Washington State University
School of Electrical Engineering and Computer Science
Fall 2018

CptS 440/540 Artificial Intelligence
Homework 6 Solution
Due: October 11, 2018 (11:59pm)

1. Translate the following sentences into propositional logic using the atomic sentences: Color(Sky,Blue), Color(Sky,Gray), Shining(Sun), Shining(Moon).

    a. If the color of the sky is blue, then the sun is shining.
    b. If the color of the sky is gray, then the sun is not shining.
    c. If the color of the sky is not blue and the color of the sky is not gray, then the sun is not shining, and the moon is shining.

    Solution:
    a. Color(Sky,Blue) $\Rightarrow$ Shining(Sun)
    b. Color(Sky,Gray) $\Rightarrow \neg$Shining(Sun)
    c. ($\neg$Color(Sky,Blue) $\wedge \neg$Color(Sky,Gray)) $\Rightarrow$ ($\neg$Shining(Sun) $\wedge$ Shining(Moon))

2. Using the following propositional knowledge base, show a resolution proof by refutation that "DayTime" is true.
    i. Cloudy(Sky) $\Rightarrow$ Color(Sky,Gray)
    ii. Color(Sky,Gray) $\Rightarrow \neg$ Shining(Moon)
    iii. Shining(Moon) $\Leftrightarrow \neg$ DayTime
    iv. Cloudy(Sky)

    a. Convert the knowledge base and negated query to CNF. Give each clause a number.
    b. Show each resolution step by indicated the two clause numbers being resolved, and the resulting clause (give it a new number).

    Solution:
    a. Knowledge Base:
        - C1: $\neg$ Cloudy(Sky) $\vee$ Color(Sky,Gray)
        - C2: $\neg$ Color(Sky,Gray) $\vee \neg$ Shining(Moon)
        - C3: $\neg$ Shining(Moon) $\vee \neg$ DayTime
        - C4: DayTime $\vee$ Shining(Moon)
        - C5: Cloudy(Sky)
        - C6: $\neg$ DayTime *// negation of query*

b. Resolution proof by refutation:
   - Resolve C6 and C4:
     - C7: Shining(Moon)
   - Resolve C7 and C2:
     - C8: ¬ Color(Sky,Gray)
   - Resolve C8 and C1:
     - C9: ¬ Cloudy(Sky)
   - Resolve C9 and C5: empty clause *// thus, DayTime must be true*

3. Translate these sentences into first-order logic using the predicates: Color(*s,c,t*), Shining(*p,t*), DayTime(*t*), NightTime(*t*), where *s* is a variable representing the Sky or Sea, *c* is a variable representing a color, *p* is a variable representing the Sun or Moon, and *t* is a variable representing a time of day.

   a. There exists a time when both the sun and the moon are shining.
   b. If the color of the sea is blue, then the sun is shining.
   c. If it is day time, then the color of the sky is either blue or gray.
   d. If the moon is shining, then it is night time.

   Solution:
   a. $\exists t\ Shining(Sun,t) \wedge Shining(Moon,t)$
   b. $\forall t\ Color(Sea,Blue,t) \Rightarrow Shining(Sun,t)$
   c. $\forall t\ DayTime(t) \Rightarrow (Color(Sky,Blue,t) \vee Color(Sky,Gray,t))$
   d. $\forall t\ Shining(Moon,t) \Rightarrow NightTime(t)$

4. Using the following first-order logic knowledge base, show a resolution proof by refutation that "$\exists t\ DayTime(t)$" is true.

   i. $\forall t\ Shining(Sun,t) \Rightarrow Color(Sky,Blue,t)$
   ii. $\forall t\ (Color(Sea,Gray,t) \vee Color(Sea,Blue,t)) \Rightarrow \neg\ NightTime(t)$
   iii. $\forall c,t\ Color(Sea,c,t) \Leftrightarrow Color(Sky,c,t)$
   iv. $\forall t\ \neg\ DayTime(t) \Rightarrow NightTime(t)$
   v. $\exists t\ Shining(Sun,t)$

   a. Convert the knowledge base and negated query to CNF. Give each clause a number. Be sure to standardize variables.
   b. Show each resolution step by indicated the two clause numbers being resolved, the resulting clause (give it a new number), and any necessary substitutions.

Solution:

a. Knowledge Base:
   - C1: ¬ Shining(Sun,t) ∨ Color(Sky,Blue,t)
   - C2: ¬ Color(Sea,Gray,t) ∨ ¬ NightTime(t)
   - C3: ¬ Color(Sea,Blue,t) ∨ ¬ NightTime(t)
   - C4: ¬ Color(Sea,c,t) ∨ Color(Sky,c,t)
   - C5: ¬ Color(Sky,c,t) ∨ Color(Sea,c,t)
   - C6: DayTime(t) ∨ NightTime(t)
   - C7: Shining(Sun,SK0)
   - C8: ¬ DayTime(t) *// negation of query*

b. Resolution proof by refutation:
   - Resolve C8 and C6:
     - Standardized C8: ¬ DayTime(t1)
     - Standardized C6: DayTime(t2) ∨ NightTime(t2)
     - C9: NightTime(t2) with {t1/t2}
   - Resolve C9 and C3:
     - Standardize C9: NightTime(t3)
     - Standardized C3: ¬ Color(Sea,Blue,t4) ∨ ¬ NightTime(t4)
     - C10: ¬ Color(Sea,Blue,t4) with {t3/t4}
   - Resolve C10 and C5:
     - Standardize C10: ¬ Color(Sea,Blue,t5)
     - Standardized C5: ¬ Color(Sky,c6,t6) ∨ Color(Sea,c6,t6)
     - C11: ¬ Color(Sky,Blue,t6) with {c6/Blue, t5/t6}
   - Resolve C11 and C1:
     - Standardize C11: ¬ Color(Sky,Blue,t7)
     - Standardized C1: ¬ Shining(Sun,t8) ∨ Color(Sky,Blue,t8)
     - C12: ¬ Shining(Sun,t8) with {t7/t8}
   - Resolve C12 and C7:
     - Standardize C12: ¬ Shining(Sun,t9)
     - Standardized C7: Shining(Sun,SK0)
     - C13: empty clause with {t9/SK0} *// thus, ∃ t DayTime(t) must be true*

5. *CptS 540 Students Only*. Create an input file for the Vampire theorem prover that can be used to solve Problem 4. Include your input file and the Vampire output, in the PDF document for your Homework 6 solution. Or, if you prefer, you can bundle your PDF along with the input and output text files into one zip file for submission. You can download the Vampire theorem prover from https://vprover.github.io/. There is a Linux binary available there that runs on the ssh1-ssh10 servers that all grad students have access to. You can run it on your own machine if you prefer.

Solution:

Vampire Input File:

```
fof(a1,axiom,
    ! [T] : (shining(sun,T) => color(sky,blue,T))).

fof(a2,axiom,
    ! [T] : ((color(sea,gray,T) | color(sea,blue,T)) => ~nighttime(T))).

fof(a3,axiom,
    ! [C,T] : (color(sky,C,T) <=> color(sea,C,T))).

fof(a4,axiom,
    ! [T] : (~daytime(T) => nighttime(T))).

fof(a5,axiom,
    ? [T] : shining(sun,T)).

fof(c1,conjecture,
    ? [T] : daytime(T)).
```

Vampire Output File (from running “vampire <inputFile>”):

```
% Refutation found. Thanks to Tanya!
% SZS status Theorem for prob4
% SZS output start Proof for prob4
1. ! [X0] : (shining(sun,X0) => color(sky,blue,X0)) [input]
2. ! [X0] : ((color(sea,blue,X0) | color(sea,gray,X0)) => ~nighttime(X0)) [input]
3. ! [X1,X0] : (color(sky,X1,X0) <=> color(sea,X1,X0)) [input]
4. ! [X0] : (~daytime(X0) => nighttime(X0)) [input]
5. ? [X0] : shining(sun,X0) [input]
6. ? [X0] : daytime(X0) [input]
7. ~? [X0] : daytime(X0) [negated conjecture 6]
8. ! [X0,X1] : (color(sky,X0,X1) <=> color(sea,X0,X1)) [rectify 3]
9. ! [X0] : (color(sky,blue,X0) | ~shining(sun,X0)) [ennf transformation 1]
10. ! [X0] : (~nighttime(X0) | (~color(sea,blue,X0) & ~color(sea,gray,X0))) [ennf
transformation 2]
11. ! [X0] : (nighttime(X0) | daytime(X0)) [ennf transformation 4]
12. ! [X0] : ~daytime(X0) [ennf transformation 7]
13. ! [X0,X1] : ((color(sky,X0,X1) | ~color(sea,X0,X1)) & (color(sea,X0,X1) |
~color(sky,X0,X1))) [nnf transformation 8]
14. ? [X0] : shining(sun,X0) => shining(sun,sK0) [choice axiom]
15. shining(sun,sK0) [skolemisation 5,14]
16. color(sky,blue,X0) | ~shining(sun,X0) [cnf transformation 9]
18. ~color(sea,blue,X0) | ~nighttime(X0) [cnf transformation 10]
19. ~color(sky,X0,X1) | color(sea,X0,X1) [cnf transformation 13]
21. daytime(X0) | nighttime(X0) [cnf transformation 11]
22. shining(sun,sK0) [cnf transformation 15]
23. ~daytime(X0) [cnf transformation 12]
24. nighttime(X0) [resolution 21,23]
25. color(sea,blue,X0) | ~shining(sun,X0) [resolution 19,16]
26. ~shining(sun,X0) | ~nighttime(X0) [resolution 25,18]
27. ~shining(sun,X0) [subsumption resolution 26,24]
29. $false [resolution 27,22]
% SZS output end Proof for prob4
% ------------------------------
% Version: Vampire 4.3.0 (commit unknown)
% Termination reason: Refutation

% Memory used [KB]: 4733
% Time elapsed: 0.001 s
% ------------------------------
% ------------------------------
```