Equations from Truth Tables, a mechanistic approach:

Intuition and pattern-matching are good, but is there a general method for producing equations from Truth Tables?

Let's Investigate...

Consider a Simple T.T. of 2 variables, A and B:

A	B	F
0	0	
0	1	
1	0	
1	1	

Now, depending on what F is, there will be a 0 or a 1 associated with each Combination of A and B (after all, this is what we mean by a Truth Table)

Something that might be less obvious is that we can describe F from the perspective of the 0s or of the 1s.

- Either approach is valid.
- There is no need to do it from both perspectives.

we usually describe F from the perspective of the 1s.

For example, if our T.T. is:

A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

we say that F is 1, only when both A and B are 0. we don't have to explicitly discuss the cases where F is 0.

if our T.T. were:

A	B	F
0	0	1
0	1	0
1	0	0
1	1	1

We would describe F as being 1 when A and B are both 0 or when A and B are both 1. Again, we ignore the cases where F=0, since they are implicitly captured.

Similarly F = 1, when A is 0 and B is 1 for the following TT:

A	B	F
0	0	0
0	1	1
1	0	0
1	1	0

, etc.

Two question now arise:

Question 1:

Can we Combine the Input Variables on each row of the T.T. So as to guarontee that their Combination = 1 ?

Question 2:

Can we Choose those Combinations from (1), for which our T.T. tells us that F=1 , and Combine them together?

Answer to Question 1:

For A=0 , B=0 : ANDing together $\bar{A}$ and $\bar{B}$ would produce a 1.

For A=0 , B=1 ANDing together $\bar{A}$ and B would produce a 1.

For A=1 , B=0 ANDing together A and $\bar{B}$ would produce a 1.

For A=1 , B=1 ANDing together A and B would produce a 1.

Recipe

if an Input variable has the value 1, we use that variable unchanged (in its original form), but,
if it has the value 0, we use the Complement of that Input Variable.

All Input variables are then ANDed together in their original or complemented form.

To emphasise: using this process, we see that

→ $\bar{A}\bar{B}$ will result in a 1 only when $A=0$ and $B=0$.

→ $\bar{A}B$ will result in a 1 only when $A=0$ and $B=1$.

→ $A\bar{B}$ will result in a 1 only when $A=1$ and $B=0$.

→ AB will result in a 1 only when $A=1$ and $B=1$.

This process extends to any number of input variables:

For a T.T. with 3 input variables, for example:

A	B	C	
0	0	0	$\bar{A}.\bar{B}.\bar{C}$ → Would result in 1 for this row
0	0	1	$\bar{A}.\bar{B}.C$ → Would result in 1 for this row
0	1	0	$\bar{A}.B.\bar{C}$ → Would result in 1 for this row
0	1	1	$\bar{A}.B.C$ → Would result in 1 for this row
1	0	0	$A.\bar{B}.\bar{C}$ → Would result in 1 for this row
1	0	1	$A.\bar{B}.C$ → Would result in 1 for this row
1	1	0	$A.B.\bar{C}$ → Would result in 1 for this row
1	1	1	$A.B.C$ → Would result in 1 for this row

Let's look at our first example again:

A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

$F = 1$ only on the row where $A = 0$ and $B = 0$.

To generate a 1 from these Inputs we use $\bar{A}.\bar{B}$.

Therefore $F = \bar{A}.\bar{B}$.

Of Course we Know from looking at the T.T. that

$$F = \overline{A+B}.$$

So is $\bar{A}.\bar{B} = \overline{A+B}$?

yes!

Convert $\bar{A}.\bar{B}$ to OR form using DeMorgan's theorem:

$$\bar{A}.\bar{B} =$$

(1) $\bar{A}+\bar{B}$ change the operator

(ii) $\bar{\bar{A}}+\bar{\bar{B}}$ Complement each Input variable

$= A+B$

(iii) $\overline{A+B}$ Complement the entire expr.

$\therefore$ $\bar{A}.\bar{B}$ Is indeed $= \overline{A+B}$

More Examples

AND :

A	B	F
0	0	0
0	1	0
1	0	0
1	1	1

$F = 1$ only when

$A = 1$ and $B = 1$

To generate a 1 from the Inputs

use $A.B$

Therefore $F = A.B$ --- trivial

So far we have been considering only examples where there was only one row in the T.T. where $F=1$.

We now look at an example where more than 1 row has a value for $F=1$. This Case addresses Question 2.

Example

EXOR :

A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

Here $F=1$ In two rows:

(i) when

$A = 0$ and $B = 1$

(ii) when

$A = 1$ and $B = 1$

To generate a 1 from the Inputs for (i), we use $\bar{A}B$ and to generate a 1 from the Inputs for (ii), we use $A\bar{B}$.

But how should we Combine them?

Since each row in a T.T. represents a separate case – describing the output corresponding to the values of the inputs in that particular case, we can simply OR together each separate case (where $F=1$) to get the final result:

$$\text{Therefore } F = \bar{A}B + A\bar{B}$$

(This should be familiar to us already)

Example

Coin:

A	B	F
0	0	1
0	1	0
1	0	0
1	1	1

$\longrightarrow \bar{A}\bar{B}$

$\longrightarrow AB$

$$F = \bar{A}\bar{B} + AB$$

Terminology

A minterm is an expression formed by ANDing together all input variables in either their original or complemented form.

There is a unique minterm associated with each combination of input variables.

Therefore, there are exactly 2^n minterms of n variables.

For 2 variables A, B:

A	B	minterm	Shorthand notation
0	0	$\bar{A}.\bar{B}$	m_0
0	1	$\bar{A}.B$	m_1
1	0	$A.\bar{B}$	m_2
1	1	$A.B$	m_3

For 3 Input Variables: A, B, C

A	B	C	minterm	Shorthand notation
0	0	0	$\bar{A}.\bar{B}.\bar{C}$	m_0
0	0	1	$\bar{A}.\bar{B}.C$	m_1
0	1	0	$\bar{A}.B.\bar{C}$	m_2
0	1	1	$\bar{A}.B.C$	m_3
1	0	0	$A.\bar{B}\bar{C}$	m_4
1	0	1	$A.\bar{B}C$	m_5
1	1	0	$AB\bar{C}$	m_6
1	1	1	ABC	m_7

The bottom Line!

To get an Equation from a Truth Table:

OR together all minterms Corresponding to those rows whose Output value = 1.

Concisely, we say F = Sum of minterms of those rows = 1 in T.T.

$$F = \sum_i m_i \quad , \text{ where o/p} = 1$$

Profound Note:

Any Combinatorial function Can be implemented using AND, OR, and NOT.

$$F = \underbrace{\sum}_{\text{OR}} \underbrace{m_i}_{\text{AND NOT}}$$