Problem 1

Part A

$$\mathbb{P}(X = 2) = 0.$$

Part B

$$\mathbb{P}(X \geq 2) = 1 - \mathbb{P}(X < 2) = e^{-20} \approx 2.061 \times 10^{-9}.$$

Part C

$$\mathbb{P}(X > 3) = e^{-30} \approx 9.358 \times 10^{-14}.$$

Part D

$$\mathbb{E}[5X] = 5 \cdot \mathbb{E}[X] = 5 \cdot \frac{1}{10} = \frac{1}{2}.$$

$$\mathrm{Var}(5X) = 25 \cdot \mathrm{Var}(X) = 25 \cdot \frac{1}{100} = \frac{1}{4}.$$

Part E

$$250 \cdot \mathbb{E}[X] = 250 \cdot \frac{1}{10} = 25\$.$$

$$250 \cdot \mathbb{E}[5X] = 1250 \cdot \frac{1}{10} = 125\$.$$

Problem 2

Part A

$f(x)$ will be a valid density if

$$\int_0^2 f(x)\mathrm{d}x = 1.$$

Therefore

$$\begin{aligned}
\int_0^2 f(x)dx &= \int_0^2 cx^3 \mathrm{d}x \\
1 &= \int_0^2 cx^3 \mathrm{d}x \\
1 &= \int_0^2 cx^3 \mathrm{d}x \\
1 &= \frac{c}{4} \cdot x^4 \Big|_0^2 \\
1 &= \frac{c}{4} \cdot 2^4 \\
1 &= 4c \implies \boxed{c = \frac{1}{4}}.
\end{aligned}$$

Part B

$$\mathbb{P}(X = 1) = \int_1^1 f(x)\mathrm{d}x = 0.$$

$$\mathbb{P}(X = 1 \text{ or } X = 2) = \mathbb{P}(X = 1) + \mathbb{P}(X = 2) = \int_1^1 f(x)\mathrm{d}x + \int_2^2 f(x)\mathrm{d}x = 0.$$

Part C

$$\mathbb{E}[X] = \int_0^2 xf(x)\mathrm{d}x = \int_0^2 \frac{x^4}{4}\mathrm{d}x = \frac{8}{5}.$$

Part D

$$\mathbb{P}(0.5 < X < 1.5) = \int_{0.5}^{1.5} \frac{x^3}{4}\mathrm{d}x = \frac{x^4}{16}\Big|_{0.5}^{1.5} = \frac{5}{16}.$$

Part E

$$\mathbb{P}(X > 1.5 | X > 0.5) = \frac{\int_{1.5}^2 x^3 \mathrm{d}x}{\int_{0.5}^2 x^3 \mathrm{d}x} = 0.6863.$$

Problem 3

Part A

$$X \sim \mathrm{Binom}(100, 0.85).$$

Part B

$$\mathbb{E}[X] = 100(0.85) = 85.$$
$$\text{Var}(X) = 100(0.85)(0.15) = 12.75.$$

Part C

$$\mathbb{P}(X \leq 80) = \sum_{n=0}^{80} \binom{100}{n} (0.85)^n (0.15)^{100-n} \approx 0.1065.$$

Part D

$$\mathbb{P}(X \leq 80) = 1 - \mathbb{P}(X > 80) = 1 - \sum_{n=81}^{100} \binom{100}{n} (0.85)^n (0.15)^{100-n}.$$

Part E

$$\mathbb{P}(X \leq 80)^2 = (0.1065)^2 = 0.01134.$$

Part F

No, since the probability of success will change between succesive trials since it improves.

Problem 4

Part A

$$\mathcal{R}_X = \mathbb{N}.$$

Part B

$$\mathbb{E}[X] = \frac{1}{0.85} \approx 1.18.$$

Part C

$$\mathbb{P}(X > 2) = 0.003375.$$

Part D

$$2 \cdot \mathbb{E}[2X] = 4 \cdot \frac{1}{0.85} \approx 9.44.$$

Problem 5

Part A

The poisson distribution can be used. Therefore

$$\mathbb{E}[X] = 100.$$

$$\mathrm{Var}(X) = 100.$$

$$\sigma = \sqrt{100} = 10.$$

Part B

$$\mathbb{P}(X = 100) = \frac{100^{100}}{100!}e^{-100} \approx 0.0399.$$

Part C

$$\mathbb{P}(X \leq 100) = 0.5266.$$

Part D

$$\mathbb{E}[3X] = 3 \cdot 100 = 300.$$

$$\mathrm{Var}(3X) = 9 \cdot 100 = 900.$$

$$\sigma = \sqrt{900} = 30.$$

Part E

$$\mathbb{P}(X = 100) \cdot (0.2)^{100} = 0.0399(0.2)^{100}.$$

Problem 6

Part A

$$\int_0^{10} cx^2 \mathrm{d}x = 1$$

$$\left.\frac{cx^3}{3}\right|_0^{10} = 1$$

$$\frac{1000c}{3} = 1 \implies \boxed{c = \frac{3}{1000}}.$$

Part B

$$\mathbb{P}(X = 5) = \mathbb{P}(X = 5 \text{ or } X = 9) = 0.$$

Part C

$$\mathbb{P}(X > 5) = \int_5^{10} cx^2 \mathrm{d}x = \frac{3}{1000} \cdot \frac{875}{3} = \frac{7}{8}.$$

Part D

$$\begin{aligned} F(a) &= \int_0^a cx^2 \mathrm{d}x \\ &= \frac{3}{1000} \frac{x^3}{3} \Big|_0^a \\ &= \frac{a^3}{1000}. \end{aligned}$$