Problem 1

Part A

X is a binomial random variable with parameters $n = 100$ and $p = 0.85$.

Part B

$$\begin{aligned} \mathrm{E}[X] &= np = 100(0.85) = 85 \\ \mathrm{Var}[X] &= npq = 100(0.85)(0.15) = 12.75. \end{aligned}$$

Part C

The R command that would solve this is `pbinom(80, 100, 0.85)` which results in a value of 0.1065.

Part D

Using the complement rule, the probability would be

$$P(X \leq 80) = 1 - P(X > 80)$$

which can be calculated using `1 - pbinom(80, 100, 0.85, lower.tail = FALSE)` which provides the same value.

Part E

Since each robot operates independently, the probability is

$$(0.1065)^2 = 0.01135.$$

Part F

Since each robot operates independently, the probability is

$$(0.10654)(1 - 0.1065443) = 0.09518.$$

Problem 2

Part A

$$\binom{52}{13} \cdot \binom{39}{13} = 5,157,850,293,780,050,462,400.$$

Part B

$$\frac{\binom{26}{13} \cdot \binom{26}{13}}{\binom{52}{13} \cdot \binom{52}{13}} = 2.68257 \cdot 10^{-10}.$$

Problem 3

From the given information, $P(H) = 0.2, P(L) = 0.3$ and $P(N) = 0.5$. Additionally,

$$\frac{1}{2}P(D|H) = P(D|L) = 2P(D|N).$$

Therefore

$$\begin{aligned}
P(H|D) &= \frac{P(D|H)P(H)}{P(D)} \\
&= \frac{P(D|H)P(H)}{P(D|H)P(H) + P(D|L)P(L) + P(D|N)P(N)} \\
&= \frac{P(D|H)P(H)}{P(D|H)P(H) + \frac{1}{2}P(D|H)P(L) + 4P(D|H)P(N)} \\
&= \frac{P(H)}{P(H) + \frac{1}{2}P(L) + 4P(N)} \\
&= \frac{0.2}{0.2 + \frac{1}{2} \cdot (0.3) + 4(0.5)} \\
&= 0.0851.
\end{aligned}$$