Problem 1

Part A

$$S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}.$$

Part B

$$S = \{1, 2, 3, 4, 5, 6, 7\}.$$

Part C

$$\begin{aligned} S = \{&(\mathbf{T}, \mathbf{T}, \mathbf{T}), (\mathbf{H}, \mathbf{H}, \mathbf{H}), \\ &(\mathbf{T}, \mathbf{T}, \mathbf{H}), (\mathbf{H}, \mathbf{T}, \mathbf{T}), \\ &(\mathbf{T}, \mathbf{H}, \mathbf{H}), (\mathbf{H}, \mathbf{H}, \mathbf{T}), \\ &(\mathbf{T}, \mathbf{H}, \mathbf{T}), (\mathbf{H}, \mathbf{T}, \mathbf{H})\} \end{aligned}$$

Part D

$$S = \{0, 1, 2, 3, 4, 5\}.$$

Part E

$$S = \{1, 2, 3, 4, 5, 6\}.$$

Part F

$$S = \{0, 1, 2, 3, 4, 5\}.$$

Problem 2

Part A

$$A = \{(J, M), (J, A), (S, M), (S, A)\}.$$

Part B

$$B = \{(J, M), (J, A), (S, M), (S, A), (M, A)\}.$$

Part C

$$A^{\complement} = \{(J, S), (M, A)\}.$$

Part D

$$A \cap B = \{(J, M), (J, A), (S, M), (S, A)\}.$$

Problem 3

Part A

A probability distribution was not used as the probability of getting a 2 is negative which is not a valid probability since it does not lie between 0 or 1.

Part B

A probability distribution was used as each probability is in the range of 0 to 1 and the sum of the probability of each element in the sample space adds up to 1.

Part C

A probability distribution was not used as the sum of the probabilities is $0.95 \neq 1$.

Problem 4

A and B cannot be mutually exclusive as the probability $P(A \cup B) = 1.10 > 1$, meaning that A and B could potentially occur at the same time. Hence they cannot be mutually exclusive.

Problem 5

Part A

$$\begin{aligned} P(A^C B^C) &= P((A \cup B)^C) \\ &= 1 - P(A \cup B) \\ &= 1 - 0.8 \\ &= 20\% \end{aligned}$$

Part B

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(AB) \\ P(AB) &= P(A) + P(B) - P(A \cup B) \\ &= 0.5 + 0.45 - 0.8 \\ &= 15\% \end{aligned}$$

Part C

$$\begin{aligned} P(AB^C) &= P(A) - P(AB) \\ &= 0.5 - 0.15 \\ &= 35\% \end{aligned}$$

Part D

The events of having a Visa card and a having a Mastercard are not mutually exlcusive since $P(AB) > 0$.

Problem 6

Part A

$$\hat{p}_1 = 0.6000$$
$$\hat{p}_2 = 0.3000$$
$$\hat{p}_3 = 0.7000$$
$$\hat{p}_4 = 0.5000$$
$$\hat{p}_5 = 0.4000$$

The true proportion of heads should be 0.5 since the coin is fair.

Part B

$$\hat{p}_1 = 0.5010$$
$$\hat{p}_2 = 0.4945$$
$$\hat{p}_3 = 0.5062$$
$$\hat{p}_4 = 0.4981$$
$$\hat{p}_5 = 0.4993$$

Problem 7

Part A

Probability student doesn't miss any days = $1 - 0.25 - 0.15 - 0.28 = 32\%$.

Part B

Probability student misses one day or less = $1 - 0.15 - 0.28 = 57\%$.

Problem 8

The meaning of a 75% chance is that if I play many games against an opponent and look at the proportion of games I won to the games I played, in the long run that proportion will converge towards 0.75 or 75%.