CPTS 553: Graph Theory
Assignment 2

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September 8, 2020

1

We know, simple graphs are those with no loops and no parallel edges. But simple graphs can have *zero* edges. Hence, among all simple graphs with 21 vertices, the minimum possible number of edges such a graph can have is **Zero**.

A simple graph with n vertices will have the maximum number of edges when it is a complete graph. We also know that

$$2|E| = \sum_{i=1}^{n}(deg(v_i))$$

For simple and complete graph with 21 vertices, $\sum_{i=1}^{21}(deg(v_i)) = 21*(21-1)$. Now,

$$2|E| = 21*(21-1)$$

This implies, $|E| = 210$. Hence, among all simple graphs with 21 vertices, the maximum possible number of edges such a graph can have is **210**.

2

The graph H can be broken down into three components:

- Component 1: Graph G
- Component 2: Graph G
- A set of edges where each edge connects the corresponding vertices of Component 1 and 2.

Total number of vertices in H is the same as the sum of #vertices in the individual components $= n+n+0 = 2n$

Total number of edges in H is the same as the sum of #edges in the individual components $= m+m+n = 2m+n$

3

3.1

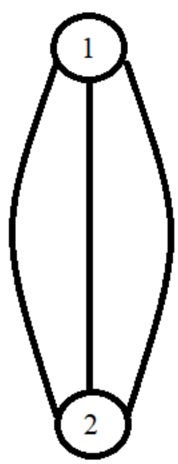
We know that,

$$2|E| = \sum_{i=1}^{n}(deg(v_i))$$

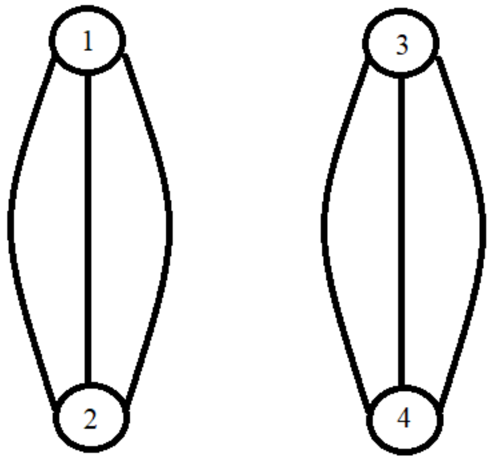
For a loopless cubic graph, every vertex in it has degree 3. Therefore, the sum of degree is $3n$ where n is the number of vertices. We know that this is equal to $2|E|$ which is an even integer. Since, $2|E| = 3n$, the right hand side has to be even as well. Hence, n is even. This proves that a loopless cubic graph must have an even number of vertices.

3.2

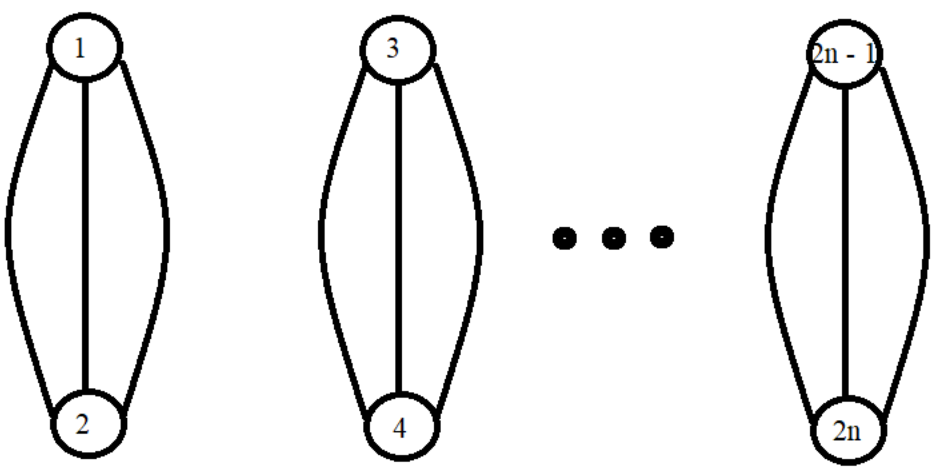
For $n = 1$, the graph will have $2n$ or 2 vertices. A loopless cubic graph with 2 vertices is shown in the figure below:



For $n = 2$, the graph will have $2n$ or 4 vertices. A loopless cubic graph with 4 vertices is shown in the figure below:

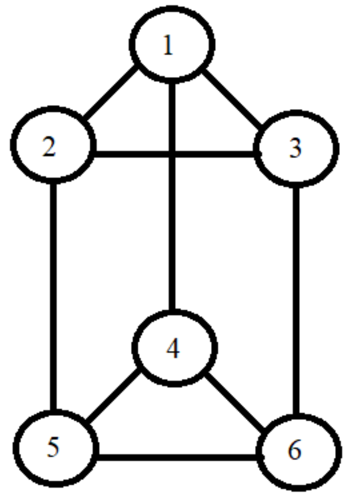


Let the graph corresponding to $n = 1$ be named G_1. We can observe that the graph for $n = 2$ (say G_2) is the same as G_1 repeated twice. Similarly, for G_n, the graph will be G_1 repeated n times.

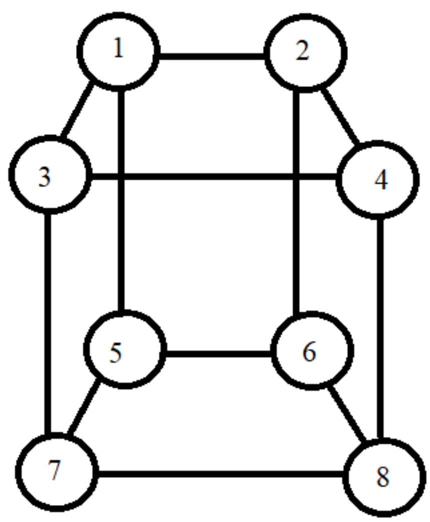


3.3

For $n = 3$, the graph will have $2n$ or vertices. A simple cubic graph with 6 vertices is shown in the figure below (let this be G_3):

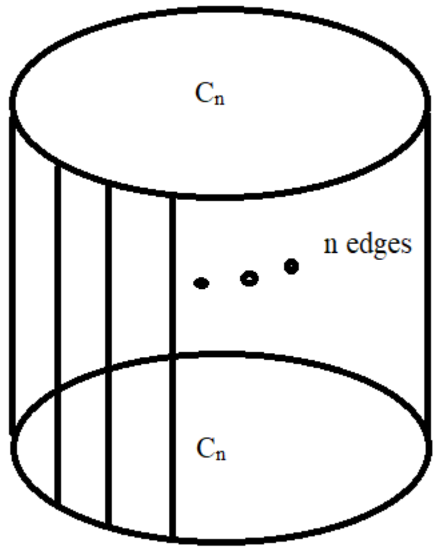


For $n = 4$, the graph will have $2n$ or 8 vertices. A simple cubic graph with 8 vertices is shown in the figure below (let this be G_4):



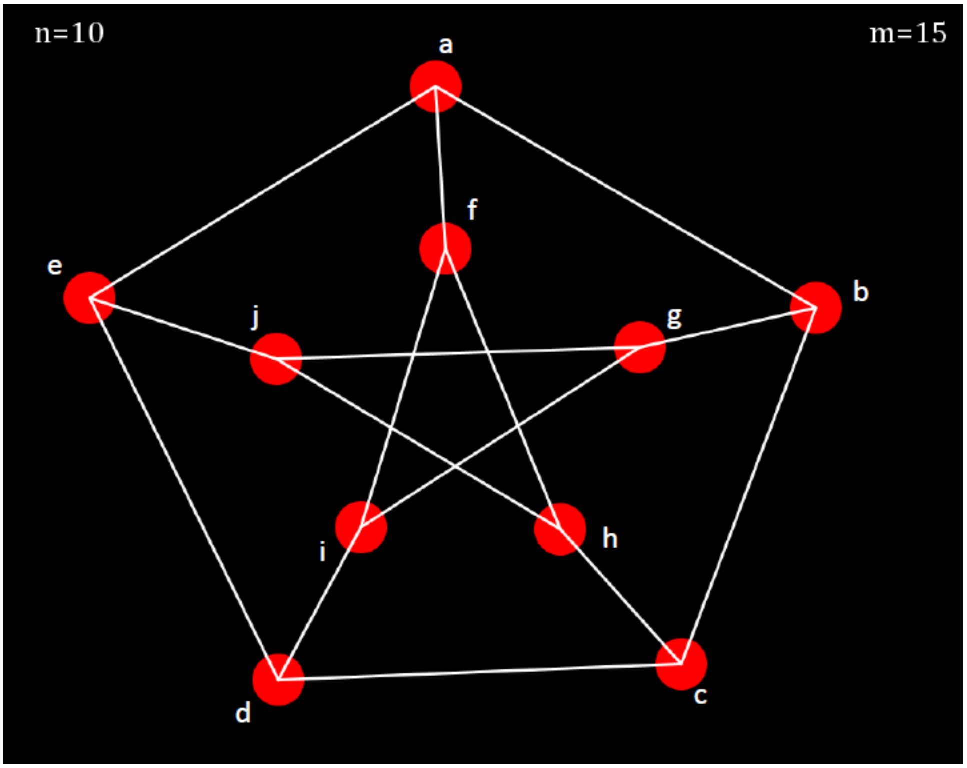
Taking inspiration from Question 2, we can see that if we consider G_3 as H, then G corresponds to C_3 which is the cycle graph with 3 vertices. If we consider G_4 as H, then G corresponds to C_4 which is the cycle graph with 4 vertices.

Similarly, G_n can be constructed by using 2 C_ns in the manner shown in Question 2.



4

The Petersen graph is shown in the figure below. In order to determine whether it is bipartite, let us assume that is indeed bipartite.



Since the graph is symmetric along the 5 outer vertices, we consider the vertex a without a loss of generality. Since our assumption is that the graph is bipartite, if the vertex a belongs to one of the vertex sets, then vertices e, f, and b will belong to the other set of vertices (since they are all adjacent to a). If we consider the vertex j, since it is adjacent to the vertex e, it will belong to the set containing a. Now if we focus on the vertex h, we see that it doesn't belong to the set containing j (since they are adjacent). h also doesn't belong to the set containing f (same reason). Therefore, h cannot be placed in either of the two sets which implies that the vertex set cannot be partitioned into two mutually exclusive sets, such that the union of the two gives the total vertex set and no pairs of vertices from the same set are adjacent to each other.

This proves that the Peterson Graph is **NOT a bipartite graph**.