 \text{ m}$	

(b) Find $B = x^2 - y$ for $x = 4, y = -3$.

Step 1	$B = (4)^2 - (-3)$	Put the numbers in brackets.
Step 2	$B = 16 + 3$	Do the arithmetic.
	$B = 19$	

Using brackets in this way may seem like extra work for you, but it is the key to avoiding mistakes when evaluating formulas.

YOUR TURN

Evaluate the following formulas.

(a)	$D = 2R$	for $R = 3.45$ cm
(b)	$W = T - C$	for $T = 1420$ kg, $C = 385$ kg
(c)	$P = 0.433H$	for $H = 11.4$ in. Round to three significant digits.
(d)	$A = bh - 2$	for $b = 1.75$ m, $h = 4.20$ m
(e)	$P = 2(8 + x)$	for $x = -3$

Use the two-step process and round your answer appropriately.

SOLUTIONS

(a) $D = 2(3.45) = 6.90$ cm

(b) $W = (1420) - (385)$
$= 1420 - 385$
$= 1035$ kg

(c) $P = 0.433(11.4)$
$= 4.9362$ or 4.94 in., rounded

(d) $A = (1.75)(4.2) - 2$
$= 7.35 - 2$
$= 5.35$ sq m

For problem (d),

1.75 ⊗ **4.2** ⊖ **2** ⊜ → *5.35*

(e) $P = 2(8 + (-3))$
$P = 2(8 - 3)$
$P = 2 \cdot 5$
$P = 10$

The order of operations for arithmetic calculations should be used when evaluating formulas. Remember:

1. Do any operations **inside brackets.**
2. Find all **powers** and **roots.**
3. Do all **divisions** or **multiplications** left to right.
4. Do all **additions** or **subtractions** left to right.

PROBLEMS

1. Evaluate the following formulas using the standard order of operations.

$a = 3, b = 4, c = 6$

(a) $3ab$ = _______ (b) $2a^2c$ = _______

(c) $2a^2 - b^2$ = _______ (d) $3a^2bc - ab$ = _______

(e) $b + 2a$ = _______ (f) $a + 3(2b - c)$ = _______

(g) $2(a + b)$ = _______ (h) $(a^2 - 1) - (2 + b)$ = _______

2. To convert a Fahrenheit (°F) temperature to a Celsius (°C) temperature, use the following formula:

$$\boxed{C = \frac{5(F - 32)}{9}}$$

To convert a Celsius (°C) temperature to a Fahrenheit (°F) temperature, use this formula:

$$\boxed{F = \frac{9C}{5} + 32}$$

Use these formulas to find the following temperatures. Round to the nearest degree.

(a) 650°F = _______ °C (b) −160°C = _______ °F

(c) 398°C = _______ °F (d) 2200°F = _______ °C

(e) 80°F = _______ °C (f) 52.5°C = _______ °F

SOLUTIONS

1. (a) $3(3)(4) = 36$

(b) $2(3)^2(6) = 2 \cdot 9 \cdot 6 = 108$

(c) $$\begin{aligned} 2(3)^2 - (4)^2 &= 2 \cdot 9 - 16 \\ &= 18 - 16 \\ &= 2 \end{aligned}$$

(d) $$\begin{aligned} 3(3)^2(4)(6) - (3)(4) &= (3 \cdot 9 \cdot 4 \cdot 6) - (3 \cdot 4) \\ &= 648 - 12 \\ &= 636 \end{aligned}$$

3 $\boxed{\times}$ **3** $\boxed{x^2}$ $\boxed{\times}$ **4** $\boxed{\times}$ **6** $\boxed{-}$ **3** $\boxed{\times}$ **4** $\boxed{=}$ $\rightarrow$ *636.*

(e) $$\begin{aligned} (4) + 2(3) &= 4 + 2 \cdot 3 \\ &= 4 + 6 \\ &= 10 \end{aligned}$$

(f) $$\begin{aligned} (3) + 3(2(4) - (6)) &= 3 + 3(2 \cdot 4 - 6) \\ &= 3 + 3(8 - 6) \\ &= 3 + 3 \cdot 2 = 3 + 6 \\ &= 9 \end{aligned}$$

(g) $$\begin{aligned} 2((3) + (4)) &= 2 \cdot (3 + 4) \\ &= 2 \cdot 7 \\ &= 14 \end{aligned}$$

(h) $$\begin{aligned} ((3)^2 - 1) - (2 + (4)) &= (9 - 1) - (2 + 4) \\ &= 8 - 6 \\ &= 2 \end{aligned}$$

2. (a) $$\begin{aligned} C &= \frac{5 \times (650 - 32)}{9} \\ &= \frac{5 \times (618)}{9} \\ &= \frac{3090}{9} \\ &\approx 343°\text{C} \end{aligned}$$

(b) $$\begin{aligned} F &= \frac{9 \times (-160)}{5} + 32 \\ &= \frac{-1440}{5} + 32 \\ &= -288 + 32 \\ &= -256°\text{F} \end{aligned}$$

(c) $$\begin{aligned} F &= \frac{9 \times 398}{5} + 32 \\ &= \frac{3582}{5} + 32 \\ &= 716.4 + 32 \\ &\approx 748°\text{F} \end{aligned}$$

(d) $$\begin{aligned} C &= \frac{5 \times (2200 - 32)}{9} \\ &= \frac{5 \times 2168}{9} \\ &= \frac{10840}{9} \\ &\approx 1204°\text{C} \end{aligned}$$

(e) $$\begin{aligned} C &= \frac{5 \times (80 - 32)}{9} \\ &= \frac{5 \times 48}{9} \\ &= \frac{240}{9} \\ &\approx 27°\text{C} \end{aligned}$$

(f) $$\begin{aligned} F &= \frac{9 \times 52.5}{5} + 32 \\ &= \frac{472.5}{5} + 32 \\ &= 94.5 + 32 \\ &\approx 127°\text{F} \end{aligned}$$

For problem (e) using a calculator.

5 $\boxed{\times}$ $\boxed{(}$ 80 $\boxed{-}$ 32 $\boxed{)}$ $\boxed{\div}$ 9 $\boxed{=}$ → **26.66666667**

CAREFUL ▶ Avoid the temptation to combine steps when you evaluate formulas. Take it slowly and carefully, follow the standard order of operations, and you will arrive at the correct answer. Rush through problems like these and you usually make mistakes. ◀

Now turn to Exercises 7-1 for a set of practice problems in evaluating formulas.

Exercises 7-1

Evaluating Formulas

A. Find the value of each of these formulas for $x = 2, y = 3, z = 4, R = 5$.

1. $A = 3x$	2. $D = 2R - y$
3. $T = x^2 + y^2$	4. $H = 2x + 3y - z$
5. $K = 3z - x^2$	6. $Q = 2xyz - 10$
7. $F = 2(x + y^2) - 3$	8. $W = 3(y - 1)$
9. $L = 3R - 2(y^2 - x)$	10. $A = R^2 - y^2 - xz$
11. $B = 3R - y + 1$	12. $F = 3(R - y + 1)$

B. Find the value of each of the following formulas. (Round to the nearest whole number if necessary.)

1. $A = 2(x + y) - 1$	for	$x = 2, y = 4$
2. $V = (L + W)(2L + W)$	for	$L = 7.5$ ft, $W = 5.0$ ft
3. $I = PRT$	for	$P = 150, R = 0.05, T = 2$
4. $H = 2(a^2 + b^2)$	for	$a = 2$ cm, $b = 1$ cm
5. $T = \dfrac{(A + B)H}{2}$	for	$A = 3.26$ m, $B = 7.15$ m, $H = 4.4$ m
6. $V = \dfrac{\pi D^2 H}{4}$	for	$\pi = 3.14, D = 6.25$ in., $H = 7.2$ in.
7. $P = \dfrac{NR(T + 273)}{V}$	for	$N = 5, R = 0.08, T = 27, V = 3$
8. $W = D(AB - \pi R^2)H$	for	$D = 9$ lb/in.^{3}, $A = 6.3$ in., $B = 2.7$ in., $\pi = 3.14, R = 2$ in., $H = 1.0$ in.
9. $V = LWH$	for	$L = 16.25$ m, $W = 3.1$ m, $H = 2.4$ m
10. $V = \pi R^2 A$	for	$\pi = 3.14, R = 3.2$ mm, $A = 0.425$ mm

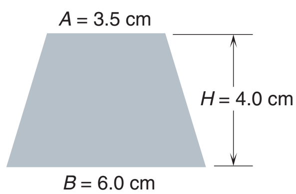
C. Practical Problems

1. The perimeter of a rectangle is given by the formula $P = 2L + 2W$, where L is the length of the rectangle and W is its width. Find P when L is $8\frac{1}{2}$ in. and W is 11 in.

2. **Electronics** The current in a simple electrical circuit is given by the formula $i = V/R$, where V is the voltage and R is the resistance of the circuit. Find the current in a circuit whose resistance is 10 ohms and which is connected across a 120 V power source.

3. **Electrical Technology** Find the power used in an electric light bulb, $P = i^2R$, if the current $i = 0.80$ A and the resistance $R = 150$ ohms. P will be in watts.

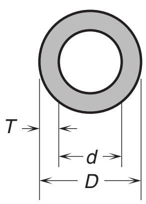
4. Find the surface area of a sphere, $A = 4\pi R^2$, when π equals roughly 3.14 and $R = 10.0$ cm. (Round to the nearest 10 cm^2.)



Problem 9

5. The Fahrenheit temperature F is related to the Celsius temperature C by the formula $F = \frac{9}{5}C + 32$. Find the Fahrenheit temperature when $C = 40°$.

6. **Machine Technology** The volume of a round steel bar depends on its length L and diameter D according to the formula $V = \pi D^2L/4$. Find the volume of a bar 20.0 cm long and 3.0 cm in diameter. Use $\pi \approx 3.14$. (Round to the nearest 10 cm^3.)



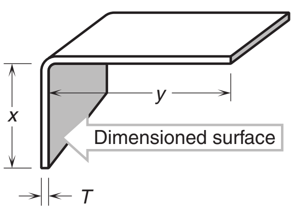
Problem 10

7. If D dollars is invested at p percent interest for t years, the amount A of the investment is

$$A = D\left(1 + \frac{pt}{100}\right)$$

Find A if $D = \$1000$, $p = 9\%$, and $t = 5$ years.

8. **Electronics** The total resistance R of two resistances a and b connected in parallel is $R = ab/(a + b)$. What is the total resistance if $a = 200$ ohms and $b = 300$ ohms?

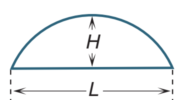


Problem 11

9. The area of a trapezoid is $T = \dfrac{(A + B)H}{2}$, where A and B are the lengths of its parallel sides and H is the height. Find the area of the trapezoid shown.

10. **Plumbing** Use the formula $T = \frac{1}{2}(D - d)$ to find the wall thickness T of tubing having the following dimensions.

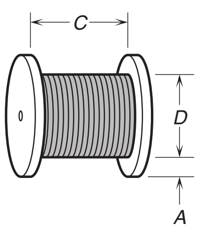
	D, outside diameter	d, inside diameter
(a)	2.125 cm	1.500 cm^3
(b)	0.785 in.	0.548 in.
(c)	1.400 cm	0.875 cm
(d)	$\frac{15}{16}$ in.	$\frac{5}{8}$ in.



Problem 12

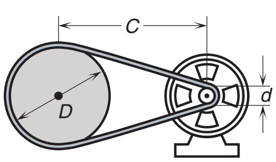
11. **Sheet-Metal Technology** To make a right-angle inside bend in sheet metal, the length of sheet used is given by the formula $L = x + y + \frac{1}{2}T$. Find L when $x = 6\frac{1}{4}$ cm, $y = 11\frac{7}{8}$ cm, and $T = \frac{1}{4}$ cm.

12. **Sheet-Metal Technology** The length of the chord of a circle can be found by using the formula $L = 2\sqrt{r^2 - (r - H)^2}$, where r is the radius and H is the height above the chord. If the portion of the circular steel disc shown has $r = 108$ mm and $H = 60$ mm, find the chord length.



Problem 13

13. **Building Technology** The rope capacity of a drum is given by the formula $L = ABC(A + D)$. How many feet of $\frac{1}{2}$ in. rope can be wound on a drum where $A = 6$ in., $C = 30$ in., $D = 24$ in., and $B = 1.05$ for $\frac{1}{2}$ in. rope? (Round to the nearest hundred inches.)



Problem 14

14. **Manufacturing** A millwright uses the following formula to find the required length of a pulley belt. (See the figure.)

$$L = 2C + 1.57(D + d) + \frac{(D + d)}{4C}.$$

Find the length of belt needed if

C = 900 mm between pulley centres
D = 600 mm follower
d = 406 mm driver

Round to the nearest 10 mm.

15. **Electrical Technology** An electrician uses a bridge circuit to locate a ground in an underground cable several kilometres long. The formula

$$\frac{R_1}{L - x} = \frac{R_2}{x} \qquad \text{or} \qquad x = \frac{R_2 L}{R_1 + R_2}$$

is used to find x, the distance to the ground. Find x if $R_1 = 750$ ohms, $R_2 = 250$ ohms, and $L = 1220$ m.

16. **Machine Technology** The cutting speed of a lathe is the rate, in feet per minute, that the revolving workpiece travels past the cutting edge of the tool. Machinists use the following formula to calculate cutting speed:

$$\text{Cutting speed, } C = \frac{3.1416DN}{12}$$

where D is the diameter of the work and N is the turning rate in rpm. Find the cutting speed if a steel shaft 3.25 in. in diameter is turned at 210 rpm. (Round to the nearest whole number.)

17. **Electronics** Find the power load P in kilowatts of an electrical circuit that takes a current I of 12 A at a voltage of 220 V if

$$P = \frac{VI}{1000}$$

18. **Aeronautical Mechanics** A jet engine developing T pounds of thrust and driving an airplane at V mph has a thrust horsepower, H, given approximately by the formula

$$H = \frac{TV}{375} \qquad \text{or} \qquad V = \frac{375H}{T}$$

Find the airspeed V if H = 16,000 hp and T = 10,000 lb.

19. **Electronics** The resistance R of a conductor is given by the formula

$$R = \frac{pL}{A} \qquad \text{or} \qquad L = \frac{AR}{p}$$

where p = coefficient of resistivity
L = length of conductor
A = cross-sectional area of the conductor

Find the length in cm of No. 16 Nichrome wire needed to obtain a resistance of 8 ohms. For this wire $p = 0.000113$ ohm-cm and $A = 0.013$ cm^2. (Round to the nearest centimetre.)

20. **Auto Mechanics** The pressure P and total force F exerted on a piston of diameter D are approximately related by the equation

$$P = \frac{1.27F}{D^2} \qquad \text{or} \qquad F = \frac{PD^2}{1.27}$$

Find the total force on a piston of diameter 3.25 in. if the pressure exerted on it is 150 lb/sq in. (Round to the nearest 50 lb.)

21. **Printing** The formula for the number of type lines L set solid in a form is

$$L = \frac{12D}{T}$$

where D is the depth in picas and T is the point size of the type. How many lines of 10-point type can be set in a space 30 picas deep?

22. **Plumbing** To determine the number N of smaller pipes that provide the same total flow as a larger pipe, use the formula

$$N = \frac{D^2}{d^2}$$

where D is the diameter of the larger pipe and d is the diameter of each smaller pipe. How many 50 mm pipes will it take to produce the same flow as a 100 mm pipe?

23. Find the volume of a sphere of radius $R = 12.7$ mm:

$$\text{Volume } V = \frac{4\pi R^3}{3} \qquad \text{Use 3.1416 for } \pi.$$

(Round to 1 mm^3.)

24. **Machine Technology** How many minutes will it take a lathe to make 17 cuts each 622.3 mm in length on a steel shaft if the tool feed F is 1.65 mm per revolution and the shaft turns at 163 rpm? Use the formula

$$T = \frac{LN}{FR}$$

where T is the cutting time (min), N the number of cuts, L the length of cut (mm), F the tool feed rate (mm/rev), and R the rpm rate of the workpiece. Round to 0.1 min.

25. **Machine Technology** Find the area of each of the following circular holes using the formula

$$A = \frac{\pi D^2}{4}$$

(a) $D = 25.4$ mm (b) $D = \frac{7}{8}$ in. (c) $D = 4.1275$ in. (d) $D = 2.0605$ cm

Use 3.1416 for π and round to 0.0001 unit.

26. Suppose that a steel band was placed tightly around the earth at the equator. If the temperature of the steel is raised 1°F, the metal will expand 0.000006 in. each inch. How much space (D) would there be between the earth and the steel band if the temperature was raised 1°F? Use the formula

$$D \text{ (in ft)} = \frac{(0.000006)(\text{diameter of the earth})(5280)}{\pi}$$

where diameter of the earth = 7917 miles. Use $\pi \approx 3.1415927$.

(Round to two decimal places.)

27. **Plumbing** When a cylindrical container is lying on its side, the following formula can be used for calculating the volume of liquid in the container:

$$V = \frac{4}{3}h^2L\sqrt{\frac{d}{h} - 0.608}$$

Use this formula to calculate the volume of water in such a tank 6.0 m long (L), 2.0 m in diameter (d), and filled to a height (h) of 0.75 m. Round to two significant digits.

28. **Meteorology** The following formula is used to calculate the wind chill factor W in degrees Celsius:

$$W = 33 - \frac{(10.45 + 10\sqrt{V} - V)(33 - T)}{22.04}$$

where T is the air temperature in degrees Celsius, and V is the wind speed in metres per second. Determine the wind chill for the following situations:

(a) Air temperature 7°C and wind speed 20 m/s (metres per second).

(b) Air temperature 28°F and wind speed 22 mph. (*Hint:* Use 1mile = approx 1.6 km, the conversion factors from Section 5-3 and the temperature formulas given earlier in this chapter to convert to the required units. Convert your final answer to degrees Fahrenheit.)

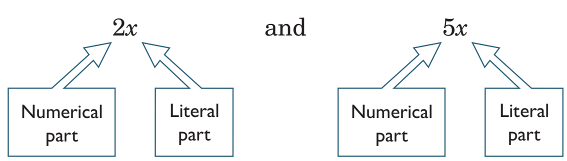
When you have completed these exercises, check your answers, then turn to Section 7-2 to learn how to add and subtract algebraic expressions.

7-2 ADDING AND SUBTRACTING ALGEBRAIC EXPRESSIONS

In the preceding section we learned how to find the value of an algebraic expression after substituting numbers for the letters. There are other useful ways of using algebra where we must manipulate algebraic expressions *without* first substituting numbers for letters. In this section we learn how to simplify algebraic expressions by adding and subtracting terms.

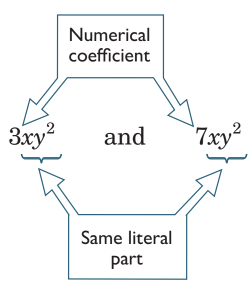
Combining Like Terms

Two algebraic terms are said to be *like terms* if they contain exactly the same literal part. For example, the terms



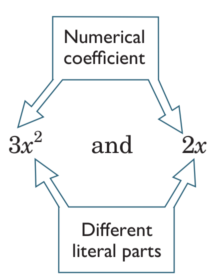
are like terms. The literal part, x, is the same for both terms.

The number multiplying the letters is called the *numerical coefficient* of the term.



are like terms.

But



are unlike terms. Their literal parts, x and x^2, are different.

You can add and subtract like terms but not unlike terms. To add or subtract like terms, add or subtract their numerical coefficients and keep the same literal part.

EXAMPLE

(a)

$2 + 3 = 5$

$2x + 3x = 5x$ We are adding like quantities

$\boxed{x} + \boxed{x} \quad + \boxed{x} + \boxed{x} + \boxed{x} = \boxed{x} + \boxed{x} + \boxed{x} + \boxed{x} + \boxed{x}$

Same literal part

(b)

$8 - 3 = 5$

$8\underbrace{a^2x} - 3\underbrace{a^2x} = 5\underbrace{a^2x}$

Same literal part

YOUR TURN

Try these problems for practice.

(a) $12d^2 + 7d^2 =$ ________ (b) $2ax - ax - 5ax =$ ________

(c) $3(y + 1) + 9(y + 1) =$ ________ (d) $8x^2 + 2xy - 2x^2 =$ ________

(e) $x - 6x + 2x =$ ________ (f) $4xy - xy + 3xy =$ ________

ANSWERS

(a) $12d^2 + 7d^2 = 19d^2$

(b) $2ax - ax - 5ax = -4ax$ The term ax is equal to $1 \cdot ax$.

(c) $3(y + 1) + 9(y + 1) = 12(y + 1)$

(d) $8x^2 + 2xy - 2x^2 = 6x^2 + 2xy$

We cannot combine the x^2-term and the xy-term because the literal parts are not the same. They are unlike terms.

(e) $x - 6x + 2x = -3x$

(f) $4xy - xy + 3xy = 6xy$

In general, to simplify a series of terms being added or subtracted, first group together like terms, then add or subtract.

EXAMPLE

$3x + 4y - x + 2y + 2x - 8y$

Be careful not to lose the negative sign on $8y$.

becomes

$(3x - x + 2x) + (4y + 2y - 8y)$ after grouping like terms,

Be careful not to lose the negative sign on x.

$(3x - x + 2x) + (4y + 2y - 8y) = 4x + (-2y)$ or $4x - 2y$

Adding a negative is the same as subtracting a positive.

It is simpler to write the final answer as a subtraction rather than as an addition.

YOUR TURN

Simplify the following expressions by adding and subtracting like terms.

(a) $5x + 4xy - 2x - 3xy$

(b) $3ab^2 + a^2b - ab^2 + 3a^2b - a^2b$

(c) $x + 2y - 3z - 2x - y + 5z - x + 2y - z$

(d) $17pq - 9ps - 6pq + ps - 6ps - pq$

(e) $4x^2 - x^2 + 2x + 2x^2 + x$

SOLUTIONS

(a) $(5x - 2x) + (4xy - 3xy) = 3x + xy$

(b) $(3ab^2 - ab^2) + (a^2b + 3a^2b - a^2b) = 2ab^2 + 3a^2b$

(c) $(x - 2x - x) + (2y - y + 2y) + (-3z + 5z - z) = -2x + 3y + z$

(d) $(17pq - 6pq - pq) + (-9ps + ps - 6ps) = 10pq - 14ps$

(e) $(4x^2 - x^2 + 2x^2) + (2x + x) = 5x^2 + 3x$

Expressions with Brackets

Brackets are used in algebra to group together terms that are to be treated as a unit. Adding and subtracting expressions usually involves working with brackets.

EXAMPLE

To add

$(a + b) + (a + d)$

First, remove all brackets: $(a + b) + (a + d)$

$$= a + b + a + d$$

Second, add like terms: $= \underbrace{a + a} + b + d$

$$= 2a + b + d$$

Removing brackets can be a tricky business. Remember these two rules:

Rule 1 If the bracket has a plus sign in front, simply remove the brackets.

EXAMPLE

(a) $1 + (3x + y) = 1 + 3x + y$

Same sign

(b) $2 + (a - 2b) = 2 + a - 2b$

Same sign

Rule 2 If the bracket has a negative sign in front, change the sign of each term inside, then remove the brackets.

EXAMPLE

Change sign from + to −

(a) $2 - (x + 2y) = 2 - x - 2y$

Change sign from + to −

Change sign

(b) $a - (2x - y) = a - 2x + y$

Change sign

Change sign

(c) $5 - (-2a + b) = 5 + 2a - b$

Change sign

NOTE ▶ In using Rule 2 you are simply rewriting subtraction as addition of the opposite. However, you must add the opposite of *all* terms inside the brackets. ◀

PROBLEMS

Simplify the following expressions by using these two rules to remove brackets.

(a) $x + (2y - a^2)$ (b) $4 - (x^2 - y^2)$

(c) $-(x + 1) + (y + a)$ (d) $ab - (a - b)$

(e) $(x + y) - (p - q)$ (f) $-(-x - 2y) - (a + 2b)$

(g) $3 + (-2p - q^2)$ (h) $-(x - y) + (-3x^2 + y^2)$

ANSWERS

(a) $x + 2y - a^2$ (b) $4 - x^2 + y^2$

(c) $-x - 1 + y + a$ (d) $ab - a + b$

(e) $x + y - p + q$ (f) $x + 2y - a - 2b$

(g) $3 - 2p - q^2$ (h) $-x + y - 3x^2 + y^2$

A third rule is needed when a multiplier is in front of the brackets.

Rule 3 If the bracket has a multiplier in front, multiply each term inside the brackets by the multiplier.

EXAMPLE

(a) $+2(a + b) = +2a + 2b$

Think of this as $(+2)(a + b) = (+2)a + (+2)b$
$= +2a + 2b$

Each term inside the brackets is multiplied by $+2$.

(b) $-2(x + y) = -2x - 2y$

Think of this as $(-2)(x + y) = (-2)x + (-2)y$

Each term inside the brackets is multiplied by -2.

(c) $-(x - y) = (-1)(x - y)$
$= (-1)(x) + (-1)(-y) = -x + y$

(d) $2 - 5(3a - 2b) = 2 + (-5)(3a - 2b) = 2 + (-5)(3a) + (-5)(-2b)$
$= 2 + (-15a) + (+10b)$ or $2 - 15a + 10b$

CAREFUL ▶ In the last example it would be incorrect first to subtract the 5 from the 2. The order of operations requires that we multiply before we subtract. ◀

When you multiply negative numbers, you may need to review the arithmetic of negative numbers in Chapter 6.

Notice that we must multiply *every* term inside the brackets by the number outside the brackets. Once the brackets have been removed, you can add and subtract like terms, as explained in operation 1.

YOUR TURN

Simplify the following expressions by removing brackets. Use the three rules.

(a) $2(2x - 3y)$ (b) $1 - 4(x + 2y)$ (c) $a - 2(b - 2x)$

(d) $x^2 - 3(x - y)$ (e) $p - 2(-y - 2x)$ (f) $3(x - y) - 2(2x^2 + 3y^2)$

ANSWERS

(a) $4x - 6y$ (b) $1 - 4x - 8y$ (c) $a - 2b + 4x$

(d) $x^2 - 3x + 3y$ (e) $p + 2y + 4x$ (f) $3x - 3y - 4x^2 - 6y^2$

Once you have simplified expressions by removing brackets, it is easy to add and subtract like terms.

EXAMPLE

$(3x - y) - 2(x - 2y) = 3x - y - 2x + 4y$ Simplify by removing brackets.

$= \underbrace{3x - 2x} - \underbrace{y + 4y}$ Group like terms.

$= x + 3y$ Combine like terms.

PROBLEMS

Try these problems for practice.

(a) $(3y + 2) + 2(y + 1)$ (b) $(2x + 1) + 3(4 - x)$

(c) $(a + b) - (a - b)$ (d) $2(a + b) - 2(a - b)$

(e) $2(x - y) - 3(y - x)$ (f) $2(x^3 + 1) - 3(x^3 - 2)$

(g) $(x^2 - 2x) - 2(x - 2x^2)$ (h) $-2(3x - 5) - 4(x - 1)$

ANSWERS

(a) $5y + 4$ (b) $-x + 13$ (c) $2b$ (d) $4b$

(e) $5x - 5y$ (f) $-x^3 + 8$ (g) $5x^2 - 4x$ (h) $-10x + 14$

Some step-by-step solutions:

(a) $(3y + 2) + 2(y + 1) = 3y + 2 + 2y + 2$
$= 3y + 2y + 2 + 2$
$= 5y + 4$

(b) $(2x + 1) + 3(4 - x) = 2x + 1 + 12 - 3x$
$= 2x - 3x + 1 + 12$
$= -x + 13$

(c) $(a + b) - (a - b) = a + b - a - (-b)$
$= a + b - a + b$
$= a - a + b + b = 0 + 2b = 2b$

$$\begin{aligned}\text{(h)}\quad -2(3x-5)-4(x-1) &= -6x-2(-5)-4x-4(-1)\\ &= -6x+10-4x+4\\ &= -6x-4x+10+4=-10x+14\end{aligned}$$

Now turn to Exercises 7-2 for additional practice on addition and subtraction of algebraic expressions.

Exercises 7-2

Adding and Subtracting Algebraic Expressions

A. Simplify by adding or subtracting like terms.

1. $3y + y + 5y$
2. $4x^2y + 5x^2y$
3. $E + 2E + 3E$
4. $ax - 5ax$
5. $9B - 2B$
6. $3m - 3m$
7. $3x^2 - 5x^2$
8. $4x + 7y + 6x + 9y$
9. $6R + 2R^2 - R$
10. $1.4A + 0.05A - 0.8A^2$
11. $x - \frac{1}{2}x - \frac{1}{4}x - \frac{1}{8}x$
12. $x + 2\frac{1}{2}x - 5\frac{1}{2}x$
13. $2 + W - 4.1W - \frac{1}{2}$
14. $q - p - 1\frac{1}{2}p$
15. $2xy + 3x + 4xy$
16. $ab + 5ab - 2ab$
17. $x^2 + x^2y + 4x^2 + 3x$
18. $1.5p + 0.3pq + 3.1p$

B. Simplify by removing brackets and, if possible, combining like terms.

1. $3x^2 + (2x - 5)$
2. $6 + (-3a + 8b)$
3. $8m + (4m^2 + 2m)$
4. $9x + (2x - 5x^2)$
5. $2 - (x + 5y)$
6. $7x - (4 + 2y)$
7. $3a - (8 - 6b)$
8. $5 - (w - 6z)$
9. $4x - (10x^2 + 7x)$
10. $12m - (6n + 4m)$
11. $15 - (3x - 8)$
12. $-12 - (5y - 9)$
13. $-(x - 2y) + (2x + 6y)$
14. $-(3 + 5m) + (11 - 4m)$
15. $-(14 + 5w) - (2w - 3z)$
16. $-(16x - 8) - (-2x + 4y)$
17. $3(3x - 4y)$
18. $4(5a + 6b)$
19. $-8(7m + 6)$
20. $-2(6x - 3)$
21. $x - 5(3 + 2x)$
22. $4y - 2(8 + 3y)$
23. $9m - 7(-2m + 6)$
24. $w - 5(4w - 3)$
25. $3 - 4(2x + 3y)$
26. $6 - 2(3a - 5b)$
27. $12 - 2(3w - 8)$
28. $2 - 11(7 + 5a)$
29. $2(3x + 4y) - 4(6x^2 - 5y^2)$
30. $-4(5a - 6b) + 7(2ab - 4b^2)$
31. $(22x - 14y) + 3(8y - 6x)$
32. $8(3w - 5z) + (9z - 6w)$
33. $6(x + y) - 3(x - y)$
34. $-5(2x - 3x^2) + 2(9x^2 + 8x)$
35. $4(x^2 - 6x + 8) - 6(x^2 + 3x - 5)$
36. $2(3a - 5b + 6ab) - 8(2a + b - 4ab)$

When you have completed these exercises, check your answers, then turn to Section 7-3 to learn how to solve algebraic equations.

7-3 SOLVING SIMPLE EQUATIONS

An arithmetic equation such as $3 + 2 = 5$ means that the number named on the left $(3 + 2)$ is the same as the number named on the right (5).

An algebraic equation such as $x + 3 = 7$ is a statement that the sum of some number x and 3 is equal to 7. If we choose the correct value for x, the number $x + 3$ will be equal to 7.

x is a *variable,* a symbol that stands for a number in an equation, a blank space to be filled. Many numbers might be put in the space, but only one makes the equation a true statement.

YOUR TURN

Find the missing numbers in the following arithmetic equations.

(a) $37 + \_\_\_\_\_\_ = 58$ (b) $\_\_\_\_\_\_ - 15 = 29$

(c) $4 \times \_\_\_\_\_\_ = 52$ (d) $28 \div \_\_\_\_\_\_ = 4$

ANSWERS

(a) $37 + 21 = 58$ (b) $44 - 15 = 29$

(c) $4 \times 13 = 52$ (d) $28 \div 7 = 4$

We could have written these equations as follows, with variables instead of blanks.

$$37 + A = 58 \qquad B - 15 = 29 \qquad 4C = 52 \qquad \frac{28}{D} = 4$$

Of course, any letters would do in place of *A, B, C,* and *D* in these algebraic equations.

How did you solve these equations? You probably "eyeballed" them—mentally juggled the other information in the equation until you found a number that made the equation true. Solving algebraic equations is very similar except that we can't "eyeball" them entirely. We need certain and systematic ways of solving the equation that will produce the correct answer quickly every time.

In this section you will learn first what a solution to an algebraic equation is—how to recognize it if you stumble over it in the dark—then how to solve linear equations.

Solution

Each value of the variable that makes an equation true is called a *solution* of the equation.

EXAMPLE

(a) The solution of $x + 3 = 7$ is $x = 4$.

☑ $(4) + 3 = 7$

(b) The solution of the equation $2x - 9 = 18 - 7x$ is $x = 3$.

☑ $2(3) - 9 = 18 - 7(3)$
$6 - 9 = 18 - 21$
$-3 = -3$

For certain equations more than one value of the variable may make the equation true.

EXAMPLE

The equation $x^2 + 6 = 5x$ is true for $x = 2$,

$$\begin{aligned}(2)^2 + 6 &= 5(2)\\ 4 + 6 &= 5 \cdot 2\\ 10 &= 10\end{aligned}$$

and it is also true for $x = 3$.

$$\begin{aligned}(3)^2 + 6 &= 5(3)\\ 9 + 6 &= 5 \cdot 3\\ 15 &= 15\end{aligned}$$

PROBLEMS

Determine whether each given value of the variable is a solution to the equation.

(a) For $4 - x = -3$ is $x = -7$ a solution?

(b) For $3x - 8 = 22$ is $x = 10$ a solution?

(c) For $-2x + 15 = 5 + 3x$ is $x = -2$ a solution?

(d) For $3(2x - 8) = 5 - (6 - 4x)$ is $x = 11.5$ a solution?

(e) For $3x^2 - 5x = -2$ is $x = \frac{2}{3}$ a solution?

ANSWERS

(a) No—the left side equals 11.

(b) Yes. (c) No—the left side equals 19, the right side equals -1.

(d) Yes. (e) Yes.

Here is the check for (e): $3\left(\frac{2}{3}\right)^2 - 5\left(\frac{2}{3}\right) = -2$

$$\begin{aligned}3\left(\frac{4}{9}\right) - 5\left(\frac{2}{3}\right) &= -2\\ \frac{4}{3} - \frac{10}{3} &= -2\\ -\frac{6}{3} &= -2\\ -2 &= -2\end{aligned}$$

Equivalent Equations

Equations with the exact same solution are called *equivalent* equations. The equations $2x + 7 = 13$ and $3x = 9$ are equivalent because substituting the value 3 for x makes them both true.

We say that an equation with the variable x is *solved* if it can be put in the form

$x = \square$ where $\square$ is some number

For example, the solution to the equation

$2x - 1 = 7$ is $x = 4$

because $2(4) - 1 = 7$

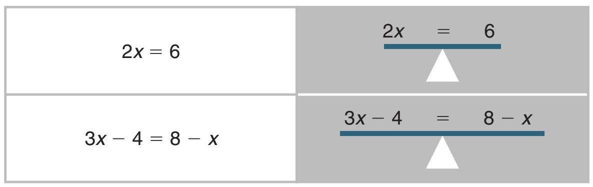
or $8 - 1 = 7$

is a true statement.

Solving Equations

Equations as simple as the previous one are easy to solve by guessing, but guessing is not a very dependable way to do mathematics. We need some sort of rule that will enable us to rewrite the equation to be solved ($2x - 1 = 7$, for example) as an equivalent solution equation ($x = 4$).

The general rule is to treat every equation as a balance of the two sides.



Any changes made in the equation must not disturb this balance.

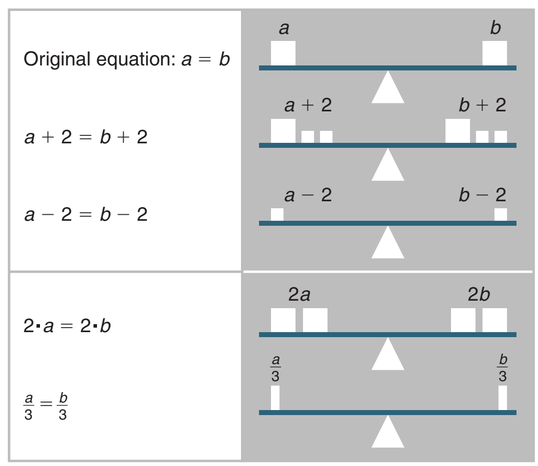
Any operation performed on one side of the equation must also be performed on the other side.

Two kinds of balancing operations may be used.

1. Adding or subtracting a number on both sides of the equation does not change the balance

and

2. Multiplying or dividing both sides of the equation by a number (but not zero) does not change the balance.



EXAMPLE

Let's work through an example.

Solve: $x - 4 = 2$.

Step 1 We want to change this equation to an equivalent equation with only x on the left, so we add 4 to each side of the equation.

$$x - 4 + 4 = 2 + 4$$

Step 2 Combine terms.

$$x \underbrace{- 4 + 4}_{0} = 2 + 4$$

$x = 6$ Solution

$(6) - 4 = 2$
$2 = 2$

YOUR TURN

Use these balancing operations to solve the equation

$8 + x = 14$

SOLUTION

Solve: $8 + x = 14$

Step 1 We want to change this equation to an equivalent equation with only x on the left, so we subtract 8 from each side of the equation.

$$8 + x - 8 = 14 - 8$$

Step 2 Combine terms.

$$x + \underbrace{8 - 8}_{0} = 14 - 8 \qquad \text{where } 8 + x = x + 8$$

$$x = 6 \quad \text{Solution}$$

✓ $8 + (6) = 14$
$14 = 14$

PROBLEMS

Solve these in the same way.

(a) $x - 7 = 10$ (b) $12 + x = 27$ (c) $x + 6 = 2$

(d) $8.4 = 3.1 + x$ (e) $6.7 + x = 0$ (f) $\frac{1}{4} = x - \frac{1}{2}$

(g) $-11 = x + 5$ (h) $x - 5.2 = -3.7$

SOLUTIONS

(a) **Solve:** $x - 7 = 10$

$$x - 7 + 7 = 10 + 7$$ Add 7 to each side.

$$x \underbrace{- 7 + 7}_{0} = 10 + 7$$ Combine terms.

$$x = 17 \quad \text{Solution}$$

✓ $(17) - 7 = 10$
$10 = 10$

(b) **Solve:** $12 + x = 27$

$$12 + x - 12 = 27 - 12$$ Subtract 12 from each side.

$$x + \underbrace{12 - 12}_{0} = 27 - 12$$ Combine terms. (Note that $12 + x = x + 12$.)

$$x = 15 \quad \text{Solution}$$

✓ $12 + (15) = 27$
$27 = 27$

(c) **Solve:** $x + 6 = 2$

$$x + 6 - 6 = 2 - 6$$ Subtract 6 from each side.

$$x \underbrace{+ 6 - 6}_{0} = 2 - 6$$ Combine terms.

$$x = -4$$ Solution

The solution is a negative number. Remember, any number, positive or negative, may be the solution of an equation.

$$(-4) + 6 = 2$$
$$2 = 2$$

(d) **Solve:** $8.4 = 3.1 + x$

The variable x is on the right side of this equation.

$$8.4 - 3.1 = 3.1 + x - 3.1$$ Subtract 3.1 from each side.

$$8.4 - 3.1 = x$$ Combine terms.

$$5.3 = x$$ Decimal numbers often appear in practical problems.

or $x = 5.3$ Solution

$5.3 = x$ is the same as $x = 5.3$.

$$8.4 = 3.1 + (5.3)$$
$$8.4 = 8.4$$

(e) **Solve:** $6.7 + x = 0$

$$6.7 + x - 6.7 = 0 - 6.7$$ Subtract 6.7 from each side.

$$x = -6.7$$ Solution

A negative-number answer is reasonable.

$$6.7 + (-6.7) = 0$$
$$6.7 - 6.7 = 0$$

(f) **Solve:** $\frac{1}{4} = x - \frac{1}{2}$

$$\underbrace{\frac{1}{4} + \frac{1}{2}}_{} = x \underbrace{- \frac{1}{2} + \frac{1}{2}}_{0}$$ Add $\frac{1}{2}$ to each side.

$$\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

$$\frac{3}{4} = x \quad \text{or} \quad x = \frac{3}{4}$$ Solution

$$\frac{1}{4} = \left(\frac{3}{4}\right) - \frac{1}{2}$$
$$\frac{1}{4} = \frac{1}{4}$$

(g) **Solve:** $-11 = x + 5$

$$-11 - 5 = x + 5 - 5$$ Subtract 5 from each side.

$$-16 = x \quad \text{or} \quad x = -16$$ Solution

$$-11 = (-16) + 5$$
$$-11 = -11$$

(h) **Solve:** $x - 5.2 = -3.7$

$$x - 5.2 + 5.2 = -3.7 + 5.2 \qquad \text{Add } 5.2 \text{ to each side.}$$

$$x = 1.5 \quad \text{Solution}$$

$(1.5) - 5.2 = -3.7$
$-3.7 = -3.7$

LEARNING HINT ▶ Notice that to solve these equations, we always do the opposite of whatever operation is being performed on the variable. If a number is being added to x in the equation, we must subtract that number in order to solve. If a number is being subtracted from x in the equation, we must add that number in order to solve. Finally, whatever we do to one side of the equation, we must also do to the other side of the equation. ◀

In the equations solved so far, the variable has appeared as simply x. In some equations the variable may be multiplied or divided by a number, so that it appears as $2x$ or $x/3$, for example. To solve such an equation, do the operation opposite of the one being performed on the variable. If the variable appears as $2x$, divide by 2; if the variable appears as $x/3$, multiply by 3, and so on.

EXAMPLE

To solve the equation $27 = -3x$ follow these steps:

$27 = -3x$ Notice that the x-term is on the right side of the equation. It is x multiplied by -3. To change $-3x$ into $1x$ or x, we must perform the opposite operation, that is, divide by -3.

Step 1 $\dfrac{27}{-3} = \dfrac{-3x}{-3}$ Divide each side by -3.

Step 2 $-9 = \left(\dfrac{-3}{-3}\right)x$

$-9 = x$ or $x = -9$ Solution

$27 = -3(-9)$
$27 = 27$

EXAMPLE

Here is a slightly different problem to solve.

$\dfrac{x}{3} = 13$ In this equation the variable x is divided by 3.

Step 1 $\dfrac{x}{3} \cdot 3 = 13 \cdot 3$ We want to change this equation to an equivalent equation with x alone on the left, so performing the opposite operation, we multiply both sides by 3.

$$\frac{3x}{3} = \left(\frac{3}{3}\right)x = x$$

Step 2 $x = 13 \cdot 3$
$x = 39$ Solution

$\dfrac{(39)}{3} = 13$
$13 = 13$

EXAMPLE

To solve $-\frac{x}{4} = 5$

Step 1 $\left(-\frac{x}{4}\right)(-4) = 5(-4)$ Multiply both sides by -4.

Step 2 $\frac{(-x)(-4)}{4} = -20$ $\qquad -\frac{x}{4} = \frac{-x}{4}$

$\frac{4x}{4} = -20$ $\qquad (-x)(-4) = +4x$

$x = -20$ $\qquad \frac{4x}{4} = \left(\frac{4}{4}\right)x = 1x = x$

$$-\frac{(-20)}{4} = 5$$
$$-(-5) = 5$$
$$5 = 5$$

NOTE ▶ When a negative sign precedes a fraction, we may move it either to the numerator or to the denominator, but not to both. In the previous problem, we placed the negative sign in the numerator in Step 2. If we had moved it to the denominator instead, we would have the following:

$$\left(\frac{x}{-4}\right)(-4) = -20 \qquad \text{or} \qquad \frac{-4x}{-4} = -20 \qquad \text{or} \qquad x = -20$$

which is exactly the same solution. ◀

YOUR TURN

Try the following problems for more practice with one-step multiplication and division equations.

(a) $-5x = -25$ (b) $\frac{x}{6} = 6$ (c) $-16 = 2x$

(d) $7x = 35$ (e) $-\frac{x}{2} = 14$ (f) $\frac{2x}{3} = 6$

(g) $-4x = 22$ (h) $\frac{x + 3}{2} = 4$ (i) $2.4x = 0.972$

SOLUTIONS

(a) **Solve:** $-5x = -25$ The variable x appears multiplied by -5.

Step 1 $\frac{-5x}{-5} = \frac{-25}{-5}$ Divide both sides by -5.

Step 2 $x = 5$ $\qquad \frac{-5x}{-5} = \left(\frac{-5}{-5}\right)x = 1x = x$

$$-5(5) = -25$$
$$-25 = -25$$

(b) **Solve:** $\dfrac{x}{6} = 6$ — The variable x appears divided by 6.

Step 1 $\left(\dfrac{x}{6}\right)(6) = 6\,(6)$ — Multiply both sides by 6.

Step 2 $x = 36$ — $\left(\dfrac{x}{6}\right)(6) = x\left(\dfrac{6}{6}\right) = x(1) = x$

$$\frac{(36)}{6} = 6$$
$$6 = 6$$

(c) **Solve:** $-16 = 2x$

Step 1 $\dfrac{-16}{2} = \dfrac{2x}{2}$ — Divide each side by 2.

Step 2 $-8 = x$ — $\dfrac{2x}{2} = \left(\dfrac{2}{2}\right)x = 1x = x$

or $x = -8$

$$-16 = 2(-8)$$
$$-16 = -16$$

(d) **Solve:** $7x = 35$

Step 1 $\dfrac{7x}{7} = \dfrac{35}{7}$ — Divide both sides by 7.

Step 2 $x = \dfrac{35}{7}$ — $\dfrac{7x}{7} = \left(\dfrac{7}{7}\right)x = x$

$x = 5$ Solution

$$7 \cdot (5) = 35$$
$$35 = 35$$

(e) **Solve:** $-\dfrac{x}{2} = 14$

Step 1 $\left(-\dfrac{x}{2}\right)\left(-2\right) = (14)\left(-2\right)$ — Multiply both sides by -2.

Step 2 $x\left(-\dfrac{1}{2}\right)(-2) = 14(-2)$ — $-\dfrac{1}{2}x = x\left(-\dfrac{1}{2}\right)$

$\underbrace{x\left(-\dfrac{1}{2}\right)(-2)}_{1} = -28$ — $\left(-\dfrac{1}{2}\right)(-2) = \dfrac{2}{2} = 1$

$x = -28$

$$-\frac{(-28)}{2} = 14$$
$$\frac{28}{2} = 14$$
$$14 = 14$$

(f) **Solve:** $\dfrac{2x}{3} = 6$

Step 1 $\left(\dfrac{2x}{3}\right)3 = (6)\,3$ — Multiply both sides by 3.

$2x = 6 \cdot 3$ — $\left(\dfrac{2x}{3}\right)3 = \dfrac{2 \cdot x \cdot \cancel{3}}{\cancel{3}} = 2 \cdot x$

$2x = 18$

Step 2 $\dfrac{2x}{\boxed{2}} = \dfrac{18}{\boxed{2}}$ Divide both sides by 2. $\dfrac{2x}{\cancel{2}} = x$

$x = 9$ Solution

✓ $\dfrac{2 \cdot (9)}{3} = 6$

$\dfrac{18}{3} = 6$

$6 = 6$

(g) **Solve:** $-4x = 22$

$\dfrac{-4x}{\boxed{-4}} = \dfrac{22}{\boxed{-4}}$ Divide both sides by -4.

$x = -5.5$ Solution

✓ $-4(-5.5) = 22$

$22 = 22$

(h) **Solve:** $\dfrac{x + 3}{2} = 4$ Multiply by 2.

$x + 3 = 8$ $\left(\dfrac{x + 3}{2}\right)2 = \dfrac{(x + 3) \cdot \cancel{2}}{\cancel{2}} = x + 3$

$x + 3 \boxed{- 3} = 8 \boxed{- 3}$ Subtract 3.

$x = 5$ Solution

✓ $\dfrac{(5) + 3}{2} = 4$

$\dfrac{8}{2} = 4$

(i) **Solve:** $2.4x = 0.972$ Divide by 2.4.

$x = 0.405$

.972 ÷ **2.4** = → *0.405*

PROBLEMS

We have now covered the four basic single-operation equations. Before moving on, try the following set of problems covering these four operations. For each problem, think carefully whether you must add, subtract, multiply, or divide in order to make the equation read $x =$ ________ or ________ $= x$.

(a) $14 = x - 8$ (b) $\dfrac{y}{6} = 3$ (c) $n + 11 = 4$

(d) $3x = 33$ (e) $7 = -\dfrac{a}{5}$ (f) $z - 2 = -10$

(g) $-8 = -16x$ (h) $26 = y + 5$ (i) $\dfrac{5x}{4} = -30$

SOLUTIONS

Each solution states the operation you should have performed on both sides and then gives the final answer.

(a) Add 8. $x = 22$. (b) Multiply by 6. $y = 18$.

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(c) Subtract 11. $n = -7$.

(d) Divide by 3. $x = 11$.

(e) Multiply by -5. $a = -35$.

(f) Add 2. $z = -8$.

(g) Divide by -16. $x = \frac{1}{2}$.

(h) Subtract 5. $y = 21$.

(i) Multiply by $\frac{4}{5}$ (or multiply by 4, divide by 5). $x = -24$.

Now turn to Exercises 7-3 for practice on solving simple equations.

Exercises 7-3

Solving Simple Equations

A. Solve the following equations.

1. $x + 4 = 13$
2. $23 = A + 6$
3. $17 - x = 41$
4. $z - 18 = 29$
5. $6 = a - 2\frac{1}{2}$
6. $73 + x = 11$
7. $y - 16.01 = 8.65$
8. $11.6 - R = 3.7$
9. $-39 = 3x$
10. $-9y = 117$
11. $13a = 0.078$
12. $\frac{x}{3} = 7$
13. $\frac{z}{1.3} = 0.5$
14. $\frac{N}{2} = \frac{3}{8}$
15. $m + 18 = 6$
16. $-34 = x-7$
17. $-6y = -39$
18. $\frac{a}{-4} = 9$
19. $22 - T = 40$
20. $79.2 = 2.2y$
21. $-5.9 = -6.6 + Q$
22. $\frac{5}{4} = \frac{Z}{8}$
23. $12 + x = 37$
24. $66 - y = 42$
25. $\frac{K}{0.5} = 8.48$
26. $-12z = 3.6$

B. Practical Problems

1. **Electrician** Ohm's law is often written in the form

$$I = \frac{E}{R}$$

where I is the current in amperes, E is the voltage, and R is the resistance in ohms. What is the voltage necessary to push a 0.80 A current through a resistance of 450 ohms?

2. **Airline Pilot** The formula

$$D = RT$$

is used to calculate the distance D travelled by an object moving at a constant average speed R during an elapsed time T. How long would it take a pilot to fly 2480 km at an average speed of 440 km/h?

3. **Wastewater Treatment Operator** A wastewater treatment operator uses the formula

$$A = 8.34FC$$

to determine the amount A of chlorine in pounds to add to a basin. The flow F through the basin is in millions of gallons per day, and the desired concentration C of chlorine is in parts per million.

If 1800 pounds of chlorine was added to a basin with a flow rate of 7.5 million gallons per day, what would be the resulting concentration?

4. **Sheet-Metal Technology** The water pressure P in pounds per square foot is related to the depth of water D in feet by the formula

$$P = 62.4D$$

where 62.4 is the weight of 1 cu ft of water. If the material forming the bottom of a tank is made to withstand 800 pounds per square foot of pressure, what is the maximum safe height for the tank?

5. **Sound Technology** The frequency f (in waves per second) and the wavelength w (in metres) of a sound travelling in air are related by the equation

$$fw = 343$$

where 343 m/s is the speed of sound in air. Find the wavelength of a musical note with a frequency of 200 waves per second.

6. At a constant temperature, the pressure P (in psi) and the volume V (in cubic feet) of a particular gas are related by the equation

$$PV = 1080$$

If the volume is 60 cu ft, what is the pressure?

7. **Electrical Technology** For a particular transformer, the voltage E in the circuits is related to the number of windings W of wire around the core by the equation

$$E = 40W$$

How many windings will produce a voltage of 840 V?

8. **Sheet-Metal Technology** The surface speed S in m/min (metres per minute) of a rotating cylindrical object is

$$S = \frac{\pi dn}{1000}$$

where d is the diameter of the object in millimetres (mm) and n is the speed of rotation in rpm (revolutions per minute). If a 200 mm grinder must have a surface speed of 1830 m/min, what should the speed of rotation be? Use $\pi = 3.14$, and round to the nearest hundred rpm.

When you have completed these exercises check your answers, then continue in Section 7-4.

7-4 SOLVING EQUATIONS INVOLVING TWO OPERATIONS

Solving many simple algebraic equations involves both kinds of operations: addition/subtraction and multiplication/division.

EXAMPLE

Solve: $2x + 6 = 14$

Step 1 We want to change this equation to an equivalent equation with only x or terms that include x on the left, so subtract 6 from both sides.

$$2x + 6 \boxed{-\ 6} = 14 \boxed{-\ 6}$$

$$2x + \underbrace{6 \boxed{-\ 6}}_{0} = 14 \boxed{-\ 6}$$

Combine terms. (Be careful. Some careless students will add $2x$ and 6 to get $8x$! You may add only *like* terms.)

$$2x = 8$$

Now change this to an equivalent equation with only x on the left.

Step 2 $\dfrac{2x}{\boxed{2}} = \dfrac{8}{\boxed{2}}$ Divide both sides of the equation by 2.

$$x = \frac{8}{2} \qquad \frac{\cancel{2}x}{\cancel{2}} = x$$

$x = 4$ Solution

$$2 \cdot (4) + 6 = 14$$
$$8 + 6 = 14$$
$$14 = 14$$

YOUR TURN

Try this one to test your understanding of the process.

Solve: $3x - 7 = 11$

SOLUTION

Solve: $3x - 7 = 11$

Step 1 $3x - 7 \boxed{+\ 7} = 11 \boxed{+\ 7}$ Add 7 to each side.

$$3x \underbrace{-\ 7 \boxed{+\ 7}}_{0} = 11 \boxed{+\ 7}$$

$$3x = 18$$

Step 2 $\dfrac{\cancel{3}x}{\boxed{\cancel{3}}} = \dfrac{18}{\boxed{3}}$ Divide both sides of the equation by 3.

$x = 6$ Solution

$$3 \cdot (6) - 7 = 11$$
$$18 - 7 = 11$$
$$11 = 11$$

YOUR TURN

Here is another two-operation equation.

Solve: $23 = 9 - \dfrac{y}{3}$

SOLUTION

Solve: $23 = 9 - \dfrac{y}{3}$

Step 1 Notice that the variable y is on the right side of the equation. We must first eliminate the 9 by subtracting 9 from both sides. Be sure to keep the negative sign in front of the $\frac{y}{3}$ term.

$$23 - 9 = 9 - \frac{y}{3} - 9$$

$$\boxed{9 - 9 = 0}$$

$$14 = -\frac{y}{3}$$

Step 2 $(14)(-3) = \left(-\dfrac{y}{3}\right)(-3)$ Multiply both sides by -3.

$$-42 = (-3)\left(-\frac{1}{3}\right)y \qquad -\frac{y}{3} = -\frac{1}{3}y$$

$$-42 = 1y \qquad (-3)\left(-\frac{1}{3}\right) = 1$$

$$\text{or} \quad y = -42$$

☑ $23 = 9 - \dfrac{(-42)}{3}$

$23 = 9 - (-14)$

$23 = 23$

NOTE ▶ In these two-step problems, we did the addition or subtraction in Step 1 and the multiplication or division in Step 2. We could have reversed this order and arrived at the correct solution, but the problem might have become more complicated to solve. Always add or subtract first, and when you multiply or divide, do so to *all* terms. ◀

To solve $\frac{1}{2}x + 4 = 8$ can I multiply by 2 and get $x + 4 = 16$?

No. When you multiply by 2 you must multiply **all** of the expression on the left by 2: $2(\frac{1}{2}x + 4) = 2(8)$ or $x + 8 = 16$ Careful.

If more than one variable term appears on the same side of an equation, combine these like terms before performing any operation to both sides.

EXAMPLE

Solve: $2x + 5 + 4x = 17$

Step 1 $(2x + 4x) + 5 = 17$ Combine the x-terms on the left side.

$6x + 5 = 17$

Step 2 $6x + 5 - 5 = 17 - 5$ Subtract 5 from each side.

$6x = 12$

Step 3 $\dfrac{6x}{6} = \dfrac{12}{6}$ Divide each side by 6.

$x = 2$

☑ $2(2) + 5 + 4(2) = 17$ Substitute 2 for x in each x-term.

$4 + 5 + 8 = 17$

$17 = 17$

YOUR TURN

Now you try one.

Solve: $32 = x - 12 - 5x$

SOLUTION

Step 1 $32 = (x - 5x) - 12$ Combine the x-terms on the right.

$32 = -4x - 12$

Step 2 $32 + 12 = -4x - 12 + 12$ Add 12 to both sides.

$44 = -4x$

Step 3 $\dfrac{44}{-4} = \dfrac{-4x}{-4}$ Divide both sides by -4.

$-11 = x$ or $x = -11$

☑ $32 = (-11) - 12 - 5(-11)$

$32 = -11 - 12 + 55$

$32 = -23 + 55$

$32 = 32$

PROBLEMS

Here are a few more two-operation equations for practice.

(a) $7x + 2 = 51$ (b) $18 - 5x = 3$ (c) $15.3 = 4x - 1.5$

(d) $\dfrac{x}{5} - 4 = 6$ (e) $11 - x = 2$ (f) $5 = 7 - \dfrac{x}{4}$

(g) $2 + \dfrac{x}{2} = 3$ (h) $2x - 9.4 = 0$ (i) $2.75 = 14.25 - 0.20x$

(j) $3x + x = 18$ (k) $12 = 9x + 4 - 5x$ (l) $6 - 3x + x = 22$

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SOLUTIONS

(a) **Solve:** $7x + 2 = 51$

Change this equation to an equivalent equation with only an x-term on the left.

Step 1 $7x + 2 - 2 = 51 - 2$ Subtract 2 from each side.

$7x + \underbrace{2 - 2}_{0} = 51 - 2$ Combine terms.

$$7x = 49$$

Step 2 $\dfrac{\cancel{7}x}{\cancel{7}} = \dfrac{49}{7}$ Divide both sides by 7.

$$x = \frac{49}{7}$$

$x = 7$ Solution

$$7 \cdot (7) + 2 = 51$$
$$49 + 2 = 51$$
$$51 = 51$$

(b) **Solve:** $18 - 5x = 3$

Step 1 $18 - 5x - 18 = 3 - 18$ Subtract 18 from each side.

$-5x + \underbrace{18 - 18}_{0} = 3 - 18$ Rearrange terms.

$$-5x = -15$$

Step 2 $\dfrac{-\cancel{5}x}{-\cancel{5}} = \dfrac{-15}{-5}$ Divide both sides by -5.

$x = 3$ Solution

$$18 - 5 \cdot (3) = 3$$
$$18 - 15 = 3$$
$$3 = 3$$

(c) **Solve:** $15.3 = 4x - 1.5$

Step 1 $15.3 + 1.5 = 4x \underbrace{- 1.5 + 1.5}_{0}$ Add 1.5 to each side.

$16.8 = 4x$ Combine terms.

$$4x = 16.8$$

Step 2 $\dfrac{\cancel{4}x}{\cancel{4}} = \dfrac{16.8}{4}$ Divide both sides by 4.

$x = 4.2$ Solution

Decimal number solutions are common in practical problems.

15.3 $\boxed{+}$ **1.5** $\boxed{=}$ $\boxed{\div}$ **4** $\boxed{=}$ $\rightarrow$ ***4.2***

$$15.3 = 4(4.2) - 1.5$$
$$15.3 = 16.8 - 1.5$$
$$15.3 = 15.3$$

(d) **Solve:** $\frac{x}{5} - 4 = 6$

Step 1 $\frac{x}{5} - 4 + 4 = 6 + 4$ Add 4 to both sides.

$$\frac{x}{5} = 10$$

Step 2 $\left(\frac{x}{5}\right) \cdot 5 = 10 \cdot (5)$ Multiply both sides by 5.

$$x = 50$$

$$\frac{(50)}{5} - 4 = 6$$

$$10 - 4 = 6$$

$$6 = 6$$

(e) **Solve:** $11 - x = 2$

$11 - x - 11 = 2 - 11$ Subtract 11 from each side.

$-x + \underbrace{11 - 11}_{0} = 2 - 11$ Combine terms.

$-x = -9$ Multiply each side by -1.

$x = 9$ Solution

$$11 - (9) = 2$$

$$2 = 2$$

(f) **Solve:** $5 = 7 - \frac{x}{4}$

Step 1 $5 - 7 = 7 - \frac{x}{4} - 7$ Subtract 7 from both sides.

$-2 = -\frac{x}{4}$ $7 - \frac{x}{4} - 7 = (7 - 7) - \frac{x}{4} = -\frac{x}{4}$

Step 2 $-2(-4) = \left(-\frac{x}{4}\right)(-4)$ Multiply both sides by -4.

$8 = x$ $\left(-\frac{x}{4}\right)(-4) = (-x)\left(\frac{-4}{4}\right) = (-x)(-1) = x$

or $x = 8$

$$5 = 7 - \frac{(8)}{4}$$

$$5 = 7 - 2$$

$$5 = 5$$

(g) **Solve:** $2 + \frac{x}{2} = 3$

$\frac{x}{2} + 2 - 2 = 3 - 2$ Subtract 2 from each side.

$$\frac{x}{2} = 1$$

$\left(\frac{x}{2}\right) \cdot 2 = 1 \cdot 2$ Multiply each side by 2.

$x = 2$ Solution

$$2 + \frac{(2)}{2} = 3$$

$$2 + 1 = 3$$

(h) **Solve:** $2x - 9.4 = 0$

$2x - 9.4 + 9.4 = 0 + 9.4$ Add 9.4 to each side.

$2x = 9.4$ Divide each side by 2.

$x = 4.7$ Solution

$$2(4.7) - 9.4 = 0$$
$$9.4 - 9.4 = 0$$

(i) **Solve:** $2.75 = 14.25 - 0.20x$

Step 1 $2.75 - 14.25 = 14.25 - 0.20x - 14.25$ Subtract 14.25 from both sides.

$-11.5 = -0.20x$

Step 2 $\dfrac{-11.5}{-0.20} = \dfrac{-0.20x}{-0.20}$ Divide both sides by -0.20.

$57.5 = x$ or $x = 57.5$

2.75 [−] **14.25** [=] [÷] **.2** [+/−] [=] → **57.5**

$$2.75 = 14.25 - 0.20(57.5)$$
$$2.75 = 14.25 - 11.5$$
$$2.75 = 2.75$$

(j) **Solve:** $3x + x = 18$

Step 1 $4x = 18$ Combine like terms on the left side.

Step 2 $\dfrac{4x}{4} = \dfrac{18}{4}$ Divide each side by 4.

$x = 4.5$

$$3(4.5) + (4.5) = 18$$
$$13.5 + 4.5 = 18$$
$$18 = 18$$

(k) **Solve:** $12 = 9x + 4 - 5x$

Step 1 $12 = 4x + 4$ Combine like terms.

Step 2 $12 - 4 = 4x + 4 - 4$ Subtract 4 from each side.

$8 = 4x$

Step 3 $\dfrac{8}{4} = \dfrac{4x}{4}$ Divide each side by 4.

$2 = x$ or $x = 2$

The check is left to you.

(l) **Solve:** $6 - 3x + x = 22$

Step 1 $6 - 2x = 22$ Combine like terms.

Careful: The $3x$-term is negative: $-3x + x = -2x$

Step 2 $6 - 2x - 6 = 22 - 6$ Subtract 6 from both sides.

$-2x = 16$ (Keep the negative sign with the $2x$-term.)

Step 3 $\dfrac{-2x}{-2} = \dfrac{16}{-2}$ Divide both sides by -2.

$x = -8$

Be sure to check your solution.

Now turn to Exercises 7-4 for more practice on solving equations.

Exercises 7-4

Solving Two-Step Equations

A. Solve.

1. $2x - 3 = 17$	2. $4x + 6 = 2$
3. $\dfrac{x}{5} = 7$	4. $-8y + 12 = 32$
5. $4.4m - 1.2 = 9.8$	6. $\dfrac{3}{4} = 3x + \dfrac{1}{4}$
7. $14 - 7n = -56$	8. $38 = 58 - 4a$
9. $3z - 5z = 12$	10. $17 = 7q - 5q - 3$
11. $23 - \dfrac{x}{4} = 11$	12. $9m + 6 + 3m = -60$
13. $-15 = 12 - 2n + 5n$	14. $2.6y - 19 - 1.8y = 1$
15. $\dfrac{1}{2} + 2x = 1$	16. $3x + 16 = 46$
17. $-4a + 45 = 17$	18. $\dfrac{x}{2} + 1 = 8$
19. $-3Z + \dfrac{1}{2} = 17$	20. $2x + 6 = 0$
21. $1 = 3 - 5x$	22. $23 = 17 - \dfrac{x}{4}$
23. $-5P + 18 = 3$	24. $5x - 2x = 24$
25. $6x + 2x = 80$	26. $x + 12 - 6x = -18$
27. $27 = 2x - 5 + 4x$	28. $-13 = 22 - 3x + 8x$

B. Practical Problems

1. **Office Services** A repair service charges \$32 for a house call and an additional \$24 per hour of work. The formula

   $T = 32 + 24H$

   represents the total charge T for H hours of work. If the total bill for a customer was \$140, how many hours of actual labour were there?

2. **Aircraft Maintenance** The air temperature T (in degrees Fahrenheit) at an altitude h (in feet) above a particular area can be approximated by the formula

   $T = -0.002h + G$

   where G is the temperature on the ground directly below. If the ground temperature is 76°F, at what altitude will the air temperature drop to freezing, 32°F?

3. Physical fitness experts sometimes use the following formula to approximate the maximum target heart rate R during exercise based on a person's age A:

$R = -0.8A + 176$

At what age should the heart rate during exercise not exceed 150 beats per minute?

4. The formula $A = p + prt$ is used to determine the total amount of money A in a bank account after an amount p is invested for t years at a rate of interest r. What rate of interest is needed for \$8000 to grow to \$12,000 after five years? (Be sure to convert your decimal answer to a percent and round to the nearest tenth of a percent.)

5. **Auto Mechanics** The formula

$P + 2T = C$

gives the overall diameter C of the crankshaft gear for a known pitch diameter P of the small gear and the height T of teeth above the pitch diameter circle. If $P = 70$ mm and $C = 80$ mm, find T.

6. **Sheet-Metal Technology** The allowance A for a Pittsburgh lock is given by

$A = 2w + 5$ mm

where w is the width of the pocket. If the allowance for a Pittsburgh lock is 18 mm, what is the width of the pocket?

7. **Plumber** A plumber's total bill A can be calculated using the formula

$A = RT + M$

where R is the hourly rate, T is the total labour time in hours, and M is the cost of materials. A plumber bids a particular job at \$1850. If materials amount to \$580, and the hourly rate is \$40 per hour, how many hours should the job take for the estimate to be accurate?

8. **Machinist** The formula

$L = 2d + 3.26(r + R)$

can be used to approximate the length L of belt needed to connect two pulleys of radii r and R if their centres are a distance d apart. How far apart can two pulleys be if their radii are 200 mm and 150 mm, and the total length of the belt connecting them is 2085 mm? (Round to the nearest mm.)

When you have completed these exercises, check your answers, then continue in Section 7-5.

7-5 SOLVING MORE EQUATIONS AND FORMULAS

Brackets in Equations

In Section 7-2 you learned how to deal with algebraic expressions involving brackets. Now you will learn to solve equations containing brackets by using these same skills.

EXAMPLE

Solve: $2(x + 4) = 27$

Step 1 Use rule 3 on page 344. Multiply each term inside the brackets by 2.

$2x + 8 = 27$

Step 2 Now solve this equation using the techniques of the previous section.

$2x + 8 - 8 = 27 - 8$ Subtract 8 from both sides.

$2x = 19$

$\frac{2x}{2} = \frac{19}{2}$ Divide both sides by 2.

$x = 9.5$

☑ $2[(9.5) + 4] = 27$

$2(13.5) = 27$

$27 = 27$

YOUR TURN

Try this similar example.

$-3(y - 4) = 36$

SOLUTION

Solve: $-3(y - 4) = 36$

Step 1 $-3y + 12 = 36$ Multiply each term inside brackets by -3.

$-3(y - 4) = -3(y) + (-3)(-4)$

Step 2 $-3y + 12 - 12 = 36 - 12$ Subtract 12 from each side.

$-3y = 24$

Step 3 $\frac{-3y}{-3} = \frac{24}{-3}$ Divide each side by -3.

$y = -8$

☑ $-3[(-8) - 4] = 36$

$-3(-12) = 36$

$36 = 36$

NOTE ▶ In each of the last two examples, you could have first divided both sides by the number in front of brackets.

Here is how each solution would have looked.

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First Example	**Second Example**
$\dfrac{2(x+4)}{2} = \dfrac{27}{2}$	$\dfrac{-3(y-4)}{-3} = \dfrac{36}{-3}$
$x + 4 = 13.5$	$y - 4 = -12$
$x + 4 - 4 = 13.5 - 4$	$y - 4 + 4 = -12 + 4$
$x = 9.5$	$y = -8$

Some students may find this technique preferable, especially when the right side of the equation is exactly divisible by the number in front of the brackets. ◀

To solve this equation: $2x - 3 = 10$
Why can't I divide both sides by 2
to get $x - 3 = 5$ so that
$x = 8$? Isn't this ok?

No. When you divide both sides of an equation by a number, you must divide **all** of both sides. Divide **all** of the expression on the left, not just part of it. $\frac{2x-3}{2} = \frac{10}{2}$ or $x - \frac{3}{2} = 5$.

Here is a more difficult equation involving brackets.

EXAMPLE

Solve: $5x - (2x - 3) = 27$

Step 1 Use rule 2 on page 343. Change the sign of each term inside the brackets and then remove them.

$5x - 2x + 3 = 27$ $\qquad -(2x - 3) = -2x + 3$

Step 2 $3x + 3 = 27$ $\qquad$ Combine the like terms on the left. $5x - 2x = 3x$

Step 3 $3x + 3 - 3 = 27 - 3$ $\qquad$ Subtract 3 from both sides.

$3x = 24$

Step 4 $\dfrac{3x}{3} = \dfrac{24}{3}$ $\qquad$ Divide both sides by 3.

$x = 8$

☑ $5(8) - [2(8) - 3] = 27$

$40 - (16 - 3) = 27$

$40 - 13 = 27$

$27 = 27$

YOUR TURN

Try this problem.

Solve: $8 - 3(2 - 3x) = 34$

SOLUTION

Step 1 Be very careful here. Some students are tempted to subtract the 3 from the 8. However, the order of operations rules on page 314 specify that multiplication must be performed before addition or subtraction. Therefore, your first step is to multiply the expression in brackets by −3.

$8 - 6 + 9x = 34$ $\quad\quad$ $8 - 3(2 - 3x) = 8 + (-3)(2) + (-3)(-3x)$

Step 2 $2 + 9x = 34$ $\quad\quad$ Combine the like terms on the left.

Step 3 $2 + 9x - 2 = 34 - 2$ $\quad\quad$ Subtract 2 from each side.

$9x = 32$

Step 4 $\dfrac{9x}{9} = \dfrac{32}{9}$

$x = 3\frac{5}{9}$

Be sure to check your answer.

PROBLEMS

Here are more equations with brackets for you to solve.

(a) $4(x - 2) = 26$ $\quad\quad$ (b) $11 = -2(y + 5)$

(c) $23 = 6 - (3n + 4)$ $\quad\quad$ (d) $4x - (6x - 9) = 41$

(e) $7 + 3(5x + 2) = 38$ $\quad\quad$ (f) $20 = 2 - 5(9 - a)$

(g) $(3m - 2) - (5m - 3) = 19$ $\quad\quad$ (h) $2(4x + 1) + 5(3x + 2) = 58$

ANSWERS

(a) $x = 8.5$ $\quad$ (b) $y = -10.5$ $\quad$ (c) $n = -7$ $\quad$ (d) $x = -16$

(e) $x = 1\frac{2}{3}$ $\quad$ (f) $a = 12.6$ $\quad$ (g) $m = -9$ $\quad$ (h) $x = 2$

Here are worked solutions to (b), (d), (f), and (h). The checks are left to you.

(b) **Solve:** $11 = -2(y + 5)$

Step 1 $11 = -2y - 10$ $\quad\quad$ Multiply each term inside brackets by −2.

Step 2 $11 + 10 = -2y - 10 + 10$ $\quad\quad$ Add 10 to each side.

$21 = -2y$

Step 3 $\dfrac{21}{-2} = \dfrac{-2y}{-2}$ Divide each side by -2.

$-10.5 = y$ or $y = -10.5$

(d) **Solve:** $4x - (6x - 9) = 41$

Step 1 $4x - 6x + 9 = 41$ Change the sign of each term inside brackets and then remove brackets.

Step 2 $-2x + 9 = 41$ Combine like terms on the left side.

Step 3 $-2x + 9 - 9 = 41 - 9$ Subtract 9 from each side.

$-2x = 32$

Step 4 $\dfrac{-2y}{-2} = \dfrac{32}{-2}$ Divide each side by -2.

$x = -16$

(f) **Solve:** $20 = 2 - 5(9 - a)$

Step 1 Multiply both terms inside brackets by -5 and then remove brackets.

$20 = 2 - 45 + 5a$ $2 - 5(9 - a)$
$= 2 + (-5)(9) + (-5)(-a)$

Step 2 $20 = -43 + 5a$ Combine like terms on the right side.

Step 3 $20 + 43 = -43 + 5a + 43$ Add 43 to both sides.

$63 = 5a$

Step 4 $\dfrac{63}{5} = \dfrac{5a}{5}$ Divide both sides by 5.

$12.6 = a$ or $a = 12.6$

(h) **Solve:** $2(4x + 1) + 5(3x + 2) = 58$

Step 1 Multiply each term inside the first brackets by 2 and each term inside the second brackets by 5.

$8x + 2 + 15x + 10 = 58$

Step 2 $23x + 12 = 58$ Combine both pairs of like terms on the left side.

Step 3 $23x + 12 - 12 = 58 - 12$ Subtract 12 from both sides.

$23x = 46$

Step 4 $\dfrac{23x}{23} = \dfrac{46}{23}$ Divide both sides by 23.

$x = 2$

Variable on Both Sides

In all the equations we have solved so far, the variable has been on only one side of the equation. Sometimes it is necessary to solve equations with variable terms on both sides.

EXAMPLE

To solve

$3x - 4 = 8 - x$

First, move all variable terms to the left side by adding x to both sides.

$3x - 4 + x = 8 - x + x$

$4x - 4 = 8 \qquad 3x + x = 4x \qquad -x + x = 0$

Next, proceed as before.

$4x - 4 + 4 = 8 + 4$ Add 4 to both sides.

$4x = 12$

$\frac{4x}{4} = \frac{12}{4}$ Divide both sides by 4.

$x = 3$

Finally, check your answer.

$3(3) - 4 = 8 - (3)$

$9 - 4 = 5$

$5 = 5$

YOUR TURN

Ready to attempt one yourself? Give this one a try, then check it with our solution.

$5y - 21 = 8y$

SOLUTION

Did you move the variable terms to the left? Here you can save a step by moving them to the right instead of to the left.

Solve: $5y - 21 = 8y$

Step 1 $5y - 21 - 5y = 8y - 5y$ Subtract $5y$ from both sides.

$-21 = 3y$

Step 2 $\frac{-21}{3} = \frac{3y}{3}$ Divide both sides by 3.

$-7 = y$ or $y = -7$

$5(-7) - 21 = 8(-7)$

$-35 - 21 = -56$

$-56 = -56$

LEARNING HINT ▶ If a variable-term is already by itself on one side of the equation, move all variable terms to this side. As in the last example, this will save a step. ◀

PROBLEMS

Now try these problems for practice in solving equations in which the variable appears on both sides.

(a) $x - 6 = 3x$

(b) $5(x - 2) = x + 4$

(c) $2(x - 1) = 3(x + 1)$

(d) $4x + 9 = 7x - 15$

(e) $4n + 3 = 18 - 2n$

(f) $2A = 12 - A$

(g) $6y = 4(2y + 7)$

(h) $8m - (2m - 3) = 3(m - 4)$

(i) $3 - (5x - 8) = 6x + 22$

(j) $-2(3x - 5) = 7 - 5(2x + 3)$

SOLUTIONS

(a) **Solve:** $x - 6 = 3x$

$x - 6 - x = 3x - x$ Subtract x from each side, so that x-terms will appear only on the right.

$-6 = 2x$ Combine terms.

or $2x = -6$ Divide by 2.

$x = -3$ Solution

✓ $(-3) - 6 = 3(-3)$

$-9 = -9$

(b) **Solve:** $5(x - 2) = x + 4$

$5x - 10 = x + 4$ Multiply each term inside the brackets by 5.

$5x - 10 - x = x + 4 - x$ Subtract x from each side.

$4x - 10 = 4$

$4x - 10 + 10 = 4 + 10$ Add 10 to each side.

$4x = 14$ Divide each side by 4.

$x = 3\frac{1}{2}$ Solution

✓ $5(3\frac{1}{2} - 2) = (3\frac{1}{2}) + 4$

$5(1\frac{1}{2}) = 7\frac{1}{2}$

$7\frac{1}{2} = 7\frac{1}{2}$

(c) **Solve:** $2(x - 1) = 3(x + 1)$ Remove brackets by multiplying.

$2x - 2 = 3x + 3$

$2x - 2 - 3x = 3x + 3 - 3x$ Subtract $3x$ from each side.

$-2 - x = 3$ Combine terms.

$-2 - x + 2 = 3 + 2$ Add 2 to each side.

$-x = 5$

or $x = -5$ Solution Solve for x, not $-x$.

Do you see that $-x = 5$ is the same as $x = -5$ Multiply both sides of the equation $-x = 5$ by -1 to get $x = -5$.

☑ $2(-5 - 1) = 3(-5 + 1)$

$2(-6) = 3(-4)$

$-12 = -12$

(d) **Solve:** $4x + 9 = 7x - 15$

$4x + 9 - 7x = 7x - 15 - 7x$ Subtract $7x$ from each side.

$-3x + 9 = -15$ Combine terms.

$-3x + 9 - 9 = -15 - 9$ Subtract 9 from each side.

$-3x = -24$ Divide each side by -3.

$x = 8$ Solution

☑ $4(8) + 9 = 7(8) - 15$
$32 + 9 = 56 - 15$
$41 = 41$

(e) **Solve:** $4n + 3 = 18 - 2n$

$4n + 3 + 2n = 18 - 2n + 2n$ Add $2n$ to both sides.

$6n + 3 = 18$ Now the n-terms appear only on the left side.

$6n + 3 - 3 = 18 - 3$ Subtract 3 from both sides.

$6n = 15$

$\frac{6n}{6} = \frac{15}{6}$ Divide by 6.

$n = 2.5$ Solution

☑ $4(2.5) + 3 = 18 - 2(2.5)$
$10 + 3 = 18 - 5$
$13 = 13$

(f) **Solve:** $2A = 12 - A$

$2A + A = 12 - A + A$ Add A to each side.

$3A = 12$ The A-terms now appear only on the left.

$\frac{3A}{3} = \frac{12}{3}$ Divide by 3.

$A = 4$ Solution

☑ $2(4) = 12 - (4)$
$8 = 8$

(g) **Solve:** $6y = 4(2y + 7)$

$6y = 8y + 28$ Multiply to remove brackets.

$6y - 8y = 8y + 28 - 8y$ Subtract $8y$ from both sides.

$-2y = 28$

$\frac{-2y}{-2} = \frac{28}{-2}$ Divide by -2.

$y = -14$ Solution

Check the solution.

(h) **Solve:** $8m - (2m - 3) = 3(m - 4)$

$8m - 2m + 3 = 3(m - 4)$	Change signs to remove brackets on the left.
$8m - 2m + 3 = 3m - 12$	Multiply by 3 to remove brackets on the right.
$6m + 3 = 3m - 12$	Combine like terms.
$6m + 3 - 3m = 3m - 12 - 3m$	Subtract $3m$ from both sides.
$3m + 3 = -12$	
$3m + 3 - 3 = -12 - 3$	Subtract 3 from both sides.
$3m = -15$	
$\dfrac{3m}{3} = \dfrac{-15}{3}$	Divide by 3.
$m = -5$ Solution	

Check the solution.

(i) **Solve:** $3 - (5x - 8) = 6x + 22$

$3 - 5x + 8 = 6x + 22$	Remove brackets by changing the signs of both terms.
$11 - 5x = 6x + 22$	Combine terms.
$11 - 5x - 6x = 6x + 22 - 6x$	Subtract $6x$ from both sides.
$11 - 11x = 22$	Combine terms.
$11 - 11x - 11 = 22 - 11$	Subtract 11 from both sides.
$-11x = 11$	Divide by -11.
$x = -1$ Solution	

Check it.

(j) **Solve:** $-2(3x - 5) = 7 - 5(2x + 3)$

$-6x + 10 = 7 - 10x - 15$	Remove brackets. Multiply each term inside brackets on the left by -2. Multiply each term inside brackets on the right by -5 (leave the 7 alone!).
$-6x + 10 = -8 - 10x$	Combine terms.
$-6x + 10 + 10x = -8 - 10x + 10x$	Add $10x$ to both sides.
$4x + 10 = -8$	Combine terms.
$4x + 10 - 10 = -8 - 10$	Subtract 10 from both sides.
$4x = -18$	Divide by 4.
$x = -4.5$ Solution	

Check it.

Remember:

1. Do only legal operations: add or subtract the same quantity from both sides of the equation; multiply or divide both sides of the equation by the same nonzero quantity.

2. Remove all brackets carefully.
3. Combine like terms when they are on the same side of the equation.
4. Use legal operations to change the equation so that you have only x by itself on one side of the equation and a number on the other side of the equation.
5. Always check your answer.

Solving Formulas

To *solve a formula* for some letter means to rewrite the formula as an equivalent formula with that letter isolated on the left of the equals sign.

For example, the area of a triangle is given by the formula

$A = \dfrac{BH}{2}$ where A is the area, B is the length of the base, and H is the height.

Solving for the base B gives the equivalent formula

$$B = \frac{2A}{H}$$

Solving for the height H gives the equivalent formula

$$H = \frac{2A}{B}$$

Solving formulas is a very important practical application of algebra. Very often a formula is not written in the form that is most useful. To use it you may need to rewrite the formula, solving it for the letter whose value you need to calculate.

To solve a formula, use the same balancing operations that you used to solve equations. You may add or subtract the same quantity on both sides of the formula and you may multiply or divide both sides of the formula by the same nonzero quantity.

EXAMPLE

To solve the formula

$S = \dfrac{R + P}{2}$ for R

First, multiply both sides of the equation by 2.

$$2 \cdot S = 2 \cdot \left(\frac{R + P}{2}\right)$$

$$2S = R + P$$

Second, subtract P from both sides of the equation.

$$2S - P = R + \underbrace{P - P}_{0}$$

$$2S - P = R$$

This formula can be reversed to read

$R = 2S - P$ We have solved the formula for R.

YOUR TURN

Solve the following formulas for the variable indicated.

(a) $V = \dfrac{3K}{T}$ for K (b) $Q = 1 - R + T$ for R

(c) $V = \pi R^2H - AB$ for H (d) $P = \dfrac{T}{A - B}$ for A

SOLUTIONS

(a) $V = \dfrac{3K}{T}$

First, multiply both sides by T to get $VT = 3K$

Second, divide both sides by 3 to get $\dfrac{VT}{3} = K$

Solved for K, the formula is $K = \dfrac{VT}{3}$

(b) $Q = 1 - R + T$

First, subtract T from both sides to get $Q - T = 1 - R$

Second, subtract 1 from both sides to get $Q - T - 1 = -R$

This is equivalent to $-R = Q - T - 1$

or $R = -Q + T + 1$

We have multiplied *all terms by* -1.

or $R = 1 - Q + T$

(c) $V = \pi R^2H - AB$

First, add AB to both sides to get $V + AB = \pi R^2H$

Second, divide both sides by πR^2 to get $\dfrac{V + AB}{\pi R^2} = H$

Notice that we divide *all* of the left side by πR^2

Solved for H, the formula is $H = \dfrac{V + AB}{\pi R^2}$

(d) $P = \dfrac{T}{A - B}$

First, multiply each side by $A - B$ to get $P(A - B) = T$

Second, multiply to remove the brackets $PA - PB = T$

Next, add PB to each side $PA = T + PB$

Finally, divide each side by P $A = \dfrac{T + PB}{P}$

Solved for A, the formula is $A = \dfrac{T + PB}{P}$

CAREFUL ▶ Remember, when using the multiplication/division rule, you must multiply or divide *all* of both sides of the formula by the same quantity. ◀

PROBLEMS

Practice solving formulas with the following problems.

Solve:

(a) $P = 2A + 3B$ for A

(b) $E = MC^2$ for M

(c) $S = \dfrac{A - RT}{1 - R}$ for A

(d) $S = \dfrac{1}{2}gt^2$ for g

(e) $P = i^2R$ for R

(f) $I = \dfrac{V}{R + a}$ for R

(g) $A = \dfrac{2V - W}{R}$ for V

(h) $F = \dfrac{9C}{5} + 32$ for C

(i) $A = \dfrac{\pi R^2 S}{360}$ for S

(j) $P = \dfrac{t^2 dN}{3.78}$ for d

(k) $C = \dfrac{AD}{A + 12}$ for D

(l) $V = \dfrac{\pi LT^2}{6} + 2$ for L

ANSWERS

(a) $A = \dfrac{P - 3B}{2}$

(b) $M = \dfrac{E}{C^2}$

(c) $A = S - SR + RT$

(d) $g = \dfrac{2S}{t^2}$

(e) $R = \dfrac{P}{i^2}$

(f) $R = \dfrac{V - aI}{I}$

(g) $V = \dfrac{AR + W}{2}$

(h) $C = \dfrac{5F - 160}{9}$

(i) $S = \dfrac{360A}{\pi R^2}$

(j) $d = \dfrac{3.78P}{t^2 N}$

(k) $D = \dfrac{CA + 12C}{A}$

(l) $L = \dfrac{6V - 12}{\pi T^2}$

USING SQUARE ROOTS IN SOLVING EQUATIONS

The equations you learned to solve in this chapter are all *linear* equations. The variable appears only to the first power—no x^2 or x^3 terms appear in the equations. You will learn how to solve more difficult algebraic equations later, but equations that look like

$x^2 = a$ where a is some positive number

can be solved easily.

To solve such an equation, simply take the square root of each side of the equation. The solution can be either

$x = \sqrt{a}$ or $x = -\sqrt{a}$

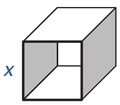
Example: Solve $x^2 = 36$.

Taking square roots, we have $x = +6$ or $x = -6$.
There are two possible solutions, one negative and one positive. Be careful, one of them, usually the negative one, may not be a reasonable answer to a practical problem.

Example: If the cross-sectional area of a square heating duct is 75 sq in., what must be the width of the duct?

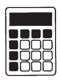
Solve $x^2 = 75$.

Taking the square root of each side, we obtain the positive solution



$x = \sqrt{75}$

$x \approx 8.7$ in. rounded



75 $\boxed{\sqrt{\ }}$ → 8.660254038

If you need to review the concept of square roots, return to Section 6-4 on page 315.

We will look at more problems like this in a later chapter.

Now turn to Exercises 7-5 for a set of practice problems on solving equations and formulas.

Exercises 7-5

Solving More Equations and Formulas

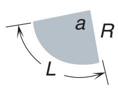
A. Solve the following equations.

1. $5(x - 3) = 30$
2. $22 = -2(y + 6)$
3. $3(2n + 4) = 41$
4. $-6(3a - 7) = 21$
5. $2 - (x - 5) = 14$
6. $24 = 5 - (3 - 2m)$
7. $6 + 2(y - 4) = 13$
8. $7 - 11(2z + 3) = 18$
9. $8 = 5 - 3(3x - 4)$
10. $7 + 9(2w + 3) = 25$
11. $6c - (c - 4) = 29$
12. $9 = 4y - (y - 2)$
13. $5x - 3(2x - 8) = 31$
14. $6a + 2(a + 7) = 8$
15. $9t - 3 = 4t - 2$
16. $7y + 5 = 3y + 11$
17. $12x = 4x - 16$
18. $22n = 16n - 18$
19. $8y - 25 = 13 - 11y$
20. $6 - 2p = 14 - 4p$
21. $9x = 30 - 6x$
22. $12 - y = y$
23. $2(3t - 4) = 10t + 7$
24. $5A = 4(2 - A)$
25. $2 - (3x - 20) = 4(x - 2)$
26. $2(2x - 5) = 6x - (5 - x)$
27. $8 + 3(6 - 5x) = 11 - 10x$
28. $2(x - 5) - 3(2x - 8) = 16 - 6(4x - 3)$

B. Solve the following formulas for the variable shown.

1. $S = LW$ for L
2. $A = \frac{1}{2}BH$ for B
3. $V = IR$ for I
4. $H = \frac{D - R}{2}$ for D
5. $S = \frac{W}{2}(A + T)$ for T
6. $V = \pi R^2 H$ for H
7. $P = 2A + 2B$ for B
8. $H = \frac{R}{2} + 0.05$ for R
9. $T = \frac{RP}{R + 2}$ for P
10. $I = \frac{E + V}{R}$ for V

C. Practical Problems



Problem 1

1. **Sheet-Metal Technology** The length of arc of a sector of a circle is given by the formula

$$L = \frac{2\pi Ra}{360}$$

R is the radius of the circle and a is the central angle in degrees.

(a) Solve for a. (b) Solve for R.

(c) Find L when $R = 250$ mm and $a = 30°$. Use $\pi \approx 3.14$.

2. **Sheet-Metal Technology** The area of the sector shown in problem 1 is $A = \pi R^2 a/360$.

(a) Solve this formula for a.

(b) Find A if $R = 305$ mm, $a = 45°$, $\pi \approx 3.14$. Round to three significant digits.

3. **Electronics** When two resistors R_1 and R_2 are put in series with a battery giving V volts, the current through the resistors is

$$i = \frac{V}{R_1 + R_2}$$

(a) Solve for R_1.

(b) Find R_2 if $V = 100$ V, $i = 0.4$ A, $R_1 = 200$ ohms.

4. **Machine Technology** Machinists use a formula known as Pomeroy's formula to determine roughly the power required by a metal punch machine.

$$P \approx \frac{t^2 dN}{3.78}$$

where P = power needed, in horsepower
t = thickness of the metal being punched
d = diameter of the hole being punched
N = number of holes to be punched at one time

(a) Solve this formula for N.

(b) Find the power needed to punch six 2 in.-diameter holes in a sheet $\frac{1}{8}$ in. thick. Round to one significant digit.

5. **Marine Technology** When a gas is kept at constant temperature and the pressure on it is changed, its volume changes in accord with the pressure–volume relationship known as Boyle's law:

$$\frac{V_1}{V_2} = \frac{P_2}{P_1}$$

where P_1 and V_1 are the beginning volume and pressure, and P_2 and V_2 are the final volume and pressure.

(a) Solve for V_1. (b) Solve for V_2.

(c) Solve for P_1. (d) Solve for P_2.

(e) Find P_1 when $V_1 = 10$ L, $V_2 = 25$ L, and $P_2 = 120$ kPa.

6. The volume of a football is roughly $V = \pi LT^2/6$, where L is its length and T is its thickness.

(a) Solve for L.

(b) Solve for T^2.

7. **Medical Technology** Nurses use a formula known as Young's rule to determine the amount of medicine to give a child under 12 years of age when the adult dosage is known.

$$C = \frac{AD}{A + 12}$$

C is the child's dose; A is the age of the child in years; D is the adult dose.

(a) Work backward and find the adult dose in terms of the child's dose. Solve for D.

(b) Find D if $C = 0.05$ g and $A = 7$.

8. **Carpentry** The projection or width P of a protective overhang of a roof is determined by the height T of the window, the height H of the header above the window, and a factor F that depends on the latitude of the construction site.

$$P = \frac{T + H}{F}$$ Solve this equation for the header height H.

9. **Electronics** For a current transformer, $\frac{i_L}{i_S} = \frac{T_P}{T_S}$

where i_L = line current
i_S = secondary current
T_P = number of turns of wire in the primary coil
T_S = number of turns of wire in the secondary coil

(a) Solve for i_L.

(b) Solve for i_S.

(c) Find i_L when $i_S = 1.5\,A$, $T_P = 1500$, $T_S = 100$.

10. **Electronics** The electrical power P dissipated in a circuit is equal to the product of the current I and the voltage V, where P is in watts, I is in amperes, and V is in volts.

(a) Write an equation giving P in terms of I and V.

(b) Solve for I.

(c) Find V when $P = 15{,}750$ W, $I = 42$ A.

11. **Electronics** The inductance L in microhenrys of a coil constructed by a ham radio operator is given by the formula

$$L = \frac{R^2N^2}{9R + 10D}$$

where R = radius of the coil
D = length of the coil
N = number of turns of wire in the coil

Find L if $R = 3$ in., $D = 6$ in., $N = 200$.

12. **Sheet-Metal Technology** A sheet-metal technician uses the following formula for calculating bend allowance, BA:

$$BA = N(0.01743R + 0.0078T)$$

where N = number of degrees in the bend
R = inside radius of the bend
T = thickness of the metal

Find BA for each of the following situations. (You'll want to use a calculator on this one.)

	N	R	T
(a)	50°	$1\frac{1}{4}$ in.	0.050 in.
(b)	65°	22 mm	0.4 mm
(c)	40°	26 mm	0.9 mm

13. **Machinist** Suppose that, on the average, 3% of the parts produced by a particular machine have proven to be defective. Then the formula

$$N - 0.03N = P$$

will give the number of parts N that must be produced in order to manufacture a total of P nondefective ones. How many parts should be produced by this machine in order to end up with 7500 nondefective ones?

14. **Office Services** The formula

$$A = P(1 + rt)$$

is used to find the total amount A of money in an account when an original amount or principal P is invested at a rate of simple interest r for t years. How long would it take \$8000 to grow to \$10,000 at 8% simple interest?

15. **Civil Engineer** The formula

$$I = 0.000014L(T - t)$$

gives the expansion I of a particular highway of length L at a temperature of T degrees Fahrenheit. The variable t stands for the temperature at which the highway was built. If a 2-mile stretch of highway was built at an average temperature of 60°F, what is the maximum temperature it can withstand if expansion joints allow for 7.5 ft of expansion? (*Hint:* The units of L must be the same as the units of I.)

When you have completed these exercises check your answers, then turn to Section 7-6 to learn about word problems.

7-6 SOLVING WORD PROBLEMS

Translating English to Algebra

Algebra is a very useful tool for solving real problems. But in order to use it you may find it necessary to translate simple English sentences and phrases into mathematical expressions or equations. In technical work especially, the formulas to be used are often given in the form of English sentences, and they must be rewritten as algebraic formulas before they can be used.

EXAMPLE

The statement

> Horsepower required to overcome vehicle air resistance is equal to the cube of the vehicle speed in mph multiplied by the frontal area in square feet divided by 150,000

translates to the formula

$$\text{hp} = \frac{\text{mph}^3 \cdot \text{area}}{150{,}000}$$

or

$$P = \frac{v^3 A}{150{,}000} \qquad \text{in algebraic form, where } v \text{ is the vehicle speed}$$

In the next few pages of this chapter we show you how to translate English statements into algebraic formulas. To begin, try the following problem.

YOUR TURN

An automotive technician found the following statement in a manual:

> The pitch diameter of a cam gear is twice the diameter of the crank gear.

Translate this sentence into an algebraic equation.

ANSWER

The equation is $P = 2C$ where P is the pitch diameter of the cam gear, and C is the diameter of the crank gear

You may use any letters you wish of course, but we have chosen letters that remind you of the quantities they represent: P for *p*itch and C for *c*rank.

Notice that the phrase "twice C" means "two times C" and is written as $2C$ in algebra.

Certain words and phrases appear again and again in statements to be translated. They are signals alerting you to the mathematical operations to be used. Here is a handy list of the *signal words* and their mathematical translations.

SIGNAL WORDS

English Term	Math Translation	Example
Equals Is, is equal to, was, are, were The same as . . . What is left is . . . The result is . . . Gives, makes, leaves, having	$=$	$A = B$
Plus, sum of Increased by, more than	$+$	$A + B$
Minus B, subtract B Less B Decreased by B, take away B Reduced by B, diminished by B B less than A B subtracted from A Difference between A and B	$-$	$A - B$
Times, multiply, of Multiplied Product of	$\times$	AB
Divide, divided by B Quotient of	$\div$	$A \div B$ or $\frac{A}{B}$
Twice, twice as much Double	$\times 2$	$2A$
Squared, square of A Cubed, cube of A		A^2 A^3

YOUR TURN

Translate the phrase "length plus 3 m" into an algebraic expression. Try it, then check your answer with ours.

SOLUTION

First, make a word equation by using brackets.

(length) (plus) (3 m)

Second, substitute mathematical symbols.

$$\begin{array}{ccc} \text{(length)} & \text{(plus)} & \text{(3 m)} \\ \downarrow & \downarrow & \downarrow \\ L & + & 3 \end{array} \quad \text{or } L + 3$$

Notice that signal words, such as "plus," are translated directly into math symbols. Unknown quantities are represented by letters of the alphabet, chosen to remind you of their meaning.

PROBLEMS

Translate the following phrases into math expressions.

(a) Weight divided by 13.6 ____________

(b) $6\frac{1}{4}$ in. more than the width ____________

(c) One-half of the original torque ____________

(d) The sum of the two lengths ____________

(e) The voltage decreased by 5 ____________

(f) Five times the gear reduction ____________

(g) 8 cm less than twice the height ____________

SOLUTIONS

(a) (weight) (divided by) (13.6)

↓ ↓ ↓

$W \quad \div \quad 13.6$ or $\dfrac{W}{13.6}$

(b) ($6\frac{1}{4}$ in.) (more than) (the width)

↓ ↓ ↓

$6\frac{1}{4} \quad + \quad W$ or $6\frac{1}{4} + W$

(c) (one-half) (of) (the original torque)

↓ ↓ ↓

$\dfrac{1}{2} \quad \times \quad T$ or $\dfrac{1}{2}T$ or $\dfrac{T}{2}$

(d) (the sum of) (the two lengths)

↓ ↓

$L_1 + L_2$

(e) (the voltage) (decreased by) (5)

↓ ↓ ↓

$V \quad - \quad 5$ or $V - 5$

(f) (five) (times) (the gear reduction)

↓ ↓ ↓

$5 \quad \times \quad G$ or $5G$

(g) 8 less than twice the height means
(twice the height) (less) (8)

↘ ↙ ↓ ↓

$2 \quad H \quad - \quad 8$ or $2H - 8$

Of course, any letters could be used in place of the ones used above.

NOTE ▶ The phrase "less than" has a meaning very different from the word "less."

"8 ***less*** 5" means $8 - 5$

"8 ***less than*** 5" means $5 - 8$ ◀

Translating Sentences to Equations

So far we have translated only phrases, pieces of sentences, but complete sentences can also be translated. An English phrase translates into an algebraic expression, and an English sentence translates into an algebraic formula or equation.

EXAMPLE

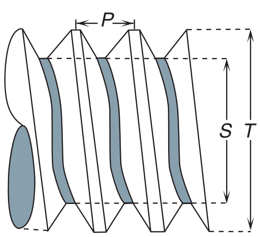
The sentence

$\underbrace{\text{The height of the duct}}_{H}\ \underbrace{\text{is equal to}}_{=}\ \underbrace{\text{its width}}_{W}$

translates to H $=$ W or $H = W$

Each word or phrase in the sentence becomes a mathematical term, letter, number, expression, or arithmetic operation sign.

YOUR TURN



Metric screw thread relation

Translate the following sentence into algebraic form as we did above.

> For metric screws, the distance between successive threads is called the pitch. To tap a hole for the screw, the drill size (S) is equal to the tap diameter ($\boldsymbol{I}$) minus the pitch (P).

SOLUTION

$S = T - P$

Follow these steps when you must translate an English sentence into an algebraic equation or formula.

Step 1	Cross out all unnecessary words.	~~The drill~~ size is equal to ~~the~~ tap diameter minus ~~the~~ pitch.
Step 2	Make a word equation using brackets.	(Size) (is equal to) (tap diameter) (minus) (pitch)
Step 3	Substitute a letter or an arithmetic symbol for each brackets.	S $=$ D $-$ P
Step 4	Combine and simplify.	$S = D - P$

In most formulas the units for the quantities involved must be given. In the formula above, D is in millimetres.

LEARNING HELP ▶ Translating English sentences or verbal rules into algebraic formulas requires that you read the sentences very differently from the way you read stories or newspaper articles. Very few people are able to write out the math formula after reading the problem only once. You should expect to read it several times, and you'll want to read it slowly. No speed reading here! ◀

The ideas in technical work and formulas are usually concentrated in a few key words, and you must find them. If you find a word you do not recognize, stop reading and look it up in a dictionary, textbook, or manual. It may be important.

Translating and working with formulas is one of the skills you must have if you are to succeed at any technical occupation.

EXAMPLE

Here is another example of translating a verbal rule into an algebraic formula:

> The electrical resistance of a length of wire is equal to the resistivity of the metal times the length of the wire divided by the square of the wire diameter.

Step 1	Eliminate all but the key words.	"~~The electrical~~ resistance ~~of a length of wire~~ is equal to ~~the~~ resistivity ~~of the metal~~ times ~~the~~ length ~~of the wire~~ divided by ~~the~~ square of ~~the wire~~ diameter."
Step 2	Make a word equation.	(Resistance) (is equal to) (resistivity) (times) (length) (divided by) (square of diameter)
Step 3	Substitute letters and symbols.	$R = \dfrac{rL}{D^2}$

If the resistivity r has units of ohms times metres, L and D will be in metres.

PROBLEMS

The more translations you do, the easier it gets. Translate each of the following technical statements into algebraic formulas.

(a) A sheet-metal worker measuring a duct cover finds that the width is 215 mm less than the height.

(b) One-quarter of a job takes $3\frac{1}{2}$ days.

(c) One-half of a coil of wire weighs $16\frac{2}{3}$ kg.

(d) The volume of an elliptical tank is approximately 0.7854 times the product of its height, length, and width.

(e) The engine speed is equal to 168 times the overall gear reduction multiplied by the speed and divided by the rolling radius of the tire.

(f) The air resistance force acting against a moving vehicle is equal to 0.0025 times the square of the speed times the frontal area of the vehicle.

(g) Two pieces of wire have a combined length of 24 m. The longer piece is five times the length of the shorter piece. (*Hint:* Write two separate equations.)

(h) Two shims are to have a combined thickness of 0.090 in. The larger shim must be 3.5 times as thick as the smaller shim. (*Hint:* Write two separate equations.)

ANSWERS

(a) $W = H - 215$

(b) $\dfrac{1}{4}J = 3\dfrac{1}{2}$ or $\dfrac{J}{4} = 3\dfrac{1}{2}$

(c) $\dfrac{1}{2}C = 16\dfrac{2}{3}$ or $\dfrac{C}{2} = 16\dfrac{2}{3}$

(d) $V = 0.7854HLW$

(e) $S = \dfrac{168Gv}{R}$

(f) $R = 0.0025v^2A$

(g) $24 = L + S$ and $L = 5S$

(h) $S + L = 0.090$ and $L = 3.5S$

Notice in problems (g) and (h) that two equations can be written. These can be combined to form a single equation.

(g) $24 = L + S$ and $L = 5S$ give $24 = 5S + S$

(h) $S + L = 0.090$ and $L = 3.5S$ give $S + 3.5S = 0.090$

General Word Problems

Now that you can translate English phrases and sentences into algebraic expressions and equations, you should be able to solve many practical word problems.

EXAMPLE

Consider this problem:

A machinist needs to use two shims with a combined thickness of 0.084 mm. One shim is to be three times as thick as the other. What are the thicknesses of the two shims?

Let x = thickness of the thinner shim

then $3x$ = thickness of the thicker shim

and the equation is

$3x + x = 0.084$ mm Combine terms.

$4x = 0.084$ mm Divide each side by 4.

$x = 0.021$ mm $3x = 3(0.021 \text{ mm}) = 0.063$ mm

The thin shim is 0.021 mm and the thicker one is 0.063 mm.

0.021 mm + 3(0.021 mm) = 0.084 mm

0.021 mm + 0.063 mm = 0.084 mm which is correct

YOUR TURN

Use your knowledge of algebra to translate the following problem to an algebraic equation and then solve it.

Two carpenters produced 42 assembly frames in one day. Carpenter 1 worked faster and produced 8 more than Carpenter 2. How many frames did each build?

SOLUTION

Let B = number of frames built by Carpenter 2
then $B + 8$ = number of frames built by Carpenter 1

and

$B + (B + 8) = 42$	Combine terms, $B + B = 2B$.
$2B + 8 = 42$	Subtract 8 from each side.
$2B = 34$	Divide both sides by 2.
$B = 17$	Carpenter 2 built 17 frames.
then $B + 8 = 25$	Carpenter 1 built 25 frames.

$17 + 25 = 42$

Some of the more difficult percent problems can be simplified using algebraic equations. One such problem, backing the tax out of a total, is commonly encountered by all those who own their own businesses.

EXAMPLE

Suppose that an auto shop has collected \$1468.63 for parts, including 15% tax (PST + GST). For accounting purposes they must determine exactly how much of the total is tax. If we let x stand for the dollar amount of the parts before tax was added, then the tax is 15% of that or $0.15x$, and we have

$$\underbrace{x}_{\text{Cost of the parts}} \quad + \quad \underbrace{0.15x}_{\text{Tax on the parts}} \quad = \quad \underbrace{\$1468.63}_{\text{Total amount}}$$

Solving for x yields

$x + 0.15x = \$1468.63$	
$1.00x + 0.15x = \$1468.63$	$(x = 1x = 1.00x)$. Combine like terms.
$1.15x = \$1468.63$	Now divide by 1.15.
$x = \$1277.07$	x is the cost of the parts.
$0.15x = 0.15(\$1277.07) = \$191.56$	Multiply by 0.15 or
$\$1468.63 - \$1277.07 = \$191.56$	Subtract from \$1468.63 to get the tax.

PROBLEMS

Practice makes perfect. Work the following set of word problems.

(a) The area of a rectangular shop floor is 400 m^2. If the width is 16 m, what is the length? (*Hint:* Area = length $\cdot$ width.)

(b) A 4270 mm-long steel rod is cut into two pieces. The longer piece is $2\frac{1}{2}$ times the length of the shorter piece. Find the length of each piece. (Ignore waste.)

(c) A carpenter wants to cut a 12 ft board into three pieces. The longest piece must have three times the length of the shortest piece, and the medium-sized piece is 2 ft longer than the shortest piece. Find the actual length of each piece. (Ignore waste in cutting.)

(d) Find the dimensions of a rectangular cover plate if its length is 60 mm longer than its width and if its perimeter is 680 mm. (*Hint:* Perimeter = $2 \cdot$ length + $2 \cdot$ width.)

(e) Two partners own a small manufacturing firm. Because Partner 1 provided more of the initial capital for the business, they have agreed that Partner 1's share of the profits should be $\frac{1}{4}$ greater than that of Partner 2. The total profit for the first quarter of this year was $17,550. How should they divide it?

(f) A plumber collected $2784.84 during the day, $1650 for GST and labour, which is not taxed, and the rest for parts, which includes 5% sales tax. Determine the total amount of sales tax that was collected.

(g) A printer knows from past experience that about 2% of a particular run of posters will be spoiled. How many posters should be printed to end up with 1500 usable posters?

SOLUTIONS

(a) Width $= 16$ m   Length $= L$

Area $= 400 \text{ m}^2 = 16 \text{ m} \cdot L$

$16L = 400$ Divide by 16.

$L = 25$ m

(b) x = shorter piece

$(2\frac{1}{2})x$ = longer piece

(1) $x + (2\frac{1}{2})x = 4270$ mm

$(3\frac{1}{2})x = 4270$ mm

or $\dfrac{7x}{2} = 4270$ Multiply by 2.

$7x = 8540$ Divide by 7.

$x = 1220$ mm   Larger piece $= 2\frac{1}{2}x = 2\frac{1}{2}(1220) = 3050$ mm

The shorter piece is 1220 mm long and the longer piece is 3050 mm long.

(c) Let x = the shortest piece

then $3x$ = the longest piece

and $x + 2$ = medium-sized piece

$x + (x + 2) + (3x) = 12$ ft   Combine terms.

$5x + 2 = 12$ Subtract 2 from each side.

$5x = 10$ Divide by 5.

$x = 2$ ft   The shortest piece is 2 ft long.

$3x = 6$ ft   The longest piece is 6 ft long.

$x + 2 = 4$ ft   The third piece is 4 ft long.

(d) Let width $= W$

then length $= W + 60$

and perimeter $= 680 \text{ mm} = 2W + 2(W + 60)$ Remove brackets, $2(W + 60) = 2W + 120$.

$680 = 2W + 2W + 120$ Combine terms.

or $4W + 120 = 680$ Subtract 120 from each side.

$4W = 560$ Divide by 4.

$W = 140$ mm

length $= W + 60 = 200$ mm

(e) Let Partner 2's share $= J$
then Partner 1's share $= J + \frac{1}{4}J$

$$\$17{,}550 = J + \left(J + \tfrac{1}{4}J\right) \qquad \text{Combine terms; remember } J = 1 \cdot J.$$
$$= 2\tfrac{1}{4}J \qquad \text{Write the mixed number as a fraction.}$$
$$= \frac{9J}{4}$$
$$\text{or} \quad \frac{9J}{4} = 17{,}550 \qquad \text{Multiply by 4.}$$
$$9J = 70{,}200 \qquad \text{Divide by 9.}$$
$$J = \$7800 \quad \text{Partner 2's share}$$
$$\text{Partner 1's share} = \$7800 + \frac{\$7800}{4}$$
$$= \$7800 + \$1950$$
$$= \$9750$$

(f) Let $x =$ cost of the parts
then $0.05x =$ amount of the sales tax

$$x + 0.05x = \$2784.84 - \$1650 \qquad \text{Subtract out the GST and labour to get the dollar amount of the parts.}$$
$$1.05x = \$1134.84$$
$$x = \$1080.80 \qquad \text{This represents the cost of the parts.}$$
$$0.05(\$1080.80) = \$54.04 \qquad \text{The total sales tax is \$54.04.}$$

(g) Let $x =$ number of posters needed to print
then $0.02x =$ number that will be spoiled

$$\underbrace{x}_{\text{the number printed}} \underset{\text{minus}}{-} \underbrace{0.02x}_{\text{the number spoiled}} = \underbrace{1500}_{\text{must total 1500}}$$

$$1.00x - 0.02x = 1500 \qquad x = 1x = 1.00x. \text{ Now subtract like terms.}$$
$$0.98x = 1500 \qquad \text{Divide by 0.98.}$$
$$x = 1531 \text{ (rounded)} \qquad \text{Print 1531 posters to end up with 1500 unspoiled ones.}$$

Now turn to Exercises 7-6 for a set of word problems.

Exercises 7-6

Solving Word Problems

A. Translate the following into algebraic equations:

1. The height of the tank is 1.4 times its width.
2. The sum of the two weights is 167 lb.
3. The volume of a cylinder is equal to $\frac{1}{4}$ of its height times π times its diameter squared.
4. The weight in kilograms is equal to 0.454 times the weight in pounds.
5. **Machine Technology** The volume of a solid bar is equal to the product of the cross-sectional area and the length of the bar.
6. The volume of a cone is $\frac{1}{3}$ times π times the height times the square of the radius of the base.

7. **Machine Technology** The pitch diameter D of a spur gear is equal to the number of teeth on the gear divided by the pitch.

8. **Machine Technology** The cutting time for a lathe operation is equal to the length of the cut divided by the product of the tool feed rate and the revolution rate of the workpiece.

9. **Machine Technology** The weight of a metal cylinder is approximately equal to 0.7854 times the height of the cylinder times the density of the metal times the square of the diameter of the cylinder.

10. The voltage across a simple circuit is equal to the product of the resistance of the circuit and the current flowing in the circuit.

B. Set up equations and solve.

1. **Wastewater Technology** Two tanks must have a total capacity of 4000 L. If one tank needs to be twice the size of the other, how many litres should each tank hold?

2. **Machine Technology** Two metal castings weigh a total of 84 kg. One weighs 12 kg more than the other. How much does each one weigh?

3. **Plumbing** The plumber wants to cut a 24 ft length of pipe into three sections. The largest piece should be twice the length of the middle piece, and the middle piece should be twice the length of the smallest piece. How long should each piece be?

4. **Electrical Technology** The sum of three voltages in a circuit is 38 V. The middle-sized one is 2 V more than the smallest. The largest is 6 V more than the smallest. What is the value of each voltage?

5. **Masonry** There are 156 concrete blocks available to make a retaining wall. The bottom three rows will all have the same number of blocks. The next six rows will each have two blocks fewer than the row below it. How many blocks are in each row?

6. **Construction Technology** A concrete mix is in volume proportions of 1 part cement, 2 parts water, 2 parts aggregate, and 3 parts sand. How many cubic metres of each ingredient are needed to make 2.8 m^3 of concrete?

7. **Office Services** A painting business has two partners. One of them is the office manager, so, receives one-third more than the other when the profits are distributed. If their profit is $85,400, how much should each of them receive?

8. **Construction Technology** A total of $560,000 is budgeted for constructing a roadway. The rule of thumb for this type of project is that the pavement costs twice the amount of the base material, and the sidewalk costs one-fourth the amount of the pavement. Using these figures, how much should each item cost?

9. **Photography** A photographer has enough liquid toner for fifty 5 in. by 7 in. prints. How many 8 in. by 10 in. prints will this amount cover? (*Hint:* The toner covers the *area* of the prints: Area = length × width.)

10. **Construction Technology** A building foundation has a length of 24,994 mm and a perimeter of 89,000 mm. What is the width? (*Hint:* Perimeter = 2 × length + 2 × width.)

11. **Roofing** A roofer earns $24/h plus $36/h overtime. During a certain week 40 h of regular time are worked. If the gross pay is $1200, how many hours of overtime were worked?

12. **Agricultural Technology** An empty avocado crate weighs 4.2 kg. How many avocados weighing 0.3 kg each can be added before the total weight of the crate reaches 15.0 kg?
13. **Machine Technology** One-tenth of the parts tooled by a machine are rejects. How many parts must be tooled to assure 4500 acceptable ones?
14. **Office Services** An auto mechanic has total receipts of $1861.16 for a certain day. $1240 of this is labour and GST. The remaining amount is for parts plus 6% sales tax on the parts only. Find the amount of the sales tax collected.
15. **Office Services** A travel agent receives a 7% commission on the base fare of a customer—that is, the fare *before* sales tax is added. If the total fare charged to a customer comes to $753.84 *including* 5.5% sales tax, and 7% GST, how much commission should the agent receive?
16. **Office Services** A newly established carpentery shop wishes to mail out letters advertising its services to the families of a small town. They have a choice of mailing the letters first class or obtaining a bulk-rate permit and mailing them at the cheaper bulk rate. The bulk-rate permit costs $80, and each piece of bulk mail then costs 16.7 cents. If the first-class rate is 48 cents, how many pieces would they have to mail in order to make bulk rate the cheaper way to go?

Check your answers, then turn to Section 7-7 to learn more algebra.

7-7 MULTIPLYING AND DIVIDING ALGEBRAIC EXPRESSIONS

Multiplying Simple Factors

Earlier we learned that only like algebraic terms, those with the same literal part, can be added and subtracted. However, any two terms, like or unlike, can be multiplied.

In order to multiply two terms such as $2x$ and $3xy$, first remember that $2x$ means 2 times x. Second, recall from arithmetic that the order in which you do multiplications does not make a difference. For example, in arithmetic

$2 \cdot 3 \cdot 4 = (2 \cdot 4) \cdot 3 = (3 \cdot 4) \cdot 2$

and in algebra

$a \cdot b \cdot c = (a \cdot c) \cdot b = (c \cdot b) \cdot a$ or $2 \cdot x \cdot 3 \cdot x \cdot y = 2 \cdot 3 \cdot x \cdot x \cdot y$

Remember that $x \cdot x = x^2$, $x \cdot x \cdot x = x^3$, and so on. Therefore,

$2x \cdot 3xy = 2 \cdot 3 \cdot x \cdot x \cdot y = 6x^2y$

EXAMPLE

The following examples show how to multiply two terms.

(a) $a \cdot 2a = a \cdot 2 \cdot a$

$= 2 \cdot \underbrace{a \cdot a}$ Group like factors together.

$= 2 \cdot a^2$

$= 2a^2$

(b) $2x^2 \cdot 3xy = 2 \cdot x \cdot x \cdot 3 \cdot x \cdot y$

$= \underbrace{2 \cdot 3} \cdot \underbrace{x \cdot x \cdot x} \cdot y$ Group like factors together.

$= 6 \cdot x^3 \cdot y$

$= 6x^3y$

(c) $3x^2yz \cdot 2xy = 3 \cdot x^2 \cdot y \cdot z \cdot 2 \cdot x \cdot y$

$$= \underbrace{3 \cdot 2}_{} \cdot \underbrace{x^2 \cdot x}_{} \cdot \underbrace{y \cdot y}_{} \cdot z$$

$$= 6 \cdot x^3 \cdot y^2 \cdot z$$

$$= 6x^3y^2z$$

Remember to group like factors together before multiplying.

If you need to review exponents, return to page 311.

YOUR TURN

Now try the following problems.

(a)	$x \cdot y =$ ________	(b)	$2x \cdot 3x =$ ________
(c)	$2x \cdot 5xy =$ ________	(d)	$4a^2b \cdot 2a =$ ________
(e)	$3x^2y \cdot 4xy^2 =$ ________	(f)	$5xyz \cdot 2ax^2 =$ ________
(g)	$3x \cdot 2y^2 \cdot 2x^2y =$ ________	(h)	$x^2y^2 \cdot 2x \cdot y^2 =$ ________
(i)	$2x^2(x + 3x^2) =$ ________	(j)	$-2a^2b(a^2 - 3b^2) =$ ________

ANSWERS

(a)	xy	(b)	$6x^2$
(c)	$10x^2y$	(d)	$8a^3b$
(e)	$12x^3y^3$	(f)	$10ax^3yz$
(g)	$12x^3y^3$	(h)	$2x^3y^4$
(i)	$2x^3 + 6x^4$	(j)	$-2a^4b + 6a^2b^3$

Were the last two problems tricky for you? Recall from Rule 3 on page 344 that when there is a multiplier in front of brackets, every term inside must be multiplied by this factor. Try it this way:

(i) $2x^2(x + 3x^2) = (2x^2)(x) + (2x^2)(3x^2)$

$$= 2 \cdot \underbrace{x^2 \cdot x}_{} + \underbrace{2 \cdot 3}_{} \cdot \underbrace{x^2 \cdot x^2}_{}$$

$$= 2 \cdot x^3 + 6 \cdot x^4$$

$$= 2x^3 + 6x^4$$

(j) $-2a^2b(a^2 - 3b^2) = (-2a^2b)(a^2) + (-2a^2b)(-3b^2)$

$$= (-2) \cdot \underbrace{a^2 \cdot a^2}_{} \cdot b + \underbrace{(-2) \cdot (-3)}_{} \cdot\ a^2 \cdot \underbrace{b \cdot b^2}_{}$$

$$= -2 \cdot a^4 \cdot b + 6 \cdot a^2 \cdot b^3$$

$$= -2a^4b + 6a^2b^3$$

Rule for Multiplication

As you did the preceding problems, you may have noticed that when you multiply like factors together, you actually add their exponents. For example,

$x^2 \cdot x^3 = x \cdot x \cdot x \cdot x \cdot x = x^5$

In general, we can state the following rule:

Rule 1 To multiply numbers written in exponential form having the same base, add the exponents.

m + n

$$x^m \cdot x^n = x^{m+n}$$

Same base

EXAMPLE

(a) $x^2 \cdot x^3 = x^{2+3} = x^5$

(b) $3^5 \times 3^2 = 3^{5+2} = 3^7$

CAREFUL ▶ 1. Notice in the second example that the like bases remain unchanged when you multiply—that is, $3^5 \times 3^2 = 3^7$, not 9^7.

2. Exponents are added during multiplication, but numerical factors are multiplied as usual. For example,

Add exponents

$$2x^2 \cdot 3x^3 = 6x^5$$

Multiply factors ◀

Dividing Simple Factors

To divide a^5 by a^2, think of it this way:

$$\frac{a^5}{a^2} = \underbrace{\frac{\cancel{a}\cdot\cancel{a}\cdot a\cdot a\cdot a}{\cancel{a}\cdot\cancel{a}}}_{\text{Cancel common factors}} = a^3$$

Notice that the final exponent, 3, is the difference between the original two exponents, 5 and 2. Remember that multiplication and division are reverse operations. It makes sense that if you add exponents when multiplying, you would subtract exponents when dividing. We can now state the following rule:

Rule 2 To divide numbers written in exponential form having the same base, subtract the exponents.

$$\frac{x^m}{x^n} = x^{m-n}$$

EXAMPLE

(a) $\dfrac{x^7}{x^3} = x^{7-3} = x^4$

(b) $\dfrac{4^5}{4^2} = 4^{5-2} = 4^3$

CAREFUL ▶

1. As with multiplication, the like bases remain unchanged when dividing with exponents. This is especially important to keep in mind when the base is a number, as in the second preceding example.
2. When dividing expressions that contain both exponential expressions and numerical factors, *subtract* the exponents but *divide* the numerical factors. For example,

Divide factors

$$\frac{10x^{10}}{2x^2} = 5x^8$$

Subtract exponents ◀

YOUR TURN

Try out the division rule on these problems:

(a) $\dfrac{x^6}{x^3}$ (b) $\dfrac{6a^4}{2a}$ (c) $\dfrac{-12m^3n^5}{4m^2n^2}$ (d) $\dfrac{5^8}{5^2}$ (e) $\dfrac{4x^5 - 8x^4 + 6x^3}{2x^2}$

ANSWERS

(a) x^3 (b) $3a^3$ (c) $-3mn^3$ (d) 5^6 (e) $2x^3 - 4x^2 + 3x$

Did you have trouble with (e)? Unlike the other problems, which have only one term in the numerator (dividend), problem (e) has *three* terms in the numerator. Just imagine that the entire numerator is enclosed in brackets and divide each of the three terms separately by the denominator (divisor). Write it out this way:

$$\frac{4x^5 - 8x^4 + 6x^3}{2x^2} = \frac{4x^5}{2x^2} - \frac{8x^4}{2x^2} + \frac{6x^3}{2x^2} = 2x^3 - 4x^2 + 3x$$

NOTE ▶ The rule for division confirms the fact stated in Section 6-4 that $a^0 = 1$. We know from arithmetic that any quantity divided by itself is equal to 1, so

$$\frac{a^n}{a^n} = 1 \quad (\text{for } a \neq 0)$$

But according to the division rule,

$$\frac{a^n}{a^n} = a^{n-n} = a^0$$ Therefore, a^0 must be equal to 1. ◀

Negative Exponents

If we apply the division rule to a problem in which the divisor has a larger exponent than the dividend, the answer will contain a negative exponent. For example,

$$\frac{x^4}{x^6} = x^{4-6} = x^{-2}$$

If we use the "cancellation method" to do this problem, we have

$$\frac{x^4}{x^6} = \frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x} = \frac{1}{x^2}$$ Therefore, $x^{-2} = \dfrac{1}{x^2}$.

In general, we define negative exponents as follows:

$$x^{-n} = \frac{1}{x^n}$$

YOUR TURN

Practice using the definition of negative exponents by giving two answers for each of the following problems—one with negative exponents, and one using fractions to eliminate the negative exponents.

(a) $\frac{a^2}{a^6}$ (b) $\frac{8x^2}{-2x^3}$ (c) $\frac{6^3}{6^5}$ (d) $\frac{5x^4y^2}{15xy^7}$ (e) $\frac{12x^6 - 9x^3 + 6x}{3x^2}$

ANSWERS

(a) a^{-4} or $\frac{1}{a^4}$

(b) $-4x^{-1}$ or $\frac{-4}{x}$

(c) 6^{-2} or $\frac{1}{6^2}$

(d) $\frac{1}{3}x^3y^{-5}$ or $\frac{x^3}{3y^5}$

(e) $4x^4 - 3x + 2x^{-1}$ or $4x^4 - 3x + \frac{2}{x}$

Now turn to Exercises 7-7 for a set of problems on multiplication and division of algebraic expressions.

Exercises 7-7

Multiplying and Dividing Algebraic Expressions

A. Simplify by multiplying.

1. $5 \cdot 4x$
2. $(a^2)(a^3)$
3. $-3R(-2R)$
4. $3x \cdot 3x$
5. $(4x^2y)(-2xy^3)$
6. $2x \cdot 2x \cdot 2x$
7. $0.4a \cdot 1.5a$
8. $x \cdot x \cdot A \cdot x \cdot 2x \cdot A^2$
9. $\frac{1}{2}Q \cdot \frac{1}{2}Q \cdot \frac{1}{2}Q \cdot \frac{1}{2}Q$
10. $(pq^2)(\frac{1}{2}pq)(2.4p^2q)$
11. $(-2M)(3M^2)(-4M^3)$
12. $2x(3x - 1)$
13. $-2(1 - 2y)$
14. $ab(a^2 - b^2)$
15. $p^2 \cdot p^4$
16. $3 \cdot 2x^2$
17. $5x^3 \cdot x^4$
18. $4x \cdot 2x^2 \cdot x^3$
19. $2y^5 \cdot 5y^2$
20. $2a^2b \cdot 2ab^2 \cdot a$
21. $xy \cdot x^2 \cdot xy^3$
22. $x(x + 2)$
23. $p^2(p + 2p)$
24. $xy(x + y)$
25. $5^6 \cdot 5^4$
26. $3^2 \cdot 3^7$
27. $10^2 \cdot 10^{-5}$
28. $2^{-6} \cdot 2^9$

29. $-3x^2(2x^2 - 5x^3)$

30. $6x^3(4x^4 + 3x^2)$

31. $2ab^2(3a^2 - 5ab + 7b^2)$

32. $-5xy^3(4x^2 - 6xy + 8y^2)$

B. Divide as indicated. Express all answers using positive exponents.

1. $\dfrac{4^5}{4^3}$

2. $\dfrac{10^9}{10^3}$

3. $\dfrac{x^4}{x}$

4. $\dfrac{y^6}{y^2}$

5. $\dfrac{10^4}{10^6}$

6. $\dfrac{m^5}{m^{10}}$

7. $\dfrac{8a^8}{4a^4}$

8. $\dfrac{-15m^{12}}{5m^6}$

9. $\dfrac{16y^2}{-24y^3}$

10. $\dfrac{-20t}{-5t^3}$

11. $\dfrac{-6a^2b^4}{-2ab}$

12. $\dfrac{36x^4y^5}{-9x^2y}$

13. $\dfrac{48m^2n^3}{-16m^6n^4}$

14. $\dfrac{15c^6d}{20c^2d^3}$

15. $\dfrac{-12a^2b^2}{20a^2b^5}$

16. $\dfrac{100xy}{-10x^2y^4}$

17. $\dfrac{6x^4 - 8x^2}{2x^2}$

18. $\dfrac{9y^3 + 6y^2}{3y}$

19. $\dfrac{12a^7 - 6a^5 + 18a^3}{-6a^2}$

20. $\dfrac{-15m^5 + 10m^2 + 5m}{5m}$

When you have completed these exercises, check your answers, then turn to Section 7-8 to learn about scientific notation.

7-8 SCIENTIFIC NOTATION

In technical work you will often deal with very small and very large numbers that may require a lot of space to write. For example, a certain computer function may take 5 nanoseconds (ns) (0.000000005 s), and in electricity, 1 kilowatt-hour (kWh) is equal to 3,600,000 joules (J) of work. Rather than take the time and space to write all the zeros, we use scientific notation to express such numbers.

Definition of Scientific Notation

In scientific notation 5 ns would be written as 5×10^{-9} s, and 1 kWh would be 3.6×10^6 J. As you can see, a number written in **scientific notation** is a product of a number between 1 and 10 and a power of 10. Stated formally, a number is expressed in scientific notation when it is in the form

$$P \times 10^k$$

where P is a number less than 10 and greater than or equal to 1, and k is an integer.

Numbers that are powers of ten are easy to write in scientific notation:

$$1000 = 1 \times 10^3$$
$$100 = 1 \times 10^2$$
$$10 = 1 \times 10^1 \qquad \text{and so on}$$

In general, to write a positive decimal number in scientific notation, first write it as a number between 1 and 10 times a power of 10, then write the power using an exponent.

EXAMPLE

$3{,}600{,}000 = 3.6 \times 1{,}000{,}000$

A multiple of 10

A number between 1 and 10

$= 3.6 \times 10^6$

$$0.0004 = 4 \times 0.0001 = 4 \times \frac{1}{10{,}000} = 4 \times \frac{1}{10^4} = 4 \times 10^{-4}$$

Converting to Scientific Notation

For a shorthand way of converting to scientific notation, follow these steps.

Step 1 If the number is given without a decimal point, rewrite it with a decimal point.

Example: $57{,}400 = 57{,}400.$ Decimal point 0.0038 Decimal point already shown

Step 2 Place a mark $_\wedge$ after the first nonzero digit.

Example: $5_\wedge 7400.$ $0.003_\wedge 8$

Step 3 Count the number of digits from the mark $_\wedge$ to the decimal point.

Example: $5_\wedge 7400.$ 4 digits $0.003_\wedge 8$ 3 digits

Step 4 Place the decimal point in the marked position and use the resulting number as the multiplier P in the scientific notation form. Discard any right end zeros on this number. Use the number of digits from the mark $_\wedge$ to the original decimal point as the exponent. If the shift from the $_\wedge$ to the original decimal point is to the right, the exponent is positive; if the shift is to the left, the exponent is negative.

Example: $5_\wedge 7400. = 5.74 \times 10^4$ Shift 4 digits right. Exponent is +4. $0.003_\wedge 8 = 3.8 \times 10^{-3}$ Shift 3 digits left. Exponent is −3.

EXAMPLE

(a) $150{,}000 = 1_\wedge 50000. = 1.5 \times 10^5$ Shift 5 digits right. Exponent is +5.

(b) $0.00000205 = 0.000002_\wedge 05 = 2.05 \times 10^{-6}$ Shift 6 digits left. Exponent is −6.

(c) $47 = 4_\wedge 7. = 4.7 \times 10^1$ Shift 1 digit right.

YOUR TURN

For practice, write the following numbers in scientific notation.

(a) 2900 (b) 0.006 (c) 1,960,100

(d) 0.0000028 (e) 600 (f) 0.0001005

SOLUTIONS

(a) $2900 = 2_{\wedge}900. = 2.9 \times 10^3$

Shift 3 digits right.

(b) $0.006 = 0.006_{\wedge} = 6 \times 10^{-3}$

Shift 3 digits left.

(c) $1,960,100 = 1_{\wedge}960100. = 1.9601 \times 10^6$

Shift 6 digits right.

(d) $0.0000028 = 0.000002_{\wedge}8 = 2.8 \times 10^{-6}$

Shift 6 digits left.

(e) $600 = 6_{\wedge}00. = 6 \times 10^2$

Shift 2 digits right.

(f) $0.0001005 = 0.0001_{\wedge}005 = 1.005 \times 10^{-4}$

Shift 4 digits left.

LEARNING HELP ▶ When a positive number greater than or equal to 10 is written in scientific notation, the exponent is positive. When a positive number less than 1 is written in scientific notation, the exponent is negative. When a number between 1 and 10 is written in scientific notation, the exponent is 0. ◀

Converting from Scientific Notation to Decimal Form

In order to convert a number from scientific notation to decimal form, shift the decimal point as indicated by the power of 10—to the right for a positive exponent, to the left for a negative exponent. Attach additional zeros as needed.

EXAMPLE

(a) Attach additional zeros.

$3.2 \times 10^4 = 3\,2000. = 32{,}000$

Shift the
decimal point
4 places right.

For a positive exponent, shift the decimal point to the right.

(b) Attach additional zeros.

$2.7 \times 10^{-3} = 0.002\,7 = 0.0027$

Shift the decimal point 3 places left.

For a negative exponent, shift the decimal point to the left.

YOUR TURN

Write each of the following numbers in decimal form.

(a) 8.2×10^{3} (b) 1.25×10^{-6}

(c) 2×10^{-4} (d) 5.301×10^{5}

ANSWERS

(a) $8.2 \times 10^{3} = 8\,200. = 8200$

(b) $1.25 \times 10^{-6} = 0.000001\,25 = 0.00000125$

(c) $2 \times 10^{-4} = 0.0002 = 0.0002$

(d) $5.301 \times 10^{5} = 5\,30100. = 530{,}100$

Multiplying and Dividing in Scientific Notation

Scientific notation is especially useful when we must multiply or divide very large or very small numbers. Although most calculators have a means of converting decimal numbers to scientific notation and can perform arithmetic with scientific notation, it is important that you be able to do simple calculations of this kind quickly and accurately without a calculator.

To multiply or divide numbers given in exponential form, use the rules given in Section 7-7.

$$a^{m} \times a^{n} = a^{m+n}$$

$$a^{m} \div a^{n} = \frac{a^{m}}{a^{n}} = a^{m-n}$$

To multiply numbers written in scientific notation, work with the decimal and exponential parts separately.

$$(A \times 10^{B}) \times (C \times 10^{D}) = (A \times C) \times 10^{B+D}$$

EXAMPLE

$26{,}000 \times 3{,}500{,}000 = ?$

Step 1 Rewrite each number in scientific notation.

$= (2.6 \times 10^4) \times (3.5 \times 10^6)$

Step 2 Regroup to work with the decimal and exponential parts separately.

$= (2.6 \times 3.5) \times (10^4 \times 10^6)$

Step 3 Multiply using the rule for multiplying exponential numbers.

$= 9.1 \times 10^{4+6}$
$= 9.1 \times 10^{10}$

When dividing numbers written in scientific notation, it may help to think of the division as a fraction.

$$(A \times 10^B) \div (C \times 10^D) = \frac{A \times 10^B}{C \times 10^D} = \frac{A}{C} \times \frac{10^B}{10^D}$$

Therefore,

$$(A \times 10^B) \div (C \times 10^D) = (A \div C) \times 10^{B-D}$$

EXAMPLE

$45{,}000 \div 0.0018 = ?$

Step 1 Rewrite. $= (4.5 \times 10^4) \div (1.8 \times 10^{-3})$

Step 2 Regroup. $= (4.5 \div 1.8) \times (10^4 \div 10^{-3})$

Step 3 Divide using the rules for exponential division.

$= 2.5 \times 10^{4-(-3)}$ $\quad 4 - (-3) = 4 + 3 = 7$
$= 2.5 \times 10^7$

If the result of the calculation is not in scientific notation, that is, if the decimal part is greater than 10 or less than 1, rewrite the decimal part so that it is in scientific notation.

EXAMPLE

$$\begin{aligned} 0.0000072 \div 0.0009 &= (7.2 \times 10^{-6}) \div (9 \times 10^{-4}) \\ &= (7.2 \div 9) \times (10^{-6} \div 10^{-4}) \\ &= 0.8 \times 10^{-6-(-4)} \quad -6 - (-4) = -6 + 4 = -2 \\ &= 0.8 \times 10^{-2} \end{aligned}$$

But 0.8 is not a number between 1 and 10, so write it as

$0.8 = 8 \times 10^{-1}$. Then, $0.8 \times 10^{-2} = 8 \times 10^{-1} \times 10^{-2}$
$= 8 \times 10^{-3}$ in scientific notation

YOUR TURN

Perform the following calculations using scientific notation, and write the answer in scientific notation.

(a) $1600 \times 350{,}000$ (b) $64{,}000 \times 250{,}000$

(c) 2700×0.0000045 (d) $15{,}600 \div 0.0013$

(e) $0.000348 \div 0.087$ (f) $0.00378 \div 540{,}000{,}000$

SOLUTIONS

(a)
$$\begin{aligned}(1.6 \times 10^3) \times (3.5 \times 10^5) &= (1.6 \times 3.5) \times (10^3 \times 10^5)\\ &= 5.6 \times 10^{3+5}\\ &= 5.6 \times 10^8\end{aligned}$$

(b)
$$\begin{aligned}(6.4 \times 10^4) \times (2.5 \times 10^5) &= (6.4 \times 2.5) \times (10^4 \times 10^5)\\ &= 16 \times 10^{4+5}\\ &= 16 \times 10^9\\ &= 1.6 \times 10^1 \times 10^9 && 16 = 1.6 \times 10^1\\ &= 1.6 \times 10^{10}\end{aligned}$$

(c)
$$\begin{aligned}(2.7 \times 10^3) \times (4.5 \times 10^{-6}) &= (2.7 \times 4.5) \times (10^3 \times 10^{-6})\\ &= 12.15 \times 10^{3+(-6)} && 3 + (-6) = -3\\ &= 12.15 \times 10^{-3}\\ &= 1.215 \times 10^1 \times 10^{-3}\\ &= 1.215 \times 10^{-2}\end{aligned}$$

(d)
$$\begin{aligned}(1.56 \times 10^4) \div (1.3 \times 10^{-3}) &= (1.56 \div 1.3) \times (10^4 \div 10^{-3})\\ &= 1.2 \times 10^{4-(-3)} && 4 - (-3) = 4 + 3 = 7\\ &= 1.2 \times 10^7\end{aligned}$$

(e)
$$\begin{aligned}(3.48 \times 10^{-4}) \div (8.7 \times 10^{-2}) &= (3.48 \div 8.7) \times (10^{-4} \div 10^{-2})\\ &= 0.4 \times 10^{-4-(-2)} && -4 - (-2) = -4 + 2 = -2\\ &= 0.4 \times 10^{-2}\\ &= 4 \times 10^{-1} \times 10^{-2} && 0.4 = 4 \times 10^{-1}\\ &= 4 \times 10^{-3}\end{aligned}$$

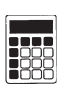
(f)
$$\begin{aligned}(3.78 \times 10^{-3}) \div (5.4 \times 10^8) &= (3.78 \div 5.4) \times (10^{-3} \div 10^8)\\ &= 0.7 \times 10^{-3-8}\\ &= 0.7 \times 10^{-11}\\ &= 7 \times 10^{-1} \times 10^{-11}\\ &= 7 \times 10^{-12}\end{aligned}$$

CALCULATORS AND SCIENTIFIC NOTATION

If a very large or very small number contains too many digits to be shown on the display of a scientific calculator, it will be converted to scientific notation. For example, if you enter the product

6480000 [×] **75000** [=]

on a calculator, the answer will be displayed as



4.86   11

The space between the 4.86 and the 11 indicates that the number is in scientific notation, and 11 is the power of 10. Interpret this as

$6{,}480{,}000 \times 75{,}000 = 4.86 \times 10^{11}$

Similarly, the division

.000006 $\boxed{\div}$ **48000000** $\boxed{=}$

gives the display **1.25 −13**.

So $0.000006 \div 48{,}000{,}000 = 1.25 \times 10^{-13}$.

You may also enter numbers in scientific notation directly into your calculator using a key labelled $\boxed{\text{EXP}}$ or $\boxed{\text{EE}}$. To enter

$0.000006 \div 48{,}000{,}000$ or $(6 \times 10^{-6}) \div (4.8 \times 10^{7})$

enter

6 $\boxed{\text{EXP}}$ **6** $\boxed{+/-}$ $\boxed{\div}$ **4.8** $\boxed{\text{EXP}}$ **7** $\boxed{=}$

and your calculator will again display **1.25 −13**.

Now turn to Exercises 7-8 for a set of problems on scientific notation.

Exercises 7-8

Scientific Notation

A. Rewrite each number in scientific notation.

1. 5000	2. 450	3. 90	4. 40,700
5. 0.003	6. 0.071	7. 0.0004	8. 0.0059
9. 6,770,000	10. 38,200	11. 0.0292	12. 0.009901
13. 1001	14. 0.0020	15. 0.000107	16. 810,000
17. 31.4	18. 0.6	19. 125	20. 0.74

21. Young's modulus for the elasticity of steel: 29,000,000 lb/in.^{2}
22. Thermal conductivity of wood: 0.00024 cal/cm·s
23. Power output: 95,500,000 W
24. Speed of light: 658,800,000 mph

B. Rewrite each number in decimal form.

1. 2×10^5	2. 7×10^6	3. 9×10^{-5}	4. 3×10^{-4}
5. 1.7×10^{-3}	6. 3.7×10^{-2}	7. 5.1×10^4	8. 8.7×10^2
9. 4.05×10^4	10. 7.01×10^{-6}	11. 3.205×10^{-3}	12. 1.007×10^3
13. 2.45×10^6	14. 3.19×10^{-4}	15. 6.47×10^5	16. 8.26×10^{-7}

C. Rewrite each of the following in scientific notation and calculate the answer in scientific notation. If necessary, convert your answer to scientific notation. Round to one decimal place if necessary.

1. $2000 \times 40{,}000$	2. 0.0037×0.0000024
3. $460{,}000 \times 0.0017$	4. $0.0018 \times 550{,}000$
5. $0.0000089 \div 3200$	6. $0.000125 \div 5000$
7. $45{,}500 \div 0.0091$	8. $12{,}450 \div 0.0083$
9. $2{,}240{,}000 \div 16{,}000$	10. $25{,}500 \div 1{,}700{,}000$

11. $0.000045 \div 0.00071$

12. $0.000086 \div 0.000901$

13. $9{,}501{,}000 \times 2410$

14. 9800×0.000066

15. $0.0000064 \div 80{,}000$

16. 1070×0.0000055

17. $\dfrac{0.000056}{0.0020}$

18. $\dfrac{0.0507}{43{,}000}$

19. $\dfrac{0.00602 \times 0.000070}{72{,}000}$

20. $\dfrac{2{,}780{,}000 \times 512{,}000}{0.000721}$

21. $64{,}000 \times 2800 \times 370{,}000$

22. $0.00075 \times 0.000062 \times 0.014$

23. $\dfrac{0.0517}{0.0273 \times 0.00469}$

24. $\dfrac{893{,}000}{5620 \times 387{,}000}$

D. Solve.

1. A brick wall 15 m by 25 m is 0.48 m thick. Under particular temperature conditions, the rate of heat flow through the wall, in calories per second, is given by the expression

$$(1.7 \times 10^{-4}) \times \left(\frac{1500 \times 2500}{48}\right)$$

Calculate the value of this quantity to the nearest tenth.

2. If an atom has a diameter of 4×10^{-8} cm and its nucleus has a diameter of 8×10^{-13} cm, find the ratio of the diameter of the nucleus to the diameter of the atom.

3. **Electrical Technology** The capacitance of a certain capacitor (in microfarads) can be found using the following expression. Calculate this capacitance.

$$(5.75 \times 10^{-8}) \times \left(\frac{8.00}{3.00 \times 10^{-3}}\right)$$

4. The energy (in joules) of a photon of visible light with a wavelength of 5.00×10^{-7} m is given by the following expression. Calculate this energy.

$$\frac{(6.63 \times 10^{-34}) \times (3.00 \times 10^{8})}{5.00 \times 10^{-7}}$$

Check your answers, then turn to Problem Set 7 for some practice problems on the work of this chapter.

PROBLEM SET 7

Basic Algebra

Answers are given in the back of the book.

A. Simplify.

1. $4x + 6x$
2. $6y + y + 5y$
3. $3xy + 9xy$
4. $6xy^3 + 9xy^3$
5. $3\frac{1}{3}x - 1\frac{1}{4}x - \frac{5}{8}x$
6. $8v - 8v$
7. $0.27G + 0.78G - 0.65G$
8. $7y - 8y^2 + 9y$
9. $3x \cdot 7x$
10. $(4m)(2m^2)$
11. $(2xy)(-5xyz)$
12. $3x \cdot 3x \cdot 3x$
13. $3(4x - 7)$
14. $2ab(3a^2 - 5b^2)$
15. $(4x + 3) + (5x + 8)$
16. $(7y - 4) - (2y + 3)$
17. $3(x + y) + 6(x - y)$
18. $(x^2 - 6) - 3(2x^2 - 5)$
19. $\dfrac{6^8}{6^4}$
20. $\dfrac{x^3}{x^8}$
21. $\dfrac{-12m^{10}}{4m^2}$
22. $\dfrac{6y}{-10y^3}$
23. $\dfrac{-16a^2b^2}{-4ab^3}$
24. $\dfrac{8x^4 - 4x^3 + 12x^2}{4x^2}$

B. Find the value of each of the following. Round to two decimal places when necessary.

1. $L = 2W - 3$ for $W = 8$

Name

Date

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2. $M = 3x - 5y + 4z$ for $x = 3, y = 5, z = 6$

3. $I = PRt$ for $P = 800, R = 0.06, t = 3$

4. $I = \dfrac{V}{R}$ for $V = 220, R = 0.0012$

5. $V = LWH$ for $L = 3\frac{1}{2}, W = 2\frac{1}{4}, H = 5\frac{3}{8}$

6. $N = (a + b)(a - b)$ for $a = 7, b = 12$

7. $L = \dfrac{s(P + p)}{2}$ for $s = 3.6, P = 38, p = 26$

8. $f = \dfrac{1}{8N}$ for $N = 6$

9. $t = \dfrac{D - d}{L}$ for $D = 12, d = 4, L = 2$

10. $V = \dfrac{gt^2}{2}$ for $g = 32.2, t = 4.1$

C. Solve the following equations. Round your answer to two decimal places if necessary.

1. $x + 5 = 17$
2. $m + 0.4 = 0.75$
3. $e - 12 = 32$
4. $a - 2.1 = -1.2$
5. $12 = 7 - x$
6. $4\frac{1}{2} - x = -6$
7. $5x = 20$
8. $-4m = 24$
9. $\frac{1}{2}y = 16$
10. $2x + 7 = 13$
11. $-4x + 11 = 35$
12. $0.75 - 5f = 6\frac{1}{2}$
13. $0.5x - 16 = -18$
14. $5y + 8 + 3y = 24$
15. $3g - 12 = g + 8$
16. $7m - 4 = 11 - 3m$
17. $3(x - 5) = 33$
18. $2 - (2x + 3) = 7$
19. $5x - 2(x - 1) = 11$
20. $2(x + 1) - x = 8$
21. $12 - 7y = y - 18$
22. $2m - 6 - 4m = 4m$
23. $9 + 5n + 11 = 10n - 25$
24. $6(x + 4) = 45$
25. $5y - (11 - 2y) = 3$
26. $4(3m - 5) = 8m + 20$

Solve the following formulas for the variable shown.

27. $A = bH$ for b

28. $R = S + P$ for P

29. $P = 2L + 2W$ for L

30. $P = \dfrac{w}{F}$ for F

31. $S = \frac{1}{2}gt - 4$ for g

32. $V = \pi R^2 H - AB$ for A

D. Rewrite in scientific notation.

1. 7500 **2.** 12,800 **3.** 0.041 **4.** 0.000236

5. 0.00572 **6.** 0.00000482 **7.** 447,000 **8.** 2,127,000

9. 80,200,000 **10.** 46,710 **11.** 0.00000705 **12.** 0.001006

Rewrite as a decimal.

13. 9.3×10^5 **14.** 6.02×10^3 **15.** 2.9×10^{-4} **16.** 3.05×10^{-6}

17. 5.146×10^{-7} **18.** 6.203×10^{-3} **19.** 9.071×10^4 **20.** 4.006×10^6

Calculate using scientific notation, and write your answer in standard scientific notation. Round to one decimal place if necessary.

21. $45{,}000 \times 1{,}260{,}000$

22. $625{,}000 \times 12{,}000{,}000$

23. 0.0007×0.0043

24. 0.0000065×0.032

25. $56{,}000 \times 0.0000075$

26. 1020×0.00055

27. $0.0074 \div 0.00006$

28. $0.000063 \div 0.0078$

29. $96{,}000 \div 3{,}400{,}000$

30. $26{,}500{,}000 \div 12{,}000$

31. $0.00089 \div 37{,}000$

32. $123{,}500 \div 0.00077$

E. Practical Problems

1. **Aeronautical Mechanics** In weight and balance calculations, airplane mechanics are concerned with the centre of gravity of an airplane. The centre of gravity may be calculated from the formula

$$\text{CG} = \frac{100(H - x)}{L}$$

where H is the distance from the datum to the empty CG, x is the distance from the datum to the leading edge of the mean aerodynamic chord (MAC), and L is the length of the MAC. CG is expressed as a percent of the MAC.

(All lengths are in inches.)

(a) Find the centre of gravity if $H = 180$, $x = 155$, and $L = 80$.

(b) Solve the formula for L, and find L when CG $= 30\%$, $H = 200$, and $x = 150$.

(c) Solve the formula for H, and find H when CG = 25%, $x = 125$, and $L = 60$.
(d) Solve the formula for x, and find x when CG = 28%, $H = 170$, and $L = 50$.

2. **Electrical Technology** Electricians use a formula which states that the level of light in a room, in foot-candles, is equal to the product of the fixture rating, in lumens, the coefficient of depreciation, and the coefficient of utilization, all divided by the area, in square feet.
(a) State this as an algebraic equation.
(b) Find the level of illumination for four fixtures rated at 2800 lumens each if the coefficient of depreciation is 0.75, the coefficient of utilization is 0.6, and the area of the room is 120 sq ft.
(c) Solve for area.
(d) Use your answer to (c) to determine the size of the room in which a level of 60 foot-candles can be achieved with 10,000 lumens, given that the coefficient of depreciation is 0.8 and the coefficient of utilization is 0.5.

3. **Sheet-Metal Technology** The length of a piece of sheet metal is four times its width. The perimeter of the sheet is 80 cm. Set up an equation relating these measurements and solve the equation to find the dimensions of the sheet.

4. **Construction Technology** Structural engineers have found that a good estimate of the crushing load for a square wooden pillar is given by the formula

$$L = \frac{25T^4}{H^2}$$ where L is the crushing load in tons, T is the thickness of the wood in inches, and H is the height of the post in feet

Find the crushing load for a 6 in.-thick post 12 ft high.

5. **Machine Technology** Two shims must have a combined thickness of 0.048 cm. One shim must be twice as thick as the other. Set up an equation and solve for the thickness of each shim.

6. **Office Services** There are three partners in a welding business. Because of the differences in their contributions, it is decided that Partner C's share of the profits will be one-and-a-half times as much as Partner A's share, and that Partner B's share will be twice as much as Partner C's share. How should a profit of $48,200 be divided?

7. **Electrical Engineering** The calculation shown is used to find the velocity of an electron at an anode. Calculate this velocity (units are in metres per second).

$$\sqrt{\frac{2(1.6 \times 10^{-19})(2.7 \times 10^2)}{9.1 \times 10^{-31}}}$$

8. **Plumbing** The formula

$$N = \sqrt{\left(\frac{D}{d}\right)^5}$$

gives the approximate number N of smaller pipes of diameter d necessary to supply the same total flow as one larger pipe of diameter D. Unlike the formula in problem 22 on page 339, this formula takes into account the extra friction caused by the smaller pipes. Use this formula to determine the number of $\frac{1}{2}$ in. pipes that will provide the same flow as one $1\frac{1}{2}$ in. pipe.

9. **Sheet-Metal Technology** The length L of the stretchout for a square pipe with a grooved seam is given by

$$L = 4s + 3w$$

where s is the side length of the square end and w is the width of the lock. Find the length of stretchout for a square pipe $1\frac{3}{4}$ in. on a side with a lock width of $\frac{3}{16}$ in.

10. **Refrigeration and Air Conditioning** In planning a solar energy heating system for a house, a contractor uses the formula

$Q = 8.33GDT$ to determine the energy necessary for heating water.

In this formula Q is the energy in BTU, G is the number of gallons heated per day, D is the number of days, and T is the temperature difference between tap water and the desired temperature of hot water. Find Q when G is 50 gallons, D is 30 days, and the water must be heated from 60° to 140°.

11. **Sheet-Metal Technology** The bend allowance for sheet metal is given by the formula

$BA = N(0.01743R + 0.0078T)$ where N is the angle of the bend in degrees, R is the inside radius of the bend, and T is the thickness of the metal.

Find BA if N is 47°, R is 18.4 mm, and T is 1.5 mm. Round to three significant digits.

12. **Machine Technology** To find the taper per millimetre of a piece of work, a machinist uses the formula

$T = \dfrac{D - d}{L}$ where D is the diameter of the large end, d is the diameter of the small end, and L is the length.

Find T if $D = 41.625$ mm, $d = 32.513$ mm, and $L = 80$ mm

13. **Construction Technology** The modulus of elasticity of a beam is 2,650,000 at a deflection limit of 360. If the modulus is directly proportional to the deflection limit, find the modulus at a deflection limit of 240. Round to three significant digits.

14. **Automotive Technology** The Static Stability Factor (SSF) is a measure of the relative rollover risk associated with an automobile.

$$\text{SSF} = \frac{1}{2} \times \frac{\text{Tire-track Width}}{\text{Height of Centre of Gravity}}$$

An SSF of 1.04 or less indicates a 40% or higher rollover risk and an SSF of 1.45 or more indicates a 10% or less rollover risk. A particular SUV has a tire-track width of 56 in. and a centre of gravity 26 in. above the road.

(a) Calculate the SSF for this SUV.

(b) By how much would auto designers need to lower the centre of gravity to improve the SSF to 1.4?