Series

1.1 Complex Sequences

The concept of a complex sequence is a direct translation of a real sequence and due to the next theorem, the can be treated as component wise sequences.

Definition 1.1 (Sequence Convergence). A sequence $\{z_n\} \subset \mathbb{C}$ converges to some $z \in \mathbb{C}$ if

$$\forall \epsilon > 0, \exists N \in \mathbb{N} \text{ s.t. } |z_n - z| < \epsilon, n > N.$$

This is written as $\lim_{n\to\infty} z_n = z$.

Theorem 1.1 (Sequence Component Convergence). A complex sequence $\{z_n\}$ converges to z if and only if

$$\lim_{n\to\infty} \operatorname{Re}\{x_n\} = \operatorname{Re} z \qquad \lim_{n\to\infty} \operatorname{Im}\{z_n\} = \operatorname{Im} z.$$

1.2 Series

Definition 1.2 (Series Convergence). A series

$$\sum_{n=0}^{\infty} z_n = z_1 + z_2 + z_3 + \dots.$$

converges to some $z \in \mathbb{C}$ if $\lim S_N = z$ where

$$S_N = \sum_{n=0}^{N} z_n$$

is the sequence of partial sums. Often, it is useful to define

$$\rho_N := z - S_N$$

which is the remainder.

Theorem 1.2 (Series Component Convergence). A infinite complex series $\sum z_n$ converges if and only if $\sum x_n$ and $\sum y_n$ converges.

1.2.1 Power Series

Definition 1.3 (Power Series). A power series is a series of the form

$$\sum_{n=0}^{\infty} a_n (z - z_0)^n$$

where $\{a_n\} \subset \mathbb{C}$ and $z_0 \in \mathbb{C}$. When the series converges, the power series is a function of z.