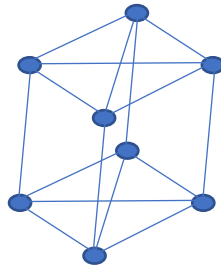Graph Theory Fall 2020

Assignment 2

Due at 5:00 pm on Tuesday, September 8

1. Among all simple graphs with 21 vertices, determine (with justification) the minimum possible and the maximum possible number of edges such a graph could have.

2. Suppose G is a simple graph (no loops, no parallel edges) with n vertices and m edges. Let H be the simple graph whose vertices take the form $(0, v)$ or $(1, v)$ for each vertex v of G. Two vertices (a, v) and (b, w) of H are adjacent if either of the following two conditions holds:

   - $a \neq b$ and $v = w$, or
   - $a = b$ and vw is an edge of G

   Later on, we will call this graph $K_2 \times G$. As an example, if G is K_4, then H is drawn below:

   In terms of n and m, how many vertices does H have and how many edges does H have?

3. Recall that a graph G is said to be **cubic** if it is 3-regular, i.e., every vertex has degree 3.

   a. Explain why a loopless cubic graph must have an even number of vertices.

   b. For each integer $n \geq 1$, construct a loopless cubic graph with $2n$ vertices.

   c. For each integer $n \geq 3$, construct a simple cubic graph with $2n$ vertices. (You could apply question #2 to this purpose.)

4. Determine, with justification, whether the Petersen graph (drawn below, with vertex set $V = \{a, b, c, d, e, f, g, h, i, j\}$) is bipartite: