Functions

1.1 Introduction to Functions

Definition 1 (Function)**.** A function f is a *'rule'* that assigns elements from a domain set A to a codomain set B with a **one-to-one** correspondence.

If a function f maps elements from a set A to a set B, then it is notated as

$$f : A \to B.$$

When considering an element $a \in A$ and its corresponding functional mapping $b \in B$, we say that $b = f(a)$ and also that

$$f : a \mapsto b.$$

which reads as f maps a to b.

1.2 Classification

There are three important classifications of functions.

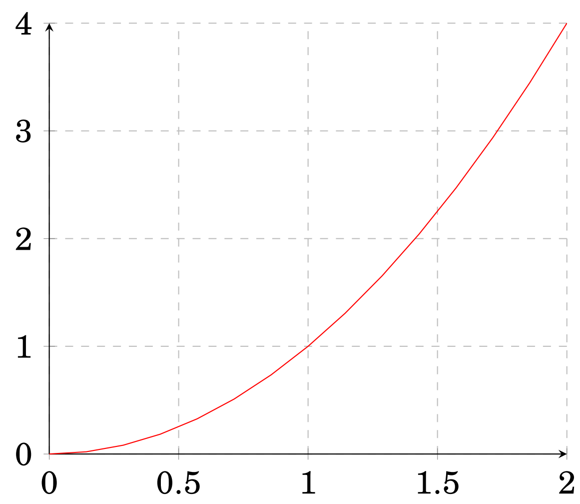
Definition 2 (Injectivity)**.** A function $f : A \to B$ is considered injective, an injection, or one-to-one if it never has the same output twice. Equivalaltenly

$$\forall a_1, a_2 \in A, f(a_1) = f(a_2) \implies a_1 = a_2.$$

Consider the function

$$f : [0,2] \to \mathbb{R} : x \mapsto x^2.$$

Graphically it is obvious that for every y-value, there is only a singularlly associated x-value. However, the function f maps to all the real numbers. Consider an output of 16. This would require an input of 4, however that is outside the range. We say that in this case that f is not *surjective*.



Theorem 1 (Function Cardinality)**.** All of the following statements are equivalent.

1. $|A| \leq |B|$
2. $\exists f : A \to B, f$ is injective
3. $\exists f : B \to A, f$ is surjective