Assignment 6 (questions about trees) is available on Blackboard; there are five questions.

Recall that a **rooted tree** $(\boldsymbol{T}, \boldsymbol{r})$ consists of a tree T and a distinguished vertex r called the **root**.

We'll look at levels of vertices of a rooted tree and the height H of rooted trees today.

Definition. Let (T, r) be a rooted tree. The **level** of a vertex v is $D(r, v)$, the distance from the root to v; equivalently, the length (in edges) of the unique r, v-path.

Observations.

- The root r is a level 0 vertex.
- For any level L vertex, any of its children are level $L+1$ vertices and its parent (if it has one) is a level $L-1$ vertex.
- If u is a level L vertex and w is a level L' vertex, then $D(u, w) \leq L + L'$.

Definition. The **height** of a rooted tree (T, r) is the maximum level among its vertices. This is also the length of a longest r, u-path as u ranges among the vertices.

Observations.

- Every level H vertex is a non-parent.
- For any two vertices u, w, $D(u, w) \leq 2H$.
- If T has n vertices and $m = n - 1$ edges, $H \leq m = n - 1$.
- If $H = n - 1$, then T is a path with the root at one endpoint.

Definition. Let (T, r) be a rooted tree and let v be any given vertex of T. Let $S(v)$ be the set of vertices that contains v and all of the descendants of v. Then the $\boldsymbol{v}$**-subtree of** $(\boldsymbol{T}, \boldsymbol{r})$ is the rooted tree (T', v) where T' is the subgraph of T induced by $S(v)$. Recall that this is the subgraph with the largest set of edges whose vertex set is $S(v)$; it contains every edge of T with both endpoints in $S(v)$.

Definition. If v is a child of the root r, then the v-subtree of T is called a **principal subtree**.

Observation. It is not necessarily true that if (T, r) has height H, then a principal subtree must have height $H - 1$.

Proposition. If (T, r) has height $H \geq 1$, then there exists a principal subtree of height $H - 1$. Furthermore, all principal subtrees have height at most $H - 1$.

Definition. If (T, r) is a rooted tree where every vertex has <u>at most</u> two children, then (T, r) is called a **binary tree**.

Definition. If (T, r) is a binary tree, then for any parent v with two children, we designate one of its children to be the **left child** and the other to be the **right child**. If v has only one child, then we allow the child to have either designation.

Definition. Given a binary tree (T, r), the principal subtree rooted at the left child of r is called the **left subtree** and the principal subtree rooted at the right child of r is called the **right subtree**.

Definition. The rooted tree (T, r) is an n-ary rooted tree if every parent has at most n children.

When $n = 3$, we call this a **ternary tree**. When $n = 1$, we call this a **unary** tree; such a tree must be a path.