

Let's try to draw the graph with the following Laplacian:

$$L = \begin{bmatrix} 2 & & & -1 & -1 & \\ & 3 & -1 & & -1 & -1 \\ & -1 & 4 & -1 & -1 & -1 \\ -1 & & -1 & 3 & & -1 \\ -1 & -1 & -1 & & 3 & \\ & -1 & -1 & -1 & & 3 \end{bmatrix}$$

For Wolfram Alpha,

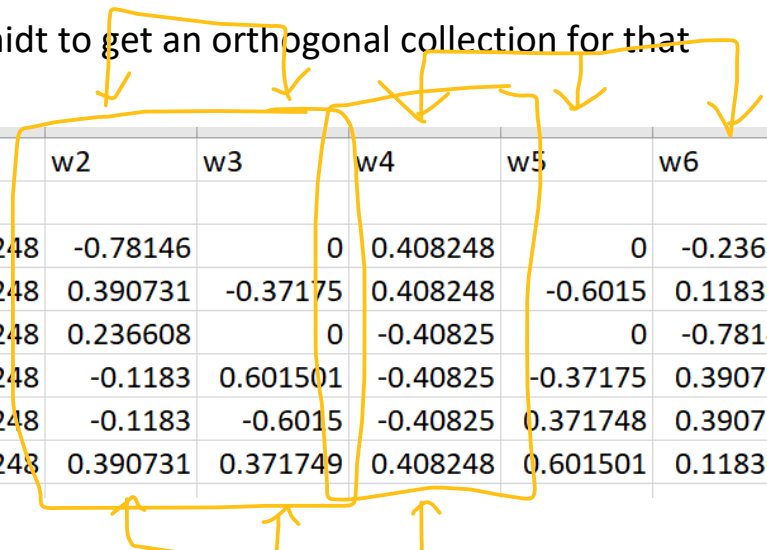
$L = \{\{2,0,0,-1,-1,0\}, \{0,3,-1,0,-1,-1\}, \{0,-1,4,-1,-1,-1\}, \{-1,0,-1,3,0,-1\}, \{-1,-1,-1,0,3,0\}, \{0,-1,-1,-1,0,3\}\}$

Eigenvalues and corresponding eigenvectors

$$0 \leq \lambda_2 = 1.697 \leq \lambda_3 = 2.382 \leq 4.000 \leq 4.618 \leq 5.303$$

	0	1.697	2.382	4	4.618	5.303
1	-2	0	1	0	-2	
1	1	-1	1	-1	1	
1	0.605551	0	-1	0	-6.60555	
1	-0.30278	1.61803	-1	-0.61803	3.30278	
1	-0.30278	-1.61803	-1	0.618034	3.30278	
1	1	1	1	1	1	1
2.44949	2.559304	2.689989	2.44949	1.662508	8.45281	

Normalized eigenvectors, i.e., given \mathbf{v} , the unit vector in the same direction is $\frac{1}{\sqrt{\mathbf{v} \cdot \mathbf{v}}} \mathbf{v}$. If there are eigenvalues with multiplicity greater than one, you have to use Gram Schmidt to get an orthogonal collection for that eigenvalue.



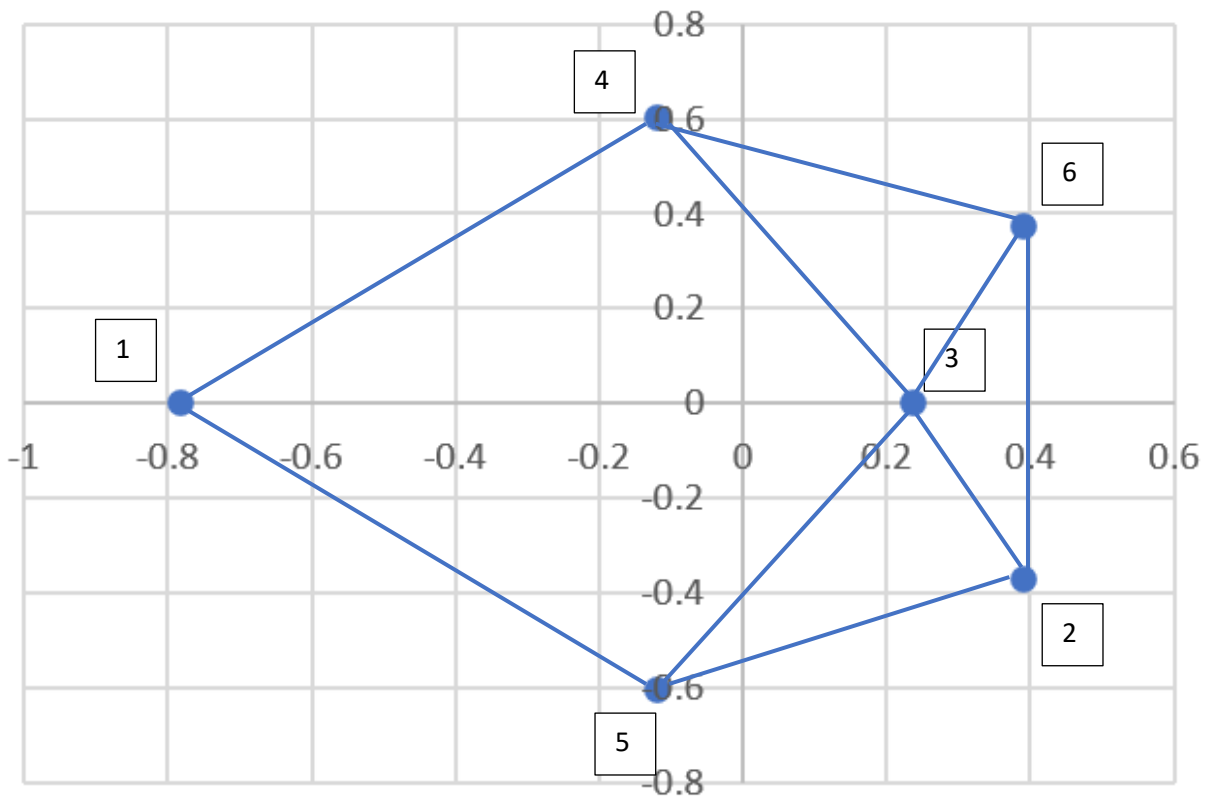
w1	w2	w3	w4	w5	w6
0.408248	-0.78146	0	0.408248	0	-0.23661
0.408248	0.390731	-0.37175	0.408248	-0.6015	0.118304
0.408248	0.236608	0	-0.40825	0	-0.78146
0.408248	-0.1183	0.601501	-0.40825	-0.37175	0.390732
0.408248	-0.1183	-0.6015	-0.40825	0.371748	0.390732
0.408248	0.390731	0.371749	0.408248	0.601501	0.118304

From the

$$x^T L x = \sum_{e=ij} (x_i - x_j)^2$$

Using the eigenvectors for the smallest positive eigenvalues to plot the vertices puts adjacent vertices close together:

v2 and v3 plot



Using the eigenvectors for the largest positive eigenvalues to plot the vertices puts adjacent vertices far apart:

v5 and v6 plot

