Null Space of Block Diagonal Matrix

Theorem 1. *Given a block diagonal matrix*

$$M = \begin{pmatrix} A_1 & & 0 \\ & \ddots & \\ 0 & & A_m \end{pmatrix}$$

then $\ker(M) = \bigcap_{i=1}^{m} \ker(A_i)$

Proof. Consider $v \in \ker(M)$. Then $Mv = 0$ and

$$Mv = \begin{pmatrix} A_1 v \\ \vdots \\ A_m v \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}.$$

Therefore $A_i v = 0$ for each A_i, hence $v \in \ker(A_i)$. □