Problem 1

Part A

The confidence interval will be of the form $\overline{X} \pm 1.96 \cdot \frac{s}{\sqrt{n}}$ with $\overline{X} = 38, s = 5, n = 36$, giving a confidence interval of

$$(36.37, 39.63).$$

Part B

The confidence interval will be of the form $\overline{X} - \overline{Y}\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{n}}$ with $\overline{X} = 38, \overline{Y} = 36, s_1 = 5, s_2 = 7, n_1 = 36, n_2 = 25$, giving a confidence interval of

$$(-1.19, 5.19).$$

Part C

We can say with 95% confidence that $\mu_1 - \mu_2$ can be equal to 0, thus we cannot claim to have evidence that compact and economy cars differ in fuel efficiency.

Problem 2

The confidence interval will be of the form $\overline{X} \pm 1.96 \cdot \sqrt{\frac{\overline{X}(1-\overline{X})}{n}}$ with $\overline{X} = \frac{710}{1000}, n = 1000$, giving a confidence interval of

$$(0.68, 0.74).$$

Problem 3

Part A

The confidence interval will be of the form $\overline{X} \pm 1.96 \cdot \frac{s}{\sqrt{n}}$ with $\overline{X} = 55, s = 7, n = 36$, giving a confidence interval of

$$(52.7, 57.2).$$

Part B

The confidence interval will be of the form $\overline{X} \pm 2.57 \cdot \frac{s}{\sqrt{n}}$ with $\overline{X} = 55, s = 7, n = 36$, giving a confidence interval of

$$(52, 58).$$

Part C

The interval with a higher confidence interval is wider, which is a consequence of needing a larger margin of error to be more certain that the true population paramter is within the interval.

Part D

We have 95% confidence that μ lies in the interval $(52.7, 57.2)$ which lies entirely above 50.

Part E

$$H_0 : \mu \leq 50$$
$$H_a : \mu > 50.$$

Problem 4

Part A

The confidence interval will be of the form $\overline{X} \pm 1.96 \cdot \frac{s}{\sqrt{n}}$ with $\overline{X} = 4.5, s = 3.6, n = 100$, giving a confidence interval of

$$(3.79, 5.21).$$

Part B

We can say with 95% confidence that the average difference between scores of nutritious and light breakfasts is between 3.79 and 5.21, hence we are 95% confident that a nutritious breakfast will have 3.79 to 5.21 more points on average.

Problem 5

$\overline{X}$ will follow an approximate normal distribution with $\mu = 2.6$ and $s = \frac{1.4}{\sqrt{100}} = 0.14$. Therefore

$$\mathbb{P}\left(\overline{X} > 3\right) = 1 - \texttt{pnorm(3, 2.6, 0.14)} \approx 0.002.$$