Graph Theory Fall 2020

Assignment 1

Due at 5:00 p.m. on Monday, August 31, 2020

Your first assignment (on a strict pass/fail basis, i.e., if you make a reasonable effort, you get full credit) is to typeset the following passage as faithfully as you can using either MSWord and the equation editor or some form of LaTEX if you're used to that system.

Cutting and pasting the image below is not in the spirit of this assignment.

I came across a result in a time series textbook the other day and have not been able to understand why it is true (the authors don't give a proof but just state it as true). I want to show that the eigenvalues of the matrix $\mathbf{G}$ given by

$$G = \begin{pmatrix} \phi_1 & \phi_2 & \phi_3 & \dots & \phi_{p-1} & \phi_p \\ 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ \vdots & & & \ddots & 0 & 0 \\ 0 & 0 & \dots & \dots & 1 & 0 \end{pmatrix}$$

correspond to the reciprocal roots of the $AR(p)$ characteristic polynomial

$$\Phi(u) = 1 - \phi_1 u - \phi_2 u^2 - \dots - \phi_p u^p$$

The one thing i was able to deduce is that the eigenvalues of $\mathbf{G}$ must satisfy

$$\lambda^p - \phi_1 \lambda^{p-1} - \phi_2 \lambda^{p-2} - \dots - \phi_{p-1} - \phi_p = 0$$