Logistic Regression

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Math 243: Stat Learning

October 25th, 2021

Outline

In today's class, we will. . .

- Review classification problems
- Discuss Logistic Regression for Classification

Section 1

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Classificaiton Problems

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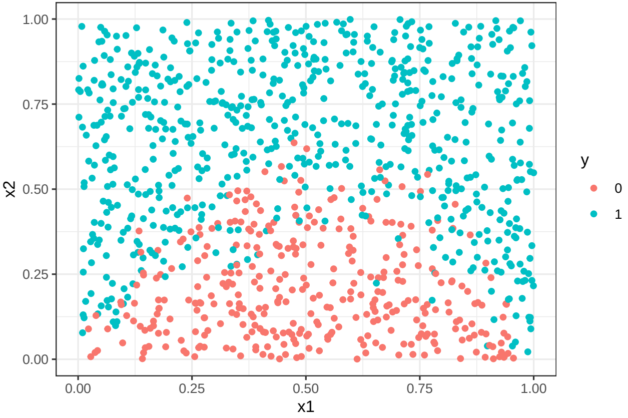
    - Example: Let Y indicate whether it is raining in Portland at noon on 10/25/21.
    - Levels: $A_1 = \text{Raining}$, $A_2 = \text{Not Raining}$.
- Goal: Build a model f to classify an observation into levels $A_1, A_2, \ldots, A_k$ based on the values of several predictors $X_1, X_2, \ldots, X_p$ (quantitative or categorical)

$$\hat{Y} = f(X_1, X_2, \ldots, X_p) \qquad \text{where } f \text{ take values in } \{A_1, \ldots, A_k\}$$

Classification Regions

Any classification model will divide predictor space into unions of regions, where each point in a region will be classified in the same way.



Different models will have different geometries for classification boundaries.

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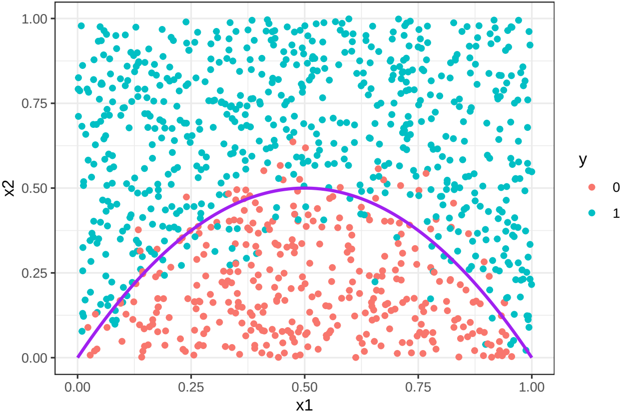

The purple line indicates the optimal decision boundary.

The Bayes Classifier and KNN

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- And our classifier model is $\hat{g}(x_0) = \mathrm{argmax}_{A_j} \hat{P}_j(x_0)$

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5. KNN requires large sample sizes (compared to alternatives)

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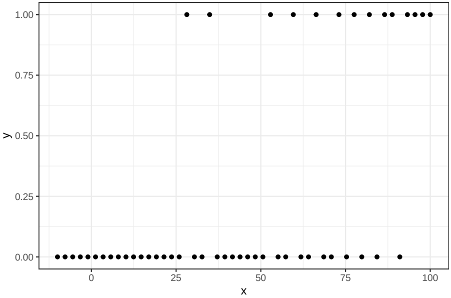

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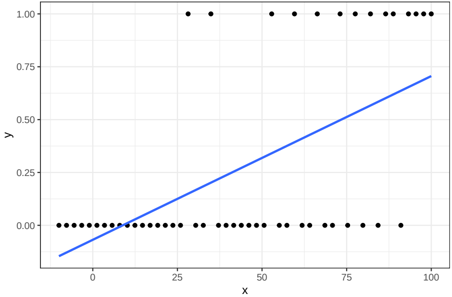

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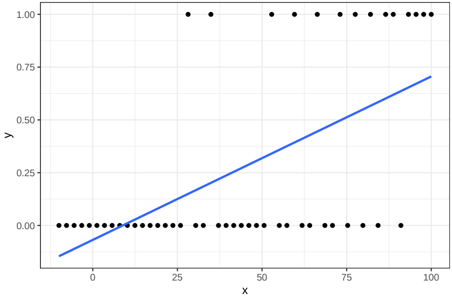

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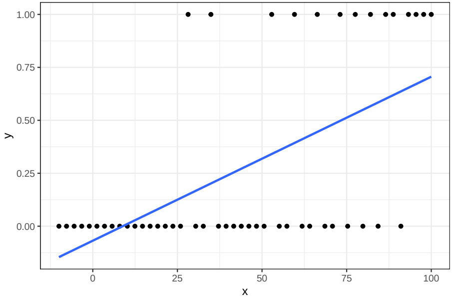

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- Predict 1 if $\hat{P}(x) \geq 0.5$, and 0 otherwise.
  - Solving the linear equation, predict 1 if $X \geq 73.4$

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3. In practice, $p(X)$ is rarely close to linear.

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  - Events that are less likely to happen than not have odds between 0 and 1, while events that are more likely to happen than not have odds between 1 and infinity.
- So instead, we consider log odds:

$$\text{log odds} = \ln \frac{p}{1-p} = \ln p - \ln(1-p)$$

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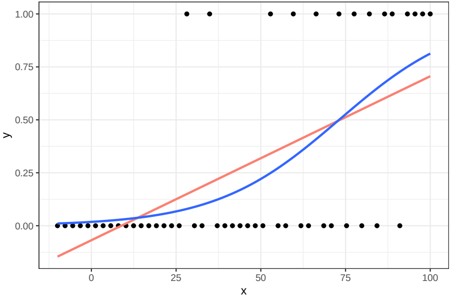

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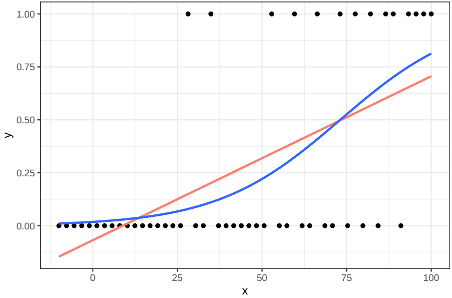

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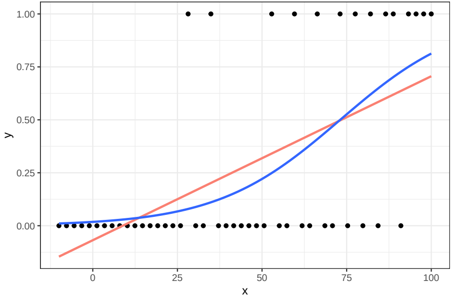

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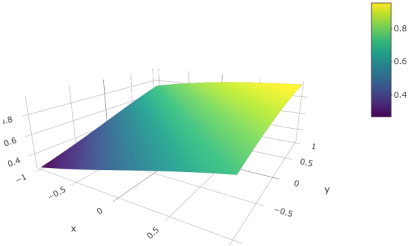

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Regression Coefficient Estimates

- Assume that the log-odds of $Y = 1$ is indeed linear in $X_1, \ldots, X_p$, so that

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- Instead, we use the method of **Maximum Likelihood** (ML)

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  - How? (Calculus or numeric methods, or R!)

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$$\ln \frac{P(Y = 1|X)}{1 - P(Y = 1|X)} = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p$$

- This implies the conditional probability is logistic:

$$P(Y = 1|X) = \frac{e^{\beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p}}{1 + e^{\beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p}}$$

Summary

- In a classification problem, we are interested a categorical response variable Y.
- We might be interested in **predicting** the class for Y based on observations, or we might be interested in **inferring** the relationships between Y and predictors.
- Ideally, we would like to estimate the conditional probability of Y given X

$$P(Y = A_j|X)$$

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- To classify, we assign a test observation the value 1 if

$$P(Y=1|X) = \frac{e^{\beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p}}{1 + e^{\beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p}} \geq 0.5$$