

Let's try to draw the graph with the following Laplacian:

$$L = \begin{bmatrix} 2 & & -1 & -1 \\ & 3 & -1 & -1 & -1 \\ -1 & -1 & 4 & -1 & -1 & -1 \\ -1 & -1 & -1 & 3 & & -1 \\ -1 & -1 & -1 & -1 & 3 & \\ -1 & -1 & -1 & -1 & -1 & 3 \end{bmatrix}$$

For Wolfram Alpha,

$L = \{\{2,0,0,-1,-1,0\},\{0,3,-1,0,-1,-1\},\{0,-1,4,-1,-1,-1\},\{-1,0,-1,3,0,-1\},\{-1,-1,-1,0,3,0\},\{0,-1,-1,-1,0,3\}\}$

Eigenvalues and corresponding eigenvectors

$$0 \leq \lambda_2 = 1.697 \leq \lambda_3 = 2.382 \leq 4.000 \leq 4.618 \leq 5.303$$

0	1.697	2.382	4	4.618	5.303
1	-2	0	1	0	-2
1	1	-1	1	-1	1
1	0.605551	0	-1	0	-6.60555
1	-0.30278	1.61803	-1	-0.61803	3.30278
1	-0.30278	-1.61803	-1	0.618034	3.30278
1	1	1	1	1	1
2.44949	2.559304	2.689989	2.44949	1.662508	8.45281

Normalized eigenvectors, i.e., given  $\mathbf{v}$ , the unit vector in the same direction is  $\frac{1}{\sqrt{\mathbf{v} \cdot \mathbf{v}}} \mathbf{v}$ . If there are eigenvalues with multiplicity greater than one, you have to use Gram Schmidt to get an orthogonal collection for that eigenvalue.

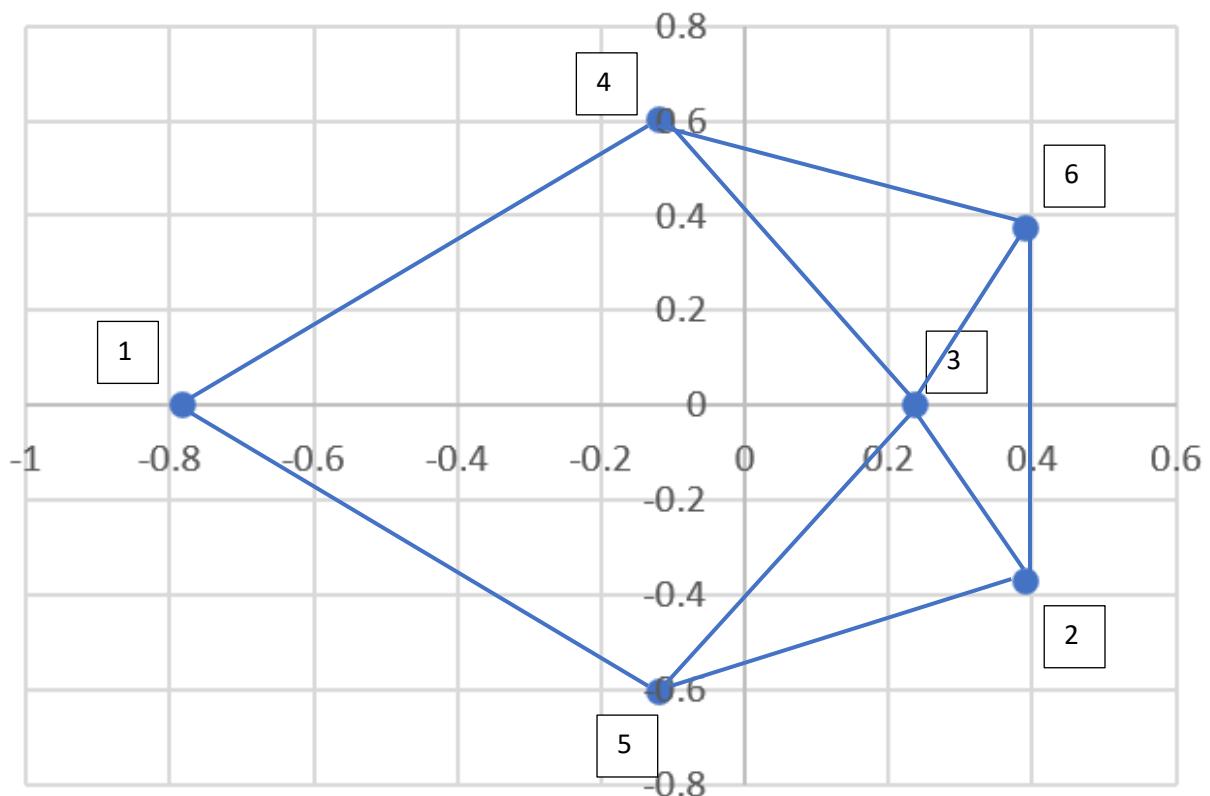
w1	w2	w3	w4	w5	w6
0.408248	-0.78146	0	0.408248	0	-0.23661
0.408248	0.390731	-0.37175	0.408248	-0.6015	0.118304
0.408248	0.236608	0	-0.40825	0	-0.78146
0.408248	-0.1183	0.601501	-0.40825	-0.37175	0.390732
0.408248	-0.1183	-0.6015	-0.40825	0.371748	0.390732
0.408248	0.390731	0.371749	0.408248	0.601501	0.118304

From the

$$x^T L x = \sum_{e=i,j} (x_i - x_j)^2$$

Using the eigenvectors for the smallest positive eigenvalues to plot the vertices puts adjacent vertices close together.

v2 and v3 plot



Using the eigenvectors for the largest positive eigenvalues to plot the vertices puts adjacent vertices far apart:

v5 and v6 plot

