









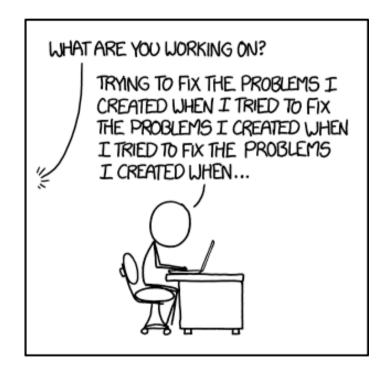
CS1117 – Introduction to Programming

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A TRADITION OF INDEPENDENT THINKING



Problems, problems, problems...





- Recursion: The definition of an operation in terms of itself.
 - Solving a problem using recursion depends on solving smaller occurrences of the same problem.

Recursive Programming:

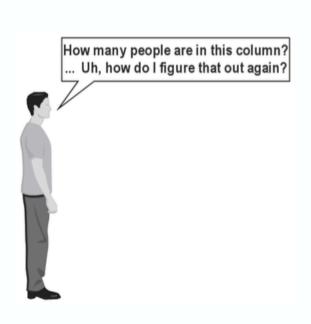
- Writing functions that call themselves to solve problems.
- We already know that in Python functions can call other functions.
- Now it's also possible for a function to call/invoke itself?



- Many functional programming languages (e.g. Haskell) use recursion only (no loops).
- A different way of thinking about problems
- + Leads to elegant, simplistic and short code (when used well)
- + Can solve certain types of problems better than iteration
- Can be slower, performance isn't always as efficient as an iterative solution.

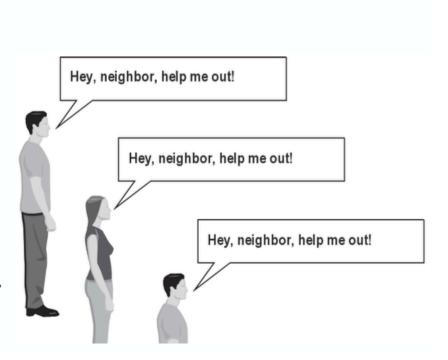


- Question: How many students in total are behind you in your "column" of the classroom?
- Assume you have poor eyesight and can't simply turn around and count them all.
- You can however see the person behind you.
- How do we solve the problem recursively?



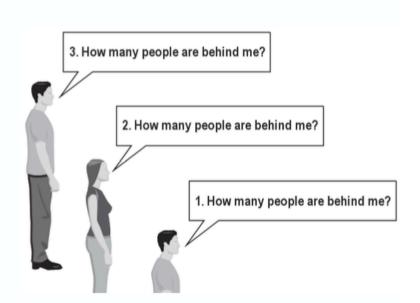


- Recursion breaks a bigger problem into smaller occurrences of the same problem
- Each person solves a small part of the problem
- What's the smallest version of the problem that's easy to answer?
- What info rom a neighbour would help me?





- If there is someone behind me, ask him/her how many people behind him/her
 - When they respond with a value N, I will answer the person in front of me with N + 1
 - -> Recursive Case
- If there's nobody behind me I will answer 0. -> Base Case





- Every recursive algorithm involves at least 2 cases:
 - Base Case: A simple occurrence that can be answered directly.
 - Recursive Case: A more complex occurrence of the problem that cannot be directly answered, but can instead be described in terms of smaller occurrences of the same problem







To solve a problem recursively:

- 1. Break into smaller problems
- 2. Solve smaller sub-problems recursively
- Divide and Conquer

3. Assemble the sub-solutions

```
recursive-algorithm(input) {
    //base-case
    if (isSmallEnough(input))
        compute the solution and return it
    else
        //recursive case
        break input into simpler instances input1, input 2,...
        solution1 = recursive-algorithm(input1)
        solution2 = recursive-algorithm(input2)
        ...
        figure out solution to this problem from solution1, solution2,...
        return solution
}
```



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Divide and Conquer



$$\sum 5 = 5 + 4 + 3 + 2 + 1$$
 OR $\sum 5 = 5 + \sum 4$
 $\sum 8 = 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$ OR $\sum 8 = 8 + \sum 7$

What's the formal definition?

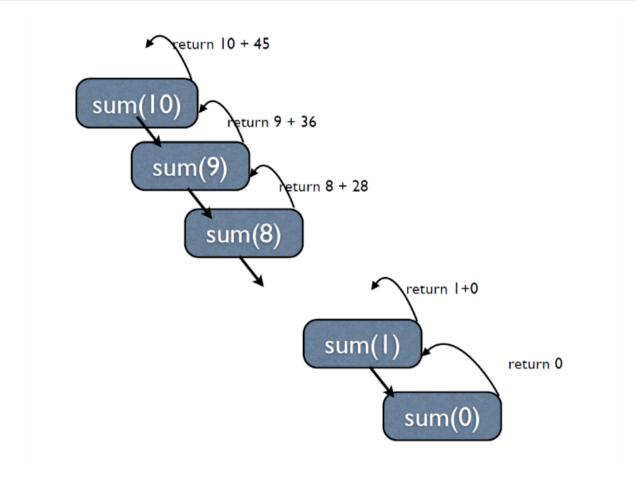


```
def summationI (n):
    sum = 0
    for count in range (1, n, 1):
        sum = sum + count
    return sum
```



```
def summationR (n):
    if n == 0:
        return 0
    else:
        sum = n + summationR (n-1)
    return sum
```





The system keeps track of the sequence of method calls that have been started but not finished yet (active calls)



$$n! = egin{cases} 1 & \text{if } n = 0, & \longrightarrow & \text{Base Line} \\ (n-1)! imes n & \text{if } n > 0 & \longrightarrow & \text{Recursive Case} \end{cases}$$



```
def factorialI (n):
    fact = 1
    for count in range (2, n+1, 1):
        fact = fact * count
    return fact
```



```
def factorialR (n):
      if n == 1:
           return 1
      else:
           return n * factorialR(n-1)
def factorialI (n):
      fact = 1
      for count in range (2, n+1, 1):
            fact = fact * count
      return fact
```



Let's look at some code...





