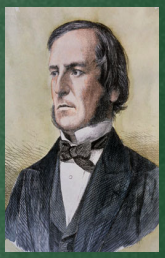
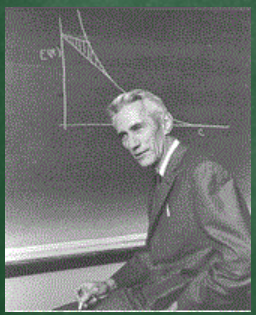
Boolean Logic

How can a 'chip', made of Silicon and wires do something that seems like it requires human thought?



To understand the answer, we need to understand Boolean Logic. This was developed by George Boole, first Professor of Mathematics at UCC in the mid 1800's

Boole's logic was used by Claude Shannon to develop Switching Theory in the 1930s



Claude Elwood Shannon is considered as the founding father of electronic communications age. He is an American mathematical engineer, whose work on technical and engineering problems within the communications industry, laying the groundwork for both the computer industry and telecommunications. After Shannon noticed the similarity between Boolean algebra and the telephone switching circuits, he applied Boolean algebra to electrical systems at the Massachusetts Institute of technology (MIT) in 1940. Later he joined the staff of Bell Telephone Laboratories in 1942. While working at Bell Laboratories, he formulated a theory explaining the communication of information and worked on the problem of most efficiently transmitting information. The mathematical theory of communication was the climax of Shannon's mathematical and engineering investigations. The concept of entropy was an important feature of Shannon's theory, which he demonstrated to be equivalent to a shortage in the information content (a degree of uncertainty) in a message.

Shannon was educated at Michigan University in 1936, where he earned his B.S. degree. Later he went to Massachusetts Institute of Technology, where he studied both electrical engineering and mathematics, receiving a master's degree and a doctorate. For his master's degree in electrical engineering, he applied George Boole's logical algebra to the problem of electrical switching. During that time Boole's system for logically manipulating 0 and 1 was little known, but it is now the nervous system of every computer in the world. Then for his doctorate, he applied mathematics to genetics. Shannon received both his master's degree and his doctorate in 1940.

Besides Shannon's theory of communication, he published a classic paper "A Symbolic Analysis of Relay and Switching Circuits." This paper point out the identity between the two "truth values" of symbolic logic and the binary values 1 and 0 of electronic circuits. Shannon showed how a "logic machine" could be built using switching circuits corresponding to the propositions of Boolean algebra.

http://www.nyu.edu/pages/linguistics/courses/v610003/shan.html

Logic Gates

A **Logic Gate** is an electronic circuit that "**Logically Combines**" inputs to produce outputs. By logically combining, we mean applying a "**logic operator**" to some operands to produce some output.

A **logic gate** implements a **logic operator**.

Boole considered 3 logic operators: **AND**, **OR**, **NOT**

Logic gates can be constructed for each of these, and for combinations of them.

With these logic gates, Boole's and Shannon's work showed that we can construct **any** electronic circuit whose outputs depend only on its inputs.

These are called **Combinatorial**, or, **Combinational Circuits**

Now, Modelling Information using numbers is one thing, "Processing" that Information is another.

By Processing, we mean transforming/combining Information to Create new Information.

Consider the expression:

$$2+3=5$$

This uses an operator, +, to map two entities, 2 and 3, into a new entity, 5. A relationship is Created between the operands of + and its result.

We could write this functionally as:

$$+(2,3) \rightarrow 5$$

In a sense, we are Creating/defining new Information by doing this. The value/utility of this new Information depends on the meaning we attach to + when it is defined.

Addition (and other arithmetic operators) allows us to equate many quantities, combined using specific rules, with a single, resultant, quantity.

There is a profundity to this that often goes unnoticed by the Casual observer:

In some sense, we can think of the result as having the same "meaning" as the combination of the operands.

For the moment, we will only consider those functions whose output (O) depends only on its inputs (I)

That is, $O = f(I)$

The work of Boole and Shannon tells us that <u>any</u> function of this type can be defined in terms of the three logic operators: AND, OR, NOT

This is remarkable when you think about it!

The actions of AND, OR, NOT can be Specified In Truth Tables, in which the effect of applying Each operator is tabulated against all possible Input Combinations

AND Truth Table

Inputs		output
A	B	A AND B
F	F	F
F	T	F
T	F	F
T	T	T

By mapping F→0 and T→1

We can write Truth Tables in terms of 0 and 1.

Shannon would write:

A	B	A AND B
0	0	0
0	1	0
1	0	0
1	1	1

Algebraically, we write A AND B as A.B or as AB

In an electronics diagram we use the Symbol:

OR Truth Table (Inclusive OR)

Inputs		Outputs
A	B	A OR B
0	0	0
0	1	1
1	0	1
1	1	1

Algebraically : $A + B$

In Circuit Symbol:

NOT Truth Table

Input	output
A	NOT A
0	1
1	0

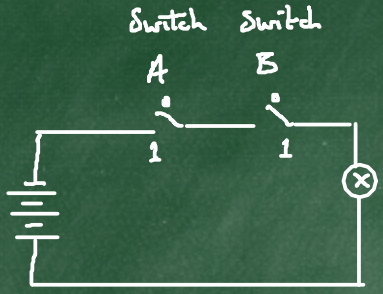
Algebraically : A' or $\overline{A}$

In Circuit Symbol:

AND, OR and NOT are known as fundamental Gates because all Combinatorial Circuits can be made from combinations of these gates. Together they form a fundamental set.

Implementing AND, OR and NOT from Electrical Circuits.

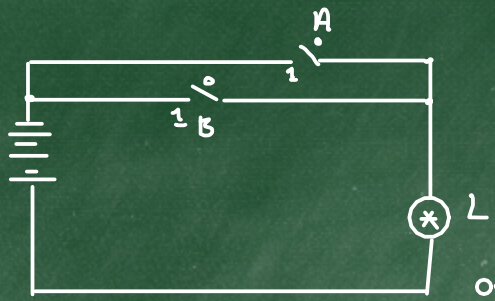
AND



A AND B
Light on = 1
Light off = 0

A	B	L
0	0	0
0	1	0
1	0	0
1	1	1

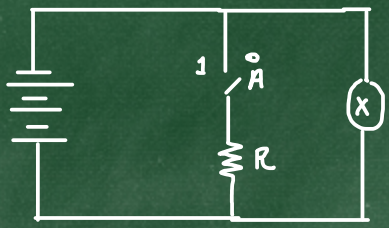
OR



A	B	L
0	0	0
0	1	1
1	0	1
1	1	1

on = 1
off = 0

NoT



Light
on = 1
off = 0

A	L
0	1
1	0