On the Staircase-Line non-Paradox in case of unequal slope

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1 Introduction

The paradox states goes as follows-

The staircase approximation of a line l with n staircases have a constant length of staircase. No matter what the value of n, on the basis of this it is claimed that as $n \to \infty$ the staircase approximates the line. Thus the line should have the same length as the staircase. which is 4 but it turns out to be $\sqrt{10}$

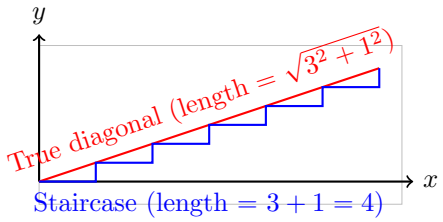

Figure 1: The line-staircase paradox with slope 1/3. As the number of steps $\to \infty$, the blue staircase converges to the red diagonal, but its length remains 4 (sum of horizontal and vertical segments) instead of $\sqrt{10}$.

The following document will be aimed at proving that such an approximation is not possible. Let us try to prove this. We are going to prove this by contradiction and the below claims are based on the same assumption

2 Looking through the lens of a function

We introduce two functions to our paper that are $Y(x)$ and $X(y)$ where $Y(x)$ is the number of different values Υ the staircase such that the point (x, Υ) lie on the staircase/line divided by the number of points in a line of unit length and similarly the value of $X(y)$ is defined. Note, that for all values in the line. In case of staircase a subscript s and for a line l will be used. Note, that for all

the constants that are achieved by the line the same values must be obtained by staircase as n tends to infinity.

2.1 The congruency of the stairs in the staircase

Now, the claim is simple in a straight line all the points have the same number of points i.e $Y_l(x_i)$ is constant $\forall x_i$ on the line. We can use this data to create a constant $R_s^u(i,j) = \frac{Y_s(x_i)}{Y_s(x_j)}$ and similarly $R_l^u = 1$. Now by this we have proven the congruency of all the stairs in the staircase.

2.2 The main proof

We will create a similar ratio now.. which is $R_l = X_l(y_i)/Y_l(x_i)$ which is equal to 1 but $R_s = \frac{dy}{dx}$ that is $R_s =$ slope of the line, which is not necessarily 1 or as we can say that in the case of all lines $l \nparallel u$ where u is the line $x = y$ with unequal x and y.