department of Computer Science
University College Cork

CS1112
Compound Logical Statements

Lecturer:

Professor Barry O’Sullivan

Office: 2-65, Western Gateway Building

email: *b.osullivan@cs.ucc.ie*

http://osullivan.ucc.ie/teaching/cs1112/

You write a piece of software, which is supposed to produce the answer "yes" whenever the input is classified as being in set A.

You write some test routines to make sure it is working. Which of the following scenarios are worth investigating?

(i) input in set A
(ii) input not in set A

(iii) output is "yes"
(iv) output is "no"

Consider a pack of two-sided cards – each card has a number on one side, and a letter on the other. If the number is odd, then the letter must be a vowel.

I have designed 4 cards to add to the pack. My cards definitely do have a single number on one side, and a single letter on the other. But I may have made a mistake with some of them over the odd/vowel relationship. You need to check which cards are OK. Which cards to you need to turn over to check? Don't turn over any more cards than you need. Place a tick in the box below each card that you think you *need* to turn over.

7	8	E	D
(i)	(ii)	(iii)	(iv)

response: 62

(i) : 52
(ii) : 23
(iii): 47
(iv): 26

(i,iv): 6 (10%)
(i,iii): 23 (37%)
(i,iv)+: 23 (37%)
(i): 5 (5%)

Compound Statements

Disjunction

Inclusive vs Exclusive OR

Conditional Statements

The Bi-Conditional

The following behaviour of the system has been verified:

- *if you are accessing from an authorised ip address and you have provided a password, then you get access*
- *if you are using an off-campus machine and the ip address is not blacklisted, or you are using a campus machine, then the ip address is authorised*

We know the following facts are true

- *the user is using a campus machine*
- *the user has provided a password*

Is the following statement true?

- *the user gets access*

Disjunction ($\vee$)

Disjunction connects two propositions. The disjunction of two propositions is true if one or both of the smaller propositions is true; otherwise it is false.

For "Disjunction" we use the symbol *v* (or sometimes "||" or OR), positioned between the two propositions we are joining.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Note: this is an inclusive "or" – it allows both propositions to be true

Note: compare this to the truth tables you saw in CS1110

Examples of Disjunction

Let p be "Dublin is the capital of Ireland"
Let q be "7+2=9"
p has value T, q has value T, and so $p \vee q$ must have value T.

In other words, it is true that
"Dublin is the capital of Ireland or 7+2=9"

q = "7+2=9" has value T,
r = "Homer Simpson lives in Cork" has value F,
so $q \vee r$ must have the value T

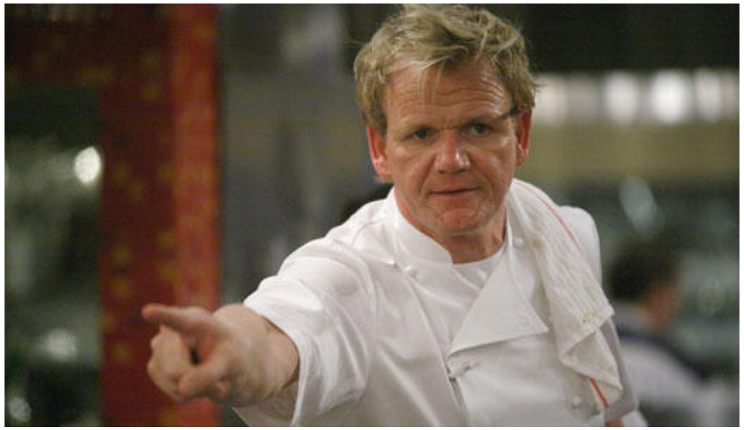
s = "7 / 2 = 19" has value F,
r = "Homer Simpson lives in Cork" has value F,
so $s \vee r$ must have the value F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Inclusive OR vs Exclusive OR

The word "or" in English has multiple interpretations.

In Hell's Kitchen, the menu says
"*First course: grilled asparagus or goose liver pate*"
So what does the menu mean?
(i) asparagus alone, or pate alone, or both together
(ii) asparagus alone, or pate alone, but not both
(iii) asparagus alone, or pate alone, or nothing



Exercise

Let *p* be "Cork is in Munster"
Let *q* be "Roses are red"
Let *r* be "Mice like to eat chili peppers"
Let *s* be "Bono is in Snow Patrol"

Express the following in symbols and connectives:
Cork is in Munster or Bono is in Snow Patrol

Express the following in English: $q \vee s$

If we now know that *p* is true, *q* is true, *r* is false, and *s* is false, what is the truth value of the following?

$p \vee q$
$p \vee r$
$r \vee s$

Conditional (→)

The conditional connects two propositions. The resulting statement is true whenever the second proposition is true, or when the first proposition is false; otherwise, the conditional statement is false.

For the "Conditional" connective, we use the symbol →, placed between the two propositions

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Whenever the first proposition is true, the second is also true; when the first proposition is false, the second could be either.

Examples of Conditional statements

Let p be "Dublin is the capital of Ireland"
Let q be "The lecturer for CS1112 in 2015/16 is O'Sullivan" p has value T, q has value T, and so $p \rightarrow q$ must have value T.

p = "Dublin is the capital of Ireland" has value T,
r = "Homer Simpson lives in Cork" has value F,
so $p \rightarrow r$ must have the value F

p = "5 / 2 = 17" has value F,
q = "Homer Simpson lives in Cork" has value F,
so $p \rightarrow q$ must have the value T

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Exercise

Let *p* be "Cork is in Munster"
Let *q* be "Roses are red"
Let *r* be "Mice like to eat chili peppers"
Let *s* be "Bono is in Snow Patrol"

If we know that *p* is true, *q* is true, *r* is false, and *s* is false, what are the truth values of the following?

$p \rightarrow q$
$p \rightarrow r$
$r \rightarrow s$
$s \rightarrow q$
$q \rightarrow s$

How do we interpret the conditional?

We are trying to analyse rational argument, and so we want some connective to say:

“since p is true, q must be true"

$\rightarrow$ is often interpreted as "if ... then ..."

so to represent "if it is raining then the road is wet", we could say *p* is the proposition "it is raining", and *q* is the proposition "the road is wet", and represent the claim as

$p \rightarrow q$

i.e. in all situations where it is raining, the road is wet, and in other situations, the road could be wet or dry.

Beware!

Do not interpret the conditional as meaning "causes" –
i.e. do not read "$p \rightarrow q$" as saying "*p* causes *q*"

All it says is that whenever *p* is true, *q* is also true, and we determine this by looking at all situations in the environment we are analysing.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Remember, if
p = "Dublin is the capital of Ireland", and
q = "O'Sullivan is the lecturer for CS1112", then

$p \rightarrow q$ is a true statement.

Dublin being capital *does not cause* O'Sullivan to lecture on CS1112

2^{nd} warning

A 2^{nd} reason why $\rightarrow$ does not mean "causes":

if p is false, then $p \rightarrow q$ is true.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Let p = "the moon is made of blue cheese"
and q = "Enda Kenny comes from Mars"

then, based on what we know about the universe, $p \rightarrow q$ is a true statement.

I cannot find a situation in which the moon is made of blue cheese and in which Enda Kenny does not come from Mars, so the conditional statement is true.

The conditional as a contract

One way to interpret a conditional statement is as a contract.

If I claim $p \rightarrow q$ is true, and p also turns out to be true, then I guarantee that q will be true; if p turns out to be false, then I am making no promises, and q could be either true or false.

E.g. p = "you get $\geq$ 40% in your CS1112 module"
q = "you get a pass for CS1112 on your student record"

and UCC asserts that $p \rightarrow q$ is true.

This is a contract – if you do get $\geq$ 40% (without cheating) for CS1112, you will get a pass; if you don't get 40%, you may or may not get a pass, depending on personal and medical circumstances.

Consider all possible cases:

(i) you get $\geq$ 40% for CS1112, and UCC gives you a pass for CS1112
- UCC has honoured its contract

(ii) you get $\geq$ 40% for CS1112, but UCC does not give you a pass for CS1112
- UCC has broken its contract

(iii) you get < 40% for CS1112, and UCC gives you a pass
- UCC has certainly not broken its contract

(iv) you get < 40% for CS1112, and UCC does not give you a pass
- UCC has not broken any contract

	p	q	$p \to q$
(i)	T	T	T
(ii)	T	F	F
(iii)	F	T	T
(iv)	F	F	T

Exercise

Write the following statements using the conditional connective:

- If I work hard, I will pass CS1112

- Whenever I bribe the bouncer, I get into this nightclub

- If you drive too fast, you will have an accident

For the following symbols and interpretations

p: the microwave is beeping

q: the food is cooked

translate the following into English:

$$p \rightarrow q$$

Biconditional ($\leftrightarrow$)

For the "Biconditional" connective, we use the symbol $\leftrightarrow$

The biconditional connects two propositions. The resulting statement is true whenever both propositions have the same truth value; otherwise, the biconditional statement is false. As before, we can represent this using a table.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Examples of Biconditional statements

Let p be "Dublin is the capital of Ireland" and q be "The lecturer for CS1112 in 2015/16 is O'Sullivan" then p takes value T and q takes value T, and so $p \leftrightarrow q$ must take value T.

p = "Dublin is the capital of Ireland" has value T,
r = "Homer Simpson lives in Cork" has value F,
so $p \leftrightarrow r$ must have the value F

p = "5 / 2 = 17" has value F,
q = "Homer Simpson lives in Cork" has value F,
so $p \leftrightarrow q$ must have the value T

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Exercise

Let *p* be "Cork is in Munster"
Let *q* be "Roses are red"
Let *r* be "Mice like to eat chili peppers"
Let *s* be "Bono is in Snow Patrol"

If we know that *p* is true, *q* is true, *r* is false, and *s* is false, what are the truth values of the following?

$p \leftrightarrow q$
$p \leftrightarrow r$
$r \leftrightarrow s$
$s \leftrightarrow q$
$q \leftrightarrow s$

How do we interpret the biconditional?

$\leftrightarrow$ is often interpreted as "... if and only if ..."

so to represent
"I am qualified to enter 2nd year if and only if I gain ≥ 50 credits in 1st year",
we could say *p* is the proposition "I am qualified to enter 2nd year",
and *q* is the proposition "I gain ≥ 50 credits in 1st year ", and
represent the claim as

$p \leftrightarrow q$

i.e. in all situations where I am qualified to enter 2nd year, I have gained ≥ 50 credits in 1st year, and in all situations where I have gained ≥50 credits in 1st year, I am qualified to enter 2nd year.

Beware!

All the same warnings about the conditional also apply to the biconditional. Do not interpret the biconditional as meaning "is the same thing as"

All it says is that *p* and *q* have the same truth values.

Remember, if
p = "Dublin is the capital of Ireland", and
q = "Ken Brown is the lecturer for CS1112 in 2014", then

p ↔ q is a true statement.

p	*q*	*p*↔*q*
T	T	T
T	F	F
F	T	F
F	F	T

We cannot say that Dublin being capital of Ireland is the same thing as O'Sullivan lecturing on CS1112

Exercise

Write the following statements using the bi conditional connective:

- I will get a BSc degree if and only if I work hard
- To win the lotto jackpot, it is necessary and sufficient to have matched 6 numbers

Next lecture ...

The formal language of propositional logic

Understanding complex statements