1.1.2

a) $(-5,7,1)-(3,-2,4)=(-8,9,-3)$. Therefore the line is $\vec{x}(t)=(3,-2,4)+t(-8,9,-3)$.

b) $(-3,-6,0)-(2,4,0)=(-5,-10,0)$. Therefore the line is $\vec{x}(t)=(2,4,0)+t(-5,-10,0)$.

c) $(3,7,-8)-(3,7,2)=(0,0,-10)$. Therefore the line is $\vec{x}(t)=(3,7,-8)+t(0,0,-10)$.

d) $(3,9,7)-(-2,-1,5)=(5,10,2)$. Therefore the line is $\vec{x}(t)=(3,9,7)+t(5,10,2)$.

1.1.3

a) Let $A=(2,-5,-1), B=(0,4,6), C=(-3,7,1)$. Then $B-A=(-2,9,7)$ and $C-A=(-5,12,2)$. Therefore the plane equation is $\vec{x}(s,t)=(2,-5,-1)+s(-2,9,7)+t(-5,12,2)$.

b) Let $A=(3,-6,7), B=(-2,0,-4), C=(5,-9,2)$. Then $B-A=(-5,6,-11)$ and $C-A=(2,-3,-5)$. Therefore the plane equation is $\vec{x}(s,t)=(3,-6,-7)+s(-5,6,-11)+t(2,-3,-5)$.

c) Let $A=(-8,2,0), B=(1,3,0), C=(6,-5,0)$. Then $B-A=(9,1,0)$ and $C-A=(14,-7,0)$. Therefore the plane equation is $\vec{x}(s,t)=(-8,2,0)+s(9,1,0)+t(14,-7,0)$.

d) Let $A=(1,1,1), B=(5,5,5), C=(-6,4,2)$. Then $B-A=(4,4,4)$ and $C-A=(-7,3,1)$. Therefore the plane equation is $\vec{x}(s,t)=(1,1,1)+s(4,4,4)+t(-7,3,1)$.

1.2.1

a) True
b) False
c) False
d) False
e) True
f) False
g) False
h) False
i) True
j) True
k) True

1.2.13

It is not a vector space. The zero vector must be $0(a_1,a_2)=(0,a_2)$ for all $a_1,a_2\in\mathbb{F}$. However since a_2 is variable, the zero vector is not unique and therefore V is not a vector space.

1.2.17

It is not a vector space. The zero vector must be $0(a_1,a_2)=(a_1,0)$ for all $a_1,a_2\in\mathbb{F}$. This fails the same way the last question does. Therefore V is not a vector space.

1.2.19

It is not a vector space. If V were a vector space, then $(2+3)\cdot(3,4) = 2\cdot(3,4)+3\cdot(3,4)$. Calculating both sides:

$$\begin{aligned}(2+3)\cdot(3,4) &= 5\cdot(3,4)\\ &= (15, \frac{4}{5})\end{aligned}$$

and

$$\begin{aligned}(2+3)\cdot(3,4) &= 2\cdot(3,4)+3\cdot(3,4)\\ &= (6,2)+(9,\frac{4}{3})\\ &= (15,\frac{10}{3})\end{aligned}$$

Since $(15, \frac{10}{3}) \neq (15, \frac{4}{5})$, distributivity doesn't hold and so V is not a vector space.

1.3.1

a) False

b) False

c) True

d) False

e) True

f) False

g) False

1.3.5

Proof. Let A be a square matrix. Then

$$(A + A^\intercal)^\intercal = A^\intercal + A = A + A^\intercal$$

Therefore since $(A + A^\intercal)^\intercal = A + A^\intercal$, $A + A^\intercal$ is a symmetric matrix. ■

1.3.12

Proof. Let $W = \{m\times n \text{ upper triangular matrices}\}$. Note that $W \subset M_{m\times n}(\mathbb{F})$. Let $w, k \in W$. $k + w$ will result in an upper triangular matrix since every element below the main diagonal is zero in both and therefore their sum below the main diagonal will be zero. Therefore $k + w \in W$. Let $c \in \mathbb{F}$. $c \cdot w$ will give an upper triangular matrix since every zero element below the main diagonal multiplied by a constant will remain zero, resulting in an upper triangular matrix. Therefore $cw \in W$. Note that the zero matrix is upper triangular and hence is in W. Since W is closed under addition and scalar multiplication and the zero matrix is in W, W is a subspace of $M_{m\times n}(\mathbb{F})$. ■

1.3.22

Let $\mathbb{F}_1$ and $\mathbb{F}_2$ be fields. First consider even functions:

Proof. Let $E = \{g \text{ is even} : g \in \mathcal{F}(\mathbb{F}_1, \mathbb{F}_2)\}$. Note that $E \subset \mathcal{F}(\mathbb{F}_1, \mathbb{F}_2)$. Let $f, g \in E$. Let $h(t) = f(t) + g(t)$ where $t \in \mathbb{F}_1$. Note that

$$h(-t) = f(-t) + g(-t) = f(t) + g(t) = h(t)$$

Therefore $h(t)$ is also an even function and hence $h \in E$. Let $c \in \mathbb{F}_2$ and $j(t) = c \cdot f(t)$. Since $j(-t) = c \cdot f(-t) = c \cdot f(t) = j(t)$, $j(t)$ is an even function and hence $j \in E$. Additionally, the zero function is even and odd so it is in E. Since E is closed under addition and scalar multiplication and contains the zero function, E is a subspace of $\mathcal{F}(\mathbb{F}_1, \mathbb{F}_2)$. ■

Next consider odd functions:

Proof. Let $O = \{g \text{ is odd} : g \in \mathcal{F}(\mathbb{F}_1, \mathbb{F}_2)\}$. Note that $O \subset \mathcal{F}(\mathbb{F}_1, \mathbb{F}_2)$. Let $f, g \in O$. Let $h(t) = f(t) + g(t)$ where $t \in \mathbb{F}_1$. Note that

$$h(-t) = f(-t) + g(-t) = -f(t) - g(t) = -(f(t) + g(t)) = -h(t)$$

Therefore $h(t)$ is also an odd function and hence $h \in O$. Let $c \in \mathbb{F}_2$ and $j(t) = c \cdot f(t)$. Note that $j(-t) = c \cdot f(-t) = -c \cdot f(t) = -j(t)$. Therefore j is an odd function and hence $j \in O$. Additionally, the zero function is even and odd so it is in O. Since O is closed under addition and scalar multiplication and contains the zero function, O is a subspace of $\mathcal{F}(\mathbb{F}_1, \mathbb{F}_2)$. ■