Problem 1

Express each of the following statements in the If .... , then .... form. There are many possible correct answers.

(a) You must eat your dinner if you want to grow.

(b) Being a multiple of 12 is a sufficient condition for a number to be even.

(c) It is necessary for you to pass your exams in order for you to obtain a degree.

(d) A triangle is equilateral only if all its sides have the same length.

Solution

(a) **If** you want to grow, **then** you must eat your dinner

(b) **If** a number is a multiple of 12, **then** it is even

(c) **If** you want to obtain a degree, **then** you need to pass your exams

(d) **If** a triangle is equalateral, **then** all its sides are of equal length

Problem 2

Suppose that Girls smell of roses and Boys have dirty hands are true statements and that The Teacher is always right is a false statment. Which of the following are true? *Hint: Label each of the given statements, and think about each of the following using connectives.*

(a) If girls smell of roses, then the Teacher is always right.

(b) If the Teacher is always right, then boys have dirty hands.

(c) If the Teacher is always right or girls smell of roses, then boys have dirty hands.

(d) If boys have dirty hands and girls smell of roses, then the Teacher is always right.

Solution

Let P = "Girls smell of roses", Q = "Boys have dirty hands", and R = "The Teacher is Always Right".

(a) $P \implies R$

- True $\implies$ False is **False**

(b) $R \implies Q$

- False $\implies$ True is **True**

(c) $(R \vee Q) \implies P$

- True $\implies$ True is **True**

(d) $(Q \wedge P) \implies R$

- True $\implies$ False is **False**

Problem 3

Write the negation (in words) of the following claim: *If Jack and Jill climb up the hill, then they fall down and like pails of water.*

Solution

Jack and Jill climb up the hill and dont fall down or dont like pails of water.

Problem 5

(a) Rewrite the following sentence using the word necessary.
If I am to get a new bicycle, I must do my homework.

(b) Rewrite the following sentence using the word sufficient.
The United States must play more soccer if it is to win the World Cup

Solution

(a) Doing my homework is **neccessary** for me to get a new bicycle

(b) To win the world cup it is **sufficient** for the United States to play more soccer

Problem 6

(a) What are the converse and the contrapositive of the statements in the previous question? Write your answers in sentences, like the originals.

(b) What are the negations of the statements in the previous question?

Solution

Part A

- Converse Statements
  (a) If I do my homework, then I can get a new bicycle
- Contrapositive Statements
  (a) If I do not do my homework, then I will not get a new bicycle

Part B

(a) I will get a new bicycle and not do my homework

Problem 7

Construct the truth tables for the propositions $P \vee (Q \wedge R)$ and $(P \vee Q) \wedge R$. Are they the same?

Solution

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

P	Q	R	$P \vee Q$	$(P \vee Q) \wedge R$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	T	T
F	T	F	T	F
F	F	T	F	F
F	F	F	F	F

The propositions are **not the same**

Problem 10

(a) Prove that $((P \vee Q) \wedge \neg P) \wedge \neg Q$ is a contradiction.

(b) Prove that $(\neg P \wedge Q) \vee (P \wedge \neg Q) \iff \neg(P \iff Q)$ is a tautology.

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$(P \vee Q) \wedge \neg P$	$((P \vee Q) \wedge \neg P) \wedge \neg Q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	F
F	F	T	T	F	F	F

Solution

Part A

A proposition is a contradiction if for every input it is false.
Since for all posible inputs P and Q the proposition is false, it is a contradiction.

Part B

Let H be the proposition in question; $H \iff (\neg P \land Q) \lor (P \land \neg Q) \iff \neg(P \iff Q)$

P	Q	$\neg P \land Q$	$P \land \neg Q$	$(\neg P \land Q) \lor (P \land \neg Q)$	$\neg(P \iff Q)$	H
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	F	F	F	F	T

Since H is always true and is the original proposition, then $(\neg P \land Q) \lor (P \land \neg Q) \iff \neg(P \iff Q)$ is a tautology.

Problem 14

Suppose that the following statements are true:

- Every Pig likes mud.
- If a creature cannot fly then it is not an astronaut.
- A creature is an astronaut if it likes mud.

Is it true that Pigs can fly? Explain your answer.

Solution

First we assume that pigs are considered creatures. Each statement can be rewritten as:

- Every pig likes mud
- Cannot fly $\implies$ Is not an astronaut
- Likes mud $\implies$ Is an astronaut

By statement 1, every pig likes mud. By statement 3, any creature that likes mud is an astronaut. Therefore every pig is an astronaut. The contrapositive of statement 2 results in "If a creature is an astronaut, then it can fly". Since the contrapositive of a proposition is logically equivalent to the original proposition, this proposition is also true. Therefore, since every pig is an astronaut and if a creature is an astronaut, it can fly, it is true that all pigs can fly.