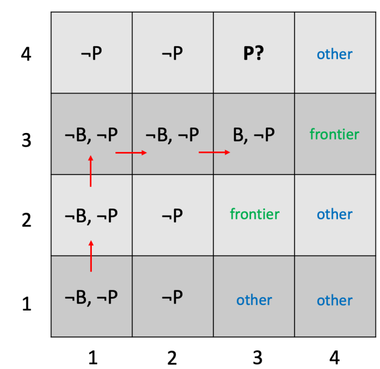
Washington State University
School of Electrical Engineering and Computer Science
Fall 2019

CptS 440/540 Artificial Intelligence
Homework 8 – Solution
Due: October 24, 2019 (11:59pm)

General Instructions: Put your answers to the following problems into a PDF document and submit as an attachment under Content → Homework 8 for the course CptS 440 Pullman (all sections of CptS 440 and 540 are merged under the CptS 440 Pullman section) on the Blackboard Learn system by the above deadline. Note that you may submit multiple times, but we will only grade the most recent entry submitted before the above deadline.

1. Suppose your agent is playing a 4x4 Wumpus world game and has visited locations (1,1), (1,2), (1,3), (2,3) and (3,3). The agent observes a breeze in (3,3), but no breeze in the other visited locations. Given this information, we want to compute the probability of a pit in (3,4). You may use $p_{x,y}$ and $\neg p_{x,y}$ as shorthand notation for $Pit_{x,y}$=true and $Pit_{x,y}$=false, respectively. Similarly, you may use $b_{x,y}$ and $\neg b_{x,y}$ as shorthand notation for $Breeze_{x,y}$=true and $Breeze_{x,y}$=false, respectively. Specifically:
   a. Define the sets: *breeze*, *known*, *frontier* and *other*.
   b. Following the method in the textbook and lecture, compute the probability *distribution* $\mathbf{P}(Pit_{3,4} \mid breeze, known)$. Show your work.



Solution: The scenario is depicted to the right.

a. Sets
   - *breeze* = breeze information we know for certain
     $= \{ \neg b_{1,1}, \neg b_{1,2}, \neg b_{1,3}, \neg b_{2,3}, b_{3,3}\}$
   - *known* = pit information we know for certain
     $= \{\neg p_{1,1}, \neg p_{1,2}, \neg p_{1,3}, \neg p_{1,4}, \neg p_{2,1}, \neg p_{2,2}, \neg p_{2,3}, \neg p_{2,4}, \neg p_{3,3}\}$
   - *frontier* = unknown pit variables for locations adjacent to known and breeze, minus *query*
     $= \{ Pit_{3,2}, Pit_{4,3} \}$
   - *other* = unknown pit variables for locations other than *known*, *frontier*, and *query*
     $= \{ Pit_{3,1}, P_{4,1}, P_{4,2}, P_{4,4}\}$

b. $\mathbf{P}(Pit_{3,4} \mid breeze, known)$

$= \mathbf{P}(P_{3,4}, breeze, known) / \mathbf{P}(breeze, known)$

$= \alpha\, \mathbf{P}(P_{3,4}, breeze, known)$

$= \alpha \sum_{unknown} \mathbf{P}(P_{3,4}, breeze, known, unknown)$

$= \alpha \sum_{frontier} \sum_{other} \mathbf{P}(P_{3,4}, breeze, known, frontier, other)$

$= \alpha \sum_{frontier} \sum_{other} \mathbf{P}(breeze \mid P_{3,4}, known, frontier, other)\, \mathbf{P}(P_{3,4}, known, frontier, other)$

Since *breeze* is independent of *other* given $P_{3,4}$, *known* and *frontier*:

$= \alpha \sum_{frontier} \sum_{other} \mathbf{P}(breeze \mid P_{3,4}, known, frontier)\, \mathbf{P}(P_{3,4}, known, frontier, other)$

$= \alpha \sum_{frontier} \mathbf{P}(breeze \mid P_{3,4}, known, frontier) \sum_{other} \mathbf{P}(P_{3,4}, known, frontier, other)$

Since $P_{3,4}$, *known, frontier, other* are independent of each other:

$= \alpha \sum_{frontier} \mathbf{P}(breeze \mid P_{3,4}, known, frontier) \sum_{other} \mathbf{P}(P_{3,4})\, P(known)\, P(frontier)\, P(other)$

$= \alpha\, \mathbf{P}(P_{3,4})\, P(known) \sum_{frontier} \mathbf{P}(breeze \mid P_{3,4}, known, frontier)\, P(frontier) \sum_{other} P(other)$

Letting $\alpha' = \alpha * P(known)$, and since $\sum_{other} P(other) = 1$:

$= \alpha'\, \mathbf{P}(P_{3,4}) \sum_{frontier} \mathbf{P}(breeze \mid P_{3,4}, known, frontier)\, P(frontier)$

$= \alpha'\, \mathbf{P}(P_{3,4}) \sum_{frontier} \mathbf{P}(breeze \mid P_{3,4}, known, frontier)\, P(frontier)$

$= \alpha' < P(p_{3,4})\, [\, \sum_{frontier} P(breeze \mid p_{3,4}, known, frontier)\, P(frontier)\,]\, ,$

$\quad P(\neg p_{3,4})\, [\, \sum_{frontier} P(breeze \mid \neg p_{3,4}, known, frontier)\, P(frontier)\,] >$

$= \alpha' < P(p_{3,4})\, [\, P(breeze \mid p_{3,4}, known, p_{3,2}, p_{4,3})\, P(p_{3,2}, p_{4,3})\, +$

$\qquad P(breeze \mid p_{3,4}, known, p_{3,2}, \neg p_{4,3})\, P(p_{3,2}, \neg p_{4,3})\, +$

$\qquad P(breeze \mid p_{3,4}, known, \neg p_{3,2}, p_{4,3})\, P(\neg p_{3,2}, p_{4,3})\, +$

$\qquad P(breeze \mid p_{3,4}, known, \neg p_{3,2}, \neg p_{4,3})\, P(\neg p_{3,2}, \neg p_{4,3})\,]\, ,$

$\quad P(\neg p_{3,4})\, [\, P(breeze \mid \neg p_{3,4}, known, p_{3,2}, p_{4,3})\, P(p_{3,2}, p_{4,3})\, +$

$\qquad P(breeze \mid \neg p_{3,4}, known, p_{3,2}, \neg p_{4,3})\, P(p_{3,2}, \neg p_{4,3})\, +$

$\qquad P(breeze \mid \neg p_{3,4}, known, \neg p_{3,2}, p_{4,3})\, P(\neg p_{3,2}, p_{4,3})\, +$

$\qquad P(breeze \mid \neg p_{3,4}, known, \neg p_{3,2}, \neg p_{4,3})\, P(\neg p_{3,2}, \neg p_{4,3})\,] >$ *not possible*

Given that $\mathbf{P}(Pit_{x,y}) = <0.2, 0.8>$:

$= \alpha' < (0.2)\, [\, (1)(0.2)(0.2) + (1)(0.2)(0.8) + (1)(0.8)(0.2) + (1)(0.8)(0.8)\,]\, ,$

$\quad (0.8)\, [\, (1)(0.2)(0.2) + (1)(0.2)(0.8) + (1)(0.8)(0.2) + (0)(0.8)(0.8)\,] >$

$= \alpha' <0.2, 0.288> \qquad \alpha' = 1 / (0.2 + 0.288) = 2.049$

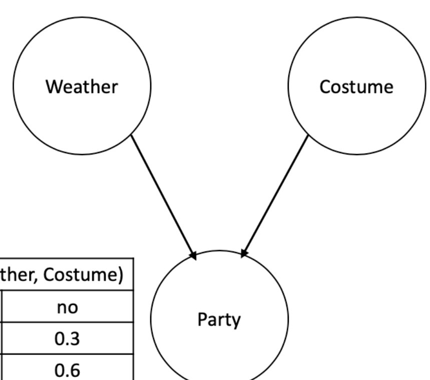
$= <0.41, 0.59>$

2. Recall the Halloween World from Homework 7. The full joint probability distribution for Halloween World is reproduced below. We are also given that Weather and Costume are independent of each other, and that Party depends on both Weather and Costume. Show a Bayesian network consistent with this information. Be sure to show all nodes, links and conditional probability tables (CPTs).

	Weather:	clear		cloudy		rain	
	Costume:	yes	no	yes	no	yes	no
Party:	yes	0.084	0.032	0.18	0.06	0.09	0.024
	no	0.036	0.048	0.12	0.14	0.09	0.096

Solution:

P(Weather)		
clear	cloudy	rain
0.2	0.5	0.3



P(Costume)	
yes	no
0.6	0.4

Weather	Costume	P(Party | Weather, Costume)	
		yes	no
clear	yes	0.7	0.3
clear	no	0.4	0.6
cloudy	yes	0.6	0.4
cloudy	no	0.3	0.7
rain	yes	0.5	0.5
rain	no	0.2	0.8

3. Using the Bayesian network in Figure 1 below, compute the following probabilities. Show your work.
   a. P(AIDone = true, Costume = false, Party = true, HaveFun = true, MakeFriends = true)
   b. P(HaveFun = true | AIDone = false, Costume = true)
   c. P(AIDone = true | HaveFun = true, MakeFriends = true)

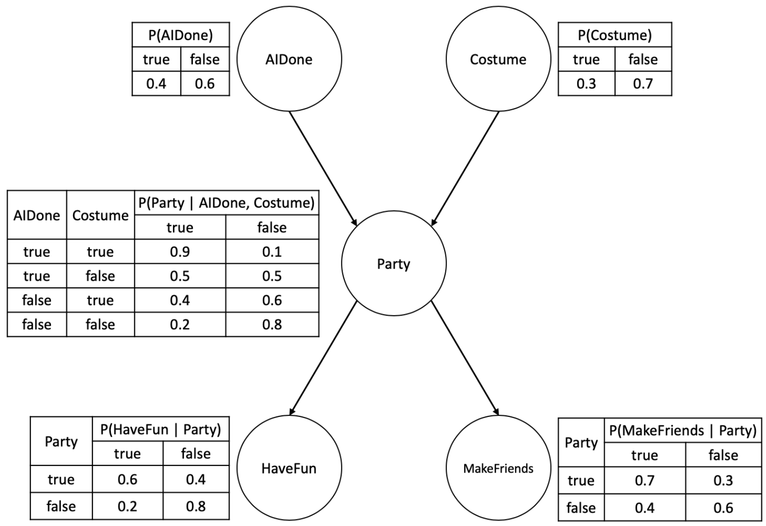


Figure 1. Bayesian Network.

Solution:

Let 'A' mean AIDone, 'a' mean AIDone=true, and '$\neg$ a' mean AIDone=false.
Let 'C' mean Costume, 'c' mean Costume=true, and '$\neg$ c' mean Costume=false.
Let 'P' mean Party, 'p' mean Party=true, and '$\neg$ p' mean Party=false.
Let 'H' mean HaveFun, 'h' mean HaveFun=true, and '$\neg$ h' mean HaveFun=false.
Let 'M' mean MakeFriends, 'm' mean MakeFriends=true, and '$\neg$ m' mean MakeFriends=false.

a. $P(a, \neg c, p, h, m) = P(a) * P(\neg c) * P(p \mid a, \neg c) * P(h \mid p) * P(m \mid p)$
$= (0.4)(0.7)(0.5)(0.6)(0.7) = 0.0588$

b. $P(h \mid \neg a, c) = P(h, \neg a, c) / P(\neg a, c) = \alpha\, \Sigma_P\, \Sigma_M\, P(h, \neg a, c, P, M)$
$= \alpha\, \Sigma_P\, \Sigma_M\, P(\neg a) * P(c) * P(P \mid \neg a, c) * P(h \mid P) * P(M \mid P)$
$= \alpha\; P(\neg a) * P(c) * \Sigma_P\, P(P \mid \neg a, c) * P(h \mid P) * \Sigma_M\, P(M \mid P)$ last term sums to 1
$= \alpha\; P(\neg a) * P(c) * [\, P(p \mid \neg a, c) * P(h \mid p) + P(\neg p \mid \neg a, c) * P(h \mid \neg p)\,]$
$= \alpha\, (0.6)\, (0.3)\, [\, (0.4)\, (0.6) + (0.6)\, (0.2)\,]$
$= \alpha\, (0.0648)$
Similarly for $P(\neg h \mid \neg a, c)$:
$= \alpha\; P(\neg a) * P(c) * [\, P(p \mid \neg a, c) * P(\neg h \mid p) + P(\neg p \mid \neg a, c) * P(\neg h \mid \neg p)\,]$
$= \alpha\, (0.6)\, (0.3)\, [\, (0.4)\, (0.4) + (0.6)\, (0.8)\,]$
$= \alpha\, (0.1152)$ $\alpha = 1 / (0.0648 + 0.1152) = 5.556$
Thus, $P(h \mid \neg a, c) = \alpha\, (0.0648) = 0.36$

c. $P(a \mid h, m) = P(a, h, m) / P(h, m) = \alpha\, \Sigma_C\, \Sigma_P\, P(a, h, m, C, P)$

$= \alpha\, \Sigma_C\, \Sigma_P\, P(a) * P(C) * P(P \mid a, C) * P(h \mid P) * P(m \mid P)$

$= \alpha\, P(a)\, \Sigma_C\, P(C)\, \Sigma_P\, P(P \mid a, C) * P(h \mid P) * P(m \mid P)$

$= \alpha\, P(a)\, [\, P(c)\, \Sigma_P\, P(P \mid a, c) * P(h \mid P) * P(m \mid P) + P(\neg c)\, \Sigma_P\, P(P \mid a, \neg c) * P(h \mid P) * P(m \mid P)\,]$

$= \alpha\, P(a)\, [\, P(c)\, \{ P(p \mid a, c) * P(h \mid p) * P(m \mid p) + P(\neg p \mid a, c) * P(h \mid \neg p) * P(m \mid \neg p) \} +$

$P(\neg c)\, \{ P(p \mid a, \neg c) * P(h \mid p) * P(m \mid p) + P(\neg p \mid a, \neg c) * P(h \mid \neg p) * P(m \mid \neg p) \}\,]$

$= \alpha\, (0.4)\, [\, (0.3)\, \{ (0.9)(0.6)(0.7) + (0.1)(0.2)(0.4) \} + (0.7)\, \{ (0.5)(0.6)(0.7) + (0.5)(0.2)(0.4) \}\,]$

$= \alpha\, (0.11632)$

Similarly, for $P(\neg a \mid h, m)$:

$= \alpha\, P(\neg a)\, [\, P(c)\, \{ P(p \mid \neg a, c) * P(h \mid p) * P(m \mid p) + P(\neg p \mid \neg a, c) * P(h \mid \neg p) * P(m \mid \neg p) \} +$

$P(\neg c)\, \{ P(p \mid \neg a, \neg c) * P(h \mid p) * P(m \mid p) + P(\neg p \mid \neg a, \neg c) * P(h \mid \neg p) * P(m \mid \neg p) \}\,]$

$= \alpha\, (0.6)\, [\, (0.3)\, \{ (0.4)(0.6)(0.7) + (0.6)(0.2)(0.4) \} + (0.7)\, \{ (0.2)(0.6)(0.7) + (0.8)(0.2)(0.4) \}\,]$

$= \alpha\, (0.10104) \qquad \alpha = 1 / (0.11632 + 0.10104) = 4.601$

Thus, $P(a \mid h, m) = \alpha\, (0.11632) = 0.535$