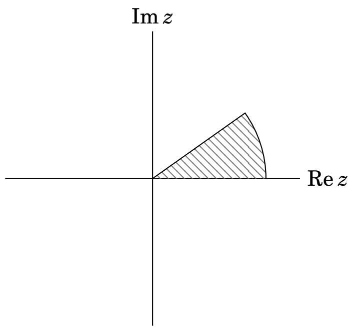
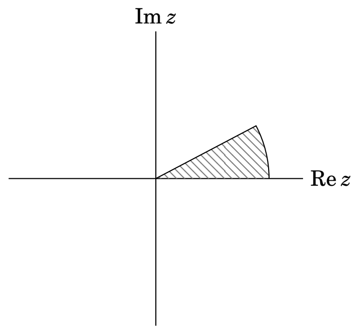
14.8

For $z = re^{i\theta}$, it follows that $z^n = r^n e^{ni\theta}$. Hence for each mapping,

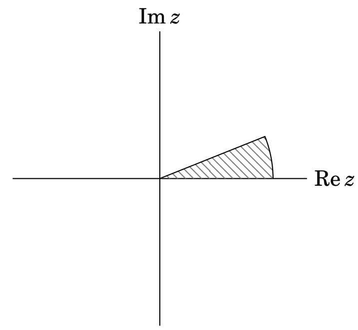
$$
\begin{aligned}
z^2 &\implies 0 \le \theta \le \frac{\pi^2}{16}, \quad 0 \le r \le 1 \\
z^3 &\implies 0 \le \theta \le \frac{\pi^3}{64}, \quad 0 \le r \le 1 \\
z^4 &\implies 0 \le \theta \le \frac{\pi^4}{256}, \quad 0 \le r \le 1
\end{aligned}
$$



(a) $w = z^2$



(b) $w = z^3$



(c) $w = z^4$

18.1

Part C

Proof. Take $\epsilon > 0$ and let $\delta = \epsilon$. Note then that for $z \in C$ in the δ deleted neighborhood of 0 (that is $z \neq 0$)

$$
\begin{aligned}
|z - 0| < \delta &\implies |z| < \delta \\
&\implies \frac{|z|^2}{|z|} < \delta \\
&\implies \frac{|\overline{z}|^2}{|z|} < \delta \\
&\implies \left| \frac{\overline{z}^2}{z} \right| < \delta \\
&\implies \left| \frac{\overline{z}^2}{z} - 0 \right| < \delta \\
&\implies \left| \frac{\overline{z}^2}{z} - 0 \right| < \epsilon
\end{aligned}
$$

Therefore $\displaystyle\lim_{z \to 0} \frac{\overline{z}^2}{z} = 0$ ■