3.3.1

a) False

b) False

c) True

d) False

e) False

f) False

g) True

h) False

3.3.2

Part A

$$\beta = \left\{ \begin{pmatrix} -3 \\ 1 \end{pmatrix} \right\}, \dim \beta = 1$$

Part B

$$\beta = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}, \dim \beta = 1$$

Part C

$$\beta = \left\{ \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \right\}, \dim \beta = 1$$

Part D

$$\beta = \left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}, \dim \beta = 1$$

Part E

$$\beta = \left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}, \dim \beta = 3$$

Part F

$$\beta = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}, \dim \beta = 0$$

Part G

$$\beta = \left\{ \begin{pmatrix} -3 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right\}, \dim \beta = 2$$

3.3.4

Part A

$$\left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & 2 & -1 \end{array}\right) \to \left(\begin{array}{cc|cc} 1 & 0 & -5 & 3 \\ 0 & 1 & 2 & -1 \end{array}\right) \implies A^{-1} = \begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix}$$

$$Ax = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \implies x = \begin{pmatrix} -5 & 3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -11 \\ 5 \end{pmatrix}$$

Part B

$$\left(\begin{array}{ccc|ccc}1&2&-1&1&0&0\\1&1&1&0&1&0\\2&-2&1&0&0&1\end{array}\right)\to\left(\begin{array}{ccc|ccc}2&-2&1&0&0&1\\1&1&1&0&1&0\\1&2&-1&1&0&0\end{array}\right)\to\left(\begin{array}{ccc|ccc}3&0&0&1&0&1\\1&1&1&0&1&0\\1&2&-1&1&0&0\end{array}\right)\to$$
$$\left(\begin{array}{ccc|ccc}3&0&0&1&0&1\\0&-1&2&-1&1&0\\3&6&-3&3&0&0\end{array}\right)\to\left(\begin{array}{ccc|ccc}3&0&0&1&0&1\\0&-1&2&-1&1&0\\0&6&-3&2&0&-1\end{array}\right)\to\left(\begin{array}{ccc|ccc}3&0&0&1&0&1\\0&-1&2&-1&1&0\\0&0&9&-4&6&-1\end{array}\right)\to$$
$$\left(\begin{array}{ccc|ccc}3&0&0&1&0&1\\0&1&-2&1&-1&0\\0&0&9&-4&6&0\end{array}\right)\to\left(\begin{array}{ccc|ccc}1&0&0&\frac{1}{3}&0&\frac{1}{3}\\0&1&0&\frac{1}{9}&\frac{1}{3}&-\frac{2}{9}\\0&0&1&-\frac{4}{9}&\frac{2}{3}&-\frac{1}{9}\end{array}\right)\implies A^{-1}=\begin{pmatrix}\frac{1}{3}&0&\frac{1}{3}\\\frac{1}{9}&\frac{1}{3}&-\frac{2}{9}\\-\frac{4}{9}&\frac{2}{3}&-\frac{1}{9}\end{pmatrix}$$

$$Ax=\begin{pmatrix}5\\1\\4\end{pmatrix}\implies x=\begin{pmatrix}\frac{1}{3}&0&\frac{1}{3}\\\frac{1}{9}&\frac{1}{3}&-\frac{2}{9}\\-\frac{4}{9}&\frac{2}{3}&-\frac{1}{9}\end{pmatrix}\begin{pmatrix}5\\1\\4\end{pmatrix}=\begin{pmatrix}3\\0\\-2\end{pmatrix}$$

3.3.8

Part A

$$\left(\begin{array}{ccc|c}1&1&0&1\\0&1&-2&3\\1&0&2&-2\end{array}\right)\to\left(\begin{array}{ccc|c}0&1&-2&3\\0&1&-2&3\\1&0&2&-2\end{array}\right)\to\left(\begin{array}{ccc|c}1&0&2&-2\\0&0&0&0\\0&0&0&0\end{array}\right)$$

Since there are infinite solutions, there does exist a vector in $\mathbb{R}^3$ that will be mapped to $\begin{pmatrix}1\\3\\-2\end{pmatrix}$ under T.

Part B

$$\left(\begin{array}{ccc|c}1&1&0&2\\0&1&-2&1\\1&0&2&1\end{array}\right)\xrightarrow{\text{Same reductions in (a)}}\left(\begin{array}{ccc|c}1&0&2&1\\0&0&0&0\\0&0&0&0\end{array}\right)$$

Since there are infinite solutions, there does exist a vector in $\mathbb{R}^3$ that will be mapped to $\begin{pmatrix}2\\1\\1\end{pmatrix}$ under T.

4.1.1

a) False

b) True

c) False

d) False (det could be negative)

e) True

4.1.2

a) 30

b) -17

c) -8

4.1.9

Let

$$A = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}, B = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix}$$

Then

$$\begin{aligned} \det AB = \det \begin{pmatrix} a_1b_1 + a_2b_3 & a_1b_2 + a_2b_4 \\ a_3b_1 + a_4b_3 & a_3b_2 + a_4b_4 \end{pmatrix} &= a_2a_3b_2b_3 - a_1a_4b_2b_3 - a_2a_3b_1b_4 + a_1a_4b_1b_4 \\ &= (a_1a_4 - a_2a_3)(b_1b_4 - b_2b_3) \\ &= \det(A) \cdot \det(B) \end{aligned}$$