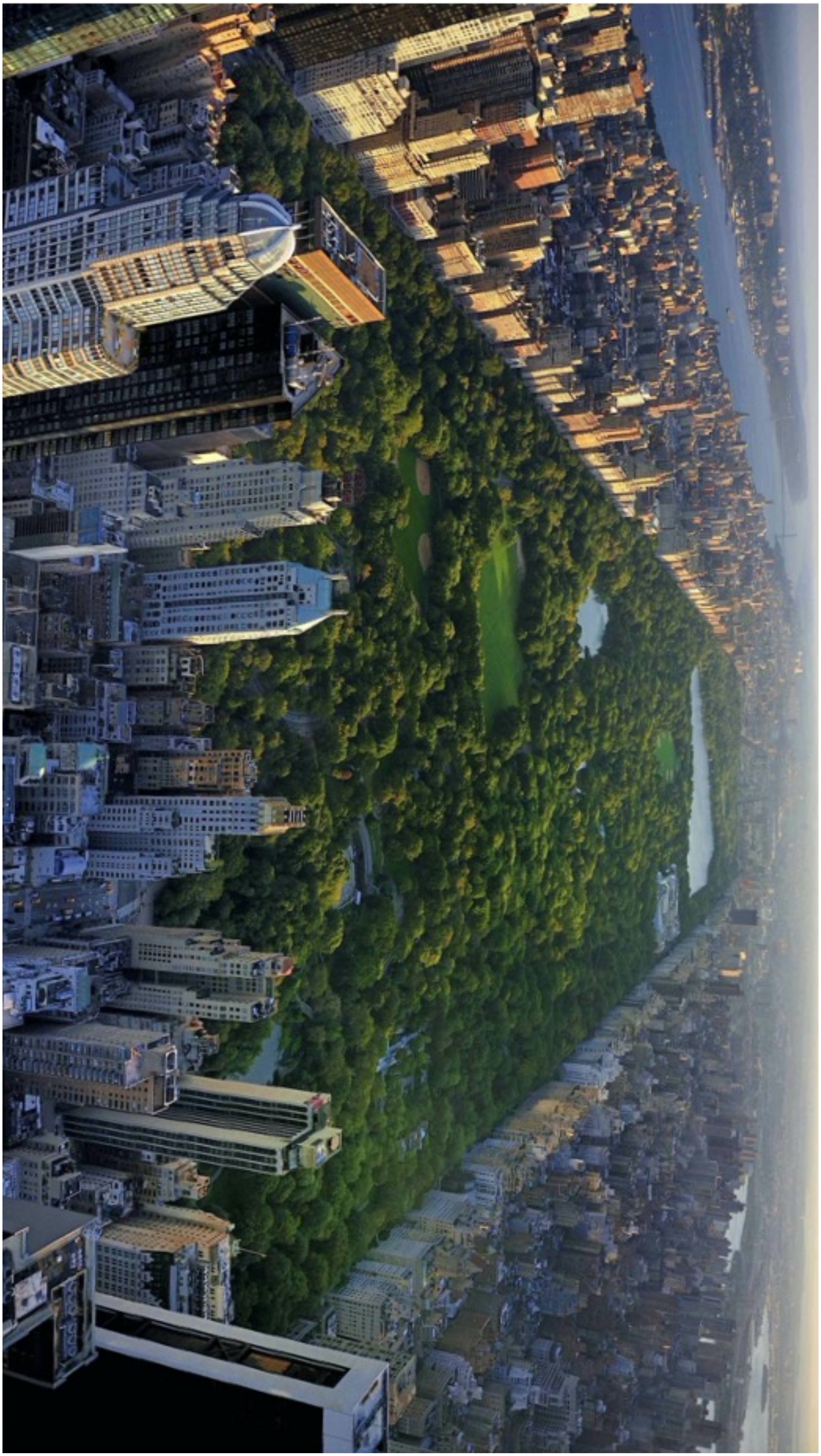
Spring 2025
Stanford CS231n 10th Anniversary

Lecture 15: 3D Vision

Jiajun Wu
May 22, 2025

Stanford University




NATIONAL GEOGRAPHIC
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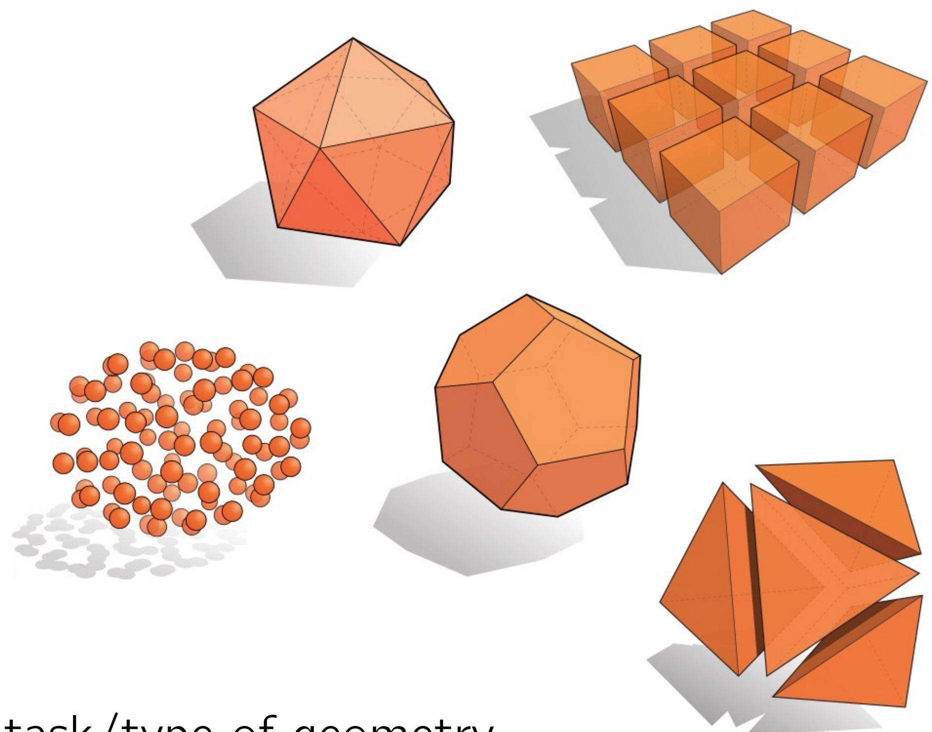
Many Ways to Represent Geometry

- Explicit
  - Point cloud
  - Polygon mesh
  - Subdivision, NURBS
  - ...
- Implicit
  - Lever sets
  - Algebraic surface
  - Distance functions
  - ...
- Each choice best suited to a different task/type of geometry

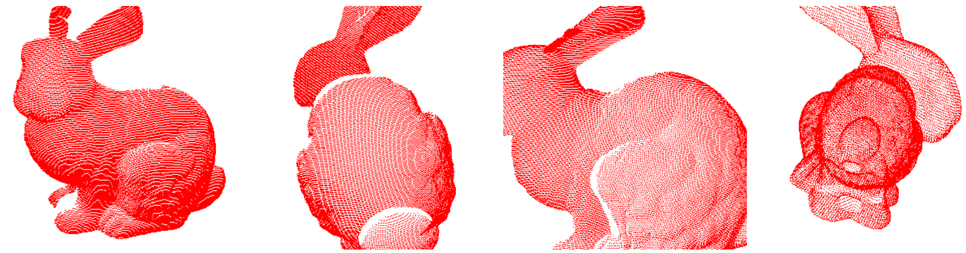


Slide credit: Ren Ng

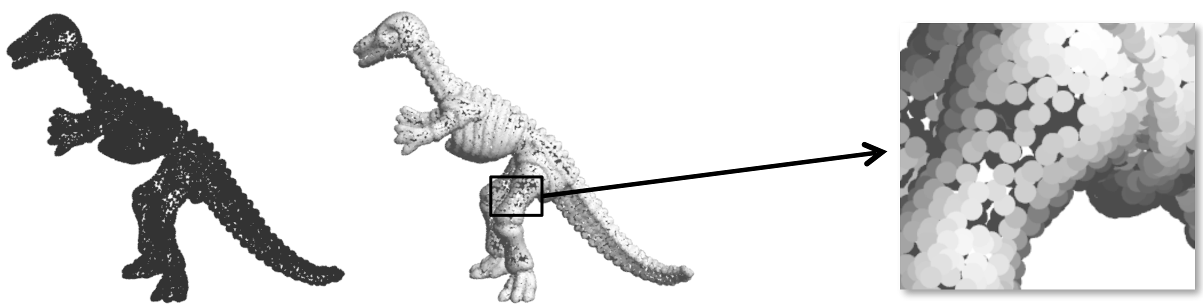
Representation Considerations

- Needs to be stored in the computer
- Creation of new shapes
  - Input metaphors, interfaces...
- Operations
  - Editing, simplification, smoothing, filtering, repairing...
- Rendering
  - Rasterization, ray tracing, neural rendering...
- Animation

Point Clouds

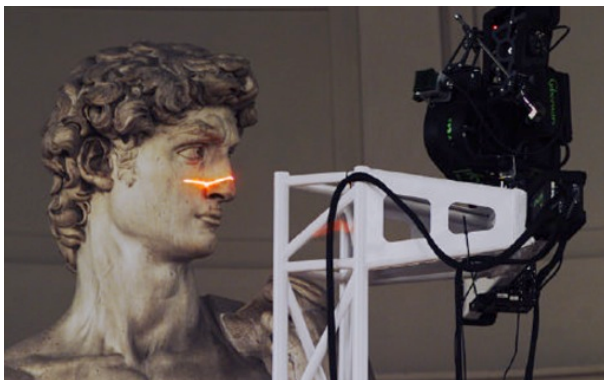


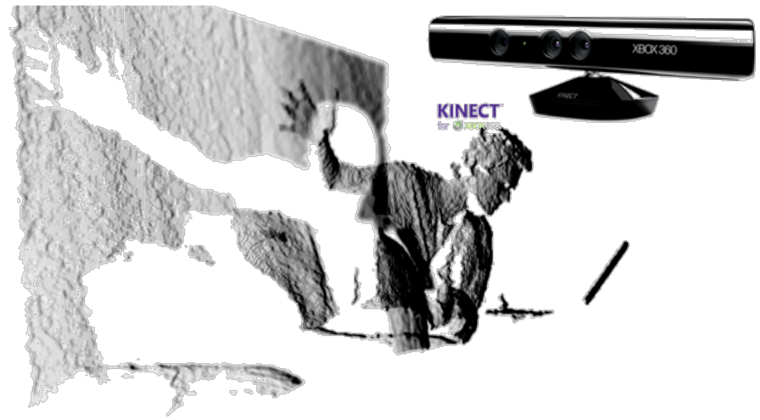
- Simplest representation: **only points**, no connectivity
- Collection of (x, y, z) coordinates, possibly with normal
- Points with orientation are called **surfels**

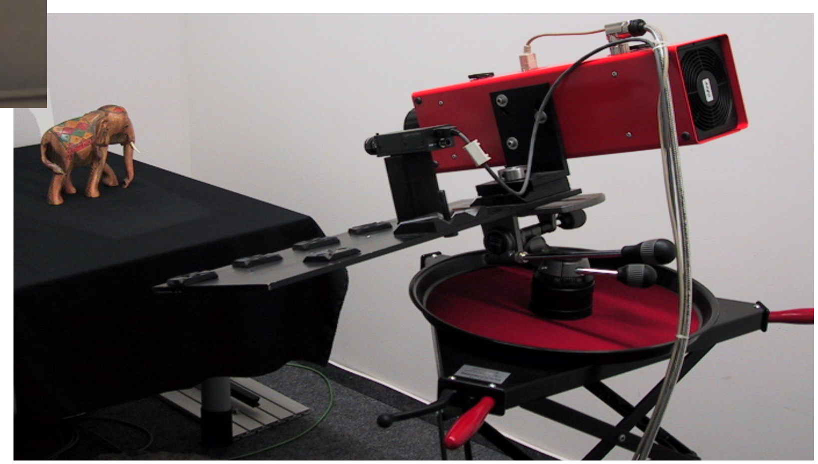


Shading needs normals!

Output of Acquisition

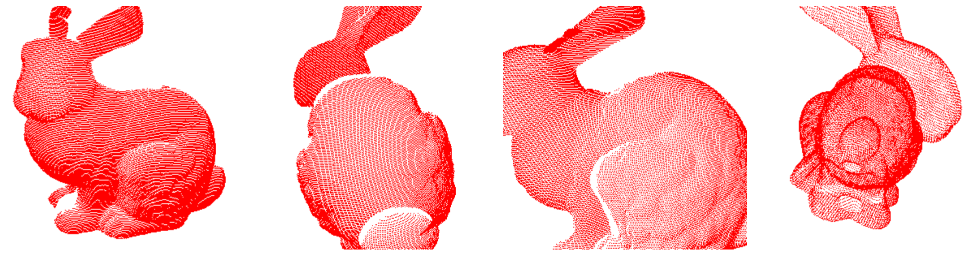






Slide credit: Hao Su

Point Clouds



- Simplest representation: **only points**, no connectivity
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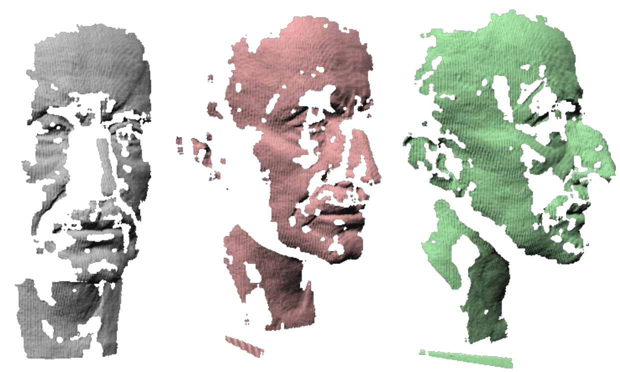
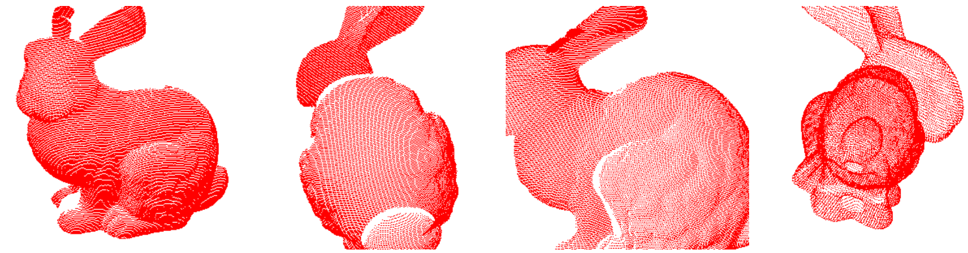


Set of raw scans

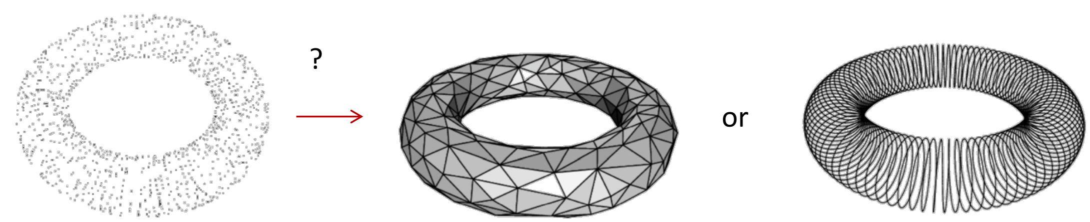
- Often results from scanners
- Potentially noisy
- Registration of multiple images

Point Clouds



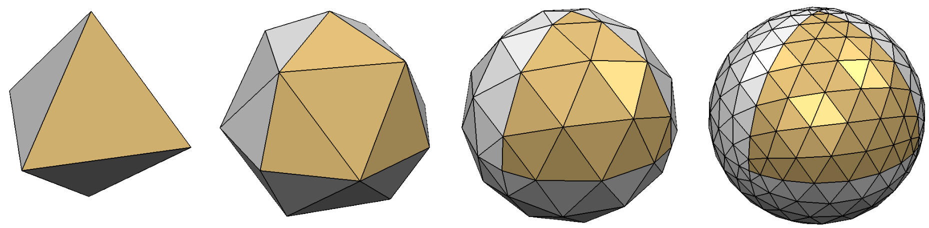
- Easily represent any kind of geometry
- Useful for large datasets
- Difficult to draw in undersampled regions

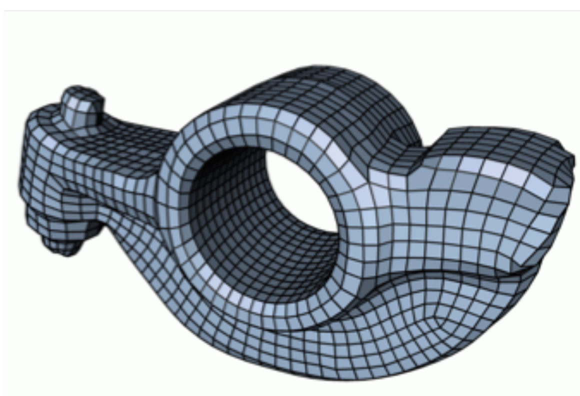
- Other limitations:
  - No simplification or subdivision
  - No direction smooth rendering
  - No topological information



Polygonal Meshes

- Boundary representations of objects

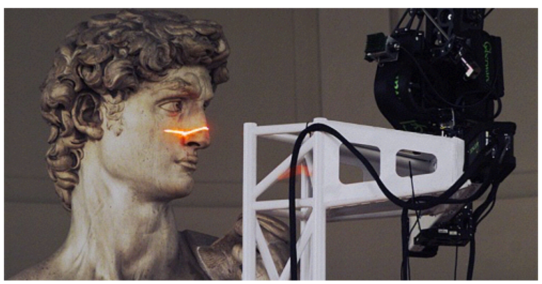


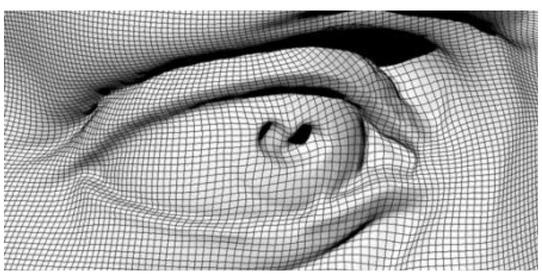


Slide credit: Hao Su

A Large Triangle Mesh

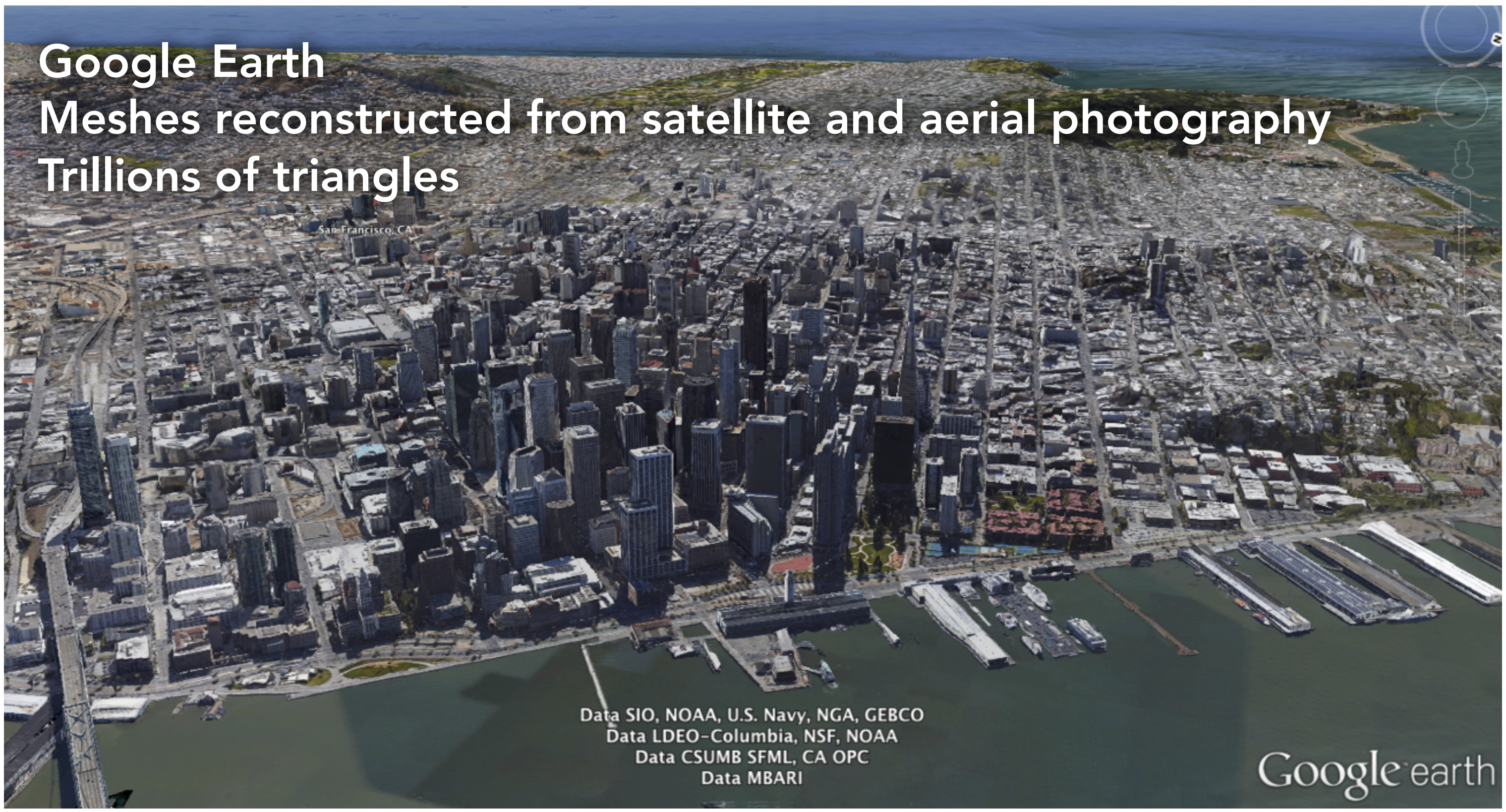
David
Digital Michelangelo Project
28,184,526 vertices
56,230,343 triangles





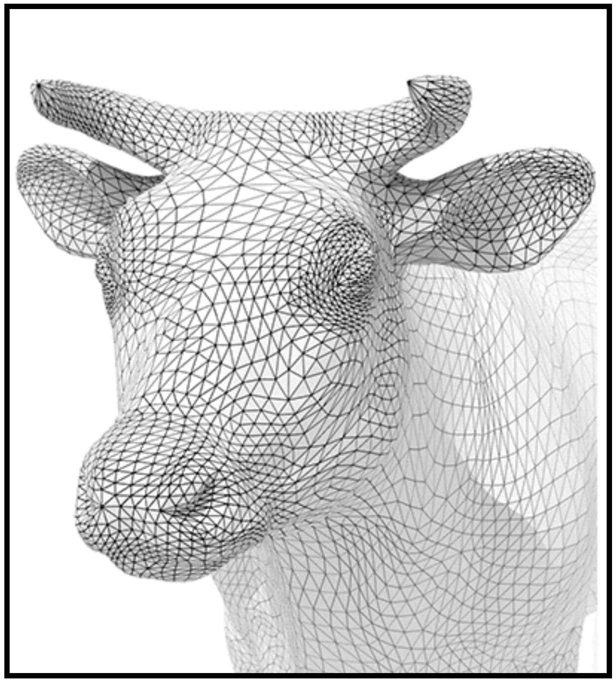
Slide credit: Ren Ng

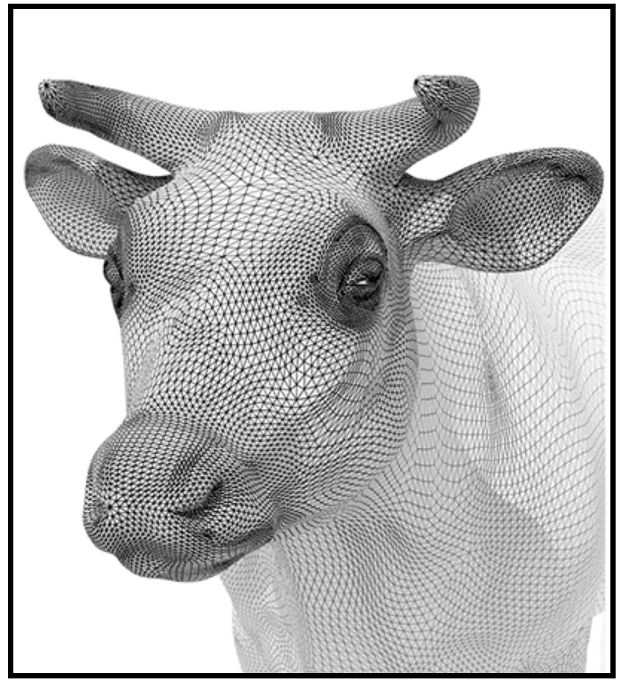
A Very Large Triangle Mesh



Slide credit: Ren Ng

Mesh Upsampling - Subdivision

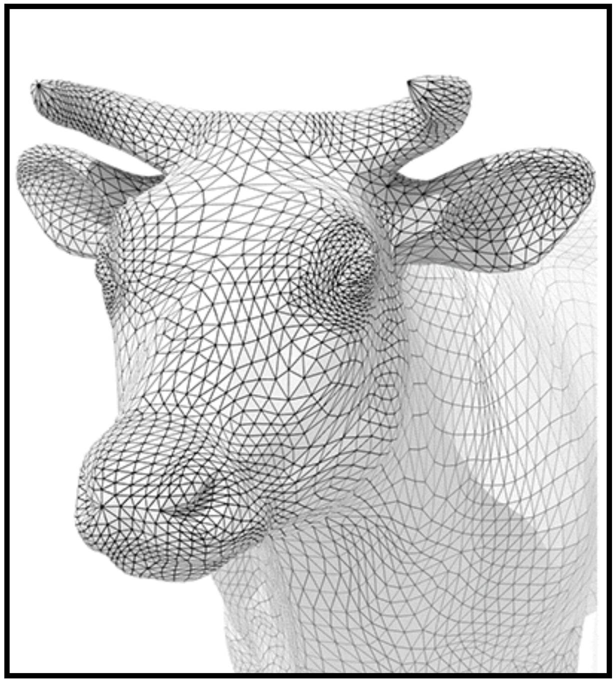


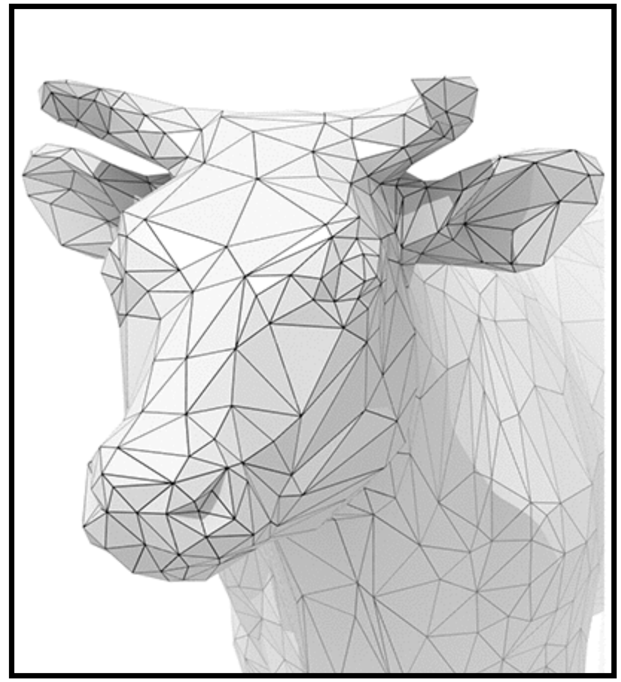


Increase resolution via interpolation

Slide credit: Ren Ng

Mesh Downsampling - Simplification

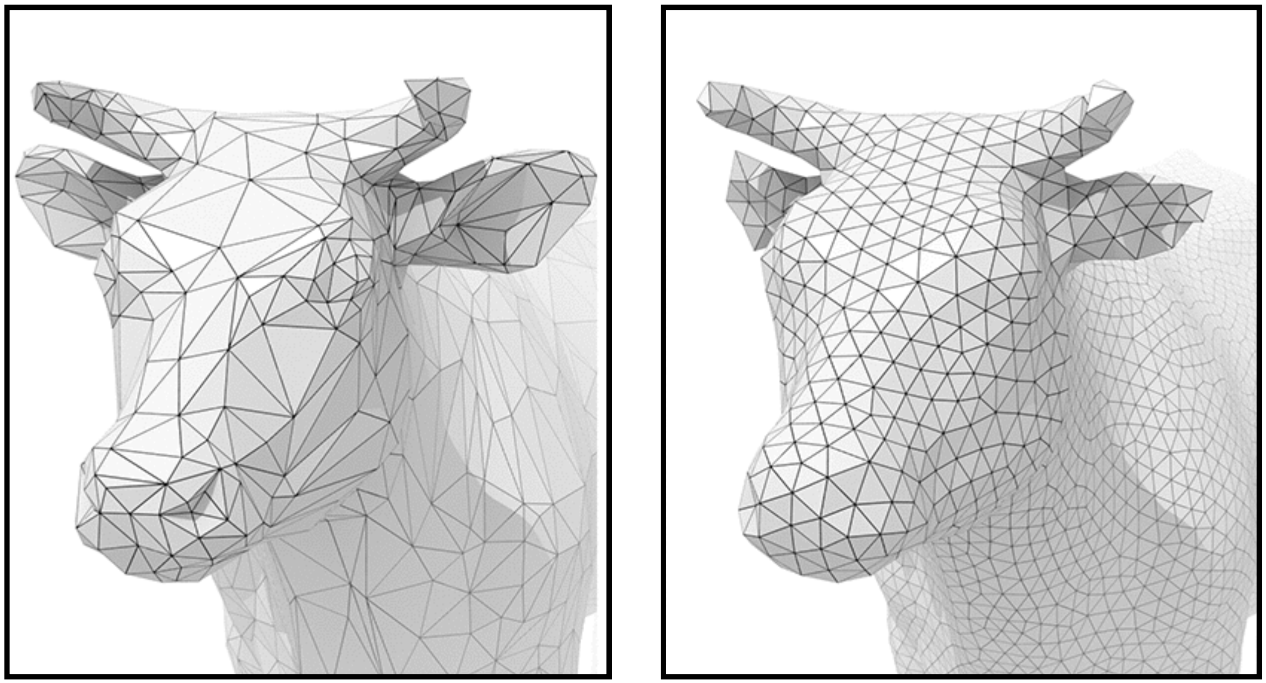




Decrease resolution; try to preserve shape/appearance

Slide credit: Ren Ng

Mesh Regularization

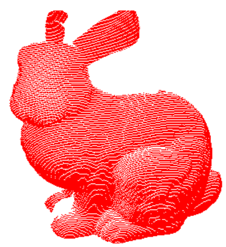


Modify sample distribution to improve quality

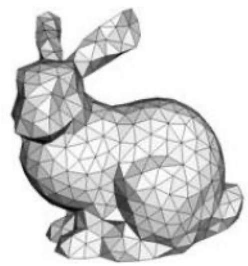
Slide credit: Ren Ng

Shape Representations

Non-parametric



Points

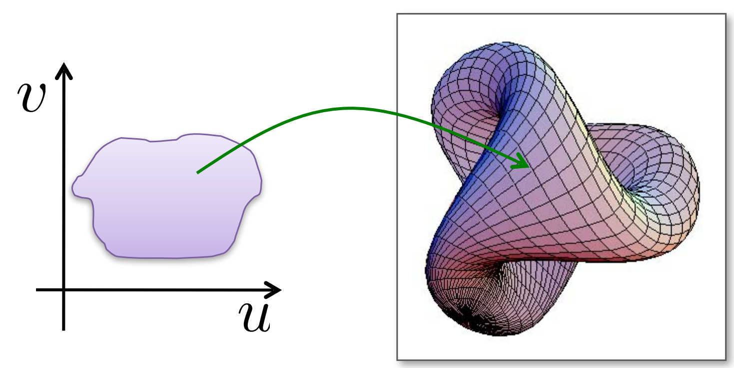


Meshes

Parametric Representation

Range of a function $f : X \to Y, X \subseteq \mathbb{R}^m, Y \subseteq \mathbb{R}^n$

Surface in 3D: $m = 2, n = 3$



$$s(u, v) = (x(u, v), y(u, v), z(u, v))$$

Slide credit: Hao Su

Parametric Curves

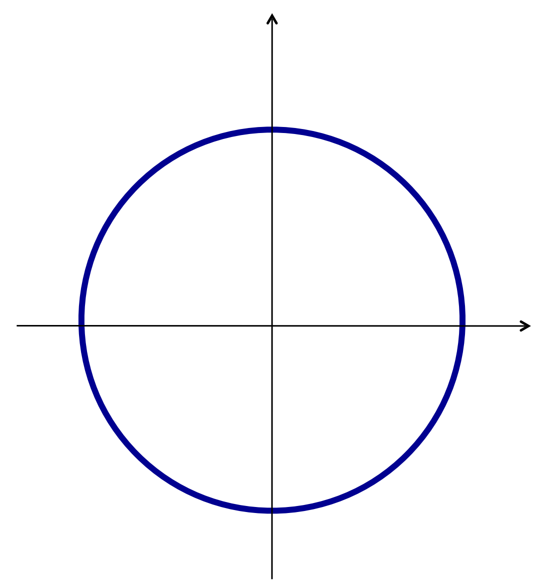
Explicit curve/circle in 2D

$\mathbf{p} : \mathbb{R} \to \mathbb{R}^2$

$t \mapsto \mathbf{p}(t) = (x(t), y(t))$

$\mathbf{p}(t) = r\,(\cos(t), \sin(t))$

$t \in [0, 2\pi)$

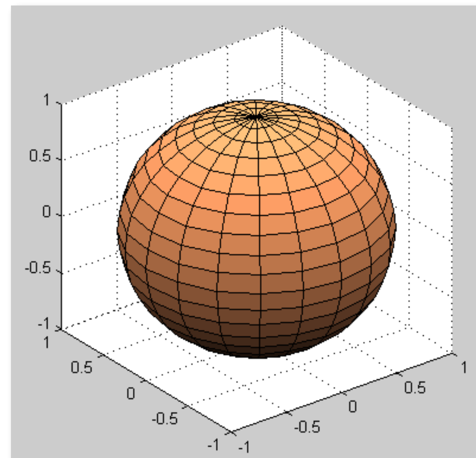


Slide credit: Hao Su

Parametric Surfaces

Sphere in 3D

$$s : \mathbb{R}^2 \to \mathbb{R}^3$$

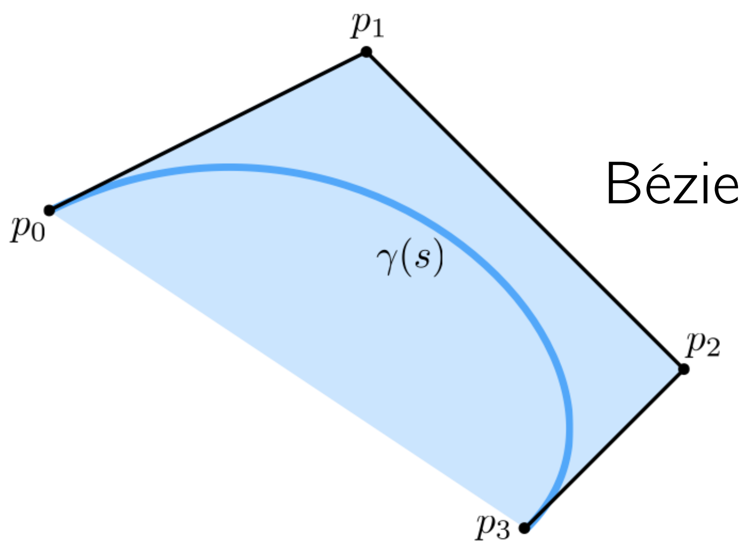


$$s(u,v) = r\,(\cos(u)\cos(v), \sin(u)\cos(v), \sin(v))$$

$$(u,v) \in [0, 2\pi) \times [-\pi/2, \pi/2]$$

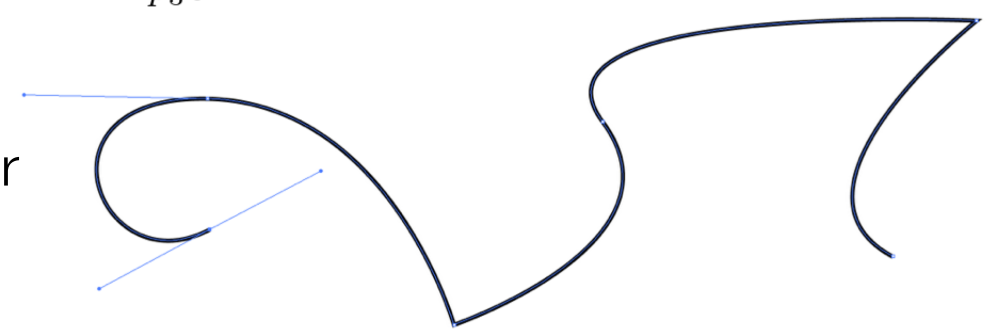
Slide credit: Hao Su

Bézier Curves



Bézier curves

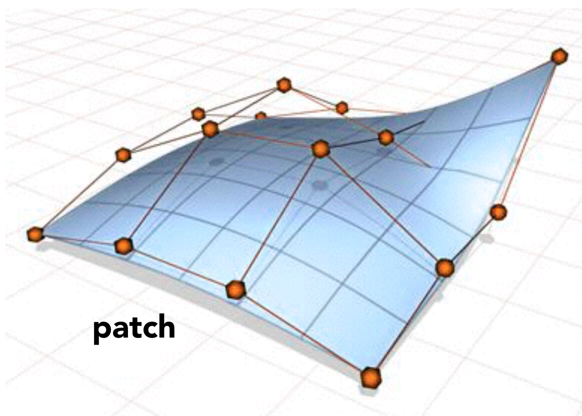
Piecewise Bézier

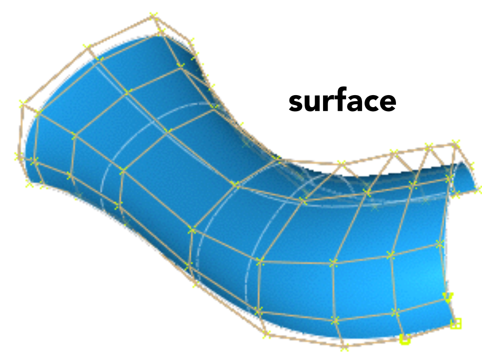


Slide credit: Ren Ng

Bézier Surfaces

Use tensor product of Bézier curves to get a patch:

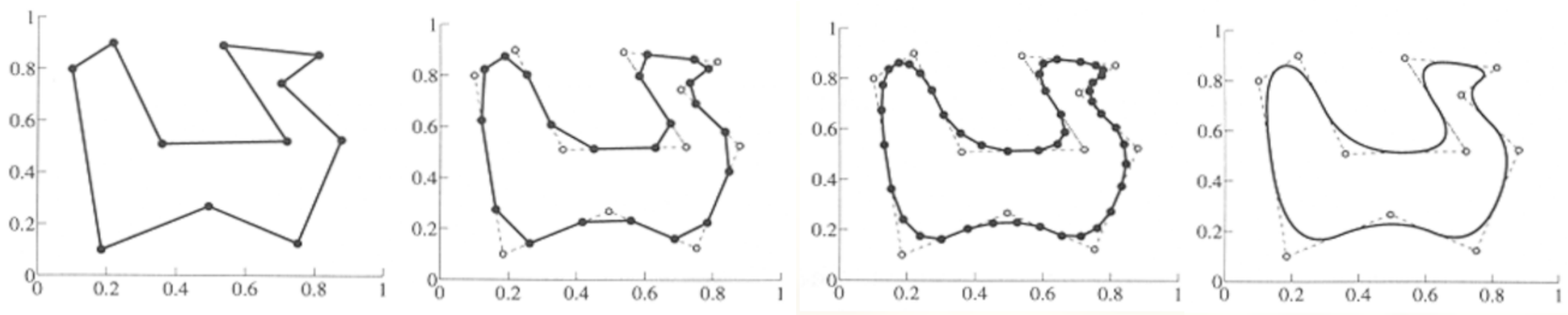




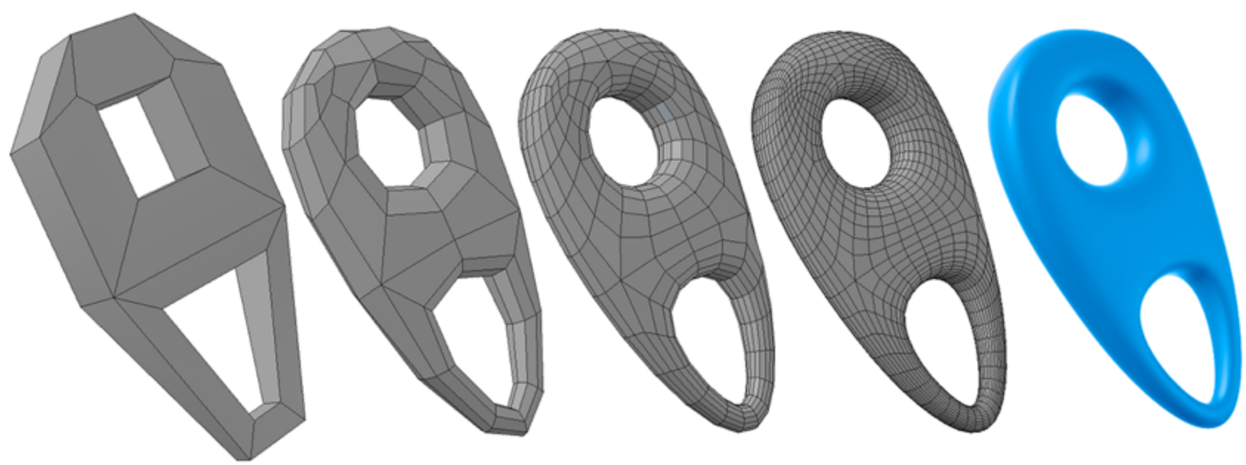
Multiple Bézier patches form a surface.

Slide credit: Ren Ng

Subdivision Curves/Surfaces



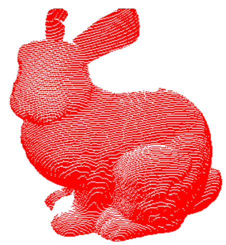
Slide cribbed from Keenan Crane, cribbed from Don Fussell.



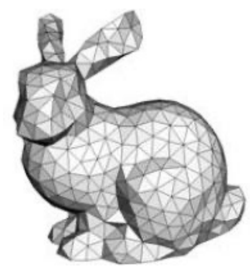
Shape Representations

Explicit

Non-parametric

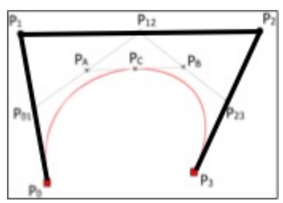


Points

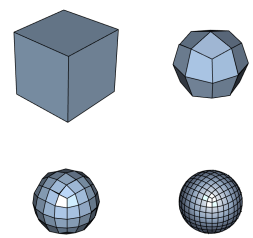


Meshes

Parametric



Splines

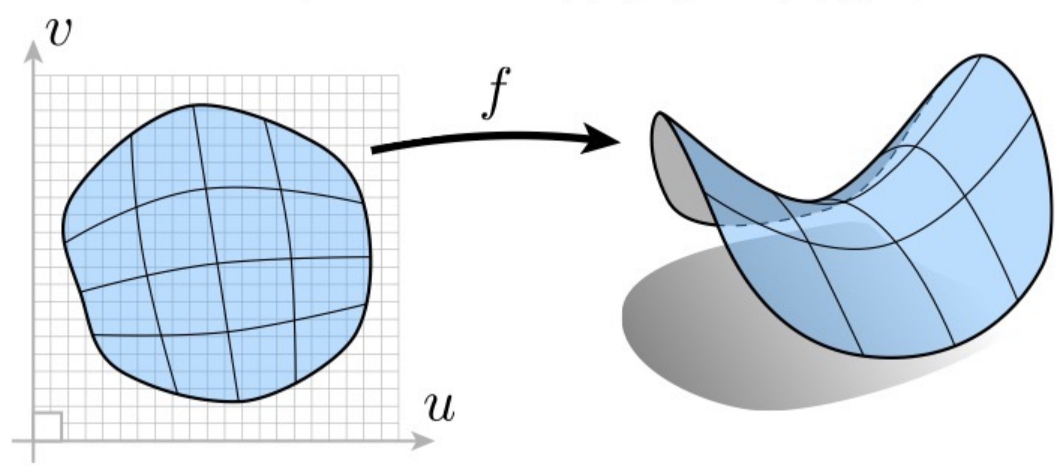


Subdivision Surfaces

“Explicit” Representations of Geometry

All points are given directly.

Generally: $f : \mathbb{R}^2 \rightarrow \mathbb{R}^3; (u, v) \mapsto (x, y, z)$



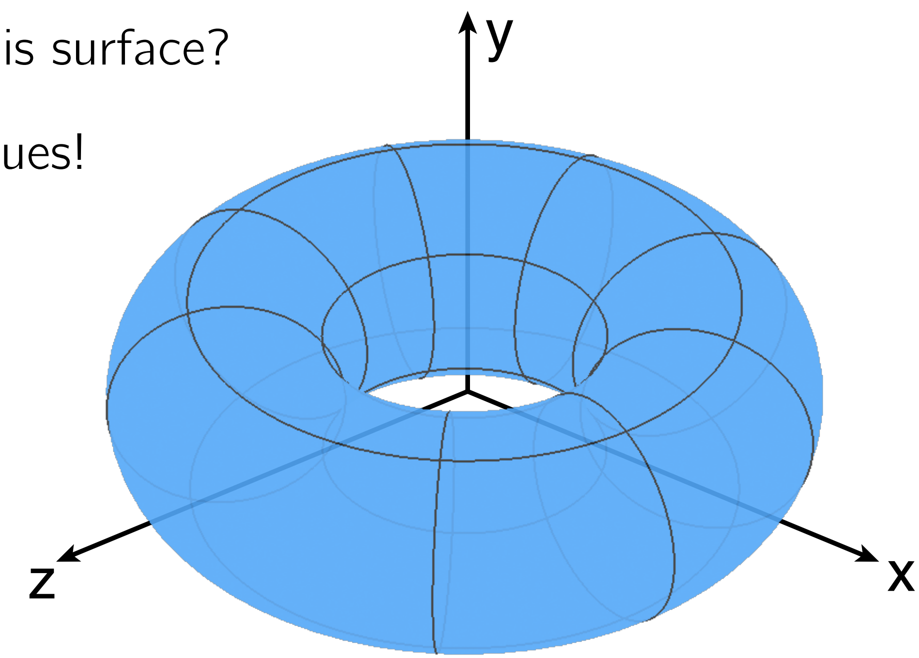
Slide credit: Ren Ng

Explicit Surface – Sampling Is Easy

$$f(u,v) = ((2+\cos u)\cos v, (2+\cos u)\sin v, \sin u)$$

What points lie on this surface?

Just plug in (u, v) values!



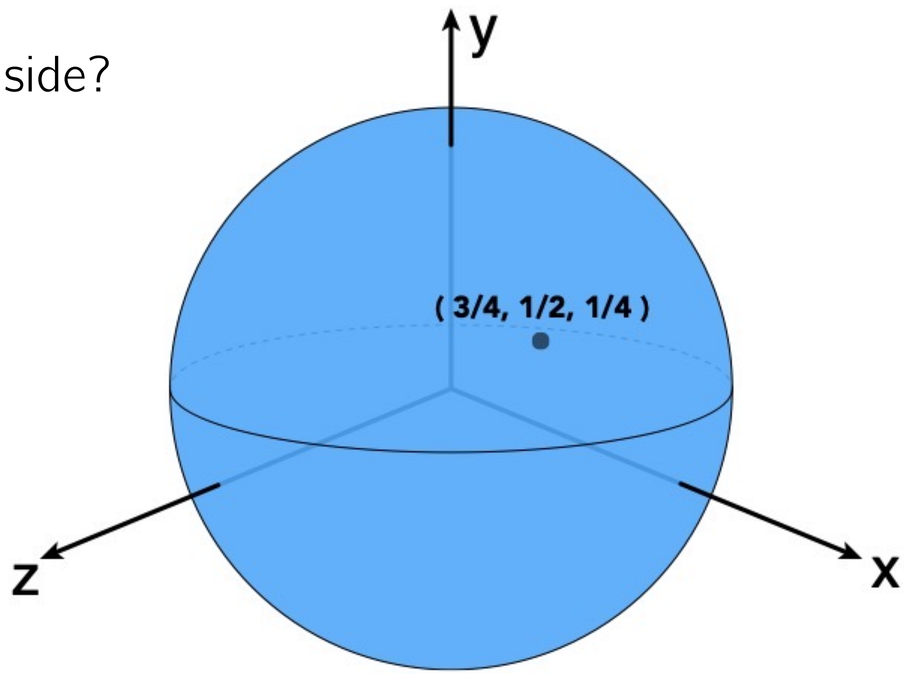
Explicit representations make some tasks easy.

Slide credit: Ren Ng

Explicit Surface – Inside/Outside Test Hard

$$f(u,v) = (\cos u \, \sin v, \sin u \, \sin v, \cos v)$$

Is $(3/4, 1/2, 1/4)$ inside?



Some tasks are hard with explicit representations.

Slide credit: Ren Ng

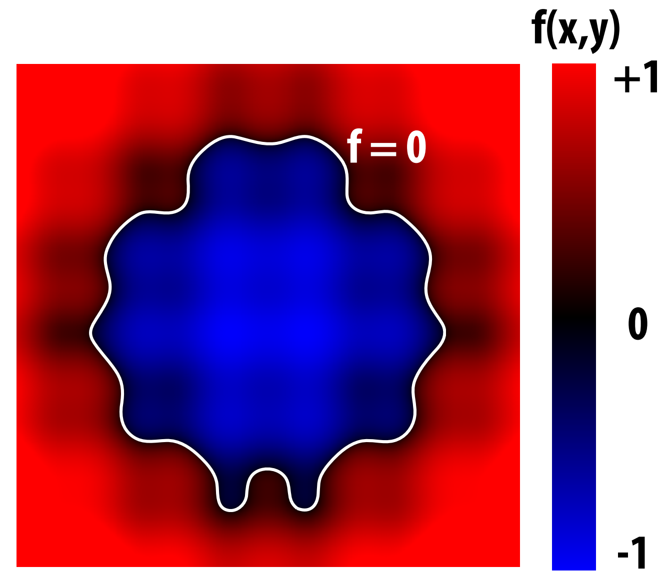
“Implicit” Representations of Geometry

Based on classifying points

- Points satisfy some specified relationship.

E.g., sphere: all points in 3D, where $x^2 + y^2 + z^2 = 1$

More generally, $f(x, y, z) = 0$

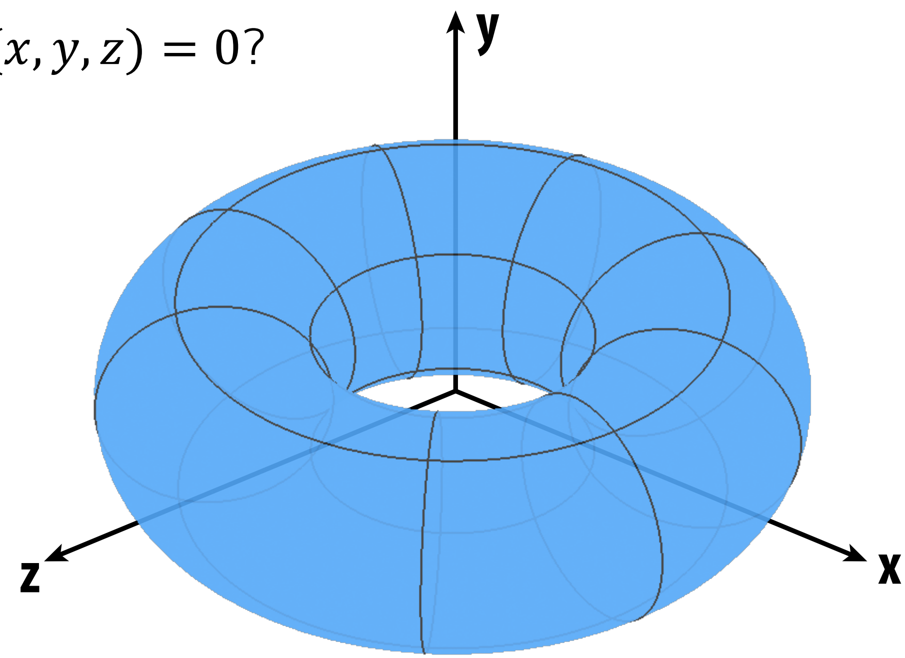


Slide credit: Ren Ng

Implicit Surface – Sampling Can Be Hard

$$f(x,y,z) = (2 - \sqrt{x^2 + y^2})^2 + z^2 - 1$$

What points lie on $f(x,y,z) = 0$?



Some tasks are hard with implicit representations.

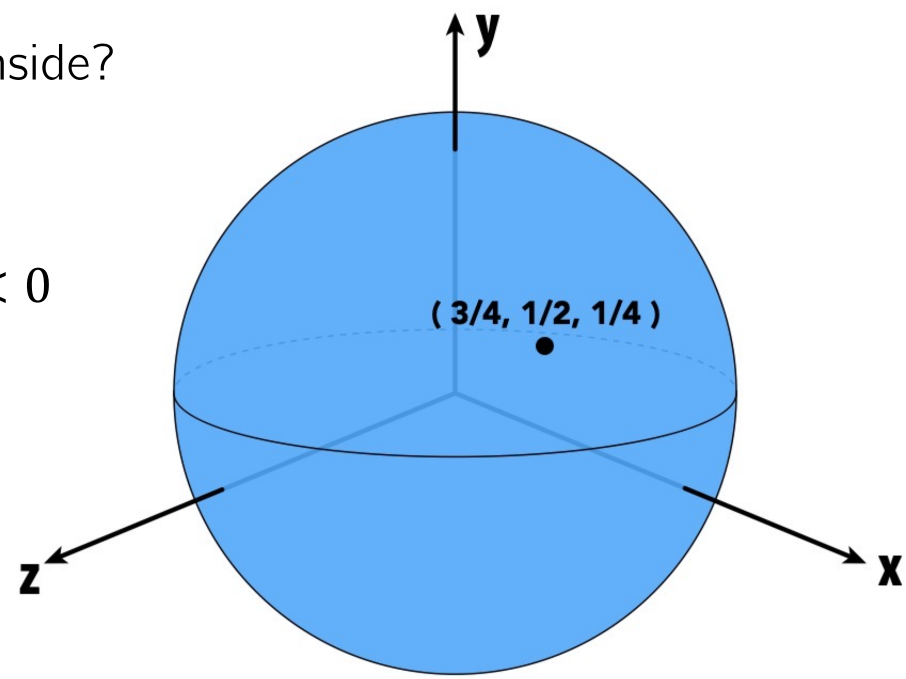
Slide credit: Ren Ng

Implicit Surface – Inside/Outside Tests Easy

$$f(x, y, z) = x^2 + y^2 + z^2 - 1$$

Is $(3/4, 1/2, 1/4)$ inside?

Just plug it in:
$f(x, y, z) = -1/8 < 0$
Yes, inside.

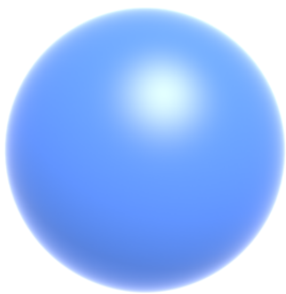


Implicit representations make some tasks easy.

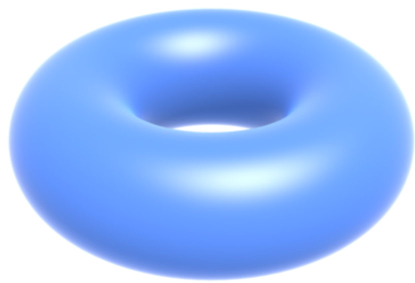
Slide credit: Ren Ng

Algebraic Surfaces (Implicit)

Surface is zero set of a polynomial in x, y, z.



$x^2+y^2+z^2=1$



$(R-\sqrt{x^2+y^2})^2+z^2=r^2$

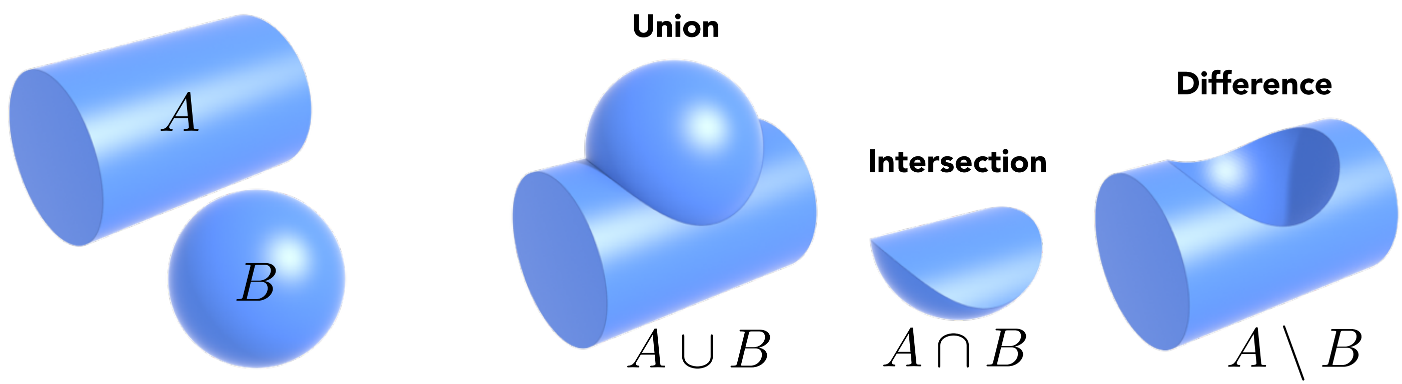
$$(x^2+\frac{9y^2}{4}+z^2-1)^3 = x^2z^3+\frac{9y^2z^3}{80}$$

More complex shapes?

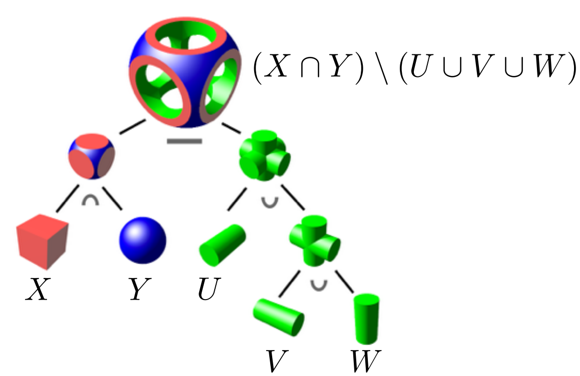
Slide credit: Ren Ng

Constructive Solid Geometry (Implicit)

Combine implicit geometry via Boolean operations



Boolean expressions:

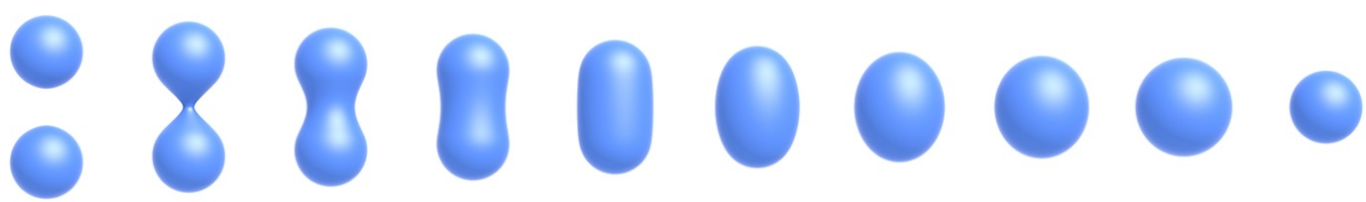


Slide credit: Ren Ng

Distance Functions (Implicit)

Instead of Boolean, gradually blend surfaces together using

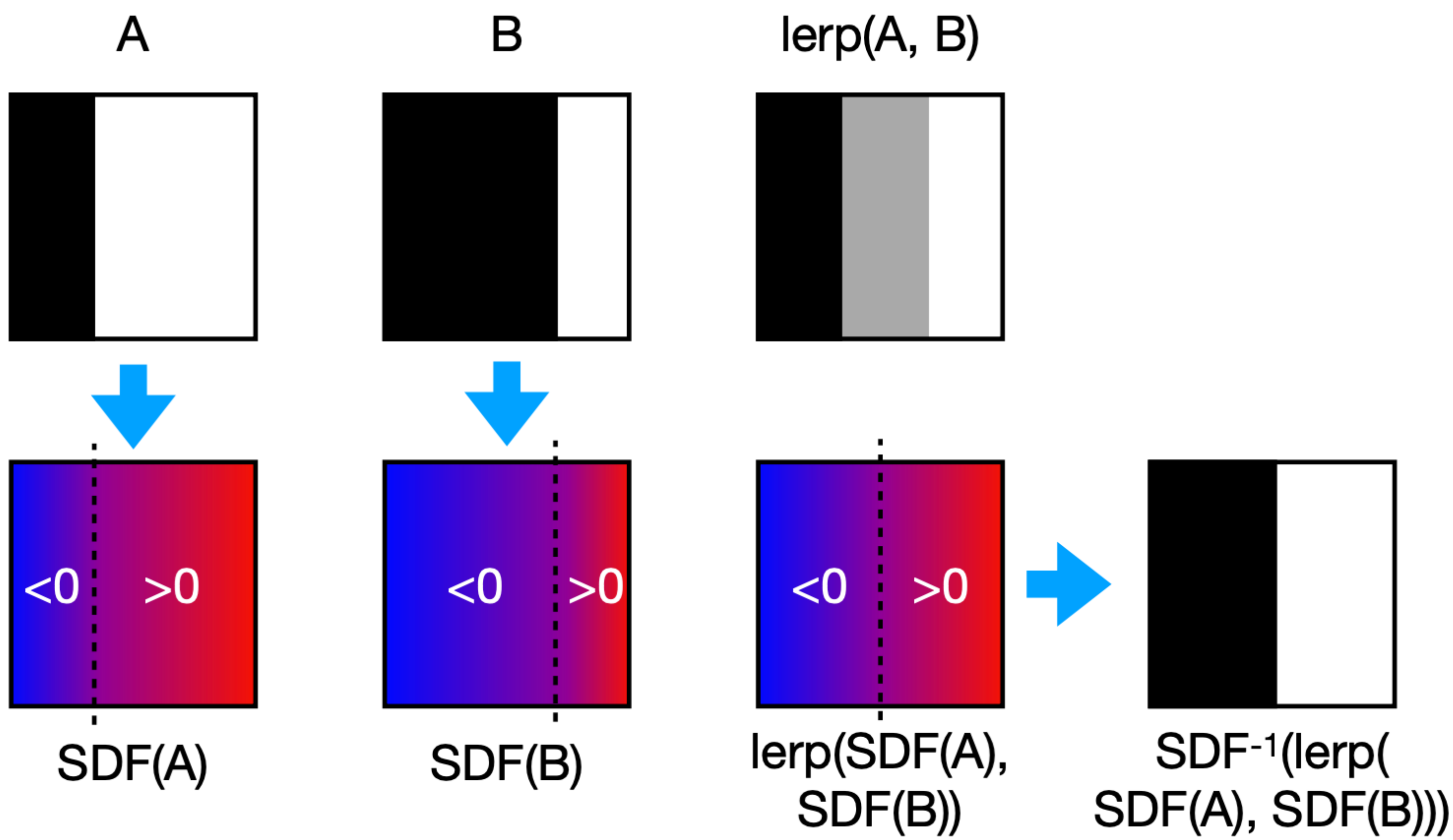
Distance functions:
Giving minimum distance (could be **signed** distance) from anywhere to object



Slide credit: Ren Ng

Distance Functions (Implicit)

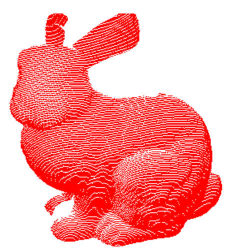
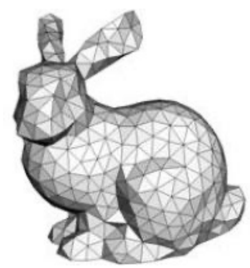
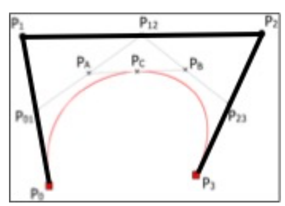
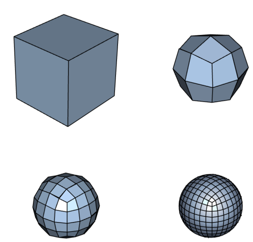
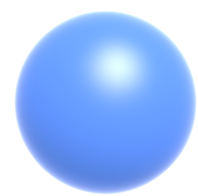
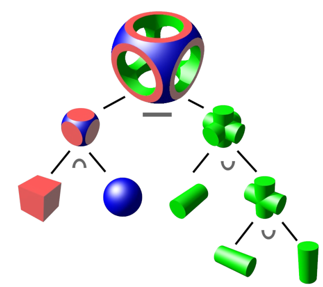
Example: Blending (linear interp.) a moving boundary



Scene of Pure Distance Functions (Not Easy!)

See http://iquilezles.org/www/material/nvscene2008/nvscene2008.htm

Shape Representations

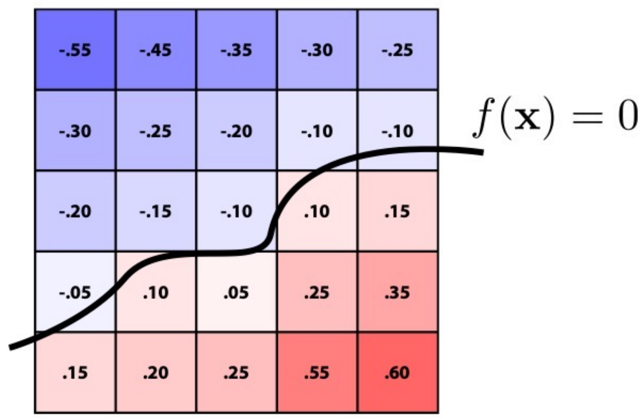
	Explicit		Implicit	
Non-parametric	 Points	 Meshes		
Parametric	 Splines	 Subdivision Surfaces	 $x^2+y^2+z^2=1$ Algebraic Surfaces	 Constructive Solid Geometry

Level Set Methods (Implicit)

Implicit surfaces have some nice features (e.g., merging/splitting).

But hard to describe complex shapes in closed form

Alternative: store a grid of values approximating function

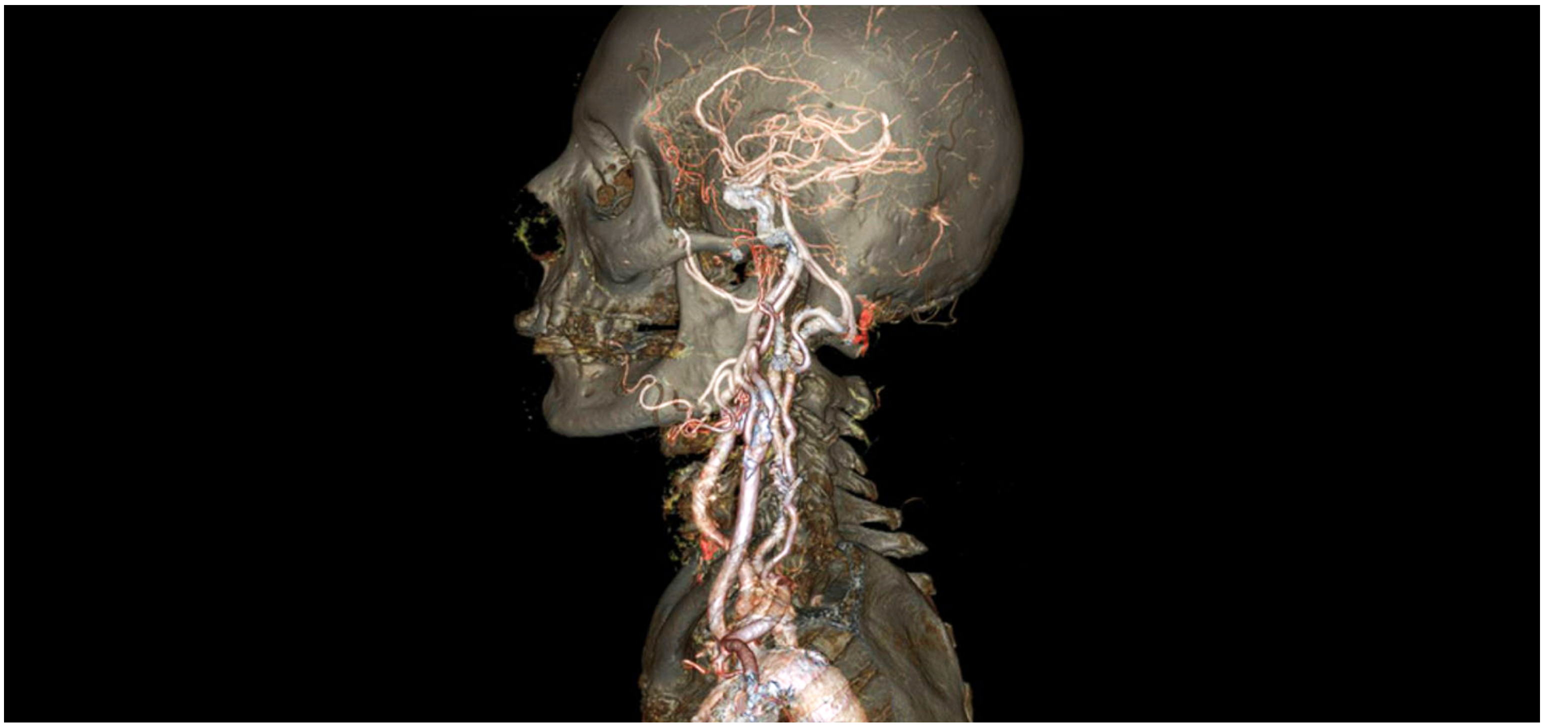


Surface is found where interpolated values equal zero.

Provides much more explicit control over shape (like a texture)

Slide credit: Ren Ng

Level Sets from Medical Data (CT, MRI, etc.)

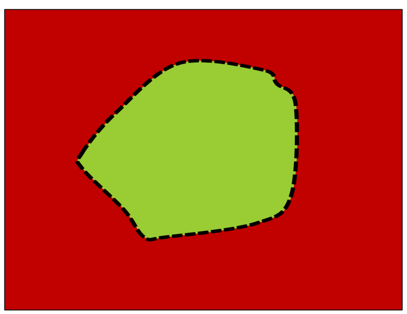


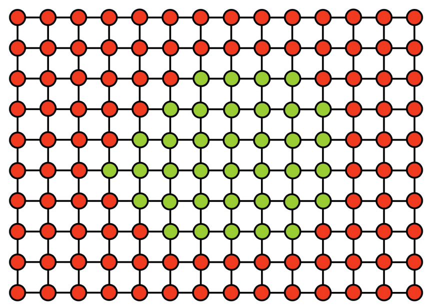
Level sets encode, e.g., constant tissue density

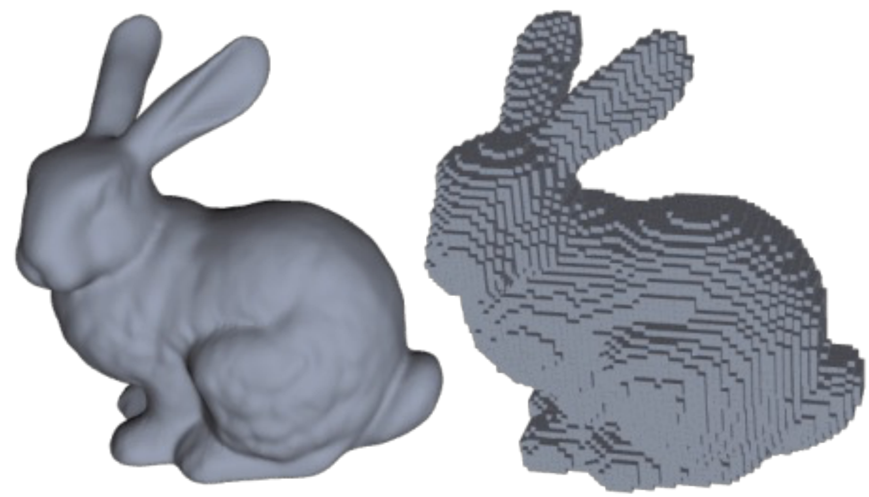
Slide credit: Ren Ng

Related Representation: Voxels

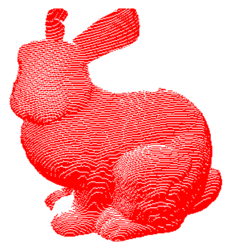
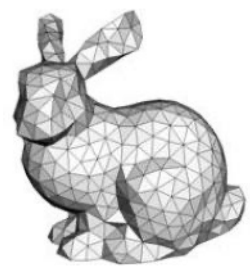
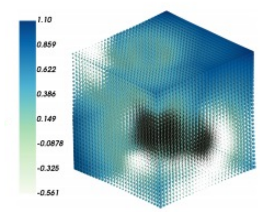
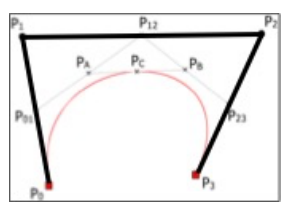
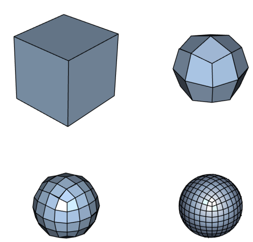
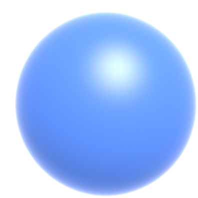
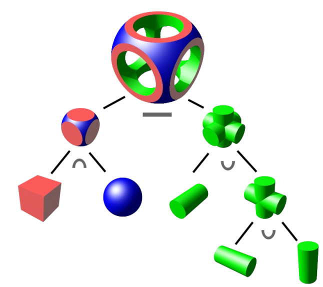
- Binary thresholding the volumetric grid







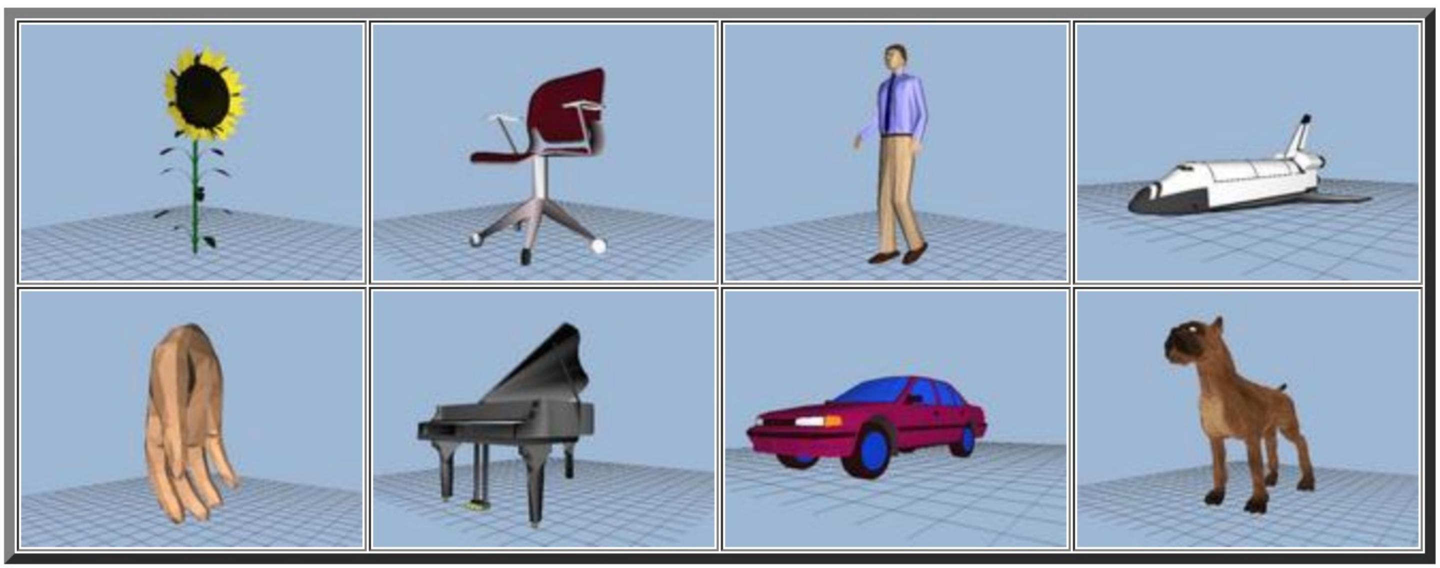
Shape Representations

	Explicit		Implicit	
Non-parametric	 Points	 Meshes	 Voxels	 Level Sets
Parametric	 Splines	 Subdivision Surfaces	 $x^2+y^2+z^2=1$ Algebraic Surfaces	 Constructive Solid Geometry

AI + Geometry: Datasets

Princeton Shape Benchmark

- 1814 Models
- 182 Categories



Shilane et al, 2004

Datasets Prior to 2014

Benchmarks	Types	# models	# classes	Avg # models per class
SHREC14LSGTB	Generic	8,987	171	53
PSB	Generic	907+907 (train+test)	90+92 (train+test)	10+10 (train+test)
SHREC12GTB	Generic	1200	60	20
TSB	Generic	10,000	352	28
CCCC	Generic	473	55	9
WMB	Watertight (articulated)	400	20	20
MSB	Articulated	457	19	24
BAB	Architecture	2257	183+180 (function+form)	12+13 (function+form)
ESB	CAD	867	45	19

Table 1. Source datasets from SHREC 2014: *Princeton Shape Benchmark (PSB)* [27], *SHREC 2012 generic Shape Benchmark (SHREC12GTB)* [16], *Toyohashi Shape Benchmark (TSB)* [29], *Konstanz 3D Model Benchmark (CCCC)* [32], *Watertight Model Benchmark (WMB)* [31], *McGill 3D Shape Benchmark (MSB)* [37], *Bonn Architecture Benchmark (BAB)* [33], *Purdue Engineering Shape Benchmark (ESB)* [9].

Datasets for 3D Objects

- Large-scale Synthetic Objects: ShapeNet, 3M models
- ModelNet: absorbed by ShapeNet
- ShapeNetCore: 51.3K models in 55 categories

...

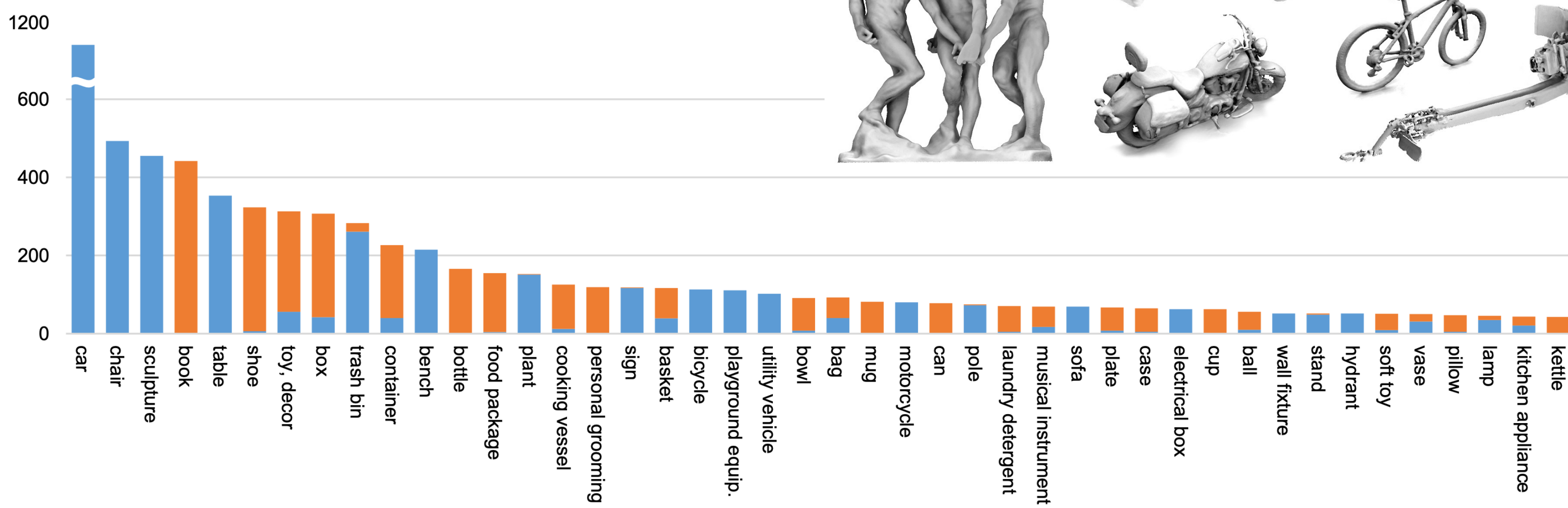
Chang et al. ShapeNet. arXiv 2015
Wu et al. 3D ShapeNets. CVPR 2015

Objverse (800K) and Objverse-XL (10M)

Deitke et al. 2022, 2023

Object Scan

- 10,933 RGBD scans
- 441 models



Choi et al, arXiv 2016

CO3D

- 19,000 videos
- 50 categories

Reizenstein et al. ICCV 2021

From Objects to Parts

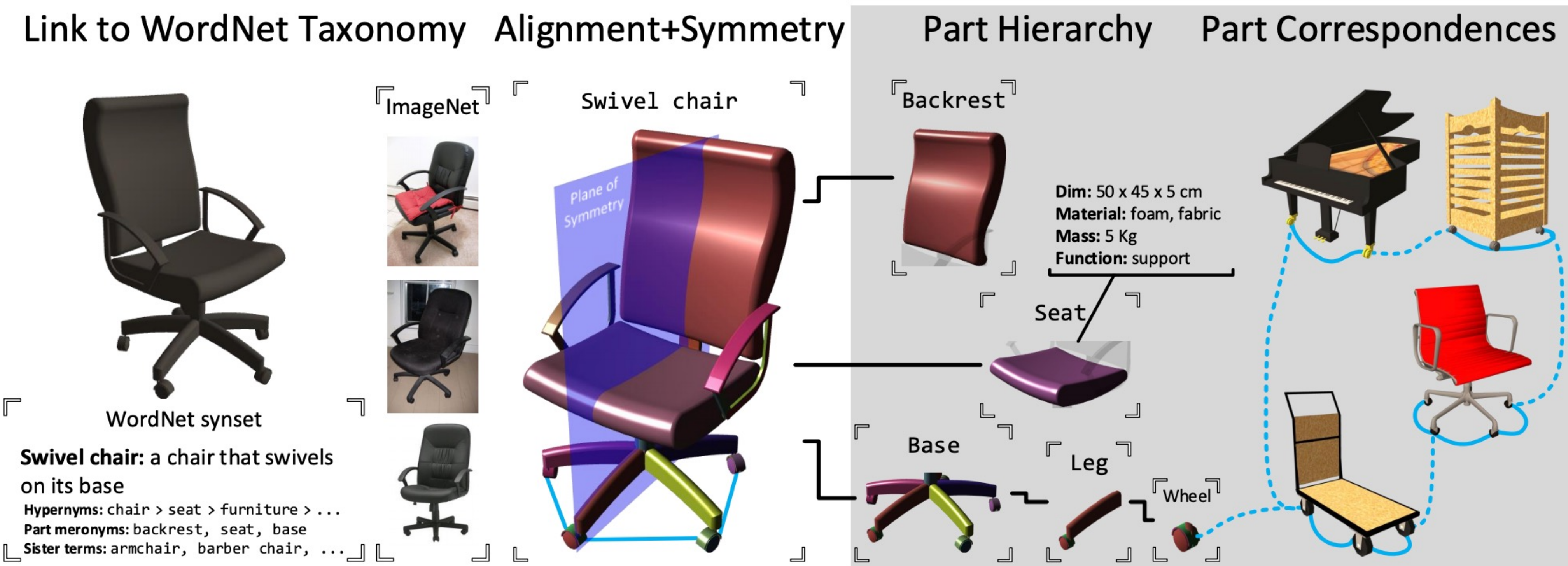
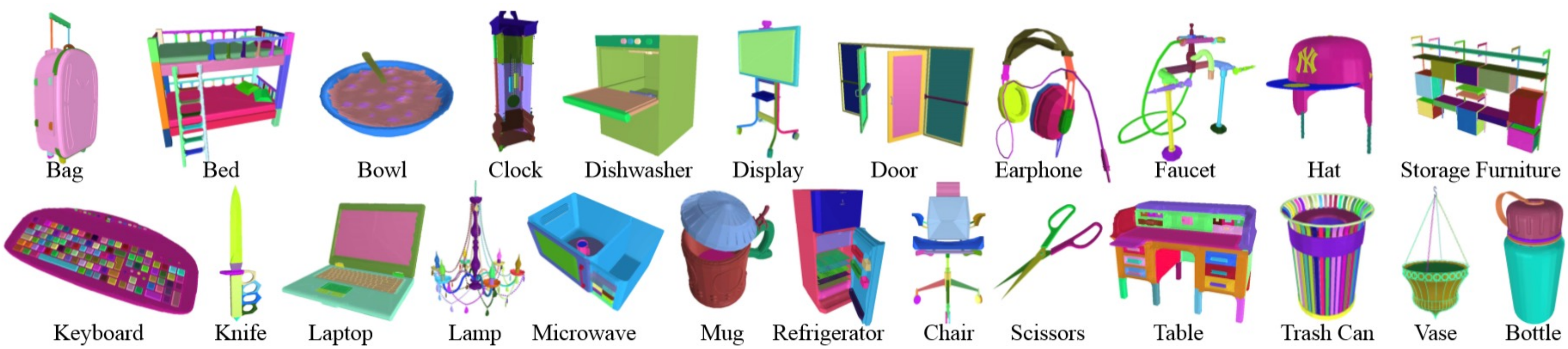


Figure from the ShapeNet paper, Chang et al. arXiv 2015

Datasets for 3D Object Parts

Fine-grained Parts: PartNet

- Fine-grained (+mobility)
- Instance-level
- Hierarchical



Mo et al. CVPR 2019. Slide credit: Hao Su

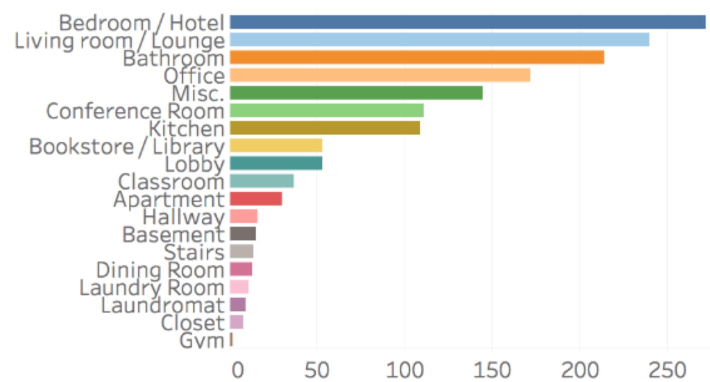
Datasets for Indoor 3D Scenes

Scanned Real Scenes: ScanNet

- 2.5M Views in 1,500 RGBD scans
- 3D camera poses
- Surface reconstructions
- Instance-level semantic segmentations

Most recently:

- ARKitScenes,
- ScanNet++ (with DSLR images)



Dai et al. CVPR 2017