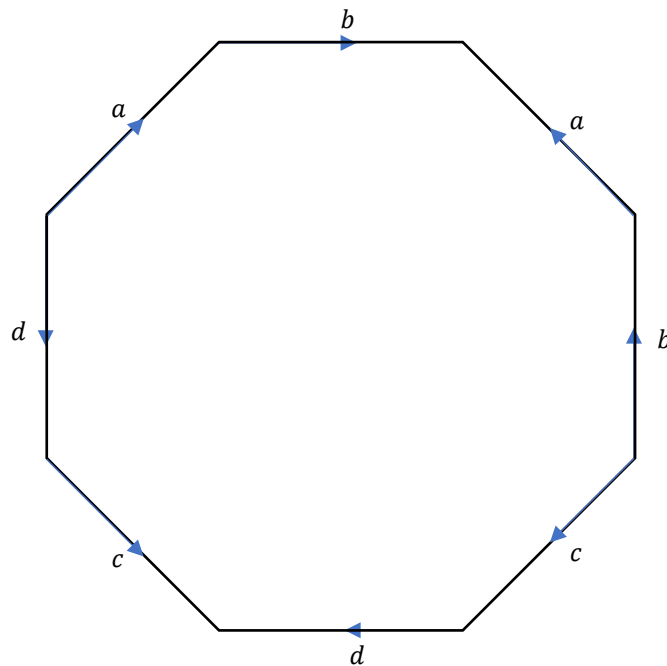


Graph Theory Fall 2020

Assignment 7

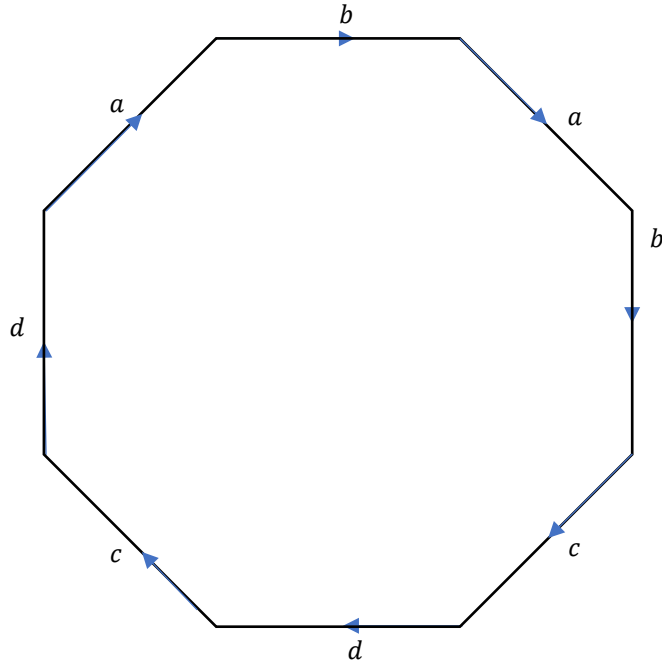
Due at 5:00 pm on Friday, November 13

1. Show that K_8 can be drawn on a 2-holed torus without edges crossing. Feel free to use the octagon model as a framework for your drawing:



2. Use an edge-counting argument to show that K_9 cannot be drawn on a 2-holed torus without edges crossing. Ingredients: For K_9 , you have $n = 9, m = \binom{9}{2} = 36$. What would r have to be? What is a lower bound on the total edge count since every region must be bounded by at least three edges?

3. If we re-orient the arcs around the diagram from #1 so they all point clockwise, what is the resulting value of $n - m + r$?



4. In terms of $n \in \{2, 3, 4, 5, 6, \dots\}$, how many tournaments are there with the node set $N = \{1, 2, 3, \dots, n\}$? This is equivalent to asking for how many ways are there to orient the edges of K_n with vertex set $\{1, 2, 3, \dots, n\}$.
5. Let $n \in \{3, 4, 5, 6, \dots\}$ be fixed. Show that there are exactly two orientations of C_n with vertex set $V = \{0, 1, 2, \dots, n - 1\}$ that are strongly connected.