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CS1113
Quantified Logic and Arguments

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Quantified Logic and Arguments

Interpreting statements with multiple quantifiers

The full language of quantified logic

Formal arguments with quantifiers

Using functions in predicate logic

Sometimes, we will want to make a statement about a function of an object rather than the object itself.

function

Suppose in the UCC database, we define a function *mentor*, which for each student designates a specific individual as a mentor.

  mentor : Students → People

Let *professor*(x) be the statement that x is professor, where x is a person in the UCC database.

predicate

We can now make a claim about the database, there is at least one student who has a professor for a mentor:

$$\exists x \; professor(mentor(x))$$

Terms in Predicate Logic Sentences

U is the universal set (or domain of discourse) of constants
V is a set of variables, such that $V \cap U = \phi$
Φ is a set of function symbols $\Phi \cap V = \Phi \cap U = \phi$

1. Every symbol representing a specific element of U is a term
2. Every variable in V is a term
3. If f is a function symbol ($f \in \Phi$), and t_1, t_2, ..., t_n are terms, then $f(t_1,t_2,\ldots,t_n)$ is a term
4. Nothing else is a term

The language of predicate logic (version 2)

Π is a set of predicate symbols such that $\Pi \cap V = \Pi \cap U = \Pi \cap \Phi = \phi$

1. if P is a predicate symbol and $t_1, t_2, \ldots, t_i$ are terms then $P(t_1, t_2, \ldots, t_i)$ is a well formed formula (wff)
2. If W is a wff, and x is a variable, then $\forall x\ W$ and $\exists x\ W$ are wffs
3. If W_1 and W_2 are wffs, then $\neg W_1$, $W_1 \wedge W_2$, $W_1 \vee W_2$, $W_1 \rightarrow W_2$ and $W_1 \leftrightarrow W_2$ are wffs
4. If W is a wff, then (W) is a wff
5. Nothing else is a wff

Use of function in our examples

If $P(x)$ and $Q(y)$ are predicates, and $f(x)$ is a function, then

$\exists x\, P(f(x))$ **is** a well formed statement in logic

(and says there is at least one value in the domain such that when you apply function f to it, and then check predicate P on the output, it is true)

$\exists x\, P(Q(x))$ is **not** a well formed statement, since you cannot have a predicate symbol as an argument for a predicate.

This is really confusing, especially if you use capital letters for functions, or lower-case for predicate. We will try not to use it in examples, but if we do, you must make it clear what names are functions.

But you will see examples of this in textbooks and in later years ...

Arguments in Predicate Logic

We need to create valid arguments in the same way that we did for propositional logic – e.g. to prove that a solution does actually meet a specification, to prove that a network or database is in a legal state, or to persuade someone that they should believe in a given conclusion.

We will use the same rules of inference as before, but we will add six new ones to handle predicates.

Rules of Inference

$$\frac{\forall x\, P(x)}{P(a) \quad \text{for any value } a \text{ in the domain}}$$

$$\frac{P(a) \quad \text{for some value } a \text{ in the domain}}{\exists x\, P(x)}$$

$$\frac{P(c) \quad \text{for an arbitrary } c \text{ in the domain}}{\forall x\, P(x)}$$

$$\frac{\exists x\, P(x)}{P(c) \quad \text{for some value } c \text{ in the domain}}$$

$$\frac{\begin{array}{l}\forall x\, (P(x) \rightarrow Q(x)) \\ P(a) \quad \text{for some value } a \text{ in the domain}\end{array}}{Q(a)}$$

$$\frac{\begin{array}{l}\forall x\, (P(x) \rightarrow Q(x)) \\ \neg Q(a) \quad \text{for some value } a \text{ in the domain}\end{array}}{\neg P(a)}$$

Example Arguments

All mice like cheese. Mickey is a mouse. Therefore Mickey likes cheese.

Let *mouse(x)* state that *x* is a mouse.
Let *likesCheese(x)* state that *x* likes cheese.

1. $\forall x\ (mouse(x) \rightarrow likesCheese(x))$	initial statement
2. $mouse(Mickey)$	initial statement
3. $likesCheese(Mickey)$	

Example Arguments

Are the following valid arguments?

All students in 2nd year CS at UCC passed CS1107. Enda Kenny is a student in 2nd year CS at UCC. Therefore Enda Kenny passed CS1107.

All students in 2nd year CS at UCC passed CS1107. Enda Kenny passed CS1107. Therefore Enda Kenny is a student in 2nd year CS at UCC.

Next Lecture

Simple Algorithms