A Spectral Approach To Meshes

Eli Griffiths
University of California, Irvine

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The Journey

Definition (Graph)

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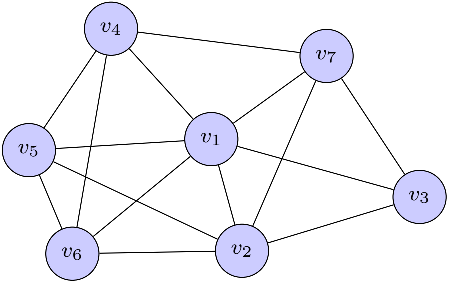

Why Graphs?

Graphs at their core encode connectivity

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- Friend networks on social media platforms

Graphs $\Leftrightarrow$ Connectivity

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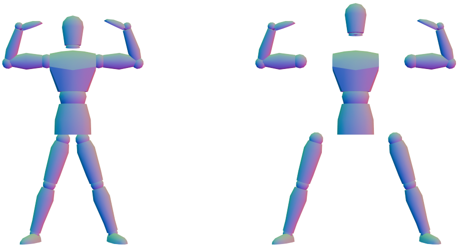
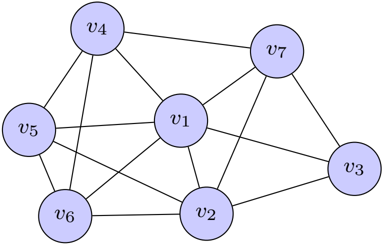
- Friend networks on social media platforms
- Molecules and bonds between atoms in structural chemistry
- Connectivity of meshes in computational geometry



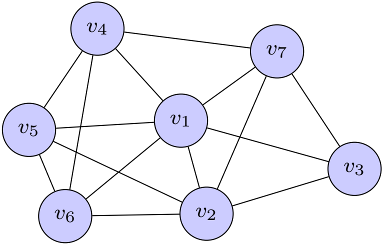
Matrix Representations

	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1							
v_2							
v_3							
v_4							
v_5							
v_6							
v_7							



Matrix Representations

	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1		✓	✓	✓	✓	✓	✓
v_2	✓		✓		✓	✓	✓
v_3	✓	✓					✓
v_4	✓				✓	✓	✓
v_5	✓	✓		✓		✓	
v_6	✓	✓		✓	✓		
v_7	✓	✓	✓	✓			



Matrix Representations

	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1		✓	✓	✓	✓	✓	✓
v_2	✓		✓		✓	✓	✓
v_3	✓	✓					✓
v_4	✓				✓	✓	✓
v_5	✓	✓		✓		✓	
v_6	✓	✓		✓	✓		
v_7	✓	✓	✓	✓			

$$\longrightarrow \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Adjacency Matrix A

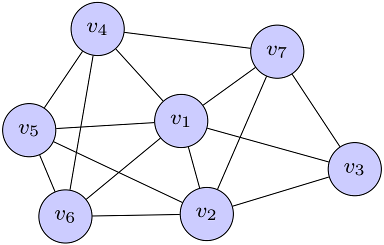
Matrix Representations

Definition (Degree Matrix)

The degree matrix of a graph G with n vertices is the $n \times n$ matrix D such that

$$(D)_{ij} = \begin{cases} \deg(v_i) & i = j \\ 0 & i \neq j \end{cases}.$$

$$\begin{pmatrix} 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$



Matrix Representations

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The Laplacian matrix of a graph G is

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Theorem

A graph has m connected components if and only if zero has algebraic multiplicity m for L

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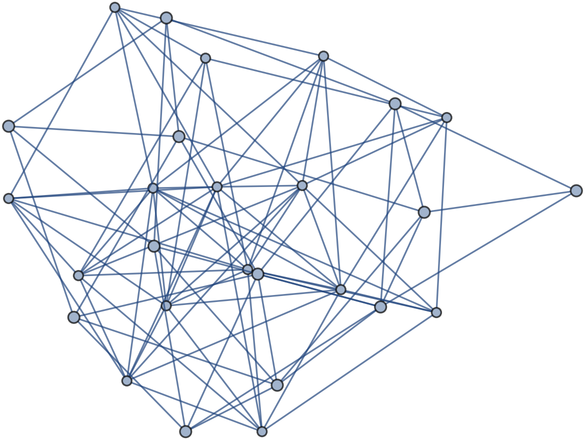
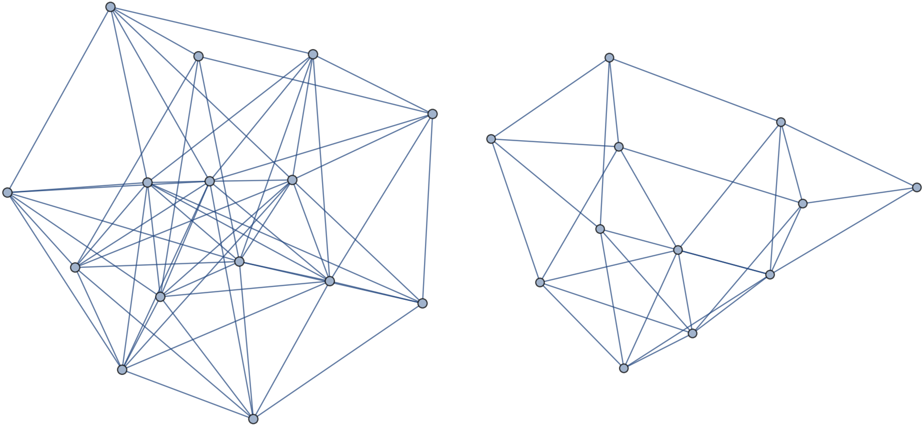
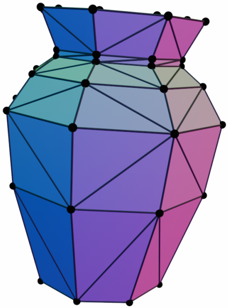
The eigenvector associated with the smallest positive eigenvalue of L (called the "Fiedler Vector") defines a well behaved partitioning scheme of a graph.





Definition (Triangular Mesh)

A **triangular mesh** is a triple $K = (V, E, F)$ such that



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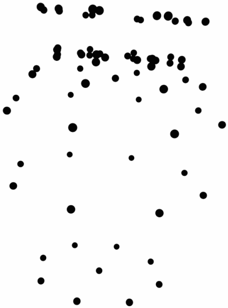

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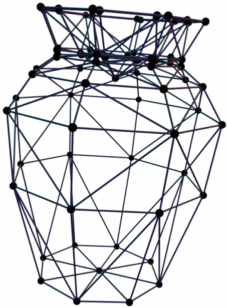

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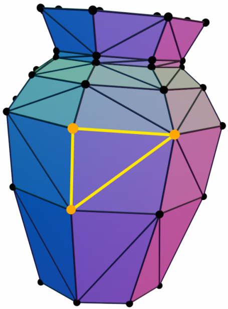
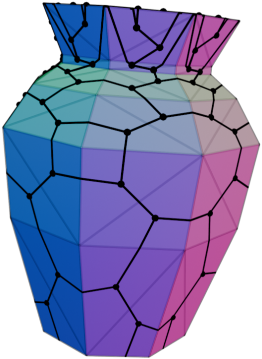
- $V \subseteq \mathbb{R}^3$ is the set of vertices
- $E \subseteq [V]^2$ is the set of representing non-intersecting edges
- $F \subseteq [E]^3$ is the set of faces such that for any $f = \{e_1, e_2, e_3\} \in F$,

$$e_1 \cap e_2 = \{v_1\}$$
$$e_2 \cap e_3 = \{v_2\}$$
$$e_3 \cap e_1 = \{v_3\}$$

for $v_1 \neq v_2 \neq v_3$.



Dual Mesh



Spectral Partitioning

Using the Fiedler vector on meshes...