Eigenvalues and Eigenvectors

1.1 Eigenvalues

Definition 1.1 (Eigenvalue). Let $A \in M_{n\times n}(\mathbb{F})$. Then $\lambda \in \mathbb{F}$ is an eigenvalue iff there is $v \in \mathbb{F}^n \setminus \{0\}$ such that

$$Av = \lambda v \Leftrightarrow \det(\lambda I_n - A) = 0.$$

Definition 1.2 (Characteristic Polynomial). The characteristic polynomial of $A \in M_{n\times n}(\mathbb{F})$ is

$$P_A(t) = \det(tI_n - A) = t^n + a_1 t^{n-1} + \ldots + a_n.$$