0.1 Different properties of Laplace Transforms

In certain cases, a function may be related to a simpler function through a change of basis via a linear transformation of the input. Examples of this are

$$f(kt) \qquad F(ks) \qquad F(as+b)$$

Consider $\mathcal{L}\{f(kt)\}$ with the restriction that $s > a \geq 0$ and $k > 0$

$$\mathcal{L}\{f(kt)\} = \int_0^\infty e^{-st} f(kt)\,\mathrm{d}t$$

Let $u = kt \implies \mathrm{d}u = k\,\mathrm{d}t$

$$= \frac{1}{k}\int_0^\infty e^{-\frac{su}{k}} f(u)\,\mathrm{d}u$$

Let $\omega = \frac{s}{k}$

$$\begin{aligned} &= \frac{1}{k}\int_0^\infty e^{-\omega u} f(u)\,\mathrm{d}u \\ &= \frac{1}{k}F(\omega) \\ &= \frac{1}{k}F\left(\frac{s}{k}\right);\ s > ka. \end{aligned}$$

0.2 Power Series

For many, many functions, they can be represented as a long form polynomial known as a power series. They have the general form of

$$f(x) = \sum_{n=1}^{\infty} a_k (x - x_0)^k.$$

Some famous power series are as follows

$$\begin{aligned} e^x &= \sum_{n=0}^{\infty} \frac{1}{n!}x^n = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \ldots \\ \cos(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{2n!}x^{2n} = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \ldots \\ \sin(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1} = x - \frac{x^3}{6} + \frac{x^5}{120} - \ldots. \end{aligned}$$