Assessing Model Accuracy

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Math 243: Stat Learning

September 4th, 2020

Outline

In today's class, we will. . .

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- Discuss the Mean Squared Error as measure of model accuracy

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- Investigate the Bias-Variance trade-off
- Analyze data from the 'guess my age' activity

Section 1

Mean Squared Error

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where $\hat{f}$ is the model, the x_i are the observed predictor values, and the y_i are the corresponding observed response values. - Under what circumstances is MSE small? - What are the problems with trying to minimize MSE on the set of observed data $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$?

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Use a model-building algorithm that uses **training data** in order to minimize MSE on a large number of previously unobserved **test data** points (x_0, y_0), i.e. minimize

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- If we have training and test data, we can construct a number of models and compare their performance on the test data in order to select the best model

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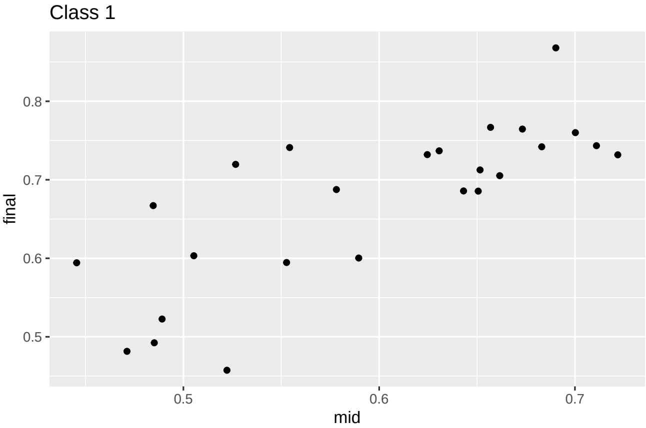
- Use the first class as training data
- Use the second class as test data

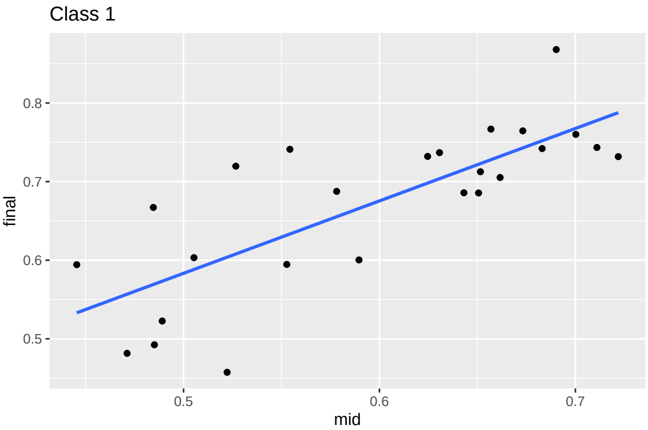
Training Set

```
scores %>% ggplot( aes(x = mid, y = final)) +
  geom_point()+labs(title = "Class 1")
```



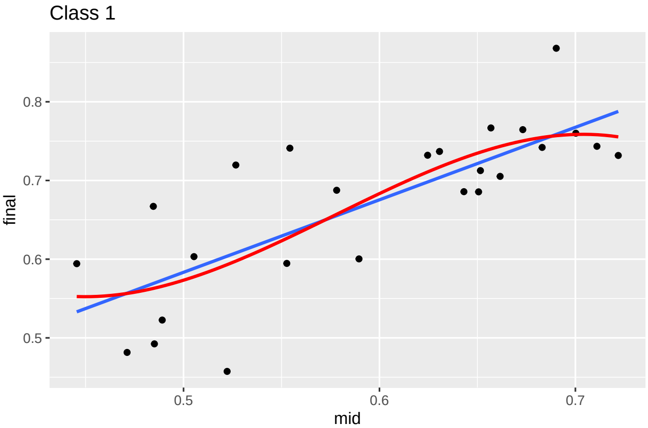
Model 1

```r
##
scores %>% ggplot( aes(x = mid, y = final)) + geom_point()+
  labs(title = "Class 1") +
  geom_smooth( method = "lm" , se = FALSE)
```

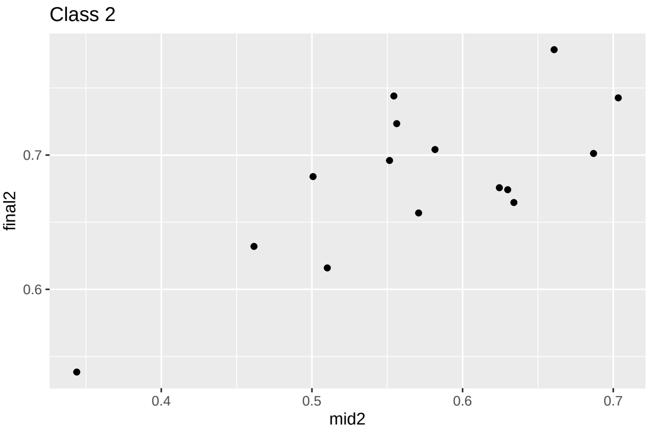


Model 1 and 2

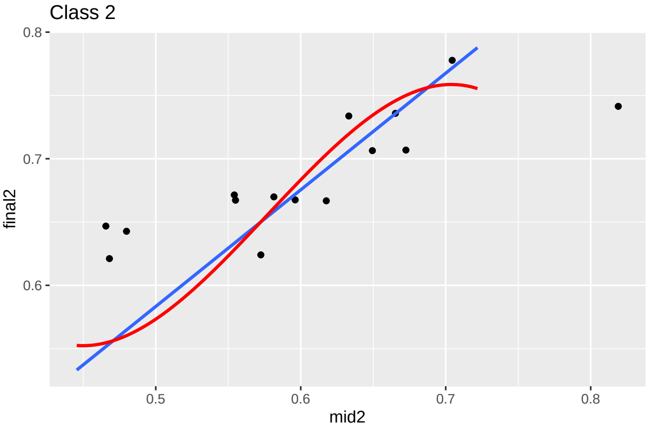
```
scores %>% ggplot( aes(x = mid, y = final)) + geom_point() +
  labs(title = "Class 1") +
  geom_smooth( method = "lm" , se = FALSE) +
  geom_smooth( method = "lm" ,formula = y ~ poly(x, 3), se = FALSE, color = "red")
```



Test Set



Test Set with models



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But no guarantee that model which minimizes MSE on training data will also do so on test data.

In fact, when selecting a complex model that minimizes MSE on the training data, the test MSE will often be very large!

Demo in RStudio

See .Rmd file (Wednesday 9-9 Demo) on the schedule page of the course website

Section 2

Bias-Variance Trade-off

Training vs Test MSE

Suppose we consider a variety of model shapes to predict Y, with each model of increasing complexity. What happens to the training MSE and the test MSE as model complexity increases?

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To minimize MSE, we need to *simultaneously* minimize both variance and bias.

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How do we solve it?