15210: Parallel and Sequential Data Structures and Algorithms

15210 Course Notes

Name: Weihan Li*, andrewid:* weihanl1

1 Introduction

Definition 1.1. Lambda Calculus a variable, a lambda abstraction , written as $(\lambda x\ .\ e)$, or an application , written as $(e1\ e2)$.

2 Work and Span

3 Sequences

4 Algorithm Techniques

4.1 Brute Force and Reduction

Definition 4.1. Brute Force Enumerating all possible **candidate solution** and checking which one is valid.

Remark. Brute Force Techniques are usually parallel, but requires gigantic amount of work.

Definition 4.2. Reduction
We typically think of reduction as a 3 step process:

- Transform the instance of problem A to one or many instances of problem B.
- Solve all instances of problem B.
- Use results to B instances to compute result to the A instance.

Remark. A reduction is **efficient** if the cost bound of A and B are asymptotically the same.

4.2 Divide and Conquer

- Divide
- Recur
- Combine

4.3 Contraction

- Contract and solve
- Expand

Remark. Contraction algorithms can be **work efficient**, if they can reduce the problem size geometrically (by a constant factor greater than 1) at each contraction step, and if the contraction and the expansions steps are efficient.

Contraction algorithms can be **span efficient**, if size of the problem instance decreases geometrically, and if contraction and expansion steps have low spans.

Example 4.1. **Span with contraction**
Scan can be viewed as applying a reduction to every prefix of the sequence and returning the results of such reductions as a sequence. Note: f is associative.

5 Randomization

Definition 5.1. Symmetry breaking An algorithm's ability to distinguish between choices that otherwise look equivalent.

Example 5.1. Select a subsequence of a sequence s.t. no two adjacent elements are selected.
Let L be the length of the subsequence. Then $E[L] = \frac{n}{4}$ where $n = |S|$.

Definition 5.2. Las Vegas Algorithm Correctness ensured with a low expected cost bound.

Definition 5.3. Monte Carlo Algorithm Cost bound ensured with a high expected correctness.

5.1 Complex analysis

Remark. Expected bounds Average case behavior

Definition 5.4. Probability bounds A bound $O(n^2)$ holds with high probability bounds if it holds with probability $p(n)$ when $\lim_{n\to\infty} p(n) = 1$.

Remark. Determining expected work is easier than expected span, because $E[X+Y] = E[X]+E[Y]$, but $E[\max(X,Y)] \neq \max(E[X], E[Y])$.

Remark. $(1 - \frac{1}{n})^n \leq \frac{1}{e}$.

Example 5.2. $E[S(\text{lucky})] = \Theta(1), E[S(\text{unlucky})] = \Theta(n)$.

$$E[S] \leq p(\text{lucky})\Theta(1) + \frac{1}{n^2}\Theta(n) \leq \Theta(1)$$