Problem 1

Part A

$$\begin{aligned} P(-) &= P(F) \cdot P(-|F) + P(F^{\complement}) \cdot P(-|F^{\complement}) \\ &= 0.1 \cdot 0.9 + 0.9 \cdot 0.05 \\ &= 0.1 \cdot 0.9 + 0.9 \cdot 0.05 \\ &= 0.135 = 13.5\%. \end{aligned}$$

Part B

$$\begin{aligned} P(F|-) &= \frac{P(-|F) \cdot P(F)}{P(-)} \\ &= \frac{0.9 \cdot 0.1}{0.135} \\ &= \frac{0.9 \cdot 0.1}{0.135} \\ &= \frac{2}{3} \approx 66.7\%. \end{aligned}$$

Part C

They are not independent since $P(F|-) \neq P(F)$.

Problem 2

$$\begin{aligned} P(\text{System Works}) &= P(\text{Comp. 1 Works}) \cdot P(\text{Comp. 2 Works}) \\ &= P(\geq 1 \text{ Unit works}) \cdot P(\geq 1 \text{ Unit works}) \\ &= P(\geq 1 \text{ Unit works})^2 \\ &= (1 - P(\text{No units work}))^2 \\ &= \left(1 - p^2\right)^2. \end{aligned}$$

Problem 3

Part A

There are 36 elements in the sample space because each ordered pair has 2 choices of any number from 1 to 6, hence there are $6^2 = 36$ choices possible. Each element is equally likely since the first outcome of a roll does not influence the outcome of the second roll. Therefore the probability of rolling a given ordered pair will be $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$.

Part B

$$\begin{aligned} P(S \leq 11) &= 1 - P(S = 12) \\ &= 1 - \frac{1}{36} \\ &= \frac{35}{36} \\ &= 0.97\overline{2} \approx 97.2\%. \end{aligned}$$

Part C

$$\begin{aligned} P(\text{Both Even}) &= 3^2 \cdot \frac{1}{36} \\ &= \frac{9}{36} \\ &= 0.25 = 25\%. \end{aligned}$$

Problem 4

The simulation results in $P(S \leq 11) = 0.9723$, which is close to the value derived in Part B.

Problem 5

$$\frac{10!}{3!3!2!} = 50,400 \text{ ways to rearrange STATISTICS.}$$

Problem 6

Part A

Each passenger can go to one of 7 floors and there are 5 passengers, therefore there are $7^5 = 16,807$ ways for the passengers to be assigned a floor.

Part B

$$\frac{7!}{2!} = 2,520 \text{ ways to assign a unique floor to each passenger.}$$

Part C

$$P(\text{Each Passenger on Unique Floor}) = \frac{2,520}{16,807} = 0.14994 \approx 15\%.$$

Problem 7

Part A

$$26^4 \cdot 10^4 = 4,569,760,000 \text{ different UCInetIDs.}$$

Part B

$(10)(26)^4(9)(8)(7) = 2,303,159,040$ different UCInetIDs without numerical repetition.

Problem 8

Part A

$$P(\text{Sophmores All Chosen}) = \frac{\binom{3}{3}\binom{9}{2}}{\binom{12}{5}} = 0.0\overline{45} \approx 4.55\%.$$

Part B

$$\begin{aligned} P(\text{At Least One Freshman}) &= 1 - P(\text{No Freshmen}) \\ &= 1 - \frac{\binom{10}{5}}{\binom{12}{5}} \\ &= 1 - 0.3\overline{18} \\ &= 0.6\overline{81} \approx 68.2\%. \end{aligned}$$