

**2.2**

Let  $z = x + iy$  such that  $\operatorname{Re}(z) = x$  and  $\operatorname{Im}(z) = y$ .

**Part A**

$$\operatorname{Re}(iz) = \operatorname{Re}(i(x + iy)) = \operatorname{Re}(ix - y) = \operatorname{Re}(-y + ix) = -y = -\operatorname{Im}(z).$$

**Part B**

$$\operatorname{Im}(iz) = \operatorname{Im}(i(x + iy)) = \operatorname{Im}(-y + ix) = x = \operatorname{Re}(z).$$

**3.1****Part A**

$$\begin{aligned} \frac{1+2i}{3-4i} + \frac{2-i}{5i} &= \frac{(1+2i)(\overline{3-4i})}{3^2 + (-4)^2} + \frac{(2-i)(\overline{5i})}{0^2 + 5^2} \\ &= \frac{(1+2i)(3+4i)}{25} + \frac{(2-i)(-5i)}{25} \\ &= \frac{3+4i+6i+8i^2}{25} + \frac{-10i+5i^2}{25} \\ &= \frac{-5+10i}{25} + \frac{-5-10i}{25} \\ &= -\frac{10}{25} = -\frac{2}{5} \end{aligned}$$

**Part B**

$$\begin{aligned} \frac{5i}{(1-i)(2-i)(3-i)} &= \frac{5i}{(2-i-2i+i^2)(3-i)} \\ &= \frac{5i}{(1-3i)(3-i)} \\ &= \frac{5i}{3-i-9i+3i^2} \\ &= \frac{5i}{-10i} \\ &= -\frac{5}{10} = -\frac{1}{2} \end{aligned}$$

**Part C**

$$(1-i)^2 = 1 - 2i + i^2 = -2i \implies (1-i)^4 = (-2i)^2 = 4i^2 = -4.$$

## 6.7

$$\begin{aligned}
|\operatorname{Re}(2 + \bar{z} + z^3)| &= \left| \frac{2 + \bar{z} + z^3 + \overline{(2 + \bar{z} + z^3)}}{2} \right| \\
&= \left| \frac{2 + \bar{z} + z^3 + 2 + z + \bar{z}^3}{2} \right| \\
&= \left| \frac{4 + z + \bar{z} + z^3 + \bar{z}^3}{2} \right| \\
&\leq \frac{2 + |z| + |\bar{z}| + |z|^3 + |\bar{z}|^3}{2} \\
&= \frac{2 + 2|z| + 2|z|^3}{2} \\
&\leq \frac{2 + 2 + 2}{2} = 3 \leq 4
\end{aligned}$$

## 6.10

**Proof.** Let  $z = x + iy$ .

$\Rightarrow$ ) Assume that  $z$  is real. That is,  $y = 0$ . Then  $z = x + 0y = x = x - 0y = \bar{z}$ . Therefore  $z = \bar{z}$

$\Leftarrow$ ) Assume that  $z = \bar{z}$ . Then

$$x + iy = x - iy.$$

Equating the imaginary components gives  $iy = -iy$  or equivalently  $y = -y$ . This is only true if  $y = 0$ . Therefore  $z = x + 0y = x$  and hence  $z$  is real.

Both directions hence prove the if and only if. ■

## 6.13

$$\begin{aligned}
|z - z_0| = R &\implies |z - z_0|^2 = R^2 \\
(z - z_0)\overline{(z - z_0)} &= R^2 \\
(z - z_0)(\bar{z} - \bar{z}_0) &= R^2 \\
z\bar{z} - z\bar{z}_0 - \bar{z}z_0 + z_0\bar{z}_0 &= R^2 \\
|z|^2 - z\bar{z}_0 - \overline{z\bar{z}_0} + |z_0|^2 &= R^2 \\
|z|^2 - (z\bar{z}_0 + \overline{z\bar{z}_0}) + |z_0|^2 &= R^2 \\
|z|^2 - 2\operatorname{Re}(z\bar{z}_0) + |z_0|^2 &= R^2
\end{aligned}$$

**9.5****Part A**

Since

$$\begin{aligned} i &\Leftrightarrow e^{i\frac{\pi}{2}} \\ 1 - i\sqrt{3} &\Leftrightarrow 2e^{-i\frac{\pi}{3}} \\ \sqrt{3} + i &\Leftrightarrow 2e^{i\frac{\pi}{6}} \end{aligned}$$

it follows that

$$\begin{aligned} i(1 - i\sqrt{3})(\sqrt{3} + i) &= e^{i\frac{\pi}{2}} \cdot 2e^{-i\frac{\pi}{3}} \cdot 2e^{i\frac{\pi}{6}} \\ &= 4e^{i(\frac{\pi}{2} - \frac{\pi}{3} + \frac{\pi}{6})} \\ &= 4e^{i\frac{\pi}{3}} \\ &= 4 \cdot \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 2 \cdot (1 + i\sqrt{3}) \end{aligned}$$

**Part B**

Since

$$\begin{aligned} 5i &\Leftrightarrow 5e^{i\frac{\pi}{2}} \\ 2 + i &\Leftrightarrow \sqrt{5}e^{i\arctan(\frac{1}{2})} \end{aligned}$$

it follows

$$\begin{aligned} \frac{5i}{2 + i} &= 5e^{i\frac{\pi}{2}} \cdot \sqrt{5}e^{-i\arctan(\frac{1}{2})} \\ &= 5\sqrt{5}e^{i(\frac{\pi}{2} - \arctan(\frac{1}{2}))} \\ &= 5\sqrt{5}e^{i(\frac{\pi}{2} - \arctan(\frac{1}{2}))} \end{aligned}$$

**Part C****Part D****9.6****9.8****11.3****11.5****12.1****12.4**