Linear Maps

1.1 Linearity

Definition 1.1 (Linear Map). A map $T : V \to W$ is linear if $T(au + bv) = aT(u) + bT(v)$ for all $a, b \in \mathbb{F}$ and $u, v \in V$.

Theorem 1.1. If $\{v_1, v_2, \ldots, v_n\}$ is a basis of V,

$$T(a_1v_1 + a_2v_2 + \ldots + a_nv_n) = a_1T(v_1) + a_2T(v_2) + \ldots a_nT(v_n)$$

A linear map can be defined just by declaring the images of a vector spaces basis vectors as the map has to obey linearity over a basis. This leads to a natural formulation of a linear map as a matrix where the columns are the images of the basis vectors under T.