Problem 1

Part A

$$
\begin{aligned}
P(A) &= 0.4 \\
P(B) &= 0.3 \\
P(AB) &= 0.2.
\end{aligned}
$$

Part B

The events of having a wireless mouse and wireless keyboard are not mutually exclusive since $P(AB) \neq 0$.

Part C

$$
\begin{aligned}
P(A \cup B) &= P(A) + P(B) - P(AB) \\
&= 0.4 + 0.2 - 0.2 \\
&= 40\%.
\end{aligned}
$$

Part D

$$
\begin{aligned}
P(A^{\complement} \cup B^{\complement}) &= P((AB)^{\complement}) \\
&= 1 - P(AB) \\
&= 1 - 0.2 \\
&= 80\%.
\end{aligned}
$$

Part E

The probability for each person is independent of the others choice, meaning for each person there is a 40% chance they have a wireless mouse. Therefore the probability that both have a wireless mouth is $(0.4)^2 = 16\%$.

Problem 2

Let the event of getting at least one 3 in 100 rolls be denoted by A. Then $A^{\complement}$ is the event of not getting any 3's in 100 rolls. The chance of not rolling a 3 each roll is $\frac{5}{6}$. Therefore $P(A^{\complement}) = \left(\frac{5}{6}\right)^{100}$, meaning that $P(A) = 1 - \left(\frac{5}{6}\right)^{100} \approx 99.9999987925\%$.

Problem 3

Part A

Proof. Let A and B be events from some sample space S. Therefore

$$\begin{aligned}P((AB)^{\mathsf{C}}) &= P(A^{\mathsf{C}} \cup B^{\mathsf{C}}) \leq P(A^{\mathsf{C}}) + P(B^{\mathsf{C}}) \\ P((AB)^{\mathsf{C}}) &\leq P(A^{\mathsf{C}}) + P(B^{\mathsf{C}}).\end{aligned}$$

Note that $P((AB)^{\mathsf{C}}) = 1 - P(AB)$, therefore

$$\begin{aligned}P((AB)^{\mathsf{C}}) &\leq P(A^{\mathsf{C}}) + P(B^{\mathsf{C}}) \\ -P(AB) &\leq -P(A) + 1 - P(B) \\ P(AB) &\geq P(A) + P(B) - 1.\end{aligned}$$

■

Part B

Since $AB \subseteq B$, then $P(AB) \leq P(B)$ meaning, therefore using that and the Bonferroni inequlity:

$$\begin{aligned}P(AB) &\geq P(A) + P(B) - 1 \\ P(AB) &\geq \frac{1}{12} \\ P(B) \geq P(AB) &\geq \frac{1}{12} \\ \frac{1}{12} \leq P(AB) &\leq \frac{1}{3}.\end{aligned}$$

Problem 4

Part A

$$P(X) = 0.2 + 0.3 + 0.1 + 0.3 + 0.1 = 1.0 \checkmark.$$

Part B

$$P(X < 3) = 0.2 + 0.3 + 0.1 = 0.6 = 60.$$

Part C

$$P((X = 0) \cup (X = 4)) = P(X = 0) + P(X = 4) = 0.2 + 0.1 = 0.3 = 30\%.$$

Part D

$$
\begin{aligned}
P(X = 1|X > 0) &= \frac{P((X = 1) \cap (X > 0))}{P(X > 0)} \\
&= \frac{P(X = 1)}{P(X > 0)} \\
&= \frac{0.3}{0.3 + 0.1 + 0.3 + 0.1} \\
&= 0.375 = 37.5\%.
\end{aligned}
$$

Part E

Let $A \Rightarrow$ patient 1 has 0 limbs injure and $B \Rightarrow$ patient 2 has 4 limbs injured. Since A and B are independent, $P(AB) = P(A) \cdot P(B)$. $P(A) = 0.2$ and $P(B) = 0.1$, therefore $P(AB) = 0.02$, meaning

$$
\begin{aligned}
P(A|B) &= \frac{P(AB)}{P(B)} \\
&= \frac{0.02}{0.1} \\
&= 0.2 = 20\%.
\end{aligned}
$$