Problem 1

Part A

$$\int_{97}^{107} \frac{1}{5\sqrt{2\pi}} e^{-\frac{(x-102)^2}{50}} \mathrm{d}x \approx 0.682689.$$

Part B

$$\mathbb{P}(X = 120) = \int_{120}^{120} \frac{1}{5\sqrt{2\pi}} e^{-\frac{(x-102)^2}{50}} \mathrm{d}x = 0.$$

$$\mathbb{P}(X < 120) = \int_{-\infty}^{120} \frac{1}{5\sqrt{2\pi}} e^{-\frac{(x-102)^2}{50}} \mathrm{d}x \approx 0.9998.$$

Part C

The 90$^{\text{th}}$ percentile is 108.41 degrees Fahrenheit.

Part D

The distribution $0.55X - 17.6$ is a normal distribution with $\mu = 38.5$ and $\sigma = 2.75$.

Part E

The distribution of the sample mean of the daily temperatures is an approximate normal distribution with a mean the same as $X \implies \mu = 102$ and a variance of $\sigma^2 = \sigma_X^2/n = 25/25 = 1$.

Part F

The sample mean in Celsius follows an approximate normal distribution with $\mu = 38.5$ and $\sigma = 0.55$.

Problem 2

Part A

The sum $\sum_{i=1}^{n} X_i$ is a binomial distribution with $p = 0.5$ and $n = 100$.

Part B

$\bar{X}$ will follow an approximate normal distribution with $\mu = 0.5$ and $\sigma = 0.05$.

Part C

$$\mathbb{P}\left(\overline{X} \geq 0.5\right) = \int_{0.5}^{\infty} \frac{1}{0.05\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-0.5}{0.05}\right)^2} \mathrm{d}x = 0.5.$$

Problem 3

Part A

$$\frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1).$$

Part B

The condifence interval is

$$(\hat{x} - z^* \cdot \frac{s}{\sqrt{n}}, \hat{x} + z^* \cdot \frac{s}{\sqrt{n}}).$$

Since this is a 95% condifence interval, $z^* = 1.96$. Therefore substituting in values resutls in

$$(5.8510, 6.7890).$$