CPTS 553: Graph Theory
Assignment 3

By: Reet Barik
WSU ID: 11630142

September 16, 2020

1

From the handshaking lemma, we know that

$$2|E| = \sum_{i=1}^{n}(deg(v_i))$$

Here, Q_n has 2^n vertices where each vertex has a degree of n. Therefore,

$$2|E| = 2^n n$$
$$\implies |E| = 2^{n-1}n$$

Therefore, number of edges in Q_n is $2^{n-1}n$

2

A non-empty simple finite graph implies that it has at least one edge. There is no restriction on a walk to not have repeating edges. Hence, for an edge $e = uv \in E$ in a simple finite graph, $u, e, v, e, u, e, v, e, u, e, v, e, u, e, v...u, e, v$ which is the same edge traversed infinite times, is a valid walk. Because there exists a walk which has an infinite length, there cannot exist a walk of maximum length.

A path is a walk that does not repeat any vertex. With this definition in mind, if we restrict the walks in a simple finite graph to not repeat any vertex, we realize that walks of infinite lengths are not possible and hence a simple finite graph has a path of maximum length.

3

Let us assume that there are no cycles in a trail that repeats a vertex. This implies that no sub-walk in that trail contain a cycle. These sub-walks are trails themselves and hence do not repeat an edge. But it has a repeating vertex. So, a trail can be said to start and end at the same (repeating) vertex making it a close walk which is essentially a cycle. This contradicts our assumption that there were no cycles. Hence, it is proved that a trail that repeats a vertex must contain a cycle.

4

Graph G and H are **NOT isomorphic**. This is so because C_3s are present in H while absent in G. In other words, if two vertices in H are adjacent to the same third vertex, then they are also adjacent to each other. This is the structural feature that is present in H but not in G which makes them non-isomorphic.

5

5.1

In the given graph, there are 10 vertices with degree 2 and 2 vertices with degree 3. Therefore, **total degree = 26**.

5.2

If we recall, deleting a vertex is nothing but removing it from the vertex set of the graph and removing all edges adjacent on it from the edge set of the graph. Here, the vertices are of two kinds: A) With degree 2 and B) With degree 3. If we delete any of the Type A vertices, the **total degree becomes 22**. If we delete any of the Type B vertices, the **total degree becomes 20**.

5.3

There are three kinds of edges in this graph: A) making two vertices of degree 2 adjacent B) making two vertices of degree 3 adjacent, and C) making a vertex of degree 2 adjacent to a vertex of degree 3. We observe that contracting any of the above kinds of edges results in the **total degree to be 24**.

5.4

There are 3 possible scenarios possible when identifying two vertices of G:
A) those two vertices are adjacent: This reduces the problem to the sane one in Section 5.3. Hence, **total degree is 24**.
B) those two vertices are not adjacent but have a common neighbor: This reduces the total number of edges in the graph by 1. Hence reduces the **total degree** by 2 making it **24**.
C) those two vertices are not adjacent and do not have a common neighbor: The total number of edges in the graph does not change. Hence the **total degree** remains **26**.