CS1112
An Introduction to Relations

Lecturer:

Professor Barry O'Sullivan

Office: 2-65, Western Gateway Building

email: *b.osullivan@cs.ucc.ie*

http://osullivan.ucc.ie/teaching/cs1112/

Introduction to Relations

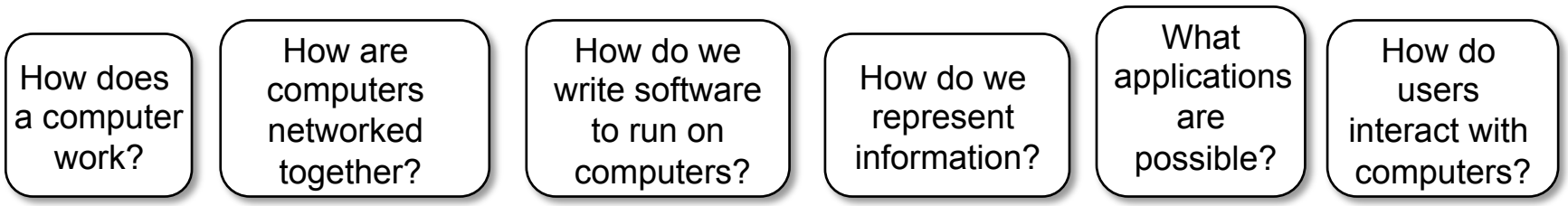
The need to represent connections between objects and connections between sets of objects

(Defn 3.1 – 3.4)

Modelling relations between objects

- Computer Science is all about models – representing them, using them, and understanding their properties
- The basics of set representation allows us to talk about collections, and about which objects are in which collection
- But we need more than this for modelling – we need to represent the connections between different objects, or the relationships between objects, and these relationships will have to be more general than functions.

Overview of Computer Science



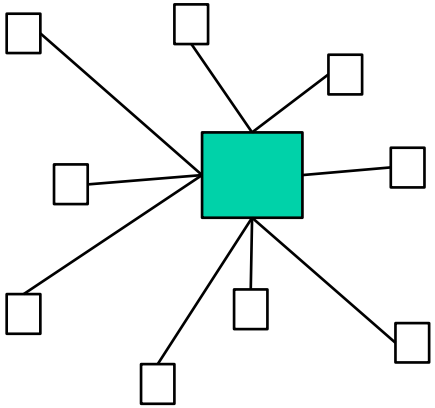
Networking

- Some processes maintained by the operating system allow communication with other devices on a network
  - Printers, scanners, etc
  - Local computers
  - Internet servers

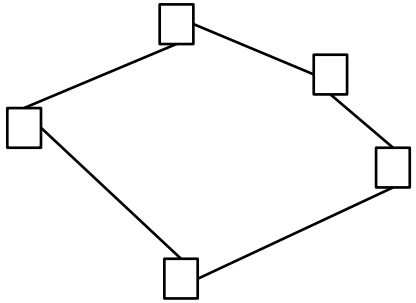
- Devices communicate using various different protocols - agreed schemes for sending and receiving messages
  - The type and content of the message
  - The packaging of the message
  - Where the message goes, and when

- Devices on a network are arranged in different patterns, or topology

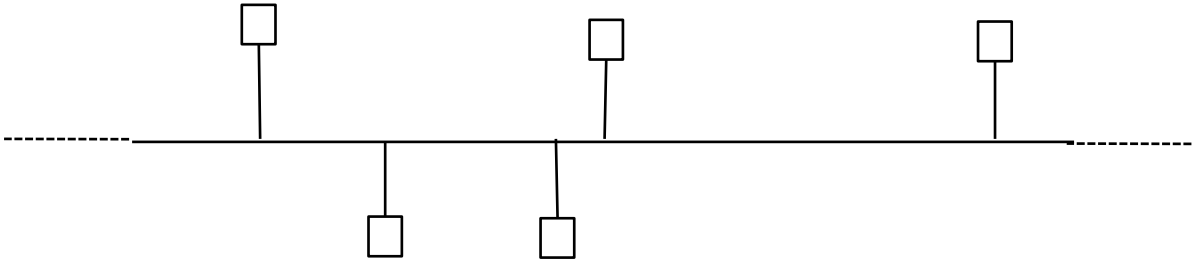
Network Topologies



Star

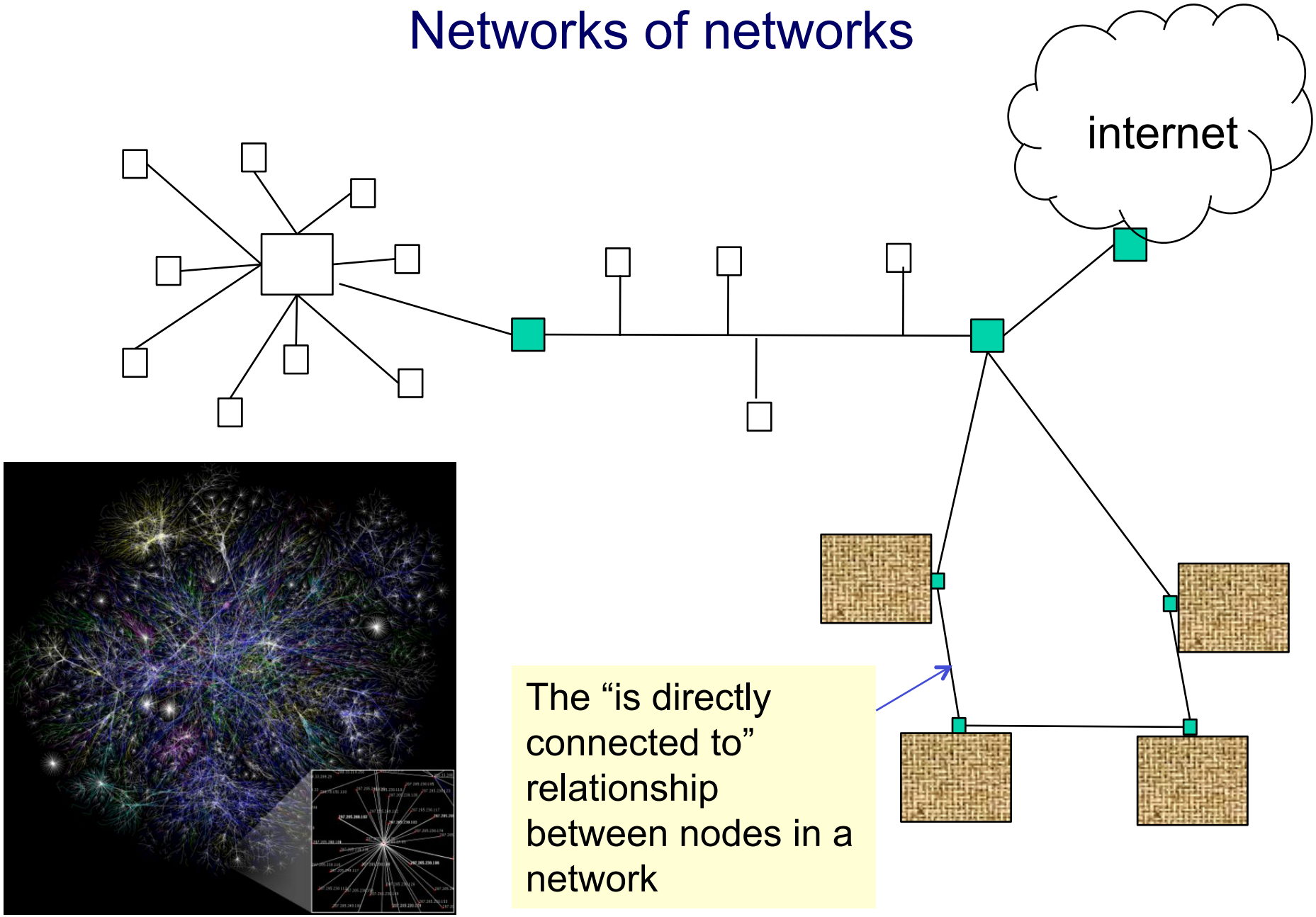


Ring



Bus

Networks of networks



The “is directly connected to” relationship between nodes in a network

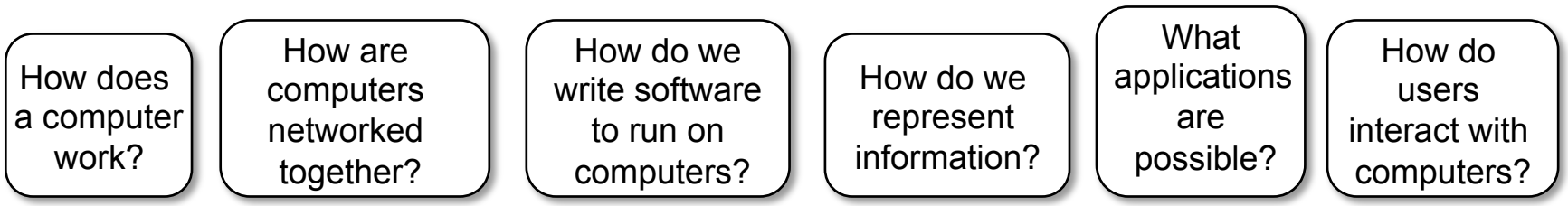
The example from lectures on sets:

U={Alice, Bob, Carol, Darragh, Eileen, Frank, Oliver, Patsy, Ronan, Susan, Ted,Una}
CS101 = {Alice, Darragh, Oliver, Ronan}
CS102 = {Bob, Carol, Eileen}
CS103 = {Eileen, Patsy, Ted}
MA101 = {Darragh, Oliver}
EC101 = {Frank, Susan, Ted}
EC102 = {Alice, Bob, Una, Patsy, Susan}

MG_1 = {Alice, Bob, Carol, Ted}
MG_2 = {Darragh, Eileen}
MG_3 = {Frank, Patsy, Ronan}
MG_4 = {Oliver, Susan, Una}

Mentors = {bertie, gordon, george}
Groups = {MG_1, MG_2, MG_3, MG_4}

Overview of Computer Science



Relational Databases

- The standard representation of data in databases is based on entities and the relationships between them
- Example: all information about students in the UCC register:

Name	Id	Address	Degree	Dept	M1	M2	...	Mentor	MId
Coveney	123	1 Main St	CK401	Computer Science	CS1100	CS1101	...	Prof. Kenny	1
McGrath	456	7 Well Rd	CK401	Computer Science	CS1100	CS1101	...	Dr. Martin	4
Lynch	789	3 Cork La.	CK401	Computer Science	CS1100	CS1101	...	Dr Gilmore	9

- Main questions
  - How do we avoid redundant information?
  - How do we ensure updates are consistent?
  - How do distinguish between different entities, or recognise that two entries refer to the same entity?

Multiple Relations

The “is a mentor of” relationship

STUDENT

Name	Id	Address	Degree
Coveney	123	1 Main St	CK401
McGrath	456	7 Well Rd	CK401
Lynch	789	3 Cork La.	CK401
…	…	…	…

DEGREE

Degree	Subject
CK401	Computer Science
CK402	Biology & Chemistry
...	...

MENTORING

MId	Id
1	123
1	124
1	125
4	456
4	457
9	789

MENTOR

Name	MId
Prof Kenny	1
Dr. Martin	4
Dr. Gilmore	9

MODULES

Id	Module
123	CS1100
123	CS1101
123	CS1102
456	CS1100
456	CS1101
...	...

Find all students registered for Computer Science

Find all students with a mentor called Prof Kenny

Query relying on consequences of multiple relationships

Relations and Databases

The Relational Model

From CS1106

UCC

Relational Model Basics

Dr. Barry O'Sullivan

Learning Objectives

What is a Data Model?

- The relational model gives us a single way to represent data: as a two-dimensional table called a relation.
- In our example column represe

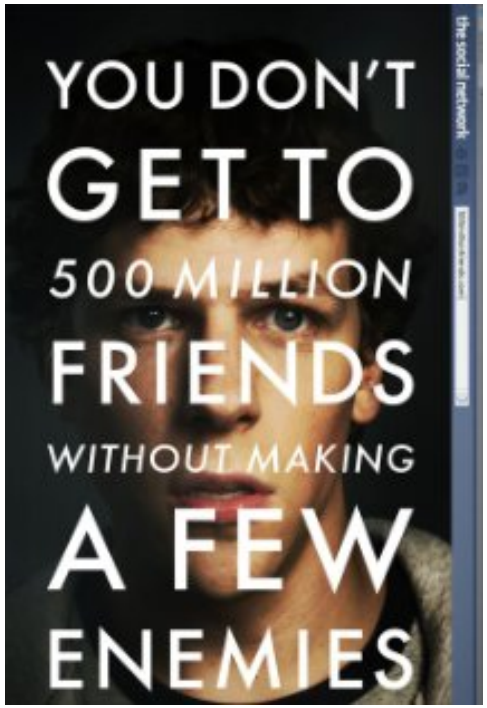
Example Relation

title

```
Gone With the
Star Wars
Wayne's World
```

title	genre
Gone with the Wind	drama
Star Wars	sci-fi
Wayne's World	comedy
WALL-E	animated
No Country for Old Men	drama
Tropic Thunder	comedy

This is a relation between 4 sets. For the moment, we will consider only relations between 2 sets.


YOU DON'T
GET TO
500 MILLION
FRIENDS
WITHOUT MAKING
A FEW
ENEMIES
the social network

- Facebook and Google+ are just databases behind a web front-end ...

The “friend” relationship between facebook users

FRIENDS

F1	F2
12345	27891
12345	92145
27891	92145

The “is_a_member” relationship between users and networks

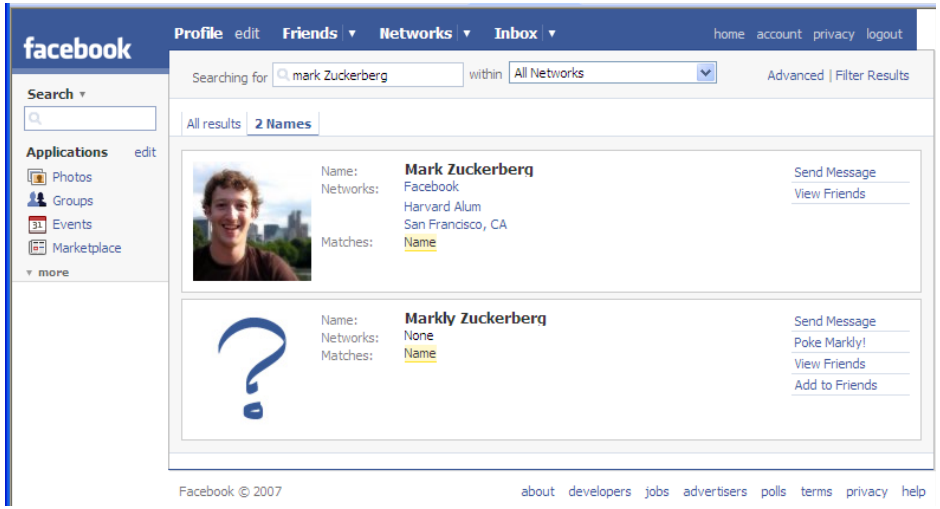
PEOPLE

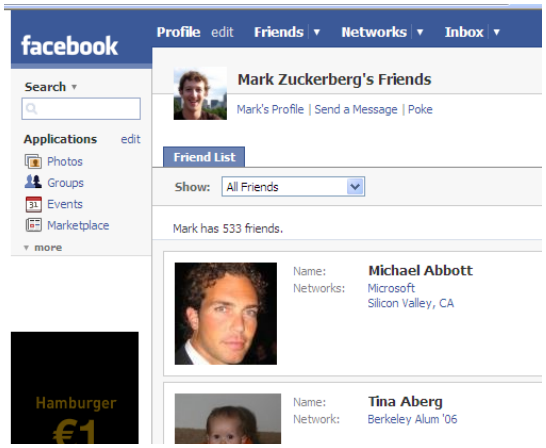
1st Name	2ndName	Id	email	Pass	...
Enda	Kenny	27891	ek@hotmail	****	...
David	Cameron	92145	dc@yahoo	****	...
Barack	Obama	12345	pres@gmail	****	...
Brian	Cowen	12643	b.cowen@gmail	****	...

NETWORKS

Id	Network
12643	Dail
92145	Westminster
27891	Dail
12345	Whitehouse

...





Modelling relations between objects (II)

We will look at:

- different ways of representing relations
- different properties of relations
- how to combine and manipulate relations, and the link to relational databases
- equivalence relations
- order relations

Examples of relations

- Enda is a friend of David on facebook
- Prof Kenny is the mentor of Mr Coveney in the database
- Alice is registered on CS101 in the UCS database
- Homer Simpson is the father of Bart Simpson
- Susan is registered for an Economics degree
- CS1112 is linked from Ken Brown's web page
- the hall is connected to the corridor in a game
- Plastering must be done before painting in construction scheduling
- CS1112 is a pre-requisite for CS2201
- Rod Flanders wants to be a friend of Bart Simpson
- Mr Coveney is mentored by Prof Kenny
- Painting must be done after plastering

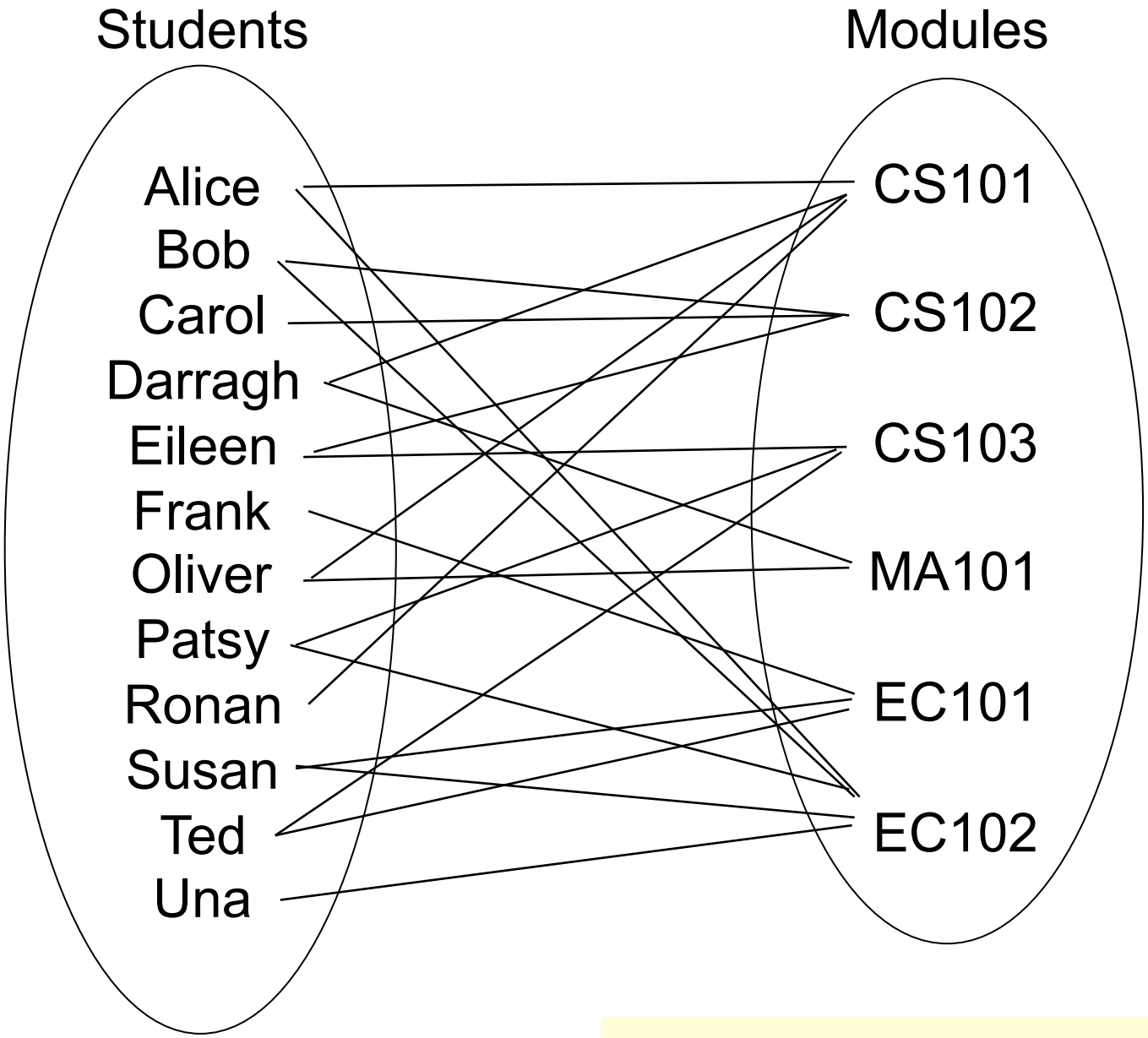
Representing relations

	Alice	Bob	Carol	Darragh	Eileen	Frank	Oliver	Patsy	Ronan	Susan	Ted	Una
CS101	X			X			X		X			
CS102		X	X		X							
CS103					X			X			X	
MA101				X			X					
EC101						X				X	X	
EC102	X		X					X		X		X

... using a matrix

... using a table

Students	Modules
Alice	CS101
Alice	EC102
Bob	CS102
Bob	EC102
Carol	CS102
Darragh	CS101
Darragh	MA101
Eileen	CS102
Eileen	CS103
Frank	EC101
Oliver	CS101
Oliver	MA101
Patsy	CS103
Patsy	EC102
Ronan	CS101
Susan	EC101
Susan	EC102
Ted	CS103
Ted	EC101
Una	EC102



... using an arrow diagram

{ (Alice, CS101), (Alice, EC02),
(Bob, CS102), (Bob, EC102),
(Darragh, CS101), (Darragh, MA101),
(Eileen, CS102), (Eileen, CS103),
(Frank, EC101),
(Oliver,CS101), (Oliver, MA101),
(Patsy, CS103), (Patsy,EC102),
(Ronan,CS101),
(Susan,EC101), (Susan, EC102),
(Ted,CS103), (Ted,EC101),
(Una, EC102) }

A set of ordered pairs over two sets is just a subset of the Cartesian product

... using a set of ordered pairs

We will use this representation as our standard

R is a relation between two sets A and B if and only if $R \subseteq A \times B$

Strictly a binary relation, since 2 sets

Example:

"is_the_square_of" $\subseteq \{0,1,2,3,4,5,6,7,8,9\} \times \{0,1,2,3,4,5,6,7,8,9\}$

"is_the_square_of" = { (0,0), (1,1), (4,2), (9,3) }

source

target

extension

To understand a binary relation completely,
we need to know all three parts:

We can write a relation expression using

	In general
• Infix notation: 2 is_a_divisor_of 6	a R b
• Prefix notation: is_a_divisor_of(2,6)	R(a,b)
• Element notation: (2,6) $\in$ is_a_divisor_of	(a,b) $\in$ R

To show some pair is not related, use negation or standard symbols:

• It is not true that 5 is_a_divisor_of 6	$\neg$R(a,b)
• (5,6) $\notin$ is_a_divisor_of	(a,b) $\notin$ R

You have seen many binary relations before:

$<$ $\leq$ $>$ $\geq$ $=$ $\neq$

Example: “$<$“ $\subseteq \mathbb{N} \times \mathbb{N}$
e.g. $3 < 723$ but $541 \not< 29$ (which we write as $541 \geq 29$)

relation	source	target
• Enda is a friend of David on facebook	users	users
• Prof Kenny is the mentor of Mr.Coveney in the DB	mentors	students
• Alice is registered on CS101 in the UCS database	students	modules
• Homer Simpson is the father of Bart Simpson	people	people
• Susan is registered for an Economics degree	students	degrees
• CS1112 is linked from Ken Brown's web page	webpages	webpages
• the hall is connected to the corridor in a game	locations	locations
• Plastering must be done before painting	tasks	tasks
• CS1112 is a pre-requisite for CS2201	modules	modules
• Rod Flanders wants to be a friend of Bart Simpson	people	people
• Mr Coveney is mentored by Prof Kenny	students	mentors
• Painting must be done after plastering	tasks	tasks

The source and target can be the same set, or can be different sets:

- Enda is a friend of David on facebook
- Prof Kenny is the mentor of Mr.Coveney in the DB
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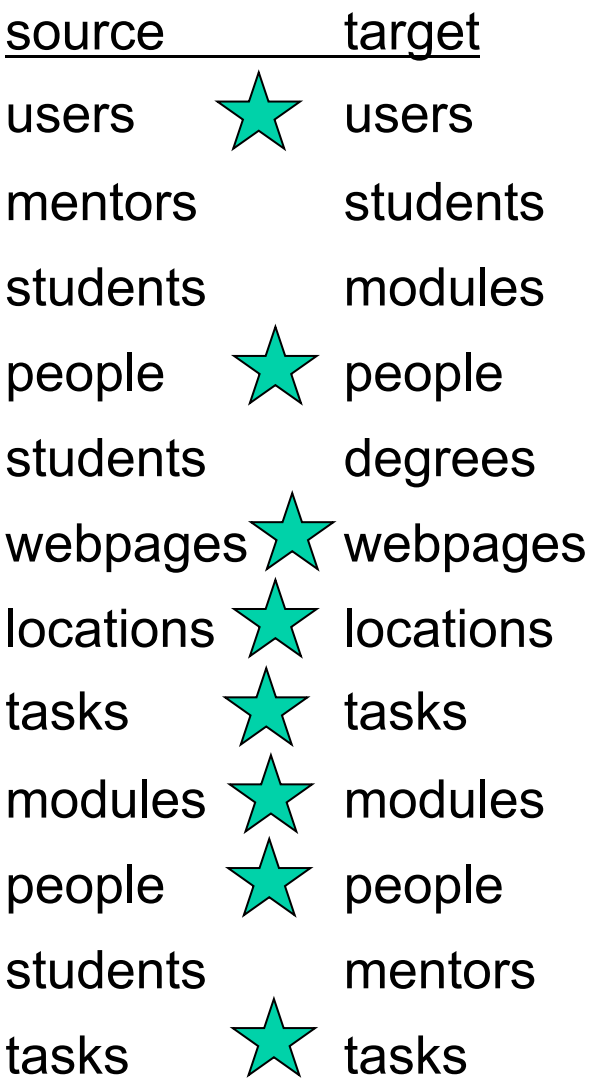

Note: a relation is ordered:

$(2,6) \in$ "is_a_divisor_of"
$(6,2) \notin$ "is_a_divisor_of"

But some relations work both ways round:

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- Prof Kenny is the mentor of Mr.Coveney in the DB
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source	target
users	users
mentors	students
students	modules
people	people
students	degrees
webpages	webpages
locations	locations
tasks	tasks
modules	modules
people	people
students	mentors
tasks	tasks

Every relation, R, has an inverse relation, R^{-1}

If $R \subseteq A \times B$, then $(a,b) \in R$ if and only if $(b,a) \in R^{-1}$

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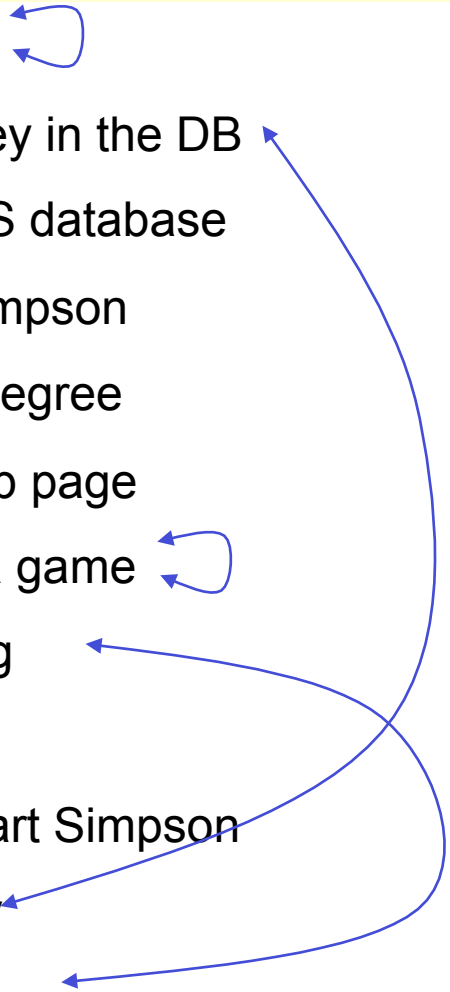


Formal Definitions and Notation

Let A and B be two sets, and let $a \in A$ and $b \in B$

R is a binary relation between A and B if and only if $R \subseteq A \times B$

We say A is the source set and B is the target set.

source(R) = A

target(R) = B

We say a is R-related to b if and only if $(a,b) \in R$
We write this as $R(a,b)$ or aRb or simply $(a,b) \in R$

To indicate that a is not R-related to b, we say $(a,b) \notin R$, or

$\neg R(a,b)$, or $a \not R b$

Relations and Functions

A *function* is a special type of relation.

If $f : A \to B : a \mapsto f(a)$

then using the ordered pair representation, we can say

$f = \{(a,b) \mid a \in A,\ b \in B,\ b = f(a)\}$

and so $f \subseteq A \times B$,

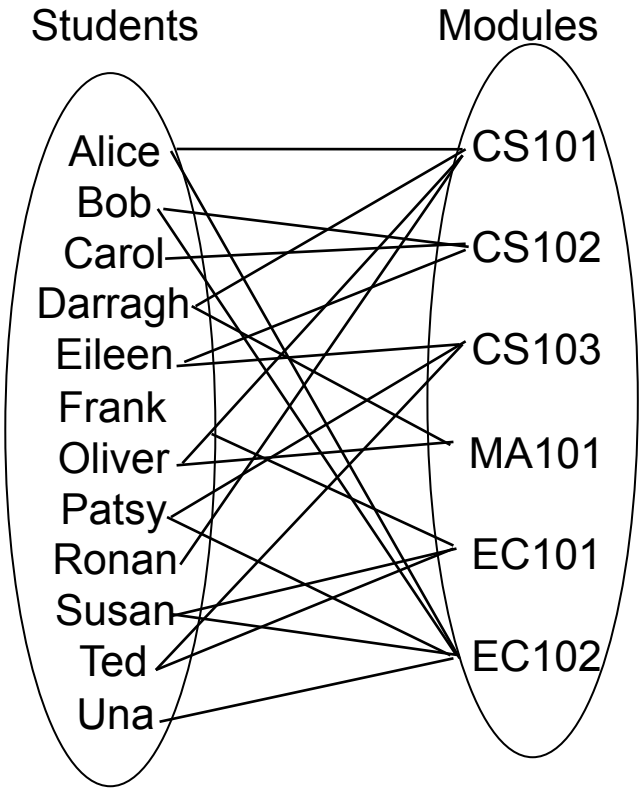
and so f is a relation between A and B.

Functions vs Relations

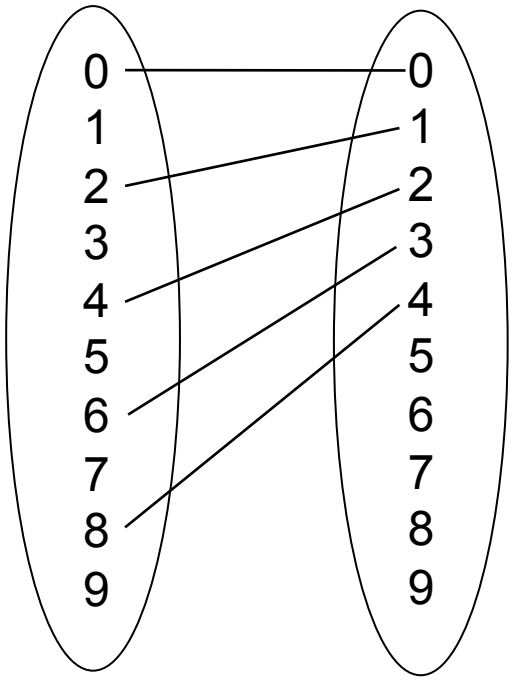
Functions f	Relations R
$\subseteq A \times B$	$\subseteq A \times B$
for every $a \in A$, there is a $b \in B$ s.t. $(a,b) \in f$.	there may be some $a \in A$ s.t. there is no $b \in B$ s.t. $(a,b) \in R$
for every $a \in A$, there is at most one b s.t. $(a,b) \in f$	there may be some $a \in A$ s.t. there are b and c with $(a,b) \in R$ and $(a,c) \in R$
We write $f(a) = b$	We write $R(a,b)$ or aRb

Functional Relations

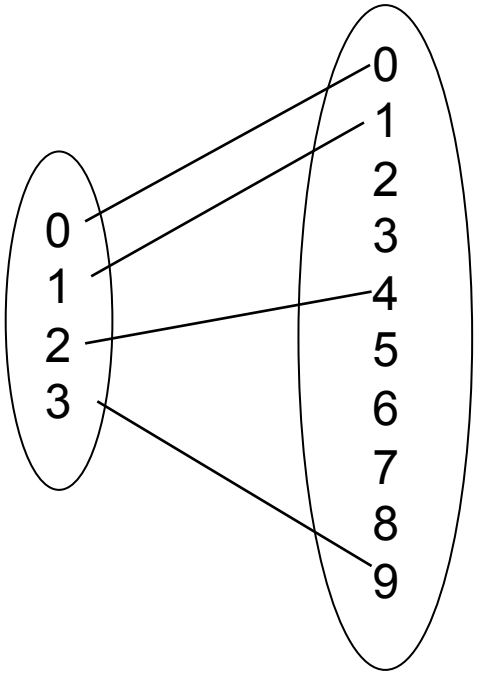
A binary relation $R \subseteq A \times B$ is functional iff for each $a \in A$, there is exactly one $b \in B$ s.t. $(a,b) \in R$.



not functional



not functional



functional

Next lecture ...

Properties of relations

Important relations

(Defn 3.1 – 3.8)