Qualifying Exam Part I
Doctor of Philosophy in Computer Science

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Abstract

Expensive atomic updates and poor cache locality are primary bottlenecks when it comes to single machine shared memory graph processing applications. One popular solution is data duplication (keeping thread local copies of all vertices) but that comes with a high memory overhead. Memory-efficient versions of this strategy comes with the added run-time cost of identifying candidate vertices. The work being reviewed in this report proposes 'RADAR' which is a novel way to address both the previously mentioned problems. This report attempts to state the problem statement, critically evaluate the proposed approach, and discuss its strengths, weaknesses, and trade-offs.

1 Introduction

With applications in path-planning, social networks analysis, graph learning, data mining, semi-supervised learning, and the likes [14] [10], graphs processing algorithms are, apart from being one of the most widely used, also one of the most workload heavy as far as data intensiveness goes. With almost half the processing time comprising of cache miss latency [1], there is a need to preserve the spatial locality while storing graphs. Otherwise the irregular memory access results in expensive atomic updates. This is another bottleneck faced by parallel graph applications as seen in [5], [6], [7], and [20]. Real world graphs presents an opportunity to address these because of the sparse presence of clusters or hubs (vertices with an inordinately high degree).

With increasing main memory capacities on single machines, graph processing algorithms are now geared more towards single machine shared memory architectures. As a result, the emphasis on atomics has increased to ensure correctness of results. One solution to getting around using atomics is data duplication wherein, thread-local copies of vertices are kept and reduction is carried out across threads to arrive at the end result. This strategy is memory intense and the way to make it memory-efficient is to do data duplication selectively. Intuitively, it can be inferred that duplication needs to be done only for hub vertices since they are accessed the most. But this memory-efficiency comes at the runtime overhead cost of identifying hub vertices. Another approach to optimize graph processing algorithms is to reorder the vertices based on degree. This assigns the hub vertices contiguous IDs in the vertex array which has shown to improve spatial locality of hub accesses [3].

The work being reviewed proposes 'RADAR' which attempts to take advantage of the mutually optimizing nature of the reordering and data duplication process. Degree sorting as a preprocessing step eliminates the runtime overhead of trying to identify the hub vertices. Carrying out data duplication for only hub vertices that meet a certain threshold decreases the memory overhead significantly. The objective of this report is to take a closer look at the problem and the motivation behind 'RADAR'. This is followed by the description of the algorithm and a discussion which involves critical evaluation of the algorithm in terms of its strength, weakness, and trade-offs.

2 Background

Graph representation in memory: The most commonly used data-structure to store graphs in shared memory frameworks is the 'Compressed Sparse Row' (CSR) representation. This format is illustrated in Figure 1. The *Coordinates* array contiguously stores the neighbor of each vertex while the *Offsets* array stores each vertex's starting offset in the Coordinates array (such that vertex i's degree can be calculated by the difference in the i-th and the $i + 1$-th entry of the Offsets array). A directed graph might be stored in the form of 2 CSRs, one for the out-neighbors and the other for the in-neighbors.

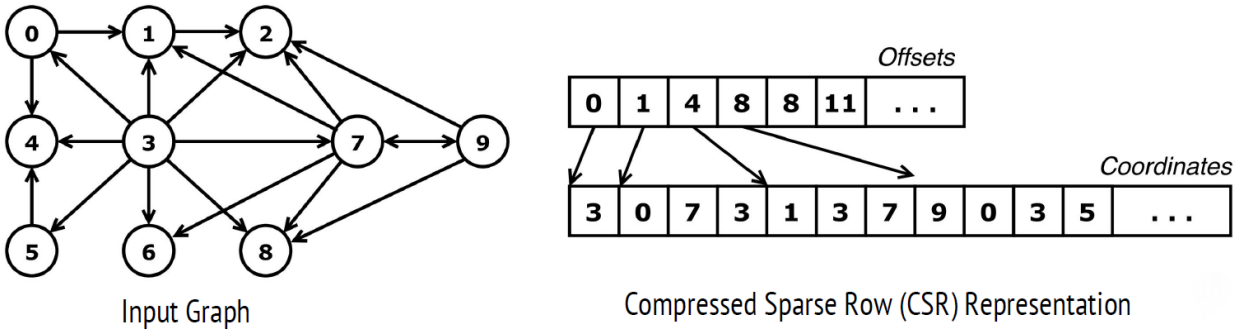


Figure 1: An illustration of the CSR representation of an example graph.

Typical Graph kernel: Graph processing algorithms are known to share a similarity as far as the core kernel is concerned. Processing of an input graph consists of visiting all vertices in the *frontier* iteratively till convergence. In shared memory frameworks, vertices are processed in parallel within iterations as shown in Algorithm 1 below:

Algorithm 1 Typical graph processing kernel

```
1: par_for src in Frontier do
2:     for dst in out_neigh(src) do
3:         AtomicUpd (vtxData[dst]), auxData[src])
```

This is called a *push phase* execution wherein, the value of a vertex is "pushed" on to its out-neighbors. This is in contrast to the *pull phase* execution which helps to eliminate the atomic updates. As shown in Algorithm 2 below, this style of execution processes a vertex by "pulling" information from its in-neighbors. This elimination of the need for atomics comes at the cost of processing redundant edges and is hence, *work-inefficient*.

Algorithm 2 Pull version of graph kernel

```
1: par_for dst in G do
2:     for src in in_neigh(dst) do
3:         if src in Frontier then
4:             Upd (vtxData[dst]), auxData[src])
```

This trade-off between the cost of atomics in push-style kernels and work-inefficiency of pull-style kernels is integrated by most graph algorithms by dynamically switching between them. For dense frontiers the pull-style kernels are preferred whereas for sparse frontiers, processing defaults to

push-style kernels (this comes with the cost of double the memory footprint since two CSRs need to be stored: one for in-neighbors and one for out-neighbors).

3 Motivation and Problem Statement

Parallel graph application frameworks in a single machine shared memory setting have been plagued by various bottlenecks. RADAR attempts to alleviate some of those. What follows makes the case for the need of something like RADAR and attempts to describe the problems it addresses:

- **Slowdown by atomics**: As shown in [20] and [6], atomic updates are a major cause of slowdown in graph applications. Figure 2 taken from [4] shows that baseline applications that replace atomics with plain loads and stores are significantly faster when compared to those with atomics.

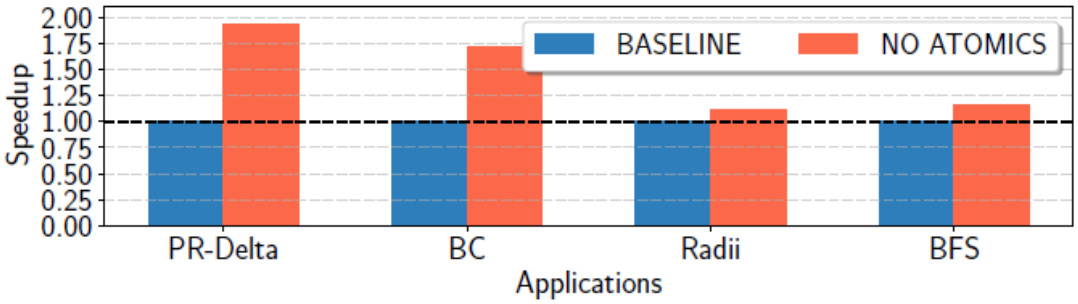


Figure 2: Net speedup obtained from replacing atomics with plain loads and stores.

- **Data Duplication as an optimization**: Data duplication has been applied as a solution for graph applications in a shared memory setting, wherein, thread-local copies of all vertex data are kept and a reduction is carried out across threads at the end to arrive at the correct result. This leads to a huge memory footprint which can be reduced by carrying out data duplication only for hub vertices (henceforth referred to as 'HUBDUP'). This reduces the memory overhead but adds the added run-time cost of having to identify hub vertices while they are being processed.

- **Locality preservation by Graph Reordering**: Graph applications suffer from irregular access to *vtxData* as shown in [5], [6], [7], and [20]. Reordering vertices by sorting them

based on degree has shown to take advantage of the power law degree distribution in real world graphs [8] and assign them contiguous IDs, thereby allowing hub vertices to fit into the cache (as shown in [3]). But as shown in Figure 3 taken from [4], a net slowdown is observed because of DegSort. This is because of *false sharing* wherein, threads comptete to share the cache lines containing the hub vertices incurring the latency of cache-coherence activity.

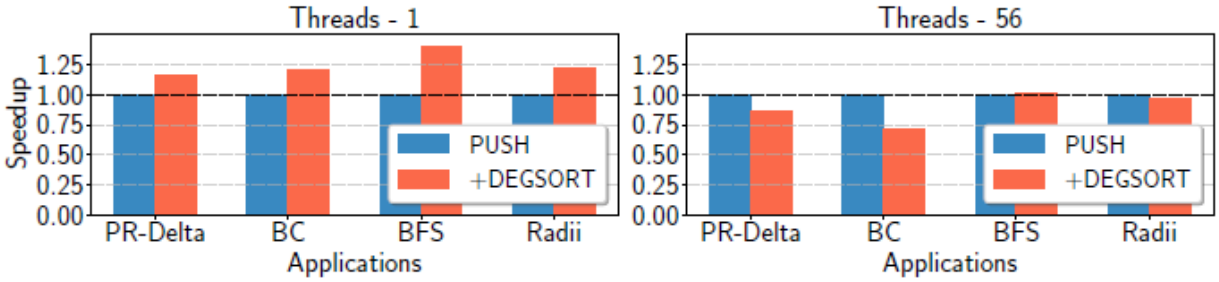


Figure 3: Net slowdown caused by DegSort in a multithreaded setting.

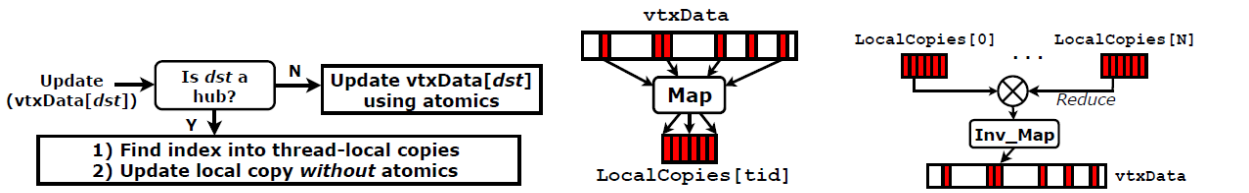
The points described above are the problems RADAR attempts to address. It does so by taking advantage of the fact that the optimizations, taken separately, though rife with their own problems are mutually enabling when made to work with each other. The following section describes how RADAR manages to do that.

4 Algorithm Description

Figure 4 describes the overall algorithm of HUBDUP. Step (a) attempts to identify if *dst* is a hub vertex on the go. If it is, thread-local copies of *dst* is made as shown in (b) and finally reduction across threads is done at the end (c) to ensure correctness of results.

With the workings of HUBDUP in mind and DegSort, the simple idea of RADAR can be summarized as follows:

$$RADAR = DegSort + HUBDUP$$



(a) Identifying hub vertices

(b) Gathering hub vertex updates in thread-local copies

(c) Reduction of values in thread-local copies

Figure 4: Conceptual description of HUBDUP

The steps for RADAR can be formulated as follows:

1. Apply DegSort on the input graph such that hub vertices get the lowest contiguous vertex IDs in the vertex array. This addresses the issue of **identifying** and **locating** the hub vertices for HUBDUP.

2. Do data duplication for HUB vertices. This is essentially the same as carrying out HUBDUP on the reordered input graph. But this time wihtout having to *map* in Step (b) or *Inv_Map* in Step (c). This is because the hub vertex IDs can be used directly as indexes into the thread-local copies (as a simple look-up).

What follows is a critique of the RADAR algorithm described above in terms of advantages, disadvantages and trade-offs.

5 Discussion

RADAR is built upon a simple idea which takes the easy identification of hub vertices by DegSort and feeds it to the memory-efficient Data Duplication scheme of HUBDUP. The advantages of the approach described in the previous section are as follows:

- **Identification of hub vertices**: Degree Sort makes it exceptionally easier to identify hub vertices by grouping them together at the beginning of the vertex array. This is a very simple way of identifying hub vertices and doesn't add to the run-time cost as was the case in naive HUBDUP.

- **Locating hub vertices**: Degree Sort assigns hub vertices contiguous IDs in the vertex array. These IDs can be directly used as indexes into the thread-local copies during hub duplication of HUBDUP without having to build a map for duplication and an inverse map during the reduction at the end.

- **Lightweight Reordering**: Degree Sort as a scheme to reorder the input graph is as lightweight a preprocessing step as they come, as opposed to heavyweight ordering schemes like [19] or [18] which would defeat the purpose of doing away with the run-time overhead of identifying and locating hub vertices.

- **Easy adaptability to varying cache sizes**: The threshold used to classify a vertex as either a hub vertex or a normal one can be used as a parameter depending on the cache size. Data duplication can be carried out only for top-k hub vertices where k is the number of vertices that can fit into a cache line (which is machine-specific).

The above advantages does come hand in hand with some disadvantages. What follows attempts to list some of the more salient ones:

- **RADAR isn't application agnostic**: Figure 5 below shows how RADAR is unequally effective for different graph algorithms. We see that it is the most effective for an application like Local Triangle Counting. This is because Local-TriCnt accesses hub vertices the most (proportionate to their degree) because hub vertices are part of most number of triangles present in the graph as opposed to normal vertices.

  As can be observed from the other two plots in Figure 5 RADAR is not as effective for applications like BFS or Radii. In fact, it is almost always as effective as standalone DegSort. This is because RADAR's locality preserving capability comes directly from DegSort. A cache line after DegSort contains hub vertices and not a single hub vertex with its neighbors. This failure to map neighborhoods in a graph to cache lines (or different levels of the cache hierarchy) by DegSort makes the *vtxData* accesses for an application like BFS, very cache-inefficient. This could be remedied by replacing DegSort with a neighborhood preserving

reordering scheme like [12], [13] or [2] which are based on community detection and graph partitioning.

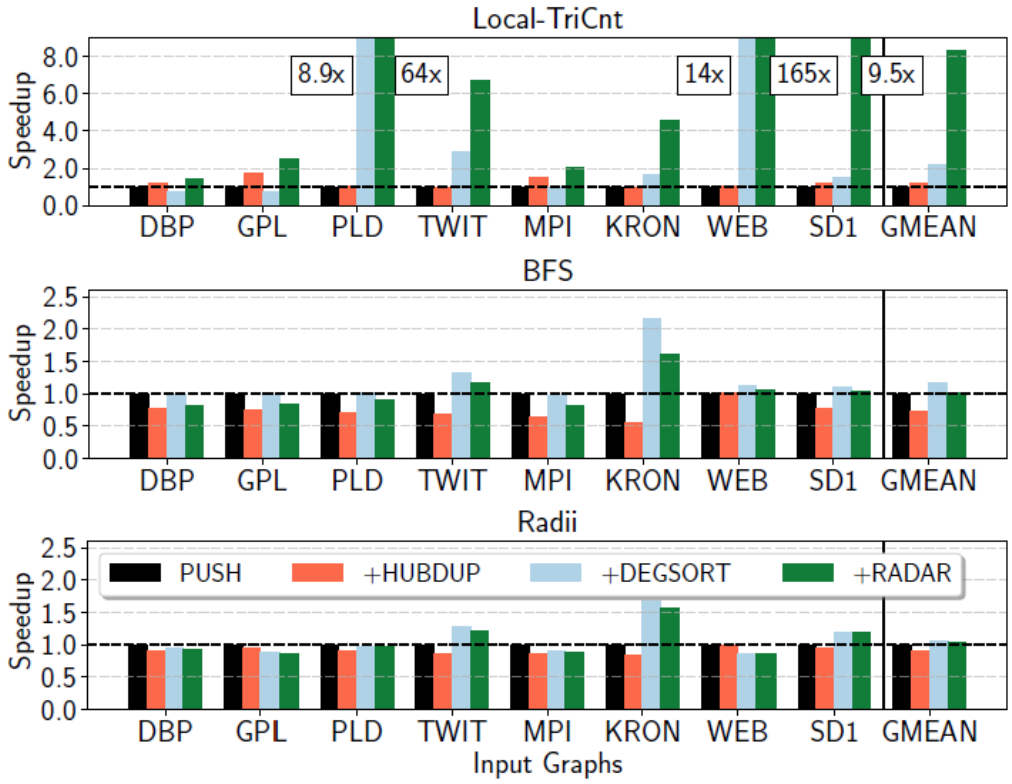


Figure 5: Varying effects of RADAR on different graph applications

- **RADAR also isn't input graph agnostic**: It can be observed from the results of [3] that lightweight reordering techniques like HubSorting and HubClustering which are minor modifications of DegSort are sensitive to the structure of the input graph. As shown in the plot in Figure 6 taken from [3], there is a direct correlation between speedup obtained from reordering techniques like DegSort and metrics like 'Packing Factor' which quantifies the graph skew and sparsity of hub vertices. It was hypothesized that input graphs meeting a certain threshold are ideal candidates for reordering with DegSort.

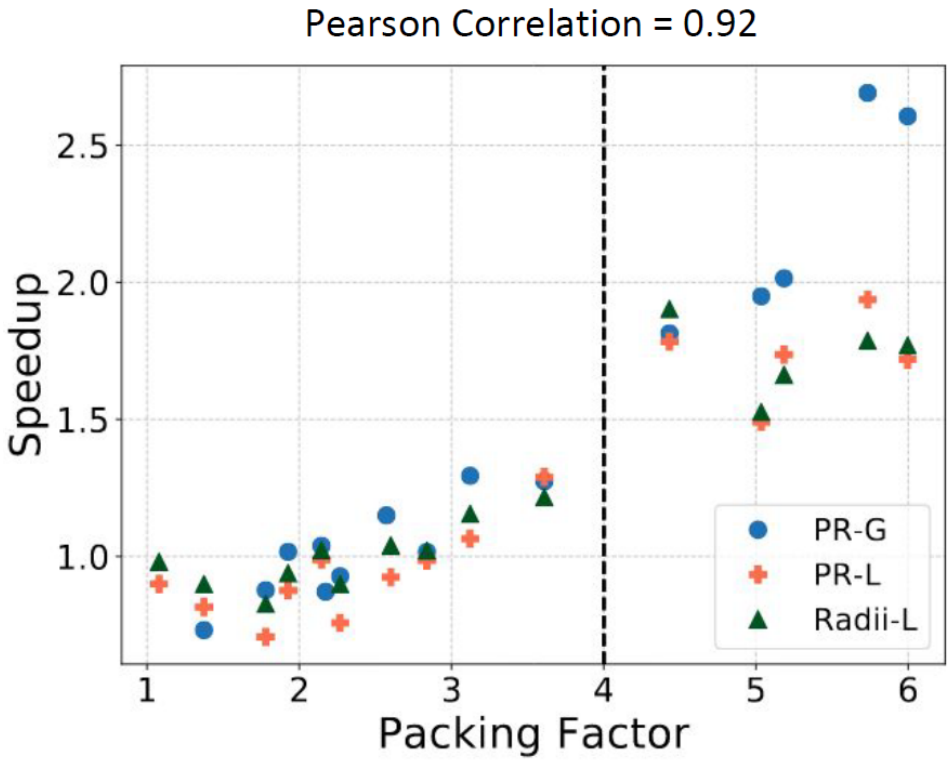


Figure 6: Correlation of Packing Factor with Speedup due to DegSort reordering.

The effectiveness of RADAR is tied very closely to the effectiveness of DegSort. Which implies RADAR performs well for those graphs which are most amenable to reordering by DegSort. This is further illustrated by the following result shown below in Figure 7.

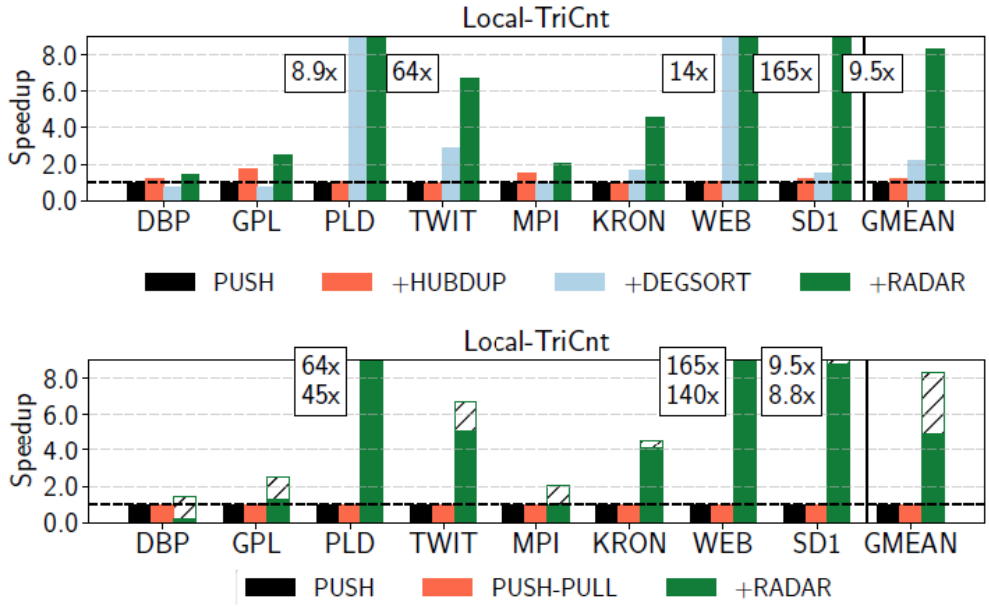


Figure 7: Effectiveness of RADAR is dependant on input graph.

For the end-application Local-TriCnt, the input graphs which show the most speedup for DegSort and RADAR as opposed to push and HUBDUP (above), are the same ones which show the most significant speedup for RADAR as opposed to Push and Push-Pull executions. This leads to the conclusion that RADAR's performance is affected by the nature of input graph.

- Another skew-aware reordering technique that is relatively as lightweight as DegSort and employs coarsegrain reordering to preserve graph structure while reducing the cache footprint of hub vertices by binning based on degree is given in [11] and could be used as an alternative to DegSort where the natural order of the input graph has been known to already preserve some form of locality.

- The end-applications that are experimented on as part of this work are traditional ones like PageRank [17], BFS [9], Triangle Counting, Radii, etc. The list of applications might be inadequate to capture the usefulness of the algorithm. Experiments on more practical end applications in the field of community detection like [15] or influence maximization like [16] might present a clearer picture of the utility of the proposed work.

6 Conclusion

This report takes a critical look at RADAR, a simple and novel idea which tries to take advantage of the mutually enabling optimizations of data duplication and graph reordering based on degree of vertices to address the bottlenecks of the same optimizations which are expensive atomics and false sharing respectively. The undeniably light-weighted-ness of RADAR, and the significant speedups observed in some of the end-applications speak to the merits of the algorithm. But a closer look suggests that the effectiveness of RADAR might be sensitive to the structure of the input graph and the access patterns of the end-applications. All in all, it provides a good starting point for the research into frameworks which try to address the issues plaguing single machine shared memory graph applications.

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Qualifying Exam Part II
Doctor of Philosophy in Computer Science

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Abstract

For most parallel graph-theoretic applications, data movement has become a primary bottleneck toward achieving scale in computing. One of the primary contributors to data movement is how the graph is stored or represented in memory. To this end, various vertex reordering schemes have been studied in the past. MinHashing has been a popular way in estimating the similarity between sets. Its use of 'sketches' makes it a powerful data analysis tool. As a result it has the potential to make for a good candidate as a solution for the vertex reordering problem.

1 Introduction

In graph processing algorithms, CPU cache performance is plagued by efficiency issues, so much so that almost half the processing time comprises of cache miss latency [1]. The problem of poor data locality is caused by irregular memory access. Real world graphs present an opportunity to improve the locality of graph applications because of the presence of clusters or hubs. One solution is to come up with something called 'vertex reordering' which is an optimal permutation among all nodes wherein, those that are frequently accessed together, are stored locally in the memory.

Of the vertex reordering schemes around, the Minimum Linear Arrangement (**MinLA**) [17] formulates the problem as one of optimization where the objective is to minimize the sum of the linear arrangement gaps (difference in ranks of vertices which share an edge). The log variant of it,

namely the Minimum Logarithmic Arrangement (**MinLogA**) is geared more towards graph compression [8] and is relevant for graph storage purposes. There are various lightweight degree based reordering techniques like **Degree Sort** which is based on the objective of assigning consecutive IDs to vertices with the highest degrees such that they fit into the same cache line. Minor variants of such a degree based approach are **Hub Sort** [19] and **Hub Clustering** [3] which attempts to take advantage of the power law degree distribution of real world graphs. **Slashburn** [12] is another degree based scheme which identifies 'caveman communities' greedily. The current state of the art **Gorder** [18] uses a window based solution and tries to preserve neighborhoods by looking at neighbor and sibling relationships between vertices. Some moderately lightweight approaches are based on community detection like **Rabbit-order** [2] which attempts to map the hierarchical community structures present in graphs to the cache hierarchy. There have also been attempts to reduce the fill of the adjacency matrices of input graphs like Reverse Cuthill-McKee **RCM** [9] and **Nested Dissection** [10].

MinHash or the min-wise independent permutations locality sensitive hashing scheme, was first introduced [6] as a way to estimate the similarity between sets. Numerous variants of it have been proposed down the years like [7]. The intuition behind the technique is that hash functions can be used to map large sets of objects to smaller ones in such a way that if two objects are close to each other semantically, then their hash values are also likely to be the same. MinHash has a lot of applications, some of which are clustering, plagiarism detection [7], eliminating near-duplicates among web documents [15], etc. It has also found widespread application in the field of bioinformatics where MinHash based algorithms like [16], [5], and [13] have found success in the problem of genome alignment and assembly.

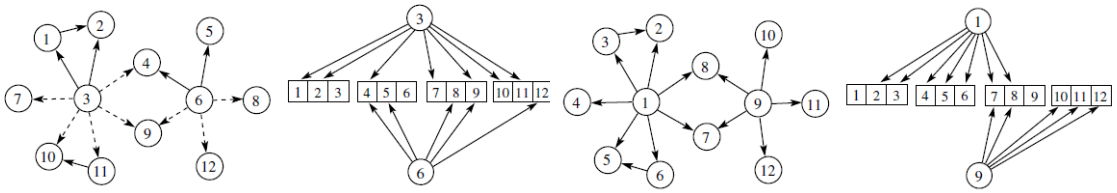
This report is an attempt to try and come up with ideas/ways to solve the vertex reordering problem by using the various transferable skills of the MinHash technique.

2 Background

This section attempts to give a general overview of vertex reordering followed by an overview of the MinHash technique.

2.1 Vertex Reordering Overview

Vertex reordering has long been a popular optimization to reduce cache latency for various graph applications. The simple example shown in Figure 1, taken from [18], shows how ordering fairs significantly better at maintaining locality than naive partitioning in case of real graphs.



(a) Partitioning without vertex re-ordering (b) Partitioning with vertex re-ordering

Figure 1: An illustration of the impact of vertex re-ordering on block partitioning of vertices.

In sub-figure (a), the edges shown as dashed lines are the edges that cross two partitions to be cut (with a total of four partitions). It can also be seen that if an algorithm accesses the out-neighbors of node 3, it needs to access all four partitions which are stored across four CPU cache lines. Similarly, for node 6, there is a need to access three partitions stored across three cache lines. Sub-figure (b) shows the partitioning after reordering. If an algorithm tries to access the out-neighbors of node 1 (topologically the same node as node 3 from the previous case), it needs to access three partitions which are stored across three CPU cache lines. Similarly, for node 9 (topologically the same node as node 6 from the previous case), there is a need to access just two partitions stored across two cache lines. The reduction in latency from naive partitioning to ordering is quite apparent.

2.2 MinHash Overview

The MinHash technique is used to estimate the similarity between sets. The similarity here, is given by the metric called **'Jaccard Coefficient'**. For two sets X and Y, it is given by:

$$J(X,Y) = \frac{|X \cap Y|}{|X \cup Y|}$$

It is essentially the ratio of the cardinality of the intersection of two sets with that of their union. It follows from the definition that $0 \leq J(X,Y) \leq 1$.

There are two ways the MinHash technique can be carried out:

- **Using k minimum values from a single hash function**: Given two sets X and Y such that high cardinalities of the two sets make the computation of $J(X,Y)$ very expensive,

  1. A universal hash function H is applied on all the elements of X and Y.
  2. After sorting, the top-k elements are obtained from X and Y. These are called *Sketches* of the sets X and Y and denoted by $S(X)$ and $S(Y)$.
  3. Since $J(S(X),S(Y)) \approx J(X,Y)$, calculating the Jaccard Coefficient of the Sketches of X and Y gives us a good estimate of the actual similarity between the two sets.

- **Using k hash functions**:

  1. k universal hash functions are applied to the elements of X and Y.
  2. Sketches $S(X)$ and $S(Y)$ are obtained by collecting the top elements from each $H_i(X)$ and $H_i(Y)$ where $1 \leq i \leq k$, such that $|S(X)| = |S(Y)| = k$.
  3. Compute $J(S(X),S(Y))$ to estimate the Jaccard similarity between X and Y.

Here, k which has a value between 1 and $min(|X|,|Y|)$ is a hyperparameter that can be tuned. Larger the k, more accurate will be the estimation of the Jaccard Coefficient.

3 Using MinHash for Vertex Reordering

This section's objective is to suggest ideas/ways how the MinHash technique can be used to solve the problem of vertex reordering. What follows are two ways suggested by the author on how that might be done.

3.1 Preserving graph neighborhoods via MinHash

Neighborhood or locality preservation is one of the main aims of vertex reordering. If there could be a way to extract neighborhoods of a certain size from an input graph, MinHash could be used to estimate the similarities between such neighborhoods and label the vertices of similar neighborhoods in a way that their IDs are closer to each other in the vertex array. Keeping this approach in mind, the following steps could be carried out for vertex reordering of an input graph using MinHash:

1. In a graph $G(V,E)$ where V is the set of vertices, and E is the set of edges, $\forall v \in V$, do a BFS from v going up to a depth d, where d is a hyperparamter than can be tuned.

   The neighborhood of a vertex v is defined by the set $S(v)$ which contains the vertices visited by the above mentioned BFS traversal.

2. Use MinHash to estimate the similarity between these sets which implicitly gives an estimate of the similarity between the neighborhoods that these sets represent.

3. Greedily order these sets based on the estimated Jaccard coefficient of each pair. This has been illustrated in Figure 2 below. Here, $J1 \approx J(A,D)$, $J2 \approx J(D,B)$, $J3 \approx J(C,B)$, and $J4 \approx J(A,C)$ such that, $J1 > J2 > J3 > J4$. This leads to the ordering where A is followed by D, B, and then C.

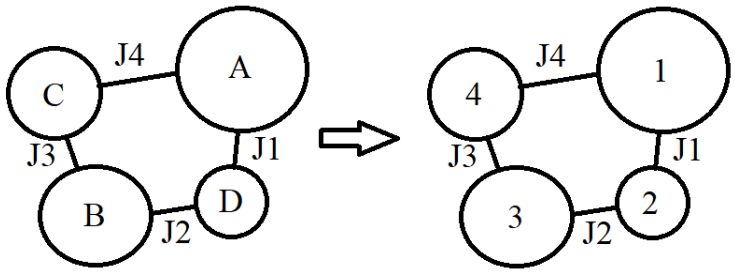


Figure 2: Greedy ordering of sets based on the pair-wise Jaccard coefficient.

4. The ordering of nodes within each set can be done conserving the BFS traversal order. This further helps to preserve the smaller neighborhoods while reordering inside the bigger sets.

3.2 Using MinHash instead of Community Detection

It has been observed as part of the work from a paper [4] under review that community detection based reordering techniques are much more adept at generating orderings that reduce the average linear arrangement score mentioned in [17]. Keeping that in mind, MinHash could be used instead of relatively heavyweight community detection methods to generate the orderings. The following is a way how that might be achieved:

1. In a graph $G(V,E)$ where V is the set of vertices, and E is the set of edges, $\forall v \in V$, do a random walk of length l, where l is a hyperparameter than can be tuned. Inspiration can be taken from 'Node2vec' [11], a random walk based graph algorithm to generate node embeddings while carrying out these walks. The random walk can be parameterized as follows.

    - During a random walk when a transition is made from a vertex u to v, the parameter q could be used to generate the probability of making the transition back from v to u.
    - During a random walk, the parameter p can be used to influence whether the transitions made one hop away will be more in a BFS, or a DFS fashion.

    Here, the random walks must be sufficiently long (l must be large enough) so that vertex

coverage is high.

2. Each random walk gives a set of vertices (large enough to make MinHash not be an overkill). These sets can be put through the MinHash treatment so as to cluster the random walks together based on similarity.

3. For pairs of sets representing random walks sharing an estimated Jaccard Coeffient higher than a set threshold, the intersection of such sets have a high probability of having vertices that might otherwise have belonged to the same community generated by a traditional community detection algorithms like [14].

4. After identification of communities, they might be ordered as illustrated in Figure 2. The only difference being, instead of using the Jaccard coefficient, one can use the number of inter-community edges as a way to determine the strength of connection between one community and the other.

5. As far as the intra-community vertex ordering is concerned, the natural order could be preserved, or each community could be used as an input graph to this whole process in a recursive fashion.

4 Discussion

The previous section was an attempt at using MinHash to address the problem of vertex reordering. The following is a discussion of some of the merits of the suggested methods:

- The approach suggested in Section 3.1 consists of a BFS traversal from each node of the input graph to generate the neighborhood. The advantages of this is:

    1. Since the BFS traversals are mutually exclusive for each vertex (because they are read-only), this step is highly parallelizable.

2. Though the size of the sets representing the neighborhoods might be large, using MinHash significantly brings down the complexity of pair-wise comparison to estimate similarity.

- The method suggested in Section 3.2 consists of doing random walks from each vertex of an input graph.

    1. Similar to the other approach suggested, this is a highly parallelizabe step (capped by the total number of threads available) and MinHash makes the pair-wise comparison of the sets representing the random walks significantly cheaper.

    2. The parameterized way to generate the random walks presents the advantage of having the opportunity to tune them based on the end graph-application. For example, if the end application is something like BFS, or Local Triangle Counting, then the random walks could be tuned to follow a more BFS way of traversal and vice versa.

5 Conclusion

To summarize, this report first takes a look at vertex reordering and MinHash separately. This is followed by the description of two suggested approaches that uses the various transferable skills of MinHash to solve the problem of vertex reordering. What follows is a brief discussion that speaks to the merits of the two suggested approcahes in terms of parallelizability and adaptability to applications. In the end, it can be said that given the inexpensive nature of MinHash and its capability of estimating the similarity between large sets of objects, using it in the field of vertex reordering remains a very fertile scope of research.

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