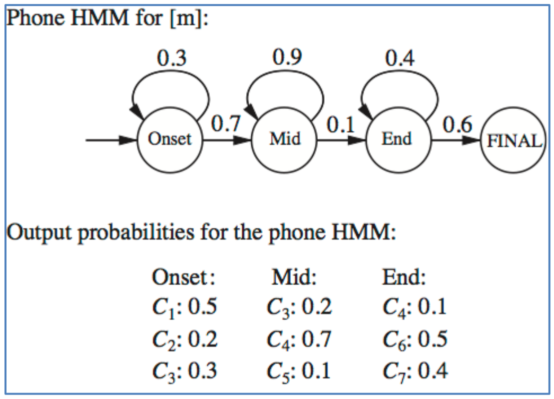
Viterbi Example

Calculate the most probable path through the HMM in Figure 23.16 (below) for the output sequence [C_1,C_2,C_3,C_4,C_5,C_5,C_6]. Also give its probability.



For this output sequence, there are two possible paths through the HMM, shown below with the probability of the path.

- Onset, Onset, Onset, Mid, Mid, Mid, End, Final
  = (0.5)(0.3*0.2)(0.3*0.3)(0.7*0.7)(0.9*0.1)(0.9*0.1)(0.1*0.5)(0.6)
  = 3.21×10^{-7}
- Onset, Onset, Mid, Mid, Mid, Mid, End, Final
  = (0.5)(0.3*0.2)(0.7*0.2)(0.9*0.7)(0.9*0.1)(0.9*0.1)(0.1*0.5)(0.6)
  = 6.43×10^{-7}

The most probable path is the second one above (Onset, Onset, Mid, Mid, Mid, Mid, End, Final) with probability 6.43×10^{-7}.

We can also use the Viterbi algorithm to find the most probable path.

$V_{1,\underline{Onset}} = P(C_1|Onset)P(Onset) = (0.5)(1.0) = 0.5$
$V_{1,Mid} = V_{1,End} = V_{1,Final} = 0$

$V_{2,Onset} = P(C_2|Onset)\max\{[P(Onset|Onset)V_{1,\underline{Onset}}], 0, 0, 0\} = (0.2)\{(0.3)(0.5)\} = 0.03$
$V_{2,Mid} = V_{2,End} = V_{2,Final} = 0$

$V_{3,Onset} = P(C_3|Onset)\max\{[P(Onset|Onset)V_{2,Onset}], 0, 0, 0\} = (0.3)\{(0.3)(0.03)\} = 0.0027$
$V_{3,Mid} = P(C_3|Mid)\max\{[P(Mid|Onset)V_{2,\underline{Onset}}], 0, 0, 0\} = (0.2)\{(0.7)(0.03)\} = 0.0042$
$V_{3,End} = V_{3,Final} = 0$

$V_{4,Onset} = 0$
$V_{4,Mid} = P(C_4|Mid)\max\{[P(Mid|Onset)V_{3,Onset}], [P(Mid|Mid)V_{3,\underline{Mid}}], 0, 0\}$
$= (0.7)\max\{(0.7)(0.0027), (0.9)(0.0042), 0, 0\}$
$= (0.7)\max\{0.00189, 0.00378, 0, 0\} = 0.002646$
$V_{4,End} = P(C_4|End)\max\{0, [P(End|Mid)V_{3,Mid}], 0, 0\} = (0.1)\{(0.1)(0.0042)\} = 0.000042$
$V_{4,Final} = 0$

$V_{5,Onset} = 0$
$V_{5,Mid} = P(C_5|Mid)\max\{0, [P(Mid|Mid)V_{4,\underline{Mid}}], 0, 0\}$
$= (0.1)\{(0.9)(0.002646)\} = 0.00023814$
$V_{5,End} = V_{5,Final} = 0$

$V_{6,Onset} = 0$
$V_{6,Mid} = P(C_5|Mid)\max\{0, P(Mid|Mid)V_{5,\underline{Mid}}, 0, 0\}$
$= (0.1)\{(0.9)(0.00023814)\} = 0.000021433$
$V_{6,End} = V_{6,Final} = 0$

$V_{7,Onset} = V_{7,Mid} = 0$
$V_{7,End} = P(C_6|End)\max\{0, [P(End|Mid)V_{6,\underline{Mid}}], 0, 0\}$
$= (0.5)\{(0.1)(0.000021433)\} = 0.000001072$
$V_{7,Final} = 0$

The final probability is $P(Final|End)V_{7,\underline{End}} = (0.6)(0.000001072) = 6.43 \times 10^{-7}$. The most probable path is the maximum V's at each stage (underlined in red above): Onset, Onset, Mid, Mid, Mid, Mid, End, Final.