Numerical Methods - Mini-Project

The Numerical Calculation Of The Double Integral

Double Integral Numerical Calculation

Given a double integral over a rectangular domain [a,b]×[c,d]

$$\int_a^b \int_c^d f(x,y)dy\,dx$$

We should compute the solution with a composite 2-point Gauss-Legendre rule with respect to x and the 3-point Gauss-Legendre rule with respect to y

2-point Gauss Legendre for x in [a,b]

$$I = \sum_{i=1}^{2} \frac{b-a}{2} * w_i * f(\frac{b-a}{2} * x_i \frac{a+b}{2})$$

3-point Gauss Legendre for y in [c,d]

$$I = \sum_{j=1}^{3} \frac{d-c}{2} * w_j * f(\frac{d-c}{2} * y_j \frac{c+d}{2})$$

For the multivariate function

$$I = \sum_{i=1}^{2} \sum_{j=1}^{3} (\frac{b-a}{2})(\frac{d-c}{2}) * w_i * w_j * f(\frac{b-a}{2} * x_i + \frac{a+b}{2}, \frac{d-c}{2} * y_j + \frac{c+d}{2})$$

Program Description

Program consists of 2 files Main.m and GaussLegendre.m
On the Main.m file we just plug in some values and test the function. Then it prints the outputs. On the second m file we have the Gauss Legendre with [a,b] interval for x and [c,d] interval for y. Next we got first, second point and weight for 2-point Gauss Legendre. Secondly, same for 3-point Gauss

Legendre and Vectors for X and Y. And at the end just compute the formula for each point.

Numerical Examples

We got 3 functions

1. $x^2 * y^2$

2. $x^3 + y^2 - x - y$

3. e^{x-y}

I chose this function because it was different to show that the program works. We got polynomial and exponential functions which give different outputs.

Numerical Tests

We got many number of Points to test the correctness of the function.
I tested the functions with matlab built-in function integral2.

Functions	Points(a,b,c,d)	My function	Integral2	Error
$x^2 * y^2$	0,1,0,1	0.1111	0.1111	0
$x^2 * y^2$	0,2,0,2	7.1111	7.1111	0
$x^2 * y^2$	0,-1,0,-1	0.1111	0.1111	0
$x^3 + y^2 - x - y$	0.5,1,-1,0	0.2760	0.2760	0

$x^3 + y^2 - x - y$	1,0.5,0,-1	0.2760	0.2760	0
e^{x-y}	-1,1,-1,1	5.5061	5.5244	0.0183
e^{x-y}	-0.5,0,-0.5,0	0.2552	0.2553	0.0001
e^{x-y}	0,-1,0,-1	1.0859	1.0862	0.0003

$$E = |\text{Integral2 - My function}|$$

As shown on the table above there is not much of an error.To conclude we can say that the function works good for computing the solution with a composite 2-point Gauss-Legendre rule with respect to x and the 3-point Gauss-Legendre rule with respect to y.