Equations from Truth Tables

Example: Suppose we have 3 inputs A, B, C, and one output F:

A	B	C	F	
0	0	0	0	m_0
0	0	1	0	m_1
0	1	0	1 *	m_2
0	1	1	1 *	m_3
1	0	0	0	m_4
1	0	1	0	m_5
1	1	0	1 *	m_6
1	1	1	1 *	m_7

F is chosen to reflect our requirement. In general, the values for F are derived from the problem description.

$$F = \Sigma(m_2, m_3, m_6, m_7) = m_2 + m_3 + m_6 + m_7$$

$$= \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC \qquad (15 \text{ Gates})$$

Optimize:

$$= \bar{A}B(\bar{C}+C) + AB(\bar{C}+C)$$
$$= \bar{A}B \cdot 1 + AB \cdot 1$$
$$= \bar{A}B + AB$$
$$= B(\bar{A}+A)$$
$$= B \cdot 1$$
$$= B$$

Example 2

A	B	C	F	
0	0	0	0	
0	0	1	1	* m_1
0	1	0	0	
0	1	1	1	* m_3
1	0	0	1	* m_4
1	0	1	0	
1	1	0	1	* m_6
1	1	1	0	

$$F = m_1 + m_3 + m_4 + m_6$$
$$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$$
$$= \bar{A}C(\bar{B}+B) + A\bar{C}(\bar{B}+B)$$
$$= \bar{A}C + A\bar{C}$$
$$= A \oplus C$$

Example : Night club Bouncer Problem.

Inputs :		Variable Names
1)	>= 18	A – age
2)	Not Drunk	D – Drunk
3)	Well-dressed	C – Clothes
4)	Male / Female	G – Gender

A = 1 => >= 18 (OK) ; A = 0 => under age (not OK)
D = 1 => Drunk (not ok) ; D = 0 => ok
C = 1 => Well-dressed (ok) ; C = 0 => not ok
G = 0 => male ; G = 1 => Female

Output Door = 1 => open i.e., entry granted
= 0 => closed i.e., entry denied.

A	D	G	C	Door	
0	0	0	0	0	
0	0	0	1	0	
0	0	1	0	0	
0	0	1	1	1	m_3
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	0	
0	1	1	1	0	
1	0	0	0	0	
1	0	0	1	1	m_9
1	0	1	0	1	m_{10}
1	0	1	1	1	m_{11}
1	1	0	0	0	
1	1	0	1	1	m_{13}
1	1	1	0	0	
1	1	1	1	1	m_{15}

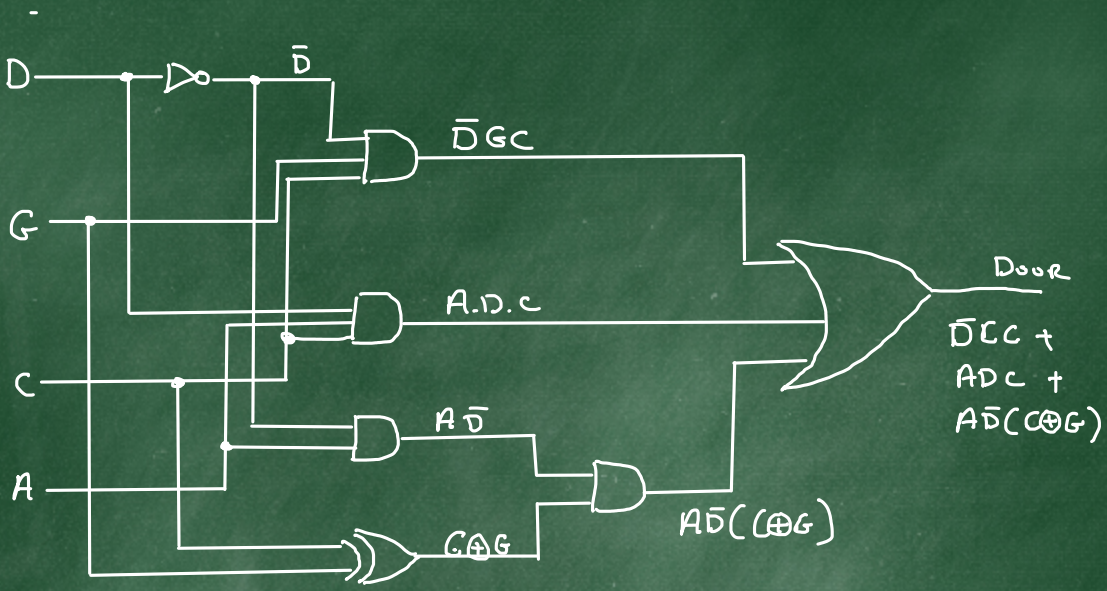
$$\text{Door} = m_3 + m_9 + m_{10} + m_{11} + m_{13} + m_{15}$$

$$= \bar{A}\bar{D}GC + A\bar{D}\bar{G}C + A\bar{D}G\bar{C} + A\bar{D}GC + AD\bar{G}\bar{C} + ADGC$$

optimize:

$$\bar{D}GC(\bar{A}+A) + ADC(\bar{G}+G) + A\bar{D}(\bar{G}C+G\bar{C})$$

$$= \bar{D}GC + ADC + A\bar{D}(G \oplus C)$$


D
G
C
A
$\bar{D}$
$\bar{D}GC$
A.D.C
$A\bar{D}$
C⊕G
$A\bar{D}(C\oplus G)$
Door
$\bar{D}CC$ +
ADC +
$A\bar{D}(C\oplus G)$