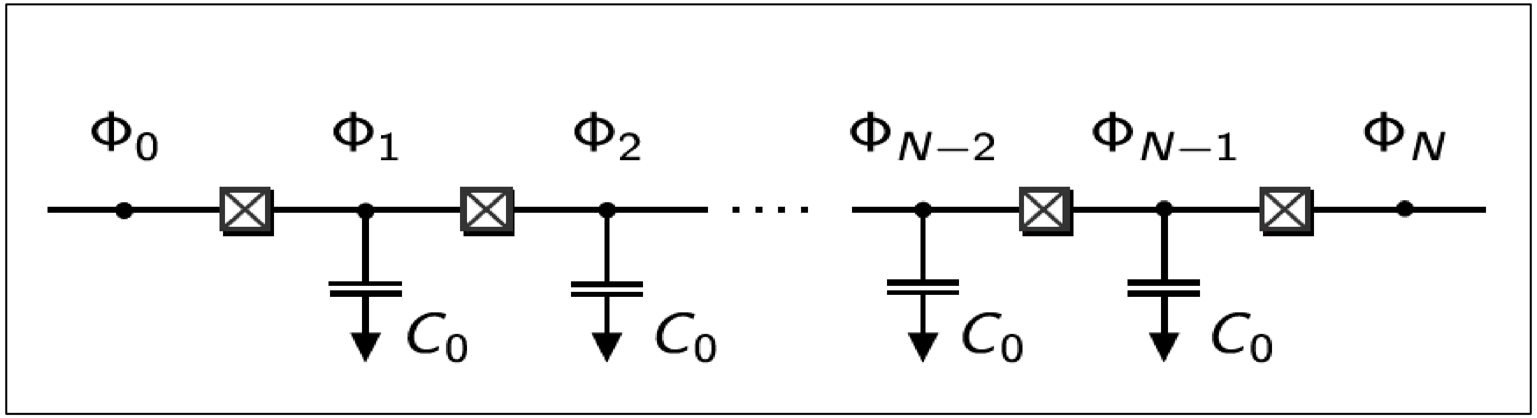
Josephson Junction Parametric Amplifier Arrays

Reet Mhaske

Guided by Prof. Ioan Pop & Nicolas Zapata

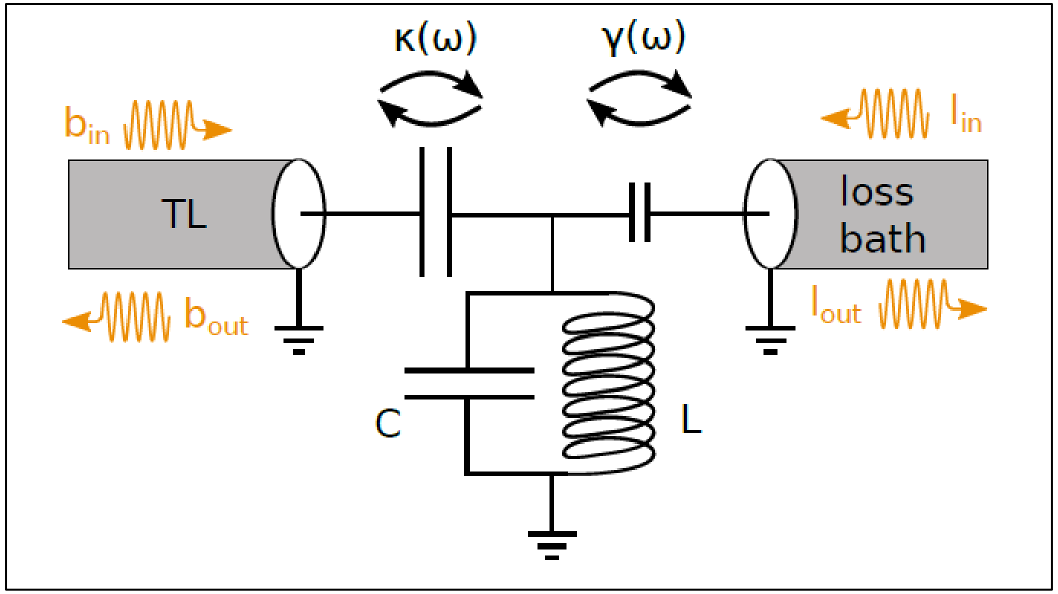


OVERVIEW

- Introducing Josephson Junction Parametric Amplifier
- The Gain-Bandwidth Product Problem
- Josephson Parametric Amplifier array & a possible Solution
- Outlook

1.1 A look at LC resonator coupled to Transmission Line

A single transmission line Parametric Amplifier (Losses modelled as another TL)



Reflection coefficient: $S_{11}(\omega) = -1 + \frac{\kappa}{i(\omega - \omega_0) + (\kappa + \gamma)/2}.$

Reference: Ivan Takmakov, Msc thesis

1.1 A look at LC resonator coupled to Transmission Line

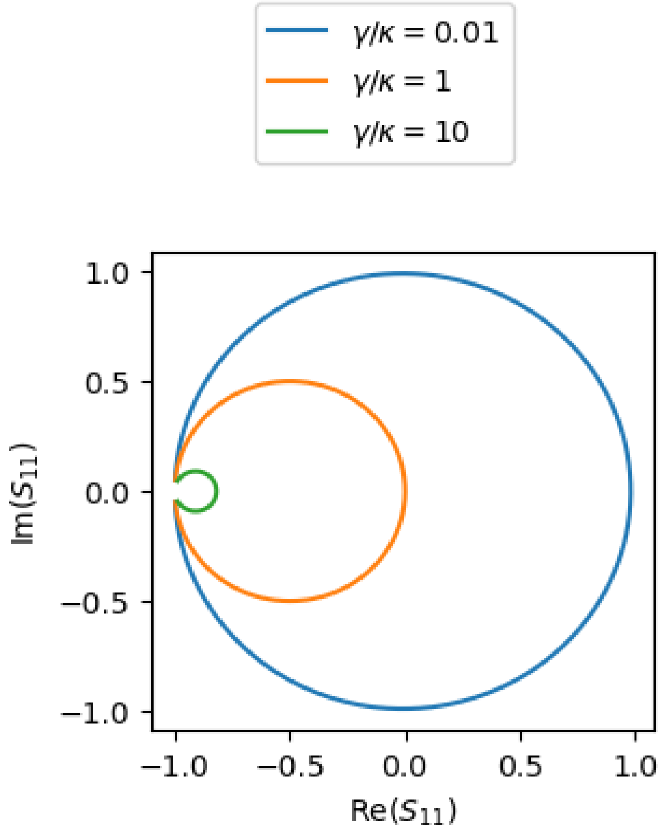


Fig 1.2 S_{11} in the complex plane for different coupling constants

A single transmission line Parametric Amplifier

The reflection coefficient is given as:

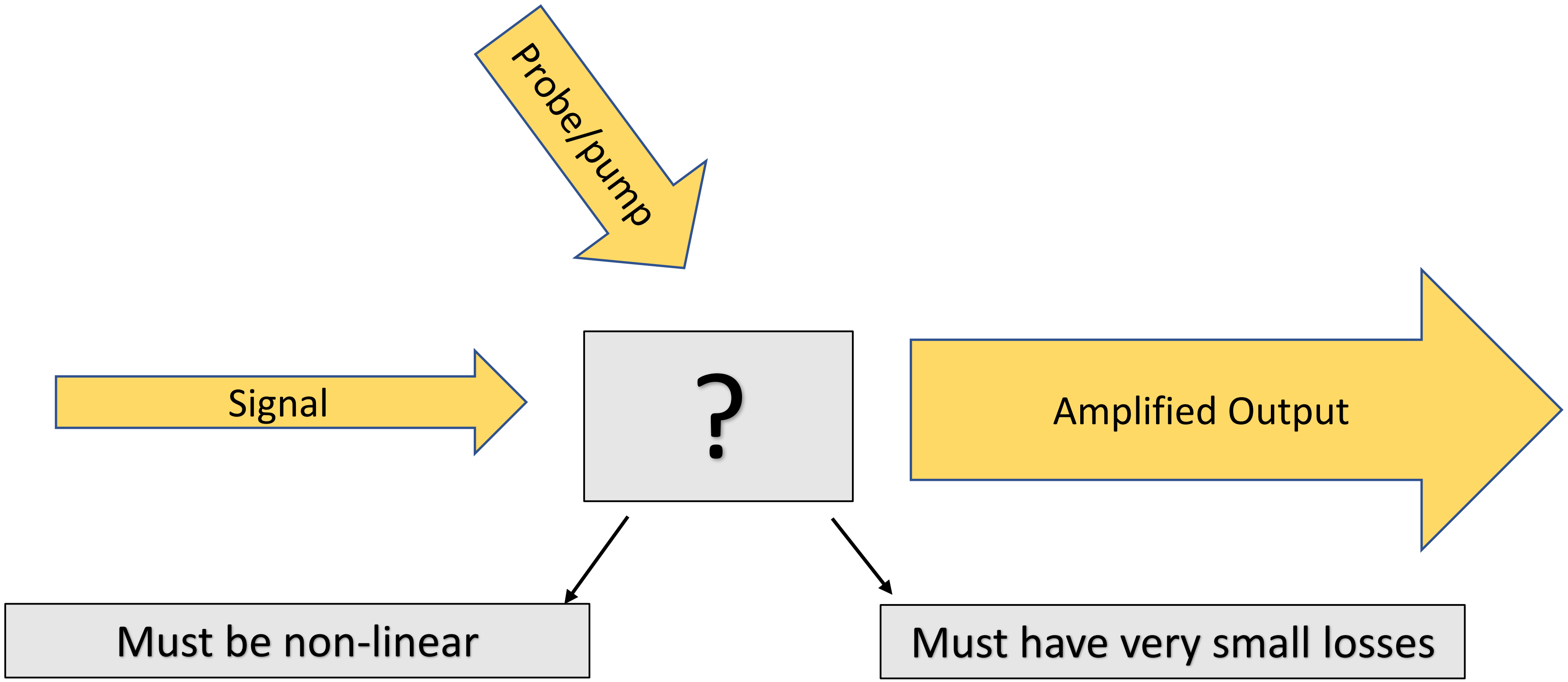
$$S_{11}(\omega) = -1 + \frac{\kappa}{i(\omega - \omega_0) + (\kappa + \gamma)/2}.$$

We observe 3 regimes:

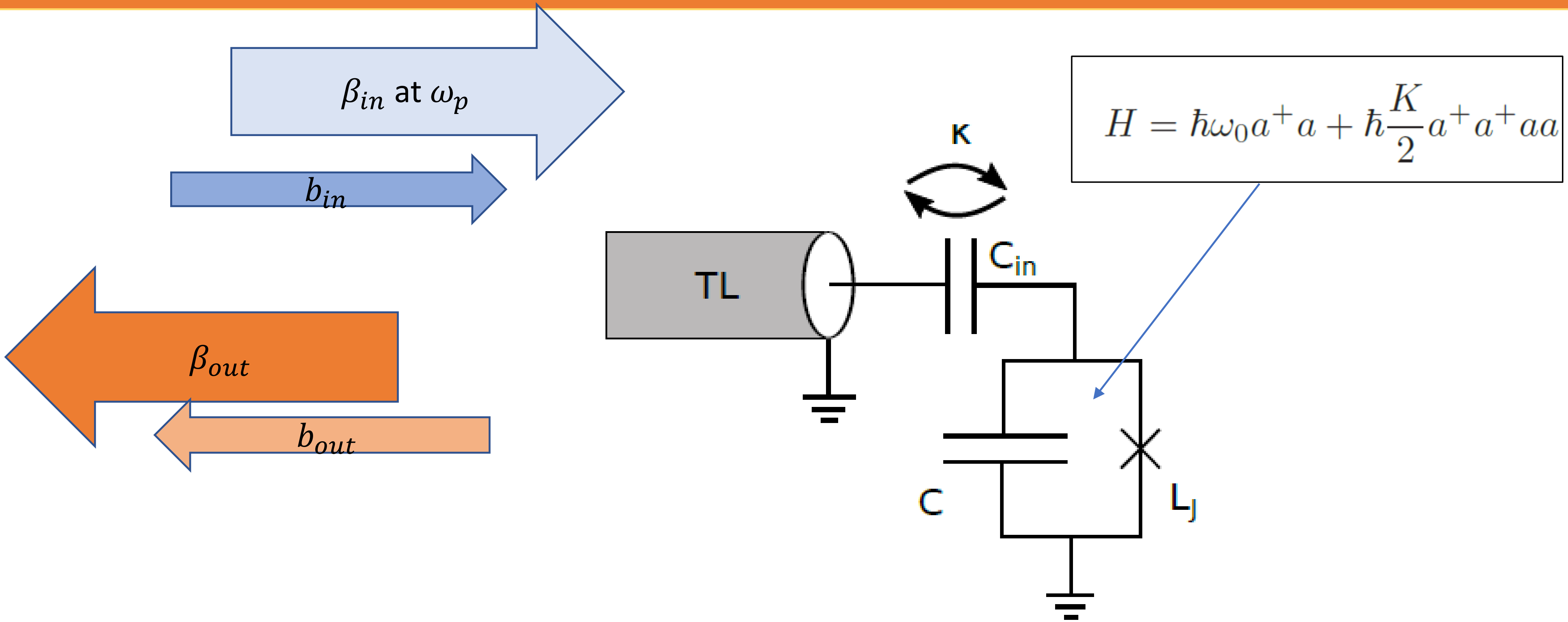
- Over-coupled ($\kappa \gg \gamma$):
- Critically coupled ($\kappa = \gamma$):
- Under-coupled ($\kappa \ll \gamma$):

1.2 Need for Parametric Amplification

Similar to an optical parametric amplifier



1.2 Josephson parametric amplifier

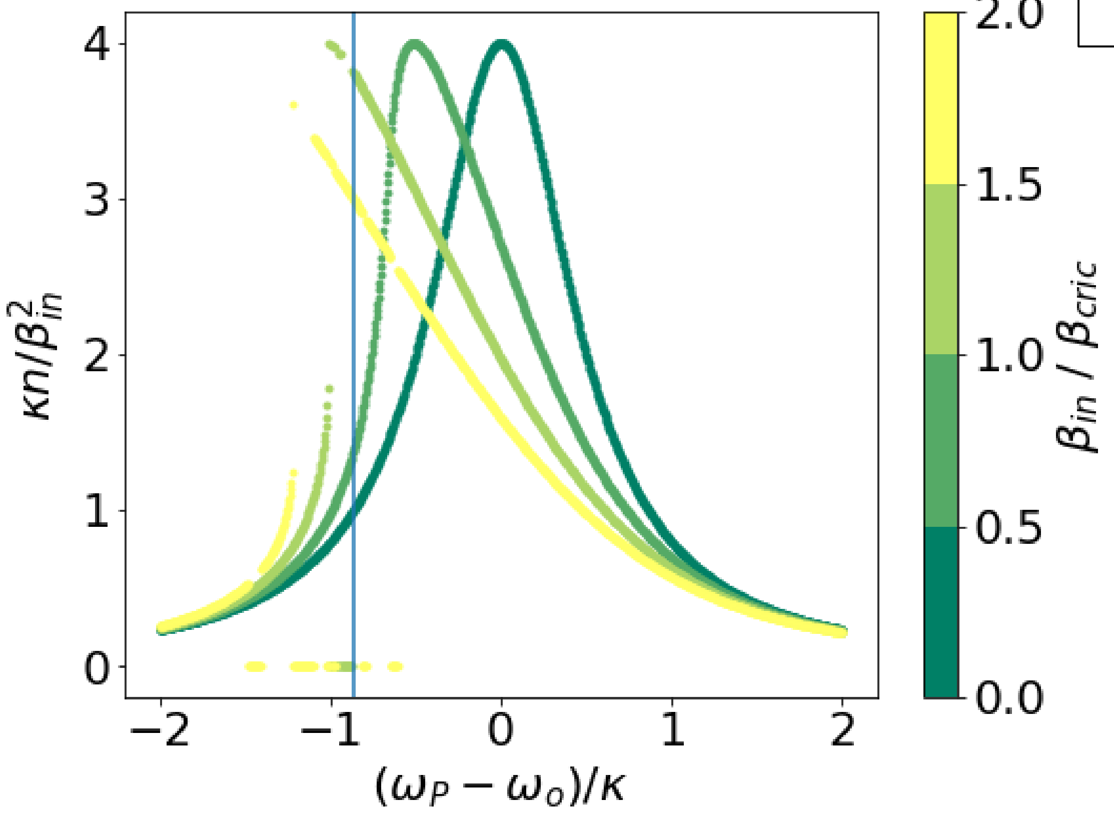


- Josephson Junction: Source of Nonlinearity (Assume $\kappa \gg K$)
- Superconducting elements: No losses ($\kappa \gg \gamma$)

Reference: Ivan Takmakov, Msc thesis

1.2 Josephson parametric amplifier

Photon Occupation number
v/s detuning for different drive
powers

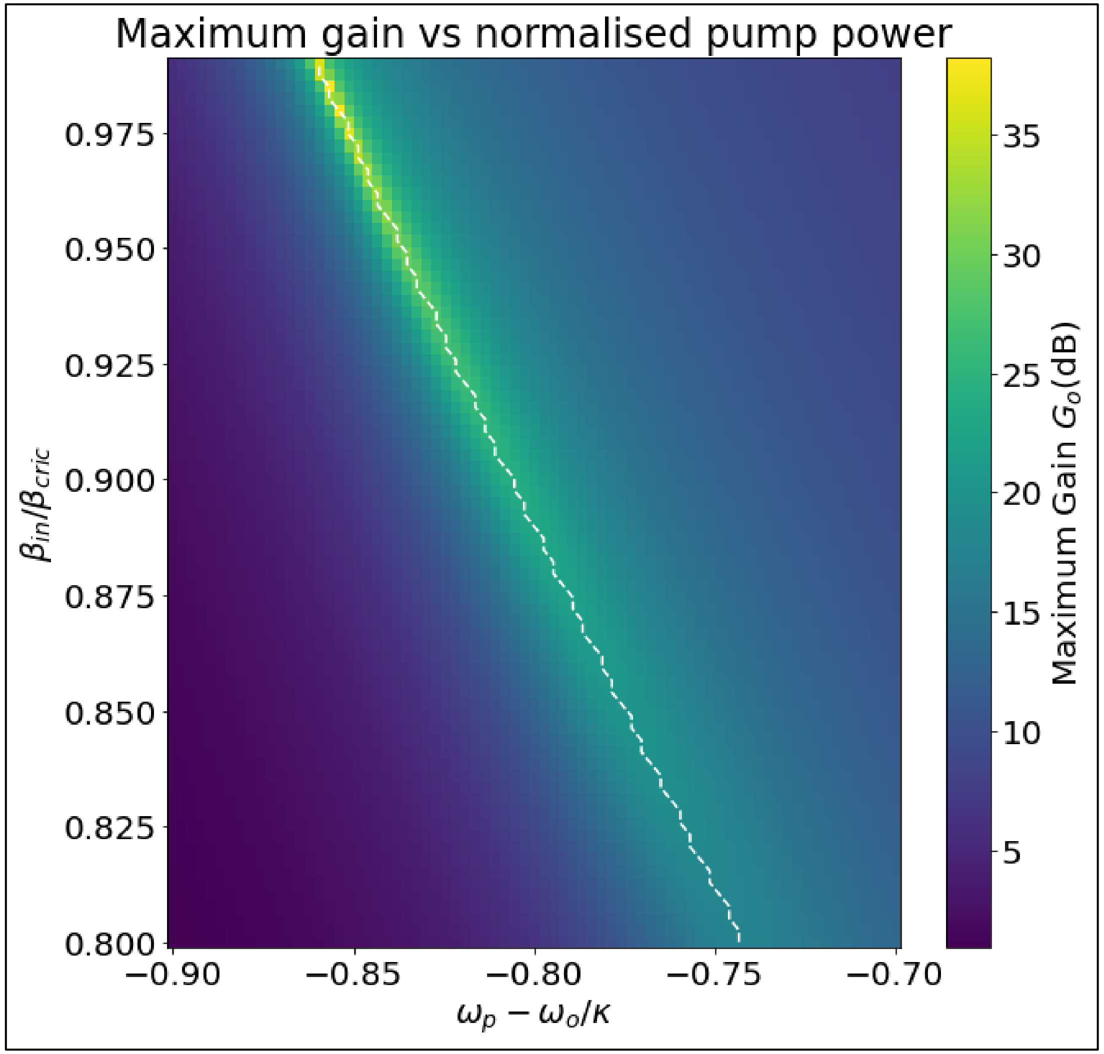


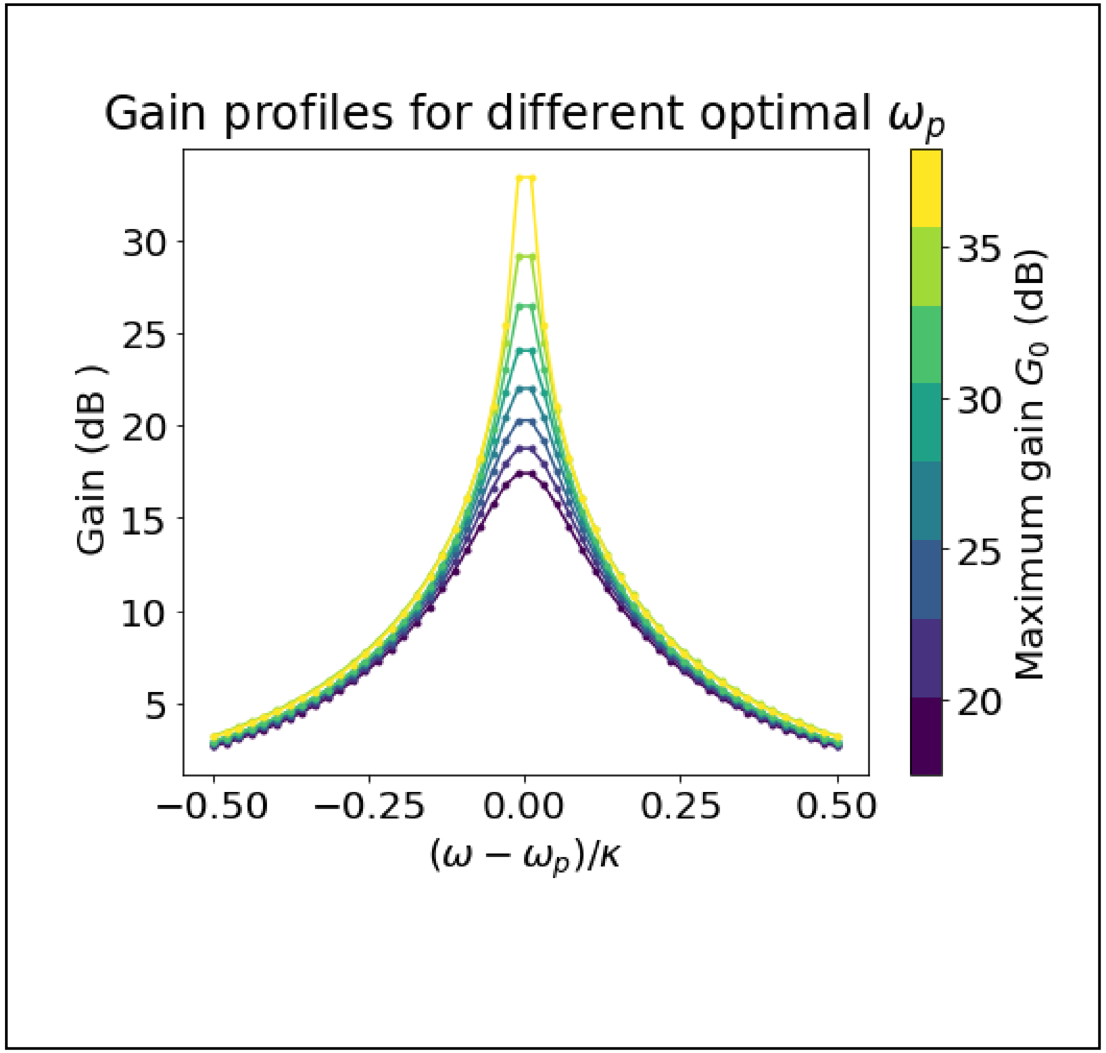
$$\left[(\omega_\mathrm{p}-\omega_0)^2+\frac{\kappa^2}{4}\right]n-2(\omega_\mathrm{p}-\omega_0)Kn^2+K^2n^3=\kappa|\beta_\mathrm{in}|^2$$

- Bifurcation after $|\beta_{cric}|^2 = \frac{K}{\sqrt{27}\kappa}$

Reference: Ivan Takmakov, Msc thesis

1.2 Josephson parametric amplifier





2. Gain-Bandwidth Product

- **Instantaneous bandwidth (BW) :** frequency range from maximum gain to 3dB lower
- As gain increases, bandwidth decreases

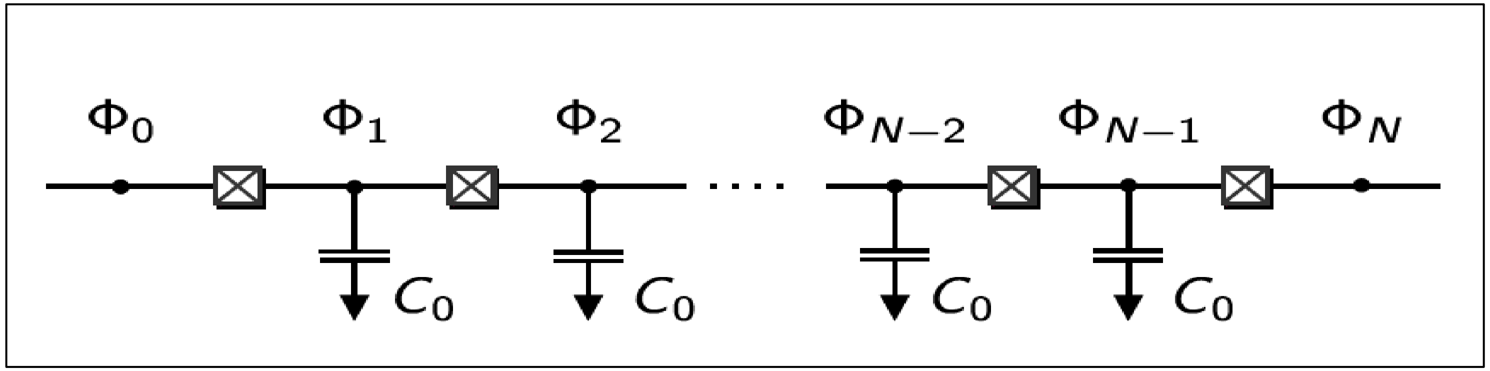
$$\mathrm{BW}\sqrt{G_0} = \kappa$$

- To increase both BW and gain, increase κ
- Increase κ is by increasing capacitance to ground
- But would it change other parameters?

3.Josephson Junction Arrays

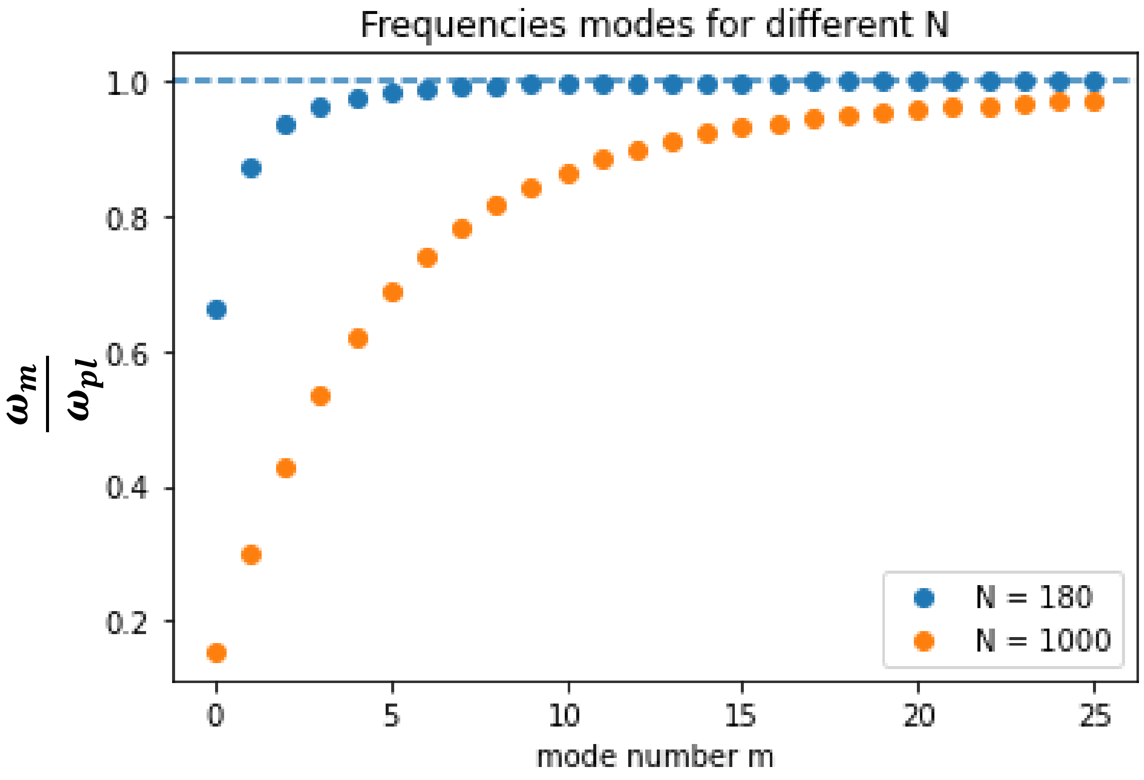
- A linear array of JJs, with capacitors to ground in between
- Before using their non-linearity, lets look at the linearized Hamiltonian:

$$\mathcal{L} = \sum_{i=1}^{N} \frac{C_0}{2}\dot{\Phi}_i^2 + \sum_{i=0}^{N} \frac{C_\mathrm{J}}{2}\left(\dot{\Phi}_{i+1} - \dot{\Phi}_i\right)^2 - \frac{1}{2L_\mathrm{J}}(\Phi_{i+1} - \Phi_i)^2.$$

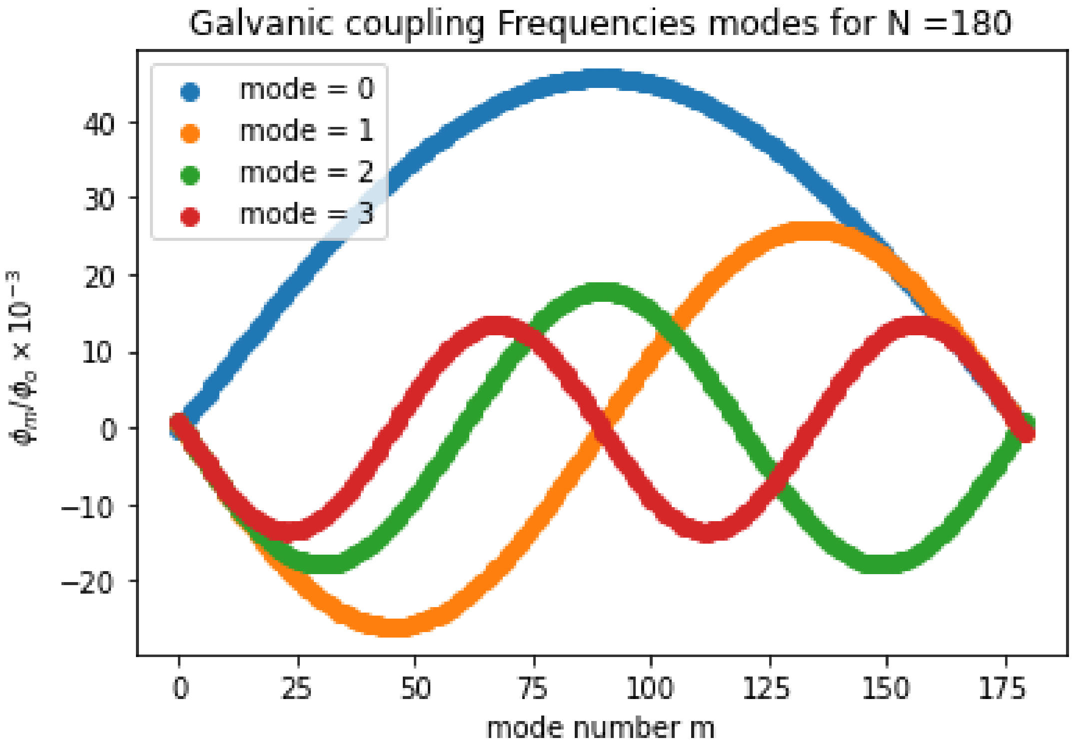


Reference: PatrickWinkel, M.Sc thesis

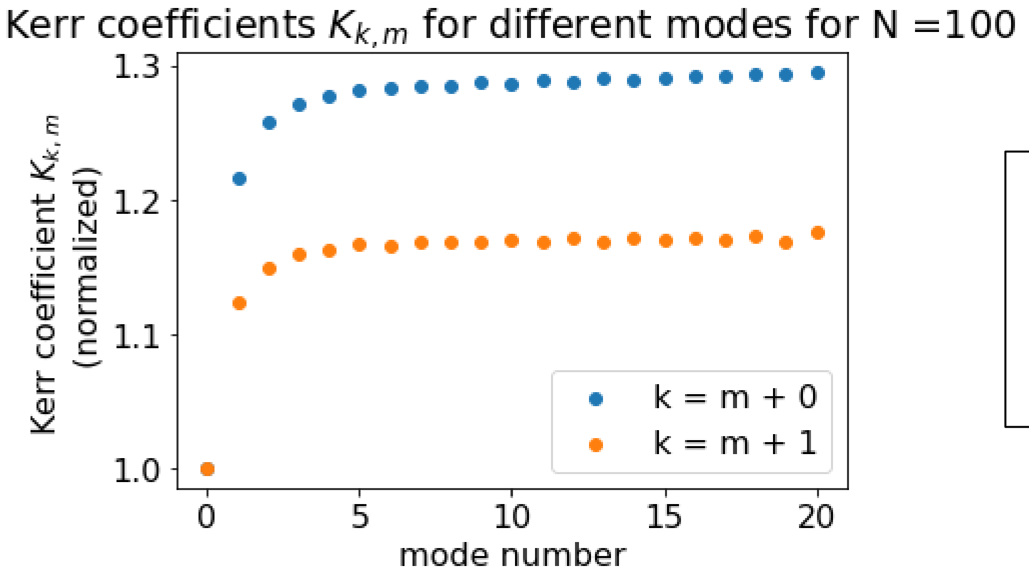
3.Josephson Junction Arrays



$$\omega_{\rm pl} = \frac{1}{\sqrt{L_{\rm J}(C_{\rm J} + C_0/4)}}$$



3.1. Josephson Junction Arrays



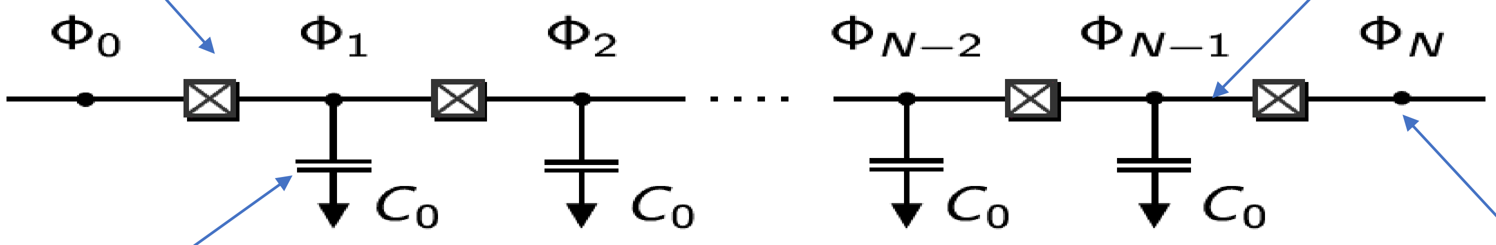
$$\mathbf{H} = \sum_{m=0}^{N-1} \hbar\omega_m \hat{a}_m^\dagger \hat{a}_m + \underbrace{\frac{\hbar}{2} K_{m,m}\omega_m \hat{a}_m^\dagger \hat{a}_m \hat{a}_m^\dagger \hat{a}_m}_{\mathrm{self-Kerr}} + \underbrace{\sum_{m,k=0}^{N-1} \frac{\hbar}{2} K_{m,k}\omega_m \hat{a}_m^\dagger \hat{a}_m \hat{a}_k^\dagger \hat{a}_k}_{\mathrm{cross-Kerr}}$$

Reference: Patrick Winkel, M.Sc thesis

3.3. Transmission formalism to find reflection coefficients

$$\tilde{T}_{\mathrm{J}} = \begin{pmatrix} 1 & \left[(j\omega L_{\mathrm{J}})^{-1} + j\omega C_{\mathrm{J}}\right]^{-1} \\ 0 & 1 \end{pmatrix}$$

$$\tilde{T}_{\mathrm{L_s}} = \begin{pmatrix} 1 & j\omega L_s \\ 0 & 1 \end{pmatrix}$$



$$\tilde{T}_{\mathrm{C_0}} = \begin{pmatrix} 1 & 0 \\ j\omega C_0 & 1 \end{pmatrix}$$

$$\tilde{T}_{\mathrm{term}} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

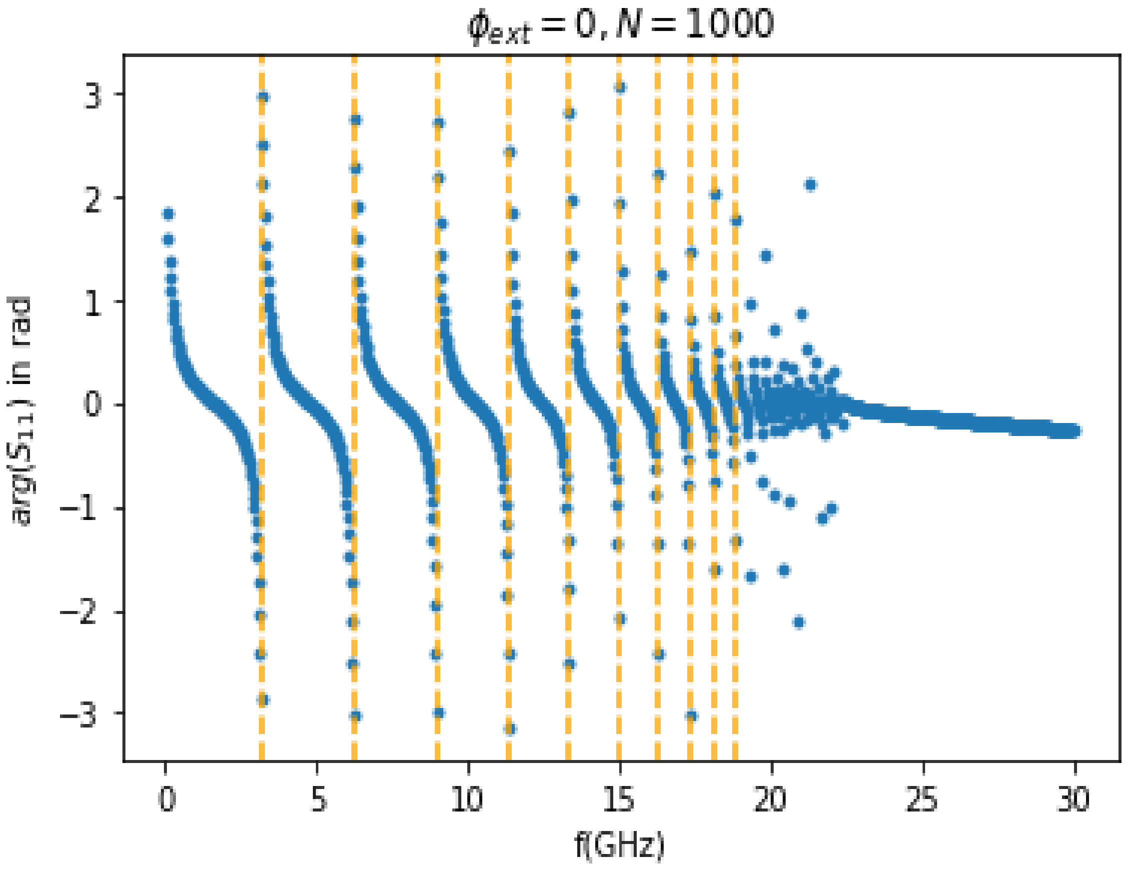
$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D}$$

$$\tilde{T} = \prod_{i=1}^{N+1} \tilde{T}_i = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

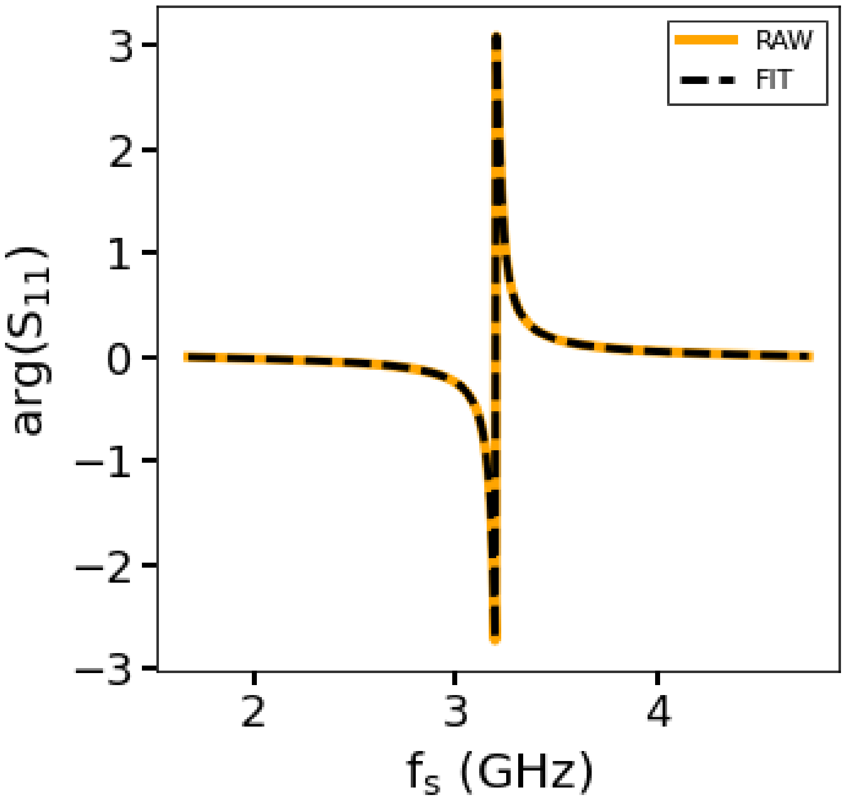
$$\tilde{T} = \left(\prod_{i=1}^{N-1} \tilde{T}_{\mathrm{J}} \tilde{T}_{\mathrm{C_0}} \tilde{T}_{\mathrm{L_s}}\right) \tilde{T}_{\mathrm{J}} \tilde{T}_{\mathrm{term}}$$

3.3. Transmission formalism to find reflection coefficients

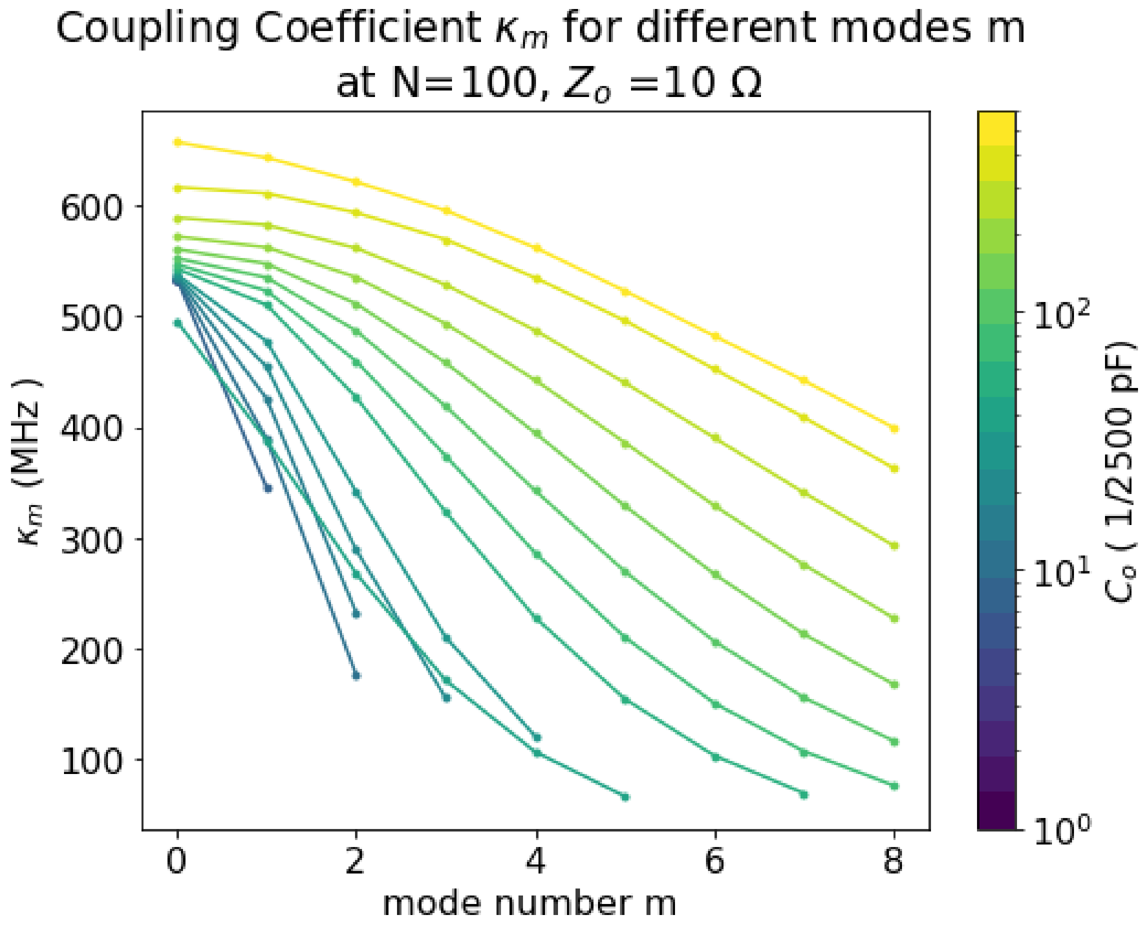
We fit $\Gamma_{\mathrm{fit},m} = \Gamma_m e^{i\phi_m} e^{is_m(\omega-\omega_m)}$ around the poles, to get coupling coefficient κ

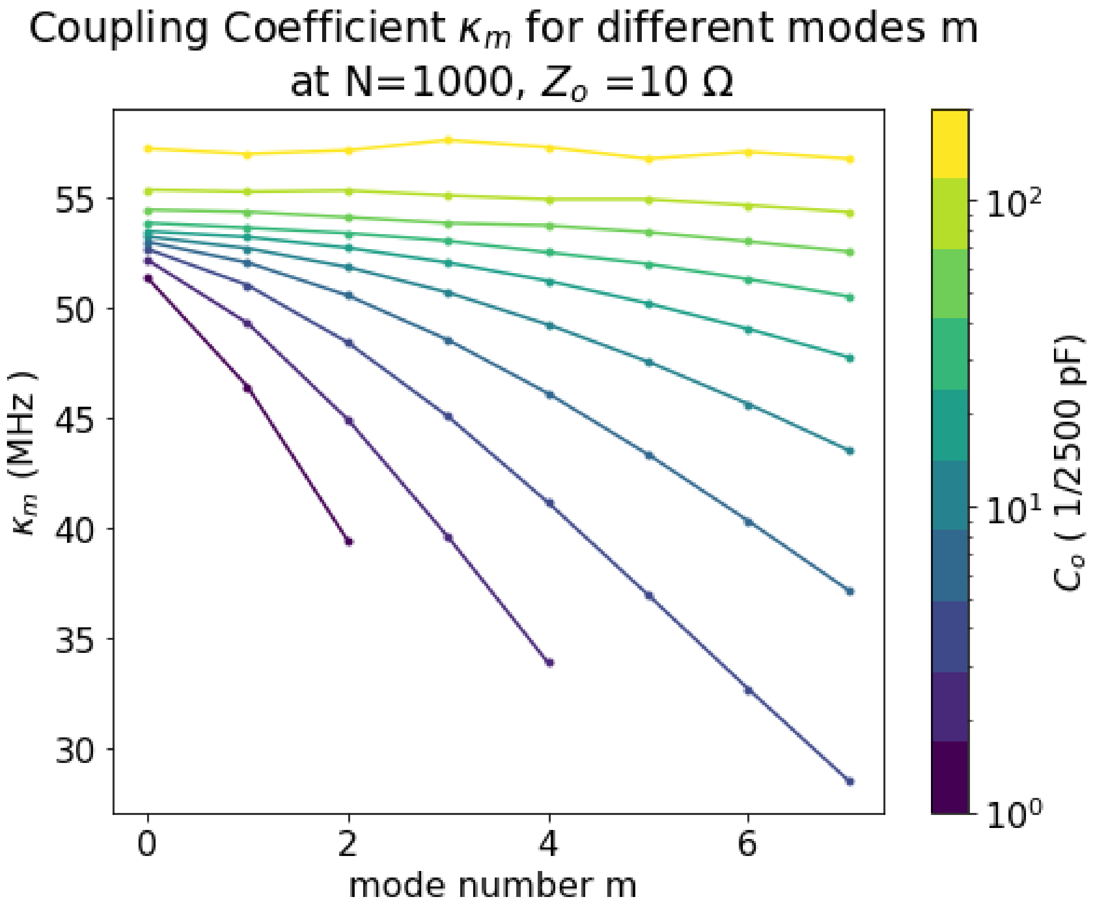


Demonstration of a fit
at N=1000, m=1, C_o= 1/2500pF, Z_o=10Ω

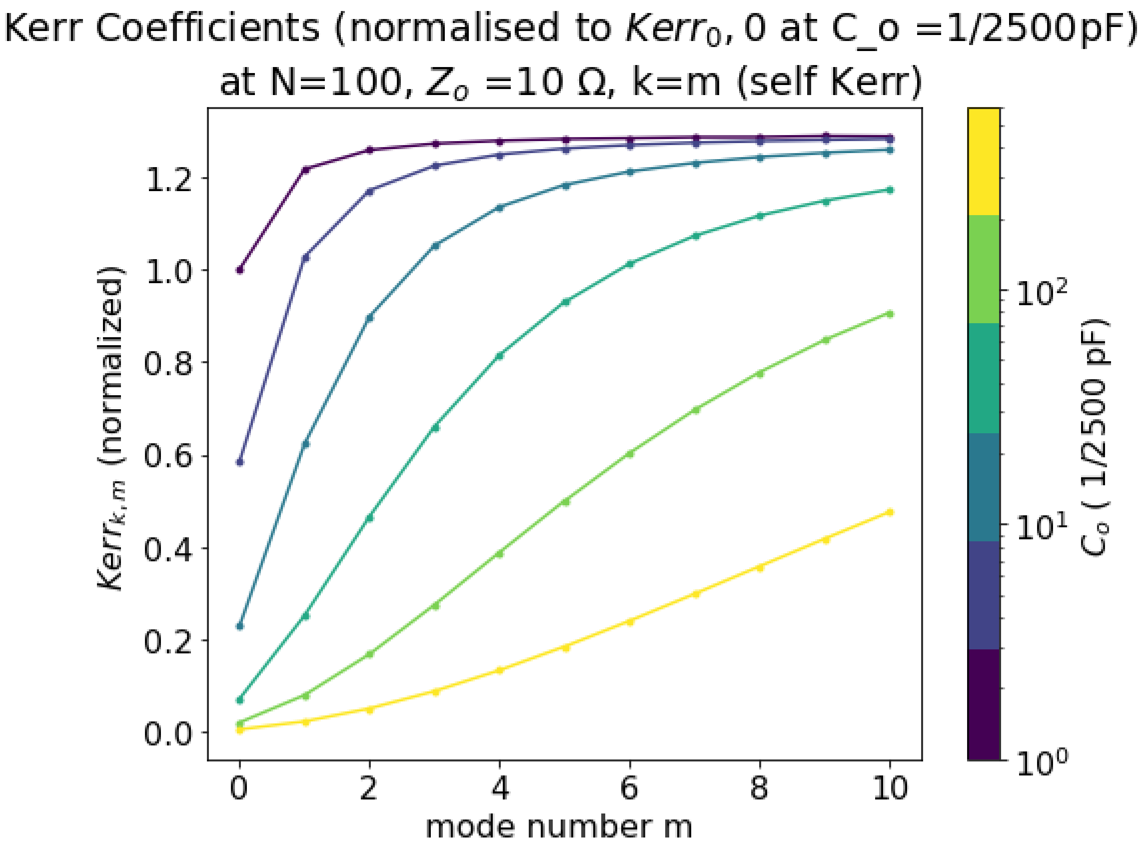


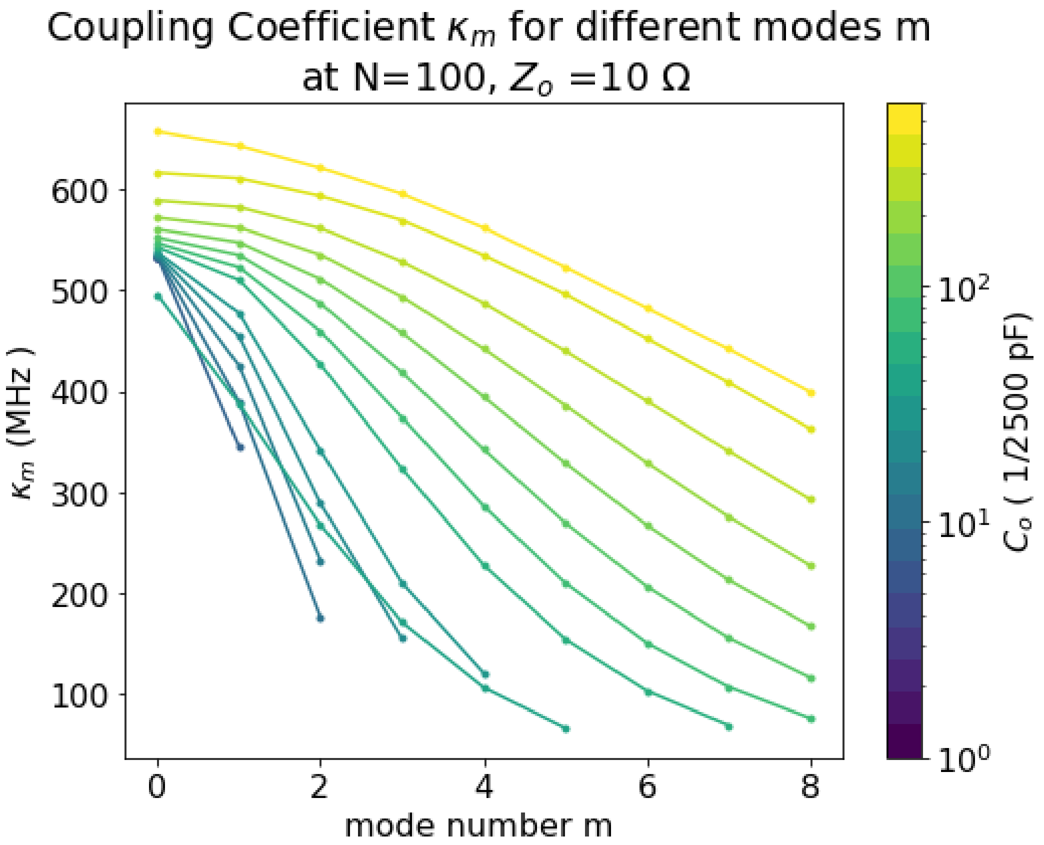
3.4. Changing κ for different array lengths by changing C_o





3.5. Comparing change in K and κ





Outlook

- What happens for Dimers?
- Changing κ by changing transmission line parameters?

Thank you!

Appendix

- Langevin equation for JPA
- Expression for Kerr coefficients of JJ parametric array
- Linear scale plots for Coupling coefficients
- Linear scale plots for Kerr coefficients
- JPA photon occupation number
- 3d plots for Kappa
- Elastic Scattering on a two-level system
- Dispersive readout of a qubit

A.1 Josephson parametric amplifier

- Treating Quantum signal as a fluctuation to a classical signal

$$A = \alpha e^{i\omega_\mathrm{p} t} + a$$

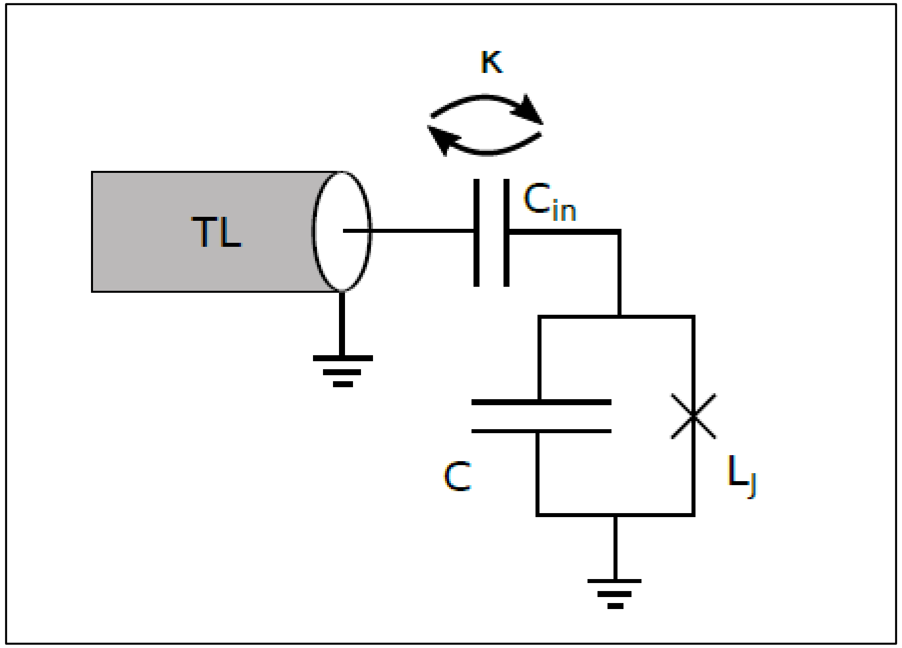
$$B_{\mathrm{in/out}} = \beta_{\mathrm{in/out}} e^{i\omega_\mathrm{p} t} + b_{\mathrm{in/out}}$$

- The Langevin equation becomes:

$$\dot{A} = -i\omega_0 A - iKA^+AA - \frac{\kappa}{2}A + \sqrt{\kappa}B_{\mathrm{in}}$$

- Which in terms of resonator photon number $n = |\alpha|^2$ becomes:

$$\left[(\omega_\mathrm{p} - \omega_0)^2 + \frac{\kappa^2}{4}\right] n - 2(\omega_\mathrm{p} - \omega_0)Kn^2 + K^2n^3 = \kappa|\beta_{\mathrm{in}}|^2$$



A.2. Backup

Now consider the non-linear Hamiltonian for small non-linearity (perturbative treatment)

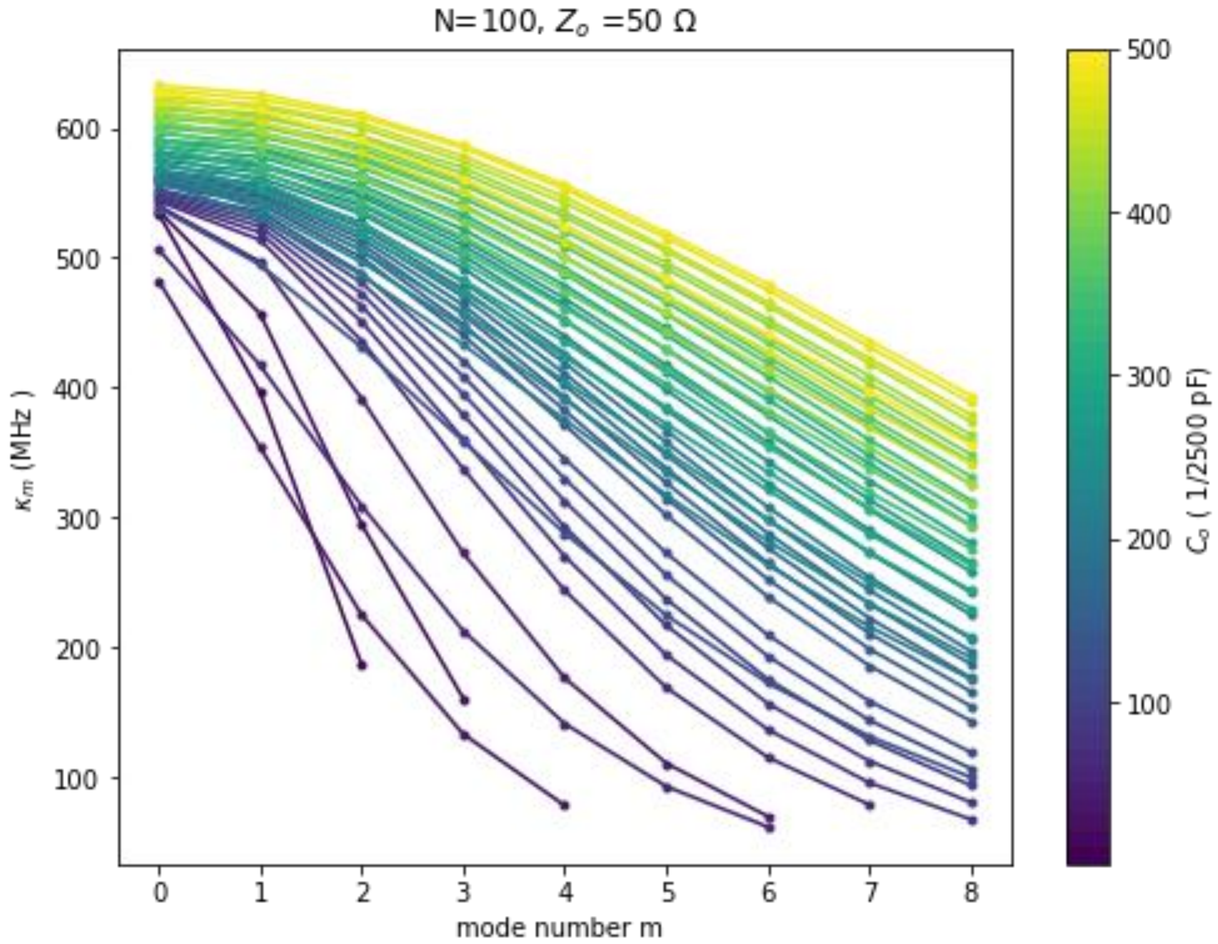
$$\mathbf{H} = \sum_{m=0}^{N-1} \hbar\omega_m \hat{a}_m^\dagger \hat{a}_m + \underbrace{\frac{\hbar}{2} K_{m,m}\omega_m \hat{a}_m^\dagger \hat{a}_m \hat{a}_m^\dagger \hat{a}_m}_{\text{self}-\text{Kerr}} + \underbrace{\sum_{m,k=0}^{N-1} \frac{\hbar}{2} K_{m,k}\omega_m \hat{a}_m^\dagger \hat{a}_m \hat{a}_k^\dagger \hat{a}_k}_{\text{cross}-\text{Kerr}}$$

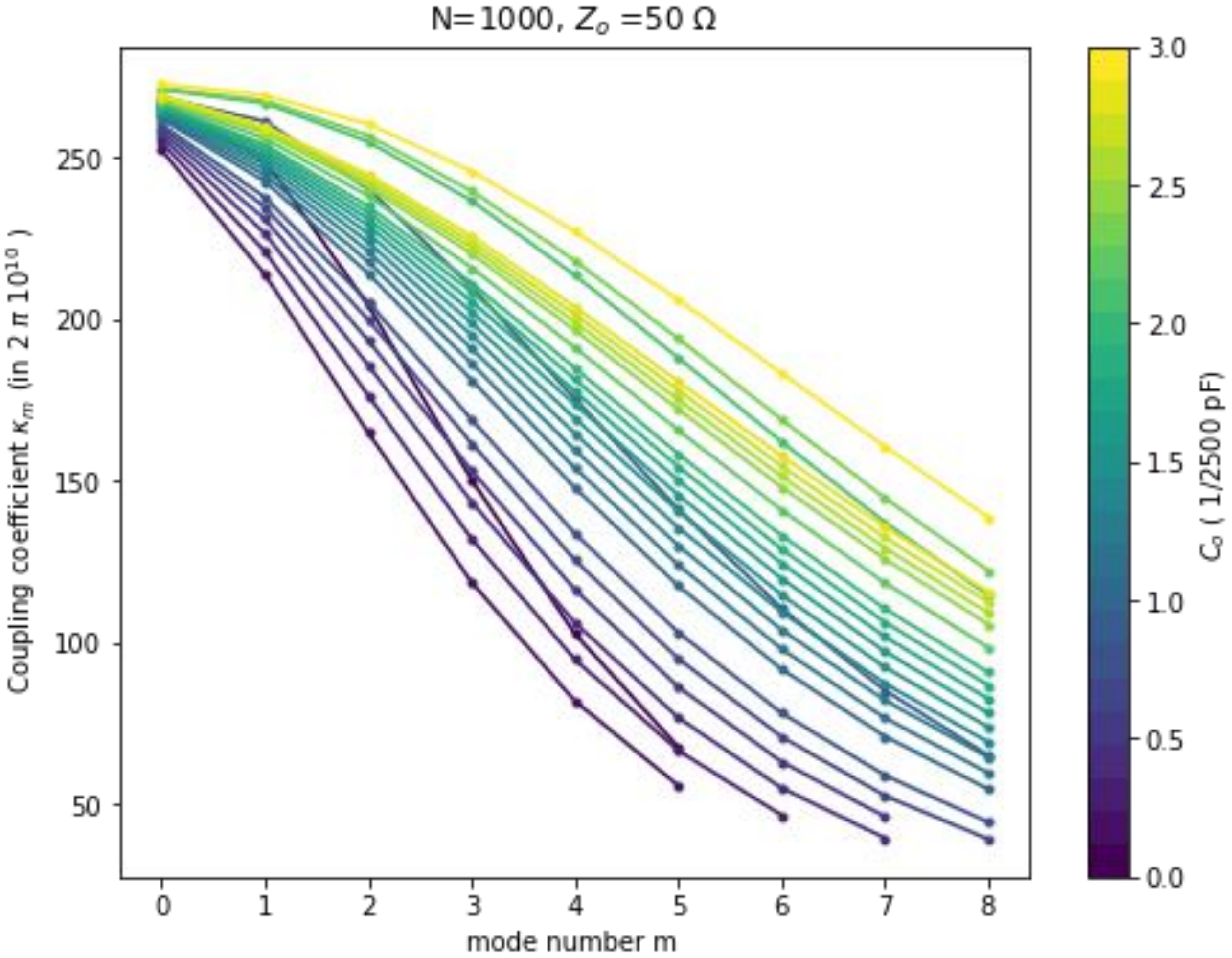
$$K_{m,m} = -\frac{2\hbar\pi^4 E_J \eta_{mmmm}}{\Phi_0^4 C_J^2 \omega_m^2}$$

$$K_{m,k} = -\frac{4\hbar\pi^4 E_J \eta_{mmkk}}{\Phi_0^4 C_J^2 \omega_m \omega_k},$$

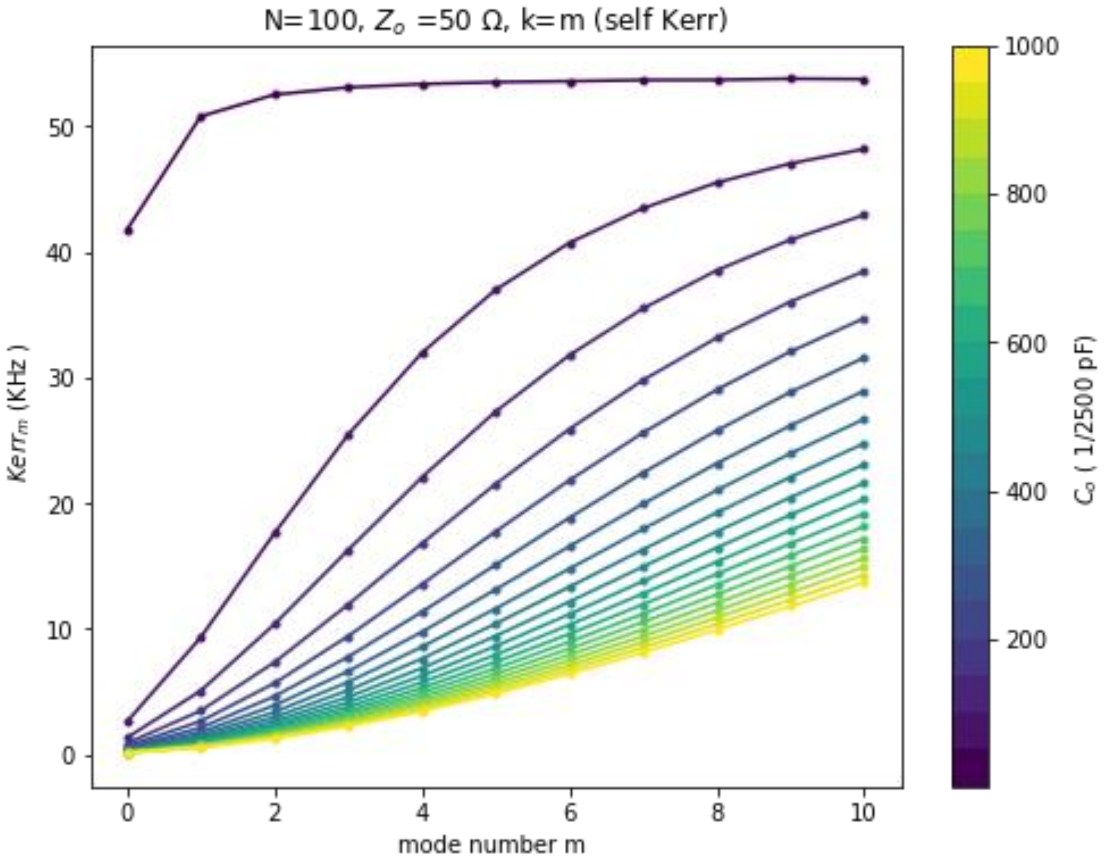
$$\eta_{mmkk} = C_J^2 \sum_{i=1}^{N} \left[\left(\sum_{j=0}^{N} \left(\tilde{C}_{i,j}^{-1/2} - \tilde{C}_{i-1,j}^{-1/2} \right) \Psi_{j,m} \right)^2 \times \left(\sum_{j=0}^{N} \left(\tilde{C}_{i,j}^{-1/2} - \tilde{C}_{i-1,j}^{-1/2} \right) \Psi_{j,k} \right)^2 \right].$$

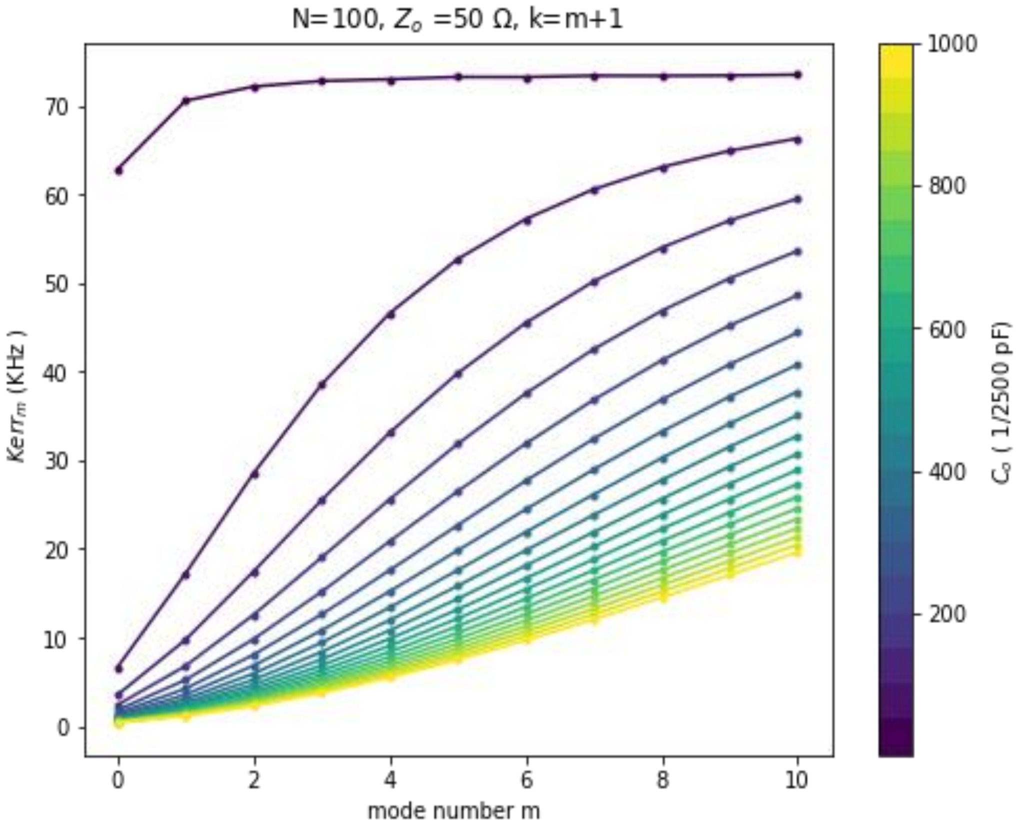
A3. Linear scale plots for coupling coefficients





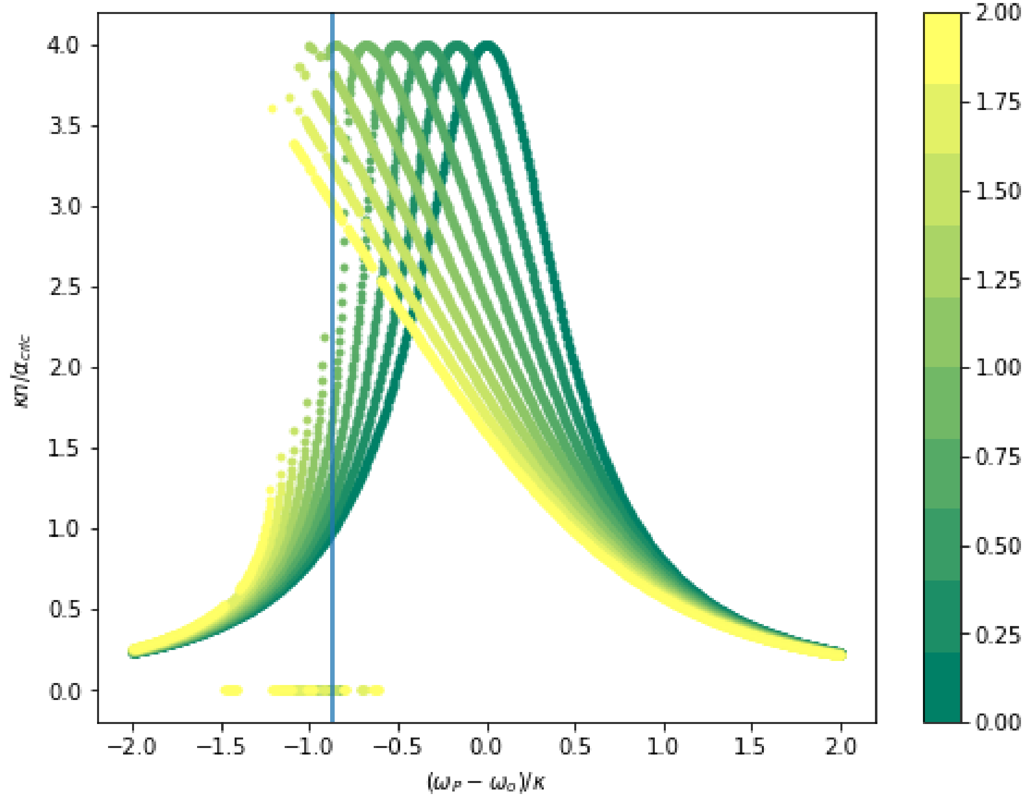
A.4 Change in Kerr coefficients (linear plots)



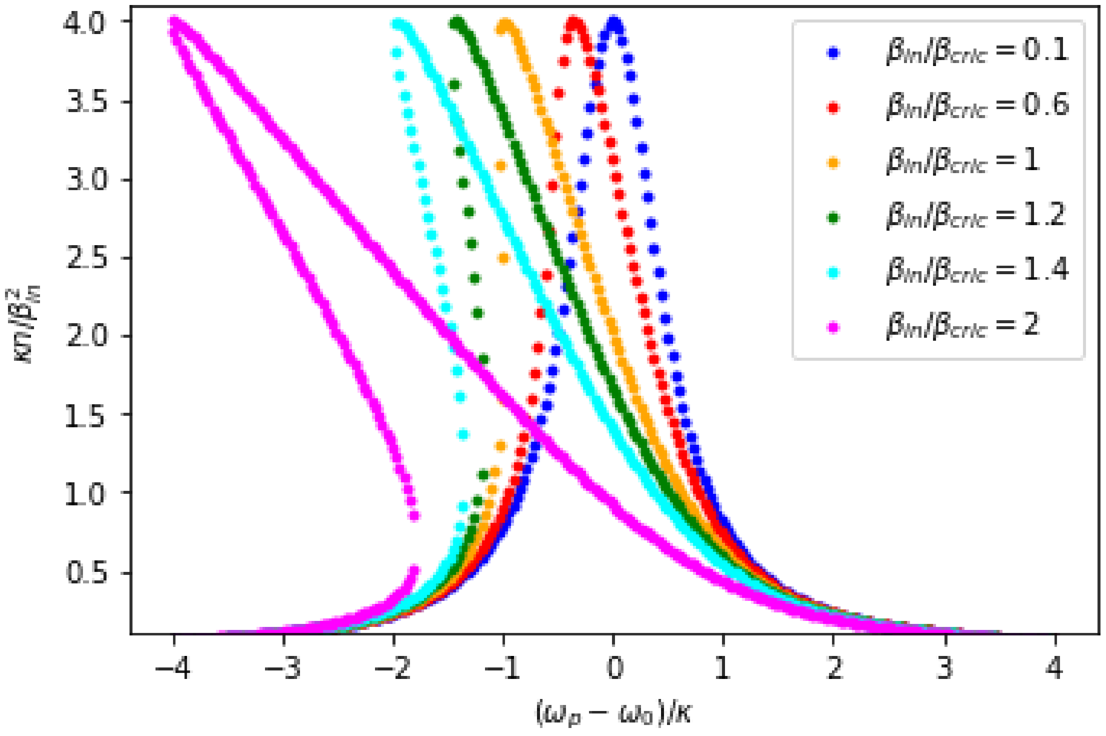


A.5 Photon occupation number in JPA (more curves)

Resonator photon number (n) as function of detuning for different drive powers



Resonator photon number (n) as function of detuning for different drive powers



A.6 Elastic scattering on a two-level system

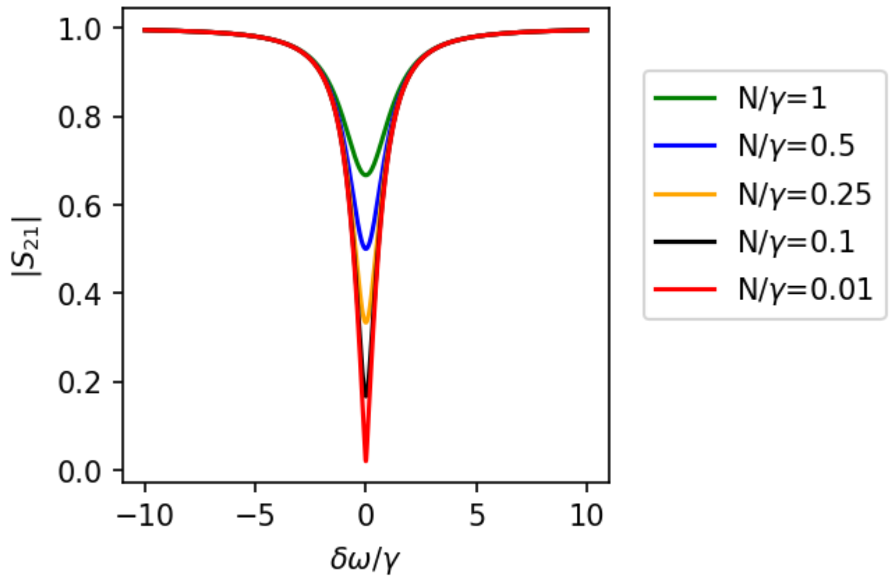
A qubit coupled to 2 transmission lines, each with coupling κ. Assume input and output to be right propagating waves. Account for intrinsic losses via γ_o. Define $\gamma = \frac{\kappa}{2} + \gamma_o$

For a driving signal with detuning $\delta\omega$ and input power Ω, the transmission coefficient is given as

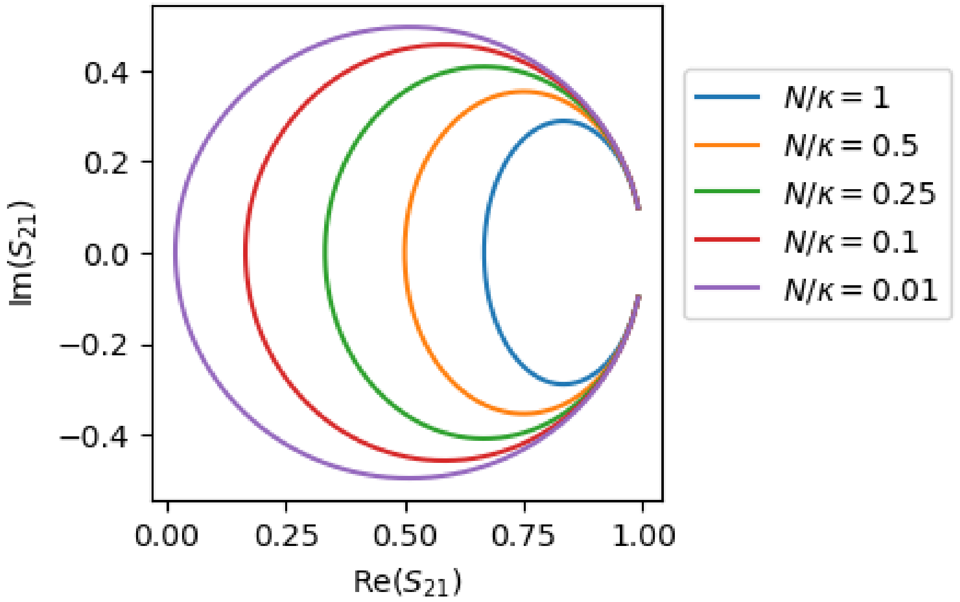
$$S_{21} = \frac{\kappa}{2\gamma} \frac{1 + i\delta\omega/\gamma}{1 + (\delta\omega/\gamma)^2 + \Omega^2/\kappa\gamma}$$

(Transmission coefficient is dependent on drive power and detuning)

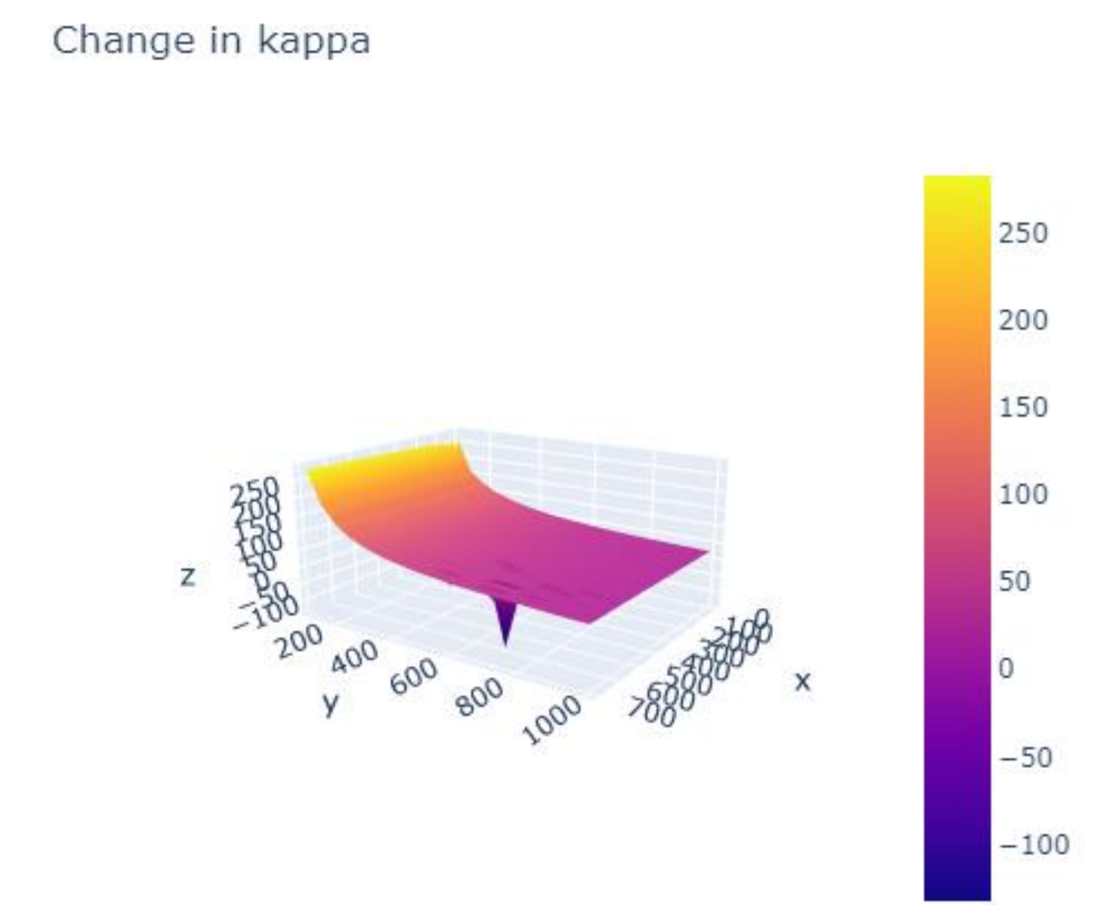
$|S_{21}|$ as function of $\delta\omega$ for different drive powers

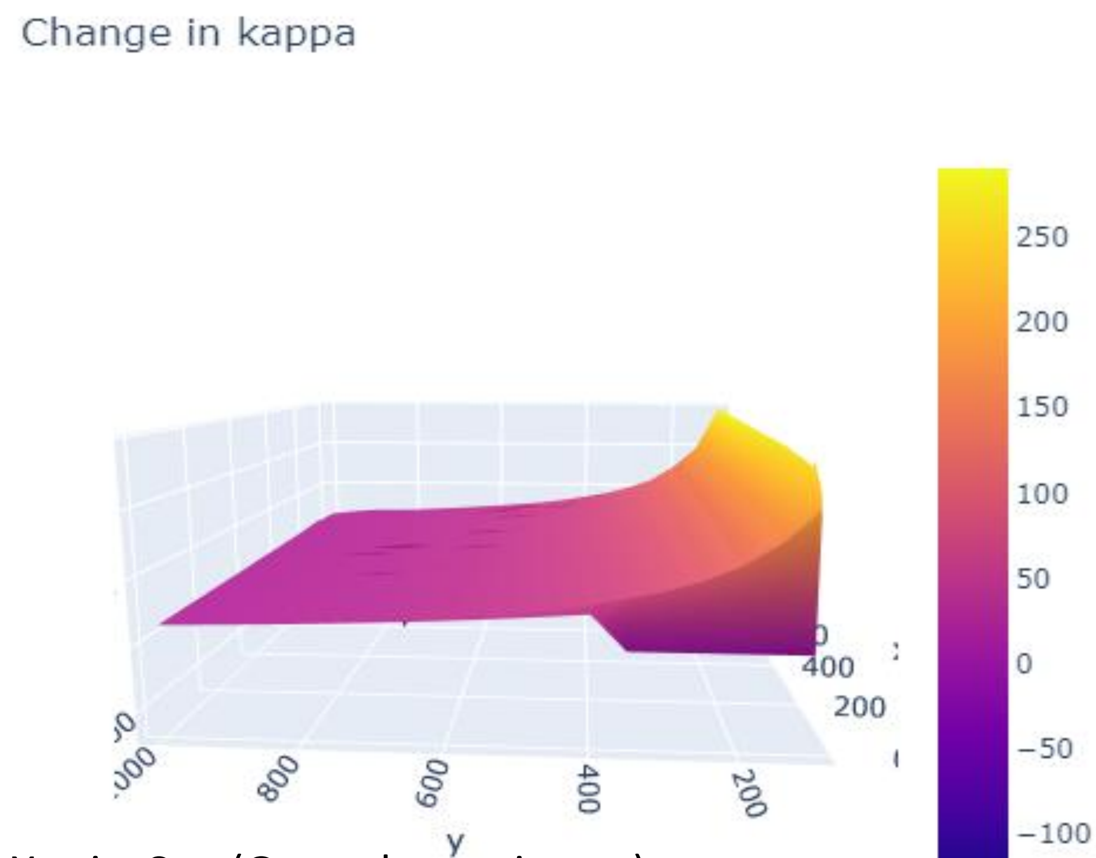


S_{21} in the complex plane for different drive powers



Coupling constants on changing C_0 and N





X axis: C_o (Ground capacitance)
Y_axis: N (Number of arrays)
Z axis: Kappa

Dispersive read-out of a qubit

The resonator is coupled to a two-level system,

$$b_{\text{out}} = \left(-1 + \frac{\kappa}{i(\omega - \omega_0 - \chi\langle\sigma_z\rangle)_q + \kappa/2}\right) b_{\text{in}};$$

(different phase for different states of the qubit)

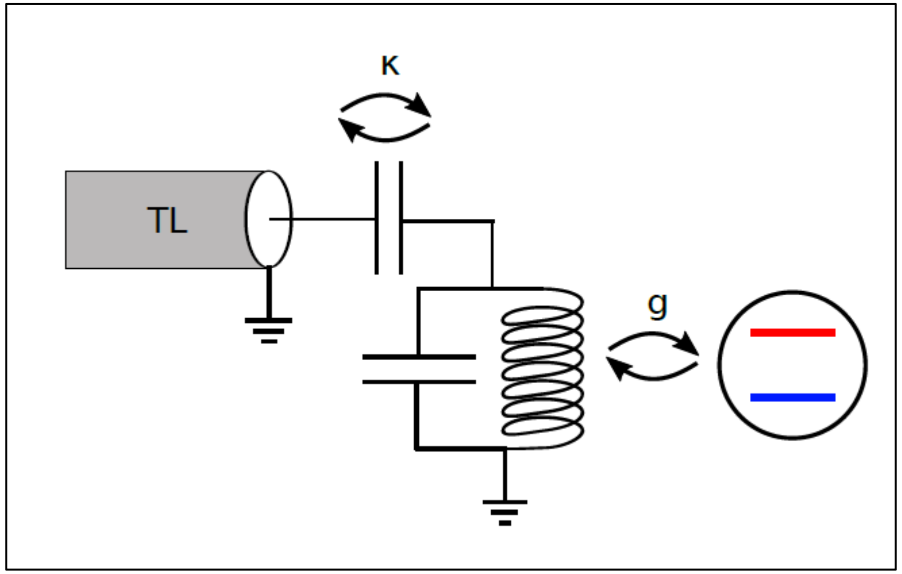
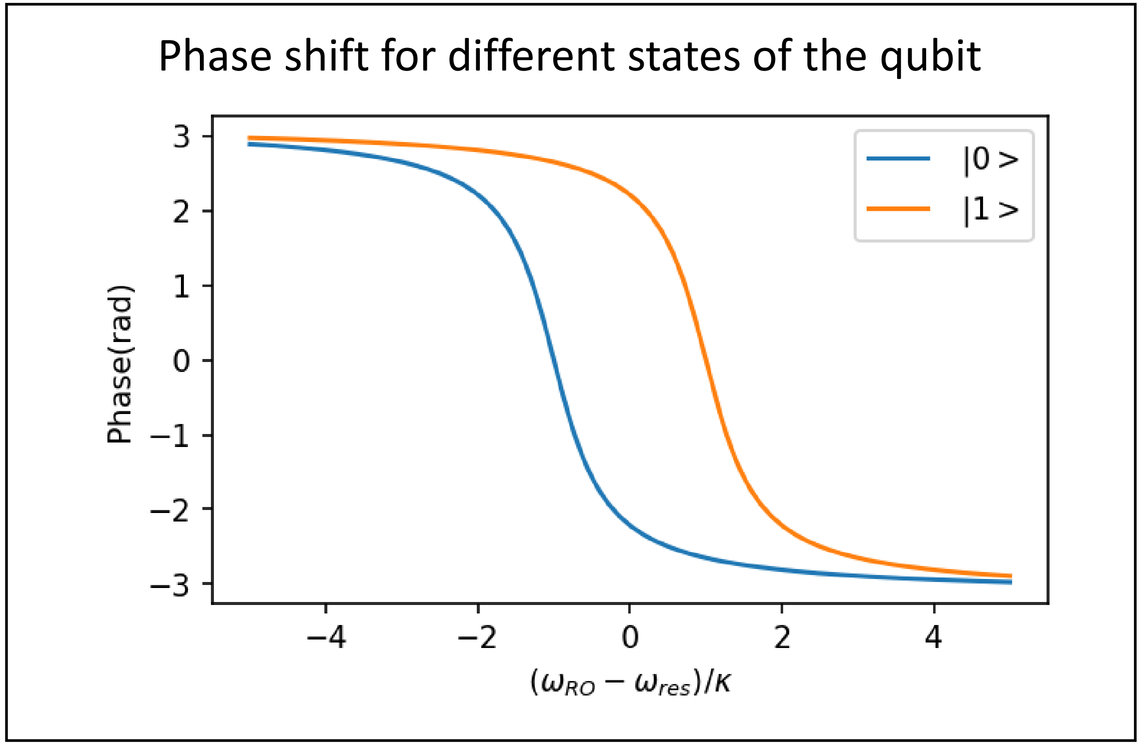


Fig2. The model of the resonator



Ref: Ivan Takmakov, Master theisis

Kerr Coefficients on changing C_o and N