Problem 1

Part A

There are 26^2 ways to choose 2 letters and 10^5 ways to choose 5 numbers. Therefore there are $26^2 \cdot 10^5 = 67,600,000$ different 7-place license plate numbers.

Part B

There are 26 ways to choose the first letter and 25 ways to choose the next letter. With a similar argument there are $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 30,240$ ways to choose 5 unique numbers. Therefore there are $26 \cdot 25 \cdot 30,240 = 19,656,000$ unique 7 place license plates.

Problem 2

There are $\binom{10}{5}$ ways to choose 5 men and $\binom{12}{5}$ ways to choose 5 women. There are 5! ways to pair up 5 men with 5 women, meaning that there are $5! \cdot \binom{10}{5} \cdot \binom{12}{5} = 23,950,080$ distinct possible results.

Problem 3

The first gift can be given to 10 children, the second gift can be given to 9 children and so forth. Therefore the number of distinct results possible is $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 604,800$.

Problem 4

$$S = \{(R,R),(R,G),(R,B),(G,R),(G,G),(G,B),(B,R),(B,G),(B,B)\}.$$

The probability associated with each point in the sample space is $\frac{1}{9} \approx 11.11\%$

Problem 5

$$S = \{(R,G),(R,B),(G,R),(G,B),(B,R),(B,G)\}.$$

Problem 6

The sample space is infinite.

$$\begin{aligned} S = \{&(\mathbf{H},\mathbf{H}),\\ &(\mathbf{T},\mathbf{H},\mathbf{H}),\\ &(\mathbf{T},\mathbf{T},\mathbf{H},\mathbf{H}),(\mathbf{H},\mathbf{T},\mathbf{H},\mathbf{H}),\\ &(\mathbf{T},\mathbf{T},\mathbf{T},\mathbf{H},\mathbf{H}),(\mathbf{H},\mathbf{T},\mathbf{T},\mathbf{H},\mathbf{H}),(\mathbf{T},\mathbf{H},\mathbf{T},\mathbf{H},\mathbf{H})\ldots\} \end{aligned}$$

There are $2^4 = 16$ possible outcomes from tossing a coin 4 times and there are 2 outcomes in the sample space that are 4 tosses, meaning the probability of the coin being tossed exactly four times if $\frac{2}{16} = \frac{1}{8} = 12.5\%$.

Problem 7

The number of ways to pick 5 people from a group of 15 is $\binom{15}{5} = 3,003$. The number of ways to pick 3 and 2 women is $\binom{6}{3} \cdot \binom{9}{2} = 720$. Therefore the probability of the committee consisteng of 3 men and 2 women is $\frac{720}{3{,}003} \approx 23.98\%$.

Problem 8

Consider the set of n balls. The group of marked balls has 1 ball in it (the marked ball) and the group of unmarked balls has $n-1$ balls in it. The total number of ways to draw k balls from n balls is $\binom{n}{k}$. The number of ways to draw k balls with the marked ball in it is $\binom{1}{1} \cdot \binom{n-1}{k-1}$. The ways to choose 1 marked ball from a set of 1 marked ball is $\binom{1}{1} = 1$ and the ways to choose the remaining $k-1$ balls is $\binom{n-1}{k-1}$. Therefore the probability of drawing the marked ball is

$$\frac{\binom{n-1}{k-1}}{\binom{n}{k}} = \frac{k}{n}.$$

Problem 9

Part 1

Since A and B are mutually exclusive, $P(AB) = 0$. Therefore

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) + P(AB) \\ &= P(A) + P(B) \\ &= 80\%. \end{aligned}$$

Part 2

$$\begin{aligned} P(AB^{\complement}) &= P(A) - P(AB) \\ &= P(A) \\ &= 30\%. \end{aligned}$$

Part 3

Since A and B are mutually exclusive, $P(AB) = 0$.

Problem 10

Firstly, the number of ways 5 cards can be dealt from a deck is $\binom{52}{5} = 2,598,960$. A straight hand will be comprised of picking 5 cards with 4 possible suits per card. Therefore given a straight range the number of possibilities is $4^5 - 4$. The substraction of 4 accounts for the 4 hands where all the cards are the same suit which would make the hand a straight flush and not a straight. There are 10 different possible ranges for a straight meaning that there are $(4^5 - 4) \cdot 10 = 10,200$ possible straight hands. Therefore the probability of being dealt a straight hand from a deck is $\frac{10{,}200}{2598960} \approx 0.392\%$.