CPTS 553: Graph Theory
Assignment 4

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1

We know $D(x, y)$ is same as $x \oplus y$. Now, $(a \oplus b) \oplus (b \oplus c) = (a \oplus c)$

1.1

- If $D(a, b)$ and $D(b, c)$ are both even:
  we know, $D(a, c) = a \oplus c$
  $\implies D(a, c) = (a \oplus b) \oplus (b \oplus c)$
  $\implies D(a, c) = D(a, b) \oplus D(b, c)$
  We know that two even numbers can differ from each other in their bit representations in only even number of places. Therefore, $D(a, b) \oplus D(b, c)$ is even. Hence, $D(a, c)$ is even.

- If $D(a, b)$ and $D(b, c)$ are both even:
  we know, $D(a, c) = D(a, b) \oplus D(b, c)$
  We also know that two odd numbers can differ from each other in their bit representations in only even number of places. Therefore, $D(a, b) \oplus D(b, c)$ is even. Hence, $D(a, c)$ is even.

1.2

We know that $D(a, c) = D(a, b) \oplus D(b, c)$
It is also known that the XOR of an even and an odd number is always odd since they differ from each other in odd number of places. Therefore, if $D(a, b)$ is even and $D(b, c)$ is odd or the other way, $D(a, c)$ is going to be odd as well.

2

All of the cycles that start and end at vertex 1 in this drawing of K_4 are as follows:

1. "1231"
2. "1241"
3. "1341"
4. "1421"
5. "1431"
6. "1321"
7. "12341"
8. "12431"
9. "14321"
10. "14231"
11. "13241"
12. "13421"

3

3.1

We notice that G_m is connected only when m is co-prime with 12. So, the values of m for which G_m is connected are: 1, 5, 7, and 11.

3.2

G_m will have different number of components based on the value of m. They are as follows:

- For m equal to 1 or 11, number of components is 1
- For m equal to 2 or 10, number of components is 2
- For m equal to 3 or 9, number of components is 3
- For m equal to 4 or 8, number of components is 4
- For m equal to 5 or 7, number of components is 1
- For m equal to 6, number of components is 6

4

We know that for an edge e in graph G, if two vertices u and v in $G - e$ is disconnected, then e is a bridge.

- Case 1: If $e1$ or $e2$ is a bridge, then for a walk to start and end from a vertex w in G containing both u and v to exist, the bridge needs to be traversed more than once. This would prevent the walk from qualifying as a cycle. Hence, no cycle can exist in G containing both u and v if either $G - e1$ or $G - e2$ disconnects u from v.

- Case 2: Let u and v be connected in $G - e1$ such that u and v becomes disconnected in $G - e1 - e2$. This makes $e2$ a bridge. Hence, a walk starting and ending from a vertex w containing both u and v in $G - e1$ will not be a cycle. Therefore, any cycle in G with u and v will contain $e1$.

  The same treatment can be done for $G - e2$ where u and v are still connected but become disconnected in $G - e2 - e1$ which makes $e1$ the bridge. This would mean any cycle in G with u and v will contain $e2$.

  Combining the above two, we see that if deleting $e1$ and $e2$ disconnects vertices u and v, then any cycle containing both u and v must contain both $e1$ and $e2$.