Problem 1

The possible values of X are

$$X = \{-2, -1, 0, 1, 2, 4\}$$

with associated probabilities

$$\begin{aligned}
\mathbb{P}(X = -2) &= \frac{\binom{8}{2}}{\binom{14}{2}} = \frac{28}{91} \\
\mathbb{P}(X = -1) &= \frac{\binom{8}{1}\binom{2}{1}}{\binom{14}{2}} = \frac{16}{91} \\
\mathbb{P}(X = 0) &= \frac{\binom{2}{2}}{\binom{14}{2}} = \frac{1}{91} \\
\mathbb{P}(X = 1) &= \frac{\binom{4}{1}\binom{8}{1}}{\binom{14}{2}} = \frac{32}{91} \\
\mathbb{P}(X = 2) &= \frac{\binom{4}{1}\binom{2}{1}}{\binom{14}{12}} = \frac{8}{91} \\
\mathbb{P}(X = 4) &= \frac{\binom{4}{2}}{\binom{14}{2}} = \frac{6}{91}.
\end{aligned}$$

Problem 2

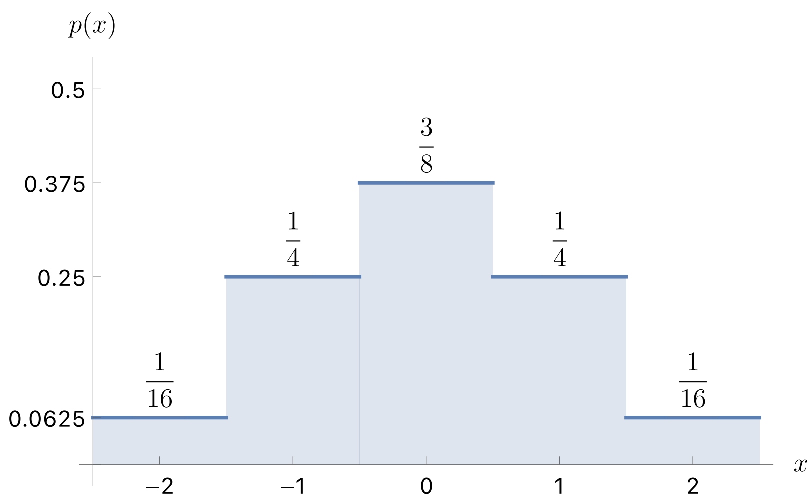
The possible values for X are

$$X = \{n - 2i : i = 0, 1, 2, \ldots, n\}.$$

Problem 3

i	$\mathbb{P}(X = i)$
1	$\frac{1}{2}$
2	$\frac{5}{10} \cdot \frac{5}{9} = \frac{5}{18}$
3	$\frac{5}{10} \cdot \frac{4}{9} \cdot \frac{5}{8} = \frac{5}{36}$
4	$\frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} \cdot \frac{5}{7} = \frac{5}{84}$
5	$\frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{5}{6} = \frac{5}{252}$.
6	$\frac{5}{10} \cdot \frac{4}{9} \cdot \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{1}{6} = \frac{1}{252}$
7	0
8	0
9	0
10	0

Problem 4



Problem 5

Let $(n_1, n_2, n_3, n_4, n_5)$ denote an order of the players where n_1 is the player with the largest number and n_5 the player with the smallest.

$$
\begin{aligned}
\mathbb{P}(X=0) = \quad & \mathbb{P}(2 \text{ beats } 1) && = \frac{(1+2+3+4)\cdot 3!}{5!} = \frac{1}{2} \\
\mathbb{P}(X=1) = \quad & \mathbb{P}((3,1,2,\ldots)) && = \frac{1}{3}\cdot\frac{1}{2} = \frac{1}{6} \\
\mathbb{P}(X=2) = \quad & \mathbb{P}((4,1,\ldots)) && = \frac{1}{4}\cdot\frac{1}{3} = \frac{1}{12} \\
\mathbb{P}(X=3) = \quad & \mathbb{P}((5,1,\ldots)) && = \frac{1}{5}\cdot\frac{1}{4} = \frac{1}{20} \\
\mathbb{P}(X=5) = \quad & \mathbb{P}((1,\ldots)) && = \frac{1}{5}
\end{aligned}
$$

Problem 6

$$\mathbb{P}(X=i|X>0) = \frac{\mathbb{P}(X=i)}{\mathbb{P}(X>0)}.$$

The probability of getting a positive amount is

$$\mathbb{P}(X>0) = \sum_{n=1}^{3}\mathbb{P}(X=i) = p(1)+p(2)+p(3) = \frac{13}{55}+\frac{1}{11}+\frac{1}{165} = \frac{1}{3}.$$

Thefore

$$
\begin{aligned}
\mathbb{P}(X=1|X>0) = \quad & \frac{\frac{13}{55}}{\frac{1}{3}} && = \frac{39}{55} \\
\mathbb{P}(X=2|X>0) = \quad & \frac{\frac{1}{11}}{\frac{1}{3}} && = \frac{3}{11} \\
\mathbb{P}(X=3|X>0) = \quad & \frac{\frac{1}{165}}{\frac{1}{3}} && = \frac{1}{55}
\end{aligned}
$$

Problem 7

$$
\begin{aligned}
\mathbb{P}(X=0) &= \binom{6}{0}\left(\frac{1}{2}\right)^6 &= \frac{1}{64} \\
\mathbb{P}(X=1) &= \binom{6}{1}\left(\frac{1}{2}\right)^6 &= \frac{6}{64} \\
\mathbb{P}(X=2) &= \binom{6}{2}\left(\frac{1}{2}\right)^6 &= \frac{15}{64} \\
\mathbb{P}(X=3) &= \binom{6}{3}\left(\frac{1}{2}\right)^6 &= \frac{20}{64} \\
\mathbb{P}(X=4) &= \binom{6}{4}\left(\frac{1}{2}\right)^6 &= \frac{15}{64} \\
\mathbb{P}(X=5) &= \binom{6}{5}\left(\frac{1}{2}\right)^6 &= \frac{6}{64} \\
\mathbb{P}(X=6) &= \binom{6}{6}\left(\frac{1}{2}\right)^6 &= \frac{1}{64}
\end{aligned}.
$$

By inspection $X = 3$ is the most likely outcome.

Problem 8

$$
p(x) = \mathbb{P}(X = x) = \binom{3}{x}(0.7)^x(0.3)^{3-x}.
$$

Problem 9

Let X denote the number of 6's rolled by 3 fair dice. Note that

$$
X \sim \text{Binom}\left(3, \frac{1}{6}\right).
$$

The probability that at most one 6 is rolled is

$$
\mathbb{P}(X \leq 1) = \mathbb{P}(X = 0) + \mathbb{P}(X = 1).
$$

Since X follows a binomial distribution,

$$
\begin{aligned}
\mathbb{P}(X \leq 1) &= \mathbb{P}(X = 0) + \mathbb{P}(X = 1) \\
&= \binom{3}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^3 + \binom{3}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^2 \\
&= \left(\frac{5}{6}\right)^3 + \frac{1}{2}\left(\frac{5}{6}\right)^2 \\
&= \frac{25}{27}.
\end{aligned}
$$

Problem 10

Let X denote the number of multiple choice questions the student gets right. Note that

$$X \sim \text{Binom}\left(5, \frac{1}{3}\right).$$

The probability that the student gets four or more questions by guessing is

$$\mathbb{P}(X \geq 4) = \mathbb{P}(X = 4) + \mathbb{P}(X = 5).$$

Since X follows a binomial distribution,

$$\begin{aligned}
\mathbb{P}(X \leq 4) &= \mathbb{P}(X = 4) + \mathbb{P}(X = 5) \\
&= \binom{5}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^1 + \binom{5}{5}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^0 \\
&= \frac{10}{243} + \frac{1}{243} \\
&= \frac{11}{243}.
\end{aligned}$$