[1] Basic Algebraic Operations

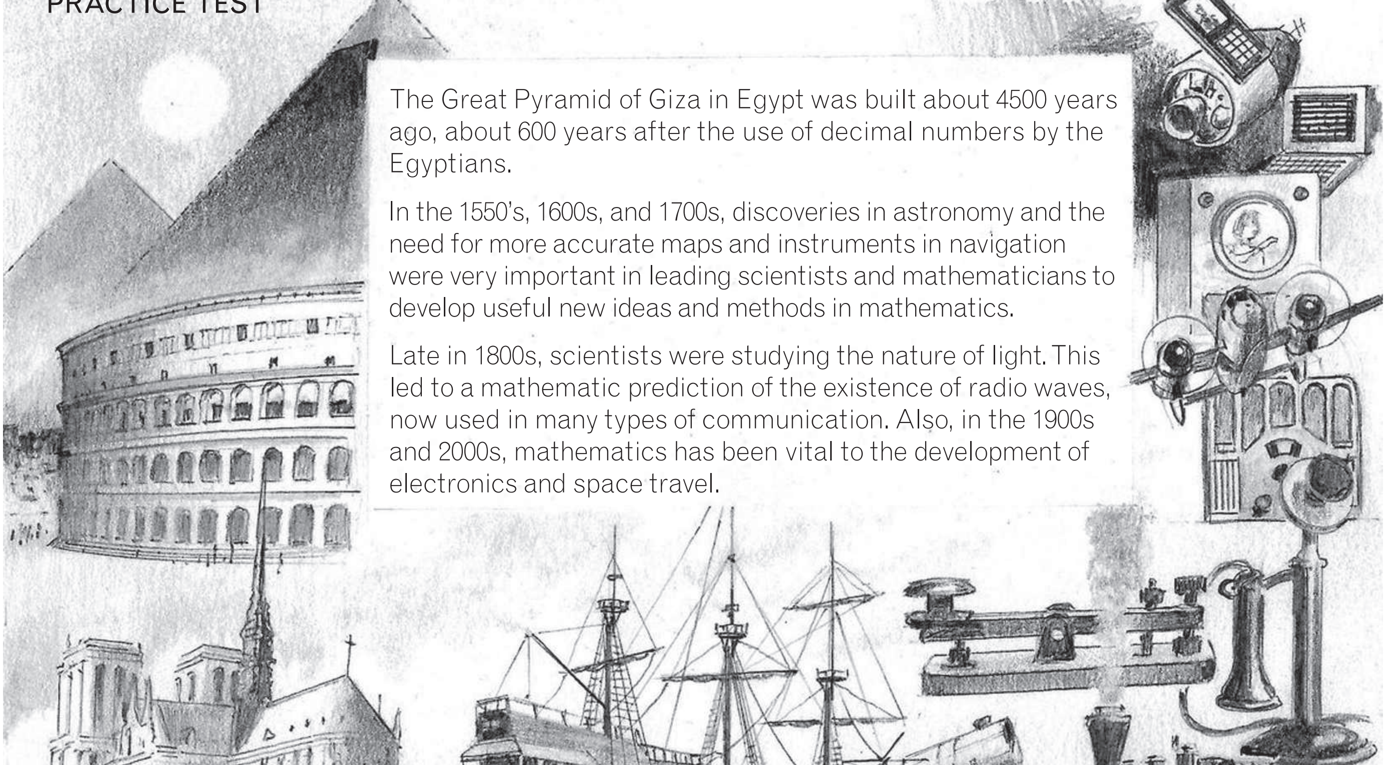
Interest in things such as the land on which they lived, the structures they built, and the motion of the planets led people in early civilizations to keep records and to create methods of counting and measuring. In turn, some of the early ideas of arithmetic, geometry, and trigonometry were developed. From such beginnings, mathematics has played a key role in the great advances in science and technology.

Often, mathematical methods were developed from studies made in sciences, such as astronomy and physics, to better describe and understand the subject being studied. Some of these methods resulted from the needs in a particular area of application.

Many people were interested in the math itself and added to what was then known. Although this additional mathematical knowledge may not have been related to applications at the time it was developed, it often later became useful in applied areas.

In the chapter introductions that follow, examples of the interaction of technology and mathematics are given. From these examples and the text material, it is hoped you will better understand the important role that math has had and still has in technology. In this text, there are applications from technologies including (but not limited to) aeronautical, business, communications, electricity, electronics, engineering, environmental, heat and air conditioning, mechanical, medical, meteorology, petroleum, product design, solar, and space. To solve the applied problems in this text will require a knowledge of the mathematics presented but will *not* require prior knowledge of the field of application.

We begin by reviewing the concepts that deal with numbers and symbols. This will enable us to develop topics in algebra, an understanding of which is essential for progress in other areas such as geometry, trigonometry, and calculus.



The Great Pyramid of Giza in Egypt was built about 4500 years ago, about 600 years after the use of decimal numbers by the Egyptians.

In the 1550's, 1600s, and 1700s, discoveries in astronomy and the need for more accurate maps and instruments in navigation were very important in leading scientists and mathematicians to develop useful new ideas and methods in mathematics.

Late in 1800s, scientists were studying the nature of light. This led to a mathematic prediction of the existence of radio waves, now used in many types of communication. Also, in the 1900s and 2000s, mathematics has been vital to the development of electronics and space travel.

1.1 Numbers

Real Number System • Number Line • Absolute Value • Signs of Inequality • Reciprocal • Denominate Numbers • Literal Numbers

In technology and science, as well as in everyday life, we use the very familiar **counting numbers,** or **natural numbers** 1, 2, 3, and so on. Because it is necessary and useful to use negative numbers as well as positive numbers in mathematics and its applications, the natural numbers are called the **positive integers,** and the numbers -1, -2, -3, and so on are the **negative integers.**

Therefore, *the* **integers** *include the positive integers, the negative integers*, and **zero,** *which is neither positive nor negative*. This means that the integers are the numbers . . . , $-3, -2, -1, 0, 1, 2, 3$ . . . and so on.

A **rational number** *is a number that can be expressed as the division of one integer* ***a*** *by another nonzero integer* ***b****, and can be represented by the fraction* $\boldsymbol{a/b}$. Here ***a*** *is the* **numerator** *and* ***b*** *is the* **denominator.** Here we have used algebra by letting letters represent numbers.

■ Irrational numbers were discussed by the Greek mathematician Pythagoras in about 540 B.C.E.

Another type of number, an **irrational number,** *cannot be written in the form of a fraction that is the division of one integer by another integer*. The following example illustrates integers, rational numbers, and irrational numbers.

EXAMPLE 1 Identifying rational numbers and irrational numbers

The numbers 5 and -19 are integers. They are also rational numbers because they can be written as $\frac{5}{1}$ and $\frac{-19}{1}$, respectively. Normally, we do not write the 1's in the denominators.

The numbers $\frac{5}{8}$ and $\frac{-11}{3}$ are rational numbers because the numerator and the denominator of each are integers.

■ For reference, $\pi = 3.14159265\ldots$

The numbers $\sqrt{2}$ and π are irrational numbers. It is not possible to find two integers, one divided by the other, to represent either of these numbers. It can be shown that square roots (and other roots) that cannot be expressed exactly in decimal form are irrational. Also, $\frac{22}{7}$ is sometimes used as an *approximation* for π, but it is not equal *exactly* to π. We must remember that $\frac{22}{7}$ is rational and π is irrational.

■ A notation that is often used for repeating decimals is to place a bar over the digits that repeat. Using this notation we can write $\frac{1121}{1665} = 0.6\overline{732}$ and $\frac{2}{3} = 0.\overline{6}$.

The decimal number 1.5 is rational since it can be written as $\frac{3}{2}$. Any such *terminating decimal* is rational. The number 0.6666 . . . , where the 6's continue on indefinitely, is rational because we may write it as $\frac{2}{3}$. In fact, any *repeating decimal* (in decimal form, a specific sequence of digits is repeated indefinitely) is rational. The decimal number 0.6732732732 . . . is a repeating decimal where the sequence of digits 732 is repeated indefinitely $(0.6732732732\ldots = \frac{1121}{1665})$. ■

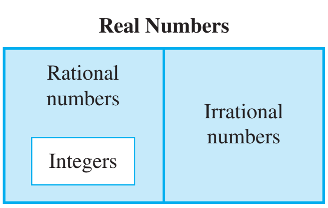


Fig. 1.1

The integers, the rational numbers, and the irrational numbers, including all such numbers that are positive, negative, or zero, make up the **real number system** (see Fig. 1.1). There are times we will encounter an **imaginary number,** *the name given to the square root of a negative number*. Imaginary numbers are not real numbers and will be discussed in Chapter 12. However, unless specifically noted, we will use real numbers. Until Chapter 12, it will be necessary to only *recognize* imaginary numbers when they occur.

Also in Chapter 12, we will consider **complex numbers,** which include both the real numbers and imaginary numbers. See Exercise 37 of this section.

■ Real numbers and imaginary numbers are both included in the *complex number system*. See Exercise 37.

EXAMPLE 2 Identifying real numbers and imaginary numbers

The number 7 is an integer. It is also rational because $7 = \frac{7}{1}$, and it is a real number since the real numbers include all the rational numbers.

The number 3π is irrational, and it is real because the real numbers include all the irrational numbers.

The numbers $\sqrt{-10}$ and $-\sqrt{-7}$ are imaginary numbers.

The number $\frac{-3}{7}$ is rational and real. The number $-\sqrt{7}$ is irrational and real.

The number $\frac{\pi}{6}$ is irrational and real. The number $\frac{\sqrt{-3}}{2}$ is imaginary. ■

■ Fractions were used by early Egyptians and Babylonians. They were used for calculations that involved parts of measurements, property, and possessions.

A **fraction** *may contain any number or symbol representing a number in its numerator or in its denominator*. The fraction indicates the division of the numerator by the denominator, as we previously indicated in writing rational numbers. Therefore, a fraction may be a number that is rational, irrational, or imaginary.

EXAMPLE 3 Fractions

The numbers $\frac{2}{7}$ and $\frac{-3}{2}$ are fractions, and they are rational.

The numbers $\frac{\sqrt{2}}{9}$ and $\frac{6}{\pi}$ are fractions, but they are not rational numbers. It is not possible to express either as one integer divided by another integer.

The number $\frac{\sqrt{-5}}{6}$ is a fraction, and it is an imaginary number. ■

The Number Line

Real numbers may be represented by points on a line. We draw a horizontal line and designate some point on it by O, which we call the **origin** (see Fig. 1.2). The integer *zero* is located at this point. Equal intervals are marked to the right of the origin, and the positive integers are placed at these positions. The other positive rational numbers are located between the integers. The points that cannot be defined as rational numbers represent irrational numbers. We cannot tell whether a given point represents a rational number or an irrational number unless it is specifically marked to indicate its value.

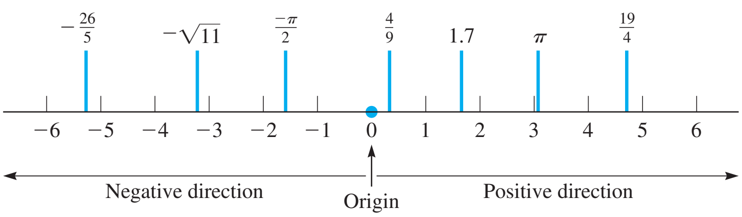


Fig. 1.2

The negative numbers are located on the number line by starting at the origin and marking off equal intervals *to the left, which is the* **negative direction.** As shown in Fig. 1.2, *the positive numbers are to the right of the origin and the negative numbers are to the left of the origin*. Representing numbers in this way is especially useful for graphical methods.

We next define another important concept of a number. *The* **absolute value** *of a positive number is the number itself, and the absolute value of a negative number is the corresponding positive number*. On the number line, we may interpret the absolute value of a number as the distance (which is always positive) between the origin and the number. Absolute value is denoted by writing the number between vertical lines, as shown in the following example.

EXAMPLE 4 Absolute value

Practice Exercises

1. $|-4.2| = ?$ **2.** $-\left|-\frac{3}{4}\right| = ?$

The absolute value of 6 is 6, and the absolute value of -7 is 7. We write these as $|6| = 6$ and $|-7| = 7$. See Fig. 1.3.

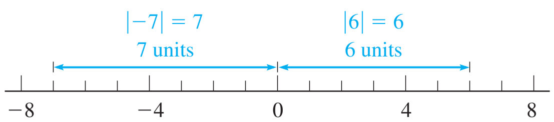


Fig. 1.3

Other examples are $\left|\frac{7}{5}\right| = \frac{7}{5}$, $|-\sqrt{2}| = \sqrt{2}$, $|0| = 0$, $-|\pi| = -\pi$, $|-5.29| = 5.29$, $-|-9| = -9$ since $|-9| = 9$. ■

■ The symbols $=$, $<$, and $>$ were introduced by English mathematicians in the late 1500s.

On the number line, *if a first number is to the right of a second number, then the first number is said to be* **greater than** *the second. If the first number is to the left of the second, it is* **less than** *the second number*. The symbol $>$ designates "is greater than," and the symbol $<$ designates "is less than." These are called **signs of inequality.** See Fig. 1.4.

Practice Exercises

Place the correct sign of inequality ($<$ or $>$) between the given numbers.

3. $-5 \quad 4$ **4.** $0 \quad -3$

EXAMPLE 5 Signs of inequality

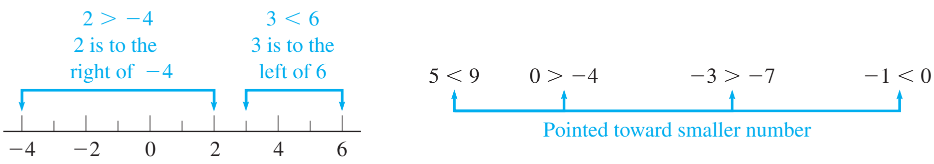


Fig. 1.4 ■

Every number, except zero, has a **reciprocal.** *The reciprocal of a number is 1 divided by the number.*

Practice Exercise

5. Find the reciprocals of

(a) -4 **(b)** $\frac{3}{8}$

EXAMPLE 6 Reciprocal

The reciprocal of 7 is $\frac{1}{7}$. The reciprocal of $\frac{2}{3}$ is

$$\frac{1}{\frac{2}{3}} = 1 \times \frac{3}{2} = \frac{3}{2} \qquad \text{invert denominator and multiply (from arithmetic)}$$

The reciprocal of 0.5 is $\frac{1}{0.5} = 2$. The reciprocal of $-\pi$ is $-\frac{1}{\pi}$. Note that the negative sign is retained in the reciprocal of a negative number.

We showed the multiplication of 1 and $\frac{3}{2}$ as $1 \times \frac{3}{2}$. We could also show it as $1 \cdot \frac{3}{2}$ or $1\left(\frac{3}{2}\right)$. We will often find the form with parentheses is preferable. ■

In applications, *numbers that represent a measurement and are written with units of measurement are called* **denominate numbers.** The next example illustrates the use of units and the symbols that represent them.

■ For reference, see Appendix B for units of measurement and the symbols used for them.

EXAMPLE 7 Denominate numbers

To show that a certain TV weighs 62 pounds, we write the weight as 62 lb.

To show that a giant redwood tree is 330 feet high, we write the height as 300 ft.

To show that the speed of a rocket is 1500 meters per second, we write the speed as 1500 m/s. (Note the use of s for second. We use s rather than sec.)

To show that the area of a computer chip is 0.75 square inch, we write the area as 0.75 in.^{2}. (We will not use sq in.)

To show that the volume of water in a glass tube is 25 cubic centimeters, we write the volume as 25 cm^3. (We will not use cu cm nor cc.) ■

Literal Numbers

It is usually more convenient to state definitions and operations on numbers in a general form. *To do this, we represent the numbers by letters, called* **literal numbers.** For example, if we want to say "If a first number is to the right of a second number on the number line, then the first number is greater than the second number," we can write "If a is to the right of b on the number line, then $a > b$." Another example of using a literal number is "The reciprocal of n is $1/n$."

Certain literal numbers may take on any allowable value, whereas other literal numbers represent the same value throughout the discussion. *Those literal numbers that may vary in a given problem are called* **variables,** and *those literal numbers that are held fixed are called* **constants.**

EXAMPLE 8 Variables and constants

(a) The resistance of an electric resistor is R. The current I in the resistor equals the voltage V divided by R, written as $I = V/R$. For this resistor, I and V may take on various values, and R is fixed. This means I and V are variables and R is a constant. For a *different* resistor, the value of R may differ.

(b) The fixed cost for a calculator manufacturer to operate a certain plant is b dollars per day, and it costs a dollars to produce each calculator. The total daily cost C to produce n calculators is

$$C = an + b$$

Here, C and n are variables, and a and b are constants, and the product of a and n is shown as an. For *another* plant, the values of a and b would probably differ.

If specific numerical values of a and b are known, say $a = \$7$ per calculator and $b = \$3000$, then $C = 7n + 3000$. Thus, constants may be numerical or literal. ■

EXERCISES 1.1

In Exercises 1–4, make the given changes in the indicated examples of this section, and then answer the given questions.

1. In the first line of Example 1, change the 5 to -3 and the -19 to 14. What other changes must then be made in the first paragraph?

2. In Example 4, change the 6 to -6. What other changes must then be made in the first paragraph?

3. In the left figure of Example 5, change the 2 to -6. What other changes must then be made?

4. In Example 6, change the $\frac{2}{3}$ to $\frac{3}{2}$. What other changes must then be made?

In Exercises 5 and 6, designate each of the given numbers as being an integer, rational, irrational, real, or imaginary. (More than one designation may be correct.)

5. $3, \quad \sqrt{-4}, \quad -\frac{\pi}{6}, \quad \frac{1}{8}$

6. $-\sqrt{-6}, \quad -2.33, \quad \frac{\sqrt{7}}{3}, \quad -6$

In Exercises 7 and 8, find the absolute value of each real number.

7. $3, \quad -4, \quad -\frac{\pi}{2}, \quad \sqrt{-1}$

8. $-0.857, \quad \sqrt{2}, \quad -\frac{19}{4}, \quad \frac{\sqrt{-5}}{-2}$

In Exercises 9–16, insert the correct sign of inequality (> or <) between the given numbers.

9. $6 \qquad 8$

10. $7 \qquad 5$

11. $-\pi \qquad -3.1416$

12. $-4 \qquad 0$

13. $-4 \qquad -|-3|$

14. $-\sqrt{2} \qquad -1.42$

15. $-\frac{1}{3} \qquad -\frac{1}{2}$

16. $-0.6 \qquad 0.2$

In Exercises 17 and 18, find the reciprocal of each number.

17. $3, \quad -\frac{4}{\sqrt{3}}, \quad \frac{y}{b}$

18. $-\frac{1}{3}, \quad 0.25, \quad 2x$

In Exercises 19 and 20, locate (approximately) each number on a number line as in Fig. 1.2.

19. $2.5, \quad -\frac{12}{5}, \quad \sqrt{3}, \quad -\frac{3}{4}$

20. $-\frac{\sqrt{2}}{2}, \quad 2\pi, \quad \frac{123}{19}, \quad -\frac{7}{3}$

In Exercises 21–44, solve the given problems. Refer to Appendix B for units of measurement and their symbols.

W **21.** Is an absolute value always positive? Explain.

W **22.** Is -2.17 rational? Explain.

23. What is the reciprocal of the reciprocal of any positive or negative number?

24. Find a rational number between -0.9 and -1.0 that can be written with a denominator of 11 and an integer in the numerator.

25. Find a rational number between 0.13 and 0.14 that can be written with a numerator of 3 and an integer in the denominator.

26. If $b > a$ and $a > 0$, is $|b - a| < |b| - |a|$?

27. List the following numbers in numerical order, starting with the smallest: $-1, 9, \pi, \sqrt{5}, |-8|, -|-3|, -3.1$.

28. List the following numbers in numerical order, starting with the smallest: $\frac{1}{5}, -\sqrt{10}, -|-6|, -4, 0.25, |-\pi|$.

29. If a and b are positive integers and $b > a$, what type of number is represented by the following?

(a) $b - a$ **(b)** $a - b$ **(c)** $\frac{b - a}{b + a}$

30. If a and b represent positive integers, what kind of number is represented by (a) $a + b$, (b) a/b, and (c) $a \times b$?

31. For any positive or negative integer: (a) Is its absolute value always an integer? (b) Is its reciprocal always a rational number?

32. For any positive or negative rational number: (a) Is its absolute value always a rational number? (b) Is its reciprocal always a rational number?

W **33.** Describe the location of a number x on the number line when (a) $x > 0$ and (b) $x < -4$.

34. Describe the location of a number x on the number line when (a) $|x| < 1$ and (b) $|x| > 2$.

35. For a number $x > 1$, describe the location on the number line of the reciprocal of x.

36. For a number $x < 0$, describe the location on the number line of the number with a value of $|x|$.

37. A *complex number* is defined as $a + bj$, where a and b are real numbers and $j = \sqrt{-1}$. For what values of a and b is the complex number $a + bj$ a real number? (All real numbers and all imaginary numbers are also complex numbers.)

38. A sensitive gauge measures the total weight w of a container and the water that forms in it as vapor condenses. It is found that $w = c\sqrt{0.1t + 1}$, where c is the weight of the container and t is the time of condensation. Identify the variables and constants.

39. In an electric circuit, the reciprocal of the total capacitance of two capacitors in series is the sum of the reciprocals of the capacitances. Find the total capacitance of two capacitances of 0.0040 F and 0.0010 F connected in series.

40. Alternating-current (ac) voltages change rapidly between positive and negative values. If a voltage of 100 V changes to −200 V, which is greater in absolute value?

41. The memory of a certain computer has a bits in each byte. Express the number N of bits in n kilobytes in an equation. (A *bit* is a single digit, and bits are grouped in *bytes* in order to represent special characters. Generally, there are 8 bits per byte. If necessary, see Appendix B for the meaning of *kilo*.)

42. The computer design of the base of a truss is x ft. long. Later it is redesigned and shortened by y in. Give an equation for the length L, in inches, of the base in the second design.

43. In a laboratory report, a student wrote "$-20°C > -30°C$." Is this statement correct? Explain.

44. After 5 s, the pressure on a valve is less than 60 lb/in.^{2} (pounds per square inch). Using t to represent time and p to represent pressure, this statement can be written "for $t > 5$ s, $p < 60$ lb/in.^{2}." In this way, write the statement "when the current I in a circuit is less than 4 A, the resistance R is greater than 12 Ω (ohms)."

Answers to Practice Exercises

1. 4.2 **2.** $-\frac{3}{4}$ **3.** $<$ **4.** $>$ **5. (a)** $-\frac{1}{4}$ **(b)** $\frac{8}{3}$

1.2 Fundamental Operations of Algebra

Fundamental Laws of Algebra • Operations on Positive and Negative Numbers • Order of Operations • Operations with Zero

The Commutative and Associative Laws

If two numbers are added, it does not matter in which order they are added. (For example, $5 + 3 = 8$ and $3 + 5 = 8$, or $5 + 3 = 3 + 5$.) This statement, generalized and accepted as being correct for all possible combinations of numbers being added, is called the **commutative law** for addition. It states that *the sum of two numbers is the same, regardless of the order in which they are added*. We make no attempt to prove this law in general, but accept that it is true.

In the same way, we have the **associative law** for addition, which states that *the sum of three or more numbers is the same, regardless of the way in which they are grouped for addition*. For example, $3 + (5 + 6) = (3 + 5) + 6$.

The laws just stated for addition are also true for multiplication. Therefore, *the product of two numbers is the same, regardless of the order in which they are multiplied,* and *the product of three or more numbers is the same, regardless of the way in which they are grouped for multiplication*. For example, $2 \times 5 = 5 \times 2$, and $5 \times (4 \times 2) = (5 \times 4) \times 2$.

The Distributive Law

Another very important law is the **distributive law.** It states that *the product of one number and the sum of two or more other numbers is equal to the sum of the products of the first number and each of the other numbers of the sum*. For example,

$$5(4 + 2) = 5 \times 4 + 5 \times 2$$

■ Note carefully the difference:
associative law: $5 \times (4 \times 2)$
distributive law: $5 \times (4 + 2)$

In this case, it can be seen that the total is 30 on each side.

In practice, these **fundamental laws of algebra** are used naturally without thinking about them, except perhaps for the distributive law.

Not all operations are commutative and associative. For example, division is not commutative, because the order of division of two numbers does matter. For instance, $\frac{6}{5} \neq \frac{5}{6}$ ($\neq$ is read "does not equal"). (Also, see Exercise 52.)

Using literal numbers, the fundamental laws of algebra are as follows:

Commutative law of addition: $a + b = b + a$

Associative law of addition: $a + (b + c) = (a + b) + c$

Commutative law of multiplication: $ab = ba$

Associative law of multiplication: $a(bc) = (ab)c$

Distributive law: $a(b + c) = ab + ac$

■ Note the meaning of *identity*.

Each of these laws is an example of an *identity,* in that the expression to the left of the = sign equals the expression to the right for any value of each of a, b, and c.

OPERATIONS ON POSITIVE AND NEGATIVE NUMBERS

When using the basic operations (addition, subtraction, multiplication, division) on positive and negative numbers, we determine the result to be either positive or negative according to the following rules.

Addition of two numbers of the same sign *Add their absolute values and assign the sum their common sign.*

EXAMPLE 1 Adding numbers of the same sign

(a) $2 + 6 = 8$ — the sum of two positive numbers is positive

(b) $-2 + (-6) = -(2 + 6) = -8$ — the sum of two negative numbers is negative

■ From Section 1.1, we recall that a positive number is preceded by no sign. Therefore, in using these rules, we show the "sign" of a positive number by simply writing the number itself.

The negative number -6 is placed in parentheses because it is also preceded by a plus sign showing addition. It is not necessary to place the -2 in parentheses. ■

Addition of two numbers of different signs *Subtract the number of smaller absolute value from the number of larger absolute value and assign to the result the sign of the number of larger absolute value.*

EXAMPLE 2 Adding numbers of different signs

(a) $2 + (-6) = -(6 - 2) = -4$ — the negative 6 has the larger absolute value

(b) $-6 + 2 = -(6 - 2) = -4$

(c) $6 + (-2) = 6 - 2 = 4$ — the positive 6 has the larger absolute value

(d) $-2 + 6 = 6 - 2 = 4$

the subtraction of absolute values ■

Subtraction of one number from another *Change the sign of the number being subtracted and change the subtraction to addition. Perform the addition.*

EXAMPLE 3 Subtracting positive and negative numbers

(a) $2 - 6 = 2 + (-6) = -(6 - 2) = -4$

Note that after changing the subtraction to addition, and changing the sign of 6 to make it -6, we have precisely the same illustration as Example 2(a).

(b) $-2 - 6 = -2 + (-6) = -(2 + 6) = -8$

Note that after changing the subtraction to addition, and changing the sign of 6 to make it -6, we have precisely the same illustration as Example 1(b).

(c) $-a - (-a) = -a + a = 0$

Subtraction of a Negative Number

This shows that subtracting a number from itself results in zero, even if the number is negative. Therefore, *subtracting a negative number is equivalent to adding a positive number of the same absolute value*.

(d) $-2 - (-6) = -2 + 6 = +4 = 4$

(e) The change in temperature from $-12°C$ to $-26°C$ is
$-26°C - (-12°C) = -26°C + 12°C = -14°C$ ■

Multiplication and division of two numbers *The product (or quotient) of two numbers of the same sign is positive. The product (or quotient) of two numbers of different signs is negative.*

Practice Exercises

Evaluate: **1.** $-5-(-8)$
2. $-5(-8)$

EXAMPLE 4 Multiplying and dividing positive and negative numbers

(a) $3(12) = 3 \times 12 = 36 \qquad \frac{12}{3} = 4$ — result is positive if both numbers are positive

(b) $-3(-12) = 3 \times 12 = 36 \qquad \frac{-12}{-3} = 4$ — result is positive if both numbers are negative

(c) $3(-12) = -(3 \times 12) = -36 \qquad \frac{-12}{3} = -\frac{12}{3} = -4$ — result is negative if one number is positive and the other is negative

(d) $-3(12) = -(3 \times 12) = -36 \qquad \frac{12}{-3} = -\frac{12}{3} = -4$ ■

ORDER OF OPERATIONS

Often, how we are to combine numbers is clear by grouping the numbers using symbols such as **parentheses,** ( ), the **bar,** ____ , between the numerator and denominator of a fraction, and **vertical lines** for absolute value. Otherwise, for an expression in which there are several operations, we use the following order of operations.

Order of Operations

1. *Operations within specific groupings are done first.*
2. *Perform multiplications and divisions (from left to right).*
3. *Then perform additions and subtractions (from left to right).*

EXAMPLE 5 Order of operations

■ Note that $20 \div (2+3) = \frac{20}{2+3}$, whereas $20 \div 2 + 3 = \frac{20}{2} + 3$.

(a) $20 \div (2+3)$ is evaluated by first adding $2+3$ and then dividing. The grouping of $2+3$ is clearly shown by the parentheses. Therefore, $20 \div (2+3) = 20 \div 5 = 4$.

(b) $20 \div 2 + 3$ is evaluated by first dividing 20 by 2 and then adding. No specific grouping is shown, and therefore the division is done before the addition. This means $20 \div 2 + 3 = 10 + 3 = 13$.

CAUTION

(c) $16 - 2 \times 3$ is evaluated by ***first multiplying*** 2 ***by*** 3 and then subtracting. We *do* **not** *first subtract* 2 *from* 16. Therefore, $16 - 2 \times 3 = 16 - 6 = 10$.

(d) $16 \div 2 \times 4$ is evaluated by first dividing 16 by 2 and then multiplying. From left to right, the division occurs first. Therefore, $16 \div 2 \times 4 = 8 \times 4 = 32$.

(e) $|3-5| - |-3-6|$ is evaluated by first performing the subtractions within the absolute value vertical bars, then evaluating the absolute values, and then subtracting. This means that $|3-5| - |-3-6| = |-2| - |-9| = 2 - 9 = -7$. ■

Practice Exercises

Evaluate: **3.** $12 - 6 \div 2$
4. $16 \div (2 \times 4)$

When evaluating expressions, it is generally more convenient to change the operations and numbers so that the result is found by the addition and subtraction of positive numbers. When this is done, we must remember that

$$a + (-b) = a - b \qquad (1.1)$$

$$a - (-b) = a + b \qquad (1.2)$$

Practice Exercises

Evaluate: **5.** $2(-3) - \frac{4-8}{2}$

6. $\frac{|5-15|}{2} - \frac{-9}{3}$

EXAMPLE 6 Evaluating numerical expressions

(a) $7 + (-3) - 6 = 7 - 3 - 6 = 4 - 6 = -2$ using Eq. (1.1)

(b) $\frac{18}{-6} + 5 - (-2)(3) = -3 + 5 - (-6) = 2 + 6 = 8$ using Eq. (1.2)

(c) $\frac{|3-15|}{-2} - \frac{8}{4-6} = \frac{12}{-2} - \frac{8}{-2} = -6 - (-4) = -6 + 4 = -2$

(d) $\frac{-12}{2-8} + \frac{5-1}{2(-1)} = \frac{-12}{-6} + \frac{4}{-2} = 2 + (-2) = 2 - 2 = 0$

In illustration (b), we see that the division and multiplication were done before the addition and subtraction. In (c) and (d), we see that the groupings were evaluated first. Then we did the divisions, and finally the subtraction and addition. ■

EXAMPLE 7 Evaluating in an application

A 3000-lb van going at 40 mi/h ran head-on into a 2000-lb car going at 20 mi/h. An insurance investigator determined the velocity of the vehicles immediately after the collision from the following calculation. See Fig. 1.5.

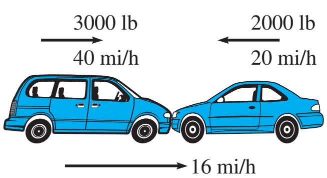


Fig. 1.5

$$\frac{3000(40) + (2000)(-20)}{3000 + 2000} = \frac{120{,}000 + (-40{,}000)}{3000 + 2000} = \frac{120{,}000 - 40{,}000}{5000}$$

$$= \frac{80{,}000}{5000} = 16 \text{ mi/h}$$

The numerator and the denominator must be evaluated before the division is performed. The multiplications in the numerator are performed first, followed by the addition in the denominator and the subtraction in the numerator. ■

OPERATIONS WITH ZERO

Because operations with zero tend to cause some difficulty, we will show them here.

If a is a real number, the operations of addition, subtraction, multiplication, and division with zero are as follows:

$$\mathbf{a + 0 = a}$$

$$\mathbf{a - 0 = a} \qquad \mathbf{0 - a = -a}$$

$$\mathbf{a \times 0 = 0}$$

$$\mathbf{0 \div a = \frac{0}{a} = 0} \qquad \textbf{(if } \mathbf{a \neq 0)}$$ (≠ means "is not equal to")

EXAMPLE 8 Operations with zero

(a) $5 + 0 = 5$ **(b)** $-6 - 0 = -6$ **(c)** $0 - 4 = -4$

(d) $\frac{0}{6} = 0$ **(e)** $\frac{0}{-3} = 0$ **(f)** $\frac{5 \times 0}{7} = \frac{0}{7} = 0$ ■

Note that there is no result defined for division by zero. To understand the reason for this, consider the results for $\frac{6}{2}$ and $\frac{6}{0}$.

$$\frac{6}{2} = 3 \quad \text{since} \quad 2 \times 3 = 6$$

If $\frac{6}{0} = b$, then $0 \times b = 6$. This cannot be true because $0 \times b = 0$ for any value of b. Thus,

NOTE ▸ ***division by zero is undefined***

(The special case of $\frac{0}{0}$ is termed *indeterminate*. If $\frac{0}{0} = b$, then $0 = 0 \times b$, which is true for any value of b. Therefore, no specific value of b can be determined.)

EXAMPLE 9 Division by zero is undefined

$$\frac{2}{5} \div 0 \text{ is undefined} \qquad \frac{8}{0} \text{ is undefined} \qquad \frac{7 \times 0}{0 \times 6} \text{ is indeterminate}$$ ■

see bottom of page 9

The operations with zero will not cause any difficulty if we remember to

CAUTION ▶ ***never divide by zero***

Division by zero is the only undefined basic operation. All the other operations with zero may be performed as for any other number.

EXERCISES 1.2

In Exercises 1–4, make the given changes in the indicated examples of this section, and then solve the resulting problems.

1. In Example 5(c), change 3 to (-3) and then evaluate.
2. In Example 6(b), change 18 to -18 and then evaluate.
3. In Example 6(d), interchange the 2 and 8 in the first denominator and then evaluate.
4. In the rightmost illustration in Example 9, interchange the 6 and the 0 above the 6. Is any other change needed?

In Exercises 5–36, evaluate each of the given expressions by performing the indicated operations.

5. $8 + (-4)$ **6.** $-4 + (-7)$ **7.** $-3 + 9$

8. $18 - 21$ **9.** $-19 - (-16)$ **10.** $-8 - (-10)$

11. $8(-3)$ **12.** $-9(3)$ **13.** $-7(-5)$

14. $\dfrac{-9}{3}$ **15.** $\dfrac{-6(20 - 10)}{-3}$ **16.** $\dfrac{-28}{-7(5 - 6)}$

17. $-2(4)(-5)$ **18.** $-3(-4)(-6)$ **19.** $2(2 - 7) \div 10$

20. $\dfrac{-64}{-2|4 - 8|}$ **21.** $-9 - |2 - 10|$ **22.** $(7 - 7) \div (5 - 7)$

23. $\dfrac{17 - 7}{7 - 7}$ **24.** $\dfrac{(7 - 7)(2)}{(7 - 7)(-1)}$ **25.** $8 - 3(-4)$

26. $-20 + 8 \div 4$ **27.** $-2(-6) + \left|\dfrac{8}{-2}\right|$ **28.** $\dfrac{|-2|}{-2} - (-2)(-5)$

29. $30(-6)(-2) \div (0 - 40)$ **30.** $\dfrac{7 - |-5|}{-1(-2)}$

31. $\dfrac{24}{3 + (-5)} - 4(-9) \div (-3)$ **32.** $\dfrac{-18}{3} - \dfrac{4 - |-6|}{-1}$

33. $-7 - \dfrac{|-14|}{2(2 - 3)} - 3|6 - 8|$ **34.** $-7(-3) + \dfrac{-6}{-3} - |-9|$

35. $\dfrac{3|-9 - 2(-3)|}{1 + (-10)}$ **36.** $\dfrac{20(-12) - 40(-15)}{98 - |-98|}$

In Exercises 37–44, determine which of the fundamental laws of algebra is demonstrated.

37. $6(7) = 7(6)$ **38.** $6 + 8 = 8 + 6$

39. $6(3 + 1) = 6(3) + 6(1)$ **40.** $4(5 \times \pi) = (4 \times 5)(\pi)$

41. $3 + (5 + 9) = (3 + 5) + 9$ **42.** $8(3 - 2) = 8(3) - 8(2)$

43. $(\sqrt{5} \times 3) \times 9 = \sqrt{5} \times (3 \times 9)$

44. $(3 \times 6) \times 7 = 7 \times (3 \times 6)$

In Exercises 45–48, for numbers a and b, determine which of the following expressions equals the given expression.

(a) $a + b$ (b) $a - b$ (c) $b - a$ (d) $-a - b$

45. $-a + (-b)$ **46.** $b - (-a)$

47. $-b - (-a)$ **48.** $-a - (-b)$

In Exercises 49–64, solve the given problems. Refer to Appendix B for units of measurement and their symbols.

49. Insert the proper sign $(=, >, <)$ to make the following true: $|5 - (-2)| \quad |-5 - |-2||$

50. Insert the proper sign $(=, >, <)$ to make the following true: $|-3 - |-7|| \quad ||-3| - 7|$

51. (a) What is the sign of the product of an even number of negative numbers? (b) What is the sign of the product of an odd number of negative numbers?

52. Is subtraction commutative? Explain.

53. Explain why the following definition of the absolute value of a real number x is either correct or incorrect (the symbol $\geq$ means "is equal to or greater than": If $x \geq 0$, then $|x| = x$; if $x < 0$, then $|x| = -x$).

54. Explain what is the error if the expression $24 - 6 \div 2 \cdot 3$ is evaluated as 27. What is the correct value?

55. Describe the values of x and y for which (a) $-xy = 1$ and (b) $\dfrac{x - y}{x - y} = 1$.

56. Describe the values of x and y for which (a) $|x + y| = |x| + |y|$ and (b) $|x - y| = |x| + |y|$.

57. Using subtraction of signed numbers, find the number of minutes from quarter of an hour to 10 min after the hour.

58. Using subtraction of signed numbers, find the difference in the altitude of the bottom of the Dead Sea, 1396 m below sea level, and the bottom of Death Valley, 86 m below sea level.

59. Some solar energy systems are used to supplement the utility company power supplied to a home such that the meter runs backward if the solar energy being generated is greater than the energy being used. With such a system, if the solar power averages 1.5 kW for a 3.0-h period and only 2.1 kW · h is used during this period, what will be the change in the meter reading for this period?

60. A baseball player's batting average (total number of hits divided by total number of at-bats) is expressed in decimal form from 0.000 (no hits for all at-bats) to 1.000 (one hit for each at-bat). A player's batting average is often shown as 0.000 before the first at-bat of the season. Is this a correct batting average? Explain.

61. The daily high temperatures (in °C) for Edmonton, Alberta in the first week in March were recorded as $-7, -3, 2, 3, 1, -4$, and -6. What was the average daily temperature for the week? (Divide the algebraic sum of readings by the number of readings.)

62. A flare is shot up from the top of a tower. Distances above the flare gun are positive and those below it are negative. After 5 s the vertical distance (in ft) of the flare from the flare gun is found by evaluating $(70)(5) + (-16)(25)$. Find this distance.

63. Find the sum of the voltages of the batteries shown in Fig. 1.6. Note the directions in which they are connected.

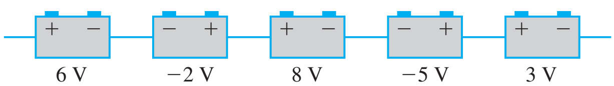


Fig. 1.6

64. A faulty gauge on a fire engine pump caused the apparent pressure in the hose to change every few seconds. The pressures (in $lb/in.^2$ above and below the set pressure were recorded as: $+7, -2, -9, -6$. What was the change between (a) the first two readings, (b) between the middle two readings, and (c) the last two readings?

65. One oil-well drilling rig drills 100 m deep the first day and 200 m deeper the second day. A second rig drills 200 m deep the first day and 100 m deeper the second day. In showing that the total depth drilled by each rig was the same, state what fundamental law of algebra is illustrated.

66. A water tank leaks 12 gal each hour for 7 h, and a second tank leaks 7 gal each hour for 12 h. In showing that the total amount leaked is the same for the two tanks, what fundamental law of algebra is illustrated?

67. Each of four persons spends 8 min browsing one website and 6 min browsing a second website. Set up the expression for the total time these persons spent browsing these websites. What fundamental law of algebra is illustrated?

68. A jet travels 600 mi/h relative to the air. The wind is blowing at 50 mi/h. If the jet travels with the wind for 3 h, set up the expression for the distance traveled. What fundamental law of algebra is illustrated?

Answers to Practice Exercises

1. 3 **2.** 40 **3.** 9 **4.** 2 **5.** -4 **6.** 8

1.3 Calculators and Approximate Numbers

Graphing Calculators • Approximate Numbers • Significant Digits • Accuracy and Precision • Rounding Off • Operations with Approximate Numbers • Estimating Results

You will be doing many of your calculations on a calculator, and a *graphing calculator* can be used for these calculations and many other operations. In this text, we will restrict our coverage of calculator use to graphing calculators because a *scientific calculator* cannot perform many of the required operations we will cover.

■ The calculator screens shown with text material are for a TI-83 or TI-84. They are intended only as an illustration of a calculator screen for the particular operation. Screens for other models may differ.

A brief discussion of the graphing calculator appears in Appendix C, and sample calculator screens appear throughout the book. Since there are many models of graphing calculators, *the notation and screen appearance for many operations will differ from one model to another*. You should *practice using your calculator and **review its manual** to be sure how it is used*. Following is an example of a basic calculation done on a graphing calculator.

EXAMPLE 1 Calculating on a graphing calculator

In order to calculate the value of $38.3 - 12.9(-3.58)$, the numbers are entered as follows. The calculator will perform the multiplication first, following the order of operations shown in Section 1.2. The sign of -3.58 is entered using the (–) key, before 3.58 is entered. The display on the calculator screen is shown in Fig. 1.7.

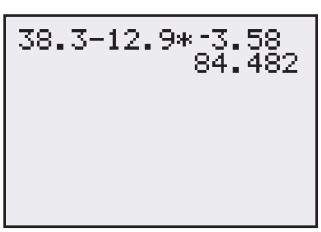


Fig. 1.7

38.3 [−] 12.9 [×] [(–)] 3.58 [ENTER]   keystrokes

This means that $38.3 - 12.9(-3.58) = 84.482$.

Note in the display that the negative sign of -3.58 is smaller and a little higher to distinguish it from the minus sign for subtraction. Also note the * shown for multiplication; the asterisk is the standard computer symbol for multiplication. ■

Looking back into Section 1.2, we see that *the minus sign is used in two different ways:* (1) to indicate subtraction and (2) to designate a negative number. This is clearly shown on a graphing calculator because there is a key for each purpose. The [−] key is used for subtraction, and the [(–)] key is used before a number to make it negative.

■ Some calculator keys on different models are labeled differently. For example, on some models, the EXE key is equivalent to the ENTER key.

■ Calculator keystrokes will generally not be shown, except as they appear in the display screens. They may vary from one model to another.

We will first use a graphing calculator for the purpose of graphing in Section 3.5. Before then, we will show some calculational uses of a graphing calculator.

APPROXIMATE NUMBERS AND SIGNIFICANT DIGITS

Most numbers in technical and scientific work are **approximate numbers,** having been determined by some *measurement*. Certain other numbers are **exact numbers,** having been determined by a *definition* or *counting process*.

EXAMPLE 2 Approximate numbers and exact numbers

One person measures the distance between two cities on a map as 36 cm, and another person measures it as 35.7 cm. However, the distance cannot be measured *exactly*.

If a computer prints out the number of names on a list of 97, this 97 is exact. We know it is not 96 or 98. Since 97 was found from precise counting, it is exact.

By definition, 60 s = 1 min, and the 60 and the 1 are exact. ■

An approximate number may have to include some zeros to properly locate the decimal point. *Except for these zeros, all other digits are called* **significant digits.**

EXAMPLE 3 Significant digits

Practice Exercises

Determine the number of significant digits.

1. 1010 **2.** 0.1010

All numbers in this example are assumed to be approximate.

34.7 has three significant digits.

0.039 has two significant digits. The zeros properly locate the decimal point.

706.1 has four significant digits. The zero is not used for the location of the decimal point. It shows the number of tens in 706.1.

CAUTION ➧ 5.90 has three significant digits. ***The zero is not necessary as a placeholder*** and should not be written unless it is significant.

1400 has two significant digits, unless information is known about the number that makes either or both zeros significant. (A temperature shown as 1400°C has two significant digits. If a price list gives all costs in dollars, a price shown as $1400 has four significant digits.) Without such information, we assume that the zeros are placeholders for proper location of the decimal point.

■ To show that zeros at the end of a whole number are significant, a notation that can be used is to place a bar over the last significant zero. Using this notation, $78{,}0\bar{0}0$ is shown to have four significant digits.

Other approximate numbers with the number of significant digits are 0.0005 (one), 960,000 (two), 0.0709 (three), 1.070 (four), and 700.00 (five). ■

From Example 3, we see that *all nonzero digits are significant. Also, zeros not used as placeholders (for location of the decimal point) are significant*.

In calculations with approximate numbers, the number of significant digits and the position of the decimal point are important. *The* **accuracy** *of a number refers to the number of significant digits it has,* whereas *the* **precision** *of a number refers to the decimal position of the last significant digit*.

EXAMPLE 4 Accuracy and precision

One technician measured the thickness of a metal sheet as 3.1 cm and another technician measured it as 3.12 cm. Here, 3.12 is more precise since its last digit represents hundredths and 3.1 is expressed only to tenths. Also, 3.12 is more accurate since it has three significant digits and 3.1 has only two.

A concrete driveway is 230 ft long and 0.4 ft thick. Here, 230 is more accurate (two significant digits) and 0.4 is more precise (expressed to tenths). ■

The last significant digit of an approximate number is not exact. It has usually been determined by estimating or *rounding off*. However, it is not off by more than one-half of a unit in its place value.

EXAMPLE 5 Meaning of the last digit of an approximate number

When we write the measured distance on the map in Example 2 as 35.7 cm, we are saying that the distance is at least 35.65 cm and no more than 35.75 cm. Any value between these, rounded off to tenths, would be 35.7 cm.

In changing the fraction $\frac{2}{3}$ to the approximate decimal value 0.667, we are saying that the value is between 0.6665 and 0.6675. ■

■ On graphing calculators, it is possible to set the number of decimal places (to the right of the decimal point) to which results will be rounded off.

To **round off** *a number to a specified number of significant digits, discard all digits to the right of the last significant digit (replace them with zeros if needed to properly place the decimal point). If the first digit discarded is 5 or more, increase the last significant digit by* 1 *(round up). If the first digit discarded is less than 5, do not change the last significant digit (round down).*

EXAMPLE 6 Rounding off

70,360 rounded off to three significant digits is 70,400. Here, 3 is the third significant digit and the next digit is 6. Because $6 > 5$, we add 1 to 3 and the result, 4, becomes the third significant digit of the approximation. The 6 is then replaced with a zero in order to keep the decimal point in the proper position.

70,430 rounded off to three significant digits, or to the nearest hundred, is 70,400. Here the 3 is replaced with a zero.

187.35 rounded off to four significant digits, or to tenths, is 187.4.

187.349 rounded off to four significant digits is 187.3. *We do not round up the* 4 *and then round up the* 3.

CAUTION ➧ 35.003 rounded off to four significant digits is 35.00. *We do not discard the zeros because they are significant* and are not used only to properly place the decimal point. ■

Practice Exercises

Round off each number to three significant digits.

3. 2015 **4.** 0.3004

OPERATIONS WITH APPROXIMATE NUMBERS

NOTE ➧ When performing operations on approximate numbers, *we must not express the result to an accuracy or precision that is not valid*. Consider the following examples.

EXAMPLE 7 Application of precision

A pipe is made in two sections. One is measured as 16.3 ft long and the other as 0.927 ft long. What is the total length of the two sections together?

	smallest values	largest values
16.3 ft	16.25 ft	16.35 ft
0.927 ft	0.9265 ft	0.9275 ft
17.227 ft	17.1765 ft	17.2775 ft

It may appear that we simply add the numbers as shown at the left. However, both numbers are approximate, and adding the smallest possible values and the largest possible values, the result differs by 0.1 (17.2 and 17.3) when rounded off to tenths. Rounded off to hundredths (17.18 and 17.28), they do not agree at all because the tenths digit is different. Thus, we get a good approximation for the total length if it is rounded off to *tenths,* the precision of the least precise length, and it is written as 17.2 ft. ■

EXAMPLE 8 Application of accuracy

We find the area of the rectangular piece of land in Fig. 1.8 by multiplying the length, 207.54 ft, by the width, 81.4 ft. Using a calculator, we find that $(207.54)(81.4) = 16{,}893.756$. This apparently means the area is 16,893.756 ft^2.

However, the area should not be expressed with this accuracy. Because the length and width are both approximate, we have

$$(207.535\text{ ft})(81.35\text{ ft}) = 16{,}882.97225\text{ ft}^2 \quad \text{least possible area}$$
$$(207.545\text{ ft})(81.45\text{ ft}) = 16{,}904.54025\text{ ft}^2 \quad \text{greatest possible area}$$

These values agree when rounded off to three significant digits (16,900 ft^2) but do not agree when rounded off to a greater accuracy. Thus, we conclude that the result is accurate only to *three* significant digits, the accuracy of the least accurate measurement, and that the area is written as 16,900 ft^2. ■

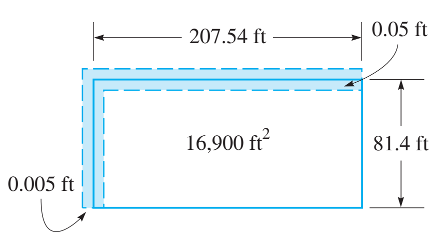


Fig. 1.8

■ The results of operations on approximate numbers shown at the right are based on reasoning that is similar to that shown in Examples 7 and 8.

The Result of Operations on Approximate Numbers

1. *When approximate numbers are added or subtracted, the result is expressed with the precision of the least precise number.*
2. *When approximate numbers are multiplied or divided, the result is expressed with the accuracy of the least accurate number.*
3. *When the root of an approximate number is found, the result is expressed with the accuracy of the number.*
4. *When approximate numbers and exact numbers are involved, the accuracy of the result is limited only by the approximate numbers.*

CAUTION ▶ ***Always express the result of a calculation with the proper accuracy or precision. When using a calculator, if additional digits are displayed, round off the final result*** (do not round off in any of the intermediate steps).

EXAMPLE 9 Adding approximate numbers

■ When rounding off a number, it may seem difficult to discard the extra digits. However, if you keep those digits, you show a number with too great an accuracy, and it is incorrect to do so.

Find the sum of the approximate numbers 73.2, 8.0627, and 93.57.

Showing the addition in the standard way and using a calculator, we have

$$\begin{array}{r l} 73.2 & \longleftarrow \text{least precise number (expressed to tenths)} \\ 8.0627 & \\ \underline{93.57} & \\ 174.8327 & \longleftarrow \text{final display must be rounded to tenths} \end{array}$$

Therefore, the sum of these approximate numbers is 174.8. ■

EXAMPLE 10 Combined operations

In finding the product of the approximate numbers 2.4832 and 30.5 on a calculator, the final display shows 75.7376. However, since 30.5 has only three significant digits, the product is 75.7.

In Example 1, we calculated that $38.3 - 12.9(-3.58) = 84.482$. We know that $38.3 - 12.9(-3.58) = 38.3 + 46.182 = 84.482$. If these numbers are approximate, we must round off the result to tenths, which means the sum is 84.5. We see that *where there is a combination of operations, the final operation determines how the final result is to be rounded off.* ■

Practice Exercise

Evaluate using a calculator.

5. $40.5 + \dfrac{3275}{-60.041}$ (Numbers are approximate.)

EXAMPLE 11 Operations with exact numbers and approximate numbers

Using the exact number 600 and the approximate number 2.7, we express the result to tenths if the numbers are added or subtracted. If they are multiplied or divided, we express the result to two significant digits. Since 600 is exact, the accuracy of the result depends only on the approximate number 2.7.

$$600 + 2.7 = 602.7 \qquad 600 - 2.7 = 597.3$$
$$600 \times 2.7 = 1600 \qquad 600 \div 2.7 = 220$$

There are 16 pieces in a pile of lumber and the average length of a piece is 482 mm. Here 16 is exact, but 482 is approximate. To get the total length of the pieces in the pile, the product $16 \times 482 = 7712$ must be rounded off to three significant digits, the accuracy of 482. Therefore, we can state that the total length is about 7710 mm. ■

NOTE ▶ A note regarding the equal sign (=) is in order. We will use it for its defined meaning of "equals exactly" and when the result is an approximate number that has been properly rounded off. Although $\sqrt{27.8} \approx 5.27$, where $\approx$ means "equals approximately," we write $\sqrt{27.8} = 5.27$, since 5.27 has been properly rounded off.

You should *make a rough estimate* of the result when using a calculator. An estimation may prevent accepting an incorrect result after using an incorrect calculator sequence, particularly if the calculator result is far from the estimated value.

EXAMPLE 12 Estimating results

In Example 1, we found that

$$38.3 - 12.9(-3.58) = 84.482 \quad \text{using exact numbers}$$

When using the calculator, if we forgot to make 3.58 negative, the display would be -7.882, or if we incorrectly entered 38.3 as 83.3, the display would be 129.482.

However, if we estimate the result as

$$40 - 10(-4) = 80$$

we know that a result of -7.882 or 129.482 cannot be correct.

When estimating, we can often use one-significant-digit approximations. If the calculator result is far from the estimate, we should do the calculation again. ■

EXERCISES 1.3

In Exercises 1–4, make the given changes in the indicated examples of this section, and then solve the given problems.

1. In Example 3, change 0.039 (the second number discussed) to 0.390. Is there any change in the conclusion?
2. In the last paragraph of Example 6, change 35.003 to 35.303 and then find the result.
3. In the first paragraph of Example 10, change 2.4832 to 2.483 and then find the result.
4. In Example 12, change 12.9 to 21.9 and then find the estimated value.

In Exercises 5–10, determine whether the given numbers are approximate or exact.

5. A car with 8 cylinders travels at 55 mi/h.
6. A computer chip 0.002 mm thick is priced at $7.50.
7. In 24 h there are 1440 min.
8. A calculator has 50 keys, and its battery lasted for 50 h of use.
9. A cube of copper 1 cm on an edge has a mass of 9 g.
10. Of a building's 90 windows, 75 were replaced 15 years ago.

In Exercises 11–16, determine the number of significant digits in each of the given approximate numbers.

11. 107; 3004; 1040 **12.** 3600; 730; 2055
13. 6.80; 6.08; 0.068 **14.** 0.8730; 0.0075; 0.0305
15. 3000; 3000.1; 3000.10 **16.** 1.00; 0.01; 0.0100

In Exercises 17–22, determine which of the pair of approximate numbers is (a) more precise and (b) more accurate.

17. 30.8; 0.010 **18.** 0.041; 7.673
19. 0.1; 78.0 **20.** 7040; 0.004
21. 7000; 0.004 **22.** 50.060; $|-8.914|$

In Exercises 23–30, round off the given approximate numbers (a) to three significant digits and (b) to two significant digits.

23. 4.936 **24.** 80.53 **25.** -50.893 **26.** 7.004
27. 9549 **28.** 30.96 **29.** 0.9449 **30.** 0.9999

In Exercises 31–40, assume that all numbers are approximate. (a) Estimate the result and (b) perform the indicated operations on a calculator and compare with the estimate.

31. $12.78 + 1.0495 - 1.633$ **32.** $3.64(17.06)$
33. $0.6572 \times 3.94 - 8.651$ **34.** $41.5 - 26.4 \div 3.7$
35. $0.0350 - \dfrac{0.0450}{1.909}$ **36.** $\dfrac{0.3275}{1.096 \times |-0.50085|}$
37. $\dfrac{23.962 \times 0.01537}{10.965 - 8.249}$ **38.** $\dfrac{0.69378 + 0.04997}{257.4 \times 3.216}$
39. $\dfrac{3872}{503.1} - \dfrac{2.056 \times 309.6}{395.2}$ **40.** $\dfrac{1}{0.5926} - \dfrac{3.6957}{1.054 - 2.935}$

In Exercises 41–44, perform the indicated operations. The first number is approximate, and the second number is exact.

41. $0.9788 + 14.9$ **42.** $17.311 - 22.98$
43. $-3.142(65)$ **44.** $8.62 \div 1728$

In Exercises 45–48, answer the given questions. Refer to Appendix B for units of measurement and their symbols.

45. The manual for a heart monitor lists the frequency of the ultrasound wave as 2.75 MHz. What are the least possible and the greatest possible frequencies?
46. A car manufacturer states that the engine displacement for a certain model is 2400 cm^3. What should be the least possible and greatest possible displacements?
47. (W) A flash of lightning struck a tower 3.25 mi from a person. The thunder was heard 15 s later. The person calculated the speed of sound and reported it as 1144 ft/s. What is wrong with this conclusion?
48. (W) A technician records 4.4 s as the time for a robot arm to swing from the extreme left to the extreme right, 2.72 s as the time for the return swing, and 1.68 s as the difference in these times. What is wrong with this conclusion?

In Exercises 49–64, perform the calculations on a calculator.

49. Evaluate: (a) $2.2 + 3.8 \times 4.5$ (b) $(2.2 + 3.8) \times 4.5$

50. Evaluate: (a) $6.03 \div 2.25 + 1.77$ (b) $6.03 \div (2.25 + 1.77)$

51. Evaluate: (a) $2 + 0$ (b) $2 - 0$ (c) $0 - 2$ (d) 2×0 (e) $2 \div 0$ Compare with operations with zero on page 9.

W **52.** Evaluate: (a) $2 \div 0.0001$ and $2 \div 0$ (b) $0.0001 \div 0.0001$ and $0 \div 0$ (c) Explain why the displays differ.

53. Enter a positive integer x (five or six digits is suggested) and then rearrange the same digits to form another integer y. Evaluate $(x - y) \div 9$. What type of number is the result?

54. Enter the digits in the order 9, 8, 7, 6, 5, 4, 3, 2, 1, 0, using between them any of the operations $(+, -, \times, \div)$ that will lead to a result of 100.

55. Show that π is not equal exactly to (a) 3.1416, or (b) $22/7$.

56. At some point in the decimal equivalent of a rational number, some sequence of digits will start repeating endlessly. An irrational number never has an endlessly repeating sequence of digits. Find the decimal equivalents of (a) $8/33$ and (b) π. Note the repetition for $8/33$ and that no such repetition occurs for π.

57. Following Exercise 56, show that the decimal equivalents of the following fractions indicate they are rational: (a) $1/3$ (b) $5/11$ (c) $2/5$. What is the repeating part of the decimal in (c)?

W **58.** Following Exercise 56, show that the decimal equivalent of the fraction $124/990$ indicates that it is rational. Why is the last digit different?

59. In 3 successive days, a home solar system produced 32.4 MJ, 26.704 MJ, and 36.23 MJ of energy. What was the total energy produced in these 3 days?

60. Two jets flew at 938 km/h and 1450 km/h, respectively. How much faster was the second jet?

61. 1 MB of computer memory has 1,048,576 bytes. How many bytes are there in 256 MB of memory? (All numbers are exact.)

62. Find the voltage in a certain electric circuit by multiplying the sum of the resistances 15.2 Ω, 5.64 Ω, and 101.23 Ω by the current 3.55 A.

63. The percent of alcohol in a certain car engine coolant is found by performing the calculation $\dfrac{100(40.63 + 52.96)}{105.30 + 52.96}$. Find this percent of alcohol. The number 100 is exact.

64. The tension (in N) in a cable lifting a crate at a construction site was found by calculating the value of $\dfrac{50.45(9.80)}{1 + 100.9 \div 23}$, where the 1 is exact. Calculate the tension.

Answers to Practice Exercises

1. 3 **2.** 4 **3.** 2020 **4.** 0.300 **5.** −14.0

1.4 Exponents

Positive Integer Exponents • Zero and Negative Exponents • Order of Operations • Evaluating Algebraic Expressions • Converting Units

In mathematics and its applications, we often have a number multiplied by itself several times. To show this type of product, we use the notation a^n, where a is the number and n is the number of times it appears. *In the expression a^n, the number a is called the* **base,** *and n is called the* **exponent;** in words, *a^n is read as "the* **nth power of a**."

EXAMPLE 1 Meaning of exponents

(a) $4 \times 4 \times 4 \times 4 \times 4 = 4^5$ the fifth power of 4

(b) $(-2)(-2)(-2)(-2) = (-2)^4$ the fourth power of −2

(c) $a \times a = a^2$ the second power of a, called "a squared"

(d) $\left(\dfrac{1}{5}\right)\left(\dfrac{1}{5}\right)\left(\dfrac{1}{5}\right) = \left(\dfrac{1}{5}\right)^3$ the third power of $\frac{1}{5}$, called "$\frac{1}{5}$ cubed" ■

We now state the basic operations with exponents using positive integers as exponents. Therefore, with m and n as positive integers, we have the following operations:

■ Two forms are shown for Eqs. (1.4) in order that the resulting exponent is a positive integer. We consider negative and zero exponents after the next three examples.

$$a^m \times a^n = a^{m+n} \tag{1.3}$$

$$\frac{a^m}{a^n} = a^{m-n} \quad (m > n, a \neq 0) \qquad \frac{a^m}{a^n} = \frac{1}{a^{n-m}} \quad (m < n, a \neq 0) \tag{1.4}$$

$$(a^m)^n = a^{mn} \tag{1.5}$$

$$(ab)^n = a^n b^n \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad (b \neq 0) \tag{1.6}$$

■ In a^3, which equals $a \times a \times a$, each a is called a factor. A more general definition of factor is given in Section 1.7.

■ Here we are using the fact that a (not zero) divided by itself equals 1, or $a/a = 1$.

■ Note that Eq. (1.3) can be verified numerically, for example, by

$2^3 \times 2^5 = 8 \times 32 = 256$

$2^3 \times 2^5 = 2^{3+5} = 2^8 = 256$

EXAMPLE 2 Illustrating Eqs. (1.3) and (1.4)

Using Eq. (1.3):

$$\underbrace{a^3 \times a^5 = a^{3+5}}_{\text{add exponents}} = a^8$$

Using the meaning of exponents:

$$a^3 \times a^5 = \underbrace{(a \times a \times a)(a \times a \times a \times a \times a)}_{(3 \text{ factors of } a)(5 \text{ factors of } a)} = \overbrace{a^8}^{8 \text{ factors of } a}$$

Using first form Eq. (1.4):

$$\frac{a^5}{a^3} = a^{5-3} = a^2 \quad (5 > 3)$$

Using the meaning of exponents:

$$\frac{a^5}{a^3} = \frac{\overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}} \times a \times a}{\underset{1}{\cancel{a}} \times \underset{1}{\cancel{a}} \times \underset{1}{\cancel{a}}} = a^2$$

Using second form Eq. (1.4):

$$\frac{a^3}{a^5} = \frac{1}{a^{5-3}} = \frac{1}{a^2} \quad (5 > 3)$$

Using the meaning of exponents:

$$\frac{a^3}{a^5} = \frac{\overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}}}{\underset{1}{\cancel{a}} \times \underset{1}{\cancel{a}} \times \underset{1}{\cancel{a}} \times a \times a} = \frac{1}{a^2}$$

■

EXAMPLE 3 Illustrating Eqs. (1.5) and (1.6)

Using Eq. (1.5):

$$\underbrace{(a^5)^3 = a^{5(3)}}_{\text{multiply exponents}} = a^{15}$$

Using the meaning of exponents:

$$(a^5)^3 = (a^5)(a^5)(a^5) = a^{5+5+5} = a^{15}$$

Using first form Eq. (1.6):

$$(ab)^3 = a^3b^3$$

Using the meaning of exponents:

$$(ab)^3 = (ab)(ab)(ab) = a^3b^3$$

Using second form Eq. (1.6):

$$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

Using the meaning of exponents:

$$\left(\frac{a}{b}\right)^3 = \left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right) = \frac{a^3}{b^3}$$

■

CAUTION ▶ When an expression involves a product or a quotient of different bases, ***only exponents of the same base may be combined.*** Consider the following example.

EXAMPLE 4 Other illustrations of exponents

(a) $(-x^2)^3 = [(-1)x^2]^3 = (-1)^3(x^2)^3 = -x^6$

(b) $ax^2(ax)^3 = \underset{\text{exponent of 1}}{a}x^2(a^3x^3) = \underset{\text{add exponents of } a}{a^4}x^5$ ← add exponents of x

(c) $\dfrac{(3 \times 2)^4}{(3 \times 5)^3} = \dfrac{3^4 2^4}{3^3 5^3} = \dfrac{3 \times 2^4}{5^3}$

(d) $\dfrac{(ry^3)^2}{r(y^2)^4} = \dfrac{r^2y^6}{ry^8} = \dfrac{r}{y^2}$

CAUTION ▶ In illustration (b), note that ax^2 ***means a times the square of x and does not mean*** a^2x^2, whereas $(ax)^3$ *does* mean a^3x^3. ■

Practice Exercises

Use Eqs. (1.3) – (1.6) to simplify the given expressions.

1. $ax^3(-ax)^2$ **2.** $\dfrac{(2c)^5}{(3cd)^2}$

EXAMPLE 5 Exponents in an application

In analyzing the amount a beam bends, the following simplification may be used. (P is the force applied to a beam of length L; E and I are constants related to the beam.)

$$\frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{2}{3}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{2}\left(\frac{PL}{4EI}\right)\left(\frac{2}{3}\right)\left(\frac{L^2}{2^2}\right)$$

$$= \frac{\overset{1}{\cancel{2}}PL(L^2)}{\underset{1}{\cancel{2}}(3)(4)(4)EI} = \frac{PL^3}{48EI}$$

In *simplifying* this expression, we combined exponents of L and divided out the 2 that was a factor common to the numerator and the denominator. ■

ZERO AND NEGATIVE EXPONENTS

If $n = m$ in Eqs. (1.4), we have $a^m/a^m = a^{m-m} = a^0$. Also, $a^m/a^m = 1$, since any nonzero quantity divided by itself equals 1. Therefore, for Eqs. (1.4) to hold for $m = n$,

$$\boxed{a^0 = 1 \qquad (a \neq 0)} \tag{1.7}$$

Equation (1.7) states that *any nonzero expression raised to the zero power is* 1. Zero exponents can be used with any of the operations for exponents.

EXAMPLE 6 Zero as an exponent

Practice Exercise

3. Evaluate: $-(3x)^0$

(a) $5^0 = 1$ **(b)** $(-3)^0 = 1$ **(c)** $-(-3)^0 = -1$ **(d)** $(2x)^0 = 1$

(e) $(ax + b)^0 = 1$ **(f)** $(a^2b^0c)^2 = a^4c^2$ **(g)** $2t^0 = 2(1) = 2$

$b^0 = 1$

We note in illustration (g) that *only t is raised to the zero power*. If the quantity $2t$ were raised to the zero power, it would be written as $(2t)^0$, as in part (d). ■

Applying both forms of Eq. (1.4) to the case where $n > m$ leads to the definition of a negative exponent. For example, applying both forms to a^2/a^7, we have

$$\frac{a^2}{a^7} = a^{2-7} = a^{-5} \quad \text{and} \quad \frac{a^2}{a^7} = \frac{1}{a^{7-2}} = \frac{1}{a^5}$$

■ Although positive exponents are generally preferred in a final result, there are some cases in which zero or negative exponents are to be used. Also, negative exponents are very useful in some operations that we will use later.

For these results to be equal, then $a^{-5} = 1/a^5$. Thus, if we define

$$\boxed{a^{-n} = \frac{1}{a^n} \qquad (a \neq 0)} \tag{1.8}$$

then all of the laws of exponents will hold for negative integers.

EXAMPLE 7 Negative exponents

Practice Exercises

Simplify: **4.** $\dfrac{-7^0}{c^{-3}}$ **5.** $\dfrac{(3x)^{-1}}{2a^{-2}}$

(a) $3^{-1} = \dfrac{1}{3}$ **(b)** $4^{-2} = \dfrac{1}{4^2} = \dfrac{1}{16}$ **(c)** $\dfrac{1}{a^{-3}} = a^3$

change signs of exponents

■ Note carefully the difference in illustrations (d) and (e).

CAUTION ▶ **(d)** $(3x)^{-1} = \dfrac{1}{3x}$ **(e)** $3x^{-1} = 3\left(\dfrac{1}{x}\right) = \dfrac{3}{x}$ **(f)** $\left(\dfrac{a^3}{x}\right)^{-2} = \dfrac{(a^3)^{-2}}{x^{-2}} = \dfrac{a^{-6}}{x^{-2}} = \dfrac{x^2}{a^6}$ ■

ORDER OF OPERATIONS

We have seen that the basic operations on numbers must be performed in a particular order. Since raising a number to a power is actually multiplication, it is performed before additions and subtractions, and in fact, before multiplications and divisions.

■ The use of exponents is taken up in more detail in Chapter 11.

ORDER OF OPERATIONS

1. *Operations within specific groupings*
2. *Powers*
3. *Multiplications and divisions (from left to right)*
4. *Additions and subtractions (from left to right)*

EXAMPLE 8 Order of operations

$$\begin{aligned} 8 - (-1)^2 - 2(-3)^2 &= 8 - 1 - 2(9) \\ &= 8 - 1 - 18 = -11 \end{aligned}$$

Because there were no specific groupings, we first squared -1 and -3. Next, we found the product $2(9)$ in the last term. Finally, the subtractions were performed. Note carefully that *we did not change the sign of* -1 *before we squared it*. ■

CAUTION ▶

EVALUATING ALGEBRAIC EXPRESSIONS

■ On many calculators, there is a specific key or key sequence to evaluate x^3.

An algebraic expression is **evaluated** *by* **substituting** *given values of the literal numbers in the expression and calculating the result*. On a calculator, the $\boxed{x^2}$ key is used to square numbers, and the $\boxed{\wedge}$ or $\boxed{x^y}$ key is used for other powers.

Calculators generally use computer symbols in the display for some of the operations to be performed. These symbols are as follows:

Multiplication: * Division: / Powers: ^

Therefore, to calculate the value of $20 \times 6 + 200/5 - 3^4$, we use the key sequence

$$20 \boxed{\times} 6 \boxed{+} 200 \boxed{\div} 5 \boxed{-} 3 \boxed{\wedge} 4$$

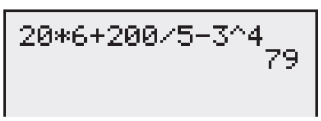


Fig. 1.9

CAUTION ▶

with the result of 79 shown in the display of Fig. 1.9. ***Note carefully that 200 is divided only by 5.*** If it were divided by $5 - 3^4$, then we would use parentheses and show the expression to be evaluated as $20 \times 6 + 200/(5 - 3^4)$.

EXAMPLE 9 Evaluating using exponents

The distance (in ft) that an object falls in 4.2 s is found by substituting 4.2 for t in the expression $16.0t^2$. We show this as

$t = 4.2$ s ⟵ substituting

$16.0(4.2)^2 = 280$ ft   estimation ⟶ $20(4)^2 = 320$

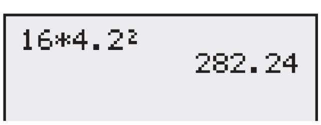


Fig. 1.10

The result is rounded off to two significant digits (the accuracy of t). The calculator will square 4.2 before multiplying. See Fig. 1.10. ■

■ When less than half of a calculator screen is needed, the figure for that calculator screen will show only a partial screen.

As noted, in evaluating an expression on a calculator, we should also estimate its value. In doing so, note that *a negative number raised to an even power gives a positive value, and a negative number raised number raised to an odd power gives a negative value.*

EXAMPLE 10 Power of a number

Using the meaning of a power of a number, we have

$$(-2)^2 = (-2)(-2) = 4 \qquad (-2)^3 = (-2)(-2)(-2) = -8$$
$$(-2)^4 = 16 \qquad (-2)^5 = -32 \qquad (-2)^6 = 64 \qquad (-2)^7 = -128$$ ■

EXAMPLE 11 Evaluating in an application

A wire made of a special alloy has a length L (in m) given by $L = a + 0.0115T^3$, where T (in °C) is the temperature (between −4°C and 4°C). Find L for $a = 8.380$ m and $T = -2.87$°C.

Substituting these values, we have

$$L = 8.380 + 0.0115(-2.87)^3 \qquad \text{estimation:}$$
$$= 8.108 \text{ m} \qquad 8.4 + 0.01(-3)^3 = 8.1$$

EXAMPLE 12 Using basic operations in converting units

■ See Appendix B for tables of values to use in converting units, and for additional examples of conversions. In particular, note Example 1 in Section B.1 and the 1 in. = 2.54 cm example that follows.

(a) A car is traveling at 65.0 mi/h. Convert this speed to km/min (kilometers per minute).

In converting units we use the basic operations on units as we do with numbers. From Appendix B we note that 1 km = 0.6214 mi, and we know that 1 h = 60 min. Using these values, we have

$$65.0\,\frac{\text{mi}}{\text{h}} = \frac{65.0 \text{ mi}}{1 \text{ h}} \times \frac{1 \text{ km}}{0.6214 \text{ mi}} \times \frac{1 \text{ h}}{60 \text{ min}} = \frac{(65.0)(1)(1) \text{ km}}{(1)(0.6214)(60) \text{ min}}$$
$$= 1.74 \text{ km/min}$$

We note that the units mi and h appear in both numerator and denominator and therefore cancel out, leaving the units km and min. Also note that the 1's and 60 are exact.

(b) The density of iron is 7.86 g/cm^3 (grams per cubic centimeter). Express this density in kg/m^3 (kilograms per cubic meter).

In the metric system 1 kg = 1000 g exactly, and 1 m = 100 cm exactly. Therefore,

$$7.86\,\frac{\text{g}}{\text{cm}^3} = \frac{7.86 \text{ g}}{1 \text{ cm}^3} \times \frac{0.001 \text{ kg}}{1 \text{ g}} \times \left(\frac{1 \text{ cm}}{0.01 \text{ m}}\right)^3 = \frac{(7.86)(0.001)(1^3) \text{ kg}}{(1)(1)(0.01^3) \text{ m}^3}$$
$$= \frac{(7.86)(0.001) \text{ kg}}{(0.01^3) \text{ m}^3} = 7860 \text{ kg/m}^3$$

Here, the units g and cm^3 are in both numerator and denominator and therefore cancel out, leaving units of kg and m^3. Also, all numbers are exact, except 7.86. ■

EXERCISES 1.4

In Exercises 1–4, make the given changes in the indicated examples of this section, and then simplify the resulting expression.

1. In Example 4(a), change $(-x^2)^3$ to $(-x^3)^2$.
2. In Example 6(d), change $(2x)^0$ to $2x^0$.
3. In Example 7(f), interchange the a^3 and x.
4. In Example 8, change $(-1)^2$ to $(-1)^3$.

In Exercises 5–48, simplify the given expressions. Express results with positive exponents only.

5. x^3x^4 6. y^2y^7 7. $2b^4b^2$ 8. $3k^5k$

9. $\dfrac{m^5}{m^3}$ 10. $\dfrac{2x^6}{-x}$ 11. $\dfrac{-n^5}{7n^9}$ 12. $\dfrac{3s}{s^4}$

13. $(P^2)^4$ 14. $(x^8)^3$ 15. $(-2\pi)^3$ 16. $(-ax)^5$

17. $(aT^2)^{30}$ 18. $(3r^2)^3$ 19. $\left(\dfrac{2}{b}\right)^3$ 20. $\left(\dfrac{F}{t}\right)^{20}$

21. $\left(\dfrac{x^2}{-2}\right)^4$ 22. $\left(\dfrac{3}{n^3}\right)^3$ 23. $(8a)^0$ 24. $-v^0$

25. $-3x^0$ 26. $-(-2)^0$ 27. 6^{-1} 28. $-w^{-5}$

29. $\dfrac{1}{R^{-2}}$ 30. $\dfrac{1}{-t^{-48}}$ 31. $(-t^2)^7$ 32. $(-y^3)^5$

33. $(2v^2)^{-6}$ 34. $-(-c^4)^{-4}$ 35. $-\dfrac{L^{-3}}{L^{-5}}$ 36. $2i^{40}i^{-70}$

37. $\dfrac{2v^4}{(2v)^4}$ 38. $\dfrac{x^2x^3}{(x^2)^3}$ 39. $\dfrac{(n^2)^4}{(n^4)^2}$ 40. $\dfrac{(3t)^{-1}}{-3t^{-1}}$

41. $(\pi^0x^2a^{-1})^{-1}$ 42. $(3m^{-2}n^4)^{-2}$ 43. $(-8g^{-1}s^3)^2$

44. $ax^{-2}(-a^2x)^3$ 45. $\left(\dfrac{4x^{-1}}{a^{-1}}\right)^{-3}$ 46. $\left(\dfrac{2b^2}{-y^5}\right)^{-2}$

47. $\dfrac{15n^2T^5}{3n^{-1}T^6}$ 48. $\dfrac{(nRT^{-2})^{32}}{R^{-2}T^{32}}$

In Exercises 49–56, evaluate the given expressions. In Exercises 51–56, all numbers are approximate.

49. $7(-4) - (-5)^2$ 50. $6 - |-2|^5 - (-2)(8)$

51. $-(-26.5)^2 - (-9.85)^3$ 52. $-0.711^2 - (-|-0.809|)^6$

53. $\dfrac{3.07(-|-1.86|)}{(-1.86)^4 + 1.596}$ 54. $\dfrac{15.66^2 - (-4.017)^4}{1.044(-3.68)}$

55. $2.38(-60.7)^2 - 2540/1.17^3 + 0.806^5(26.1^3 - 9.88^4)$

56. $0.513(-2.778) - (-3.67)^3 + 0.889^4/(1.89 - 1.09^2)$

In Exercises 57–68, perform the indicated operations.

57. Does $\left(\dfrac{1}{x^{-1}}\right)^{-1}$ represent the reciprocal of x?

58. Does $\left(\dfrac{0.2 - 5^{-1}}{10^{-2}}\right)^{0}$ equal 1? Explain.

59. If $a^3 = 5$, then what does a^{12} equal?

60. Is $a^{-2} < a^{-1}$ for any negative value of a? Explain.

61. If a is a positive integer, simplify $(x^a \cdot x^{-a})^5$.

62. If a and b are positive integers, simplify $(-y^{a-b} \cdot y^{a+b})^2$.

63. In developing the "big bang" theory of the origin of the universe, the expression $(kT/(hc))^3(GkThc)^2c$ arises. Simplify this expression.

64. In studying planetary motion, the expression $(GmM)(mr)^{-1}(r^{-2})$ arises. Simplify this expression.

65. In designing a cam for a fire engine pump, the expression $\pi\left(\dfrac{r}{2}\right)^3\left(\dfrac{4}{3\pi r^2}\right)$ is used. Simplify this expression.

66. For a certain integrated electric circuit, it is necessary to simplify the expression $\dfrac{gM(2\pi fM)^{-2}}{2\pi fC}$. Perform this simplification.

67. If \$2500 is invested at 4.2% interest, compounded quarterly, the amount in the account after 6 years is $2500(1 + 0.042/4)^{24}$. Calculate this amount (the 1 is exact).

68. In designing a building, it was determined that the forces acting on an I beam would deflect the beam an amount (in cm), given by $\dfrac{x(1000 - 20x^2 + x^3)}{1850}$, where x is the distance (in m) from one end of the beam. Find the deflection for $x = 6.85$ m. (The 1000 and 20 are exact.)

69. Calculate the value of $\left(1 + \dfrac{1}{n}\right)^n$ for $n = 1, 10, 100, 1000$ on a calculator. (For even larger values of n, the value will never exceed 2.7183. The limiting value is called e. See page 348.)

70. For computer memory 1 MB $= 2^{10}$ bytes, 1 GB $= 2^{10}$ MB, and 1 TB $= 2^{10}$ GB. How many bytes are there in 1 TB? (MB is megabyte, GB is gigabyte, TB is terabyte)

In Exercises 71–74, make the indicated unit conversions.

71. Express 2.5 ft in inches

72. Express 0.225 km in feet

73. Express 65.2 m/s in ft/h

74. Express 7.25 g/cm^2 in kg/m^2

75. A car gets 25.0 mi/gal of gas. Express this in km/L.

76. The acceleration due to gravity is 32.0 ft/s^2. Express this in m/min^2.

Answers to Practice Exercises

1. a^3x^5 **2.** $\dfrac{2^5c^3}{3^2d^2} = \dfrac{32c^3}{9d^2}$ **3.** -1 **4.** $-c^3$ **5.** $\dfrac{a^2}{6x}$

1.5 Scientific Notation

Meaning of Scientific Notation • Changing Numbers to and from Scientific Notation • Scientific Notation on a Calculator

■ Television was invented in the 1920s and first used commercially in the 1940s.

The use of fiber optics was developed in the 1950s.

X-rays were discovered by Roentgen in 1895.

In technical and scientific work, we encounter numbers that are inconvenient to use in calculations. Examples are: radio and television signals travel at 30,000,000,000 cm/s; the mass of Earth is 6,600,000,000,000,000,000,000 tons; a fiber in a fiber-optic cable has a diameter of 0.000005 m; some X-rays have a wavelength of 0.000000095 cm. Although calculators and computers can handle such numbers, a convenient and useful notation, called *scientific notation*, is used to represent these or any other numbers.

A number in ***scientific notation*** *is expressed as the product of a number greater than or equal to* 1 *and less than* 10, *and a power of* 10, *and is written as*

$$P \times 10^k$$

where $1 \le P < 10$ and k is an integer. (The symbol $\le$ means "is less than or equal to.")

EXAMPLE 1 Scientific notation

(a) $34{,}000 = 3.4 \times 10{,}000 = 3.4 \times 10^4$ **(b)** $6.82 = 6.82 \times 1 = 6.82 \times 10^0$

between 1 and 10

(c) $0.00503 = \dfrac{5.03}{1000} = \dfrac{5.03}{10^3} = 5.03 \times 10^{-3}$ ■

From Example 1, we see how a number is changed from ordinary notation to scientific notation. *The decimal point is moved so that only one nonzero digit is to its left. The number of places moved is the power of* 10 *(k), which is positive if the decimal point is moved to the left and negative if moved to the right*. To change a number from scientific notation to ordinary notation, this procedure is reversed. The next two examples illustrate these procedures.

NOTE ▶

EXAMPLE 2 Changing numbers to scientific notation

(a) $34{,}000 = 3.4 \times 10^4$
4 places to left

(b) $6.82 = 6.82 \times 10^0$
0 places

(c) $0.00503 = 5.03 \times 10^{-3}$
3 places to right ■

EXAMPLE 3 Changing numbers to ordinary notation

(a) To change 5.83×10^6 to ordinary notation, we move the decimal point six places to the right, including additional zeros to properly locate the decimal point.

$$5.83 \times 10^6 = 5{,}830{,}000$$
6 places to right

(b) To change 8.06×10^{-3} to ordinary notation, we must move the decimal point three places to the left, again including additional zeros to properly locate the decimal point.

$$8.06 \times 10^{-3} = 0.00806$$
3 places to left ■

Practice Exercises

1. Change 2.35×10^{-3} to ordinary notation.
2. Change 235 to scientific notation.

Scientific notation provides a practical way of handling very large or very small numbers. First, all numbers are expressed in scientific notation. Then the calculation can be done with numbers between 1 and 10, using the laws of exponents to find the power of ten of the result. Thus, scientific notation gives an important use of exponents.

EXAMPLE 4 Scientific notation in calculations

■ See Exercise 41 of Exercises 1.1 for a brief note on computer data.

The processing rate of a computer processing 803,000 bytes of data in 0.0000025 s is

$$\frac{803{,}000}{0.00000525} = \frac{8.03 \times 10^5}{5.25 \times 10^{-6}} = \left(\frac{8.03}{5.25}\right) \times \overbrace{10^{11}}^{5-(-6)=11} = 1.53 \times 10^{11}\text{ bytes/s}$$

■ Another important use of scientific notation is shown by the metric system use of prefixes as shown in Appendix B.

As shown, it is proper to leave the result (*rounded off*) in scientific notation. This method is useful when using a calculator and then estimating the result. In this case, the estimate is $(8 \times 10^5) \div (5 \times 10^{-6}) = 1.6 \times 10^{11}$. ■

Another advantage of scientific notation is that the precise number of significant digits of a number can be shown directly, even when the final significant digit is 0.

EXAMPLE 5 Scientific notation and significant digits

In determining the gravitational force between two stars 750,000,000,000 km apart, it is necessary to evaluate $750{,}000{,}000{,}000^2$. If 750,000,000,000 has *three* significant digits, we can show this by writing

$$750{,}000{,}000{,}000^2 = (7.50 \times 10^{11})^2 = 7.50^2 \times 10^{2\times 11} = 56.3 \times 10^{22}$$

■ The number 7.50×10^{11} cannot be entered on a scientific calculator in the form 750000000000. It could be entered in this form on a graphing calculator.

Since 56.3 is not between 1 and 10, we can write this result in scientific notation as

$$56.3 \times 10^{22} = (5.63 \times 10)(10^{22}) = 5.63 \times 10^{23}$$ ■

We can enter numbers in scientific notation on a calculator, as well as have the calculator give results automatically in scientific notation. See the next example.

EXAMPLE 6 Scientific notation on a calculator

The wavelength λ (in m) of the light in a red laser beam can be found from the following calculation. Note the significant digits in the numerator.

$$\lambda = \frac{3{,}000{,}000}{4{,}740{,}000{,}000{,}000} = \frac{3.00 \times 10^6}{4.74 \times 10^{12}} = 6.33 \times 10^{-7}\text{ m}$$

The key sequence is 3 EE 6 ÷ 4.74 EE 12 ENTER. See Fig. 1.11. ■

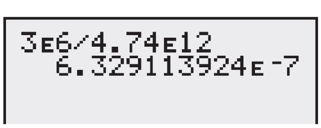


Fig. 1.11

EXERCISES 1.5

In Exercises 1 and 2, make the given changes in the indicated examples of this section, and then rewrite the number as directed.

1. In Example 3(b), change the exponent -3 to 3 and then write the number in ordinary notation.

2. In Example 5, change the exponent 2 to -1 and then write the result in scientific notation.

In Exercises 3–10, change the numbers from scientific notation to ordinary notation.

3. 4.5×10^4 **4.** 6.8×10^7 **5.** 2.01×10^{-3} **6.** 9.61×10^{-5}

7. 3.23×10^0 **8.** 8×10^0 **9.** 1.86×10 **10.** 1×10^{-1}

In Exercises 11–18, change the numbers from ordinary notation to scientific notation.

11. 4000 **12.** 56,000 **13.** 0.0087 **14.** 0.7

15. 6.09 **16.** 10 **17.** 1 **18.** 0.0000908

In Exercises 19–22, perform the indicated calculations using a calculator and by first expressing all numbers in scientific notation.

19. 28,000(2,000,000,000) **20.** 50,000(0.006)

21. $\dfrac{88{,}000}{0.0004}$ **22.** $\dfrac{0.00003}{6{,}000{,}000}$

In Exercises 23–26, perform the indicated calculations and then check the result using a calculator. Assume that all numbers are exact.

23. $2 \times 10^{-35} + 3 \times 10^{-34}$ **24.** $5.3 \times 10^{12} - 3.7 \times 10^{10}$

25. $(1.2 \times 10^{29})^3$ **26.** $(2 \times 10^{-16})^{-5}$

In Exercises 27–34, perform the indicated calculations using a calculator. All numbers are approximate.

27. 1280(865,000)(43.8) **28.** 0.0000569(3,190,000)

29. $\dfrac{0.0732(6710)}{0.00134(0.0231)}$ **30.** $\dfrac{0.00452}{2430(97{,}100)}$

31. $(3.642 \times 10^{-8})(2.736 \times 10^5)$ **32.** $\dfrac{(7.309 \times 10^{-1})^2}{5.9843(2.5036 \times 10^{-20})}$

33. $\dfrac{(3.69 \times 10^{-7})(4.61 \times 10^{21})}{0.0504}$ **34.** $\dfrac{(9.9 \times 10^7)(1.08 \times 10^{12})^2}{(3.603 \times 10^{-5})(2054)}$

In Exercises 35–44, change numbers in ordinary notation to scientific notation or change numbers in scientific notation to ordinary notation. See Appendix B for an explanation of symbols used.

35. The power plant at Grand Coulee Dam produces 6,500,000 kW.

36. A certain computer has 60,000,000,000,000 bytes of memory.

37. A fiber-optic system requires 0.000003 W of power.

38. A red blood cell measures 0.0075 mm across.

39. The frequency of a certain TV signal is 2,000,000,000 Hz.

40. In 2010 it was estimated that the global reserves of recoverable shale oil was about 5×10^{11} m^3.

41. The Gulf of Mexico oil spill in 2010 covered more than 12,000,000,000 m^2 of ocean surface.

42. A *parsec,* a unit used in astronomy, is about 3.086×10^{16} m.

43. The power of the signal of a laser beam probe is 1.6×10^{-12} W.

44. The electrical force between two electrons is about 2.4×10^{-43} times the gravitational force between them.

In Exercises 45–50, solve the given problems.

45. *Engineering notation* expresses a number as $P \times 10^k$, where P has one, two, or three digits to the left of the decimal point, and k is an integral multiple of 3. Write the following numbers in engineering notation. (a) 2300 (b) 0.23 (c) 23 (d) 0.00023

46. See Exercise 45. Write the following numbers in engineering notation. (a) 8,090,000 (b) 809,000 (c) 0.0809

47. A *googol* is defined as 1 followed by 100 zeros. (a) Write this number in scientific notaion. (b) A *googolplex* is defined as 10 to the googol power. Write this number using powers of 10, and not the word *googol*. (Note the name of the Internet company.)

48. The number of electrons in the universe has been estimated at 10^{79}. How many times greater is a googol than the estimated number of electrons in the universe? (See Exercise 47.)

49. The diameter of the sun, 1.4×10^7 m, is about 110 times the diameter of Earth. Express the diameter of Earth in scientific notation.

50. GB means *gigabyte* where *giga* means *billion*, or 10^9. Actually, 1 GB $= 2^{30}$ bytes. Use a calculator to show that the use of *giga* is a reasonable choice of terminology.

In Exercises 51–54, perform the indicated calculations.

51. A computer can do an addition in 7.5×10^{-15} s. How long does it take to perform 5.6×10^6 additions?

52. Uranium is used in nuclear reactors to generate electricity. About 0.000000039% of the uranium disintegrates each day. How much of 0.085 mg of uranium disintegrates in a day?

53. A TV signal travels 1.86×10^5 mi/s for 4.57×10^4 mi from the station transmitter to a satellite and then to a receiver dish. How long does it take the signal to go from the transmitter to the dish?

54. (a) Determine the number of seconds in a day in scientific notation. (b) Using the result of part (a), determine the number of seconds in a century (assume 365.24 days/year).

In Exercises 55–58, perform the indicated calculations by first expressing all numbers in scientific notation.

55. One *atomic mass unit* (amu) is 1.66×10^{-27} kg. If one oxygen atom has 16 amu (an exact number), what is the mass of 125,000,000 oxygen atoms?

56. The rate of energy radiation (in W) from an object is found by evaluating the expression kT^4, where T is the thermodynamic temperature. Find this value for the human body, for which $k = 0.000000057$ W/K^4 and $T = 303$ K.

57. In a microwave receiver circuit, the resistance R of a wire 1 m long is given by $R = k/d^2$, where d is the diameter of the wire. Find R if $k = 0.00000002196\ \Omega \cdot \text{m}^2$ and $d = 0.00007998$ m.

58. The average distance between the sun and Earth, 149,600,000 km, is called an *astronomical unit* (AU). If it takes light 499.0 s to travel 1 AU, what is the speed of light? Compare this with the speed of the TV signal in Exercise 53.

Answers to Practice Exercises

1. 0.00235 **2.** 2.35×10^2

1.6 Roots and Radicals

Principal *n*th Root • Simplifying Radicals • Using a Calculator • Imaginary Numbers

At times, we have to find the *square root* of a number, or maybe some other root of a number, such as a *cube root*. This means we must find a number that when squared, or cubed, and so on equals some given number. For example, to find the square root of 9, we must find a number that when squared equals 9. In this case, either 3 or -3 is an answer. Therefore, *either 3 or -3 is a square root of 9 since* $3^2 = 9$ and $(-3)^2 = 9$.

To have a general notation for the square root and have it represent *one* number, *we define the* **principal square root** *of a to be positive if a is positive and represent it by* $\sqrt{a}$. This means $\sqrt{9} = 3$ and not -3.

■ Unless we state otherwise, when we refer to the root of a number, it is the principal root.

The general notation for the **principal *n*th root** *of a is* $\sqrt[n]{a}$. (When $n = 2$, do not write the 2 for n.) *The* $\sqrt{\ }$ *sign is called a* **radical sign.**

EXAMPLE 1 Roots of numbers

(a) $\sqrt{2}$ (the square root of 2) **(b)** $\sqrt[3]{2}$ (the cube root of 2)

(c) $\sqrt[4]{2}$ (the fourth root of 2) **(d)** $\sqrt[7]{6}$ (the seventh root of 6) ■

NOTE ▸ To have a single defined value for all roots (not just square roots) and to consider only real-number roots, *we define the* **principal *n*th root** *of a to be positive if a is positive and to be negative if a is negative and n is odd*. (If a is negative and n is even, the roots are not real.)

EXAMPLE 2 Principal *n*th root

(a) $\sqrt{169} = 13 \quad (\sqrt{169} \neq -13)$ **(b)** $-\sqrt{64} = -8$

(c) $\sqrt[3]{27} = 3$ since $3^3 = 27$ **(d)** $\sqrt{0.04} = 0.2$ since $0.2^2 = 0.04$

odd

(e) $-\sqrt[4]{256} = -4$ **(f)** $\sqrt[3]{-27} = -3$ **(g)** $-\sqrt[3]{27} = -(+3) = -3$ ■

Another property of square roots is developed by noting illustrations such as $\sqrt{36} = \sqrt{4 \times 9} = \sqrt{4} \times \sqrt{9} = 2 \times 3 = 6$. In general, this property states that *the square root of a product of positive numbers is the product of their square roots*.

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \qquad (a \text{ and } b \text{ positive real numbers}) \tag{1.9}$$

This property is used in simplifying radicals. It is most useful if either a or b is a **perfect square,** *which is the square of a rational number*.

EXAMPLE 3 Simplifying square roots

(a) $\sqrt{8} = \sqrt{(4)(2)} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

perfect squares simplest form

(b) $\sqrt{75} = \sqrt{(25)(3)} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$

(c) $\sqrt{4 \times 10^2} = \sqrt{4}\sqrt{10^2} = 2(10) = 20$

(Note that the square root of the square of a positive number is that number.) ■

■ Try this one on your calculator: $\sqrt{12345678987654321}$

In order to represent the square root of a number *exactly*, use Eq. (1.9) to write it in simplest form. However, a decimal *approximation* is often acceptable, and we use the $\boxed{\sqrt{x}}$ key on a calculator. (We will show another way of finding the root of a number in Chapter 11.)

EXAMPLE 4 Using a calculator in an application

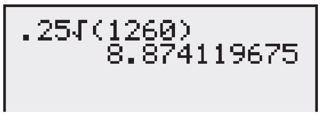


Fig. 1.12

After reaching its greatest height, the time (in s) for a rocket to fall h ft is found by evaluating $0.25\sqrt{h}$. Find the time for the rocket to fall 1260 ft.

The calculator evaluation is shown in Fig. 1.12. From the display, we see that the rocket takes 8.9 s to fall 1260 ft. The result is rounded off to two significant digits, the accuracy of 0.25 (an approximate number). ■

In simplifying a radical, *all operations under a radical sign must be done before finding the root*. The horizontal bar groups the numbers under it.

EXAMPLE 5 More on simplifying square roots

(a) $\sqrt{16+9} = \sqrt{25}$ first perform the addition $16 + 9$

$= 5$

CAUTION ➧ However, $\mathbf{\sqrt{16+9}}$ **is *not*** $\mathbf{\sqrt{16} + \sqrt{9} = 4 + 3 = 7}$.

(b) $\sqrt{2^2+6^2} = \sqrt{4+36} = \sqrt{40} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$, but

$\sqrt{2^2+6^2}$ is ***not*** $\sqrt{2^2} + \sqrt{6^2} = 2 + 6 = 8$. ■

Practice Exercises

Simplify:

1. $\sqrt{12}$ 2. $\sqrt{36+144}$

In defining the principal square root, we did not define the square root of a negative number. However, in Section 1.1, we defined the square root of a negative number to be an **imaginary number.** More generally, *the even root of a negative number is an imaginary number, and the odd root of a negative number is a negative real number.*

EXAMPLE 6 Imaginary roots and real roots

$\overset{\text{even}}{\downarrow}$ $\sqrt{-64}$ is imaginary $\overset{\text{even}}{\downarrow}$ $\sqrt[4]{-243}$ is imaginary $\overset{\text{odd}}{\downarrow}$ $\sqrt[3]{-64} = -4$ (a real number) ■

Practice Exercise

3. Is $\sqrt[3]{-8}$ real or imaginary? If it is real, evaluate it.

A much more detailed coverage of roots, radicals, and imaginary numbers is taken up in Chapters 11 and 12.

EXERCISES 1.6

In Exercises 1–4, make the given changes in the indicated examples of this section and then solve the given problems.

1. In Example 2(b), change the square root to a cube root and then evaluate.
2. In Example 3(b), change $\sqrt{(25)(3)}$ to $\sqrt{(15)(5)}$ and explain whether or not this would be a better expression to use.
3. In Example 5(a), change the $+$ to $\times$ and then evaluate.
4. In the first illustration of Example 6, place a $-$ sign before the radical. Is there any other change in the statement?

In Exercises 5–36, simplify the given expressions. In each of 5–9 and 12–21, the result is an integer.

5. $\sqrt{81}$ 6. $\sqrt{225}$ 7. $-\sqrt{121}$ 8. $-\sqrt{36}$
9. $-\sqrt{49}$ 10. $\sqrt{0.25}$ 11. $\sqrt{0.09}$ 12. $-\sqrt{900}$
13. $\sqrt[3]{125}$ 14. $\sqrt[4]{16}$ 15. $\sqrt[3]{-216}$ 16. $-\sqrt[5]{-32}$
17. $(\sqrt{5})^2$ 18. $(\sqrt[3]{31})^3$ 19. $(-\sqrt[3]{-47})^3$ 20. $(\sqrt[5]{-23})^5$
21. $(-\sqrt[4]{53})^4$ 22. $-\sqrt{32}$ 23. $\sqrt{1200}$ 24. $\sqrt{50}$
25. $2\sqrt{84}$ 26. $\dfrac{\sqrt{108}}{2}$ 27. $\sqrt{\dfrac{80}{|3-7|}}$ 28. $\sqrt{81 \times 10^2}$
29. $\sqrt[3]{-8^2}$ 30. $\sqrt[4]{9^2}$ 31. $\dfrac{7^2\sqrt{81}}{(-3)^2\sqrt{49}}$ 32. $\dfrac{2^5\sqrt[5]{-243}}{-3\sqrt{144}}$
33. $\sqrt{36+64}$ 34. $\sqrt{25+144}$
35. $\sqrt{3^2+9^2}$ 36. $\sqrt{8^2-4^2}$

In Exercises 37–44, find the value of each square root by use of a calculator. Each number is approximate.

37. $\sqrt{85.4}$ 38. $\sqrt{3762}$ 39. $\sqrt{0.4729}$ 40. $\sqrt{0.0627}$
41. (a) $\sqrt{1296+2304}$ (b) $\sqrt{1296} + \sqrt{2304}$
42. (a) $\sqrt{10.6276+2.1609}$ (b) $\sqrt{10.6276} + \sqrt{2.1609}$

43. **(a)** $\sqrt{0.0429^2 - 0.0183^2}$ **(b)** $\sqrt{0.0429^2} - \sqrt{0.0183^2}$

44. **(a)** $\sqrt{3.625^2 + 0.614^2}$ **(b)** $\sqrt{3.625^2} + \sqrt{0.614^2}$

In Exercises 45–56, solve the given problems.

45. The speed (in mi/h) of a car that skids to a stop on dry pavement is often estimated by $\sqrt{24s}$, where s is the length (in ft) of the skid marks. Estimate the speed if $s = 150$ ft.

46. The resistance in an amplifier circuit is found by evaluating $\sqrt{Z^2 - X^2}$. Find the resistance for $Z = 5.362\ \Omega$ and $X = 2.875\ \Omega$.

47. The speed (in m/s) of sound in seawater is found by evaluating $\sqrt{B/d}$ for $B = 2.18 \times 10^9$ Pa and $d = 1.03 \times 10^3\ \text{kg/m}^3$. Find this speed, which is important in locating underwater objects using sonar.

48. The terminal speed (in m/s) of a skydiver can be approximated by $\sqrt{40m}$, where m is the mass (in kg) of the skydiver. Calculate the terminal speed (after reaching this speed, the skydiver's speed remains fairly constant before opening the parachute) of a 75-kg skydiver.

49. A TV screen is 52.3 in. wide and 29.3 in. high. The length of a diagonal (the dimension used to describe it—from one corner to the opposite corner) is found by evaluating $\sqrt{w^2 + h^2}$, where w is the width and h is the height. Find the diagonal.

50. A car costs \$38,000 new and is worth \$24,000 2 years later. The annual rate of depreciation is found by evaluating $100\left(1 - \sqrt{V/C}\right)$, where C is the cost and V is the value after 2 years. At what rate did the car depreciate? (100 and 1 are exact.)

51. A *tsunami* is a very high ocean tidal wave (or series of waves) often caused by an earthquake. An Alaskan tsunami in 1958 measured over 500 m high; an Asian tsunami in 2004 killed over 230,000 people; a tsunami in Japan in 2011 killed over 10,000 people. An equation that approximates the speed v (in m/s) of a tsunami is $v = \sqrt{gd}$, where $g = 9.8\ \text{m/s}^2$ and d is the average depth (in m) of the ocean floor. Find v (in km/h) for $d = 3500$ m (valid for many parts of the Indian Ocean and Pacific Ocean).

52. The greatest distance (in km) a person can see from a height h (in m) above the ground is $\sqrt{1.27 \times 10^4 h + h^2}$. What is this distance for the pilot of a plane 9500 m above the ground?

W **53.** Is it always true that $\sqrt{a^2} = a$? Explain.

54. For what values of x is (a) $x > \sqrt{x}$, (b) $x = \sqrt{x}$, and (c) $x < \sqrt{x}$?

55. A graphing calculator has a specific key sequence to find cube roots. Using a calculator, find $\sqrt[3]{2140}$ and $\sqrt[3]{-0.214}$.

56. A graphing calculator has a specific key sequence to find nth roots. Using a calculator, find $\sqrt[7]{0.382}$ and $\sqrt[7]{-382}$.

57. A graphing calculator can determine if a number is real or imaginary. Use the $a + bi$ setting for numbers (see Exercise 37 on page 6), where i represents $\sqrt{-1}$ on a calculator. Use a calculator to determine if the following numbers are real or imaginary: (a) $\sqrt{-64}$ (b) $\sqrt[3]{-64}$

58. Use a calculator to determine if the following numbers are real or imaginary (see Exercise 57): (a) $\sqrt[5]{-32}$ (b) $\sqrt[4]{-64}$

Answers to Practice Exercises

1. $2\sqrt{3}$ **2.** $6\sqrt{5}$ **3.** real, -2

1.7 Addition and Subtraction of Algebraic Expressions

Algebraic Expressions • Terms • Factors • Polynomials • Similar Terms • Simplifying • Symbols of Grouping

Because we use letters to represent numbers, we can see that all operations that can be used on numbers can also be used on literal numbers. In this section, we discuss the methods for adding and subtracting literal numbers.

Addition, subtraction, multiplication, division, and taking of roots are known as **algebraic operations.** *Any combination of numbers and literal symbols that results from algebraic operations is known as an* **algebraic expression.**

When an algebraic expression consists of several parts connected by plus signs and minus signs, each part (along with its sign) is known as a **term** *of the expression. If a given expression is made up of the product of a number of quantities, each of these quantities, or any product of them, is called a* **factor** *of the expression.*

CAUTION ➧ It is very important to ***distinguish clearly between terms and factors,*** because some operations that are valid for terms are not valid for factors, and conversely. Some of the common errors in handling algebraic expressions occur because these operations are not handled properly.

EXAMPLE 1 Terms and factors

In the study of the motion of a rocket, the following algebraic expression may be used.

terms

$$gt^2 - 2vt + 2s$$

factors

This expression has three terms: gt^2, $-2vt$, and $2s$. The first term, gt^2, has a factor of g and two factors of t. Any product of these factors is also a factor of gt^2. This means other factors are gt, t^2, and gt^2 itself. ■

EXAMPLE 2 Terms and factors

$7a(x^2 + 2y) - 6x(5 + x - 3y)$ is an expression with terms $7a(x^2 + 2y)$ and $-6x(5 + x - 3y)$.

The term $7a(x^2 + 2y)$ has individual factors of 7, a, and $(x^2 + 2y)$, as well as products of these factors. The factor $x^2 + 2y$ has two terms, x^2 and $2y$.

The term $-6x(5 + x - 3y)$ has factors 2, 3, x, and $(5 + x - 3y)$. The negative sign in front can be treated as a factor of -1. The factor $5 + x - 3y$ has three terms, 5, x, and $-3y$. ■

■ In Chapter 11, we will see that roots are equivalent to noninteger exponents.

A **polynomial** *is an algebraic expression with only nonnegative integer exponents on one or more variables, and has no variable in a denominator. The* **degree of a term** *is the sum of the exponents of the variables of that term, and the* **degree of the polynomial** *is the degree of the term of highest degree.*

A **multinomial** *is any algebraic expression of more than one term.* Terms like $1/x$ and $\sqrt{x}$ can be included in a multinomial, but not in a polynomial. (Since $1/x = x^{-1}$, the exponent is negative.

EXAMPLE 3 Polynomials

Some examples of polynomials are as follows:

(a) $4x^2 - 5x + 3$ (degree 2) **(b)** $2x^6 - x$ (degree 6) **(c)** $3x$ (degree 1)

(d) $xy^3 + 7x - 3$ (degree 4) (add exponents of x and y)

(e) -6 (degree 0) $(-6 = -6x^0)$

From (c), note that a single term can be a polynomial, and from (e), note that a constant can be a polynomial. The expressions in (a), (b), and (d) are also multinomials.

The expression $x^2 + \sqrt{y + 2} - 8$ is a multinomial, but not a polynomial because of the square root term. ■

■ In practice, the numerical coefficient is usually called the coefficient. However, a more general definition is: *any factor of a product is the coefficient of the other factors.*

A polynomial with one term is called a **monomial.** *A polynomial with two terms is called a* **binomial,** *and one with three terms is called a* **trinomial.** *The numerical factor is called the* **numerical coefficient** (or simply **coefficient**) *of the term. All terms that differ at most in their numerical coefficients are known as* **similar** *or* **like** *terms.* That is, similar terms have the same variables with the same exponents.

EXAMPLE 4 Monomial, binomial, trinomial

(a) $7x^4$ is a monomial. The numerical coefficient is 7.

(b) $3ab - 6a$ is a binomial. The numerical coefficient of the first term is 3, and the numerical coefficient of the second term is -6. Note that the sign is attached to the coefficient.

(c) $8cx^3 - x + 2$ is a trinomial. The coefficients of the first two terms are 8 and -1.

(d) $x^2y^2 - 2x + 3y - 9$ is a polynomial with four terms (no special name). ■

EXAMPLE 5 Similar terms

(a) $8b - 6ab + 81b$ is a trinomial. The first and third terms are similar because they differ only in their numerical coefficients. The middle term is not similar to the others because it has a factor of a.

(b) $4x^2 - 3x$ is a binomial. The terms are not similar since the first term has two factors of x, and the second term has only one factor of x.

(c) $3x^2y^3 - 5y^3x^2 + x^2 - 2y^3$ is a polynomial. The commutative law tells us that $x^2y^3 = y^3x^2$, which means the first two terms are similar. ■

In adding and subtracting algebraic expressions, we combine similar terms into a single term. The ***simplified*** expression will contain only terms that are not similar.

EXAMPLE 6 Simplifying expressions

■ Some calculators can display algebraic expressions and perform algebraic operations.

(a) $3x + 2x - 5y = 5x - 5y$ Add similar terms—result has unlike terms

(b) $6a^2 - 7a + 8ax$ cannot be simplified since there are no like terms.

(c) $6a + 5c + 2a - c = 6a + 2a + 5c - c$ commutative law

$= 8a + 4c$ add like terms ■

To group terms in an algebraic expression, we use **symbols of grouping.** In this text, we use **parentheses,** ( ), **brackets,** [ ], and **braces,** { }. The **bar,** which is used with radicals and fractions, also groups terms. In earlier sections, we used parentheses and the bar.

CAUTION ▸ In simplifying an expression using the distributive law, to remove the symbols of grouping *if a* ***MINUS sign*** *precedes the grouping,* ***change the sign of EVERY term in the grouping,*** *or if a* **plus sign** *precedes the grouping* ***retain the sign of every term.***

EXAMPLE 7 Symbols of grouping

(a) $2(a + 2x) = 2a + 2(2x)$ use distributive law

$= 2a + 4x$

(b) $-(+a - 3c) = (-1)(+a - 3c)$ treat $-$ sign as -1

$= (-1)(+a) + (-1)(-3c)$

$= -a + 3c$ note change of signs

Normally, $+a$ would be written simply as a. ■

Practice Exercises

Use the distributive law.

1. $3(2a + y)$ **2.** $-3(-2r + s)$

In Example 7(a), we see that $2(a + 2x) = 2a + 4x$. This is true for any values of a and x, and is therefore an *identity* (see page 7).

EXAMPLE 8 Simplifying: signs before parentheses

Practice Exercise

3. Simplify $2x - 3(4y - x)$

$+$ sign before parentheses

(a) $3c + (2b - c) = 3c + 2b - c = 2b + 2c$ use distributive law

$2b = +2b$ signs retained

$-$ sign before parentheses

(b) $3c - (2b - c) = 3c - 2b + c = -2b + 4c$ use distributive law

$2b = +2b$ signs changed

■ Note in each case that the parentheses are removed and the sign before the parentheses is also removed.

(c) $3c - (-2b + c) = 3c + 2b - c = 2b + 2c$ use distributive law

signs changed

(d) $y(3 - y) - 2(y - 3) = 3y - y^2 - 2y + 6$ note the $-2(-3) = +6$

$= -y^2 + y + 6$ ■

EXAMPLE 9 Simplifying in an application

In designing a certain machine part, it is necessary to perform the following simplification.

$$16(8 - x) - 2(8x - x^2) - (64 - 16x + x^2) = 128 - 16x - 16x + 2x^2 - 64 + 16x - x^2$$
$$= 64 - 16x + x^2$$ ■

At times, we have expressions in which more than one symbol of grouping is to be removed in the simplification. Normally, *when several symbols of grouping are to be removed, it is more convenient to* ***remove the innermost symbols first.***

NOTE ▸

One of the most common errors made is changing the sign of only the first term when removing symbols of grouping preceded by a minus sign. *Remember*, ***if the symbols are preceded by a minus sign, we must change the sign of every term.***

CAUTION ▸

EXAMPLE 10 Several symbols of grouping

(a) $3ax - [ax - (5s - 2ax)] = 3ax - [ax - 5s + 2ax]$ remove parentheses

$= 3ax - ax + 5s - 2ax$ remove brackets

$= 5s$

(b) $3a^2b - \{[a - (2a^2b - a)] + 2b\} = 3a^2b - \{[a - 2a^2b + a] + 2b\}$ remove parentheses

$= 3a^2b - \{a - 2a^2b + a + 2b\}$ remove brackets

$= 3a^2b - a + 2a^2b - a - 2b$ remove braces

$= 5a^2b - 2a - 2b$ ■

Calculators use only parentheses for grouping symbols, and we often need to use one set of parentheses within another set. These are called **nested parentheses.** In the next example, note that the innermost parentheses are removed first.

EXAMPLE 11 Nested parentheses

■ See Appendix C.4, Example 1, for an advanced graphing calculator, TI89, display of this simplification.

$$2 - (3x - 2(5 - (7 - x))) = 2 - (3x - 2(5 - 7 + x))$$
$$= 2 - (3x - 10 + 14 - 2x)$$
$$= 2 - 3x + 10 - 14 + 2x = -x - 2$$ ■

EXERCISES 1.7

In Exercises 1–4, make the given changes in the indicated examples of this section, and then solve the resulting problems.

1. In Example 6(a), change $2x$ to $2y$.

2. In Example 8(a), change the sign before $(2b - c)$ from $+$ to $-$.

3. In Example 10(a), change $[ax - (5s - 2ax)]$ to $[(ax - 5s) - 2ax]$.

4. In Example 10(b), change $\{[a - (2a^2b - a)] + 2b\}$ to $\{a - [2a^2b - (a + 2b)]\}$.

In Exercises 5–54, simplify the given algebraic expressions.

5. $5x + 7x - 4x$

6. $6t - 3t - 4t$

7. $2y - y + 4x$

8. $-4C + L - 6C$

9. $2F - 2T - 2 + 3F - T$

10. $x - 2y - 3x - y + z$

11. $a^2b - a^2b^2 - 2a^2b$

12. $-xy^2 - 3x^2y^2 + 2xy^2$

13. $s + (3s - 4 - s)$

14. $5 + (3 - 4n + p)$

15. $v - (4 - 5x + 2v)$

16. $-2a - \frac{1}{2}(b - a)$

17. $2 - 3 - (4 - 5a)$

18. $\sqrt{A} + (h - 2\sqrt{A}) - 3\sqrt{A}$

19. $(a - 3) + (5 - 6a)$

20. $(4x - y) - (-2x - 4y)$

21. $-(t - 2u) + (3u - t)$

22. $-2(6x - 3y) - (5y - 4x)$

23. $3(2r + s) - (-5s - r)$

24. $3(a - b) - 2(a - 2b)$

25. $-7(6 - 3j) - 2(j + 4)$

26. $-(5t + a^2) - 2(3a^2 - 2st)$

27. $-[(6 - n) - (2n - 3)]$

28. $-[(A - B) - (B - A)]$

29. $2[4 - (t^2 - 5)]$

30. $-3[-3 - \frac{2}{3}(-a - 4)]$

31. $-2[-x - 2a - (a - x)]$

32. $-2[-3(x - 2y) + 4y]$

33. $aZ - [3 - (aZ + 4)]$

34. $9v - [6 - (-v - 4) + 4v]$

35. $8c - \{5 - [2 - (3 + 4c)]\}$

36. $7y - \{y - [2y - (x - y)]\}$

37. $5p - (q - 2p) - [3q - (p - q)]$

38. $-(4 - \sqrt{LC}) - [(5\sqrt{LC} - 7) - (6\sqrt{LC} + 2)]$

39. $-2\{-(4 - x^2) - [3 + (4 - x^2)]\}$

40. $-\{-[-(x - 2a) - b] - (a - x)\}$

41. $5V^2 - (6 - (2V^2 + 3))$

42. $-2F + 2((2F - 1) - 5)$

43. $-(3t - (7 + 2t - (5t - 6)))$

44. $a^2 - 2(x - 5 - (7 - 2(a^2 - 2x) - 3x))$

45. $-4[4R - 2.5(Z - 2R) - 1.5(2R - Z)]$

46. $-3\{2.1e - 1.3[-f - 2(e - 5f)]\}$

47. In determining the size of a V belt to be used with an engine, the expression $3D - (D - d)$ is used. Simplify this expression.

48. When finding the current in a transistor circuit, the expression $i_1 - (2 - 3i_2) + i_2$ is used. Simplify this expression. (The numbers below the i's are *subscripts*. Different subscripts denote different variables.)

49. Research on a plastic building material leads to $\left[\left(B + \frac{4}{3}\alpha\right) + 2\left(B - \frac{2}{3}\alpha\right)\right] - \left[\left(B + \frac{4}{3}\alpha\right) - \left(B - \frac{2}{3}\alpha\right)\right]$. Simplify this expression.

50. One car goes 30 km/h for $t - 1$ hours, and a second car goes 40 km/h for $t + 2$ hours. Find the expression for the sum of the distances traveled by the two cars.

51. A shipment contains x film cartridges for 15 exposures each and $x + 10$ cartridges for 25 exposures each. What is the total number of photographs that can be taken with the film from this shipment?

52. Each of two suppliers has $2n + 1$ bundles of shingles costing \$30 each and $n - 2$ bundles costing \$20 each. How much more is the total value of the \$30 bundles than the \$20 bundles?

53. For the expressions $2x^2 - y + 2a$ and $3y - x^2 - b$ find (a) the sum, and (b) the difference if the second is subtracted from the first.

54. For the following expressions, subtract the third from the sum of the first two: $3a^2 + b - c^3, 2c^3 - 2b - a^2, 4c^3 - 4b + 3$.

In Exercises 55–60, answer the given questions.

W **55.** Is the following simplification correct? Explain.

$$\begin{aligned} 2x - 3y + 5 - (4x - y + 3) &= 2x - 3y + 5 - 4x - y + 3 \\ &= -2x - 4y + 8 \end{aligned}$$

W **56.** Is the following simplification correct? Explain.

$$\begin{aligned} 2a - 3b - 4c - (-5a + 3b - 2c) &= 2a - 3b - 4c + 5a - 3b - 2c \\ &= 7a - 6b - 6c \end{aligned}$$

57. If $3 - 5x < 0$, simplify $5x - 3 - |3 - 5x|$.

58. If $4 - x < 0$, simplify $|3 - x| - |2 - x|$.

W **59.** For any real numbers a and b, is it true that $|a - b| = |b - a|$? Explain.

W **60.** Is subtraction associative? That is, in general, does $(a - b) - c$ equal $a - (b - c)$? Explain.

Answers to Practice Exercises

1. $6a + 3y$ **2.** $6r - 3s$ **3.** $5x - 12y$

1.8 Multiplication of Algebraic Expressions

Multiplying Monomials • Products of Monomials and Polynomials • Powers of Polynomials

To find the product of two or more monomials, we multiply the numerical coefficients to find the numerical coefficient of the product, and multiply the literal numbers, remembering that *the exponents may be combined only if the base is the same*.

EXAMPLE 1 Multiplying monomials

(a) $3c^5(-4c^2) = -12c^7$ multiply numerical coefficients and add exponents of c

(b) $(-2b^2y^3)(-9aby^5) = 18ab^3y^8$ add exponents of same base

(c) $2xy(-6cx^2)(3xcy^2) = -36c^2x^4y^3$ ■

NOTE ▸ If a product contains a monomial that is raised to a power, *we must first raise it to the indicated power* before proceeding with the multiplication.

EXAMPLE 2 Product containing power of a monomial

(a) $-3a(2a^2x)^3 = -3a(8a^6x^3) = -24a^7x^3$

(b) $2s^3(-st^4)^2(4s^2t) = 2s^3(s^2t^8)(4s^2t) = 8s^7t^9$ ■

We find the product of a monomial and a polynomial by using the distributive law, which states that we *multiply each term of the polynomial by the monomial.* In doing so, we must be careful to give the correct sign to each term of the product.

EXAMPLE 3 Product of monomial and polynomial

(a) $2ax(3ax^2 - 4yz) = 2ax(3ax^2) + (2ax)(-4yz) = 6a^2x^3 - 8axyz$

(b) $5cy^2(-7cx - ac) = (5cy^2)(-7cx) + (5cy^2)(-ac) = -35c^2xy^2 - 5ac^2y^2$ ■

It is generally not necessary to write out the middle step as it appears in the preceding example. We write the answer directly. For instance, Example 3(a) would appear as $2ax(3ax^2 - 4yz) = 6a^2x^3 - 8axyz$.

Practice Exercises

Perform the indicated multiplications.

1. $2a^3b(-6ab^2)$ **2.** $-5x^2y^3(2xy - y^4)$

We find the product of two polynomials by using the distributive law. The result is that *we multiply each term of one polynomial by each term of the other and add the results.* Again we must be careful to give each term of the product its correct sign.

EXAMPLE 4 Product of polynomials

■ Note that, using the distributive law, $(x-2)(x+3) = (x-2)(x) + (x-2)(3)$ leads to the same result.

$$(x-2)(x+3) = x(x) + x(3) + (-2)(x) + (-2)(3)$$
$$= x^2 + 3x - 2x - 6 = x^2 + x - 6$$ ■

Finding the power of a polynomial is equivalent to using the polynomial as a factor the number of times indicated by the exponent. It is sometimes convenient to write the power of a polynomial in this form before multiplying.

EXAMPLE 5 Power of a polynomial

two factors

(a) $(x+5)^2 = (x+5)(x+5) = x^2 + 5x + 5x + 25$
$= x^2 + 10x + 25$

(b) $(2a-b)^3 = (2a-b)(2a-b)(2a-b)$ the exponent 3 indicates three factors
$= (2a-b)(4a^2 - 2ab - 2ab + b^2)$
$= (2a-b)(4a^2 - 4ab + b^2)$
$= 8a^3 - 8a^2b + 2ab^2 - 4a^2b + 4ab^2 - b^3$
$= 8a^3 - 12a^2b + 6ab^2 - b^3$

Practice Exercises

Perform the indicated multiplications.

3. $(2s-5t)(s+4t)$ **4.** $(3u+2v)^2$

■ See Appendix C.4, Example 1(b), for an advanced graphing calculator, TI89, display of this multiplication.

We should note in illustration (a) that

CAUTION

$(x+5)^2$ is not equal to $x^2 + 25$

because the term $10x$ is not included. We must follow the proper procedure and not simply square each of the terms within the parentheses. ■

EXAMPLE 6 Products in an application

An expression used with a lens of a certain telescope is simplified as shown.

$$\begin{aligned} &a(a+b)^2 + a^3 - (a+b)(2a^2 - s^2) \\ &= a(a+b)(a+b) + a^3 - (2a^3 - as^2 + 2a^2b - bs^2) \\ &= a(a^2 + ab + ab + b^2) + a^3 - 2a^3 + as^2 - 2a^2b + bs^2 \\ &= a^3 + a^2b + a^2b + ab^2 - a^3 + as^2 - 2a^2b + bs^2 \\ &= ab^2 + as^2 + bs^2 \end{aligned}$$ ■

EXERCISES 1.8

In Exercises 1–4, make the given changes in the indicated examples of this section, and then solve the resulting problems.

1. In Example 2(b), change the factor $(-st^4)^2$ to $(-st^4)^3$.
2. In Example 3(a), change the factor $2ax$ to $-2ax$.
3. In Example 4, change the factor $(x+3)$ to $(x-3)$.
4. In Example 5(b), change the exponent 3 to 2.

In Exercises 5–66, perform the indicated multiplications.

5. $(a^2)(ax)$ **6.** $(2xy)(x^2y^3)$ **7.** $-ac^2(acx^3)$
8. $-2cs^2(-4cs)^2$ **9.** $(2ax^2)^2(-2ax)$ **10.** $6pq^3(3pq^2)^2$
11. $i^2(R+2r)$ **12.** $2x(-p-q)$ **13.** $-3s(s^2-5t)$
14. $-3b(2b^2-b)$ **15.** $5m(m^2n+3mn)$ **16.** $a^2bc(2ac-3b^2c)$
17. $3M(-M-N+2)$ **18.** $-4c^2(-9gc-2c+g^2)$
19. $ax(cx^2)(x+y^3)$ **20.** $-2(-3st^3)(3s-4t)$
21. $(x-3)(x+5)$ **22.** $(a+7)(a+1)$
23. $(x+5)(2x-1)$ **24.** $(4t_1+t_2)(2t_1-3t_2)$
25. $(2a-b)(-2b+3a)$ **26.** $(-3+4w^2)(3w^2-1)$
27. $(2s+7t)(3s-5t)$ **28.** $(5p-2q)(p+8q)$
29. $(x^2-1)(2x+5)$ **30.** $(3y^2+2)(2y-9)$

31. $(x - 2y - 4)(x - 2y + 4)$

32. $(2a + 3b + 1)(2a + 3b - 1)$

33. $2(a + 1)(a - 9)$ **34.** $-5(y - 3)(y + 6)$

35. $-3(3 - 2T)(3T + 2)$ **36.** $2n(-n + 5)(6n + 5)$

37. $2L(L + 1)(4 - L)$ **38.** $ax(x + 4)(7 - x^2)$

39. $(2x - 5)^2$ **40.** $(x - 3y)^2$ **41.** $(x_1 + 3x_2)^2$

42. $(-7m - 1)^2$ **43.** $(xyz - 2)^2$ **44.** $(-6x^2 + b)^2$

45. $2(x + 8)^2$ **46.** $3(3R - 4)^2$

47. $(2 + x)(3 - x)(x - 1)$ **48.** $(-c^2 + 3x)^3$

49. $3T(T + 2)(2T - 1)$ **50.** $[(x - 2)^2(x + 2)]^2$

51. Let $x = 3$ and $y = 4$ to show that (a) $(x + y)^2 \neq x^2 + y^2$ and (b) $(x - y)^2 \neq x^2 - y^2$. ($\neq$ means "does not equal")

52. Evaluate the product $(98)(102)$ by expressing it as $(100 - 2)(100 + 2)$.

53. Is the following simplification correct?
$(x^2)(x^4) + (x^3)^5 = x^8 + x^8 = 2x^8$
If not, what is the correct result?

54. Is the following simplification correct?
$[(x^3)(x^2)]^2 + [x(x^4)]^2 = 2x^7$
If not, what is the correct result?

W **55.** Square an integer between 1 and 9 and subtract 1 from the result. Explain why the result is the product of the integer before and the integer after the one you chose.

W **56.** Explain how, by appropriate grouping, the product $(x - 2)(x + 3)(x + 2)(x - 3)$ is easier to find. Find the product.

57. By multiplication, show that $(x + y)^3$ is not equal to $x^3 + y^3$.

58. By multiplication, show that $(x + y)(x^2 - xy + y^2) = x^3 + y^3$.

59. In finding the value of a certain savings account, the expression $P(1 + 0.01r)^2$ is used. Multiply out this expression.

60. A savings account of \$1000 that earns $r\%$ annual interest, compounded quarterly, has a value of $1000(1 + 0.0025r)^2$ after six months. Perform the indicated multiplication.

61. In using aircraft radar, the expression $(2R - X)^2 - (R^2 + X^2)$ arises. Simplify this expression.

62. In calculating the temperature variation of an industrial area, the expression $(2T^3 + 3)(T^2 - T - 3)$ arises. Perform the indicated multiplication.

63. In a particular computer design containing n circuit elements, n^2 switches are needed. Find the expression for the number of switches needed for $n + 100$ circuit elements.

64. Simplify the expression $(T^2 - 100)(T - 10)(T + 10)$, which arises when analyzing the energy radiation from an object.

65. In finding the maximum power in part of a microwave transmitter circuit, the expression $(R_1 + R_2)^2 - 2R_2(R_1 + R_2)$ is used. Multiply and simplify.

66. In determining the deflection of a certain steel beam, the expression $27x^2 - 24(x - 6)^2 - (x - 12)^3$ is used. Multiply and simplify.

Answers to Practice Exercises

1. $-12a^4b^3$ **2.** $-10x^3y^4 + 5x^2y^7$

3. $2s^2 + 3st - 20t^2$ **4.** $9u^2 + 12uv + 4v^2$

1.9 Division of Algebraic Expressions

Dividing Monomials • Dividing by a Monomial • Dividing One Polynomial by Another

To find the quotient of one monomial divided by another, we use the laws of exponents and the laws for dividing signed numbers. Again, the *exponents may be combined only if the base is the same*.

EXAMPLE 1 Dividing monomials

(a) $\dfrac{3c^7}{c^2} = 3c^{7-2} = 3c^5$

(b) $\dfrac{16x^3y^5}{4xy^2} = \dfrac{16}{4}(x^{3-1})(y^{5-2}) = 4x^2y^3$

divide coefficients

subtract exponents

(c) $\dfrac{-6a^2xy^2}{2axy^4} = -\left(\dfrac{6}{2}\right)\dfrac{a^{2-1}x^{1-1}}{y^{4-2}} = -\dfrac{3a}{y^2}$

As shown in illustration (c), we use only positive exponents in the final result unless there are specific instructions otherwise. ■

From arithmetic, we may show how a multinomial is to be divided by a monomial. When adding fractions $\left(\text{say } \frac{2}{7} \text{ and } \frac{3}{7}\right)$, we have $\dfrac{2}{7} + \dfrac{3}{7} = \dfrac{2 + 3}{7}$.

Looking at this from *right to left,* we see that *the quotient of a multinomial divided by a monomial is found by dividing each term of the multinomial by the monomial and adding the results*. This can be shown as

$$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$$

■ This is an identity and is valid for all values of a and b, and all values of c except zero (which would make it undefined).

CAUTION ▶ Be careful: Although $\dfrac{a + b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$, we must note that $\dfrac{c}{a + b}$ ***is not*** $\dfrac{c}{a} + \dfrac{c}{b}$.

EXAMPLE 2 Dividing by a monomial

(a) $$\frac{4a^2 + 8a}{2a} = \frac{4a^2}{2a} + \frac{8a}{2a} = 2a + 4$$

each term of numerator divided by denominator

(b) $$\frac{4x^3y - 8x^3y^2 + 2x^2y}{2x^2y} = \frac{4x^3y}{2x^2y} - \frac{8x^3y^2}{2x^2y} + \frac{2x^2y}{2x^2y}$$
$$= 2x - 4xy + 1$$

Practice Exercise

Divide: **1.** $\dfrac{4ax^2 - 6a^2x}{2ax}$

■ Until you are familiar with the method, it is recommended that you do write out the middle steps.

We usually do not write out the middle step as shown in these illustrations. The divisions of the terms of the numerator by the denominator are usually done by inspection (mentally), and the result is shown as it appears in the next example.

NOTE ▸ Note carefully the last term 1 of the result. When all factors of the numerator are the same as those in the denominator, we are dividing a number by itself, which gives result of 1. ■

EXAMPLE 3 Application of dividing by a monomial

The expression $\dfrac{2p + v^2d + 2ydg}{2dg}$ is used when analyzing the operation of an irrigation pump. Performing the indicated division, we have

$$\frac{2p + v^2d + 2ydg}{2dg} = \frac{p}{dg} + \frac{v^2}{2g} + y$$ ■

DIVISION OF ONE POLYNOMIAL BY ANOTHER

■ This is similar to long division of numbers.

To divide one polynomial by another, use the following steps. (1) Arrange the dividend (the polynomial to be divided) and the divisor in descending powers of the variable. (2) Divide the first term of the dividend by the first term of the divisor. The result is the first term of the quotient. (3) Multiply the entire divisor by the first term of the quotient and ***subtract*** the product from the dividend. (4) Divide the first term of this difference by the first term of the divisor. This gives the second term of the quotient. (5) Multiply this term by the entire divisor and ***subtract*** the product from the first difference. (6) Repeat this process until the remainder is zero or a term of lower degree than the divisor.

EXAMPLE 4 Dividing one polynomial by another

Perform the division $(6x^2 + x - 2) \div (2x - 1)$.

(This division can also be indicated in the fractional form $\dfrac{6x^2 + x - 2}{2x - 1}$.)

We set up the division as we would for long division in arithmetic. Then, following the procedure outlined above, we have the following:

$$\begin{array}{r|l} & 3x + 2 \\ \hline 2x - 1 & 6x^2 + x - 2 \\ & \underline{6x^2 - 3x} \\ & \quad\quad 4x - 2 \\ & \quad\quad \underline{4x - 2} \\ & \quad\quad\quad\quad 0 \end{array}$$

$\frac{6x^2}{2x}$ divide first term of dividend by first term of divisor

$3x(2x - 1)$

CAUTION ▸ subtract

$6x^2 - 6x^2 = 0$

$x - (-3x) = 4x$

subtract

Practice Exercise

Divide: **2.** $(6x^2 + 7x - 3) \div (3x - 1)$

The remainder is zero and the quotient is $3x + 2$. Note that when we subtracted $-3x$ from x, we obtained $4x$. ■

EXAMPLE 5 Quotient with a remainder

Perform the division $\dfrac{8x^3 + 4x^2 + 3}{4x^2 - 1}$. Because there is no x-term in the dividend, we should leave space for any x-terms that might arise (which we will show as $0x$).

$$
\begin{array}{llrl}
 & & 2x + 1 & \\
\text{divisor} & 4x^2 - 1 & \overline{)\,8x^3 + 4x^2 + 0x + 3} & \text{dividend} \qquad \dfrac{8x^3}{4x^2} = 2x \\
 & & \underline{8x^3 \qquad\quad - 2x \qquad} & \text{subtract} \\
 & & 4x^2 + 2x + 3 & \qquad\qquad\quad \dfrac{4x^2}{4x^2} = 1 \\
 & 0x - (-2x) = 2x & & \\
 & & \underline{4x^2 \qquad\quad - 1} & \text{subtract} \\
 & & 2x + 4 & \text{remainder}
\end{array}
$$

■ See Appendix C.4, Example 2, for an advanced graphing calculator, TI89, display of this division.

Because the degree of the remainder $2x + 4$ is less than that of the divisor, we now show the quotient in this case as $2x + 1 + \frac{2x + 4}{4x^2 - 1}$. ■

EXERCISES 1.9

In Exercises 1–4, make the given changes in the indicated examples of this section and then perform the indicated divisions.

1. In Example 1(c), change the denominator to $-2a^2xy^5$.
2. In Example 2(b), change the denominator to $2xy^2$.
3. In Example 4, change the dividend to $6x^2 - 7x + 2$.
4. In Example 5, change the sign of the middle term of the numerator from $+$ to $-$.

In Exercises 5–24, perform the indicated divisions.

5. $\dfrac{8x^3y^2}{-2xy}$ 6. $\dfrac{-18b^7c^3}{bc^2}$ 7. $\dfrac{-16r^3t^5}{-4r^5t}$ 8. $\dfrac{51mn^5}{17m^2n^2}$

9. $\dfrac{(15x^2)(4bx)(2y)}{30bxy}$ 10. $\dfrac{(5sT)(8s^2T^3)}{10s^3T^2}$ 11. $\dfrac{6(ax)^2}{-ax^2}$

12. $\dfrac{12a^2b}{(3ab^2)^2}$ 13. $\dfrac{3a^2x + 6xy}{3x}$ 14. $\dfrac{2m^2n - 6mn}{-2m}$

15. $\dfrac{3rst - 6r^2st^2}{3rs}$ 16. $\dfrac{-5a^2n - 10an^2}{5an}$

17. $\dfrac{4pq^3 + 8p^2q^2 - 16pq^5}{4pq^2}$ 18. $\dfrac{a^2x_1x_2^2 + ax_1^3 - ax_1}{ax_1}$

19. $\dfrac{2\pi fL - \pi fR^2}{\pi fR}$ 20. $\dfrac{9(aB)^4 - 6aB^4}{-3aB^3}$

21. $\dfrac{-3ab^2 + 6ab^3 - 9a^2b^2}{-9a^2b^2}$ 22. $\dfrac{2x^{n+2} + 4ax^n}{2x^n}$

23. $\dfrac{6y^{2n} - 4ay^{n+1}}{2y^n}$ 24. $\dfrac{3a(F + T)b^2 - (F + T)}{a(F + T)}$

In Exercises 25–42, perform the indicated divisions. Express the answer as shown in Example 5 when applicable.

25. $(2x^2 + 7x + 3) \div (x + 3)$ 26. $(3t^2 - 7t + 4) \div (t - 1)$
27. $(x^2 - 3x + 2) \div (x - 2)$ 28. $(2x^2 - 5x - 7) \div (x + 1)$
29. $(x - 14x^2 + 8x^3) \div (2x - 3)$
30. $(6 + 7y + 6y^2) \div (2y + 1)$
31. $(4Z^2 - 5Z - 7) \div (4Z + 3)$
32. $(6x^2 - 5x - 9) \div (-4 + 3x)$

33. $\dfrac{x^3 + 3x^2 - 4x - 12}{x + 2}$ 34. $\dfrac{3x^3 + 19x^2 + 13x - 20}{3x - 2}$

35. $\dfrac{2a^4 + 4a^2 - 16}{a^2 - 2}$ 36. $\dfrac{6T^3 + T^2 + 2}{3T^2 - T + 2}$

37. $\dfrac{x^3 + 8}{x + 2}$ 38. $\dfrac{D^3 - 1}{D - 1}$

39. $\dfrac{x^2 - 2xy + y^2}{x - y}$ 40. $\dfrac{3r^2 - 5rR + 2R^2}{r - 3R}$

41. $\dfrac{x^2 - y^2 + 2yz - z^2}{x + y - z}$ 42. $\dfrac{a^4 + b^4}{a^2 - 2ab + 2b^2}$

In Exercises 43–54, perform the indicated divisions.

43. When $2x^2 - 9x - 5$ is divided by $x + c$, the quotient is $2x + 1$. Find c.
44. When $6x^2 - x + k$ is divided by $3x + 4$, the remainder is zero. Find k.
45. By division show that $\dfrac{x^4 + 1}{x + 1}$ is not equal to x^3.
46. By division show that $\dfrac{x^3 + y^3}{x + y}$ is not equal to $x^2 + y^2$.
47. Find k such that the remainder of the division $(6x^3 - x^2 - 14x + k) \div (3x - 2)$ is -2.
48. The area of a certain rectangle can be represented by $6x^2 + 19x + 10$. If the length is $2x + 5$, what is the width? (Divide the area by the length.)
49. In the optical theory dealing with lasers, the following expression arises: $\dfrac{8A^5 + 4A^3\mu^2E^2 - A\mu^4E^4}{8A^4}$. ($\mu$ is the Greek letter mu.)

50. In finding the total resistance of the resistors shown in Fig. 1.13, the following expression is used.

$$\frac{6R_1 + 6R_2 + R_1R_2}{6R_1R_2}$$

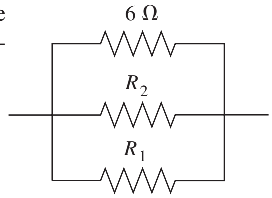


Fig. 1.13

51. When analyzing the potential energy associated with gravitational forces, the expression $\frac{GMm[(R + r) - (R - r)]}{2rR}$ arises. Perform the indicated division.

52. A computer model shows that the temperature change T in a certain freezing unit is found by using the expression $\frac{3T^3 - 8T^2 + 8}{T - 2}$. Perform the indicated division.

53. In analyzing the displacement of a certain valve, the expression $\frac{s^2 - 2s - 2}{s^4 + 4}$ is used. Find the reciprocal of this expression and then perform the indicated division.

54. In analyzing a rectangular computer image, the area and width of the image vary with time such that the length is given by the expression $\frac{2t^3 + 94t^2 - 290t + 500}{2t + 100}$. By performing the indicated division, find the expression for the length.

Answers to Practice Exercises

1. $2x - 3a$ **2.** $2x + 3$

1.10 Solving Equations

Types of Equations • Solving Basic Types of Equations • Checking the Solution • First Steps • Ratio and Proportion

In this section, we show how algebraic operations are used in solving equations. In the following sections, we show some of the important applications of equations.

An **equation** *is an algebraic statement that two algebraic expressions are equal.* Any value of the literal number representing the **unknown** that produces equality when **substituted** in the equation is said to **satisfy** the equation.

EXAMPLE 1 Valid values for equations

The equation $3x - 5 = x + 1$ is true only if $x = 3$. Substituting 3 for x in the equation, we have $3(3) - 5 = 3 + 1$, or $4 = 4$; substituting $x = 2$, we have $1 = 3$, which is not correct.

This equation is valid for only one value of the unknown. *An equation valid only for certain values of the unknown is a* **conditional equation.** In this section, nearly all equations we solve will be conditional equations that are satisfied by only one value of the unknown. ■

EXAMPLE 2 Identity and contradiction

■ We have previously noted *identities* on pages 7 and 28.

(a) The equation $x^2 - 4 = (x - 2)(x + 2)$ is true for all values of x. For example, substituting $x = 3$ in the equation, we have $3^2 - 4 = (3 - 2)(3 + 2)$, or $5 = 5$. Substituting $x = -1$, we have $(-1)^2 - 4 = (-1 - 2)(-1 + 2)$, or $-3 = -3$. *An equation valid for all values of the unknown is an* **identity.**

(b) The equation $x + 5 = x + 1$ is not true for any value of x. For any value of x we try, we find that the left side is 4 greater than the right side. *Such an equation is called a* **contradiction.** ■

■ Equations can be solved on most graphing calculators. An estimate (or guess) of the answer may be required to find the solution. See Exercises 45 and 46.

To **solve** *an equation, we find the values of the unknown that satisfy it*. There is one basic rule to follow when solving an equation:

Perform the same operation on both sides of the equation.

We do this to isolate the unknown and thus to find its value.

By performing the same operation on both sides of an equation, the two sides remain equal. Thus,

■ Although we may multiply both sides of an equation by zero, this produces $0 = 0$, which is not useful in finding the solution.

we may add the same number to both sides, subtract the same number from both sides, multiply both sides by the same number, or divide both sides by the same number (not zero).

EXAMPLE 3 Basic operations used in solving

In each of the following equations, we may isolate x, and thereby solve the equation, by performing the indicated operation.

■ The word *algebra* comes from Arabic and means "a restoration." It refers to the fact that when a number has been added to one side of an equation, the same number must be added to the other side to maintain equality.

$x - 3 = 12$	$x + 3 = 12$	$\frac{x}{3} = 12$	$3x = 12$
add 3 to both sides	subtract 3 from both sides	multiply both sides by 3	divide both sides by 3
$x - 3 + 3 = 12 + 3$	$x + 3 - 3 = 12 - 3$	$3\left(\frac{x}{3}\right) = 3(12)$	$\frac{3x}{3} = \frac{12}{3}$
$x = 15$	$x = 9$	$x = 36$	$x = 4$

NOTE ▸ Each solution should be checked by substitution in the original equation. ■

EXAMPLE 4 Operations used for solution; checking

Solve the equation $2t - 7 = 9$.

We are to perform basic operations to both sides of the equation to finally isolate t on one side. The steps to be followed are suggested by the form of the equation.

$$2t - 7 = 9 \quad \text{original equation}$$
$$2t - 7 + 7 = 9 + 7 \quad \text{add 7 to both sides}$$
$$2t = 16 \quad \text{combine like terms}$$
$$\frac{2t}{2} = \frac{16}{2} \quad \text{divide both sides by 2}$$
$$t = 8 \quad \text{simplify}$$

■ Note that the solution generally requires a combination of basic operations.

NOTE ▸ Therefore, we conclude that $t = 8$. Checking *in the original equation*, we have

$$2(8) - 7 \stackrel{?}{=} 9, \qquad 16 - 7 \stackrel{?}{=} 9, \qquad 9 = 9$$

The solution checks. ■

EXAMPLE 5 First remove parentheses

Solve the equation $x - 7 = 3x - (6x - 8)$.

■ With simpler numbers, many basic steps are done by inspection and not actually written down.

$$x - 7 = 3x - 6x + 8 \quad \text{parentheses removed}$$
$$x - 7 = -3x + 8 \quad x\text{-terms combined on right}$$
$$4x - 7 = 8 \quad 3x \text{ added to both sides} \quad \text{—by inspection}$$
$$4x = 15 \quad \text{7 added to both sides} \quad \text{—by inspection}$$
$$x = \tfrac{15}{4} \quad \text{both sides divided by 4} \quad \text{—by inspection}$$

Checking in the original equation, we obtain (after simplifying) $-\frac{13}{4} = -\frac{13}{4}$. ■

Practice Exercises

Solve for x.

1. $3x + 4 = x - 6$

2. $2(5 - x) = x - 8$

CAUTION ▸ Note that we ***always check in the original equation***. This is done since errors may have been made in finding the later equations.

From these examples, we see that the following steps are used in solving the basic equations of this section.

■ Many other types of equations require more advanced methods for solving. These are considered in later chapters.

PROCEDURE FOR SOLVING EQUATIONS

1. *Remove grouping symbols (distributive law).*
2. *Combine any like terms on each side (also after step 3).*
3. *Perform the same operations on both sides until $x =$ result is obtained.*
4. *Check the solution in the original equation.*

NOTE ➧ If an equation contains numbers not easily combined by inspection, the best procedure is to *first solve for the unknown and then perform the calculation.*

EXAMPLE 6 First solve for unknown—then calculate

When finding the current i (in A) in a certain radio circuit, the following equation and solution are used.

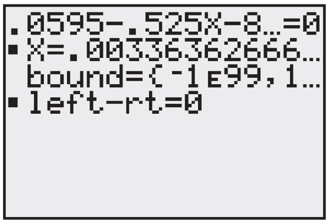


Fig. 1.14

$$0.0595 - 0.525i - 8.85(i + 0.00316) = 0$$

$$0.0595 - 0.525i - 8.85i - 8.85(0.00316) = 0 \quad \text{note how the above procedure is followed}$$

$$(-0.525 - 8.85)i = 8.85(0.00316) - 0.0595$$

$$i = \frac{8.85(0.00316) - 0.0595}{-0.525 - 8.85} \quad \text{evaluate}$$

$$= 0.00336 \text{ A}$$

The calculator solution of this equation, using the *Solver* feature, is shown in Fig. 1.14. When doing the calculation indicated above in the solution for i, be careful to group the numbers in the denominator for the division. Also, be sure to round off the result as shown above, but do not round off values before the final calculation. ■

Ratio and Proportion

The quotient a/b is also called the **ratio** *of a to b. An equation stating that two ratios are equal is called a* **proportion.** Because a proportion is an equation, if one of the numbers is unknown, we can solve for its value as with any equation. Usually, this is done by noting the denominators and multiplying each side by a number that will clear the fractions.

EXAMPLE 7 Ratio

If the ratio of x to 8 equals the ratio of 3 to 4, we have the proportion

$$\frac{x}{8} = \frac{3}{4}$$

We can solve this equation by multiplying both sides by 8. This gives

$$8\left(\frac{x}{8}\right) = 8\left(\frac{3}{4}\right), \quad \text{or} \quad x = 6$$

Substituting $x = 6$ into the original proportion gives the proportion $\frac{6}{8} = \frac{3}{4}$. Because these ratios are equal, the solution checks. ■

Practice Exercise

3. If the ratio of 2 to 5 equals the ratio of x to 30, find x.

EXAMPLE 8 Applied proportion

The supports for a roof are triangular trusses for which the longest side is $8/5$ as long as the shortest side for all of the trusses. If the shortest side of one of the trusses is 3.80 m, what is the length of the longest side of that truss?

If we label the longest side L, since the ratio of sides is $8/5$, we have

$$\frac{L}{3.80} = \frac{8}{5}$$

$$3.80\left(\frac{L}{3.80}\right) = 3.80\left(\frac{8}{5}\right)$$

$$L = 6.08 \text{ m}$$

■ Generally, units of measurement will not be shown in intermediate steps. The proper units will be shown with the data and final result.

■ If the result is required to be in feet, we have the following change of units (see P. A. 5):

$$6.08 \text{ m}\left(\frac{1 \text{ ft}}{0.3048 \text{ m}}\right) = 19.9 \text{ ft}$$

Checking, we note that $6.08/3.80 = 1.6$ and $8/5 = 1.6$. The solution checks. ■

The meanings of *ratio* and *proportion* (particularly ratio) will be of importance when studying trigonometry in Chapter 4. A detailed discussion of ratio and proportion is found in Chapter 18. A general method of solving equations involving fractions, such as we found in Examples 7 and 8, is given in Chapter 6.

EXERCISES 1.10

In Exercises 1–4, make the given changes in the indicated examples of this section and then solve the resulting problems.

1. In Example 3, change 12 to -12 in each of the four illustrations and then solve.
2. In Example 4, change $2t - 7$ to $7 - 2t$ and then solve.
3. In Example 5, change $(6x - 8)$ to $(8 - 6x)$ and then solve.
4. In Example 8, change $8/5$ to $7/4$ and then solve.

In Exercises 5–40, solve the given equations.

5. $x - 2 = 7$ **6.** $x - 4 = -1$ **7.** $x + 5 = 4$

8. $s + 6 = -3$ **9.** $-\dfrac{t}{2} = 5$ **10.** $\dfrac{x}{-4} = 2$

11. $4E = -20$ **12.** $2x = 12$ **13.** $3t + 5 = -4$

14. $5D - 2 = 13$ **15.** $5 - 2y = -3$ **16.** $-5t + 8 = 18$

17. $3x + 7 = x$ **18.** $6 + 4L = 5 - 3L$

19. $2(s - 4) = s$ **20.** $3(4 - n) = -n$

21. $-(r - 4) = 6 + 2r$ **22.** $-(x + 2) + 5 = 5x$

23. $2(x - 3) = -x$ **24.** $4(7 - F) = -7$

25. $0.1x - 0.5(x - 2) = 2$ **26.** $1.5x - 0.3(x - 4) = 6$

27. $-4 - 3(1 - 2p) = -7 + 2p$ **28.** $3 - 6(2 - 3t) = t - 5$

29. $\dfrac{4x - 2(x - 4)}{3} = 8$ **30.** $2x = \dfrac{-5(7 - 3x) + 2}{4}$

31. $|x| - 1 = 8$ **32.** $2 - |x| = 4$

33. $|2x - 3| = 5$ **34.** $|7 - x| = 1$

In Exercises 35–42, all numbers are approximate.

35. $5.8 - 0.3(x - 6.0) = 0.5x$ **36.** $1.9t = 0.5(4.0 - t) - 0.8$

37. $-0.24(C - 0.50) = 0.63$ **38.** $27.5(5.17 - 1.44x) = 73.4$

39. $\dfrac{x}{2.0} = \dfrac{17}{6.0}$ **40.** $\dfrac{3.0}{7.0} = \dfrac{R}{42}$

41. $\dfrac{165}{223} = \dfrac{13V}{15}$ **42.** $\dfrac{276x}{17.0} = \dfrac{1360}{46.4}$

In Exercises 43–54, solve the given problems.

43. Identify each of the following equations as a conditional equation, an identity, or a contradiction.
(a) $2x + 3 = 3 + 2x$ (b) $2x - 3 = 3 - 2x$

W **44.** For what values of a is the equation $2x + a = 2x$ a conditional equation? Explain.

45. Solve the equation of Example 5 by using the Equation Solver of a graphing calculator.

46. Solve the equation of Example 6 by using the Equation Solver of a graphing calculator.

47. To find the amount of a certain investment of x dollars, it is necessary to solve the equation $0.03x + 0.06(2000 - x) = 96$. Solve for x.

48. In finding the rate v (in km/h) at which a polluted stream is flowing, the equation $15(5.5 + v) = 24(5.5 - v)$ is used. Find v.

49. In finding the maximum operating temperature T (in °C) for a computer integrated circuit, the equation $1.1 = (T - 76)/40$ is used. Find the temperature.

50. To find the voltage V in a circuit in a TV remote-control unit, the equation $1.12V - 0.67(10.5 - V) = 0$ is used. Find V.

51. In blending two gasolines of different octanes, in order to find the number n of gallons of one octane needed, the equation $0.14n + 0.06(2000 - n) = 0.09(2000)$ is used. Find n, given that 0.06 and 0.09 are exact and the first zero of 2000 is significant.

52. In order to find the distance x such that the weights are balanced on the lever shown in Fig. 1.15, the equation $210(3x) = 55.3x + 38.5(8.25 - 3x)$ must be solved. Find x. (3 is exact.)

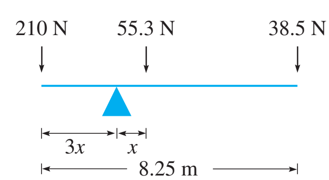


Fig. 1.15

53. The manufacturer of a certain car powered partly by gas and partly by batteries (a hybrid car) claims it can go 1250 mi on one full tank of 15 gal. How far can the car go on 5.5 gal?

54. A person 1.8 m tall is photographed with a 35-mm camera, and the film image is 20 mm. Under the same conditions, how tall is a person whose film image is 16 mm?

Answers to Practice Exercises

1. -5 **2.** 6 **3.** 12

1.11 Formulas and Literal Equations

Formulas • Literal Equations • Subscripts • Solve for Symbol before Substituting Numerical Values

Einstein published his first paper on relativity in 1905.

An important application of equations is in the use of *formulas* that are found in geometry and nearly all fields of science and technology. *A* **formula** *is an equation that expresses the relationship between two or more related quantities*. For example, Einstein's famous formula $E = mc^2$ shows the equivalence of energy E to the mass m of an object and the speed of light c.

We can solve a formula for a particular symbol just as we solve any equation. That is,

NOTE ▶ we *isolate the required symbol by using algebraic operations on literal numbers.*

EXAMPLE 1 Solving for symbol in formula

In Einstein's formula $E = mc^2$, solve for m.

$$\frac{E}{c^2} = m \qquad \text{divide both sides by } c^2$$

$$m = \frac{E}{c^2} \qquad \text{switch sides to place } m \text{ at left}$$

The required symbol is usually placed on the left, as shown. ■

EXAMPLE 2 Symbol with subscript in formula

A formula relating acceleration a, velocity v, initial velocity v_0, and time is $v = v_0 + at$. Solve for t.

■ The subscript $_0$ makes v_0 a different literal symbol from v. (We have used subscripts in a few of the earlier exercises.)

$$v - v_0 = at \qquad v_0 \text{ subtracted from both sides}$$

$$t = \frac{v - v_0}{a} \qquad \text{both sides divided by } a \text{ and then sides switched}$$ ■

EXAMPLE 3 Symbol in capital and in lowercase

■ See Appendix C.4, Example 3, for an advanced graphing calculator, TI89, display of this solution.

In the study of the forces on a certain beam, the equation $W = \dfrac{L(wL + 2P)}{8}$ is used. Solve for P.

CAUTION ▶ Be careful. Just as subscripts can denote different literal numbers, a capital letter and the same letter in lowercase are different literal numbers. In this example, W and w are different literal numbers. This is shown in several of the exercises in this section.

$$8W = \frac{8L(wL + 2P)}{8} \qquad \text{multiply both sides by 8}$$

$$8W = L(wL + 2P) \qquad \text{simplify right side}$$

$$8W = wL^2 + 2LP \qquad \text{remove parentheses}$$

$$8W - wL^2 = 2LP \qquad \text{subtract } wL^2 \text{ from both sides}$$

$$P = \frac{8W - wL^2}{2L} \qquad \text{divide both sides by } 2L \text{ and switch sides}$$ ■

EXAMPLE 4 Formula with groupings

The effect of temperature on measurements is important when measurements must be made with great accuracy. The volume V of a special precision container at temperature T in terms of the volume V_0 at temperature T_0 is given by $V = V_0[1 + b(T - T_0)]$, where b depends on the material of which the container is made. Solve for T.

Because we are to solve for T, we must isolate the term containing T. This can be done by first removing the grouping symbols.

$$V = V_0[1 + b(T - T_0)] \qquad \text{original equation}$$

$$V = V_0[1 + bT - bT_0] \qquad \text{remove parentheses}$$

$$V = V_0 + bTV_0 - bT_0V_0 \qquad \text{remove brackets}$$

$$V - V_0 + bT_0V_0 = bTV_0 \qquad \text{subtract } V_0 \text{ and add } bT_0V_0 \text{ to both sides}$$

$$T = \frac{V - V_0 + bT_0V_0}{bV_0} \qquad \text{divide both sides by } bV_0 \text{ and switch sides}$$ ■

Practice Exercises

Solve for the indicated letter. Each comes from the indicated area of study.

1. $\theta = kA + \lambda$, for λ (robotics)
2. $P = n(p - c)$, for p (economics)

To determine the values of any literal number in an expression for which we know

NOTE ▶ values of the other literal numbers, we should *first solve for the required symbol and then evaluate.*

EXAMPLE 5 Solve for symbol before substituting

The volume V (in mm^3) of a copper sphere changes with the temperature T (in °C) according to $V = V_0 + V_0\beta T$, where V_0 is the volume at 0°C. For a given sphere, $V_0 = 6715\text{ mm}^3$ and $\beta = 5.10 \times 10^{-5}/°\text{C}$. Evaluate T for $V = 6908\text{ mm}^3$.

We first solve for T and then substitute the given values.

$$V = V_0 + V_0\beta T$$
$$V - V_0 = V_0\beta T$$
$$T = \frac{V - V_0}{V_0\beta}$$

Now substituting, we have

$$T = \frac{6908 - 6715}{(6715)(5.10 \times 10^{-5})}$$
$$= 564°\text{C} \quad \text{rounded off}$$

estimation

$$\frac{200}{(5 \times 10^{-5})(70)} = \frac{200}{0.35} = 600$$

(Copper melts at about 1100°C.)

EXERCISES 1.11

In Exercises 1–4, solve for the given letter from the indicated example of this section.

1. For the formula in Example 2, solve for a.
2. For the formula in Example 3, solve for w.
3. For the formula in Example 4, solve for T_0.
4. For the formula in Example 5, solve for β. (Do not evaluate.)

In Exercises 5–40, each of the given formulas arises in the technical or scientific area of study shown. Solve for the indicated letter.

5. $E = IR$, for R (electricity)
6. $PV = nRT$, for T (chemistry)
7. $rL = g_2 - g_1$, for g_1 (surveying)
8. $W = S_dT - Q$, for Q (air conditioning)
9. $Q = SLd^2$, for L (machine design)
10. $P = 2\pi Tf$, for T (mechanics)
11. $p = p_a + dgh$, for g (hydrodynamics)
12. $2Q = 2I + A + S$, for I (nuclear physics)
13. $A = \dfrac{Rt}{PV}$, for V (jet engine design)
14. $u = -\dfrac{eL}{2m}$, for L (spectroscopy)
15. $ct^2 = 0.3t - ac$, for a (medical technology)
16. $2p + dv^2 = 2d(C - W)$, for W (fluid flow)
17. $T = \dfrac{c + d}{v}$, for d (traffic flow)
18. $B = \dfrac{\mu_0 I}{2\pi R}$, for R (magnetic field)
19. $\dfrac{K_1}{K_2} = \dfrac{m_1 + m_2}{m_1}$, for m_2 (kinetic energy)
20. $f = \dfrac{F}{d - F}$, for d (photography)
21. $a = \dfrac{2mg}{M + 2m}$, for M (pulleys)
22. $v = \dfrac{V(m + M)}{m}$, for M (ballistics)
23. $C_0^2 = C_1^2(1 + 2V)$, for V (electronics)
24. $A_1 = A(M + 1)$, for M (photography)
25. $N = r(A - s)$, for s (engineering)
26. $T = 3(T_2 - T_1)$, for T_1 (oil drilling)
27. $T_2 = T_1 - \dfrac{h}{100}$, for h (air temperature)
28. $p_2 = p_1 + rp_1(1 - p_1)$, for r (population growth)
29. $Q_1 = P(Q_2 - Q_1)$, for Q_2 (refrigeration)
30. $p - p_a = dg(y_2 - y_1)$, for y_1 (fire science)
31. $N = N_1T - N_2(1 - T)$, for N_1 (machine design)
32. $t_a = t_c + (1 - h)t_m$, for h (computer access time)
33. $L = \pi(r_1 + r_2) + 2x_1 + x_2$, for r_1 (pulleys)
34. $I = \dfrac{VR_2 + VR_1(1 + \mu)}{R_1R_2}$, for μ (electronics)
35. $P = \dfrac{V_1(V_2 - V_1)}{gJ}$, for V_2 (jet engine power)
36. $W = T(S_1 - S_2) - Q$, for S_2 (refrigeration)
37. $C = \dfrac{2eAk_1k_2}{d(k_1 + k_2)}$, for e (electronics)

38. $d = \dfrac{3LPx^2 - Px^3}{6EI}$, for L (beam deflection)

39. $V = C\left(1 - \dfrac{n}{N}\right)$, for n (property deprecation)

40. $\dfrac{p}{P} = \dfrac{AI}{B + AI}$, for B (atomic theory)

In Exercises 41–46, find the indicated values.

41. For a car's cooling system, the equation $p(C - n) + n = A$ is used. If $p = 0.25$, $C = 15.0$ L, and $A = 13.0$ L, solve for n (in L).

42. A formula used in determining the total transmitted power P_t in an AM radio signal is $P_t = P_c(1 + 0.500\,m^2)$. Find P_c if $P_t = 680$ W and $m = 0.925$.

43. A formula relating the Fahrenheit temperature F and the Celsius temperature C is $F = \frac{9}{5}C + 32$. Find the Celsius temperature that corresponds to 90.2°F.

44. In forestry, a formula used to determine the volume V of a log is $V = \frac{1}{2}L(B + b)$, where L is the length of the log and B and b are the areas of the ends. Find b (in ft^2) if $V = 38.6$ ft^3, $L = 16.1$ ft, and $B = 2.63$ ft^2. See Fig. 1.16.

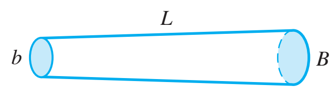


Fig. 1.16

45. The voltage V_1 across resistance R_1 is $V_1 = \dfrac{VR_1}{R_1 + R_2}$, where V is the voltage across resistances R_1 and R_2. See Fig. 1.17. Find R_2 (in Ω) if $R_1 = 3.56$ Ω, $V_1 = 6.30$ V, and $V = 12.0$ V.

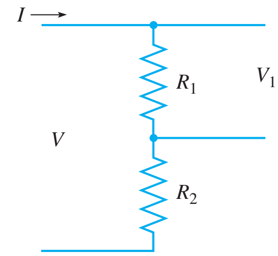


Fig. 1.17

46. The efficiency E of a computer multiprocessor compilation is given by $E = \dfrac{1}{q + p(1 - q)}$, where p is the number of processors and q is the fraction of the compilation that can be performed by the available parallel processors. Find p for $E = 0.66$ and $q = 0.83$.

In Exercises 47 and 48, set up the required formula and solve for the indicated letter.

47. One missile travels at a speed of v_2 mi/h for 4 h, and another missile travels at a speed of v_1 for $t + 2$ hours. If they travel a total of d mi, solve the resulting formula for t.

48. A microwave transmitter can handle x telephone communications, and 15 separate cables can handle y connections each. If the combined system can handle C connections, solve for y.

Answers to Practice Exercises

1. $\theta - kA$ **2.** $\dfrac{P + nc}{n}$

1.12 Applied Word Problems

Procedure for Solving Word Problems • Identifying the Unknown Quantities • Setting Up the Proper Equation • Examples of Solving Word Problems

Many applied problems are at first word problems, and we must put them into mathematical terms for solution. Usually, the most difficult part in solving a word problem is identifying the information needed for setting up the equation that leads to the solution. To do this, you must read the problem carefully to be sure that you understand all of the terms and expressions used. Following is an approach you should use.

■ See Appendix A, page A-1, for a variation to the method outlined in these steps. You might find it helpful.

Procedure for Solving Word Problems

1. *Read the statement of the problem.* First, read it quickly for a general overview. Then *reread* slowly and carefully, *listing the information* given.
2. *Clearly identify the unknown quantities* and then *assign an appropriate letter to represent one of them,* stating this choice clearly.
3. *Specify the other unknown quantities* in terms of the one in step 2.
4. *If possible, make a sketch* using the known and unknown quantities.
5. *Analyze the statement* of the problem and *write the necessary equation.* This is often the most difficult step because ***some of the information may be implied and not explicitly stated.*** Again, a very careful reading of the statement is necessary.

CAUTION

6. *Solve the equation,* clearly stating the solution.
7. *Check the solution with the original statement* of the problem.

Read the following examples very carefully and note just how the outlined procedure is followed.

EXAMPLE 1 Sum of forces on a beam

A 17-lb beam is supported at each end. The supporting force at one end is 3 lb more than at the other end. Find the forces.

Since the force at each end is required, we write

$$\text{let } F = \text{the smaller force (in lb)} \qquad \text{step 2}$$

■ Be sure to carefully identify your choice for the unknown. In most problems, there is really a choice. Using the word *let* clearly shows that a specific choice has been made.

as a way of establishing the unknown for the equation. Any appropriate letter could be used, and we could have let it represent the larger force.

Also, since the other force is 3 lb more, we write

$$F + 3 = \text{the larger force (in lb)} \qquad \text{step 3}$$

■ The statement after "let x (or some other appropriate letter) =" should be clear. It should completely define the chosen unknown.

We now draw the sketch in Fig. 1.18. step 4

Fig. 1.18

Since the forces at each end of the beam support the weight of the beam, we have the equation

$$F + (F + 3) = 17 \qquad \text{step 5}$$

This equation can now be solved:

$$2F = 14$$
$$F = 7 \text{ lb} \qquad \text{step 6}$$

Thus, the smaller force is 7 lb, and the larger force is 10 lb. This checks with the original statement of the problem. step 7 ■

CAUTION Always check a verbal problem with the ***original statement*** of the problem, not the first equation, because it was derived from the statement.

EXAMPLE 2 Office complex energy-efficient lighting

In designing an office complex, an architect planned to use 34 energy-efficient ceiling lights using a total of 1000 W. Two different types of lights, one using 25 W and the other using 40 W, were to be used. How many of each were planned?

Since we want to find the number of each type of light, we

$$\text{let } x = \text{number of 25 W lights}$$

■ "Let x = 25 W lights" is incomplete. We want to find out how many there are.

Also, since there are 34 lights in all,

$$34 - x = \text{number of 40 W lights}$$

We also know that the total wattage of all lights is 1000 W. This means

$$\underbrace{\overset{\text{25 W each}}{25}\,\overset{\text{number}}{x}}_{\text{total wattage of 25 W lights}} + \underbrace{\overset{\text{40 W each}}{40}(\overset{\text{number}}{34 - x})}_{\text{total wattage of 40 W lights}} = \overset{\text{total wattage of all lights}}{1000}$$

■ See Appendix A, page A-1, for a "sketch" that might be used with this example.

$$25x + 1360 - 40x = 1000$$
$$-15x = -360$$
$$x = 24$$

Therefore, there are 24 25-W lights and 10 40-W lights. The total wattage of these lights is $24(25) + 10(40) = 600 + 400 = 1000$. We see that this checks with the statement of the problem. ■

EXAMPLE 3 Number of medical slides

A medical researcher finds that a given sample of an experimental drug can be divided into 4 more slides with 5 mg each than with 6 mg each. How many slides with 5 mg each can be made up?

We are asked to find the number of slides with 5 mg, and therefore we

$$\text{let } x = \text{number of slides with 5 mg}$$

Because the sample may be divided into 4 more slides with 5 mg each than of 6 mg each, we know that

$$x - 4 = \text{number of slides with 6 mg}$$

Because *it is the same sample that is to be divided,* the total mass of the drug on each set of slides is the same. This means

Practice Exercise

1. Solve the problem in Example 3 by letting y = number of slides with 6 mg.

$$\underbrace{\overset{\text{5 mg each}}{5}\overset{\text{number}}{x}}_{\substack{\text{total mass}\\ \text{5-mg slides}}} = \underbrace{\overset{\text{6 mg each}}{6}\overset{\text{number}}{(x - 4)}}_{\substack{\text{total mass}\\ \text{6-mg slides}}}$$

$$5x = 6x - 24$$

$$-x = -24 \quad \text{or} \quad x = 24$$

Therefore, the sample can be divided into 24 slides with 5 mg each, or 20 slides with 6 mg each. Since the total mass, 120 mg, is the same for each set of slides, the solution checks with the statement of the problem. ■

EXAMPLE 4 Distance traveled—space travel

A space shuttle maneuvers so that it may "capture" an already orbiting satellite that is 6000 km ahead. If the satellite is moving at 27,000 km/h and the shuttle is moving at 29,500 km/h, how long will it take the shuttle to reach the satellite? (All digits shown are significant.)

First, we let t = the time for the shuttle to reach the satellite. Then, using the fact that the shuttle must go 6000 km farther in the same time, we draw the sketch in Fig. 1.19. Next, we use the formula *distance* = *rate* × *time* ($d = rt$). This leads to the following equation and solution.

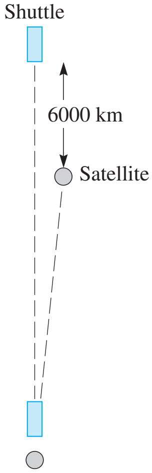


Fig. 1.19

$$\underbrace{\overset{\text{speed of shuttle}}{29{,}500}\overset{\text{time}}{t}}_{\substack{\text{distance traveled}\\ \text{by shuttle}}} = \underbrace{6000}_{\substack{\text{distance between}\\ \text{at beginning}}} + \underbrace{\overset{\text{speed of satellite}}{27{,}000}\overset{\text{time}}{t}}_{\substack{\text{distance traveled}\\ \text{by satellite}}}$$

$$2500t = 6000$$

$$t = 2.400 \text{ h}$$

This means that it will take the shuttle 2.400 h to reach the satellite. In 2.400 h, the shuttle will travel 70,800 km, and the satellite will travel 64,800 km. We see that the solution checks with the statement of the problem. ■

■ A maneuver similar to the one in this example was used on several servicing missions to the Hubble space telescope from 1999 to 2009.

EXAMPLE 5 Mixture—gasoline and methanol

A refinery has 7250 L of a gasoline-methanol blend that is 6.00% methanol. How much pure methanol must be added so that the resulting blend is 10.0% methanol?

■ "Let x = methanol" is incomplete. We want to find out the volume (in L) that is to be added.

First, let x = the number of liters of methanol to be added. The total volume of methanol in the final blend is the volume in the original blend plus that which is added. This total volume is to be 10.0% of the final blend. See Fig. 1.20.

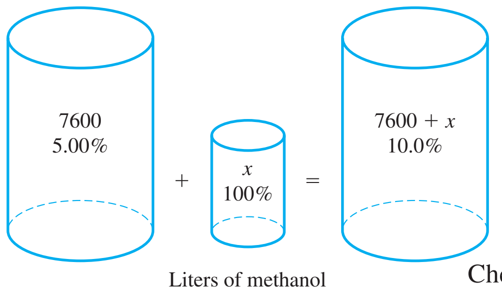


Fig. 1.20

$$\underbrace{\overset{6.00\%}{0.0600}(\overset{\text{original volume}}{7250})}_{\text{methanol in original mixture}} + \underbrace{x}_{\text{methanol added}} = \underbrace{\overset{10\%}{0.100}(\overset{\text{final volume}}{7250 + x})}_{\text{methanol in final mixture}}$$

$$435 + x = 725 + 0.100x$$

$$0.900x = 290, \quad x = 322\text{ L} \quad \text{to be added}$$

Checking (to three sig. digits), there would be 757 L of methanol of a total 7570 L. ■

EXERCISES 1.12

In Exercises 1–4, make the given changes in the indicated examples of this section and then solve the resulting problems.

1. In Example 2, in the first line, change 34 to 37.
2. In Example 3, in the second line, change "4 more slides" to "3 more slides."
3. In Example 4, in the second line, change 27,000 km/h to 27,500 km/h.
4. In Example 5, change "pure methanol" to "of a blend with 50.0% methanol."

In Exercises 5–34, set up an appropriate equation and solve. Data are accurate to two sig. digits unless greater accuracy is given.

5. A certain new car costs $5000 more than the same model new car cost 6 years ago. Together a new model today and six years ago cost $64,000. What was the cost of each?
6. The flow of one stream into a lake is 1700 ft^3/s more than the flow of a second stream. In 1 h, 1.98×10^7 ft^3 flow into the lake from the two streams. What is the flow rate of each?
7. Approximately 6.9 million wrecked cars are recycled in two consecutive years. There were 500,000 more recycled the second year than the first year. How many are recycled each year?
8. A business website was accessed as often on the first day of a promotion as on the next two days combined. The first day hits were four times those on the third. The second day hits were 4×10^4 more than the third. How many were there each day?
9. Petroleum rights to 140 acres of land are leased for $37,000. Part of the land leases for $200 per acre, and the reminder for $300 per acre. How much is leased at each price?
10. A vial contains 2000 mg, which is to be used for two dosages. One patient is to be administered 660 mg more than another. How much should be administered to each?
11. After installing a pollution control device, a car's exhaust contained the same amount of pollutant after 5.0 h as it had in 3.0 h. Before the installation the exhaust contained 150 ppm/h (parts per million per hour) of the pollutant. By how much did the device reduce the emission?
12. Three meshed spur gears have a total of 107 teeth. If the second gear has 13 more teeth than the first and the third has 15 more teeth than the second, how many teeth does each have?
13. In the design of a bridge, an engineer determines that four fewer 18-m girders are needed for the span length than 15-m girders. How long is the span?
14. A fuel oil storage depot had an 8-week supply on hand. However, cold weather caused the supply to be used in 6 weeks when 5000 gal extra were used each week. How many gallons were in the original supply?
15. The sum of three electric currents that come together at a point in a circuit is zero. If the second current is twice the first and the third current is 9.2 μA more than the first, what are the currents? (The sign indicates the direction of flow.)
16. A delivery firm uses one fleet of trucks on daily routes of 8 h. A second fleet, with five more trucks than the first, is used on daily routes of 6 h. Budget allotments allow for 198 h of daily delivery time. How many trucks are in each fleet?
17. A natural gas pipeline feeds into three smaller pipelines, each of which is 2.6 km longer than the main pipeline. The total length of the four pipelines is 35.4 km. How long is each section?
18. At 100% efficiency two generators would produce 750 MW of power. At efficiencies of 65% and 75%, they produce 530 MW. At 100% efficiency, what power would each produce?
19. A wholesaler sells three types of GPS systems. A dealer orders twice as many economy systems at $40 each, and 75 more econoplus systems at $80 each, than deluxe systems at $140 each, for $42,000. How many of each were ordered?

20. A person won a state lottery prize of \$20,000, from which 25% was deducted for taxes. The remainder was invested, partly for a 40% gain, and the rest for a 10% loss. How much was each investment if there was a \$2000 net investment gain?

21. Train A is 520 ft long and traveling at 60.0 mi/h. Train B is 440 ft long and traveling at 40 mi/h in the opposite direction of train A on an adjacent track. How long does it take for the trains to completely pass each other? (*Footnote*: A law was once actually passed by the Wisconsin legislature that included "whenever two trains meet at an intersection . . . , neither shall proceed until the other has.")

22. A family has \$3850 remaining of its monthly income after making the monthly mortgage payment, which is 23.0% of the monthly income. How much is the mortgage payment?

23. A ski lift takes a skier up a slope at 50 m/min. The skier then skis down the slope at 150 m/min. If a round trip takes 24 min, how long is the slope?

24. Before being put out of service, the supersonic jet Concorde made a trip averaging 120 mi/h less than the speed of sound for 0.1 h, and 410 mi/h more than the speed of sound for 3.0 h. If the trip covered 3990 mi, what is the speed of sound?

25. Trains at each end of the 50.0-km-long Eurotunnel under the English Channel start at the same time into the tunnel. Find their speeds if the train from France travels 8.0 km/h faster than the train from England and they pass in 17.0 min. See Fig. 1.21.

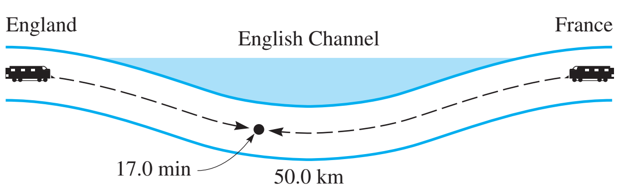


Fig. 1.21

26. An executive would arrive 10.0 min early for an appointment if traveling at 60.0 mi/h, or 5.0 min early if traveling at 45.0 mi/h. How much time is there until the appointment?

27. One lap at the Indianapolis Speedway is 2.50 mi. In a race, a car stalls and then starts 30.0 s after a second car. The first car travels at 260 ft/s, and the second car travels at 240 ft/s. How long does it take the first car to overtake the second, and which car will be ahead after eight laps?

28. A computer chip manufacturer produces two types of chips. In testing a total of 6100 chips of both types, 0.50% of one type and 0.80% of the other type were defective. If a total of 38 defective chips were found, how many of each type were tested?

29. Two gasoline distributors, A and B, are 228 mi apart on Interstate 80. A charges \$3.40/gal and B charges \$3.20/gal. Each charges 0.2¢/gal per mile for delivery. Where on Interstate 80 is the cost to the customer the same?

30. An outboard engine uses a gasoline-oil fuel mixture in the ratio of 15 to 1. How much gasoline must be mixed with a gasoline-oil mixture, which is 75% gasoline, to make 8.0 L of the mixture for the outboard engine?

31. A car's radiator contains 12 L of antifreeze at a 25% concentration. How many liters must be drained and then replaced by pure antifreeze to bring the concentration to 50% (the manufacturer's "safe" level)?

32. How much sand must be added to 250 lb of a cement mixture that is 22% sand to have a mixture that is 25% sand?

33. To pass a 20-m long semitrailer traveling at 70 km/h in 10 s, how fast must a 5.0-m long car go?

34. An earthquake emits primary waves moving at 8.0 km/s and secondary waves moving at 5.0 km/s. How far from the epicenter of the earthquake is the seismic station if the two waves arrive at the station 2.0 min apart?

Answer to Practice Exercise

1. 24 with 5 mg, 20 with 6 mg

CHAPTER 1 EQUATIONS

Commutative law of addition: $a + b = b + a$

Commutative law of multiplication: $ab = ba$

Associative law of addition: $a + (b + c) = (a + b) + c$

Associative law of multiplication: $a(bc) = (ab)c$

Distributive law: $a(b + c) = ab + ac$

$$a + (-b) = a - b \quad (1.1)$$

$$a - (-b) = a + b \quad (1.2)$$

$$a^m \times a^n = a^{m+n} \quad (1.3)$$

$$\frac{a^m}{a^n} = a^{m-n} \quad (m > n, a \neq 0), \qquad \frac{a^m}{a^n} = \frac{1}{a^{n-m}} \quad (m < n, a \neq 0) \quad (1.4)$$

$$(a^m)^n = a^{mn} \quad (1.5)$$

$$(ab)^n = a^n b^n, \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad (b \neq 0) \quad (1.6)$$

$$a^0 = 1 \quad (a \neq 0) \quad (1.7)$$

$$a^{-n} = \frac{1}{a^n} \quad (a \neq 0) \quad (1.8)$$

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \quad (a \text{ and } b \text{ positive real numbers}) \quad (1.9)$$

CHAPTER 1 QUICK CHAPTER REVIEW

Determine each of the following as being either ***true*** or ***false***.

1. The absolute value of any real number is positive.
2. $16 - 4 \div 2 = 14$
3. For approximate numbers, $26.7 - 15 = 11.7$.
4. $2a^3 = 8a^3$
5. $0.237 = 2.37 \times 10^{-1}$
6. $-\sqrt{-4} = 2$
7. $4x - (2x + 3) = 2x + 3$
8. $(x - 7)^2 = 49 - 14x + x^2$
9. $\dfrac{6x + 2}{2} = 3x$
10. If $5x - 4 = 0$, $x = 5/4$
11. If $a - bc = d$, $c = (d - a)/b$
12. In setting up the solution to a word problem involving numbers of gears, it would be sufficient to "let $x =$ the first gear."

CHAPTER 1 REVIEW EXERCISES

In Exercises 1–12, evaluate the given expressions.

1. $-2 + (-5) - 3$
2. $6 - 8 - (-4)$
3. $\dfrac{(-5)(6)(-4)}{(-2)(3)}$
4. $\dfrac{(-9)(-12)(-4)}{24}$
5. $-5 - |2(-6)| + \dfrac{-15}{3}$
6. $3 - 5|-3 - 2| - \dfrac{|-4|}{-4}$
7. $\dfrac{18}{3 - 5} - (-4)^2$
8. $-(-3)^2 - \dfrac{-8}{(-2) - |-4|}$
9. $\sqrt{16} - \sqrt{64}$
10. $-\sqrt{81 + 144}$
11. $(\sqrt{7})^2 - \sqrt[3]{8}$
12. $-\sqrt[4]{16} + (\sqrt{6})^2$

In Exercises 13–20, simplify the given expressions. Where appropriate, express results with positive exponents only.

13. $(-2rt^2)^2$
14. $(3a^0b^{-2})^3$
15. $-3mn^{-5}t(8m^{-3}n^4)$
16. $\dfrac{15p^4q^2r}{5pq^5r}$
17. $\dfrac{-16N^{-2}(NT^2)}{-2N^0T^{-1}}$
18. $\dfrac{-35x^{-1}y(x^2y)}{5xy^{-1}}$
19. $\sqrt{45}$
20. $\sqrt{9 + 36}$

In Exercises 21–24, for each number, (a) determine the number of significant digits and (b) round off each to two significant digits.

21. 8000
22. 21,490
23. 9.050
24. 0.7000

In Exercises 25–28, evaluate the given expressions. All numbers are approximate.

25. $37.3 - 16.92(1.067)^2$
26. $\dfrac{8.896 \times 10^{-12}}{-3.5954 - 6.0449}$
27. $\dfrac{\sqrt{0.1958 + 2.844}}{3.142(65)^2}$
28. $\dfrac{1}{0.03568} + \dfrac{37,466}{29.63^2}$

In Exercises 29–60, perform the indicated operations.

29. $a - 3ab - 2a + ab$
30. $xy - y - 5y - 4xy$
31. $6LC - (3 - LC)$
32. $-(2x - b) - 3(-x - 5b)$
33. $(2x - 1)(5 + x)$
34. $(C - 4D)(D - 2C)$
35. $(x + 8)^2$
36. $(2r - 9s)^2$
37. $\dfrac{2h^3k^2 - 6h^4k^5}{2h^2k}$
38. $\dfrac{4a^2x^3 - 8ax^4}{-2ax^2}$
39. $4R - [2r - (3R - 4r)]$
40. $-3b - [3a - (a - 3b)] + 4a$
41. $2xy - \{3z - [5xy - (7z - 6xy)]\}$
42. $x^2 + 3b + [(b - y) - 3(2b - y + z)]$
43. $(2x + 1)(x^2 - x - 3)$
44. $(x - 3)(2x^2 + 1 - 3x)$
45. $-3y(x - 4y)^2$
46. $-s(4s - 3t)^2$
47. $3p[(q - p) - 2p(1 - 3q)]$
48. $3x[2y - r - 4(x - 2r)]$
49. $\dfrac{12p^3q^2 - 4p^4q + 6pq^5}{2p^4q}$
50. $\dfrac{27s^3t^2 - 18s^4t + 9s^2t}{-9s^2t}$
51. $(2x^2 + 7x - 30) \div (x + 6)$
52. $(4x^2 - 41) \div (2x + 7)$
53. $\dfrac{3x^3 - 7x^2 + 11x - 3}{3x - 1}$
54. $\dfrac{w^3 + 7w - 4w^2 - 12}{w - 3}$
55. $\dfrac{4x^4 + 10x^3 + 18x - 1}{x + 3}$
56. $\dfrac{8x^3 - 14x + 3}{2x + 3}$
57. $-3\{(r + s - t) - 2[(3r - 2s) - (t - 2s)]\}$
58. $(1 - 2x)(x - 3) - (x + 4)(4 - 3x)$
59. $\dfrac{2y^3 - 7y + 9y^2 + 5}{2y - 1}$
60. $\dfrac{6x^2 + 5xy - 4y^2}{2x - y}$

In Exercises 61–72, solve the given equations.

61. $3x + 1 = x - 8$
62. $4y - 3 = 5y + 7$
63. $\dfrac{5x}{7} = \dfrac{3}{2}$
64. $\dfrac{2(4 - N)}{-3} = \dfrac{5}{4}$
65. $-6x + 5 = -3(x - 4)$
66. $-2(-4 - y) = 3y$
67. $2s + 4(3 - s) = 6$
68. $2|x| - 1 = 3$
69. $3t - 2(7 - t) = 5(2t + 1)$
70. $-(8 - x) = x - 2(2 - x)$
71. $2.7 + 2.0(2.1x - 3.4) = 0.1$
72. $0.250(6.721 - 2.44x) = 2.08$

In Exercises 73–82, change numbers in ordinary notation to scientific notation or change numbers in scientific notation to ordinary notation. (See Appendix B for an explanation of the symbols that are used.)

73. A certain computer has 60,000,000,000,000 bytes of memory.
74. The escape velocity (the velocity required to leave the earth's gravitational field) is about 25,000 mi/h.
75. When pictures of the surface of Mars were transmitted to the earth from the Pathfinder mission in 1997, Mars was about 192,000,000 km from earth.
76. Police radar has a frequency of 1.02×10^9 Hz.
77. Among the stars nearest the earth, Centaurus A is about 2.53×10^{13} mi away.
78. Before its destruction in 2001, the World Trade Center had nearly 10^7 ft^2 of office space. (See Exercise 36, page 128.)

79. The faintest sound that can be heard has an intensity of about 10^{-12} W/m^2.

80. An optical coating on glass to reduce reflections is about 0.00000015 m thick.

81. The maximum safe level of radiation in the air of a home due to radon gas is 1.5×10^{-1} Bq/L. (Bq is the symbol for bequerel, the metric unit of radioactivity, where 1 Bq = 1 decay/s.)

82. A certain virus was measured to have a diameter of about 0.00000018 m.

In Exercises 83–96, solve for the indicated letter. Where noted, the given formula arises in the technical or scientific area of study.

83. $V = \pi r^2 L$, for L (oil pipeline volume)

84. $R = \dfrac{2GM}{c^2}$, for G (astronomy: black holes)

85. $P = \dfrac{\pi^2 EI}{L^2}$, for E (mechanics)

86. $f = p(c - 1) - c(p - 1)$, for p (thermodynamics)

87. $Pp + Qq = Rr$, for q (moments of forces)

88. $V = IR + Ir$, for R (electricity)

89. $d = (n - 1)A$, for n (optics)

90. $mu = (m + M)v$, for M (physics: momentum)

91. $N_1 = T(N_2 - N_3) + N_3$, for N_2 (mechanics: gears)

92. $Q = \dfrac{kAt(T_2 - T_1)}{L}$, for T_1 (solar heating)

93. $R = \dfrac{A(T_2 - T_1)}{H}$, for T_2 (thermal resistance)

94. $Z^2\left(1 - \dfrac{\lambda}{2a}\right) = k$, for λ (radar design)

95. $d = kx^2[3(a + b) - x]$, for a (mechanics: beams)

96. $V = V_0[1 + 3a(T_2 - T_1)]$, for T_2 (thermal expansion)

In Exercises 97–102, perform the indicated calculations.

97. A computer's memory is 5.25×10^{13} bytes, and that of a model 30 years older is 6.4×10^4 bytes. What is the ratio of the newer computer's memory to the older computer's memory?

98. The time (in s) for an object to fall h feet is given by the expression $0.25\sqrt{h}$. How long does it take a person to fall 66 ft from a sixth-floor window into a net while escaping a fire?

99. The CN Tower in Toronto is 0.553 km high. The Willis Tower (formerly the Sears Tower) in Chicago is 442 m high. How much higher is the CN Tower than the Willis Tower?

100. The time (in s) it takes a computer to check n cells is found by evaluating $(n/2650)^2$. Find the time to check 4.8×10^3 cells.

101. The combined electric resistance of two parallel resistors is found by evaluating the expression $\dfrac{R_1R_2}{R_1 + R_2}$. Evaluate this for $R_1 = 0.0275\ \Omega$ and $R_2 = 0.0590\ \Omega$.

102. The distance (in m) from the earth for which the gravitational force of the earth on a spacecraft equals the gravitational force of the sun on it is found by evaluating $1.5 \times 10^{11}\sqrt{m/M}$, where m and M are the masses of the earth and sun, respectively. Find this distance for $m = 5.98 \times 10^{24}$ kg and $M = 1.99 \times 10^{30}$ kg.

In Exercises 103–106, simplify the given expressions.

103. One transmitter antenna is $(x - 2a)$ ft long, and another is $(x + 2a)$ yd long. What is the sum, in feet, of their lengths?

104. In finding the value of an annuity, the expression $(Ai - R)(1 + i)^2$ is used. Multiply out this expression.

105. A computer analysis of the velocity of a link in an industrial robot leads to the expression $4(t + h) - 2(t + h)^2$. Simplify this expression.

106. When analyzing the motion of a communications satellite, the expression $\dfrac{k^2r - 2h^2k + h^2rv^2}{k^2r}$ is used. Perform the indicated division.

In Exercises 107–118, solve the given problems.

107. Does the value of $3 \times 18 \div (9 - 6)$ change if the parentheses are removed?

108. Does the value of $(3 \times 18) \div 9 - 6$ change if the parentheses are removed?

W **109.** In solving the equation $x - (3 - x) = 2x - 3$, what conclusion can be made?

W **110.** In solving the equation $7 - (2 - x) = x + 2$, what conclusion can be made?

111. For $x = 2$ and $y = -4$, evaluate (a) $2|x| - 2|y|$; (b) $2|x - y|$.

112. If $a < 0$, write the value of $|a|$ without the absolute value symbols.

113. If $3 - x < 0$, solve $|3 - x| + 7 = 2x$ for x.

114. Solve $|x - 4| + 6 = 3x$ for x. (Be careful!)

115. Show that $(x - y)^3 = -(y - x)^3$.

116. Is division associative? That is, is it true (if $b \neq 0, c \neq 0$) that $(a \div b) \div c = a \div (b \div c)$?

117. What is the ratio of 8×10^{-3} to 2×10^4?

118. What is the ratio of $\sqrt{4 + 36}$ to $\sqrt{4}$?

In Exercises 119–134, solve the given problems. All data are accurate to two significant digits unless greater accuracy is given.

119. A certain engine produces 250 hp. What is this power in kilowatts (kW)?

120. The pressure gauge for an automobile tire shows a pressure of 32 lb/in^2. What is this pressure in N/m^2?

121. Two computer software programs cost \$190 together. If one costs \$72 more than the other, what is the cost of each?

122. A sponsor pays a total of \$9500 to run a commercial on two different TV stations. One station charges \$1100 more than the other. What does each charge to run the commercial?

123. Three chemical reactions each produce oxygen. If the first produces twice that of the second, the third produces twice that of the first, and the combined total is 560 cm^3, what volume is produced by each?

124. In testing the rate at which a polluted stream flows, a boat that travels at 5.5 mi/h in still water took 5.0 h to go downstream between two points, and it took 8.0 h to go upstream between the same two points. What is the rate of flow of the stream?

125. The voltage across a resistor equals the current times the resistance. In a microprocessor circuit, one resistor is 1200 Ω greater than another. The sum of the voltages across them is 12.0 mV. Find the resistances if the current is 2.4 μA in each.

126. An air sample contains 4.0 ppm (parts per million) of two pollutants. The concentration of one is four times the other. What are the concentrations?

127. One road crew constructs 450 m of road bed in 12 h. If another crew works at the same rate, how long will it take them to construct another 250 m of road bed?

128. The fuel for a two-cycle motorboat engine is a mixture of gasoline and oil in the ratio of 15 to 1. How many liters of each are in 6.6 L of mixture?

129. A ship enters Lake Superior from Sault Ste. Marie, moving toward Duluth at 17.4 km/h. Two hours later, a second ship leaves Duluth moving toward Sault Ste. Marie at 21.8 km/h. When will the ships pass, given that Sault Ste. Marie is 634 km from Duluth?

130. A helicopter used in fighting a forest fire travels at 105 mi/h from the fire to a pond and 70 mi/h with water from the pond to the fire. If a round-trip takes 30 min, how long does it take from the pond to the fire? See Fig. 1.22.

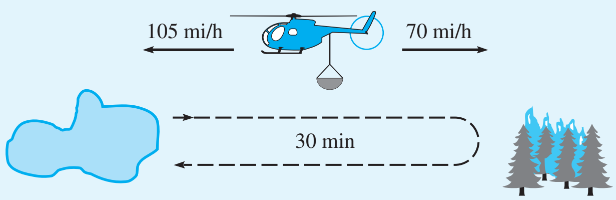


Fig. 1.22

131. One grade of oil has 0.50% of an additive, and a higher grade has 0.75% of the additive. How many liters of each must be used to have 1000 L of a mixture with 0.65% of the additive?

132. Each day a mining company crushes 18,000 Mg of shale-oil rock, some of it 72 L/Mg and the rest 150 L/Mg of oil. How much of each type of rock is needed to produce 120 L/Mg?

133. An architect plans to have 25% of the floor area of a house in ceramic tile. In all but the kitchen and entry, there are 2200 ft^2 of floor area, 15% of which is tile. What area can be planned for the kitchen and entry if each has an all-tile floor?

134. A *karat* equals 1/24 part of gold in an alloy (for example, 9-karat gold is 9/24 gold). How many grams of 9-karat gold must be mixed with 18-karat gold to get 200 g of 14-karat gold?

Writing Exercise

135. In calculating the simple interest earned by an investment, the equation $P = P_0 + P_0rt$ is used, where P is the value after an initial principal P_0 is invested for t years at interest rate r. Solve for r, and then evaluate r for $P = \$7625$, $P_0 = \$6250$, and $t = 4.000$ years. Write a paragraph or two explaining (a) your method for solving for r, and (b) the calculator steps used to evaluate r, noting the use of parentheses.

CHAPTER 1 PRACTICE TEST

In Problems 1–5, evaluate the given expressions. In Problems 3 and 5, the numbers are approximate.

1. $\sqrt{9 + 16}$ **2.** $\dfrac{(7)(-3)(-2)}{(-6)(0)}$ **3.** $\dfrac{3.372 \times 10^{-3}}{7.526 \times 10^{12}}$

4. $\dfrac{(+6)(-2) - 3(-1)}{|2 - 5|}$ **5.** $\dfrac{346.4 - 23.5}{287.7} - \dfrac{0.944^3}{(3.46)(0.109)}$

In Problems 6–12, perform the indicated operations and simplify. When exponents are used, use only positive exponents in the result.

6. $(2a^0b^{-2}c^3)^{-3}$ **7.** $(2x + 3)^2$ **8.** $3m^2(am - 2m^3)$

9. $\dfrac{8a^3x^2 - 4a^2x^4}{-2ax^2}$ **10.** $\dfrac{6x^2 - 13x + 7}{2x - 1}$

11. $(2x - 3)(x + 7)$ **12.** $3x - [4x - (3 - 2x)]$

13. Solve for y: $5y - 2(y - 4) = 7$

14. Solve for x: $3(x - 3) = x - (2 - 3d)$

15. Express 0.0000036 in scientific notation.

16. List the numbers -3, $|-4|$, $-\pi$, $\sqrt{2}$, and 0.3 in numerical order.

17. What fundamental law is illustrated by
$3(5 + 8) = 3(5) + 3(8)$?

18. (a) How many significant digits are in the number 3.0450?

(b) Round it off to two significant digits.

19. If P dollars is deposited in a bank that compounds interest n times a year, the value of the account after t years is found by evaluating $P(1 + i/n)^{nt}$, where i is the annual interest rate. Find the value of an account for which $P = \$1000$, $i = 5\%$, $n = 2$, and $t = 3$ years (values are exact).

20. In finding the illuminance from a light source, the expression $8(100 - x)^2 + x^2$ is used. Simplify this expression.

21. The equation $L = L_0[1 + \alpha(t_2 - t_1)]$ is used when studying thermal expansion. Solve for t_2.

22. An alloy weighing 20 lb is 30% copper. How many pounds of another alloy, which is 80% copper, must be added for the final alloy to be 60% copper?