Permutations

1.1 Groups of Permutations

Theorem 1.1 (Symmetric Groups). Let A be a set and define $S_A = \{\phi : \phi : A \to A,\ \text{one-to-one and onto}\}$. With S_A equipped with the binary operation of composition, $\langle S_A, \circ \rangle$ is a group.

Consider the basic example where $A = \{1, 2, 3\}$. Consider an example element $\phi \in S_A$. It can be defined in the following way

$$\begin{aligned} \phi(1) &\to 1 \\ \phi(2) &\to 3 \\ \phi(3) &\to 2. \end{aligned}$$

Something of interest is a map from S_A can also be naturally expressed as a matrix like the following

$$\phi = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}.$$