① Dividing $x^3 + x^2 + 1$ by $x - \alpha$:

$$
\begin{array}{r|l}
x^3 + x^2 + 1 & x - \alpha \\
-(x^3 - \alpha x^2) & x^2 + (1+\alpha)x + (\alpha^2+\alpha) \\
\hline
(1+\alpha)x^2 & \\
-((1+\alpha)x^2 - (\alpha^2+\alpha)x) & \\
\hline
(\alpha^2+\alpha)x + 1 & \\
-[(\alpha^2+\alpha)x - (\alpha^3+\alpha^2)] & \\
\hline
1 - (\alpha^3+\alpha^2) & \\
\hline
0 &
\end{array}
$$

$\alpha^3 = \alpha^2 + 1$

$\downarrow$

$\alpha^3 + \alpha^2 = 1$

② Dividing $x^2 + (1+\alpha)x + (\alpha^2+\alpha)$ by $x - \alpha^2$:

$$
\begin{array}{r|l}
x^2 + (1+\alpha)x + (\alpha^2+\alpha) & x - \alpha^2 \\
x^2 - \alpha^2 x & x + (1+\alpha+\alpha^2) \\
\hline
(1+\alpha+\alpha^2)x & \\
(1+\alpha+\alpha^2)x - \alpha^2(1+\alpha+\alpha^2) & \\
\hline
\alpha^2+\alpha+\alpha^2+\alpha^3+\alpha^4 = 0 &
\end{array}
$$