Scientific Notation

In technical and scientific publications, you will encounter either very small numbers or very large numbers...it becomes more convenient to write them in shorthand notation...scientific notation

$$P \times 10^{k}$$

Where $1 \leq P < 10$ for positive numbers
Where $-10 < P \leq -1$ for negative numbers

$29000 = 2.9 \text{ x } 10000 = 2.9 \text{ x } 10^4$

$$0.00834 = \frac{8.34}{1000} = \frac{8.34}{10^3} = 8.34 \times 10^{-3}$$

Engineering Notation

Engineering notation is similar to scientific notation with the stipulation that the power of 10 be divisible by a factor of 3

$$P \times 10^{k}$$

Where $1 \leq P < 1000$ and k is an integer multiple of 3

$19680000000 = 19.68 \times 10^{9}$

$0.45 = 450 \times 10^{-3}$

$5.36 \times 10^{4} = 53.6 \times 10^{3}$

Scientific Notation

Due to the limits of measurement for some electronic instruments, 3 significant digits is typical

764832 expressed in scientific notation and with 3 significant digits becomes

7.65×10^5

For numbers greater than 1, move the decimal point to the left and the exponent for the power of 10 becomes positive

29308 1654736

For numbers less than 1, move the decimal point to the right and the exponent for the power of 10 becomes negative

0.0001678 0.0000009735

Scientific Notation

Using the concepts learned from power of 10, let's represent the following number using positive and negative exponents

470. = 47.0 x 10 = 4.70 x 100 = 0.470 x 1000

470. = 47.0 x 10^1 = 4.70 x 10^2 = 0.470 x 10^3

For every 1 increase in exponent (multiply by 10), the decimal of the number is moved to the left 1 place value

470. = 4700/10 = 47000/100 = 470000/1000

470. = 4700/10^1 = 47000/10^2 = 470000/10^3

470. = 4700 x 10^{-1} = 47000 x 10^{-2} = 470000 x 10^{-3}

For every 1 decrease in exponent (divide by 10), the decimal of the number is moved to the right 1 place value

Addition and Subtraction in Scientific Notation

As long as the exponent for the powers of 10 are the same, adding and subtracting will follow normal algorithms for such operations

$3.1 \times 10^2 + 2.7 \times 10^2 =$

$3.8 \times 10^3 + 4.3 \times 10^2 =$

Since the powers of 10 are not equal value, change the power of 10 of one of the numbers to match the other before adding or subtracting

Problems with Complex Denominators

Simplify the numerator and denominator separately, then simplify. Final answer should be in scientific notation.

$$\frac{25 \times 3.3 \times 10^{-2}}{3.3 \times 10^{3} + 4.7 \times 10^{4}}$$

Review Questions

Review question set 2

Reciprocals

Reciprocal of a number is that number divided by 1

$$\frac{1}{27\times10^{3}}=\frac{1}{27}\times\frac{1}{10^{3}}=0.037\times10^{-3}=3.7\times10^{-5}$$

The general rule for sum and product fractions. Please note the distinction between the two types

$$\frac{1}{a+b}\neq\frac{1}{a}+\frac{1}{b} \qquad \frac{1}{ab}=\left(\frac{1}{a}\right)\left(\frac{1}{b}\right)$$

$$\frac{1}{22\times10^{2}}+\frac{1}{47\times10^{3}}$$

Raising a Power of 10 to another Power

When raising a power to a power, multiply the exponents

$$\left(10^{3}\right)^{2} = 10^{3} \times 10^{3} = 10^{6}$$

$$\left(10^{-2}\right)^{3} = 10^{-2} \times 10^{-2} \times 10^{-2} = 10^{-6}$$

The general rule for raising a power to another power

$$\left(10^{a}\right)^{b} = 10^{ab}$$

$$\left(\frac{10^{3}}{10^{-7}}\right)^{2}$$

Squares or nth Powers

Two possible ways to solve this problem

$$\left(0.0236\times10^{2}\right)^{2}=2.36\times2.36=5.57$$

$$\left(0.0236\times10^{2}\right)^{2}=0.0236^{2}\times\left(10^{2}\right)^{2}=0.000557\times10^{4}=5.57$$

The general rule for sum and product of powers. Please note the distinction between the two types

$$\left(a+b\right)^{n}\neq a^{n}+b^{n}$$

$$\left(ab\right)^{n}=\left(a^{n}\right)\left(b^{n}\right)$$

$$\left(0.00873\times10^{4}\right)^{-2}$$

Square Roots or nth Roots

Simplify as much as possible before taking square root

$$\sqrt{2.69\times10^{-6}}=\sqrt{2.69}\times\sqrt{10^{-6}}=2.69^{1/2}\times\left(10^{-6}\right)^{1/2}=1.64\times10^{-3}$$

Root of any degree can be represented as a fractional power

$$\sqrt{a}=a^{1/2} \qquad \sqrt[m]{a^n}=a^{n/m}$$

Please note the distinction between the two types

$$\sqrt{a+b}\neq\sqrt{a}+\sqrt{b} \qquad \sqrt{ab}=\left(\sqrt{a}\right)\left(\sqrt{b}\right)$$

$$\sqrt{4.91\times10^{-6}} \qquad \sqrt[4]{6.82\times10^{8}}$$

Review Questions

Review question set 3