Rings and Fields

1.1 Rings

Definition 1.1 (Ring). A ring $\langle R, +, \cdot \rangle$ is a set $\mathcal{R}$ together with two binary operations $+$ and $\cdot$ such that

1. $\langle R, + \rangle$ is an abelian group
2. $\cdot$ is associative
3. The left and right distributivity laws hold
   (a) $\forall a, b, c \in R \rightarrow c \cdot (a + b) = ca + cb$
   (b) $\forall a, b, c \in R \rightarrow (a + b) \cdot c = ac + bc$