ADDING 2 bits

Understanding the problem:

Consider the two bits 0, 1 and add them together in all possible combinations

a	0	0	1	1
b	0	1	0	1
C_{out}	0	0	0	1
Sum	0	1	1	0

Every addition produces a half-sum (s) and a Carry out (C_{out}).

Let us now reorder our information in the form of two Truth Tables:

a	b	s
0	0	0
0	1	1
1	0	1
1	1	0

≡ XOR

a	b	C_{out}
0	0	0
0	1	0
1	0	0
1	1	1

≡ AND

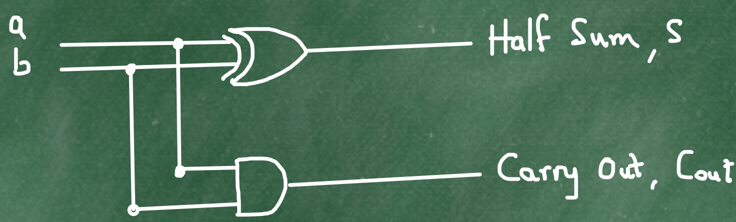
Thus adding a to b ($a + b$) can be done with logic gates!

$$\text{Sum} = a \oplus b$$

$$C_{out} = a \cdot b$$

ADDING IS BEING EXPRESSED USING LOGIC

The logic circuit to add 2 bits is thus:



This circuit is called a half-adder

Looking More closely at the 'Carry' process

Decimal Example : add 796 to 248

$$
\begin{array}{cccc}
 & 7 & 9 & 6 \\
1 & 2_1 & 4_1 & 8_0 \\
\hline
1 & 0 & 4 & 4
\end{array}
$$

We see that if we wish to perform multidigit addition, we need to add 3 things in each step: a Carry-in, a Sumend, and an augend.

Each step results in a Sum and a Carry-out (which becomes the Carry-in to the next, higher-order addition)

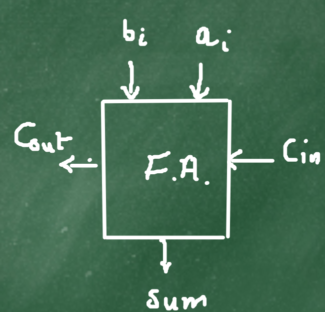
6 ← Augend — a_i

Sumend — b_i

Cout

Carry-out → 1 8 0 ← Carry-In — C_{in}

4 ← Sum



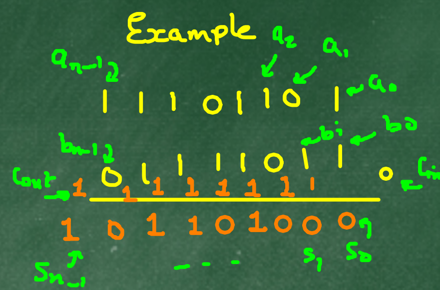
We call this Component a

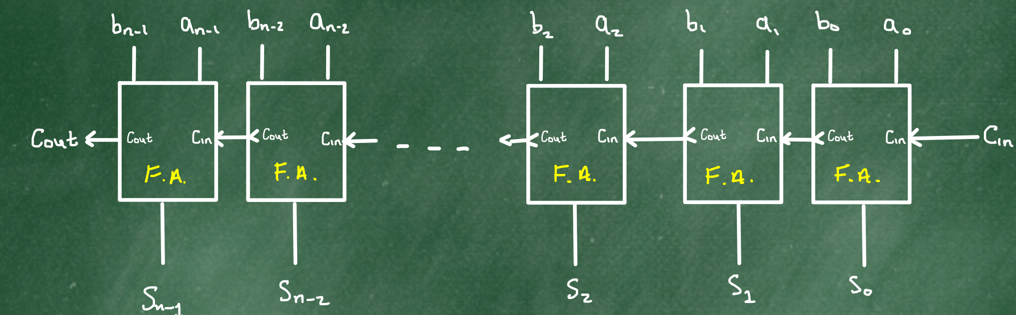
Full-ADDER (F.A.)

We Can Combine full-adders to make multidigit ADDERS

In general

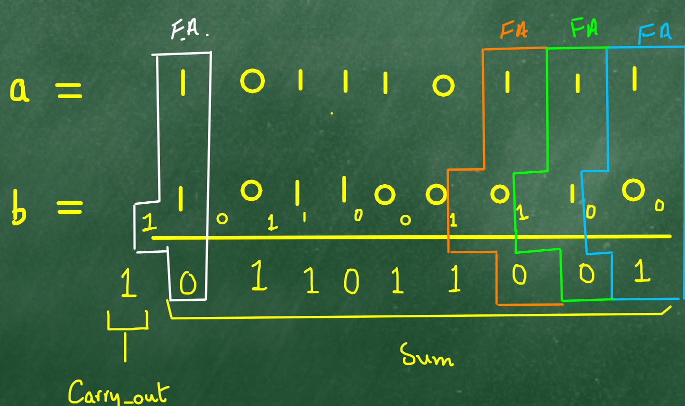
$$\begin{array}{lccccccc} & a_{n-1} & a_{n-2} & \cdots & a_2 & a_1 & a_0 & \\ C_{out} & b_{n-1} & b_{n-2}\,{}_{C_{n-3}} & \cdots & b_2\,{}_{C_1} & b_1\,{}_{C_0} & b_0 & 0 \\ \hline & S_{n-1} & S_{n-2} & & S_2 & S_1 & S_0 & \end{array}$$





An n-bit multidigit Binary Ripple-Carry Adder.

The Carry ripples through the Circuit, giving the device its name.



Building a Logic Circuit to Implement a Full-ADDER

Construct a Truth Table for a Full-ADDER by tabulating all possible Combinations of Inputs against their associated outputs

Inputs			Outputs	
C_{in}	a	b	Sum	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$C_{in} = 0$

$Sum = a \oplus b$

$C_{out} = a \cdot b$

$C_{in} = 1$

$Sum = a \odot b$

$C_{out} = a + b$

Later, we will Learn a mechanistic way of Creating Equation and Circuits from Truth Tables.

For now, we are going to Create a Circuit by Intuition and Pattern matching (i.e, Comparing what we see with what we already Know)

In doing this,
we can Consider this Truth Table as Consisting of 2 Parts: Part 1, where $C_{in} = 0$ and Part 2, where $C_{in} = 1$.