CS1112
Properties of Relations

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Properties of Relations

Operations on relations

Relations over multiple sets

(Defn 3.1 – 3.8)

Relations: a reminder

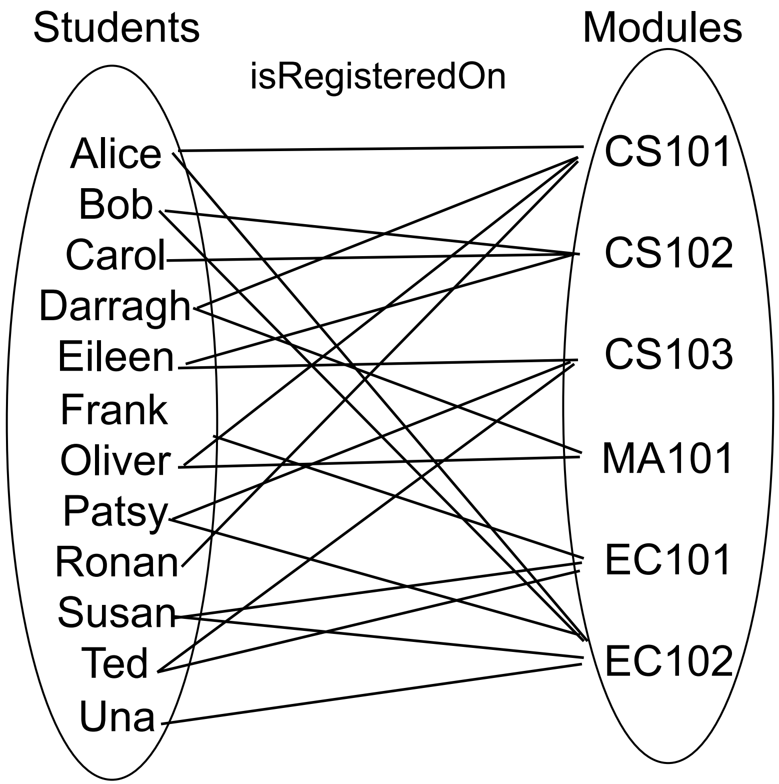
Let A and B be two sets, and let $a \in A$ and $b \in B$

R is a binary relation between A and B
if and only if $R \subseteq A \times B$

Note: this means
the relation R is
a set of ordered pairs

We say a is R-related to b
if and only if $(a,b) \in R$

We write this as $R(a,b)$ or aRb
or simply $(a,b) \in R$



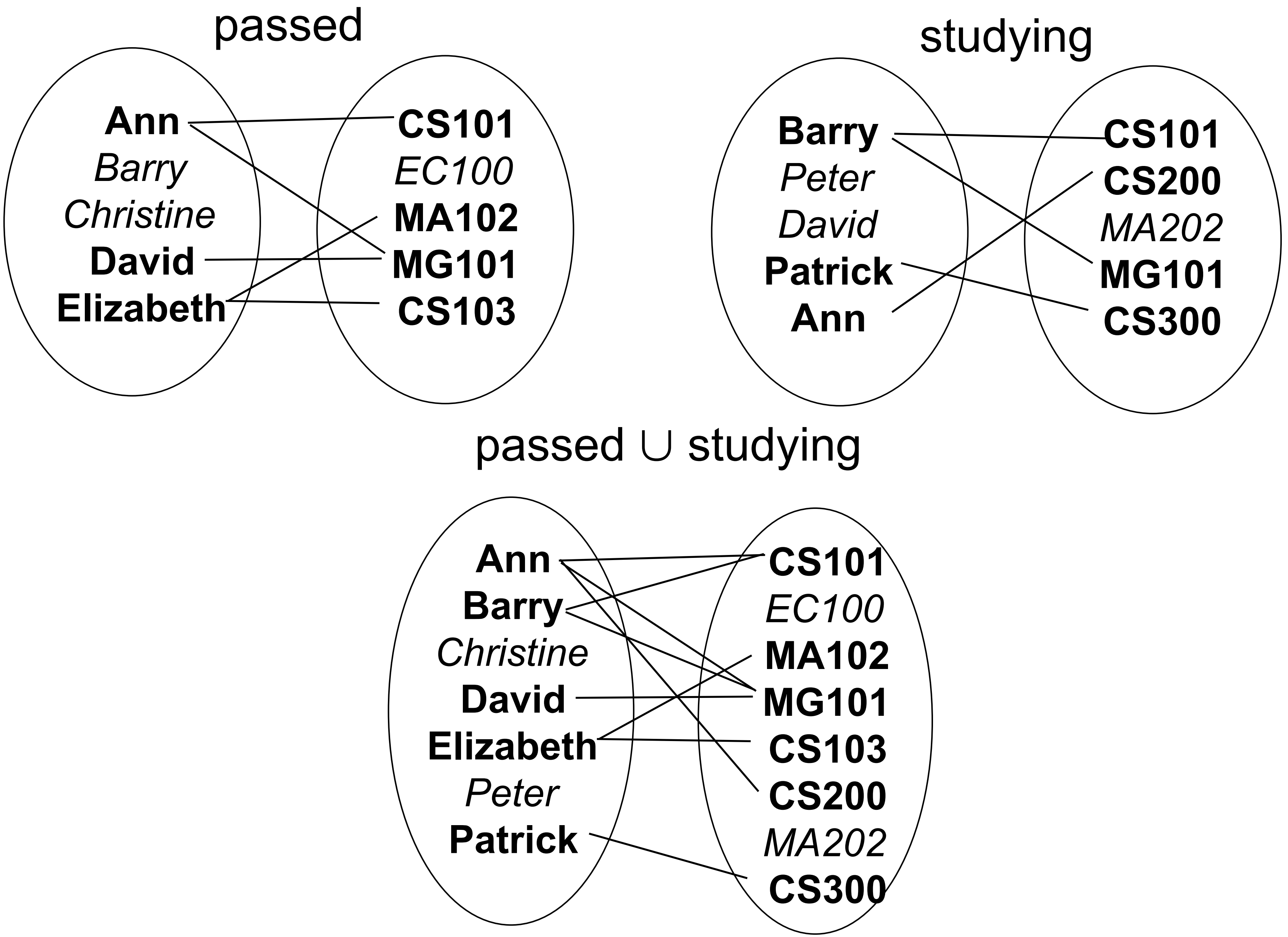
Operations on relations

Standard set operations on relations produce new relations

Let $R \subseteq A \times B$ and $S \subseteq C \times D$ be two relations.
Then $R \cup S$, $R \cap S$, $R \backslash S$, $R \times S$ and R' are all relations

Example: $R \cup S \subseteq (A \cup C) \times (B \cup D)$
$(x,y) \in R \cup S$ if and only if $(x,y) \in R$ or $(x,y) \in S$

Example: $R' \subseteq A \times B$, where $U = A \times B$
$(x,y) \in R'$ if and only if $(x,y) \notin R$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CS300

Relations are ordered

- It is not always true that *R(a,b)* if and only if *R(b,a)*
  - even if *A* and *B* are the same set, it is not always true

- Example: the relation "is less than", which we write as "<“, over the set {0,1,2,3,4}

R= {(0,1), (0,2), (0,3), (0,4), (1,2), (1,3), (1,4), (2,3), (2,4), (3,4)}
$0<1$, $0<2$, $0<3$, $0<4$, $1<2$, $1<3$, $1<4$, $2<3$, $2<4$, $3<4$

but

(1,0) $\notin$ *R* (since $1 \geq 0$, not $1 < 0$)
(2,0) $\notin$ *R*
(3,0) $\notin$ *R*, etc.

Relation inverse

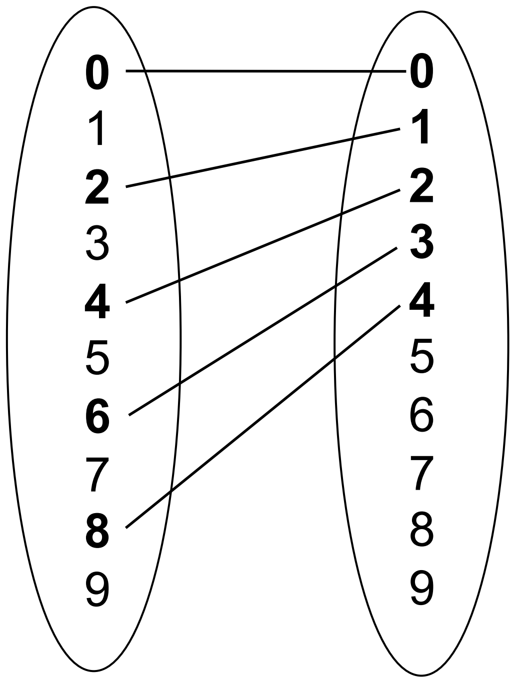
The inverse of a relation is simply all the original pairs, but in reverse order

Let $R \subseteq A \times B$. Then the inverse, $R^{-1} \subseteq B \times A$

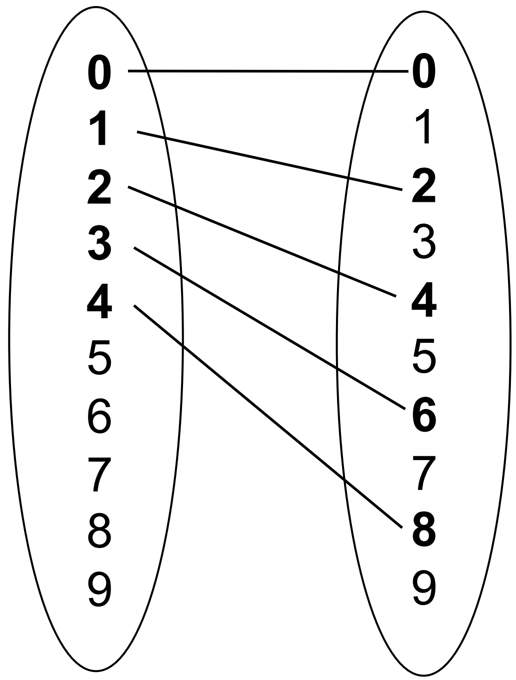
$(x,y) \in R$ if and only if $(y,x) \in R^{-1}$

$(R^{-1})^{-1} = R$

Example

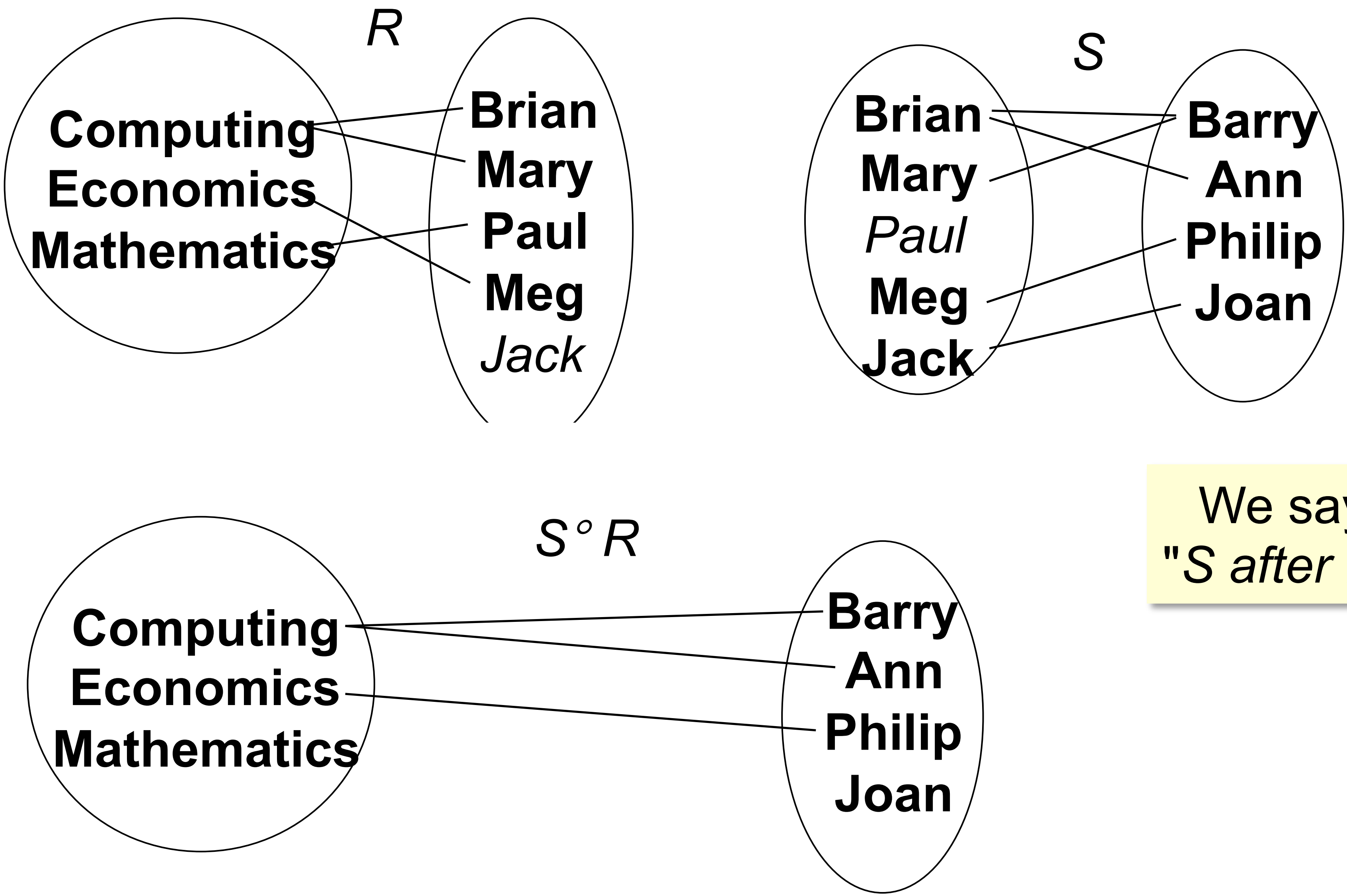


R



R^{-1}

Composing two relations together



We say "*S after R*"

Composing relations together

Let $R \subseteq A \times B$ and $S \subseteq B \times C$ be two relations. Then we can apply one after the other to get a new relation $S \circ R \subseteq A \times C$

$(a,c) \in S \circ R$ if and only if there is some $b \in B$ such that
$(a,b) \in R$ and $(b,c) \in S$

We say "*S after R*"

$(S \circ R)^{-1} = R^{-1} \circ S^{-1}$

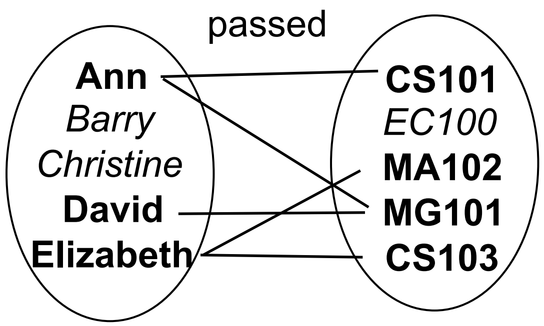
Two simple standard relations

A null relation between two sets is the empty set (i.e. nothing in first is related to anything in second)
(to get useful information out of this, we still need to know the source and target)

A universal relation between two sets is just the cartesian product of the sets (i.e. everything in first is related to everything in second)

Relations over more than 2 sets

So far, we have considered relations between two sets. E.g.



But we started by talking about an example from CS1106 that involved 4 sets:

Example Relation

title	*year*	*length*	*genre*
Gone With the Wind	1939	231	drama
Star Wars	1977	124	sciFi
Wayne's World	1992	95	comedy

So what is a relation over multiple sets?

n-ary relations

Let $A_1, A_2, ..., A_n$ be sets.

R is a relation over $A_1, A_2, ..., A_n$ if and only if
$R \subseteq A_1 \times A_2 \times ... \times A_n$

Any element of R is an *n*-tuple $(a_1, a_2, ..., a_n)$, with each $a_i \in A_i$.

Example: a relation between 5 sets. There are three 5-tuples in this relation.

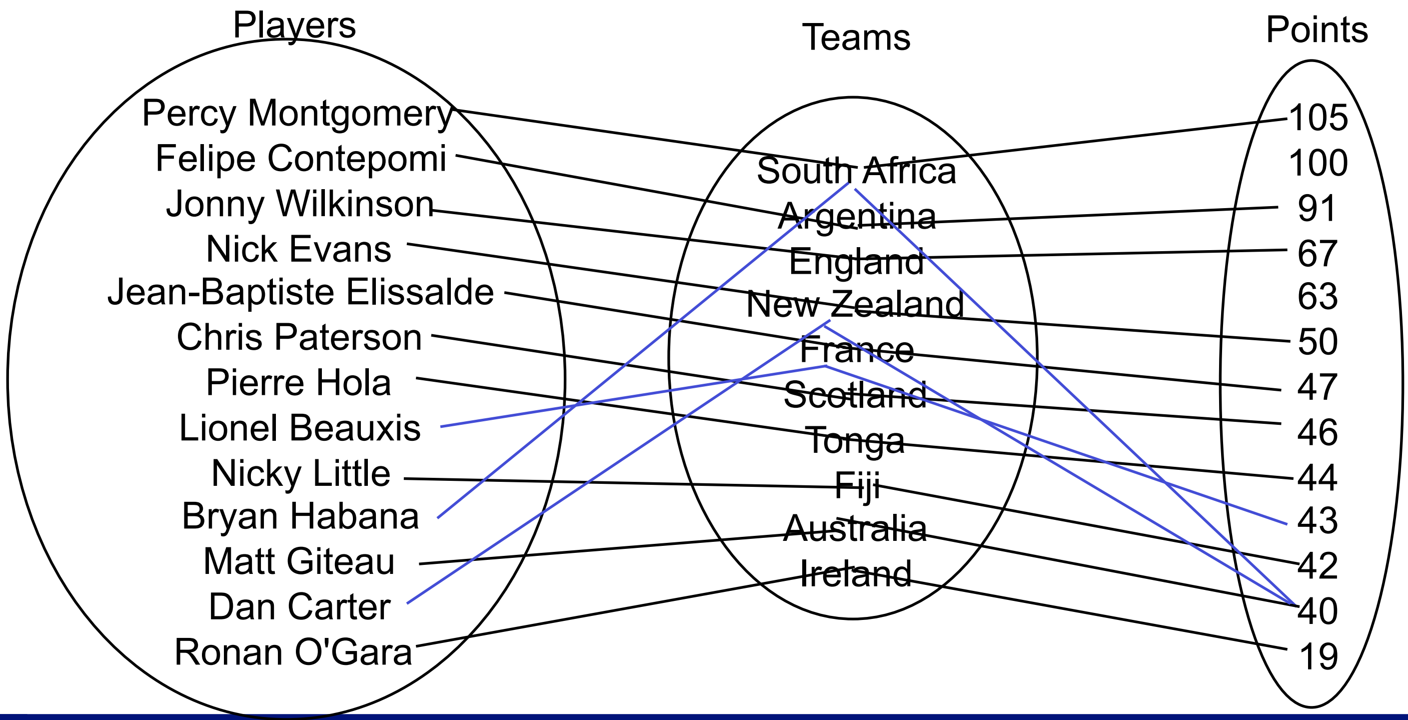
STUDENT

Name	Id	Address	Degree	MId
Coveney	123	1 Main St	CK401	1
McGrath	456	7 Well Rd	CK401	4
Lynch	789	3 Cork La.	CK401	9

Example: points scored in the Rugby World Cup 2007

R: Players x Teams x Points

R = {(Percy Montgomery, South Africa,105),(Felipe Contepomi,Argentina,91),
(Jonny Wilkinson,England, 67), (Nick Evans,New Zealand,50),
(Jean-Baptiste Elissalde,France,47), (Chris Paterson, Scotland,46),
(Pierre Hola,Tonga,44), (Lionel Beauxis,France,43), (Nicky Little,Fiji,42),
(Bryan Habana, South Africa,40), (Matt Giteau, Australia, 40),
(Dan Carter, New Zealand,40),(Ronan O'Gara,Ireland,19) ,... }



Example Relation

title	*year*	*length*	*genre*
Gone With the Wind	1939	231	drama
Star Wars	1977	124	sciFi
Wayne's World	1992	95	comedy

Four sets:

S: all possible character strings (*title*)

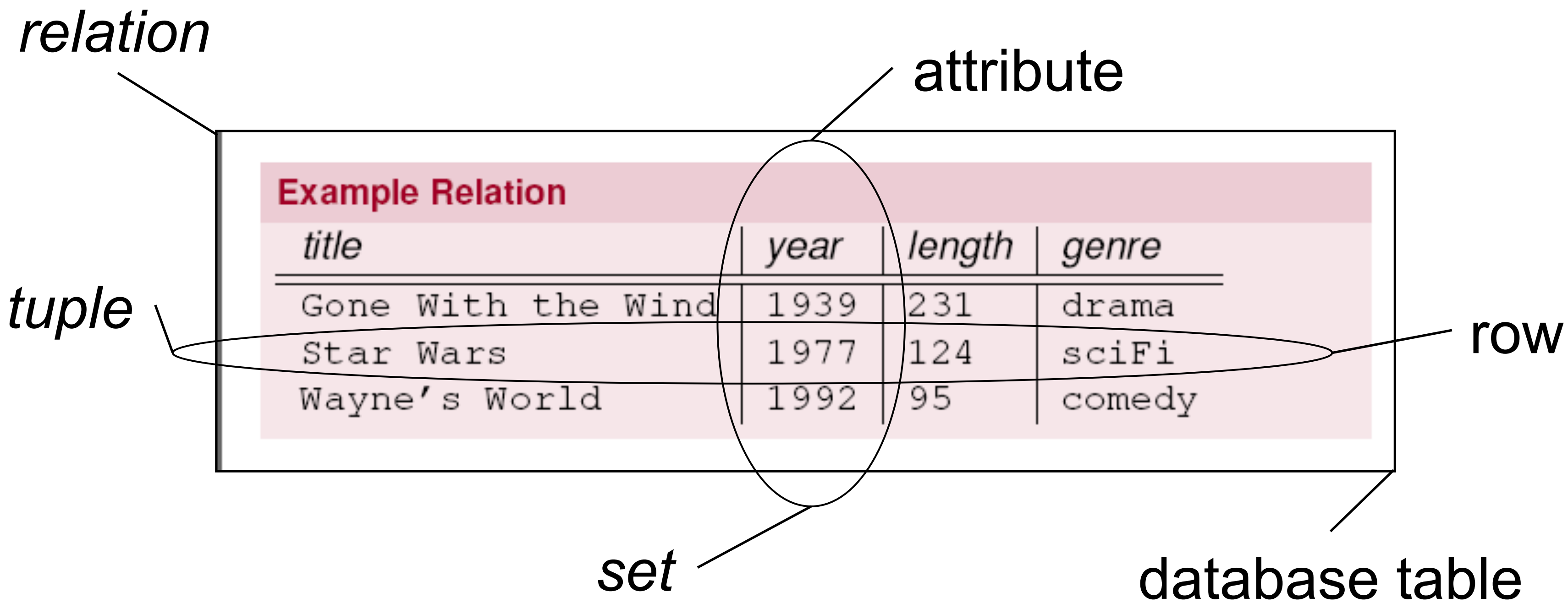
N_+: positive integers, representing calendar years (*year*)

N_+: positive integers, representing lengths in minutes (*length*)

S: all possible character strings (*genre*)

Terminology

Relations	Databases
Set	attribute, data type
position in cartesian product	column, field
tuple	record, row



Selection

We can *select* all tuples from a relation R that satisfy some condition C (and this defines a new relation that is a subset of R).

E.g. select film table records that satisfy "year > 1970"

{ ("Gone with the wind", 1939,231,drama),
("Star Wars", 1977, 124, sciFi),
("Wayne's World", 1992, 95, comedy)}

becomes

{ ("Star Wars", 1977, 124, sciFi),
("Wayne's World", 1992, 95, comedy)}

Projection

We can *project* a relation R over a subset of its attributes by throwing away all other components of each tuple (and this gives a new relation)

E.g. projecting the film table over (title,genre) :

{ (“Gone with the wind”, 1939,231,drama),
(“Star Wars”, 1977, 124, sciFi),
(“Wayne’s World”, 1992, 95, comedy) }

becomes

{ (“Gone with the wind”,drama),
(“Star Wars”, sciFi), (“Wayne’s World”,comedy) }

Formal definitions

R is a relation over $A_1, A_2, \ldots, A_n$

If C is some condition, then the *selection from R that satisfies C* is written $\text{select}_C(R)$.

$(a_1,a_2,\ldots,a_n) \in \textit{select}_C(R)$ if and only if $(a_1,a_2,\ldots,a_n)$ satisfies C and $(a_1,a_2,\ldots,a_n) \in R)$

For the *projection over* $A_{i1}, A_{i2}, \ldots, A_{im}$, where $1 \leq i_1 \leq i_2 \leq \ldots \leq i_m \leq n$, we will write $\textit{project}_{i1\ i2 \ldots\ im}(R)$.

If $(a_1, a_2, \ldots, a_n) \in R$ then $(a_{i1}, a_{i2}, \ldots, a_{im}) \in P_{i1\ i2\ldots\ im}(R)$

Exercise

People: { *alan, barry, christine, david, elizabeth*,…}
Tables: $\{t_1, t_2, t_3, \ldots, t_n\}$ Seats: $\{s_1, s_2, s_3, \ldots, s_m\}$ Diet: {*any, veg, no-nut*}
SitsAt: People x Seats = {(*alan*,s_1), (*christine*,s_2), …}
IsAtTable: Seats x Tables = $\{(s_1,t_1),(s_2,t1),(s_3,t_1),(s_4,t_1),(s_5,t_2),\ldots\}$
IsWith: People x People = {(*alan, christine*), (*barry,elizabeth*), (*barry,david*),…}
Requires: People x Diets = {(*david,veg*), …}

Formulate the following questions in terms of operations on relations:

1. Are all seats allocated?
2. Does everybody have a seat?
3. What seat is christine sitting at?
4. What table is david sitting at?
5. Who is sitting at seat s_6?
6. Who is with the person sitting at seat s_6?
7. Which seats require vegetarian meals?
8. What is the table assignment?
(i.e. who sits at what table?)

Next lecture ...

Relations on a single set

(Defn 3.9 – 3.21)