

CPTS 553: Graph Theory

Assignment 8

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1

1.1

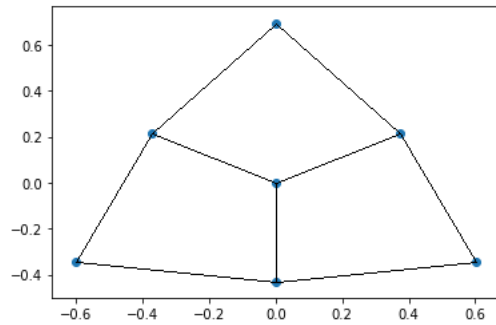
The Eigenvalues are as follows: 0, 1.3820, 1.3820, 2.5858, 3.6180, 3.6180, 5.4142

1.2

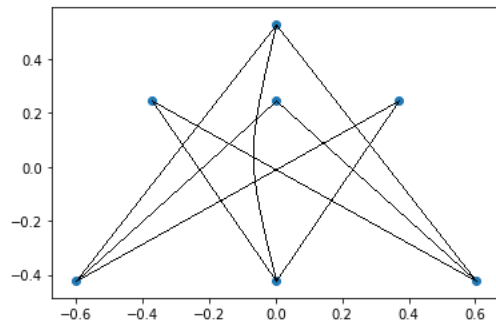
Eigenvectors \mathbf{v} and \mathbf{w} corresponding to λ_2 and λ_3 are
$$\begin{pmatrix} 1.61803 \\ -1.61803 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} -1 \\ 0 \\ 0.61803 \\ 0 \\ -0.61803 \\ 1 \\ 0 \end{pmatrix}$$

The computed vector \mathbf{z} is
$$\begin{pmatrix} -0.50 \\ -0.49 \\ 0.309 \\ 0.309 \\ -0.618 \\ 1 \\ 0 \end{pmatrix}$$

1.3



1.4



1.5

From observation, one can see that this is a bipartite graph. Hence, chromatic number is 2.

2

2.1

$$T^2 = \begin{pmatrix} 0 & 0 & 1 & 2 & 3 & 1 & 2 \\ 1 & 0 & 1 & 2 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 2 & 1 & 2 & 2 & 0 & 0 & 1 \\ 1 & 1 & 3 & 4 & 4 & 0 & 1 \\ 0 & 1 & 1 & 2 & 3 & 0 & 0 \end{pmatrix} \quad T^4 = \begin{pmatrix} 7 & 6 & 11 & 14 & 11 & 3 & 7 \\ 5 & 4 & 9 & 12 & 11 & 4 & 8 \\ 3 & 3 & 6 & 8 & 7 & 0 & 2 \\ 1 & 2 & 4 & 6 & 7 & 0 & 1 \\ 1 & 1 & 4 & 8 & 13 & 7 & 9 \\ 9 & 5 & 11 & 14 & 11 & 9 & 14 \\ 7 & 3 & 7 & 8 & 3 & 4 & 7 \end{pmatrix}$$

$$T^8 = \begin{pmatrix} 213 & 174 & 379 & 528 & 515 & 177 & 323 \\ 197 & 152 & 337 & 470 & 457 & 176 & 308 \\ 83 & 77 & 170 & 246 & 261 & 78 & 142 \\ 49 & 48 & 112 & 170 & 197 & 64 & 107 \\ 171 & 113 & 268 & 380 & 379 & 197 & 309 \\ 325 & 233 & 507 & 684 & 613 & 261 & 462 \\ 181 & 135 & 283 & 374 & 319 & 118 & 227 \end{pmatrix}$$

$$T^{16} = \begin{pmatrix} 341029 & 261078 & 582099 & 814808 & 798871 & 317445 & 546935 \\ 314001 & 240120 & 534689 & 747494 & 730809 & 290296 & 501032 \\ 154695 & 3117881 & 263538 & 369294 & 362797 & 145778 & 249903 \\ 111373 & 284224 & 188352 & 263538 & 257921 & 104876 & 179435 \\ 248311 & 216249 & 479908 & 668260 & 647487 & 257921 & 447021 \\ 463993 & 356289 & 791851 & 1106364 & 1080629 & 425253 & 736794 \\ 240949 & 185943 & 413651 & 579046 & 568099 & 222234 & 384863 \end{pmatrix}$$

2.2

2.3

The relative values of the column entries for the principal eigenvector are: $\begin{pmatrix} 1.4133 \\ 1.29576 \\ 0.642843 \\ 0.460018 \\ 1.1574 \\ 1.9124 \\ 1 \end{pmatrix}$. Hence,

the raking is as follows: 6, 1, 2, 5, 7, 3, 4.

2.4

Player 5 was ranked higher than Player 7 despite winning 2 matches which is lesser than Player 7's 3 wins. This is because one of Player 5's wins came against Player 6 who is the highest ranked player and the other one against Player 7.