I want to finish discussing binary sort and why it's $O(n \log n)$. The extra credit assignment is on Blackboard.

Then we'll head into graph topology, i.e., planar and non-planar graphs.

Recall that to conduct a binary algorithm with N possible outcomes, we need a binary tree with N leaves, and so the height of that tree must be at least $H \geq \lceil \log_2 N \rceil$. To sort a list of n distinct items, i.e.,

$$\{a_1, a_2, a_3, \dots, a_n\},$$

we need to accommodate $n!$ possible outcomes. Hence, we need our binary tree to satisfy

$$H \geq \lceil \log_2(n!) \rceil.$$

Observations.

$$\log_2 k = \frac{\ln k}{\ln 2} = \frac{1}{\ln 2} \ln k.$$

$$\begin{aligned} \log_2(n!) &= \frac{1}{\ln 2} \ln(n!) = \frac{1}{\ln 2} \ln(1 \cdot 2 \cdot 3 \cdot \dots \cdot n) \\ &= \frac{1}{\ln 2} (\ln 1 + \ln 2 + \ln 3 + \cdots + \ln n) \end{aligned}$$

We're down to analyzing

$$\frac{1}{\ln 2} \sum_{k=1}^{n} \ln k.$$

$$\sum_{k=1}^{n} \ln k < \int_1^{n+1} \ln x \; dx < \sum_{k=1}^{n+1} \ln k.$$

As a consequence,

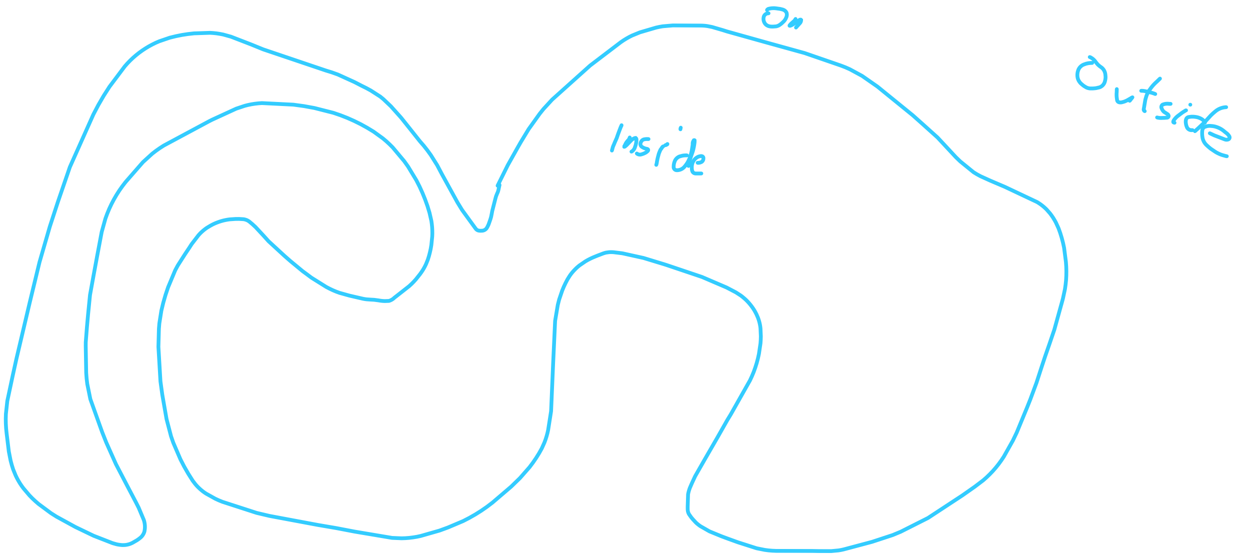
$$n \ln n - (n-1) < \sum_{k=1}^{n} \ln k < (n+1)\ln(n+1) - n.$$

This implies

$$\log_2(n!) = O(n \log n).$$

Chapter IV. Graph topology

Planar or non-planar graphs



Intuitively, the Jordan curve theorem says if you have a simple closed curve C in the plane $\mathbb{R}^2$, then the plane is partitioned into three connected parts, the part “inside” C, the part “outside” C, and C itself.

Intuitive definition. A **planar graph** is a graph that can be drawn in the plane $\mathbb{R}^2$ without edges crossing.

When we say “drawn”, we treat our vertices as elements of $\mathbb{R}^2$, the edges will be non-self-intersecting curves (often, they’ll be line segments) whose endpoints are the vertices drawn as elements of $\mathbb{R}^2$. to typeset $\mathbb{R}$, use \doubleR in the equation editor.

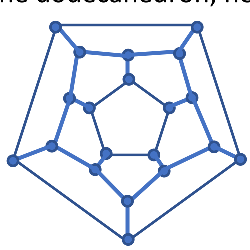
Definition. A **drawn graph** is a graph whose vertices are points in $\mathbb{R}^2$ and whose edges are non-self-intersecting curves whose endpoints are the vertices drawn as points.

Definition. A **plane graph** is a drawn graph whose edges do not cross.

Definition. A **planar graph** is a graph that is isomorphic to a plane graph.

For a non-drawn graph, consider Q_3 whose vertices are ordered triples of $\{0,1\}$ where two vertices are adjacent if and only if they differ in exactly one coordinate.

Here is a drawn graph for the dodecahedron; hence, the dodecahedron is planar.



Easy examples of planar graphs: Trees. Q_3. Cycles. $K_4, K_{2,n}$

Examples of nonplanar graphs: K_5, $K_{3,3}$, K_6

Claim. If G is planar and H is a subgraph of G, then H is planar.

Contrapositive. If H is not planar and H is a subgraph of G, then G is not planar.