Problem 1

Part A

X is a geometric random variable with parameter $p = 0.2$.

Part B

$$\mathbb{E}(X) = \frac{1}{p} = \frac{1}{0.2} = 5.$$

Part C

$$\begin{gathered}\operatorname{Var}(X) = \frac{1-p}{p^2} = \frac{0.8}{0.04} = 20 \\ \sigma = \sqrt{\operatorname{Var}(X)} = \sqrt{20}.\end{gathered}$$

Part D

$$\mathbb{P}(X \leq 4) = \texttt{pgeom(3,0.2)} = 0.5904.$$

Part E

Since the events are independent,

$$P(\text{Both hits} \leq 4) = P(X \leq 4)^2 = (0.5904)^2 = 0.3486.$$

Part F

Let H_1 and H_2 be the number of attempts it takes to hit the target a first time then the target a second. Then

$$\mathbb{E}(H_1 + H_2) = \mathbb{E}(H_1) + \mathbb{E}(H_2) = 2 \cdot \mathbb{E}(X) = 10.$$

Problem 2

Part A

Since the distribution is a poisson distribution with parameter $\lambda = 10$,

$$\begin{aligned}\text{Expected number of cars in 1 hour } &= \mathbb{E}(X) = 10 \\ \text{Expected number of cars in 3 hours } &= \mathbb{E}(3X) = 3 \cdot \mathbb{E}(X) = 3 \cdot 10 = 30.\end{aligned}$$

Part B

$$\mathbb{P}(X \leq 15) = e^{-10} \sum_{n=0}^{15} \frac{10^n}{n!} = \texttt{ppois(15,10)} = 0.9513.$$

Part C

This problem corresponds to a poisson distribution with parameter $\lambda = 30$. Therefore

$$\mathbb{P}(X \leq 45) = e^{-30} \sum_{n=0}^{45} \frac{30^n}{n!} = \texttt{ppois(45,30)} = 0.996.$$

Part D

$$\begin{aligned}
\mathbb{P}(\text{10 Arrive and all pass}) &= \mathbb{P}(\text{10 Arrive}) \cdot \mathbb{P}(\text{All pass}|\text{10 Arrive}) \\
&= \mathbb{P}(X = 10) \cdot \mathbb{P}(10\text{Successes}) \\
&= \left(\frac{10^{10} \cdot e^{-10}}{10!} \cdot \left(\frac{1}{2}\right)^{10} \right) \\
&= \texttt{dpois(10,10)*dbinom(10,10,0.5)} \\
&= 0.0001222.
\end{aligned}$$

Problem 3

Part A

m	1	2	3	4	5	6
$\mathbb{P}(M = m)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{3}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

Part B

$$\mathbb{E}(M) = 1 \cdot \frac{1}{36} + 2 \cdot \frac{3}{36} + 3 \cdot \frac{5}{36} + 4 \cdot \frac{7}{36} + 5 \cdot \frac{9}{36} + 6 \cdot \frac{11}{36} = 4.47222.$$

Part C

$$\begin{aligned}
\mathrm{Var}(M) &= \mathbb{E}\left(M^2\right) - (\mathbb{E}(M))^2 \\
&= 1^2 \cdot \frac{1}{36} + 2^2 \cdot \frac{3}{36} + 3^2 \cdot \frac{5}{36} + 4^2 \cdot \frac{7}{36} + 5^2 \cdot \frac{9}{36} + 6^2 \cdot \frac{11}{36} - 4.47222 \\
&= 1.9716.
\end{aligned}$$

Part D

These are the events in which the max is a 4,

$$\begin{aligned}
&(1,4), (2,4), (3,4), \\
&(4,1), (4,2), (4,3), \\
&(4,4).
\end{aligned}$$

Of the events, there are 2 that have a roll of 2, therefore the probability is $\frac{2}{7}$