

Data Representation

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Vocabulary

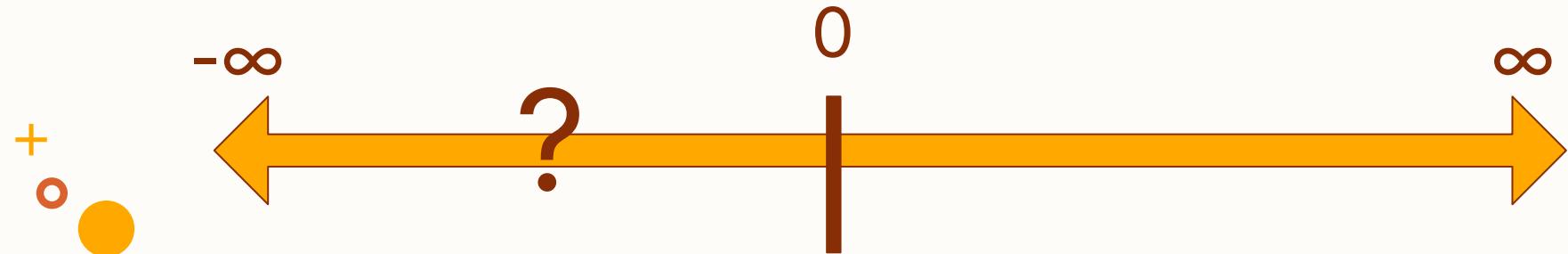
- Bit = 1/0
- Byte
 - Refers to 8 contiguous bits
- Nibble
 - 4 Contiguous bits
- 4 bytes → 32 bits





Data Representation

- We can represent $0 \rightarrow \infty$ in binary
 - Practically, we only usually do 2^{64}
- But there are more numbers
- How about negatives





Signed Magnitude

- You have a few options
- Simplest approach
- Have a bit represent the sign
- If MSB == 1
 - Negative
- If MSB == 0
 - Positive





Signed Magnitude

- Signed magnitude representation
 - MSB = Sign
 - Everything else = Magnitude
 - Distance from 0

1000011





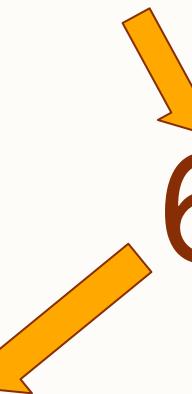
Signed Magnitude

1	0	0	0	1	1	0
---	---	---	---	---	---	---



-

6



-6





Signed Magnitude

- This has a few problems
 - Different bit lengths = different numbers
 - What is this number?

[1 0 0 0]

= -0 ?





Signed Magnitude

- Signed Magnitude addition
 - Completely broken

$$\begin{array}{r} 1010 \\ + 0010 \\ \hline 1100 \end{array} \quad \begin{array}{r} - 2 \\ + 2 \\ \hline = -4 ? \end{array}$$

+

o



2's Complement

- Is there a better option?
- Yes
 - 2's complement binary
- Similar representation
- MSB = Sign Bit
- Remaining = Count down instead





2's Complement

- Same bit length problem as before
 - 4-bit representation
 - Different from 8-bit
- Consider the range of this system
- $-8 \rightarrow +7$

1000	-8
1001	-7
1010	-6
1011	-5
1100	-4
1101	-3
1110	-2
1111	-1
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7



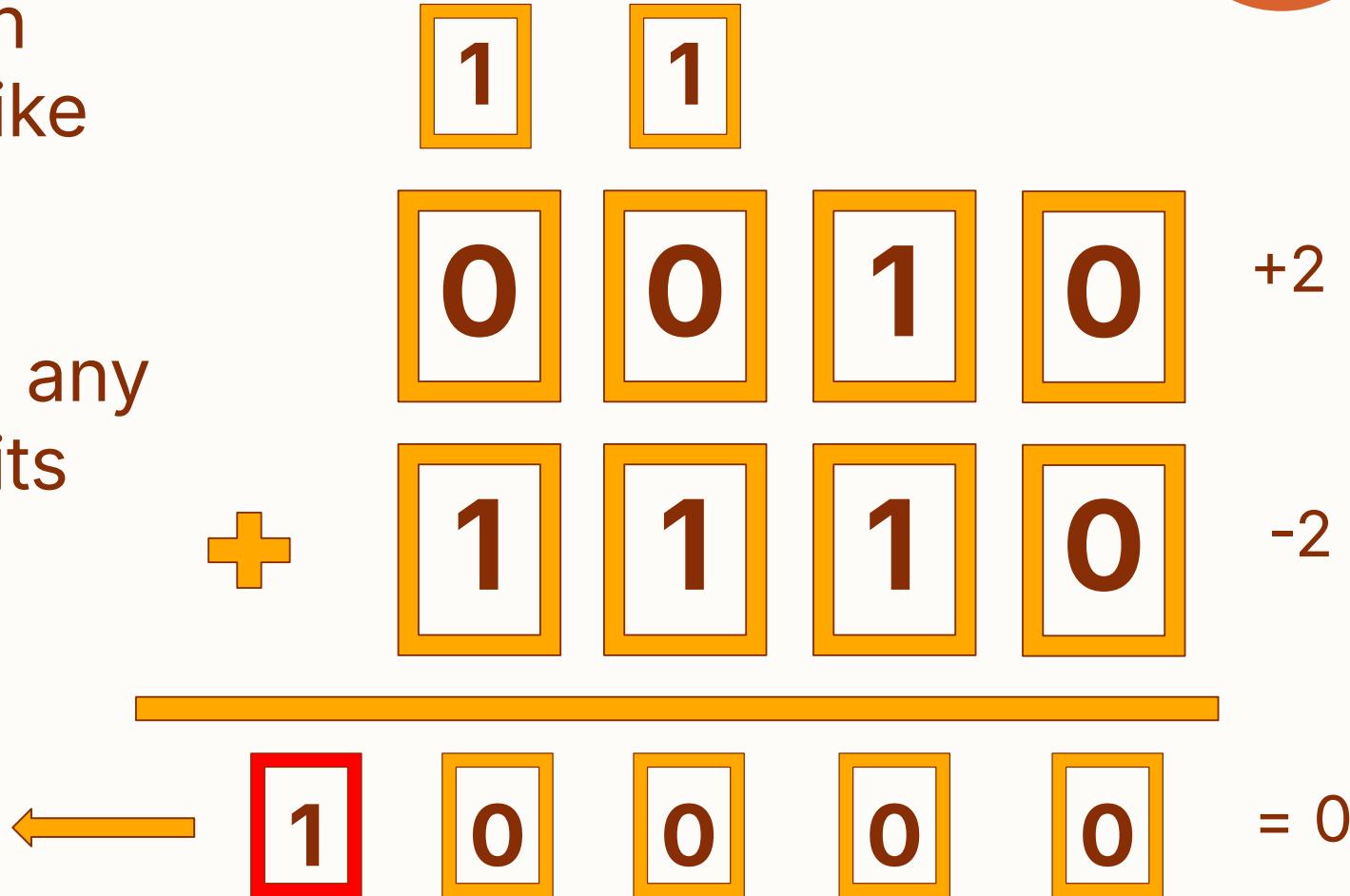


Signed Magnitude

- 2's Complement Addition
 - Works as intended
- Lets see that in action
 - Use previous example



- Addition works like normal
- Discard any carry bits





2's Complement

- How to go from Decimal to 2's Complement Binary
 - **Choose a bit length first**
 - If number is positive
 - Just convert like normal
 - What to do if negative?





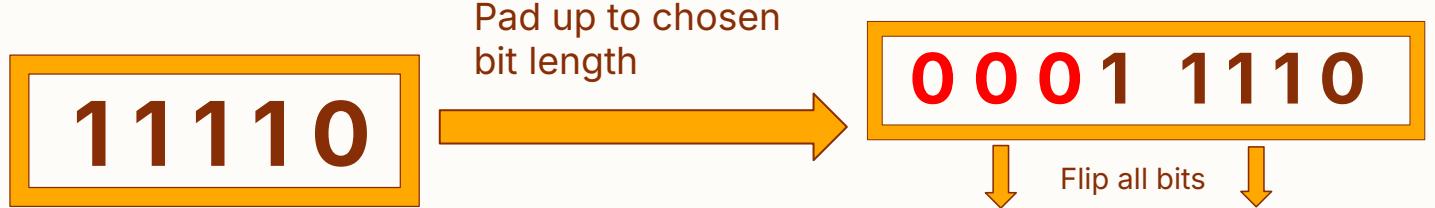
2's Complement

- Negative Decimal to 2's complement
 - **Chose a bit length**
 - Take abs. value of number
 - Convert to binary like normal
 - Flip all the bits
 - Add 1_2





2's Complement



Flip all bits
1110 0001

Add 1
(pad with 0s as necessary) + 0000 0001

Final Answer → 1110 0010





2's Complement

- Example together
 - -26_{10}
- Whiteboard example
 - -23_{10} 8-bit two's complement
 - Hint: $23_{10} = 10111_2$





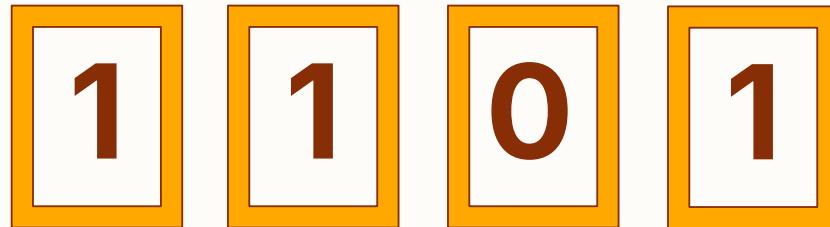
2's Complement

- What about the other direction?
- 2's complement bin. → Decimal
- Let's reframe how we think of the MSB
 - It is effectively a sign bit
- However
 - It really represents the negative version of the next exponents
 - Let's see that



- MSB is really the “number of -8s”
- Or whatever the MSB is

-8 4 2 1



+

o

●

$$-8 + 4 + 0 + 1 = -3$$



2's Complement

- You can use this same process to convert
- Let's do an example
- Whiteboard: 1011 0111





Negation

- How do we go turn a positive 2's complement number into a negative?
- Simple
 - Flip bits
 - Add 1

1000	-8	
1001	-7	
1010	-6	
1011	-5	
1100	-4	
1101	-3	
1110	-2	
1111	-1	
0111	7	
0110	6	
0101	5	
0100	4	
0011	3	
0010	2	
0001	1	
0000	0	





Negation

- We have already done this when converting
- What are we really doing there?
 - Flip the sign
 - Account for off by 1
- Please Remember:
 - Different bit lengths = Different output



Other Number Representations

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Other Representations

- Data Representation
 - Binary = a number
- Could we do more?
 - What other things could we represent?
- More numbers
- Colors
- Characters + more





Real numbers

- We can represent all integers
- What about real numbers
 - Decimals, etc
- Limited number of bits
- Infinite amount of numbers
- Compromises



Fixed Point

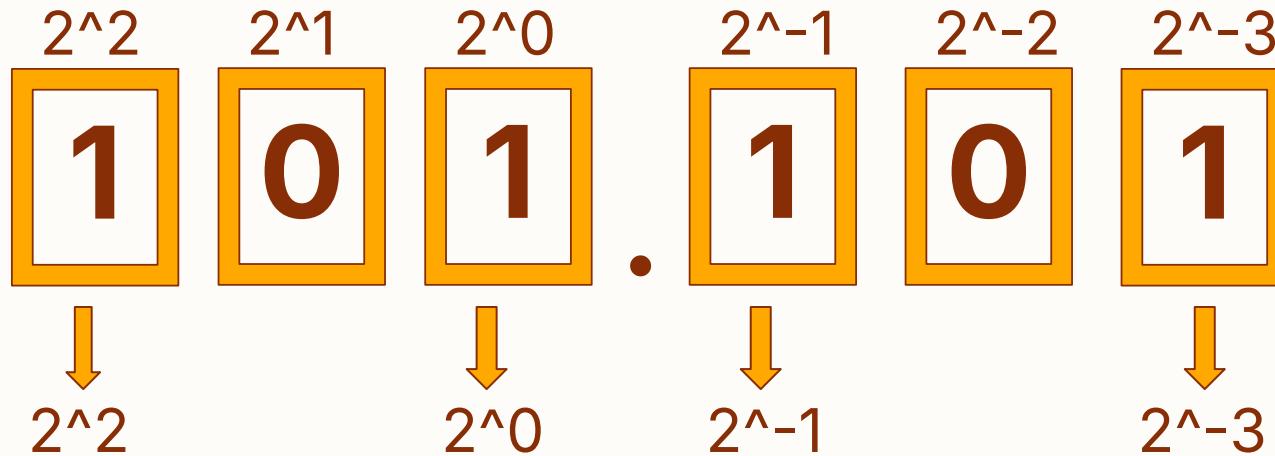
- We could just allocate some bits to the decimal
- Continue having 2^N
- Use negative exponents

2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
1	0	1	. 0	0	1





Fixed Point



A diagram showing the addition of binary numbers. It consists of two rows of digits. The top row contains the binary digits: 4, +, 1, +, 0.5, +, 0.125. The bottom row contains the corresponding powers of 2: 2^2 , 2^1 , 2^0 , \cdot , 2^{-1} , 2^{-2} , and 2^{-3} . The sum of these binary numbers is shown as 5.625.

+
o

4 + 1 + 0.5 + 0.125

2^2 2^1 2^0 \cdot 2^{-1} 2^{-2} 2^{-3}

5.625



Fixed Point

- Decimal Point \Rightarrow Binary Point
- Consider the range of a 6-bit systems
 - Only integers
 - 0-64
- What's our new range?
 - 0-8
 - With some added precision





Fixed Point

- When we lock out “binary point” into a single position
- We lose a significant amount of range
- How do we fix this?
 - Make out point “floating” instead of being fixed in place





Floating Point

- Many ways to do this
- IEEE 745
 - Defines the “floating point” standard
 - Every programming language uses this
- Makes a lot of compromises, but fixes range problem
- Let's see how it works





Floating Point

- Scientific notation
 - Provides a quick way of writing very very large numbers
- Uses exponents to shorten things
- Floating points also takes advantage of this





Floating Point

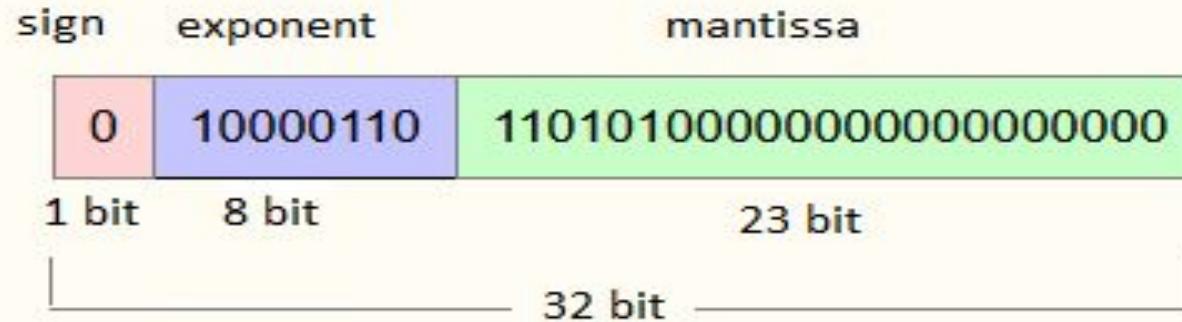
Floating Point Representaion

$12762 = 1.2762 \times 10^4$

The diagram illustrates the components of a floating-point number. The number 12762 is shown as 1.2762×10^4 . A green arrow points from the label "Exponent" to the power of 4. Another green arrow points from the label "Significand (Mantissa)" to the decimal fraction 1.2762. A third green arrow points from the label "Base" to the digit 10.



- Sign bit (1-bit)
 - Sign of the number
- Exponent
 - 10^{\wedge} whatever power
- Mantissa
 - Number to multiply by





Floating Point

- A lot more clever than we will go into
- Has some problems
 - Let's add some number $(0.1) \times 3$
- Floating point math errors
- We cannot represent all real numbers
- Compromises
 - Math is sometimes off
 - NaN, -0, 1.0 / +-0.0





Floating Point

- Designing this is really hard
- Kind of the best we got
- Can be expanded
 - Often uses 64 bits instead of 32
- Problems arise from
 - Limited bits
 - Infinite numbers





Other Representations

- Characters
 - How to represent them?
- Could define a mapping
 - Number₂ ⇒ Character
 - ~Logical mapping
- Encoding Standard
 - Any way of turning data into something meaningful





ASCII

- ASCII
 - American Standard Code for Information Interchange
- 1961
 - Up until now, everyone was kind of just using their own “standard”





ASCII

- Simple Standard
 - 7-bit standard
 - 128 possibilities
 - 0-128 in binary
- 65-90 Uppercase Characters
- 97-122 Lowercase Character



Dec	Hx	Oct	Char		Dec	Hx	Oct	Html	Chr		Dec	Hx	Oct	Html	Chr		Dec	Hx	Oct	Html	Chr
0	0	000	NUL	(null)	32	20	040	 	Space		64	40	100	@	Ø		96	60	140	`	'
1	1	001	SOH	(start of heading)	33	21	041	!	!		65	41	101	A	A		97	61	141	a	a
2	2	002	STX	(start of text)	34	22	042	"	"		66	42	102	B	B		98	62	142	b	b
3	3	003	ETX	(end of text)	35	23	043	#	#		67	43	103	C	C		99	63	143	c	c
4	4	004	EOT	(end of transmission)	36	24	044	$	\$		68	44	104	D	D		100	64	144	d	d
5	5	005	ENQ	(enquiry)	37	25	045	%	%		69	45	105	E	E		101	65	145	e	e
6	6	006	ACK	(acknowledge)	38	26	046	&	&		70	46	106	F	F		102	66	146	f	f
7	7	007	BEL	(bell)	39	27	047	'	'		71	47	107	G	G		103	67	147	g	g
8	8	010	BS	(backspace)	40	28	050	((72	48	110	H	H		104	68	150	h	h
9	9	011	TAB	(horizontal tab)	41	29	051))		73	49	111	I	I		105	69	151	i	i
10	A	012	LF	(NL line feed, new line)	42	2A	052	*	*		74	4A	112	J	J		106	6A	152	j	j
11	B	013	VT	(vertical tab)	43	2B	053	+	+		75	4B	113	K	K		107	6B	153	k	k
12	C	014	FF	(NP form feed, new page)	44	2C	054	,	,		76	4C	114	L	L		108	6C	154	l	l
13	D	015	CR	(carriage return)	45	2D	055	-	-		77	4D	115	M	M		109	6D	155	m	m
14	E	016	SO	(shift out)	46	2E	056	.	.		78	4E	116	N	N		110	6E	156	n	n
15	F	017	SI	(shift in)	47	2F	057	/	/		79	4F	117	O	O		111	6F	157	o	o
16	10	020	DLE	(data link escape)	48	30	060	0	0		80	50	120	P	P		112	70	160	p	p
17	11	021	DC1	(device control 1)	49	31	061	1	1		81	51	121	Q	Q		113	71	161	q	q
18	12	022	DC2	(device control 2)	50	32	062	2	2		82	52	122	R	R		114	72	162	r	r
19	13	023	DC3	(device control 3)	51	33	063	3	3		83	53	123	S	S		115	73	163	s	s
20	14	024	DC4	(device control 4)	52	34	064	4	4		84	54	124	T	T		116	74	164	t	t
21	15	025	NAK	(negative acknowledge)	53	35	065	5	5		85	55	125	U	U		117	75	165	u	u
22	16	026	SYN	(synchronous idle)	54	36	066	6	6		86	56	126	V	V		118	76	166	v	v
23	17	027	ETB	(end of trans. block)	55	37	067	7	7		87	57	127	W	W		119	77	167	w	w
24	18	030	CAN	(cancel)	56	38	070	8	8		88	58	130	X	X		120	78	170	x	x
25	19	031	EM	(end of medium)	57	39	071	9	9		89	59	131	Y	Y		121	79	171	y	y
26	1A	032	SUB	(substitute)	58	3A	072	:	:		90	5A	132	Z	Z		122	7A	172	z	z
27	1B	033	ESC	(escape)	59	3B	073	;	:		91	5B	133	[[123	7B	173	{	{
28	1C	034	FS	(file separator)	60	3C	074	<	<		92	5C	134	\	\		124	7C	174	|	
29	1D	035	GS	(group separator)	61	3D	075	=	=		93	5D	135]]		125	7D	175	}	}
30	1E	036	RS	(record separator)	62	3E	076	>	>		94	5E	136	^	^		126	7E	176	~	~
31	1F	037	US	(unit separator)	63	3F	077	?	?		95	5F	137	_	_		127	7F	177		DEL



ASCII

- Lots of legacy characters
- Cool tricks
 - Flip 1-bit to convert from different cases
- Extremely limited
- English is primary language of computing
 - We still need to use other languages





ASCII

- Unicode
 - Fixes this limited problem
 - Supports different languages
 - Universal standard (kinda)
 - Supports emojis!
- “Unicode Miracle”





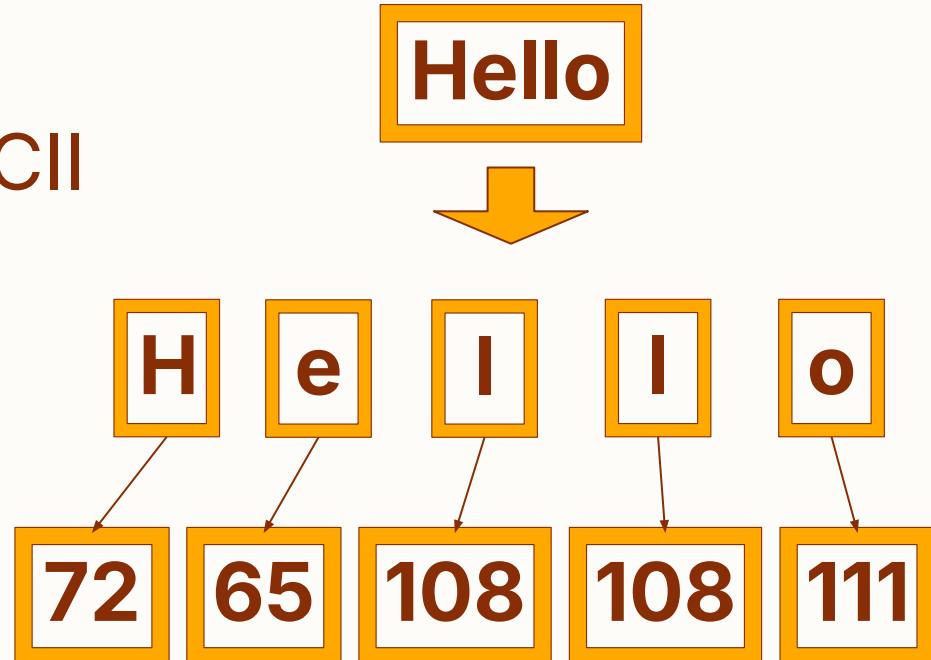
ASCII

- The first 128 unicode characters = ASCII
- ASCII is a subset of Unicode
- Generally, people just consider plaintext ASCII



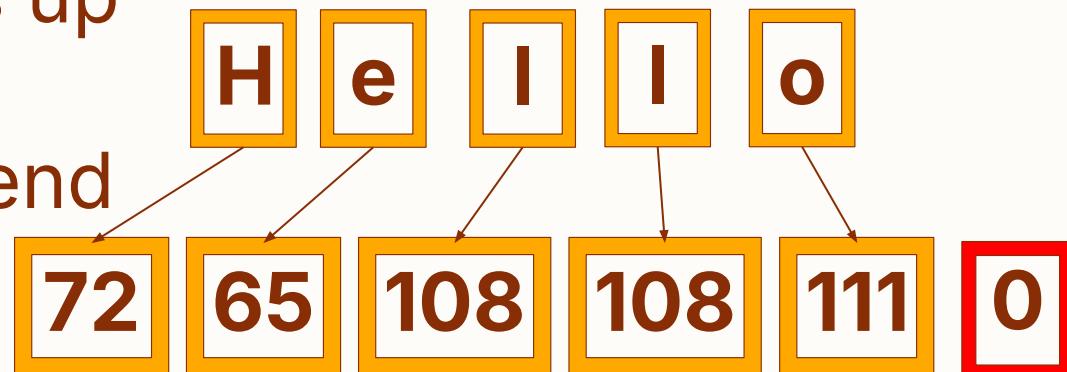
ASCII

- Convert a message to ASCII
- Lookup in table
- This is CASE SENSITIVE



ASCII

- List of characters
- Is called a String
 - This comes up later
- Strings often end with a NULL character





Colors

- We can use bits to represent theoretically anything
 - ~Real numbers
 - Characters
- Colors is also a common one
 - “8-bit color”





Colors

- Color and light is really complicated
 - Bits are very simple
 - Compromises
- Additive colors
- Mix different light colors
 - Make more complex colors





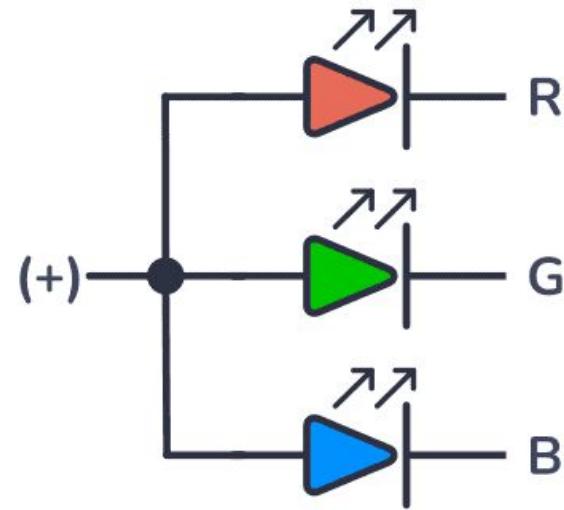
Colors

- Additive Colors
 - Red Channel
 - Green Channel
 - Blue Channel
- Allocate bits to each
- You could then map these to physical lights



Colors

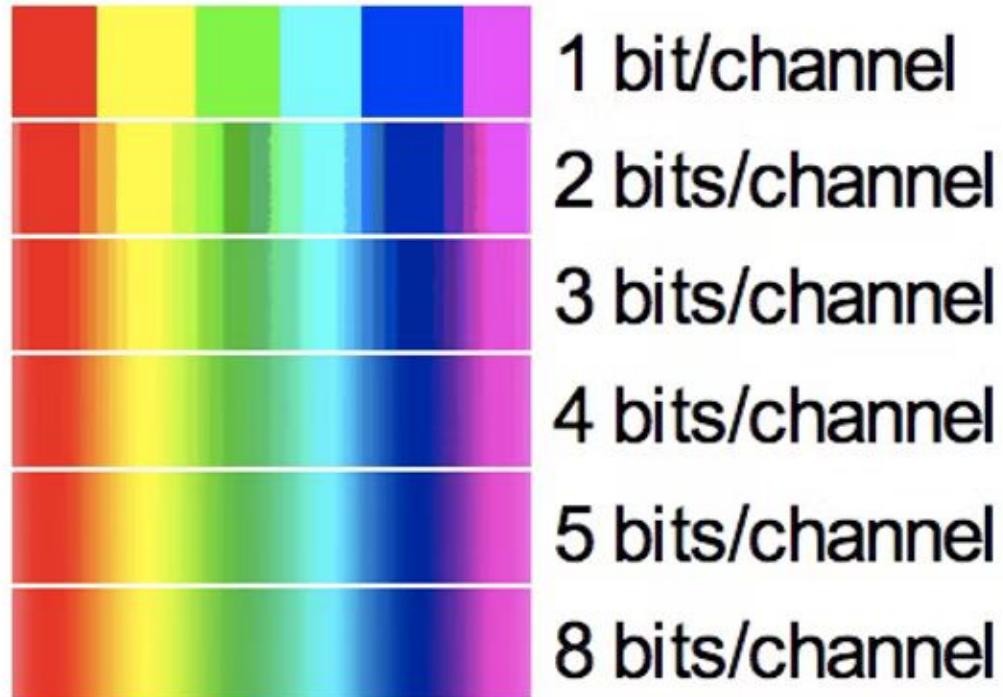
- Physical hardware
- Mapped to each channel
- Your screen has 1000s of these





Colors

- More bits = more colors
- 8 bits is usually good enough
- 256 values per channel





Colors

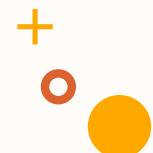
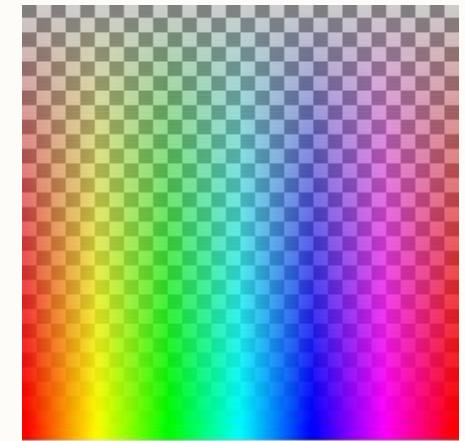
- This lets us describe color using only 3 numbers
- Color = (R, G, B)
- E.g. Green = (0, 255, 0)
Purple = (255, 0, 255)
- Let's use a color picker
- There are different color formats as well (e.g CMYK)





Colors

- We use these colors to encode images
- What about transparency?
- Sometimes a 4th channel is added
- “Alpha” value → RGBA
- Let's see that





What are we doing?

- Taken binary
 - Easy for computers to work with
- Developed “standards”
- Different ways of **encoding** information with bits





Encodings

- Unsigned binary integers
- Signed magnitude
- Two's Complement
- Fixed Point
- Floating Point
- RGB(A)





Encodings

- All represent different potentially useful ideas with bits
- What are standards?
- Who defines them?
- Who gets to choose which bit goes where?





Standards

- “Standards” are really just documents
- Let's look at some
- IEEE ⇒ Floating point

- GS1 ⇒ Barcode



Logic (Gates)

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Logic Gates

- Everything we've done so far has been "theoretical"
- Bits are how we *could* interact with a computer
- Next, we'll dive into how a computer **DOES** interact with bits





Logic Gates

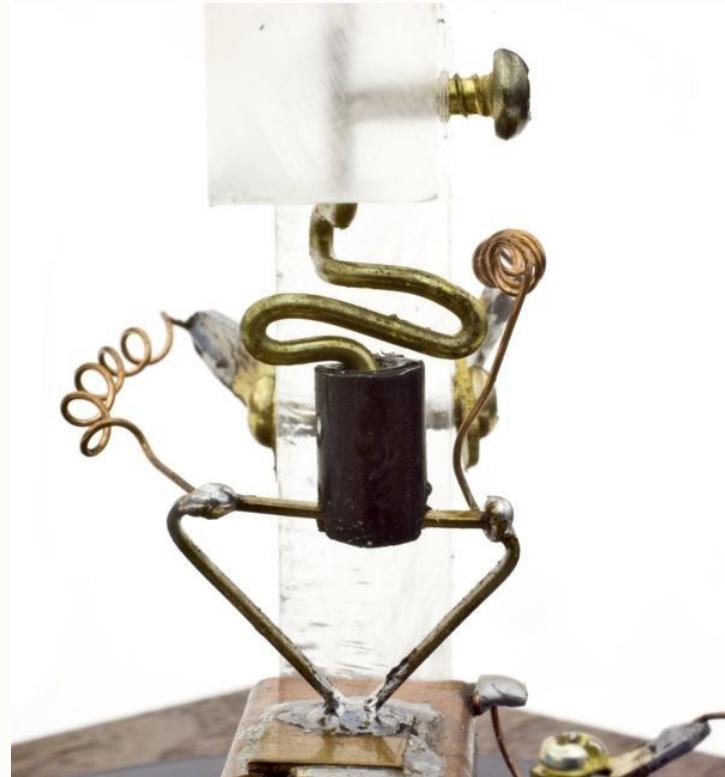
- Bell Labs (AT&T)
 - 1947
- Scientists wanted to build a re-usable amplifier
- Vacuum tubes were horrible at the time
- Wanted easy, solid, reusable circuit





Logic Gates

- The Transistor is born
- Bulky at the time
- Size reduced dramatically over time





Logic Gates



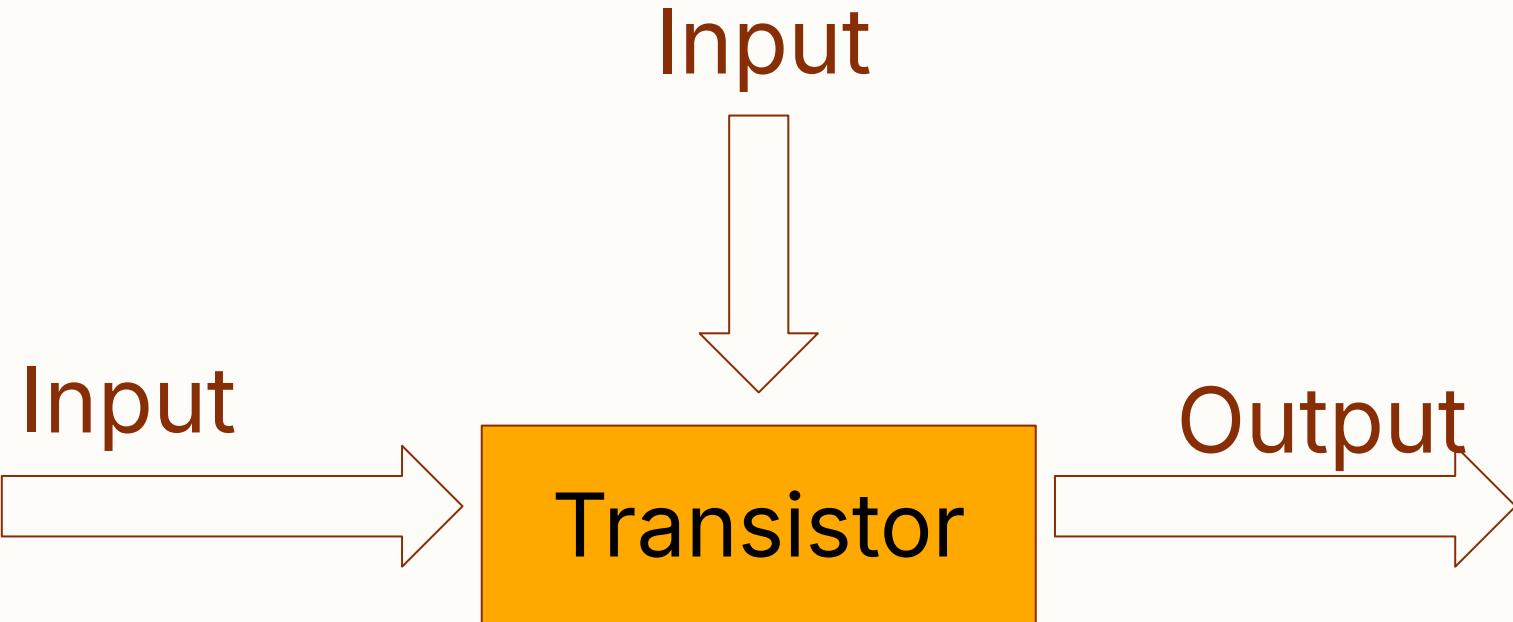


Logic Gates

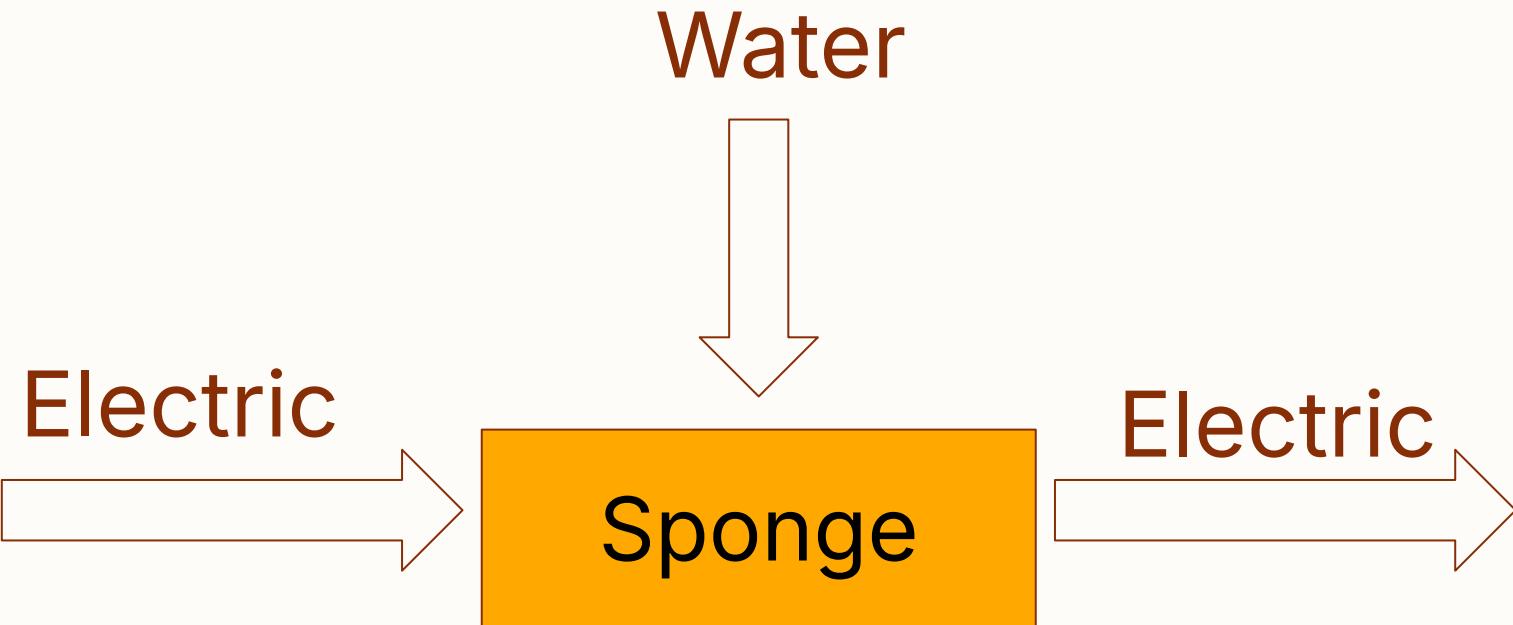
- So what actually is a transistor?
- Semi-conductor
- An analogy
 - Dry Sponge
 - Wet Sponge
- 2 inputs, 1 output
- Let's look at that



Logic Gates



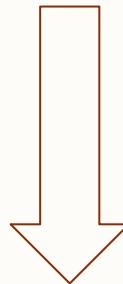
Logic Gates





Logic Gates

No Water



Power



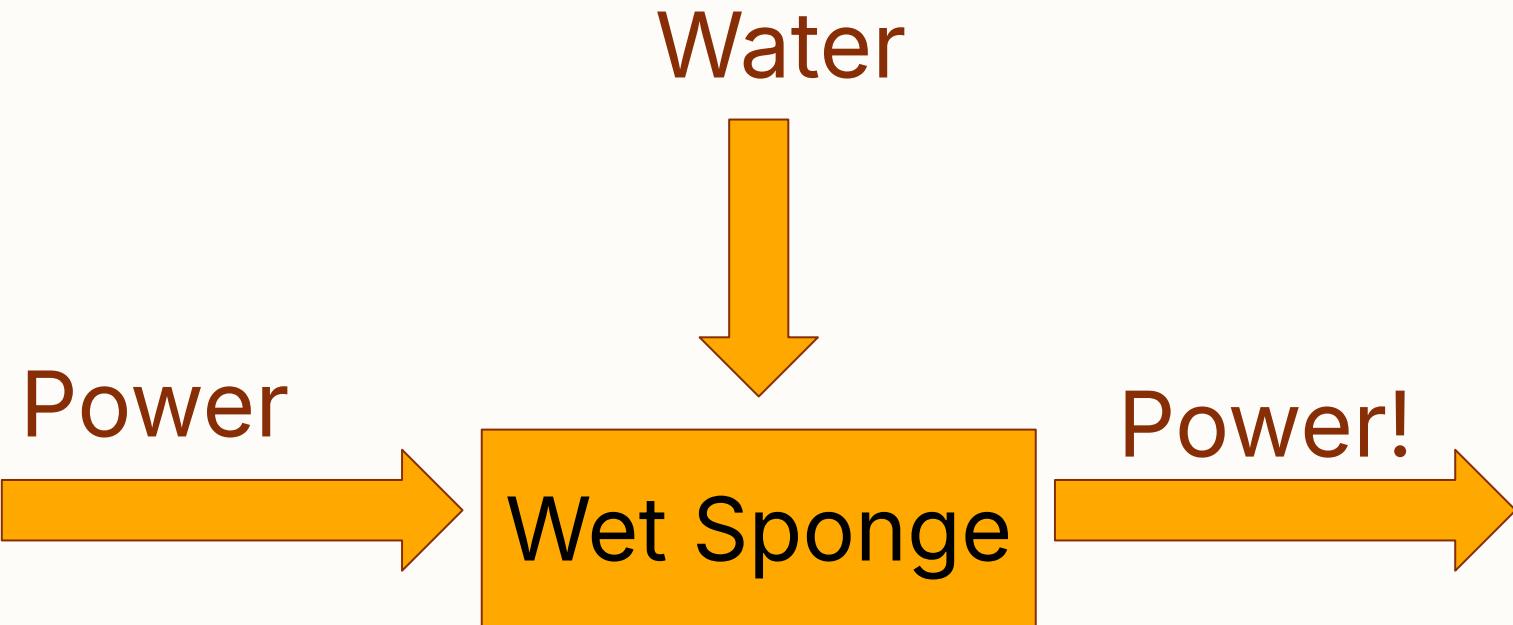
Dry Sponge

No Power





Logic Gates





Logic Gates

- Today, these are incredibly tiny
 - Power/No Power
 - Water/ No Water
 - Examples of bits!
-
- Think back to that RBG device we saw last week

