A Little Theory (it will be very Practical)

Boolean Algebra (we have already been using these ...)

Postulates:

P 1: There are two operators . and + , operating on pairs of elements to produce a result belonging to the set : Closure rule.

Essentially . and + operate on 0 and 1 values to give a 1 or a 0 value.

P 2: The operators . and + are commutative

$$A \cdot B = B \cdot A$$
$$A + B = B + A$$

P 3. The operators . and + are distributive

$$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$$
$$A + (B \cdot C) = (A + B) \cdot (A + C)$$

P 4. Two identity elements exist, such that

$$1 \cdot A = A$$
$$0 + A = A$$

P5. For each A, there is an Inverse $\bar{A}$ (A') such that

$$A \cdot \bar{A} = 0 \quad ; \quad A + \bar{A} = 1$$

Theorems

T_1: $A.0 = 0$

T_2: $A+1 = 1$

T_3: $A.A = A$

T_4: $A+A = A$

T_5: $A+(A.B) = A$

T_6: $A+(\bar{A}.B) = A+B$

T_7: $A.B.C = A.(B.C) = (A.B).C$

T_8: $A+B+C = A+(B+C) = (A+B)+C$

T_9: $\overline{A.B} = \bar{A}+\bar{B}$ (De Morgan)

T_{10}: $\overline{A+B} = \bar{A}.\bar{B}$ (De Morgan)

T_{11}: $\bar{\bar{A}} = A$

These Theorems can be proven by Perfect Induction or by algebraic manipulation.

Prove T_5 by Perfect Induction

$$T_5 : \quad A + (A.B) = A$$

A	B	A.B	A+(A.B)
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

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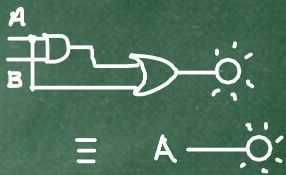
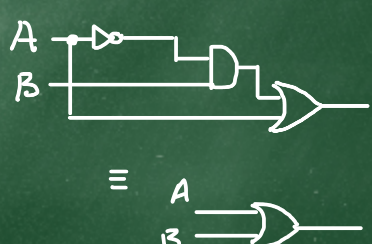


We've eliminate 2 gates from our circuit

Prove T_6 by Algebraic Manipulation

$$T_6 : \quad \underbrace{A + (\bar{A}.B)}_{L.H.S} = \underbrace{A+B}_{R.H.S}$$

$$\begin{aligned} LHS &= A + (\bar{A}.B) \\ &= (A+\bar{A}).(A+B) && \text{by } P3 \\ &= 1 \, . \, (A+B) && \text{by } P_5 \\ &= (A+B) && \text{by } P_4 \\ &= R.H.S. \end{aligned}$$



We've eliminated 2 gates

De Morgan's Theorems

These theorems provide a means by which the $\cdot$ operator can be replaced by the + operator, and visa versa, by forming the Inverse of the expression.

this Is useful for manipulating logic expressions into a form Suitable for Implementation.

Method :

(1) Change the operator
(2) Complement each Input variable
(3) Complement the entire expression

(note: the Complement of A is $\bar{A}$, etc)

Example :

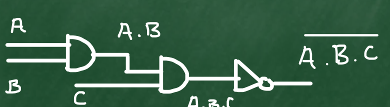
Change $\overline{A.B.C}$ Into 'OR' form

Apply the method :

(1) $\overline{A + B + C}$

(2) $\overline{\bar{A} + \bar{B} + \bar{C}}$

(3) $\overline{\overline{\bar{A} + \bar{B} + \bar{C}}} = \bar{A} + \bar{B} + \bar{C}$



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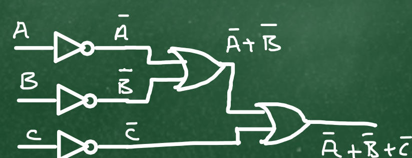

We could use perfect Induction to show that

$$\overline{A.B.C} = \bar{A} + \bar{B} + \bar{C}$$

A	B	C	A.B.C	$\overline{A.B.C}$	$\bar{A}$	$\bar{B}$	$\bar{C}$	$\bar{A}+\bar{B}+\bar{C}$
0	0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	0	1
0	1	0	0	1	1	0	1	1
0	1	1	0	1	1	0	0	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	0	1	0	1
1	1	0	0	1	0	0	1	1
1	1	1	1	0	0	0	0	0

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We can apply De Morgan to part of an expression if we wish, this can be useful if part of the expression is already in the desired form.

Example:

Convert $A+(B.C)$ to AND form

method: treat the subexpression, we wish to leave unchanged, as a single variable:

$$A+(B.C) = A+X \quad \text{, where } X = B.C$$

Convert $A+X$ to AND form:

(1) $A.X$

(2) $\bar{A}.\bar{X}$

(3) $\overline{\bar{A}.\bar{X}}$

Now replace the subexpression represented by X:

$$\overline{\bar{A}.\bar{X}} = \overline{\bar{A}.\overline{(B.C)}}$$

Building Circuits using NAND or NOR gates only

We see that De Morgan's Theorems allow us to eliminate either OR or AND operators from our equations, and hence OR or AND gates from our Circuits.

In other words, we can use De Morgan's Theorems to construct equations and Circuits with only AND and NOT or with only OR and NOT.

The following questions present arise : Since every equation (circuit) can be expressed (constructed) using AND and NOT only — Can they be expressed using NAND only?

Similarly, since every equation (circuit) can be expressed (constructed) using OR and NOT only — Can they be expressed (constructed) using NOR only?

Let's Investigate:

NAND only

(1) Express OR using NAND

$$A + B = \overline{\bar{A} \cdot \bar{B}}$$

using De Morgan, we get an equation with NAND and NOT

So, Can we express $\bar{A}$ (or $\bar{B}$) using NAND only?

A	B	$\overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

Notice: when both Inputs have the Same Value, the output Is the Inverse (NOT) of the Input.

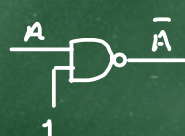
Therefore,

(2) Expressing NOT as NAND

For Completeness, we also note that if one Input to NAND is 1, the output will be the Inverse of the other Input:

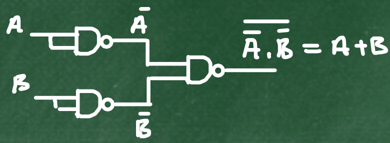
A	B	$\overline{A \cdot B}$
0	0	1
0	1	1
1	0	1
1	1	0

Thus



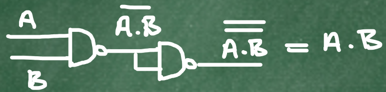
Therefore, to complete expressing OR in terms of NAND:

$$A + B = \overline{\overline{A} . \overline{B}} \equiv$$



(3) Express AND in terms of NAND:

$$A.B = \overline{\overline{A.B}}$$



NOR only

(1) Express AND using NOR

$$A.B = \overline{\bar{A} + \bar{B}}$$

using De Morgan, we get an equation with NOR and NOT

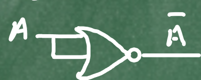
So, Can we express $\bar{A}$ (or $\bar{B}$) using NOR only?

A	B	$\overline{A+B}$
0	0	1
0	1	1
1	0	1
1	1	0

Notice: when both Inputs have the Same Value, the output Is the Inverse (NOT) of the Input.
—Just as with NAND

Therefore,

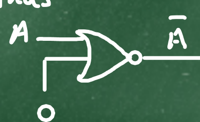
(2) Expressing NOT as NOR



For Completeness, we also note that if one Input to NOR is 0, the output will be the Inverse of the other Input:

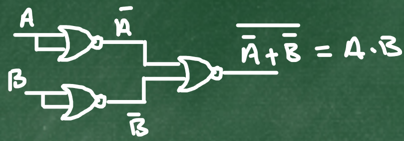
A	B	$\overline{A.B}$
0	0	1
0	1	0
1	0	0
1	1	0

Thus



Therefore, to Complete expressing AND in terms of NOR:

$$A \cdot B = \overline{\overline{A}+\overline{B}} \equiv$$



(3) Express OR In terms of NOR :

$$A+B = \overline{\overline{A+B}}$$