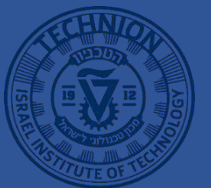


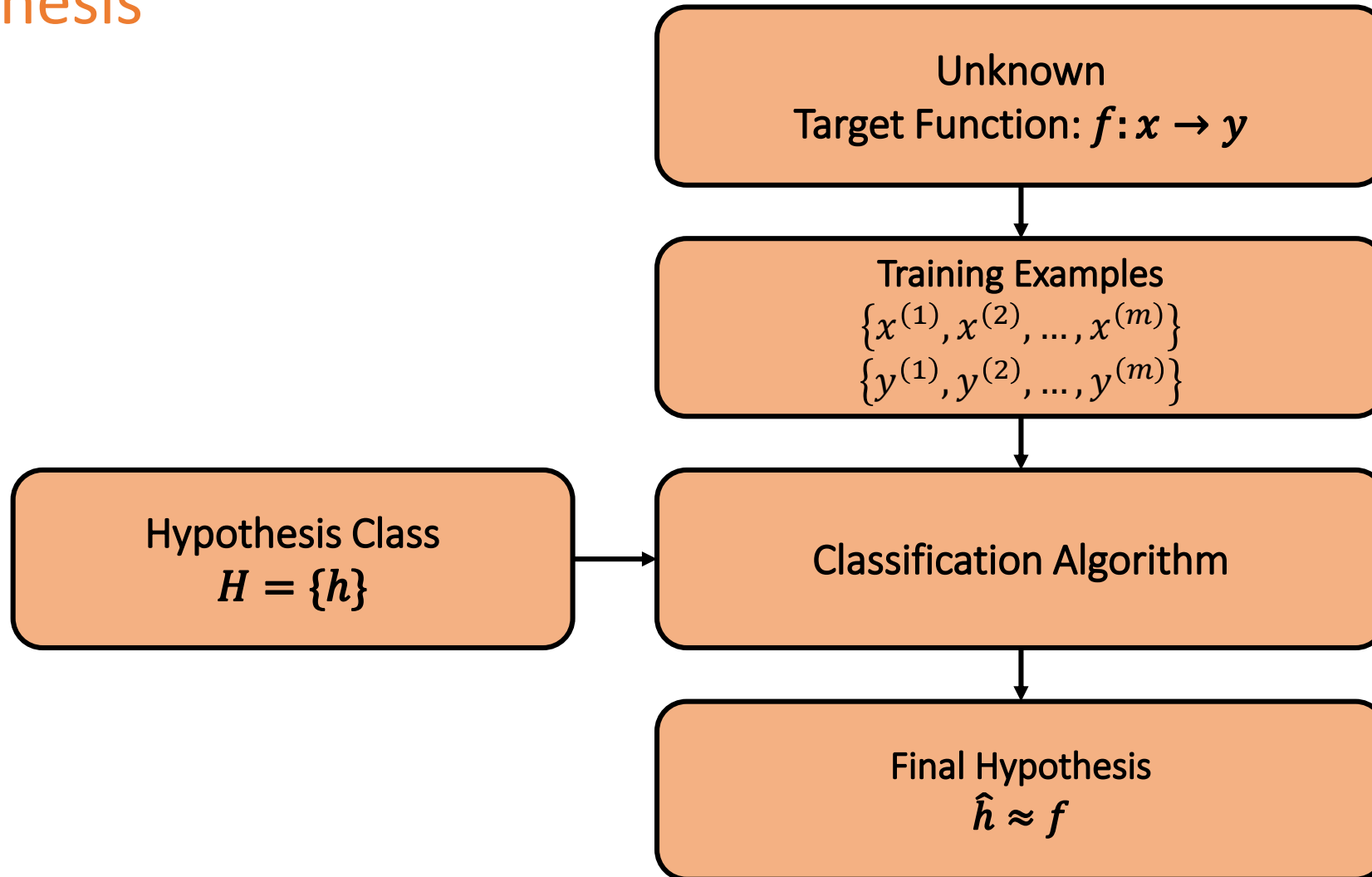
#L07-Practical consideration on training a model

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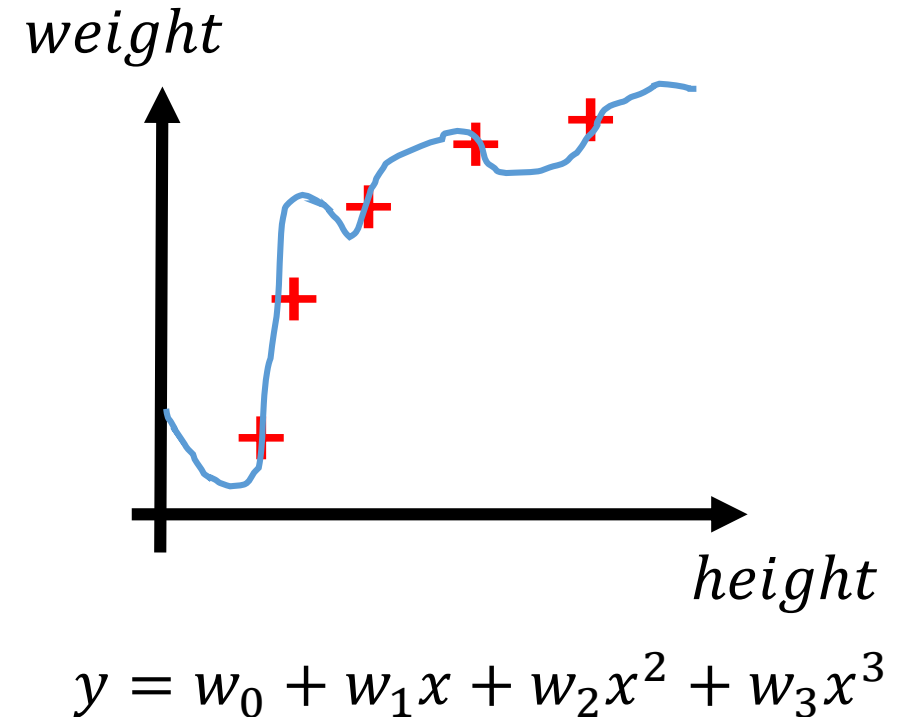
Hypothesis



Evaluating a model

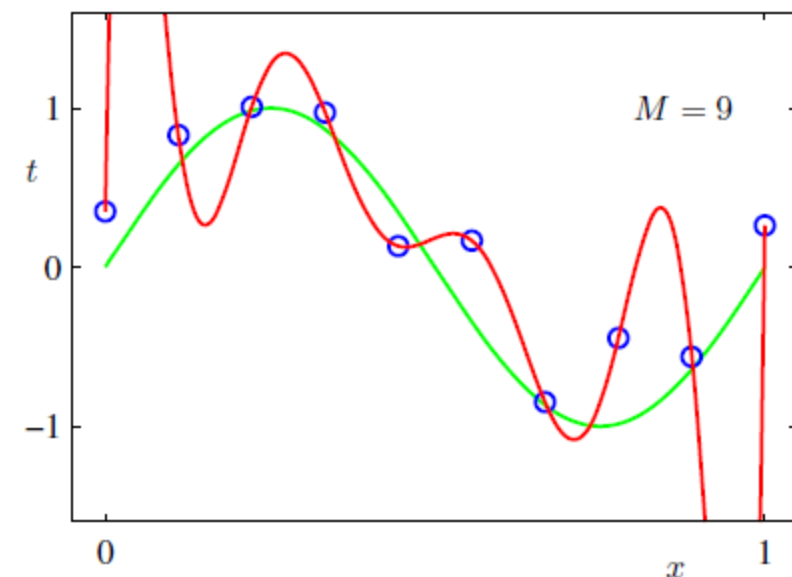
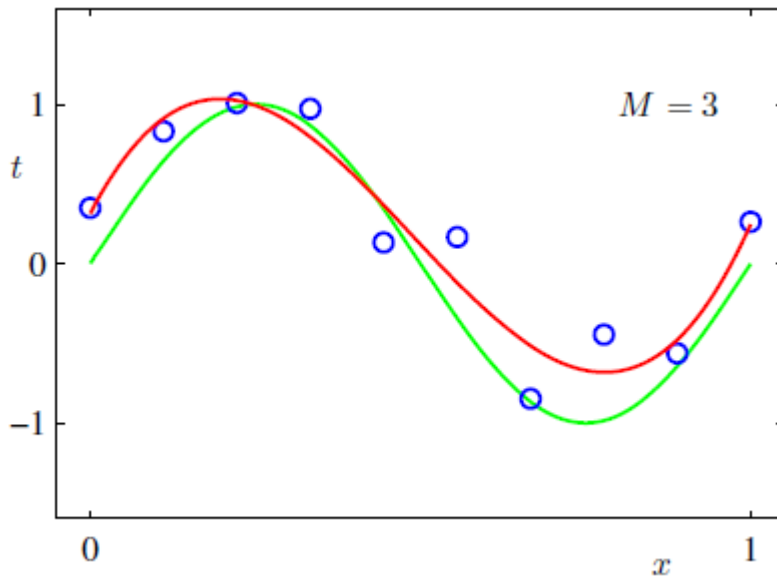
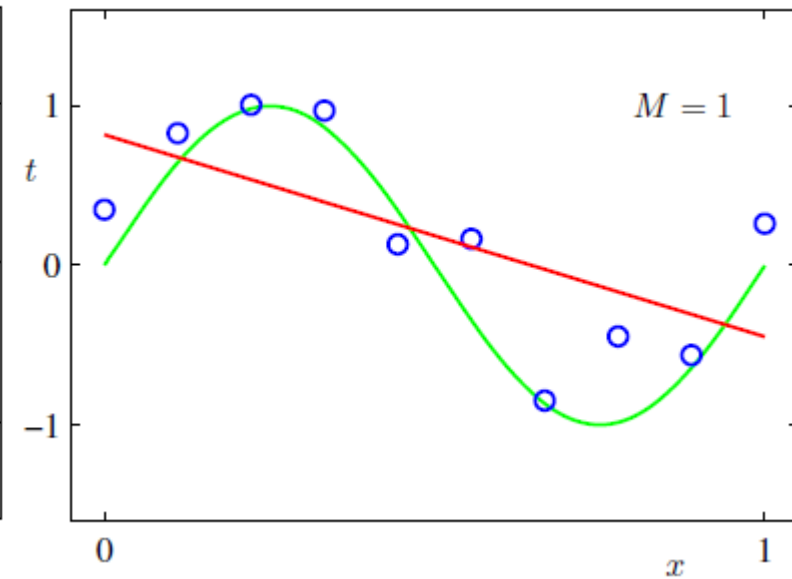
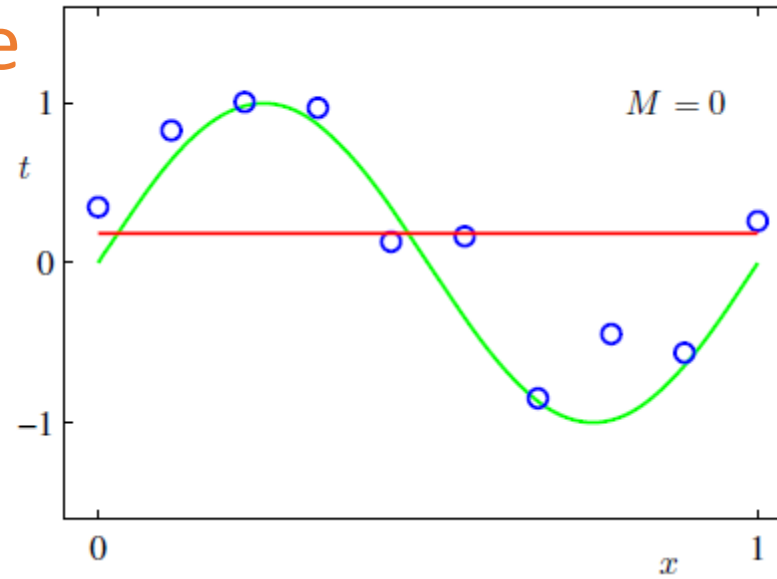
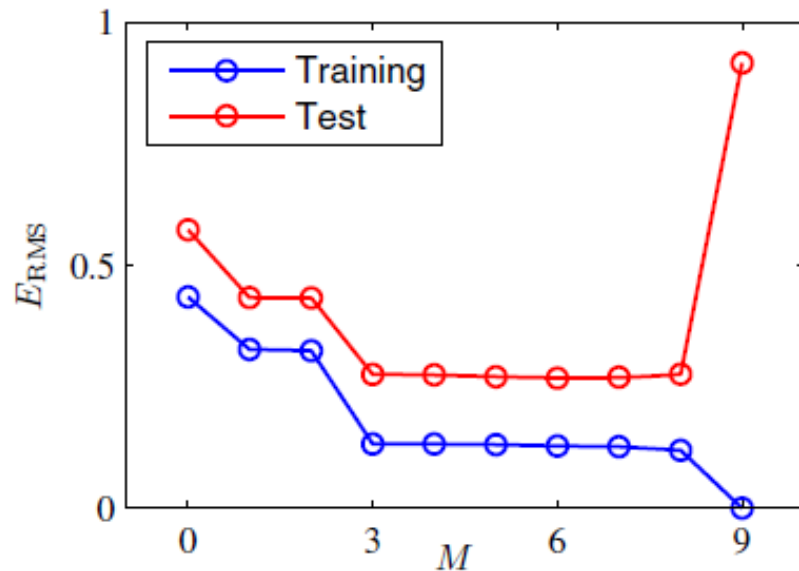
Overfitting

- You trained a model with its $J \rightarrow 0$. You feel very proud!
- Then you go out in the real world and start making predictions. Surprise, results are not as expected! What happened?
- Very likely your model is overfitting the training examples leading to bad generalization.



Reminder: bias-variance

- Under fitting (high bias),
- Overfitting (high variance).

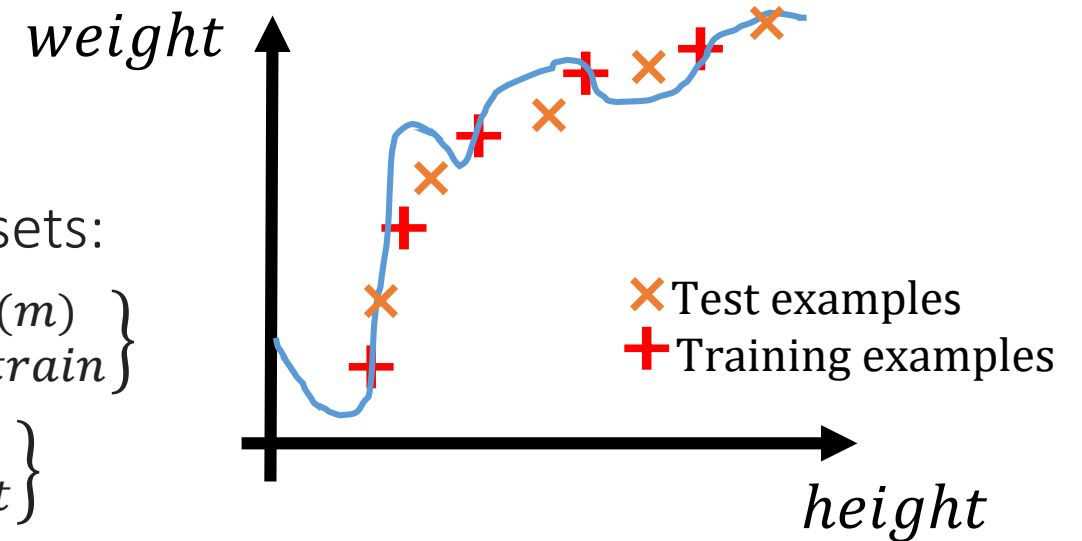


Test set

- How can we evaluate our model?
- Training set** and **Test set**.
- We divide our dataset into training and test sets:

$$\left\{x_{train}^{(1)}, x_{train}^{(2)}, \dots, x_{train}^{(m)}\right\}, \left\{y_{train}^{(1)}, y_{train}^{(2)}, \dots, y_{train}^{(m)}\right\}$$
$$\left\{x_{test}^{(1)}, x_{test}^{(2)}, \dots, x_{test}^{(p)}\right\}, \left\{y_{test}^{(1)}, y_{test}^{(2)}, \dots, y_{test}^{(p)}\right\}$$

- Typically ~70% training and ~30% test.
- Going back to our history from slide 1. What would have happened if we had used a training and test set?
 - E.g. $J_{train} \rightarrow 0$ and $J_{test} \rightarrow 2.3$
 - We would have observed a gap between the training and test errors!



$$y = w_0 + w_1x + w_2x^2 + w_3x^3$$

Diagnosing a model

- Say your model does not get the performance that you were expecting.
 - Do you need more data?
 - Is your model too simple/complex?
 - Do you need to regularize it?
 - Do you need to design new features?
- How do you figure out where to invest your time?

Model selection

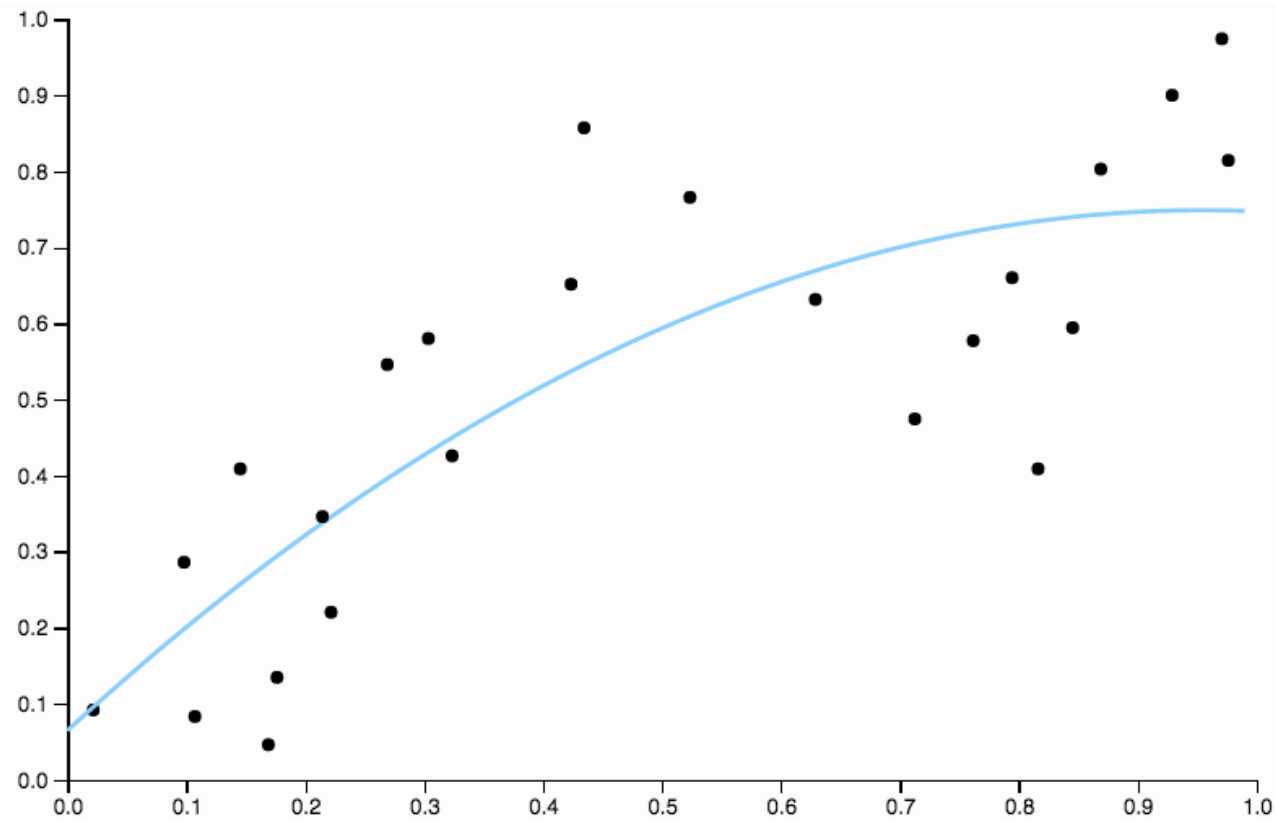
Diagnosing a Model

- Consider we evaluated our model and it does not generalize well i.e. $J_{Test} \gg J_{Train}$.
- How can we improve?
 1. Degree of the polynomial d ? \rightarrow complexity of the hypothesis class.
 2. Value of the regularization parameter λ ? \rightarrow dealing with overfitting.
 3. Do we need to increase the amount of data m ? \rightarrow learning curve.
- We will see how to explore which avenue is the most promising. But first let's clarify again the difference between the notion of **model complexity** and **regularization**.

**Tradeoff
bias-variance**

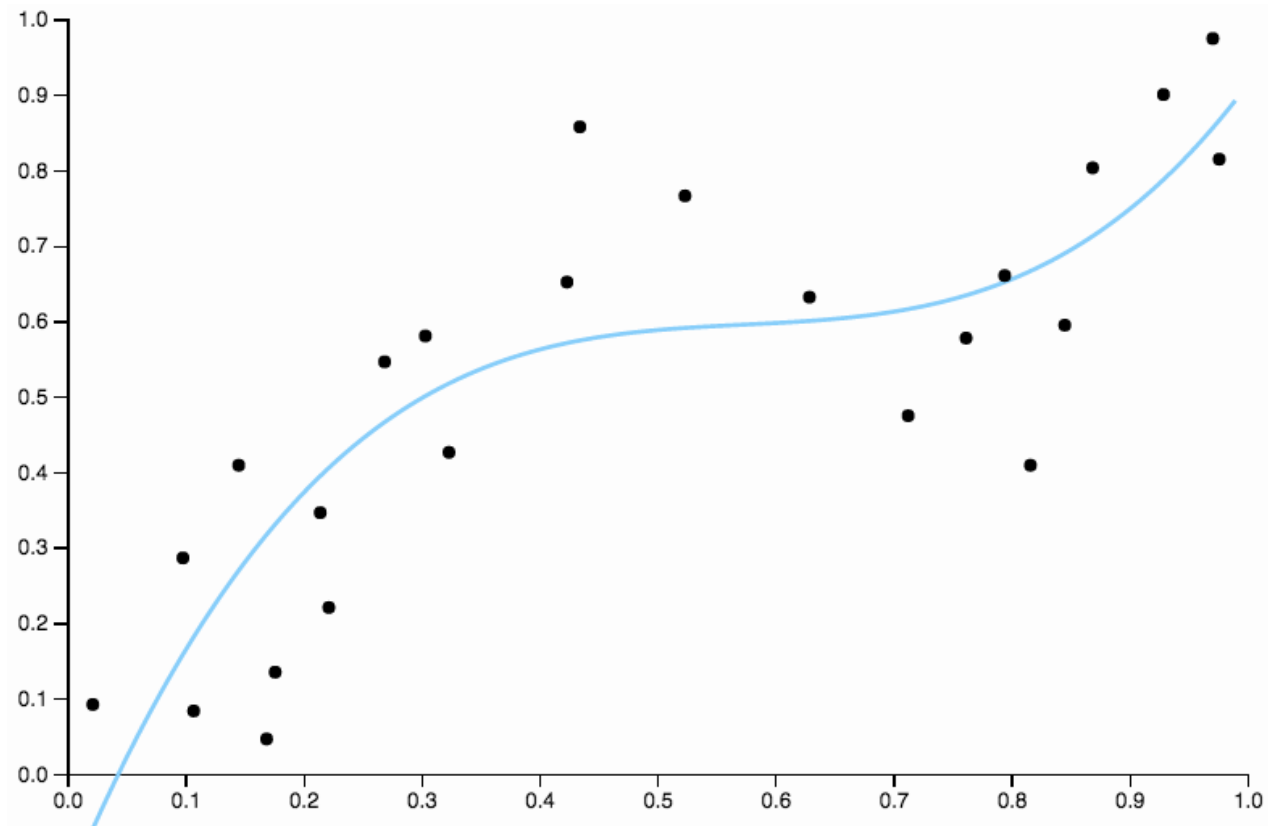
Tradeoff bias-variance

- Low d , model is too simple.



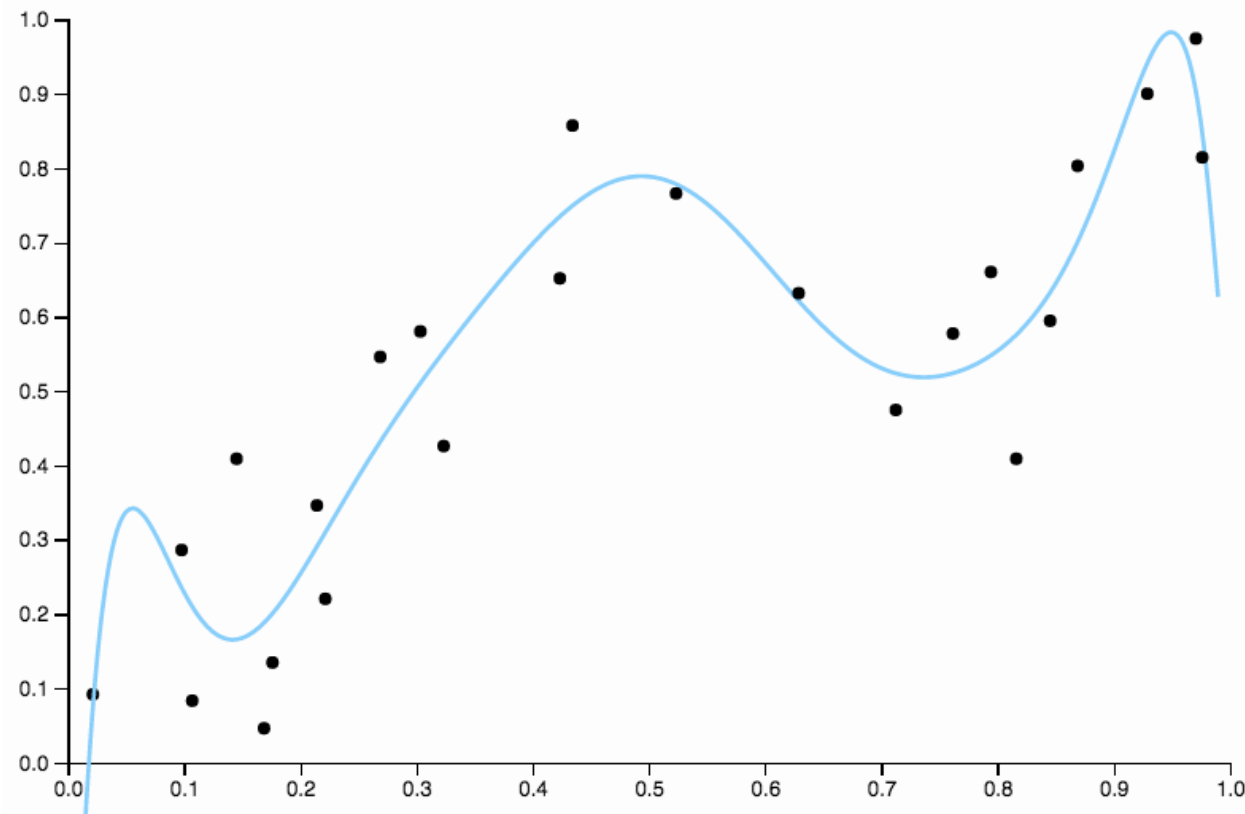
Tradeoff bias-variance

- Increase d . Better, still not enough!



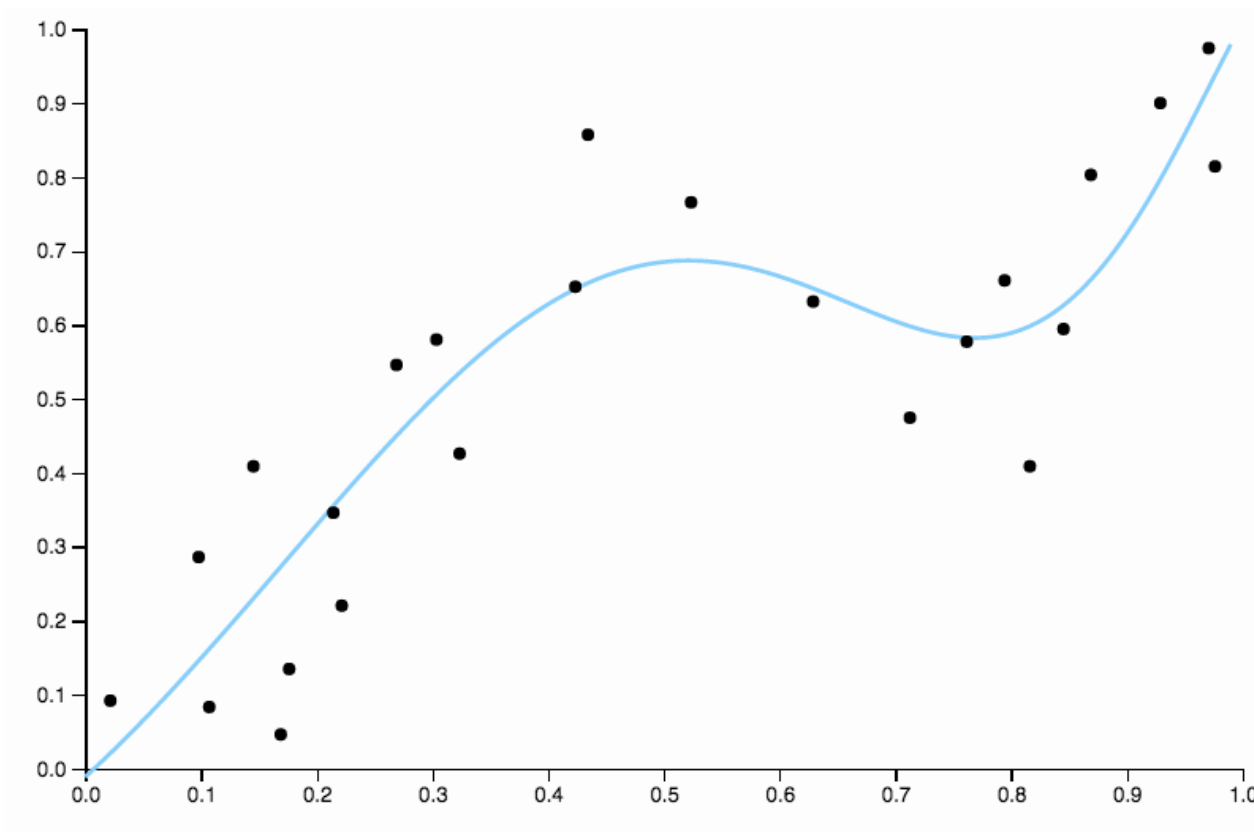
Tradeoff bias-variance

- We increase more d . But we now get overfitting!



Tradeoff bias-variance

- We keep the same d but now use some regularization. Sweet spot!





Tradeoff bias-variance

- Choose d so that the hypothesis class is about right.
- Then use regularization to use the power of a more complex model while not using it to its “full extend”.
- This will provide you with a **tradeoff between bias and variance**.

Diagnosing a model

- Consider we evaluated our model and it does not generalize well i.e. $J_{Test} \gg J_{Train}$.
- How can we improve?
 1. Degree of the polynomial d ? \rightarrow complexity of the hypothesis class.
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 3. Do we need to increase the amount of data m ? \rightarrow learning curve.

**Tradeoff
bias-variance**

Validation set

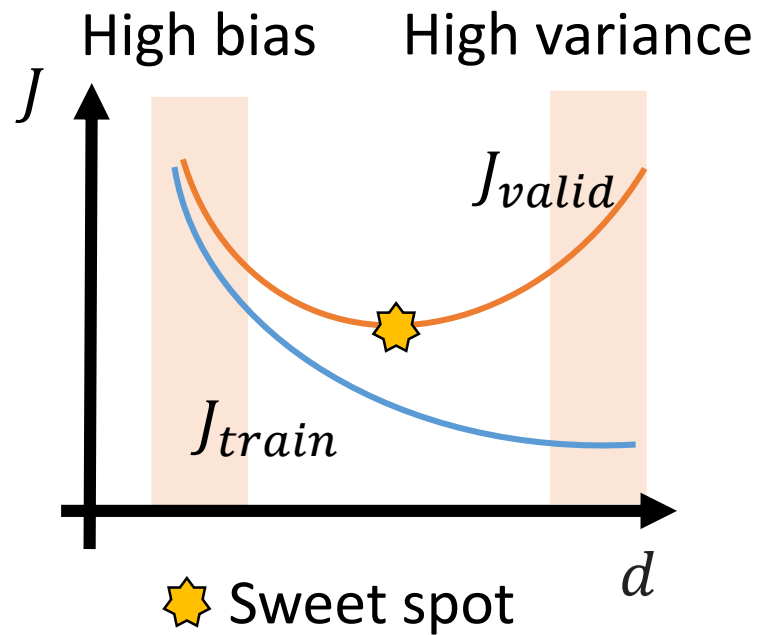
- Degree of the polynomial d ? → complexity of the hypothesis class.
 - $h_w(x)$ assumed to be a polynomial of degree d .
 - Compute $J_{train}(w)$ for different values of d .
 - Select d based on best performance on the test set $J_{test}(w)$.
 - However, how do we ensure good generalization? Indeed, we just used the test set to tune our model.
 - We call this problem information leakage.

Validation set

- Degree of the polynomial d ? → complexity of the hypothesis class.
 - We need to divide our model in three parts (instead of two like before):
 - Training, validation and test sets.
 - A typical split is 60%-20%-20%.
 - We use the validation set to tune the d parameter (by minimizing $J_{valid}(w)$) and reporting the final error of the model on the test set $J_{test}(w)$.
 - This is the good practice!

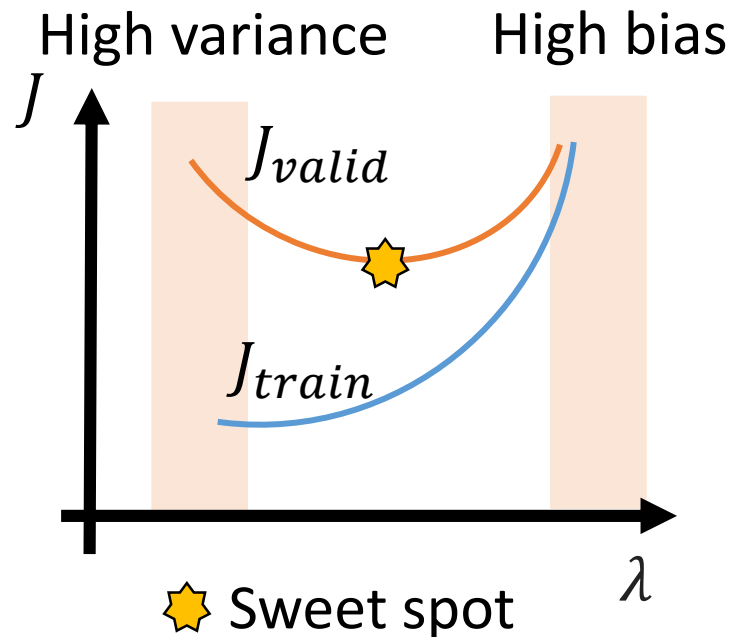
Validation set

- Degree of the polynomial d ? → complexity of the hypothesis class.



Validation set

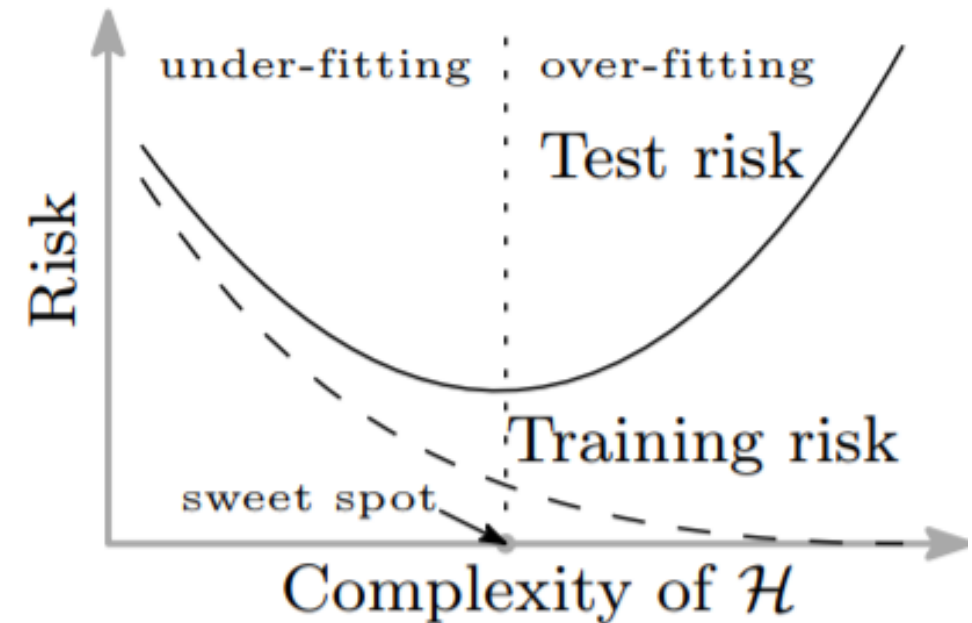
- Value of the regularization parameter λ ? → dealing with overfitting.



- $J(w) = \frac{1}{2m} \left[\sum_{i=1}^m (h_w(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n w_j^2 \right]$
 - Blue: the regularization term.
 - λ : Regularization parameter. It controls the tradeoff bias-variance.
 - $\lambda \rightarrow 0$: no regularization.
 - $\lambda \rightarrow \infty$: underfitting ($h_w(x) = w_0$).

Bias-variance tradeoff

- The control of the **function class complexity** is used to balance between bias and variance and find the **sweet spot** that is a **good tradeoff**.
- The control of the function class complexity may be **explicit via the choice of the hypothesis class** or implicitly through regularization.



(a) U-shaped “bias-variance” risk curve

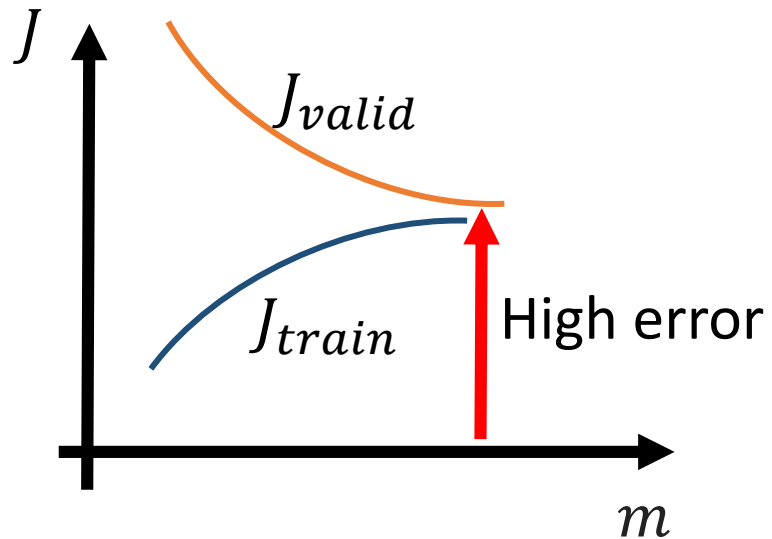
Learning curves

Learning curves

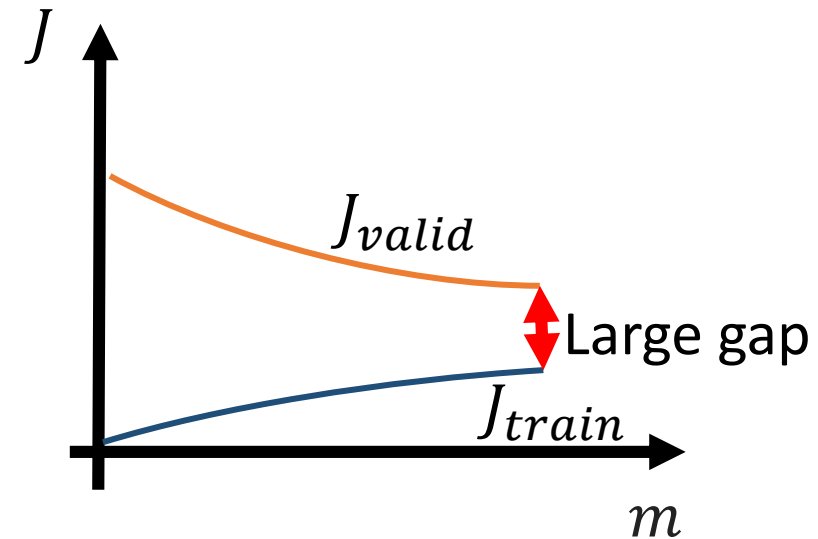
- Do we need to increase the amount of data m ? → learning curve
 - Do we have enough examples for building our model?
 - **Learning curve:** graph that is used to compare the performance of a model on training and validation datasets over a varying number of training examples.
 - Performance will generally improve as the number of training examples increases.
 - We want to graph $J_{train}(w)$ and $J_{valid}(w)$ as a function of m .

Learning curves

- Do we need to increase the amount of data m ? → learning curve



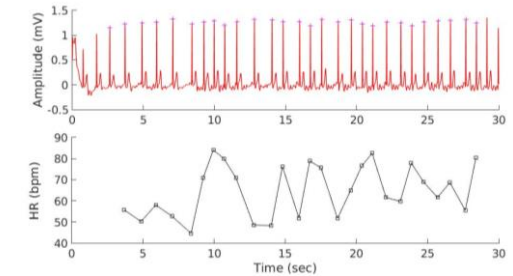
Diagnosis: High bias
→ Getting more data will not help.



Diagnosis: High variance
→ Getting data might improve.

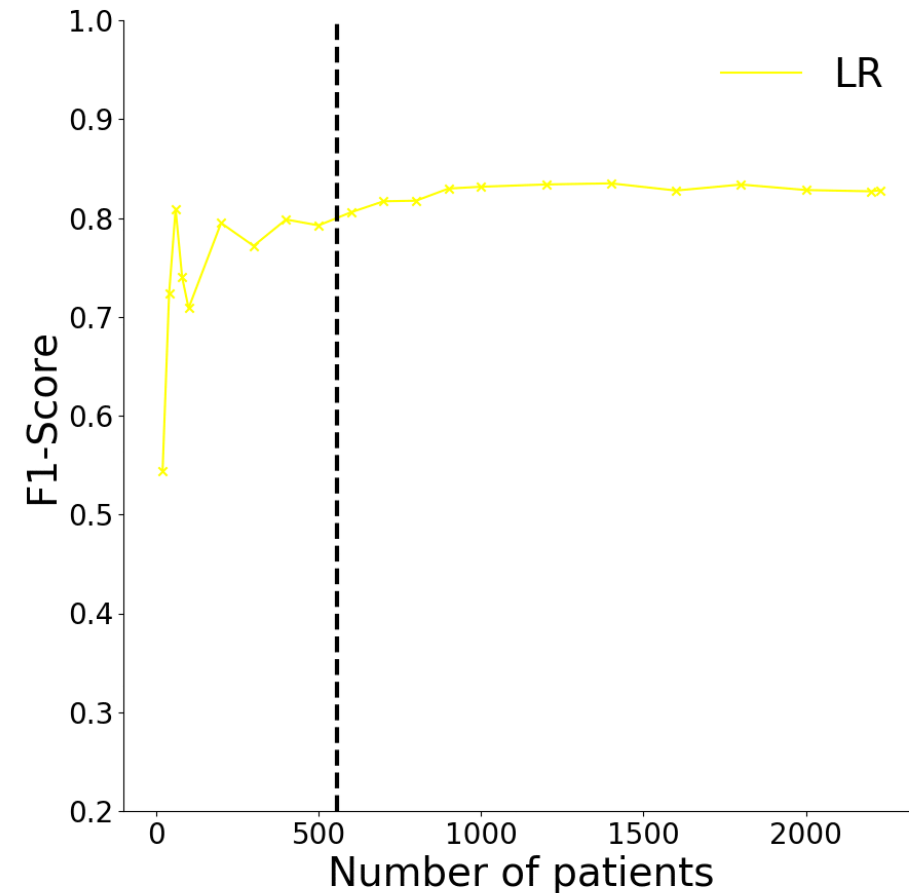
Learning curves: example

- Binary classification: AF, non-AF.
- A total of $n=2,891$ unique patients recording.
 - 2,612 non-AF and 279 AF.
- Each recording is 24h long, totaling 68,800h.
- Model elaboration:
 - 80-20% train-test set split.
 - 5-fold cross validation on the training set.



Learning curves: example

- We create the learning curve while adding patients one by one.
- When adding patients we alternate with AF and non-AF so that we have 50-50%.
- We do this until we exhaust the AF examples - dotted black vertical line.
- Will we gain by increasing the number of patients?



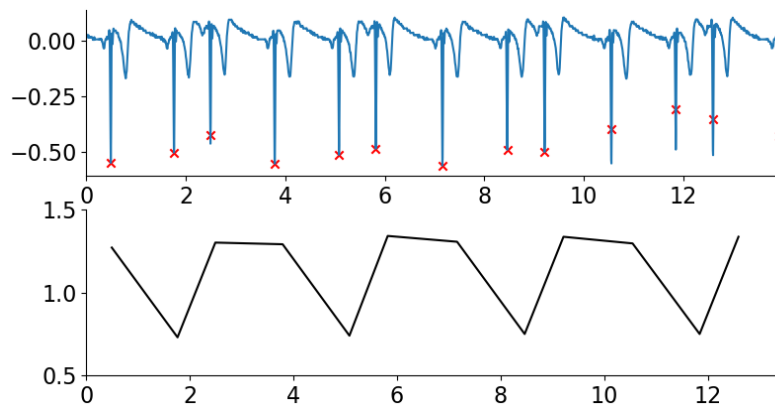
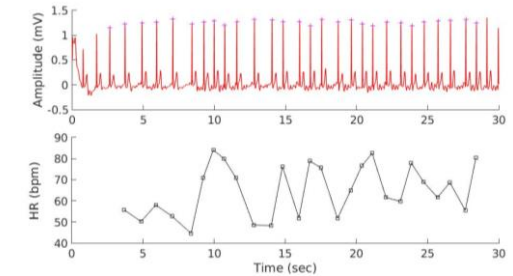
Error analysis

Error analysis

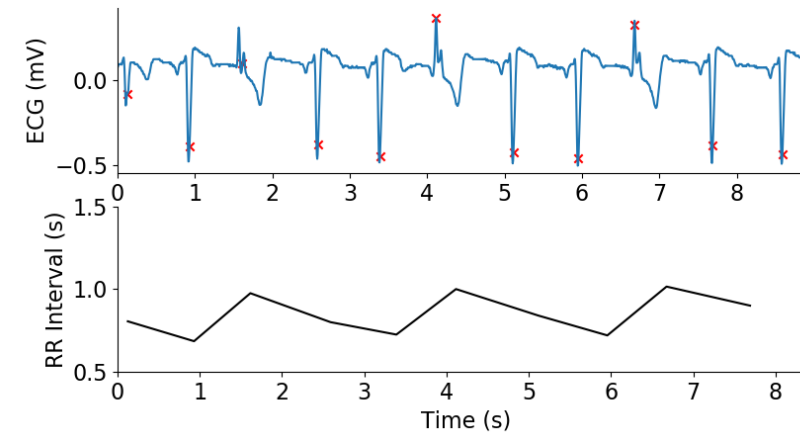
- What if the problem actually comes from the fact that the features are not descriptive enough/missing some characteristic of the data?
- Manually examine the examples in the validation set that your algorithm classify incorrectly. This is called “**error analysis**”:
 - Look for systematic trends the algorithm is making errors on.
 - Try to categorize.

Error analysis

- E.g. Atrial fibrillation:
 - Is the misclassification due to the presence of noise?
 - Some specific non-AF rhythms that we do not learn well?
- We can start by looking at single examples:



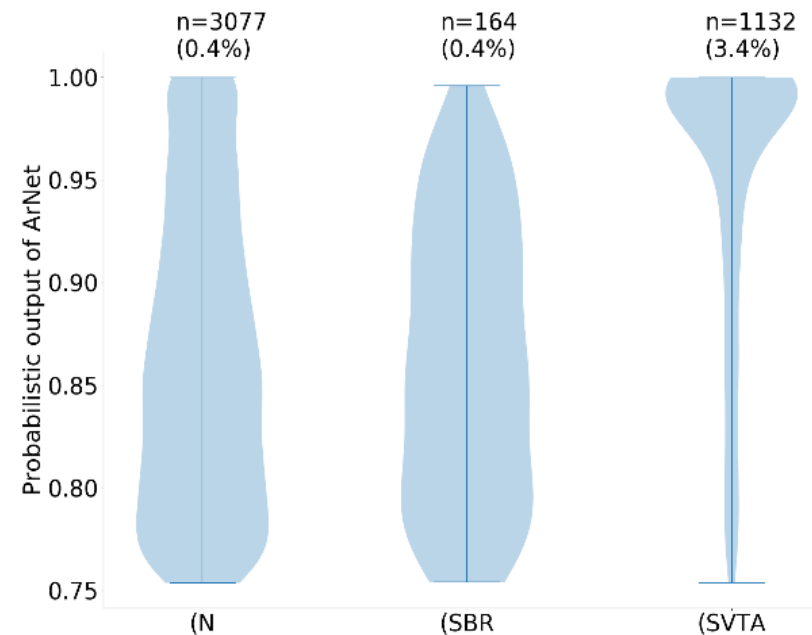
Atrial premature contractions



Trigeminy

Error analysis: examples

- Aggregation of examples:



- If you spot that some specific type of recurrent error then you can look at:
 - Adding features that may discriminate them from the other examples.
 - Augmenting their representation in the dataset if they are not well represented.

Generalization performance

Generalization performance

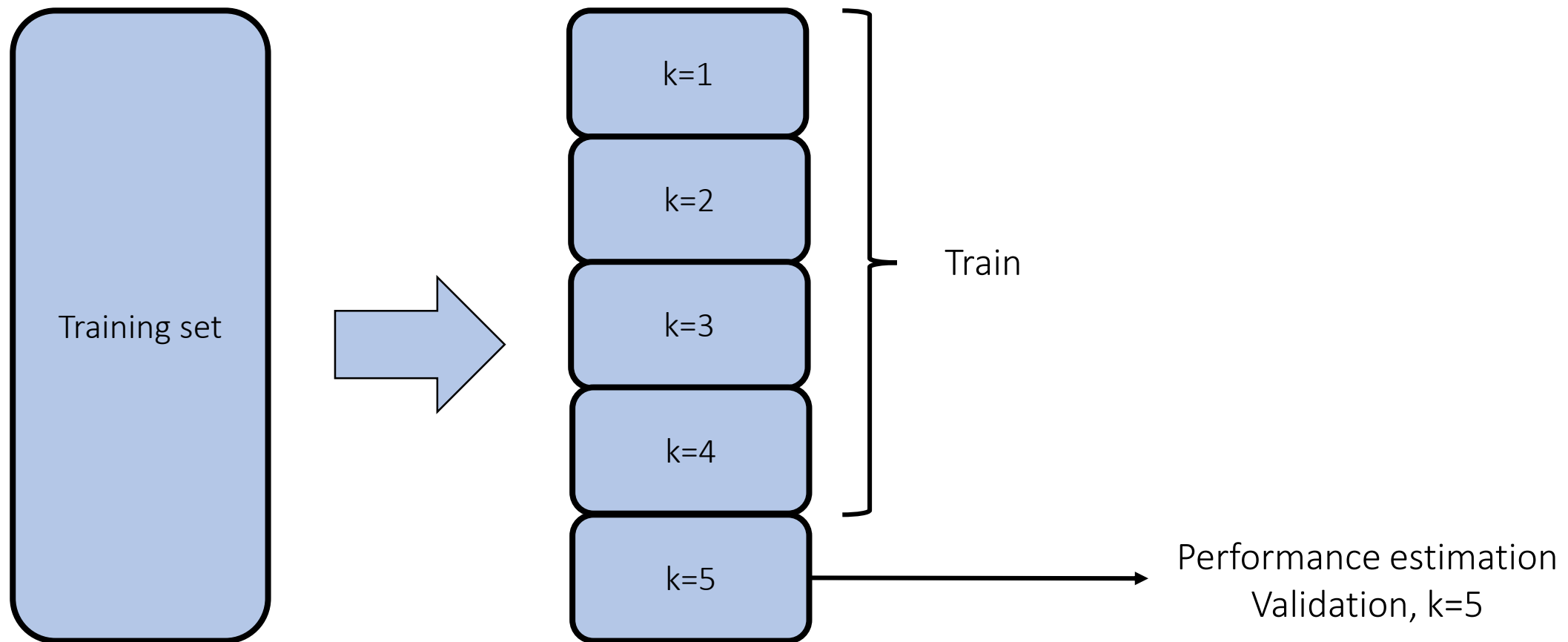
- **Generalization performance** relates to how accurately the algorithm is able to predict outcome values for previously unseen data.
- In order to set our hyperparameters we created a validation set to assess how the model will generalize to an independent dataset. This is called **cross-validation**.
- However, in practice **we want to use multiple rounds of cross-validation** to reduce variability. It will provide a more accurate estimate of model prediction performance.
- How do we do that?

Generalization Performance

- Recall: for training our model we:
 - Use a **training set** from which the model learns.
 - Use a **validation set** that enables us to perform hyperparameters selection.
 - Use a **test set** which we use to assess the final model performance. This set is not used to train the model or tune its hyperparameters.
- How can we assess **generalization performance** to best decide on our hyperparameters?

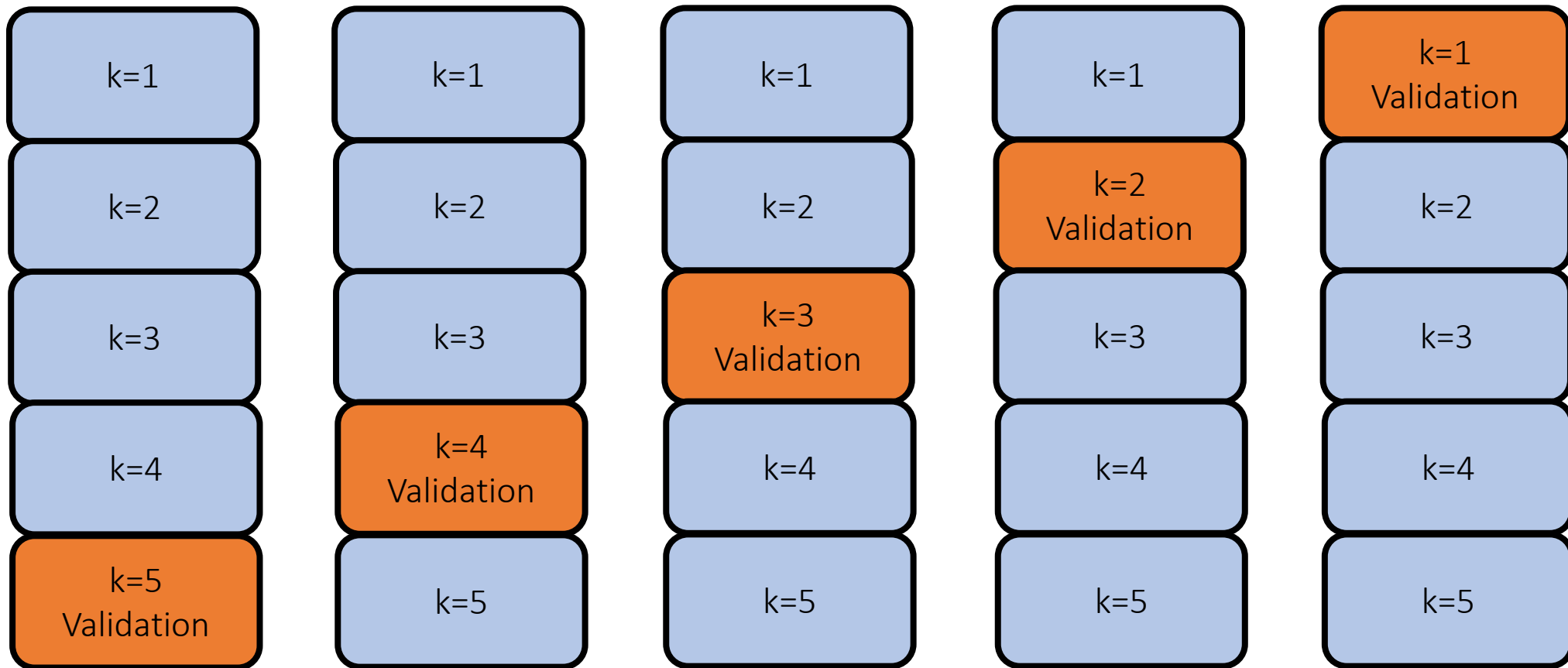
K-fold cross validation

- Divide the training set into k folds. Say here $k = 5$



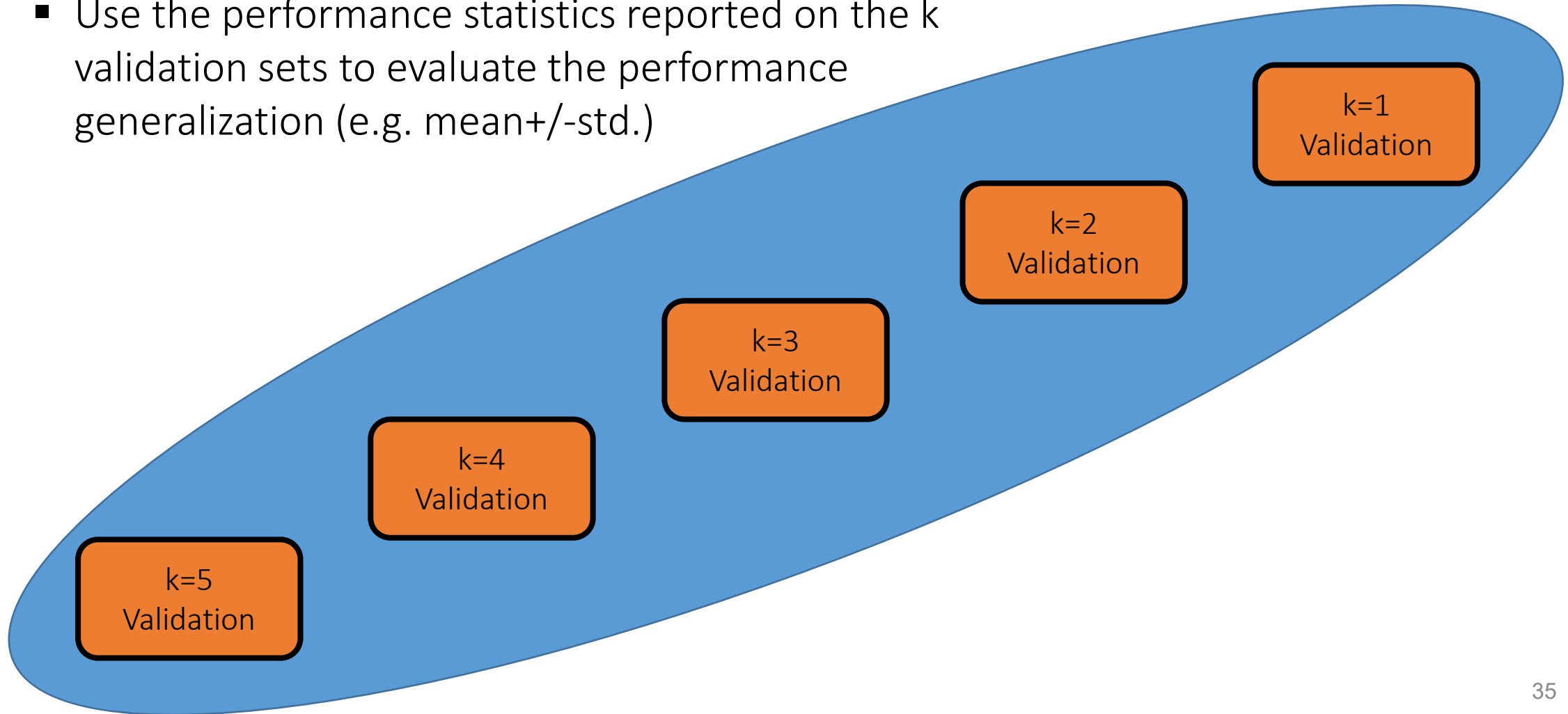
K-fold cross validation

- Repeat on each of the k folds. Each time consider 4 as training and 1 as validation:



K-fold cross Validation

- Use the performance statistics reported on the k validation sets to evaluate the performance generalization (e.g. mean \pm std.)



Cross validation

- Cross validation has different flavors.
 - K-fold cross validation.
 - What we just saw.
 - Leave-one-out cross-validation.
 - Can be computationally expensive.
 - Repeated random sub-sampling validation.
 - May not cover all the training examples if not enough iterations.
- **Important note:** to be able to compare performance statistics accross multiple experiments, the split train-validation-test must be the same – write a script to be able to reproduce this exact split.

Table 1

Study database. The diagnosis is based on the ICSD-3 and AASM 2017 guidelines and using the recommended rule for hypopnea.

Diagnosis	Number	Percentage (%)
Non-OSA	503	56.7
Mild OSA	206	23.2
Moderate OSA	103	11.6
Severe OSA	75	8.5
Total	887	

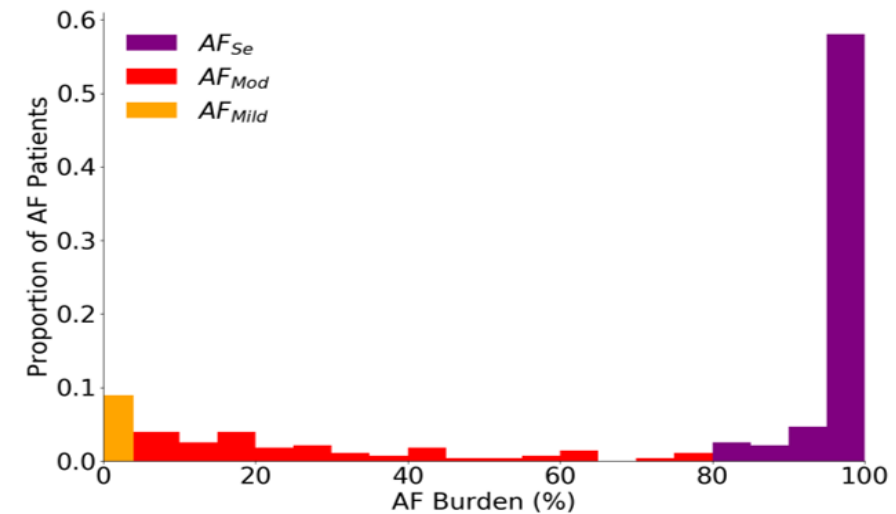
Stratification

- When doing cross validation consider performing **stratification** to ensure the different folds have the same proportion of each classes.
- Examples:

Table 1

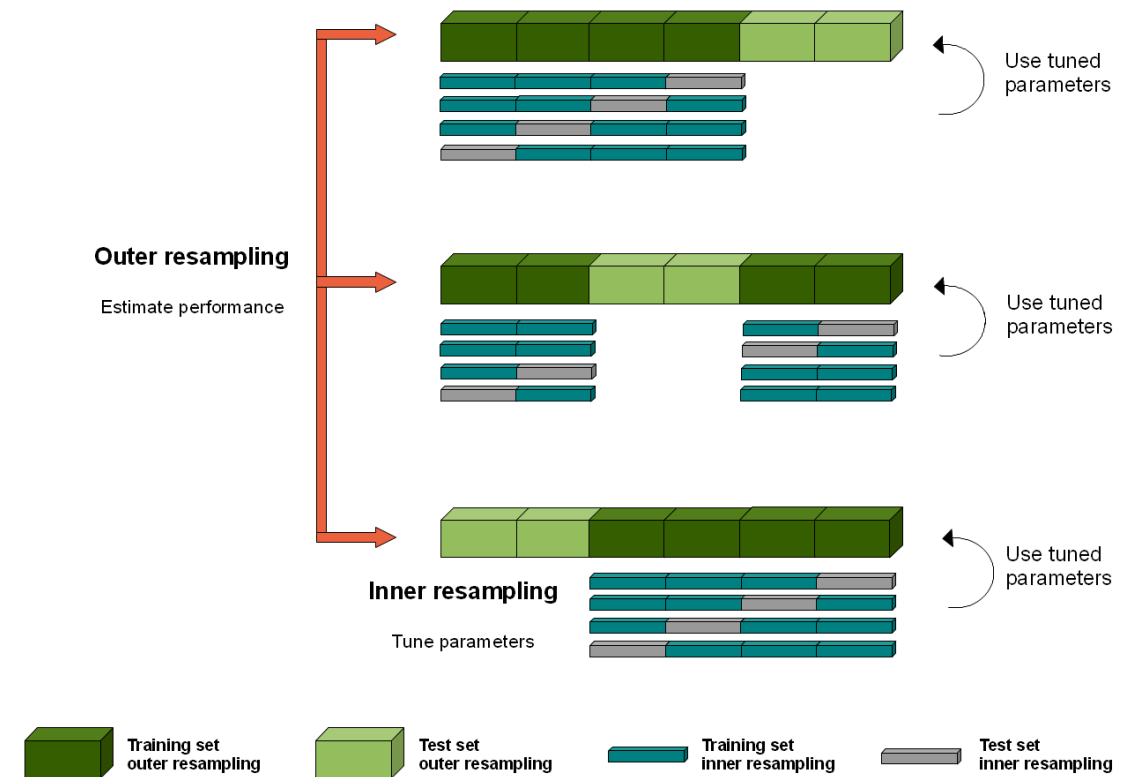
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Total	887	



Nested cross validation

- What if we want to estimate the **generalization error** of our model?
- **Nested cross validation** consists of:
 - An inner loop: fit a model to each training set, select hyperparameters by maximizing the performance over the validation set.
 - The outer loop where you estimate the generalization error by averaging test set scores over several dataset splits.



Nested cross validation

- Nested Cross Validation will provide you with a good idea of the generalization capacity of your model by:
 - Analyzing the generalization error: e.g. is the standard deviation of the error very high?
 - Analyzing model settings consistency (e.g. features selected, value of the weights in LR, hyperparameters values).

Nested cross validation

Feature extraction

4 SpO₂ features

8 demographic features

Machine Learning Optimization → Gradient Descent

Outer loop : 5 folds → use tuned hyperparameters in each fold and optimize parameters across the 5 folds.

Inner loop : 100 sub-folds → select hyperparameters so that statistics F_1 is maximized across the 100 sub-folds.

Table 5

Performance of the models (average and standard deviation for the test sets) evaluated against the AHI R2017.

Statistics/model	AUROC	Ac	F_1	NPV	PPV	Se	Se-mild	Se-moderate	Se-severe	Sp
NoSAS	0.83 ± 0.03	0.72 ± 0.03	0.58 ± 0.07	0.69 ± 0.03	0.81 ± 0.04	0.46 ± 0.08 (176/384)	0.33 ± 0.04 (67/206)	0.54 ± 0.14 (57/103)	0.69 ± 0.14 (52/75)	0.92 ± 0.02 (463/503)
STOP-BANG	0.77 ± 0.04	0.72 ± 0.02	0.65 ± 0.04	0.73 ± 0.03	0.70 ± 0.03	0.61 ± 0.05 (233/384)	0.47 ± 0.06 (97/206)	0.71 ± 0.13 (75/103)	0.81 ± 0.11 (61/75)	0.81 ± 0.02 (405/503)
LR-SB	0.87 ± 0.04	0.75 ± 0.04	0.76 ± 0.04	0.89 ± 0.04	0.66 ± 0.04	0.90 ± 0.04 (345/384)	0.84 ± 0.04 (172/206)	0.97 ± 0.03 (100/103)	0.97 ± 0.06 (73/75)	0.64 ± 0.05 (324/503)
LR-ODI	0.92 ± 0.01	0.85 ± 0.03	0.83 ± 0.03	0.87 ± 0.02	0.84 ± 0.04	0.82 ± 0.03 (315/384)	0.70 ± 0.04 (143/206)	0.94 ± 0.04 (97/103)	1.00 ± 0.00 (75/75)	0.88 ± 0.03 (443/503)
LR-SpO ₂	0.92 ± 0.02	0.85 ± 0.02	0.82 ± 0.03	0.87 ± 0.03	0.82 ± 0.01	0.83 ± 0.05 (317/384)	0.70 ± 0.07 (143/206)	0.96 ± 0.04 (99/103)	1.00 ± 0.00 (75/75)	0.86 ± 0.01 (435/503)
OxyDOSa	0.94 ± 0.02	0.86 ± 0.03	0.84 ± 0.04	0.90 ± 0.03	0.82 ± 0.03	0.87 ± 0.04 (335/384)	0.77 ± 0.05 (158/206)	0.99 ± 0.02 (102/103)	1.00 ± 0.00 (75/75)	0.85 ± 0.03 (428/503)

Four sets of classifiers were evaluated for comparison. These are denoted: LR-SB for which classifiers were trained using all the demographic features used for the STOP-BANG questionnaire; LR-ODI for which classifiers were trained using the oxygen desaturation index as the sole feature; LR-SpO₂ for which classifiers were trained using all the oxygen saturation features; OxyDOSa for which classifiers were trained using features selected from all oxygen saturation and the demographic features available from the STOP-BANG questionnaire. Statistics are reported for the test sets.

Important notes on performance generalization

- Summary:
 - **Training set:** What you develop the model on.
 - **Validation set:** Data excluded from model development but used in model hyperparameters selection.
 - **Test set:** Data excluded from ALL development, and used once to evaluate the final model.
- Be VERY aware of **information leakage** i.e. accidental use of information from the test set. You want to avoid that!

Important notes on generalization

- Examples of information leakage pitfalls:
 - Normalizing using all the data, then splitting into train/test.
 - Using data from the same patient in train and test.
 - Training a model, evaluating on test, iterating.

Take Home

- Tools for diagnosing a model:
 - Complexity of the hypothesis class,
 - Dealing with overfitting through regularization,
 - Dataset-size and learning curve.
 - Error analysis.

Take home

- To search for adequate hyperparameters and evaluate your model **performance** and **generalization performance** divide your data into: **train, validation and test sets**:
 - Use a **training set** from which the model learns.
 - Use a **validation set** that enables to perform hyperparameters selection.
 - Use a **test set** which we use to assess the final model performance. This set is not used to train the model or tune its hyperparameters.
- Use a flavor of **cross-validation** when optimizing model hyperparameters.
 - K-fold cross validation.
 - Leave-one-out cross-validation.
 - Repeated random sub-sampling validation.
 - Nested Cross Validation.

Take Home

- Perform **stratification** to ensure the different folds have the same proportion of each classes
- A practical approach:
 - Start with a simple algorithm, obtain results on cross-validation. Then plot learning curves and get an idea of where you can improve.
 - Avoid “premature optimization” and let evidence guide your design.
- Do not use information from your test set, this is **information leakage**, that will lead to bad generalization of the model performance.

References

- [1] Machine Learning Basic Concepts. CDT Lectures Notes 2013. Alistair Johnson.
- [2] Coursera, Andrew Ng. Advice for applying machine learning.
- [3] Belkin, Mikhail, et al. "Reconciling modern machine learning and the bias-variance trade-off." arXiv preprint arXiv:1812.11118 (2018).