My plan is to start in on section I.B. on page 9. "Main ingredients for a graph: Vertices and Edges"

For HW 1 using equation editor: To input a matrix, I first start with "ALT =" for the "type equation here" prompt. In that, I type in a pair of brackets followed by a space bar. This gets you a box inside a square bracket pair. Next, type in "\matrix(@@@&&&&)<space>" where the number of @ symbols is one less than the number of rows and the & symbols is one less than the number of columns. Then you can type in each of the boxes:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & a_{1,n+1} \\ & & & \vdots & \\ & & & \ddots & \\ & & & & \end{bmatrix}$$

I've set up office hour Zoom meetings Tues and Thursdays starting at 10:00 a.m.

What is a graph? Ingredients of a graph G are called "vertices" (singular form is "vertex") and "edges". The set of vertices is denoted $V(G)$; if we understand what graph is being looked at, we'll simply write V for this set. The set V can contain *any sort of objects*.

Conventions:

- Most authors do not allow V to be the empty set.
- It's okay for V to be infinite.
- Usually, the letters toward the English alphabet such as u, v, w, x, y, z are used for vertices. When we discuss digraphs (directed graphs), we'll have special uses for s, t.
- If there are several vertices, we'll use indices, such as $v_0, v_1, v_2, v_3, \ldots$

The quantity $n = |V|$, i.e., the cardinality (number of elements) of V is called the **order** of the graph.

The quantity $m = |E|$, i.e., the cardinality with multiplicity of E is called the **size** of the graph.

The collection of edges $E(G)$ is a multiset; a multiset is different from a set because you keep track of the number of occurrences of an element in a multiset. As sets,

$$\{a\} = \{a, a, a, a, a\}$$

but as multisets, they are different. In $\{a\}$, we say the "multiplicity" of a is 1 whereas in $\{a, a, a, a, a\}$, the multiplicity of a is 5. In the notes, the symbol $\mu_E(x)$ denotes the multiplicity of the element x in the multiset E. Two multisets A, B are equal if and only for all $x \in U$,

$$\mu_A(x) = \mu_B(x).$$

Conventions for edges:

- We'll use letters near e for edges.
- The multiset E is allowed to be empty.
- It's quite rare for $\mu_E(e)$ to be infinite, so we'll assume $\mu_E(e) \in \mathbb{Z}^{\geq 0}$.

Formally, an edge is an unordered pair of (not necessarily distinct) vertices. There is no distinction between the edge $e = \{u, v\}$ and the edge $e = \{v, u\}$; These represent the same edge. For brevity, I will write $e = uv = vu$ instead of the parenthetical notation above. An edge of the form uu is called a **loop.**

Terminology:

Definition. Each of the following statements conveys the same information about an edge $e \in E$:

- $e = \{u, v\}$
- $e = uv$
- $e = vu$
- u and v are the **endpoints** or **end vertices** of e
- e is **incident** with u and with v
- u and v are **incident** with e
- e **joins** u and v

WARNING: We don't typically use the word "connect" in this context.

If e_1 and e_2 have the same set of endpoints, but are different instances in the multiset E, then e_1 and e_2 are said to be **parallel** edges.

If e_1 and e_2 share an endpoint, then they are **adjacent** edges.

When there is an edge joining u and v, we will say that u and v are **adjacent** vertices.

Each of the following statements conveys the same information about two vertices u and v:

- $uv \in E$
- u is **adjacent** to v
- v is adjacent to u
- u and v are adjacent
- u and v are **neighbors**

If u is adjacent to itself, then the edge $e = uu$ is called a **loop**.

If v has no neighbors, then v is called an **isolated** vertex.

Graphs can have various properties:

- A **loopless** graph has no loops.
- A **null** graph has no vertices. This means $V = \emptyset$ and $n = 0$. Quite often, null graphs are not allowed; many theorems in graph theory flow more naturally by disallowing null graphs.
- A **simple** graph is not a null graph, has no loops, and no parallel edges
- An **empty** graph has no edges, i.e., $E = \emptyset$
- A **trivial** graph is simple and has exactly one vertex.
- An empty and nontrivial graph is said to be **totally disconnected**.
- A **finite** graph is one where V is a finite set.
- An **infinite** graph is one where V is an infinite set.