

CPTS 553: Graph Theory

Assignment 1

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I came across a result in a time series textbook the other day and have not been able to understand why it is true (the authors don't give a proof but just state it as true). I want to show that the eigenvalues of the matrix \mathbf{G} given by

$$G = \begin{pmatrix} \phi_1 & \phi_2 & \phi_3 & \cdots & \phi_{p-1} & \phi_p \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & & & \ddots & & \\ 0 & 0 & \cdots & \cdots & 1 & 0 \end{pmatrix}$$

correspond to the reciprocal roots of the $\mathbf{AR}(p)$ characteristic polynomial

$$\Phi(u) = 1 - \phi_1 u - \phi_2 u^2 - \cdots - \phi_p u^p$$

The one thing that I was able to deduce is that eigenvalues of \mathbf{G} must satisfy

$$\lambda^p - \phi_1 \lambda^{p-1} - \phi_2 \lambda^{p-2} - \cdots - \phi_{p-1} - \phi_p = 0$$