CPTS 553: Graph Theory
Assignment 7

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November 13, 2020

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Let us assume that K_9 can be drawn on a 2-holed torus without edges crossing. We know that $n = 9$, $m = 36$, and $n - m + r = -2$.
Therefore, $r = 25$.
Since every region must be bounded by at least three edges, we can arrive at the following inequality: $2m \geq 3r$. Now, $m = 36$ and $r = 25$.
Therefore, we get, $72 \geq 75$ which is impossible.
Hence, our assumption that K_9 can be drawn on a 2-holed torus without edges crossing, is wrong.

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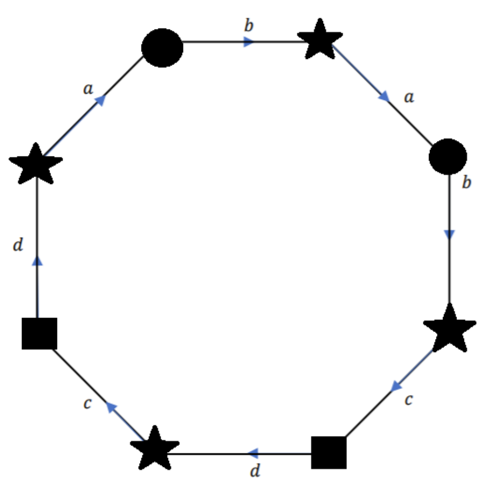

The diagram can be redrawn with the nodes classified as shown above. Here, we can see that nodes

shown with similar markings (star, square, circle) are topologically similar. Since there are three distinct points, $n = 3$.
The distinct arcs possible are star-to-circle, circle-to-star, star-to-square, and square-to-star. Hence, $m = 4$.
The interior of the figure described by the arcs, consist of one region, hence $r = 1$.
Therefore, $n - m + r = 0$.

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In K_n, there are $\binom{n}{2}$ edges. And each edge can be oriented in two ways. Therefore, the total number of orientations possible are $2^{\binom{n}{2}}$.

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For a fixed $n \in \{3, 4, 5, 6, \dots\}$, each vertex in C_n has a total degree of 2. Now,

- Case 1: Indegree = 0, Outdegree = 2: This makes that vertex unreachable from any other part of the graph since it has no incoming edges. The graph is not strongly connected. Hence, this is not possible.
- Case 2: Indegree = 2, Outdegree = 0: There is no way to reach any other part of the graph from this vertex since there are no outgoing edges. So the graph is not strongly connected. Hence, this is also an impossible case.
- Case 3: Indegree = 1, Outdegree = 1: For each vertex to have 1 incoming and 1 outgoing edge, there are only 2 possible orientations.
    1. All the arcs are oriented in a clockwise manner, i.e. there is an edge from v_i to v_{i+1}
    2. All the arcs are oriented in a counter-clockwise manner, i.e. there is an edge from v_{i+1} to v_i.

Therefore, it can be observed that For a fixed $n \in \{3, 4, 5, 6, \dots\}$, there are exactly two orientations of C_n with vertex set $V = \{0, 1, 2, \dots, n-1\}$ that are strongly connected.