6.1.5

Once at Denver, I have 6 choices of flights to get to San Francisco from which I have an independent choice of 7 more flights to get to New York. Therefore there are $6 \cdot 7 = 42$ ways for me to get to New York from Denver.

6.1.7

There are 26 choices for each letter meaning there are $26^3 = 17576$ three letter initials.

6.1.9

There is only 1 choice for the first letter (the letter A) and then 26 choices for each remaining initial, hence there are $1 \cdot 26^2 = 676$ such initials.

6.1.23

a) $\lfloor \frac{999}{7} \rfloor - \lceil \frac{100}{7} \rceil + 1 = 128$

b) $(999 - 100 + 1) - \left(\lfloor \frac{999}{7} \rfloor - \lceil \frac{100}{7} \rceil + 1\right) = 450$

c) $\frac{999}{111} = 9$

d) $(999 - 100 + 1) - \left(\lfloor \frac{999}{4} \rfloor - \lceil \frac{100}{4} \rceil + 1\right) = 900 - (249 - 25 + 1) = 675$

e) By the inclusion exclusion principle,

$$\left[\left\lfloor \frac{999}{3} \right\rfloor - \left\lceil \frac{100}{3} \right\rceil + 1\right] + \left[\left\lfloor \frac{999}{4} \right\rfloor - \left\lceil \frac{100}{4} \right\rceil + 1\right] - \left[\left\lfloor \frac{999}{12} \right\rfloor - \left\lceil \frac{100}{12} \right\rceil + 1\right] = 450.$$

f) $(999 - 100 + 1) - 450 = 450$

g) $\left[\lfloor \frac{999}{3} \rfloor - \lceil \frac{100}{3} \rceil + 1\right] - \left[\lfloor \frac{999}{12} \rfloor - \lceil \frac{100}{12} \rceil + 1\right] = 225$

h) $\left[\lfloor \frac{999}{12} \rfloor - \lceil \frac{100}{12} \rceil + 1\right] = 75$

6.1.27

There are 3 choices of representative for each of the 50 states, hence 3^{50} choices.

6.1.29

There are $26^2 \cdot 10^4 + 10^2 \cdot 26^4 = 52,457,600$ such license plates.

6.1.33

a) $(26-5)^8 = 37,822,859,361$

b) $\frac{21!}{(21-8)!} = 8,204,716,800$

c) $5 \cdot 26^7 = 40,159,050,880$

d) $5 \cdot \frac{25!}{(25-7)!} = 12,113,640,000$

e) $26^8 - 21^8 = 171,004,205,215$

f) $8 \cdot 5 \cdot 21^7 = 72,043,541,640$

g) $26^7 - 21^7 = 6,230,721,635$

h) $26^6 - 21^6 = 223,149,655$

6.1.35

a) There are none

b) $5! = 120$

c) $\frac{6!}{(6-5)!} = 6! = 720$

d) $\frac{7!}{(7-5)!} = 2520$

6.3.1

$$\begin{array}{ccc} \{a,b,c\} & \{b,a,c\} & \{c,a,b\} \\ \{c,b,a\} & \{b,c,a\} & \{a,c,b\} \end{array}$$

6.3.3

There are $6! = 720$ permutations.

6.3.5

a) $P(6,3) = \frac{6!}{3!} = 120$

b) $P(6,5) = 6! = 720$

c) $P(8,1) = 8$

d) $P(8,5) = 8(7)(6)(5)(4) = 6720$

e) $P(8,8) = 8! = 40,320$

f) $P(10,9) = 10! = 3,628,800$

6.3.9

There are $P(12,3) = 12(11)(10) = 1320$ podium placements.

6.3.13

There are $n!$ to arrange the row of men and $n!$ the row of women, meaning interleaving them gives $(n!)^2$ possible seating arrangements.

6.3.17

This is the same as the number of total subsets minus the number of subsets with $0,1,2$ elements. There is 1 subset with 0 elements, $C(100,1) = 100$ subsets with 1 element, and $C(100,2) = 4950$ subsets with 2 elements. Therefore there are

$$2^{100} - 4950 - 100 - 1 = 2^{100} - 5051$$

subsets with more than 2 elements.

6.3.21

a) The remaining letters have 4! permutations and the string "BCD" has 5 possible positions, hence there are $5 \cdot 4! = 120$ possible strings

b) The remaining letters have 3! permutations and the string "CFGA" has 4 possible positions, hence there are $4 \cdot 3! = 24$ possible strings

c) The remaining letters have 3! permutations and the two strings have $\frac{5!}{3!} = 20$ possible positions, hence there are $20 \cdot 3! = 120$ possible strings

d) The remaining letters have $2! = 2$ permutations and the two strings have $\frac{4!}{2!}$, hence there are $2 \cdot \frac{4!}{2!} = 24$ possible strings

e) Since each string has a common letter, the only possible strings are those with "ABCDE" in it, which is $\frac{3!}{2!} = 3$ possible strings

f) There are no strings since the substrings have a shared letter that isn't at the beginning or end

6.3.23

There are 9 possible places that the women can go after the men arrange themselves. Therefore there are $8! \cdot P(9,5) = 8! \cdot \frac{9!}{4!} = 609,638,400$ arrangements.

6.3.29

a) There are $C(25,4) = \frac{25!}{4!(21!)} = 12650$ ways to form the committee

b) There are $P(25,4) = \frac{25!}{21!} = 303,600$ ways to pick the roles

6.4.1

$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4.$$

6.4.7

By the binomial theorem,

$$(2-x)^{19} = \sum_{k=0}^{19} \binom{19}{k} 2^{19-k} x^k (-1)^k.$$

Therefore the coefficient of x^9 is $\binom{19}{9} 2^{19-9}(-1)^9 = -94,595,072$

6.4.9

By the binomial theorem,

$$(2x-3y)^{200} = \sum_{k=0}^{200} \binom{200}{k} (2n)^{200-k} (3y)^k (-1)^k.$$

Therefore the coefficient of $x^{101}y^{99}$ is $-\binom{200}{99} 2^{101} 3^{99}$

6.4.11

$$\left(3x^4 - 2y^3\right)^5 = 243x^{20} - 810x^{16}y^3 + 1080x^{12}y^6 - 720x^8y^9 + 240x^4y^{12} - 32y^{15}.$$

9.1.1

a) $R = \{(0,0),(1,1),(2,2),(3,3)\}$

b) $R = \{(1,3),(3,1),(2,2),(4,0)\}$

c) $R = \left\{\begin{matrix} (1,0), & (2,0), & (3,0), & (4,0) \\ (2,1), & (3,1), & (4,1), & \\ (3,2), & (4,2), & & \\ (4,3) & & & \end{matrix}\right\}$

d) $R = \left\{\begin{matrix} (1,0), & (1,1), & (1,2), & (1,3) \\ (2,0), & (2,2), & & \\ (3,0), & (3,3) & & \end{matrix}\right\}$

e) $R = \left\{\begin{matrix} (0,1), & & & \\ (1,0), & (1,1), & (1,2), & (1,3), \\ (2,1), & (2,3), & & \\ (3,1), & (3,2), & & \\ (4,1), & (4,3) & & \end{matrix}\right\}$

f) $R = \{(1,2),(2,1),(2,2)\}$

9.1.3

a) Transitive

b) Reflexive, symmetric, transitive

c) Symmetric

d) Antisymmetric

e) Reflexive, symmetric, antisymmetric, transitive

f) None

9.1.7

a) Symmetric

b) Symmetric, transitive

c) Symmetric

d) Reflexive, symmetric, transitive

e) Reflexive, transitive

f) Reflexive, symmetric, transitive

g) Antisymmetric

h) Antisymmetric, transitive

9.1.9

Since the empty set has no elements, any statement over the elements of it is true. Therefore it satisfies being reflexive, symmetric, and transitive.

9.1.53

Proof. Consider the two directions.

$\Rightarrow$) Assume that R is symmetric. Then for $(a, b) \in R$ we have $(b, a) \in R$. Therefore $(a, b) \in R^{-1}$, hence $R \subseteq R^{-1}$. By the same logic, $R^{-1} \subseteq R$ meaning $R = R^{-1}$

$\Leftarrow$) Assume that $R = R^{-1}$. If $(a, b) \in R$, then $(a, b) \in R^{-1}$ meaning $(b, a) \in R$. Therefore R is symmetric.

Both directions hence establish the if and only if. ■

9.1.55

Proof. Consider the two directions.

$\Rightarrow$) Assume that R is reflexive. Then $(a, a) \in R$. Since (a, a) equals itself, $(a, a) \in R^{-1}$. Therefore R^{-1} is reflexive.

$\Leftarrow$) The same logic as the forward direction applies with the role of R and R^{-1} swapped.

Both directions hence establish the if and only if. ■

9.1.57

Proof. We proceed with induction. The base of $n = 1$ holds since $R^1 = R$. Fix $n \in \mathbb{N} \geq 1$ and assume that $R^n = R$. Note that $R^n \subseteq R$. Let $(a, b) \in R$. By the inductive hypothesis, we have $(a, b) \in R^n$ and since R is reflexive, $(b, b) \in R^{n+1}$. Therefore $(a, b) \in R^{n+1}$ meaning $R \subseteq R^{n+1}$. Hence $R = R^{n+1}$ which was to be shown. ■

9.2.1

$$R = \{(1, 2, 3), (1, 2, 4), (1, 3, 4), (2, 3, 4)\}.$$

9.2.5

Airline and flight number, airline and departure time

9.2.7

a) Yes since it is a managed unique identifier

b) No

c) No since it is subject to uncontrollable change

9.2.9

a) Social Security Number

b) There are no two people with the same name and address

c) There are no two people living in the same place

9.2.11

(Nadir, 122, 34, Detroit, 08:10), (Nadir, 199, 13, Detroit, 08:47), (Nadir, 322, 34, Detroit, 09:44)

9.2.17

Airline	Destination
Nadir	Detroit
Acme	Denver
Acme	Anchorage
Acme	Honolulu

9.2.19

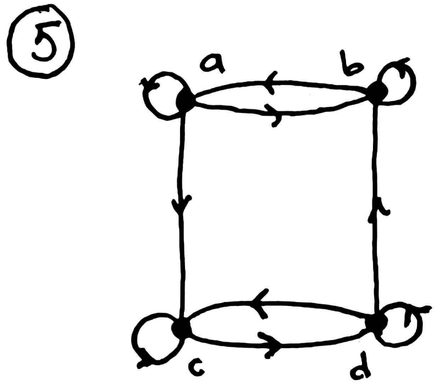
Supplier	Part #	Project	Quantity	Color Code
23	1092	1	2	2
23	1101	3	1	1
23	9048	4	12	2
31	4975	3	6	2
31	3477	2	25	2
32	6984	4	10	1
32	9191	2	80	4
33	1001	1	14	8

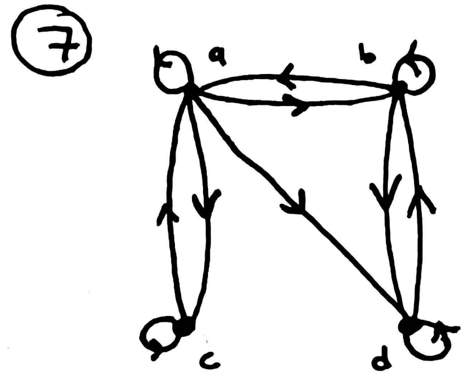
9.4.1

a) $R = \{(0,0),(1,1),(2,2),(3,3),(0,1),(1,1),(1,2),(2,0),(2,2),(3,0)\}$

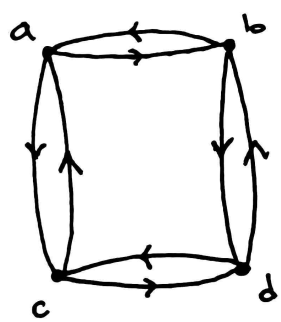
b) $R = \{(0,1),(1,1),(1,2),(2,0),(2,2),(3,0),(1,0),(2,1),(0,2),(0,3)\}$

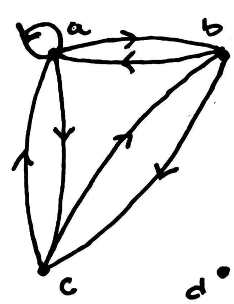
9·4·{5,7}

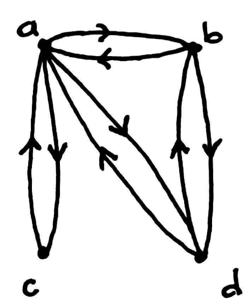




9·4·9







9·4·19

a) $R^2 = \{(1,1),(1,5),(2,3),(3,3),(3,1),(3,2),(3,4),(4,1),(4,5),(5,3),(5,4)\}$

b) $R^3 = \left\{\begin{array}{llll} (1,1), & (1,2), & (1,3), & (1,4), \\ (2,1), & (2,5), & & \\ (3,1), & (3,3), & (3,4), & (3,5), \\ (4,1), & (4,2), & (4,3), & (4,4), \\ (5,1), & (5,3), & (5,5) & \end{array}\right\}$

c) $R^4 = \left\{\begin{array}{lllll} (1,1), & (1,3), & (1,4), & (1,5), & \\ (2,1), & (2,2), & (2,3), & (2,4), & \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), \\ (4,1), & (4,3), & (4,4), & (4,5), & \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5) \end{array}\right\}$

d) $R^5 = \begin{Bmatrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), \\ (2,1), & (2,3), & (2,4), & (2,5), & \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5) \end{Bmatrix}$

e) $R^6 = \begin{Bmatrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5) \end{Bmatrix}$

f) $R^* = \begin{Bmatrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5) \end{Bmatrix}$

9.4.25

a) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

9.4.27

a) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

9.5.1

a) Equivalence relation

b) Not reflexive and not transitive

c) Equivalence relation

d) Not transitive

e) Not symmetric, not transitive

9.5.9

Part A

Proof. Examining if R is an equivalence relation necessitates checking 3 conditions

1. If $x \in A$, note that $f(x) = f(y)$ and therefore $(x, x) \in R$. Hence R is reflexive.
2. Let $x, y \in A$. If $(x, y) \in R$, then $f(x) = f(y)$. Since equality is symmetric, $(y, x) \in R$ and R is hence symmetric.
3. Let $x, y, z \in A$ with $(x, y) \in R$ and $(y, z) \in R$. Then $f(x) = f(y)$ and $f(y) = f(z)$. Therefore $f(x) = f(z)$ meaning $(x, z) \in R$. Hence R is transitive.

Since R is reflexive, symmetric, and transitive, it is an equivalence relation. ■

Part B

Its the set of all pre-images of elements in the co-domain

9.5.27

a) The sets of people of the same age

b) The sets of people with the same two parents

9.6.1

a) Partial ordering

b) Not antisymmetric, not transitive

c) Partial ordering

d) Partial ordering

e) Not antisymmetric, not transitive

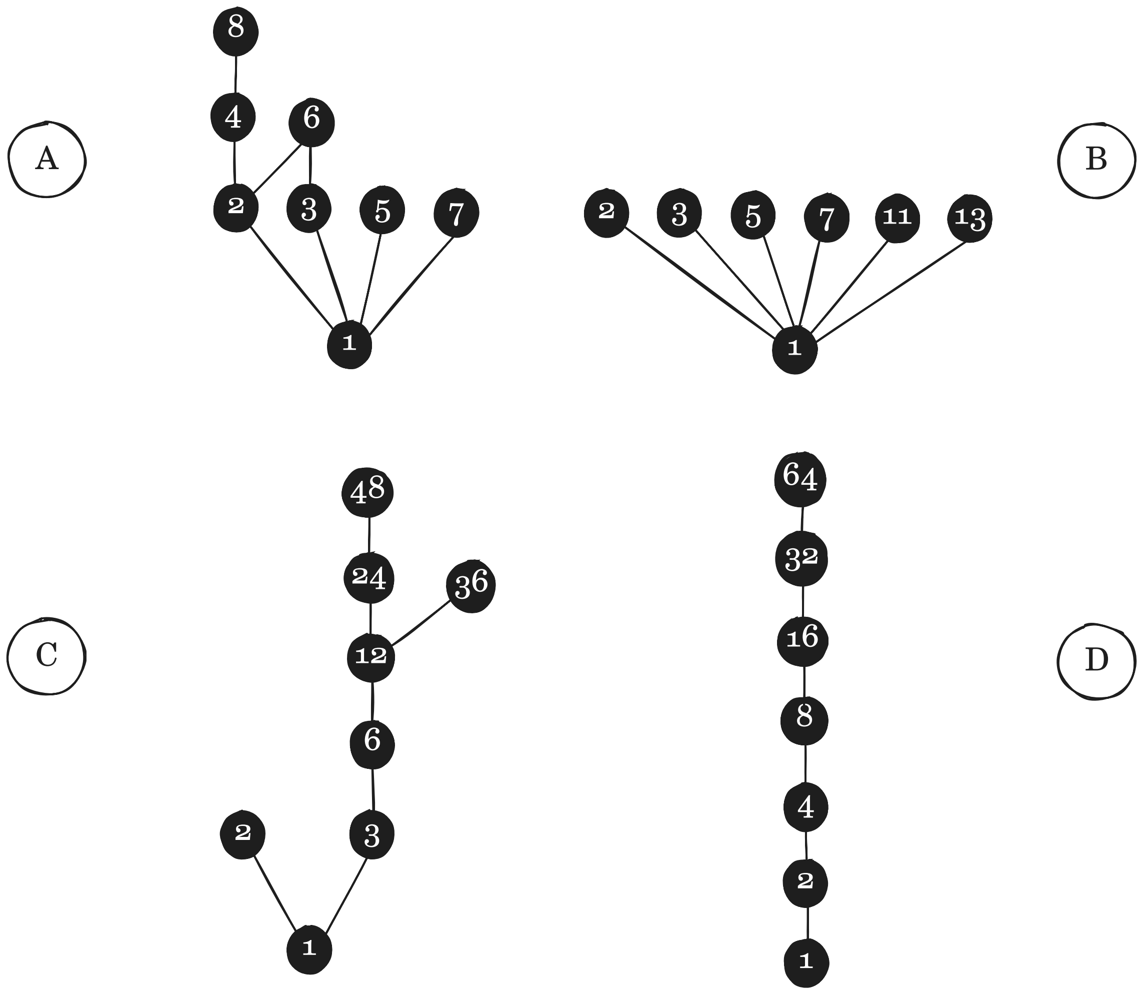
9.6.7

a) Not antisymmetric $\implies$ No

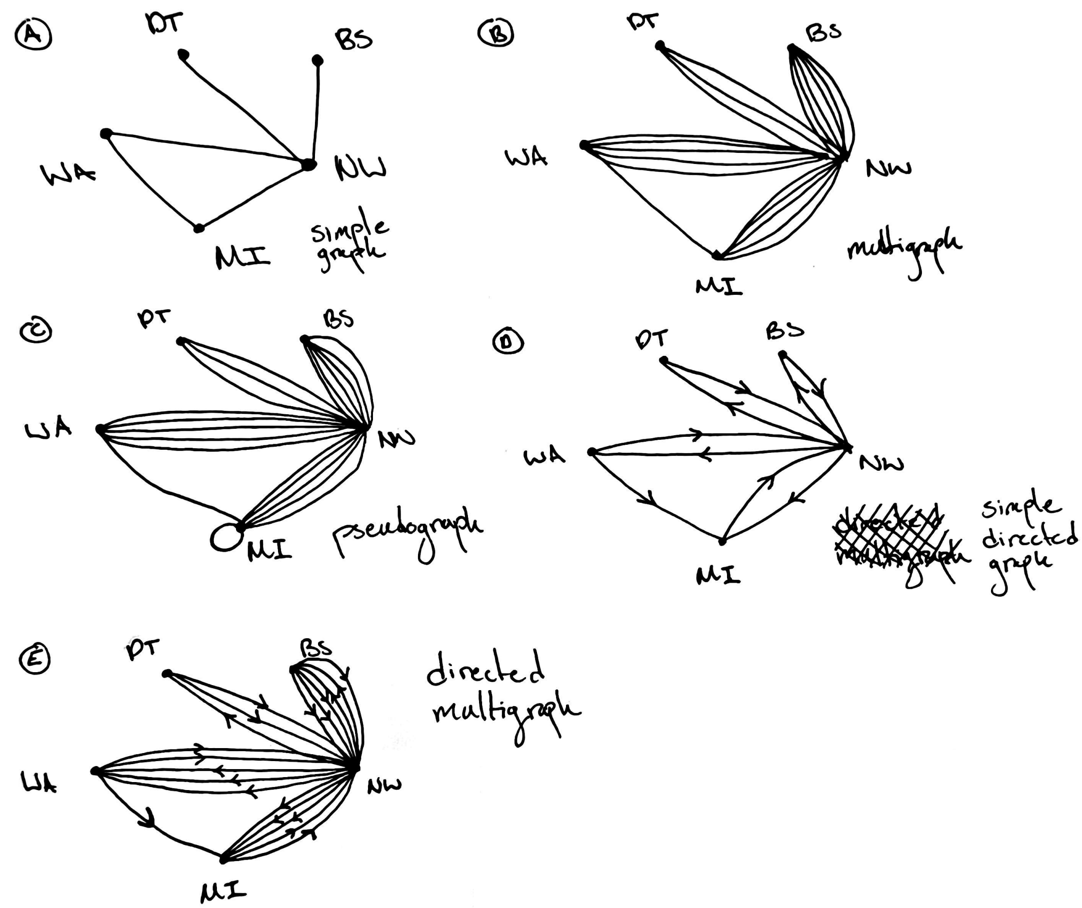
b) Yes

c) No transitive $\implies$ No

9.6.23



10.1.1



10.1.5

The edges are undirected, there are multiple edges, and there are loops. Therefore the graph is a pseudograph

10.1.9

The edges are directed, there are multiple edges, and there are loops. Therefore the graph is a directed multigraph.