Pg1 Home Work 1

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Exercises 1.1 Page 10-12
2, 4, 8, 10, 14, 16, 28, 38, 40, 48, 50, 52, 54

Linear Equations

determine whether the following equations are linear in the variables x and y

2) $3x - 4xy = 0$ Not Linear - on left hand side "$4xy$" has the products of the variables; linear equations cannot.

4) $x^2 + y^2 = 4$ Not Linear - x and y are raised to a power of 2, linear equations cannot have variables w/ exponents.

Parametric Representation

find a parametric representation of the solution set of the linear equation

8) $3x - \frac{1}{2}y = 9$

$x = \frac{1}{3}(\frac{1}{2}y + 9) = \frac{y}{6} + 3$, set y to be free variable

$\boxed{y = t \rightarrow x = \frac{t}{6} + 3 \quad t \in \mathbb{R}}$

10) $12x_1 + 24x_2 - 36x_3 = 12$

we will let x_2 & x_3 be free variables

$x_1 = -2x_2 + 3x_3 + 1$ let $x_3 = t$ & $x_2 = s$

$\boxed{x_1 = 1 - 2s + 3t, \quad s, t \in \mathbb{R}}$

1. Graph 2 lines, find intersections
2. repeat for line pairs
3. What basic types of Solutions are Possible

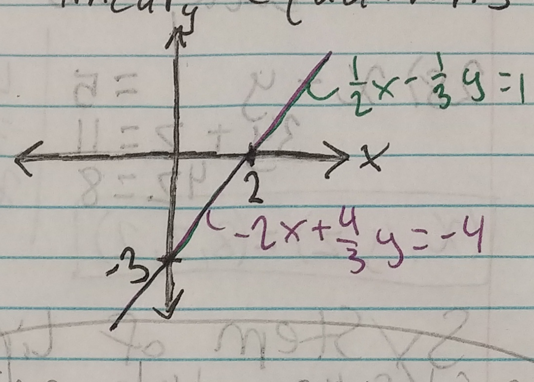
$-4\left(\frac{1}{2}x - \frac{1}{3}y - 1\right) = -2x + \frac{4}{3}y + 4$

Graphical Analysis

Graph system of linear equations. Solve & interpret.

14) $\frac{1}{2}x - \frac{1}{3}y = 1$

$-2x + \frac{4}{3}y = -4$



$x=0 \rightarrow \begin{cases} y = -3 \\ y = -3 \end{cases}$

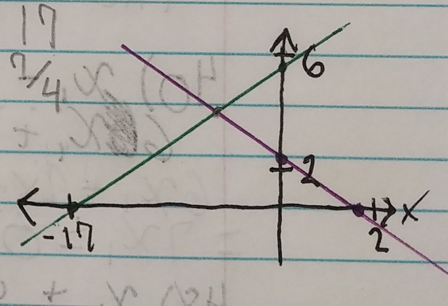
$y=0 \rightarrow \begin{cases} x = 2 \\ x = 2 \end{cases}$

The two equations represent the same line. ∞ many Solutions.

16) $-x + 3y = 17 = E_1$

$4x + 3y = 7 = E_2$

$x=0 \rightarrow y = \frac{17}{3}$, $y = \frac{7}{3}$ $y=0 \rightarrow x = -17$, $x = \frac{7}{4}$



to Solve by elimination we will $E_2 - E_1$

$4x - (-x) + 3y - 3y = 7 - 17$

$5x = -10 \rightarrow x = -2$

$-x + 3y = 17 \rightarrow 3y = 17 + x \rightarrow \frac{17+x}{3} = y = f(x)$

$f(-2) = (17-2)/3 = 5$ $(x,y) = (-2,5)$

equation is linear & intersects @ Point $(-2,5)$ exactly one solation

28 & More $\longrightarrow$

Back-Substitution

use back sub to solve eqn.

28) $x - y = 5$
$3y + z = 11$
$4z = 8$

$4z = 8 \rightarrow z = 2$
$3y + z = 11 \rightarrow y = (11-2)/3 = 3$
$x - y = 5 \rightarrow x = 5 + 3 = 8$

$\boxed{(x, y, z) = (8, 3, 2)}$

System of linear Equations

solve these yo.

38) $3x + 2y = 2 = E_1$
$6x + 4y = 14 = E_2$

$E_2 - 2E_1 = 6x + 4y - 6x - 4y = 14 - 4$
this gives us $0 = 10$, but $0 \neq 10$.
The System $\boxed{\text{has no solution}}$

40) $x_1 - 2x_2 = 0 = E_1$, lets use Elimination by $E_2 + E_1 =$
$6x_1 + 2x_2 = 0 = E_2$

$6x_1 + 2x_2 + x_1 - 2x_2 = 0 + 0$
$= 7x_1 = 0 \rightarrow x_1 = 0 \rightarrow x_2 = 0$

$\boxed{\text{System has one solution } (x_1, x_2) = (0, 0)}$

48) $x + y + z = 2 = E_1$
$-x + 3y + 2z = 8 = E_2$
$4x + y = 4 = E_3$

from E_3: $y = 4 - 4x$
$E_1 = -3x + z = -2$ Eliminate z with $E_2 - 2E_1$
$E_2 = -13x + 2z = -4$
$E_2 - 2E_1 = -13x + 2z + 6x - 2z = -4 + 4$
$= -7x = 0 \rightarrow x = 0 \rightarrow y = 4 \rightarrow z = 2 - 0 - 4 = -2$

$\boxed{(x, y, z) = (0, 4, -2)}$

Back-Sub Cont.

50) $5x_1 - 3x_2 + 2x_3 = 3 = E_1$
$2x_1 + 4x_2 - x_3 = 7 = E_2$
$x_1 - 11x_2 + 4x_3 = 3 = E_3$

I need to eliminate 1 variable, I'll use Subistution of x_3 using E_2
$x_3 = -2x_1 + 4x_2 - 7$. Now let's rewrite sys

$E_1 = 5x_1 - 3x_2 + 2(2x_1 + 4x_2 - 7) = 3$
$E_3 = x_1 - 11x_2 + 4(2x_1 + 4x_2 - 7) = 3$

$E_1 = 9x_1 + 5x_2 = 17$
$E_3 = 9x_1 + 5x_2 = 31$

$\boxed{17 \neq 31 \text{ System is inconsistent, NO SOLUTION}}$

52) $x_1 + 4x_3 = 13 = E_1$
$4x_1 - 2x_2 + x_3 = 7 = E_2$
$2x_1 - 2x_2 - 7x_3 = -19 = E_3$

using E_1 sub x_1 in terms of x_3
$x_1 = -4x_3 + 13$

we can eliminate x_2 with $E_2 - E_3$
$4x_1 - \cancel{2x_2} + x_3 - 2x_1 + \cancel{2x_2} + 7x_3 = 7 + 19$
$E_4 = 2x_1 + 8x_3 = 26 \to 2(-4x_3 + 13) + 8x_3 = 26$
$-8x_3 + 8x_3 = 26 - 26 = 0$. Okay so that didn't help.
From E_4 we know $x_1 = -4x_3 - 13$
So subbing in $E_1 \to -\cancel{4x_3} - 13 + 4x_3 = 13$
andddd we get $0 = 26$, which isn't true in this universe IDK where I went wrong :/ RETRYING!!!

$E_2 - 4E_1 = \cancel{4x_1} - 2x_2 + x_3 - \cancel{4x_1} - 16x_3 = 7 - 52$
$= -2x_2 - 15x_3 = -45$
$E_3 - 2E_1 = \cancel{2x_1} - 2x_2 - 7x_3 - \cancel{2x_1} - 8x_3 = -19 - 26$
$= -2x_2 - 15x_3 = -45$, $E_2 = E_3$

Go to Back

52 cont)

$E_1 = x_1 \quad + 4x_3 = 13$ | now we have 3
$E_2 = E_3 = \quad -2x_2 - 15x_3 = -45$ | vars & 2 eqs,
meaning we have 1 free variable

$x_1 = -4x_3 + 13, \quad x_3 = t$

from E_2 $\quad x_2 = -\frac{15}{2}\cancel{t}x_3 + \frac{45}{2}$

$$\boxed{x_3 = t, \; x_2 = \frac{45}{2} - \frac{15}{2}t, \; x_1 = -4t + 13 \quad t \in \mathbb{R}}$$

54) $x_1 - 2x_2 + 5x_3 = 2 = E_1$
$\quad 3x_1 + 2x_2 - x_3 = -2 = E_2$

free variable

$E_1 + E_2 = 4x_1 + 4x_3 = 0 \Rightarrow x_1 = -x_3 = -t$
$E_1 = x_1 - 2x_2 + 5x_1 = 2 = \cancel{4}x_1 - 2x$
$E_2 = 3x_1 + 2x_2 + x_1 = -2 = \cancel{3}4x_1 + 2x_2 = -2$

$E_2 \Rightarrow 2x_2 = 4x_1 - 2 \Rightarrow x_2 = 2x_1 - 1 = 2t - 1$

$$\boxed{(x_1, x_2, x_3) = (-t, 2t-1, t) \quad t \in \mathbb{R}}$$