I'll put up links to the shared videos of the full screen shared versions of the lectures on the same Blackboard page as the assignments, so hopefully that will alleviate access difficulties some are having with the Zoom cloud recordings.

Today, I want to finish Chapter I by looking at "collapsing/identifying vertices", "splitting vertices", "subdividing edges or merging edges".

In Chapter II, we discuss what it means for a graph to be connected in terms of walks, paths, trails, ...

I'll put forth guidelines for the project next week.

To "subdivide" an edge uv, visually, it looks we "deposit" a vertex w along uv, splitting it into edges uw and wv. More precisely, delete the edge uv, create a new w and create the edges uw and wv."

When one discusses "subdivisions of a graph G", that means you've done a sequence of subdivisions of the edges of G.

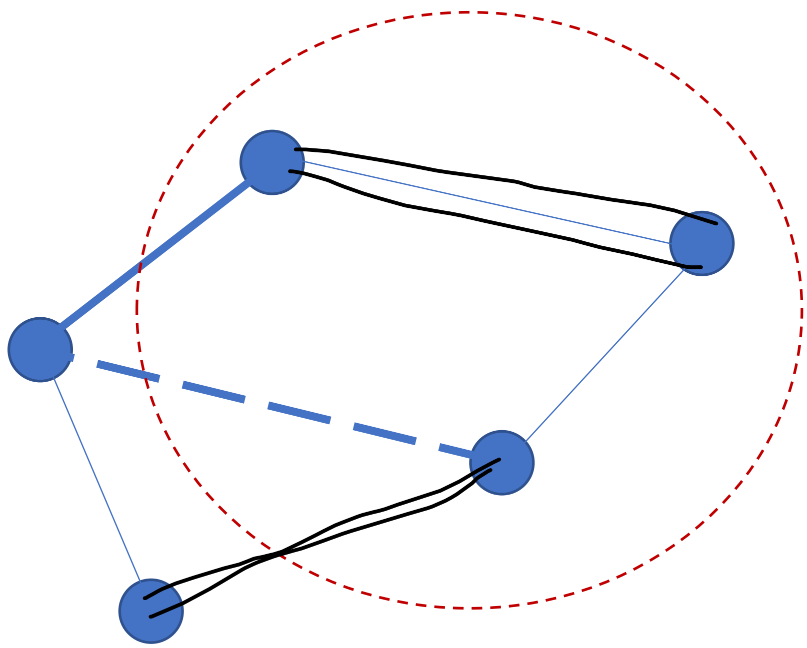
The reverse process is called "merging edges." The process works if you have a vertex w of degree 2 whose neighbors are u and v. You delete the vertex w and you then create an edge joining u and v.

The next operation is "splitting a vertex."

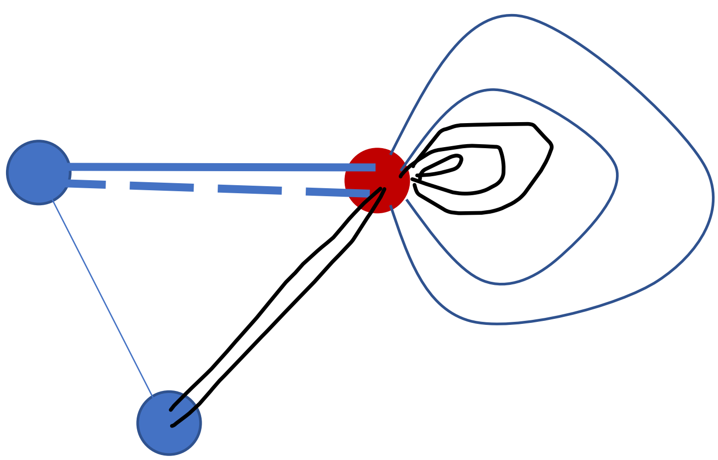
Given a graph $G = (V, E)$, and a vertex v that we wish to split. If we split it once, we create a new vertex w not already in V. For each edge of the form uv, we create an edge of the form uw as well. Every "copy" of v created this way has exactly the same neighbors as v in the new graph.

The final operation is sort of a reverse of this operation, but is more general.

"Collapsing a set of vertices $W \subseteq V$" is more complicated. We "treat the set of vertices W" as a single vertex.

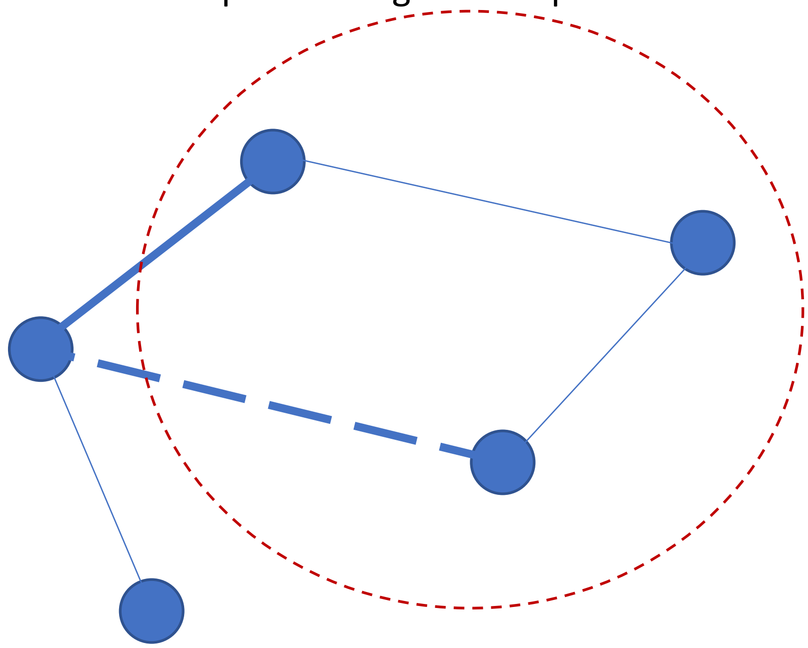


We treat the collection of vertices inside the dashed set as a single vertex:

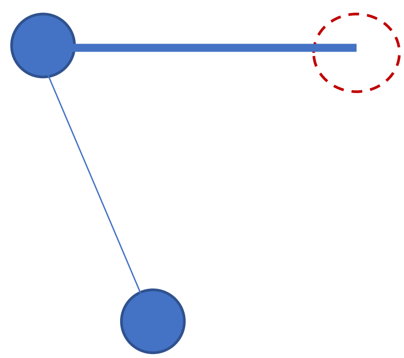


This operation "collapsing" requires that all edges be accounted for, i.e., every edge in the original graph must have a corresponding edge in the collapsed graph.

The operation of “identifying” a set of vertices (in simple graphs) is similar, only we do not include parallel edges or loops.



Here, identifying rather than collapsing the three vertices indicated results in a P_3:



“Contracting an edge” is the same as identifying its endpoints.

Chapter II. Connectivity.

To “not be connected” means “you can’t get there from here by following edges.”

The concept that makes this precise is a “walk”:

Definition. Given two vertices u and v in a graph G, a u, v- **walk** W is a sequence that alternates between vertices and edges

$$W:\ u = w_0, e_1, w_1, e_2, w_2, e_3, w_3, \dots, w_{k-1}, e_k, w_k = v$$

such that the endpoints of e_i are w_{i-1} and w_i.

There is no prohibition against $u = v$; there is no prohibition against $k = 0$.