COMP4041-LDO Linear and Discrete Optimization
Coursework – Semester 1, Session 2021-2022

INSTRUCTIONS

This coursework is **worth 25%** of the overall module assessment. The electronic submission via Moodle consists of the following files (please upload separate files, not a compressed file):

1. One Excel file for the spreadsheet model, please name the file *cw-mc-ID* replacing ID with your own 8-digit student ID.
2. One LP-Solve file for the algebraic model, please name the file *cw-mc-ID* replacing ID with you own 8-digit student ID.
3. One demonstration video for the presentation of your optimization models, please name the file *cw-mc-ID* replacing ID with you own 8-digit student ID.

The **normal submission deadline is 05 January 2022 at 16:00 hrs**. If submitting after this date, a penalty of 5 marks (the standard 5% absolute) out of the 100 marks available will be applied for each late working day.

The **late submission deadline is 12 January 2022 at 16:00 hrs**. Submissions after this date will only be accepted if supported through the extenuating circumstances procedure, https://www.nottingham.ac.uk/studentservices/services/extenuating-circumstances.aspx.

Students are reminded of the **Policy on Academic Integrity and Misconduct** and must ensure that all material from other sources is clearly quoted and acknowledged. See: https://www.nottingham.ac.uk/studyingeffectively/studying/integrity/index.aspx.

COURSEWORK DESCRIPTION

Refer to the **Milk Collection** optimization problem described in section 12.23 of the following book in the reading list (available online from the library):

Model Building in Mathematical Programming. H.P. Williams, Wiley, 5th edition, 2013.

The optimization model for this problem is given in section 13.23 while the optimal solution is described in section 14.23 respectively. Study the description, optimization model and optimal solution for this problem to make sure you understand them well.

Develop an **Excel spreadsheet model** to solve the given optimization problem. The spreadsheet model should execute with no errors. Make sure to include appropriate labels and comments in the spreadsheet model to clarify the approach. Also include comments in the spreadsheet model to clearly illustrate the corresponding algebraic model being implemented, whether it follows the model given in the book or not. Good principles of spreadsheet modelling should be followed whenever possible.

Develop an **LP-Solve model** to solve the above optimization problem. The LP-Solve model should execute with no errors. Make sure to include appropriate comments in the LP-Solve model to clarify the algebraic formulation. Also include the algebraic compact notation as comments in the LP-Solve model to clearly illustrate the correspondence between each algebraic expression and the constraints implemented in the LP-Solve model. Explain the correspondence between your LP-Solve model and the algebraic model provided in the book.

Develop a short **demonstration video** of maximum 15 minutes duration that describes the design and use of your spreadsheet model, the implementation of the LP-Solve model as well as the correspondence between the two models. This demonstration video should include explaining how the spreadsheet model was designed (layout, calculations, solver settings, etc.) and how it can be used to understand the solution found. It may also describe any issues, additional insights, reflexions and clarifications about your work. There is no need to describe the given optimization problem but references to it and the LP-Solve algebraic model might be needed. Any appropriate software may be used for producing the video, but please make sure the video file can be played in standard media players and/or Internet browsers. Also, please aim to keep the size of the file as small as possible while still ensuring good viewing quality. The maximum file size allowed for the video is 150MB. Make sure to select in advance suitable software to record your video without exceeding the maximum file size.

MARKING CRITERIA

The purpose of this coursework is to assess your ability to understand and interpret an optimization problem and to implement the corresponding optimization model. If there is any element of the problem that is not entirely clear to you, please attempt to interpret such element in the best way you can and explain your rationale. The given algebraic model for the problem might not be described in full detail and hence you will have to achieve an understanding of the model. If your optimization model or solution do not follow the ones given in the book, then please explain as appropriate. Although you should endeavour to provide the correct model, this does not mean that all marks will be lost because of your model not finding the correct optimal solution. Marks are awarded for correctness but also for quality of the work as follows:

Correct Spreadsheet Model (30 marks): this refers to the spreadsheet model being fully correct in terms of modelling and solving the optimization problem, any innovative modelling mechanisms implemented, and the correspondence to the LP-Solve model.

Quality of Spreadsheet Model (20 marks): this refers to layout and presentation of the spreadsheet model for clarity and usability, any additional features developed to enhance the visualisation of the model and the solution, any additional features developed to enhance the implementation and usability of the model.

Correct and Clear LP-Solve Model (20 marks): this refers to the LP-solve model being fully correct and clear in terms of modelling, the lp-solve model solving the optimization problem correctly, and the correspondence to the spreadsheet model.

Quality of Demonstration Video (30 marks): this refers to the effectiveness and the visual quality of the video in explaining the optimization models, the optimal solutions obtained, any issues/insights/reflections that enhance the demonstration.

12.23 Milk collection

A small milk processing company is committed to collecting milk from 20 farms and taking it back to the depot for processing. The company has one tanker lorry with a capacity for carrying 80 000 litres of milk. Eleven of the farms are small and need a collection only every other day. The other nine farms need a collection every day. The positions of the farms in relation to the depot (numbered 1) are given in Table 12.16 together with their collection requirements.

Find the optimal route for the tanker lorry on each day, bearing in mind that it has to (i) visit all the ‘every day’ farms, (ii) visit some of the ‘every other day’ farms and (iii) work within its capacity. On alternate days, it must again visit the ‘every day’ farms and also visit the ‘every other day’ farms not visited on the previous day.

For convenience, a map of the area considered is given in Figure 12.7.

Table 12.16

Farm	Position 10 miles East	North	Collection frequency	Collection requirement (1000 l)
1 (Depot)	0	0	–	–
2	−3	3	Every day	5
3	1	11	Every day	4
4	4	7	Every day	3
5	−5	9	Every day	6
6	−5	−2	Every day	7
7	−4	−7	Every day	3
8	6	0	Every day	4
9	3	−6	Every day	6
10	−1	−3	Every day	5
11	0	−6	Every other day	4
12	6	4	Every other day	7
13	2	5	Every other day	3
14	−2	8	Every other day	4
15	6	10	Every other day	5
16	1	8	Every other day	6
17	−3	1	Every other day	8
18	−6	5	Every other day	5
19	2	9	Every other day	7
20	−6	−5	Every other day	6
21	5	−4	Every other day	6

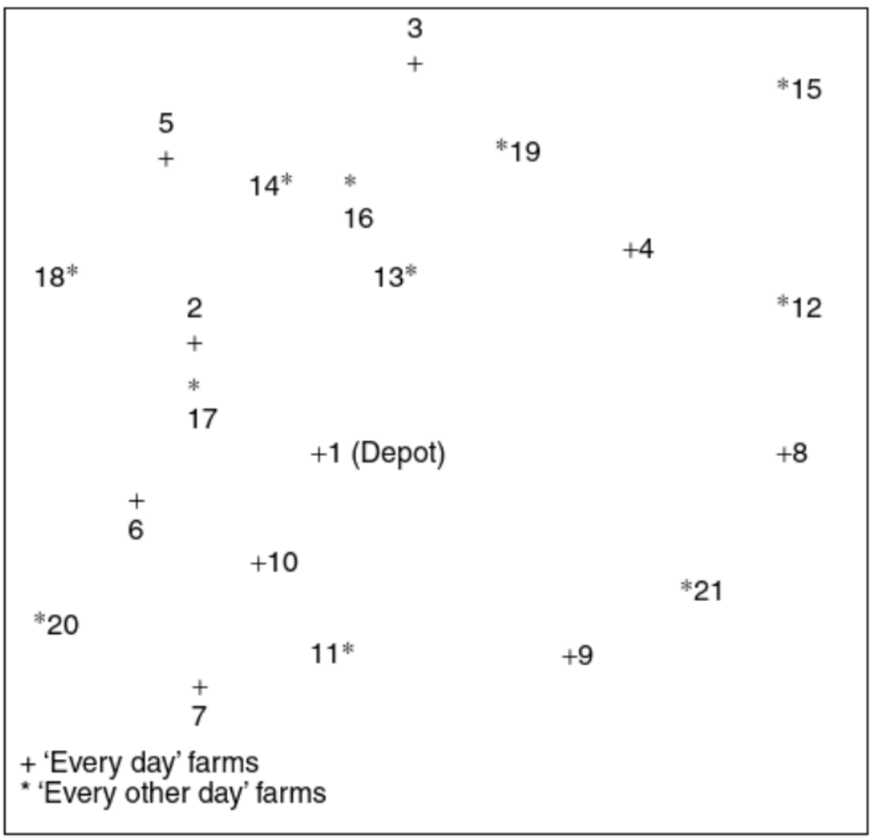

Figure 12.7

13.23 Milk collection

This problem is an extension of the travelling salesman problem whose formulation is discussed in Section 9.5. We extend the formulation given there.

13.23.1 Variables

$$
\begin{aligned}
x_{ijk} &= 1 \text{ if the tour on day } k \text{ goes directly between farms } i \text{ and } j \\
&\quad \text{(in either direction) for } i < j, k = 1, 2, \\
&= 0 \text{ otherwise;} \\
y_{ik} &= 1 \text{ if farm } i \text{ is visited on the tour on day } k \text{ for } i = 11 \text{ to } 21, k = 1, 2, \\
&= 0 \text{ otherwise.}
\end{aligned}
$$

13.23.2 Constraints

The limited tanker capacity gives

$$\sum_i K_i y_{ik} \leq C \quad \text{for } k = 1, 2,$$

where K_i is the daily pickup requirement from farm i and C is the tanker capacity.

The limit on visiting some farms only every other day gives

$$y_{i1} + y_{i2} = 1 \quad \text{for } i = 11 \text{ to } 21.$$

The need to visit the 'every day' farms on each tour gives

$$\sum_{j:j>i} x_{ijk} + \sum_{j:j<i} x_{jik} = 2 \quad \text{for } i = 1 \text{ to } 10,\ k = 1, 2.$$

The need to visit the 'every other day' farms on the chosen day gives

$$\sum_{j:j>i} x_{ijk} + \sum_{j:j<i} x_{jik} - 2y_{ik} = 0 \quad \text{for } i = 11 \text{ to } 21,\ k = 1, 2.$$

Taking into account the considerations discussed in Chapter 10, these constraints imply those below for which the associated linear programming relaxation is more constrained, making the model much easier to solve:

$$x_{ijk} - y_{ik} \leq 0 \quad \text{for } i = 11 \text{ to } 21, j = 11 \text{ to } 21, \quad j > i, k = 1, 2;$$

$$x_{jik} - y_{ik} \leq 0 \quad \text{for } i = 11 \text{ to } 21, j = 1 \text{ to } 21, \quad i > j, k = 1, 2.$$

In order to prevent unnecessary (and computationally more costly) symmetric alternative solutions (switching days of visiting farms), it is convenient to set $y_{11,1}$ to 1, forcing farm 11 to be visited on the first day.

13.23.3 Objective

$$\text{Minimize} \quad \sum_{\substack{i,\ j \\ i<j}} c_{ij} x_{ij}$$

where c_{ij} is the distance between farm i and farm j.

This model has 65 constraints and 442 variables (all 0–1 integer).

As it is only a *relaxation* of the true model (the subtour elimination constraints have been ignored), it will almost certainly be necessary to add these on an 'as needed' basis during the course of optimization in a similar manner as for the travelling salesman problem, as described in Section 9.5.

14.23 Milk collection

The optimal solution is given in Figure 14.9 with dashes representing the first day's routes and dotted lines those of the second day. The total distance covered is 1229 miles.

In the formulation given in Section 13.23, this first solution produced subtours on day 1 around farms 2, 5 and 18; around 3, 16, 13, 4 and 19 and around 1, 8, 21, 9, 11, 7, 6 and 10 and on day 2, around farms 6, 7 and 20 and around all the other farms. This infeasible solution covers 1214 miles (there are alternative, equally good solutions). Subtour elimination constraints were then introduced to prevent the subtours (on both days to prevent them re-arising on the other day), resulting in another solution with no subtours on day 1 but subtours on day 2 around farms 1, 2, 17, 6, 7 and 10; around 4, 15, 3, 5, 14, 16 and 13 and around 8, 9 and 21. Subtour elimination constraints were introduced to prevent these subtours (on both days). This resulted in the optimal solution (no subtours) given in Figure 14.9.

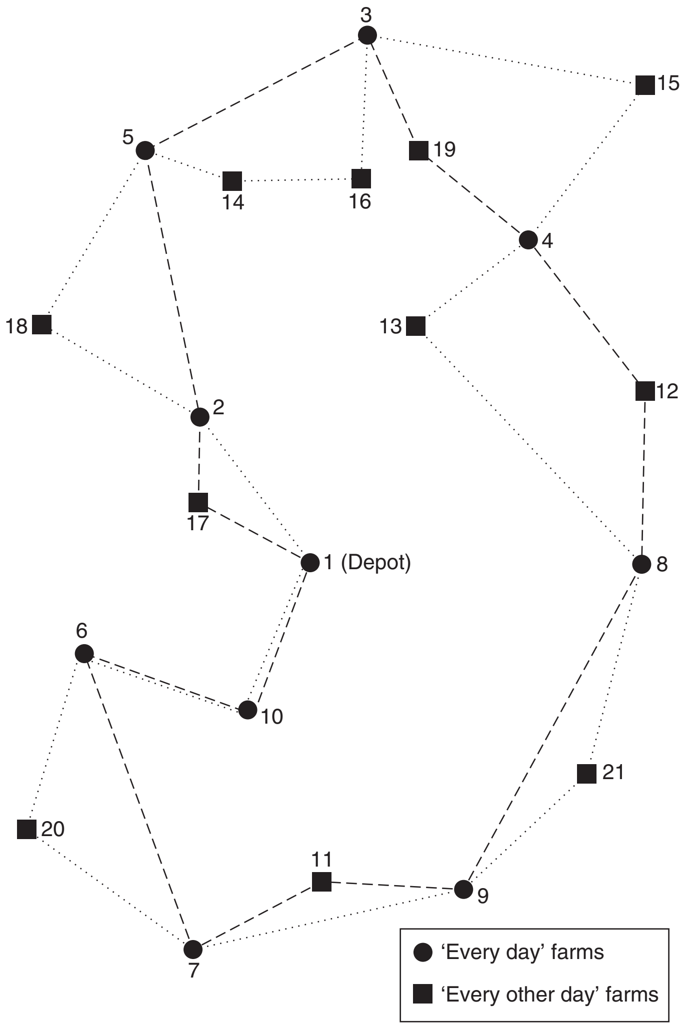

Figure 14.9

These stages required 34, 13 and 272 nodes, respectively. Out of interest, not introducing the 'disaggregated' constraints described in Section 13.23 resulted in the first stage taking 1707 nodes.

This problem is based on a larger one described by Butler, Williams and Yarrow (1997). That larger problem required a more sophisticated solution approach using generalizations of known results concerning the structure of the travelling salesman polytope.