Washington State University
School of Electrical Engineering and Computer Science
Fall 2018

CptS 440/540 Artificial Intelligence
Homework 8 Solution
Due: October 25, 2018 (11:59pm)

General Instructions: Put your answers to the following problems into a PDF document and submit as an attachment under Content → Homework 8 for the course CptS 440 Pullman (all sections of CptS 440 and 540 are merged under the CptS 440 Pullman section) on the Blackboard Learn system by the above deadline. Note that you may submit multiple times, but we will only grade the most recent entry submitted before the above deadline.

1. The table below shows the joint probability distribution over three random variables: Sea $\in$ {blue, gray}, Sun $\in$ {true, false}, and Sky $\in$ {clear, cloudy, overcast}. Based on this distribution, and the fact that Sun and Sea are conditionally independent given Sky, draw a Bayesian network consistent with this information. Be sure to show all nodes, links and conditional probability tables (CPTs). All CPT entries should use two decimal place precision. Also, the CPTs should have columns for all possible values of the variables. For example, the CPT for the Sun node should have entries for when Sun=true, and entries for when Sun=false.

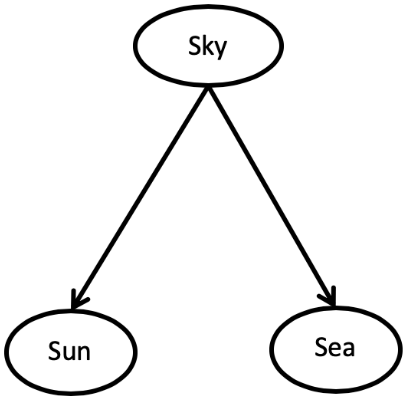
Sea		blue		gray	
Sun		true	false	true	false
Sky	clear	0.10	0.09	0.08	0.06
	cloudy	0.07	0.10	0.07	0.09
	overcast	0.05	0.09	0.08	0.12

Solution:

P(Sky=clear) = 0.10 + 0.09 + 0.08 + 0.06 = 0.33
P(Sky=cloudy) = 0.07 + 0.10 + 0.07 + 0.09 = 0.33
P(Sky=overcast) = 0.05 + 0.09 + 0.08 + 0.12 = 0.34
P(Sun=true | Sky=clear) = P(Sun=true, Sky=clear) / P(Sky=clear) = (0.10 + 0.08) / 0.33 = 0.55
P(Sun=true | Sky=cloudy) = P(Sun=true, Sky=cloudy) / P(Sky=cloudy) = (0.07 + 0.07) / 0.33 = 0.42
P(Sun=true | Sky=overcast) = P(Sun=true, Sky=overcast) / P(Sky=overcast) = (0.05 + 0.08) / 0.34 = 0.38
P(Sun=false | Sky) = 1 – **P**(Sun=true | Sky)
P(Sea=blue | Sky=clear) = P(Sea=blue, Sky=clear) / P(Sky=clear) = (0.10 + 0.09) / 0.33 = 0.58
P(Sea=blue | Sky=cloudy) = P(Sea=blue, Sky=cloudy) / P(Sky=cloudy) = (0.07 + 0.10) / 0.33 = 0.52
P(Sea=blue | Sky=overcast) = P(Sea=blue, Sky=overcast) / P(Sky=overcast) = (0.05 + 0.09) / 0.34 = 0.41
P(Sea=gray | Sky) = 1 – **P**(Sea=blue | Sky)

Final Bayesian network shown below.

P(Sky)		
clear	cloudy	overcast
0.33	0.33	0.34



	P(Sun | Sky)	
Sky	true	false
clear	0.55	0.45
cloudy	0.42	0.58
overcast	0.38	0.62

	P(Sea | Sky)	
Sea	blue	gray
clear	0.58	0.42
cloudy	0.52	0.48
overcast	0.41	0.59

2. Using the Bayesian network in Figure 1 below, compute the following probabilities. Show your work.
   a. P(Time = day, Sky = clear, Sun = true, Moon = false, Sea = blue)
   b. P(Moon = true | Time = night, Sky = cloudy)
   c. P(Time = day | Moon = true)
   d. P(Sea = blue | Time = day, Sky = clear)
   e. P(Time = day | Sea = blue, Moon = false)

Solution:

a. P(Time = day, Sky = clear, Sun = true, Moon = false, Sea = blue)
= P(Time=day) * P(Sky=clear) * P(Sun=true | Time=day, Sky=clear) *
P(Moon=false | Time=day, Sky=clear) * P(Sea=blue | Sun=true)
= (0.5)(0.5)(0.9)(0.8)(0.8) = 0.14

b. P(Moon = true | Time = night, Sky = cloudy) = 0.5 *(from Moon's CPT)*

c. P(Time = day | Moon = true)
= α P(Time=day, Moon=true)
Since, Sun and Sea are not ancestors of Time or Moon, they can be ignored:
= α < Σ_{Sky} P(Time=day, Moon=true, Sky), Σ_{Sky} P(Time=night, Moon=true, Sky) >
= α < Σ_{Sky} P(Time=day, Moon=true, Sky), Σ_{Sky} P(Time=night, Moon=true, Sky) >
= α < Σ_{Sky} P(Time=day) * P(Sky) * P(Moon=true | Time=day, Sky) ,
Σ_{Sky} P(Time=night) * P(Sky) * P(Moon=true | Time=night, Sky) >
= α < P(Time=day) * [P(Sky=clear) * P(Moon=true | Time=day, Sky=clear) +
P(Sky=cloudy) * P(Moon=true | Time=day, Sky=cloudy) +
P(Sky=overcast) * P(Moon=true | Time=day, Sky=overcast)] ,
P(Time=night) [P(Sky=clear) * P(Moon=true | Time=night, Sky=clear) +
P(Sky=cloudy) * P(Moon=true | Time=night, Sky=cloudy) +

P(Sky=overcast) * P(Moon=true | Time=night, Sky=overcast)] >
= α < (0.5) * [(0.5)(0.2) + (0.3)(0.1) + (0.2)(0.0)] , (0.5) * [(0.5)(0.9) + (0.3)(0.5) + (0.2)(0.1)] >
= α <0.065, 0.31>
= <0.17, 0.83>
So, P(Time=day | Moon=true) = 0.17

d. P(Sea = blue | Time = day, Sky = clear)
= α P(Sea=blue, Time=day, Sky=clear)
Since Moon is not an ancestor of Sea, Time or Sky, it can be ignored:
= α < Σ_{Sun} P(Sea=blue, Time=day, Sky=clear, Sun), Σ_{Sun} P(Sea=gray, Time=day, Sky=clear, Sun) >
= α < Σ_{Sun} P(Time=day) * P(Sky=clear) * P(Sun | Time=day, Sky=clear) * P(Sea=blue | Sun) ,
Σ_{Sun} P(Time=day) * P(Sky=clear) * P(Sun | Time=day, Sky=clear) * P(Sea=gray | Sun) >
= α < P(Time=day) * P(Sky=clear) * Σ_{Sun} P(Sun | Time=day, Sky=clear) * P(Sea=blue | Sun) ,
P(Time=day) * P(Sky=clear) * Σ_{Sun} P(Sun | Time=day, Sky=clear) * P(Sea=gray | Sun) >
= α < (0.5)(0.5) * [P(Sun=true | Time=day, Sky=clear) * P(Sea=blue | Sun=true) +
P(Sun=false | Time=day, Sky=clear) * P(Sea=blue | Sun=false)] ,
(0.5)(0.5)* [P(Sun=true | Time=day, Sky=clear) * P(Sea=gray | Sun=true) +
P(Sun=false | Time=day, Sky=clear) * P(Sea=gray | Sun=false)] >
= α < (0.5)(0.5) * [(0.9)(0.8) + (0.1)(0.3)] , (0.5)(0.5) * [(0.9)(0.2) + (0.1)(0.7)] >
= α <0.1875, 0.0625>
= <0.75, 0.25>
So, P(Sea = blue | Time = day, Sky = clear) = 0.75

e. P(Time = day | Sea = blue, Moon = false)
= α P(Time=day, Sea=blue, Moon=false)
= α < Σ_{Sky} Σ_{Sun} P(Time=day, Sea=blue, Moon=false, Sky, Sun) ,
Σ_{Sky} Σ_{Sun} P(Time=night, Sea=blue, Moon=false, Sky, Sun) >
= α < Σ_{Sky} Σ_{Sun} P(Time=day) * P(Sky) * P(Sun | Time=day, Sky) * P(Moon=false | Time=day, Sky) *
P(Sea=blue | Sun) ,
Σ_{Sky} Σ_{Sun} P(Time=night)*P(Sky)*P(Sun | Time=night, Sky)*P(Moon=false | Time=night, Sky) *
P(Sea=blue | Sun) >
= α < P(Time=day) Σ_{Sky} P(Moon=false | Time=day, Sky) P(Sky) Σ_{Sun} P(Sun | Time=day, Sky) *
P(Sea=blue | Sun) ,
P(Time=night) Σ_{Sky} P(Moon=false | Time=night, Sky) P(Sky) Σ_{Sun} P(Sun | Time=night, Sky) *
P(Sea=blue | Sun) >
= α < (0.5) { [P(Moon=false | Time=day, Sky=clear) P(Sky=clear) *
Σ_{Sun} P(Sun | Time=day, Sky=clear) P(Sea=blue | Sun)] +
[P(Moon=false | Time=day, Sky=cloudy) P(Sky=cloudy) *
Σ_{Sun} P(Sun | Time=day, Sky=cloudy) P(Sea=blue | Sun)] +
[P(Moon=false | Time=day, Sky=overcast) P(Sky=overcast) *
Σ_{Sun} P(Sun | Time=day, Sky=overcast) P(Sea=blue | Sun)] } ,
(0.5) { [P(Moon=false | Time=night, Sky=clear) P(Sky=clear) *
Σ_{Sun} P(Sun | Time=night, Sky=clear) P(Sea=blue | Sun)] +

[P(Moon=false | Time=night, Sky=cloudy) P(Sky=cloudy) *
Σ_{Sun} P(Sun | Time=night, Sky=cloudy) P(Sea=blue | Sun)] +
[P(Moon=false | Time=night, Sky=overcast) P(Sky=overcast) *
Σ_{Sun} P(Sun | Time=night, Sky=overcast) P(Sea=blue | Sun)] } >
= α < (0.5) { [P(Moon=false | Time=day, Sky=clear) P(Sky=clear) *
(P(Sun=true | Time=day, Sky=clear) P(Sea=blue | Sun=true) +
P(Sun=false | Time=day, Sky=clear) P(Sea=blue | Sun=false))] +
[P(Moon=false | Time=day, Sky=cloudy) P(Sky=cloudy) *
(P(Sun=true | Time=day, Sky=cloudy) P(Sea=blue | Sun=true) +
P(Sun=false | Time=day, Sky=cloudy) P(Sea=blue | Sun=false))] +
[P(Moon=false | Time=day, Sky=overcast) P(Sky=overcast) *
(P(Sun=true | Time=day, Sky=overcast) P(Sea=blue | Sun=true) +
P(Sun=false | Time=day, Sky=overcast) P(Sea=blue | Sun=false))] } ,
(0.5) { [P(Moon=false | Time=night, Sky=clear) P(Sky=clear) *
(P(Sun=true | Time=night, Sky=clear) P(Sea=blue | Sun=true) +
P(Sun=false | Time=night, Sky=clear) P(Sea=blue | Sun=false))] +
[P(Moon=false | Time=night, Sky=cloudy) P(Sky=cloudy) *
(P(Sun=true | Time=night, Sky=cloudy) P(Sea=blue | Sun=true) +
P(Sun=false | Time=night, Sky=cloudy) P(Sea=blue | Sun=false))] +
[P(Moon=false | Time=night, Sky=overcast) P(Sky=overcast) *
(P(Sun=true | Time=night, Sky=overcast) P(Sea=blue | Sun=true) +
P(Sun=false | Time=night, Sky=overcast) P(Sea=blue | Sun=false))] } >
= α < (0.5) { [(0.8)(0.5) * ((0.9)(0.8) + (0.1)(0.3))] +
[(0.9)(0.3) * ((0.6)(0.8) + (0.4)(0.3))] +
[(1.0)(0.2) * ((0.2)(0.8) + (0.8)(0.3))] } ,
(0.5) { [(0.1)(0.5) * ((0.0)(0.8) + (1.0)(0.3))] +
[(0.5)(0.3) * ((0.0)(0.8) + (1.0)(0.3))] +
[(0.9)(0.2) * ((0.0)(0.8) + (1.0)(0.3))] } >
= α <0.271, 0.057>
= <0.83, 0.17>
So, P(Time=day | Sea=blue, Moon=false) = 0.83

3. Using the Bayesian network in Figure 1 below, answer the following questions.
   a. Given that Time=day, what is the most likely sample produced by Direct Sampling?
   b. Given that Time=night and Sky=overcast, what is the most likely sample produced by Direct Sampling?

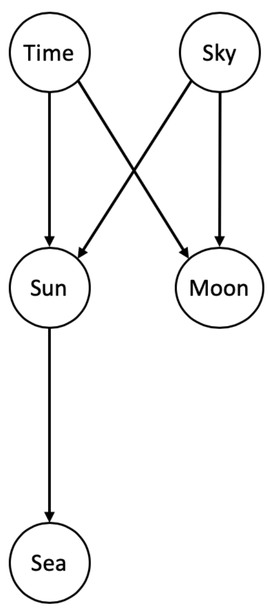
Solution:
   a. <Time=day, Sky=clear, Sun=true, Moon=false, Sea=blue>
   b. <Time=night, Sky=overcast, Sun=false, Moon=false, Sea=gray>

4. *CptS 540 Students Only*. Based on the Bayesian network in Figure 1 below, answer the following questions.
   a. How many entries would there be in the full joint probability distribution table for this problem?
   b. What is the minimum number of probabilities needed to fully specify the information in the full joint probability distribution.

   Solution:
   a. Multiplying number of values for each variable: 2 * 3 * 2 * 2 * 2 = 48
   b. Need the entries in all but one column of each CPT: 1 + 2 + 6 + 6 + 2 = 17

P(Time)	
day	night
0.5	0.5

P(Sky)		
clear	cloudy	overcast
0.5	0.3	0.2



Time	Sky	P(Sun | Time, Sky)	
		true	false
day	clear	0.9	0.1
	cloudy	0.6	0.4
	overcast	0.2	0.8
night	clear	0.0	1.0
	cloudy	0.0	1.0
	overcast	0.0	1.0

Time	Sky	P(Moon | Time, Sky)	
		true	false
day	clear	0.2	0.8
	cloudy	0.1	0.9
	overcast	0.0	1.0
night	clear	0.9	0.1
	cloudy	0.5	0.5
	overcast	0.1	0.9

Sun	P(Sea | Sun)	
	blue	gray
true	0.8	0.2
false	0.3	0.7

Figure 1. Bayesian Network.