GENERATING SPIN SPIRALS USING RYDBERG ATOMS

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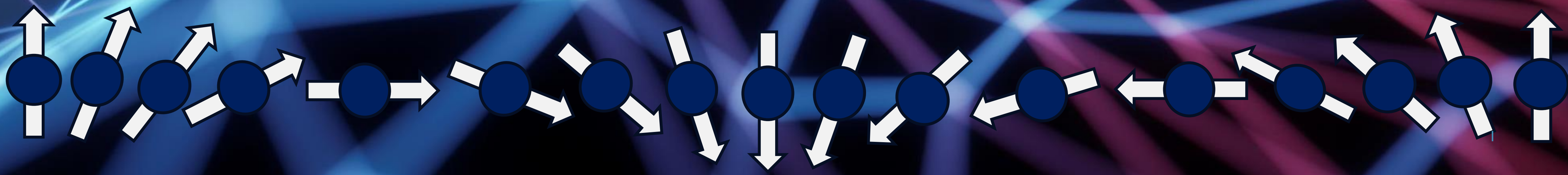
Supervisor: Prof Matthias Weidemuller

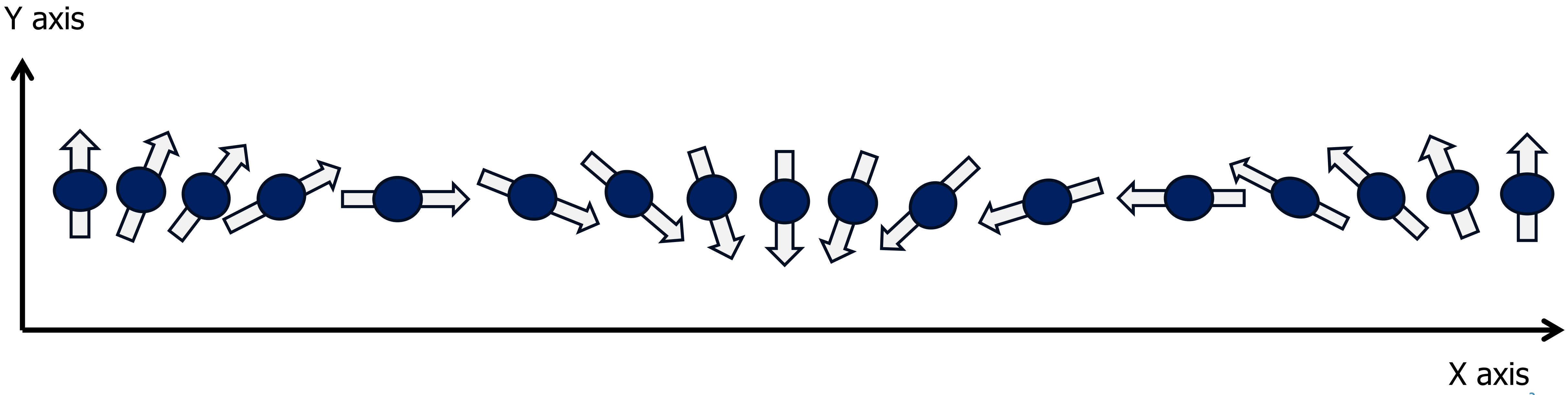


OVERVIEW

- What are spiral states?
- Why Spin Spiral?
- How to create spin spirals?
- Mapping to a spin Hamiltonian

1. WHAT ARE SPIN SPIRAL STATES?

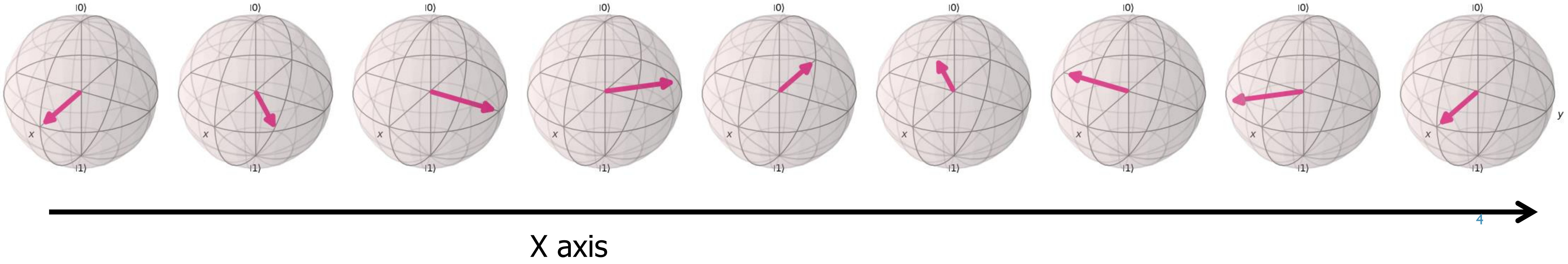
$$\left|\Psi_{spiral}\right\rangle = \prod_i \sin\left(\frac{2\pi}{\lambda}x_i\right)\hat{x}_i + \cos\left(\frac{2\pi}{\lambda}x_i\right)\hat{y}_i$$



1. WHAT ARE SPIN SPIRAL STATES?

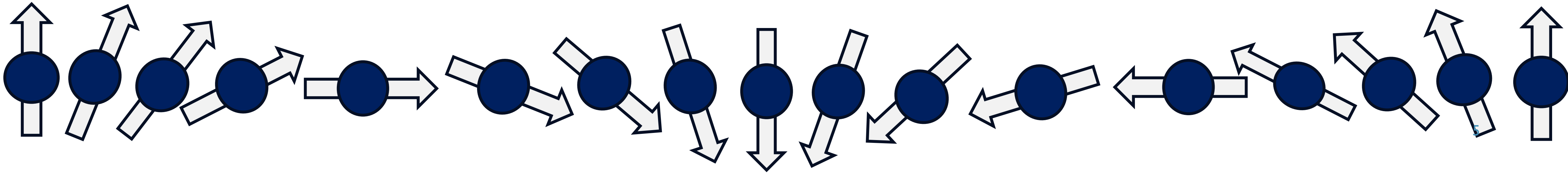
$$\left|\Psi_{spiral}\right\rangle = \prod_i |\downarrow\rangle_i + e^{\frac{i2\pi}{\lambda}x_i}|\uparrow\rangle_i$$

$|\downarrow\rangle + |\uparrow\rangle$ $\quad |\downarrow\rangle + e^{i2\pi\frac{1}{8}}|\uparrow\rangle$ $\quad |\downarrow\rangle + e^{i2\pi\frac{2}{8}}|\uparrow\rangle$ $\quad \cdots \quad \cdots \quad \cdots$ $\quad |\downarrow\rangle + e^{i2\pi\frac{7}{8}}|\uparrow\rangle$ $\quad |\downarrow\rangle + e^{i2\pi\frac{8}{8}}|\uparrow\rangle$

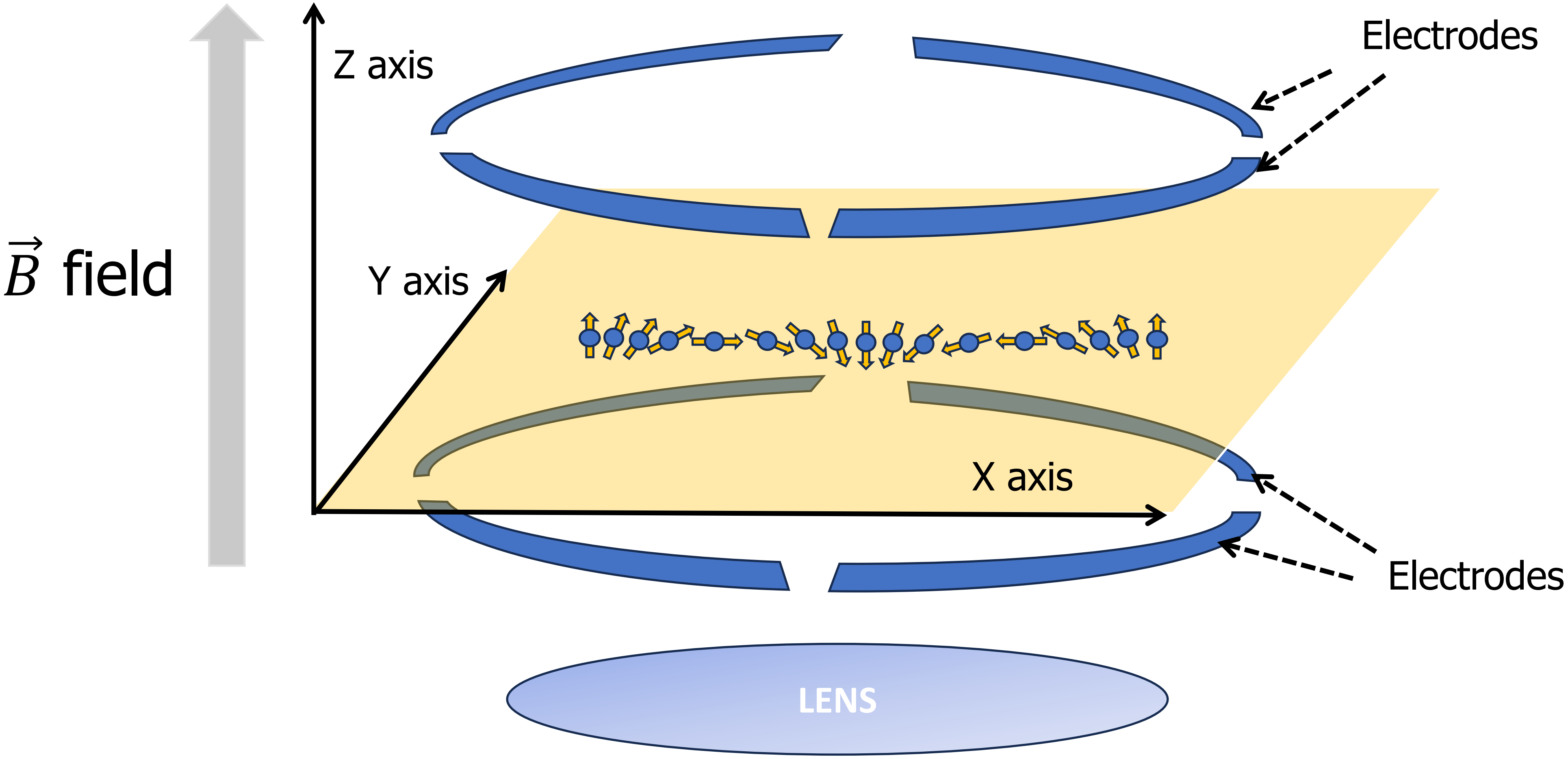


2. WHY SPIN SPIRALS ?

- Introduces a new length scale (λ) to the system
- Can study spin-spin correlation
- Understand spin dynamics better
- How far-from equilibrium systems relax



“SEEING” SPIRALS USING FLUORESCENCE IMAGING



3. HOW TO CREATE SPIN SPIRALS?

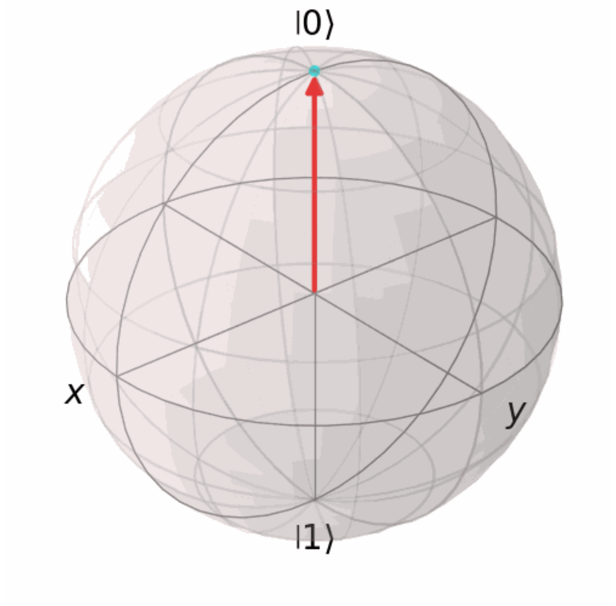
- The Protocol
- Choosing the two energy levels
- Setting the electrode voltages

3. HOW TO CREATE SPIN SPIRALS

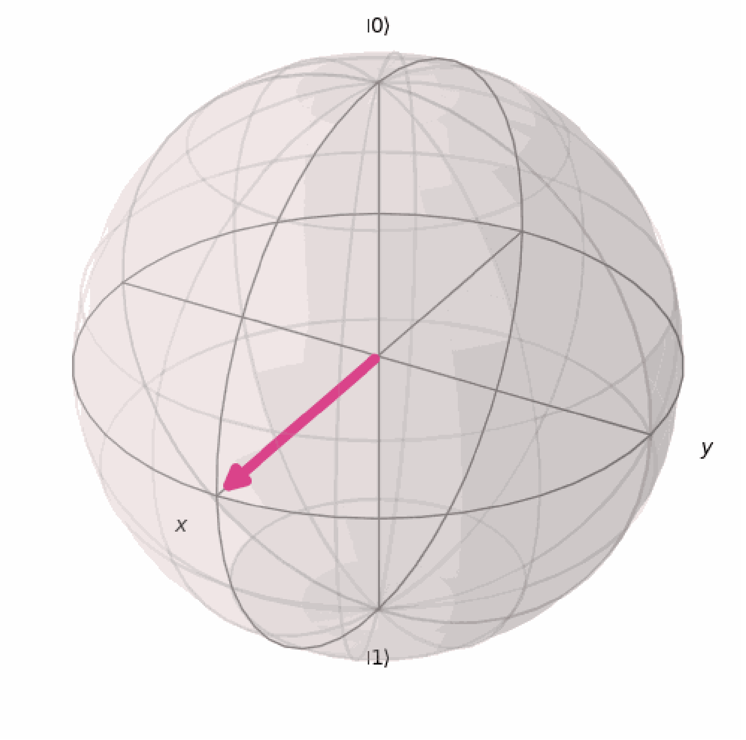
A. THE PROTOCOL

3A. THE PROTOCOL

Consider a single qubit



under $H = |{\downarrow}\rangle\langle{\downarrow}| + \hbar\omega\, |{\uparrow}\rangle\langle{\uparrow}|$

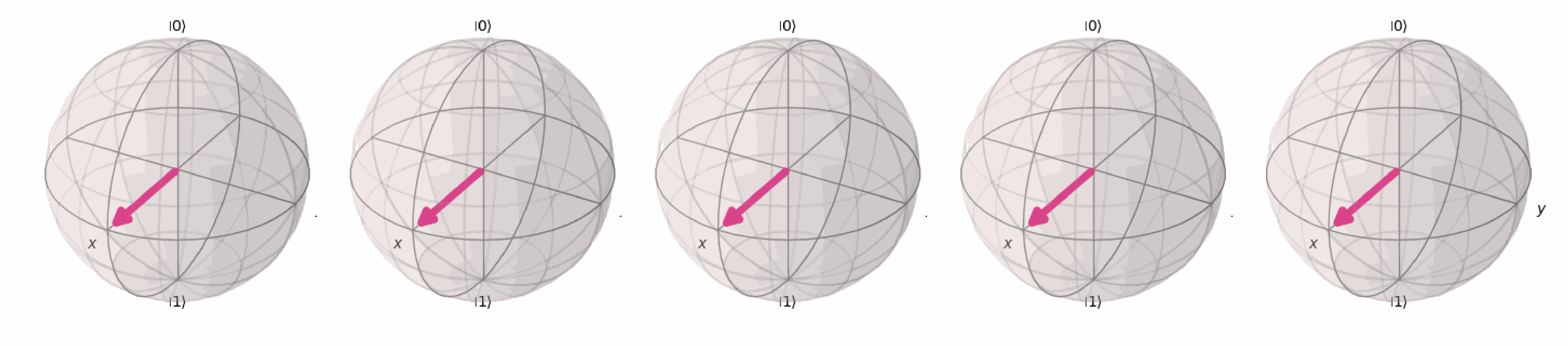


A single qubit initialized in $|{\uparrow}\rangle + e^{i\theta}|{\downarrow}\rangle$

Evolves as $|{\uparrow}\rangle + e^{i(\theta+\omega t)}|{\downarrow}\rangle$

3A. THE PROTOCOL

For multiple qubits, create position dependent differences in ω

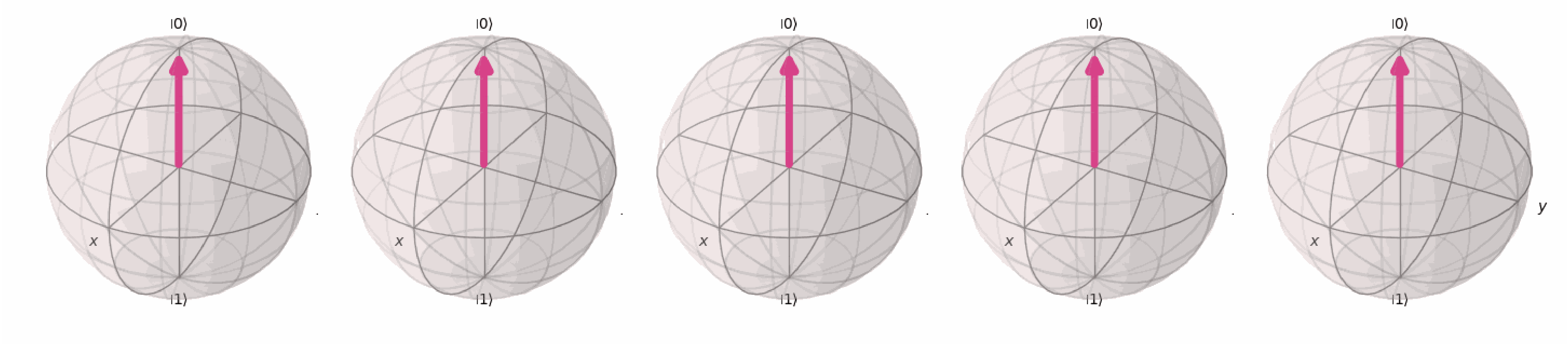


Energy gaps: $\boldsymbol{\omega_o}$ $\omega_0 + \delta\omega$ $\omega_0 + 2\delta\omega$ $\omega_0 + 3\delta\omega$ $\omega_0 + 4\delta\omega$

X axis

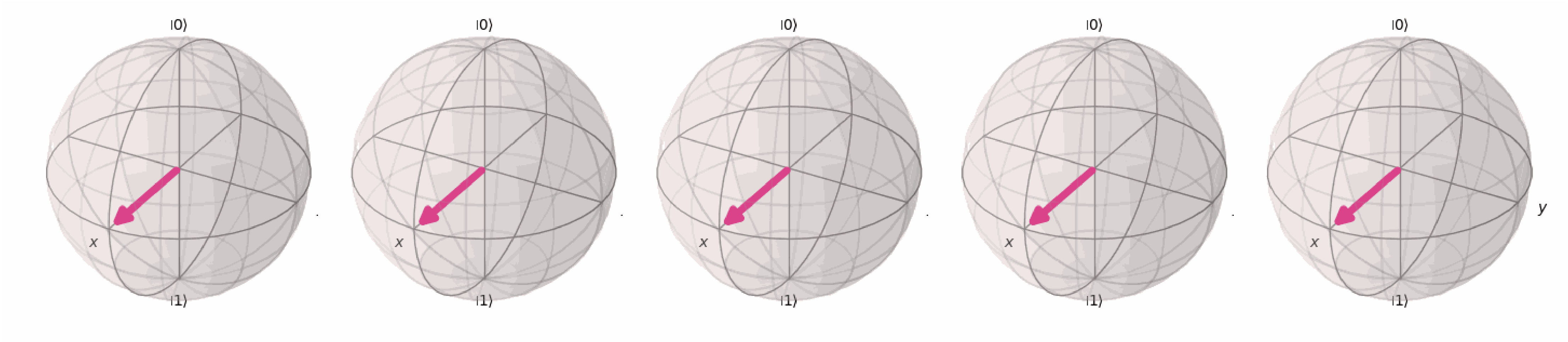
SO THE PROTOCOL TO CREATE SPIRAL STATES WOULD BE…

1. Initialize all spins in x-y plane



SO THE PROTOCOL TO CREATE SPIRAL STATES WOULD BE…

2. Create the position dependent frequency difference

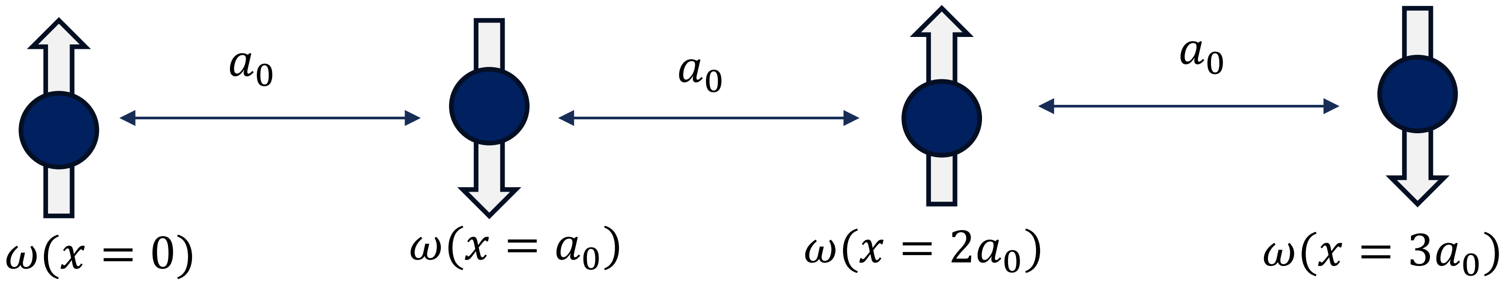


3. Remove the position dependent frequency difference, when desired state is achieved (after τ_{spiral})

SPIRALIZATION TIME

Time to reach desired state (τ_{spin})
Maximum τ_{spin} needed:

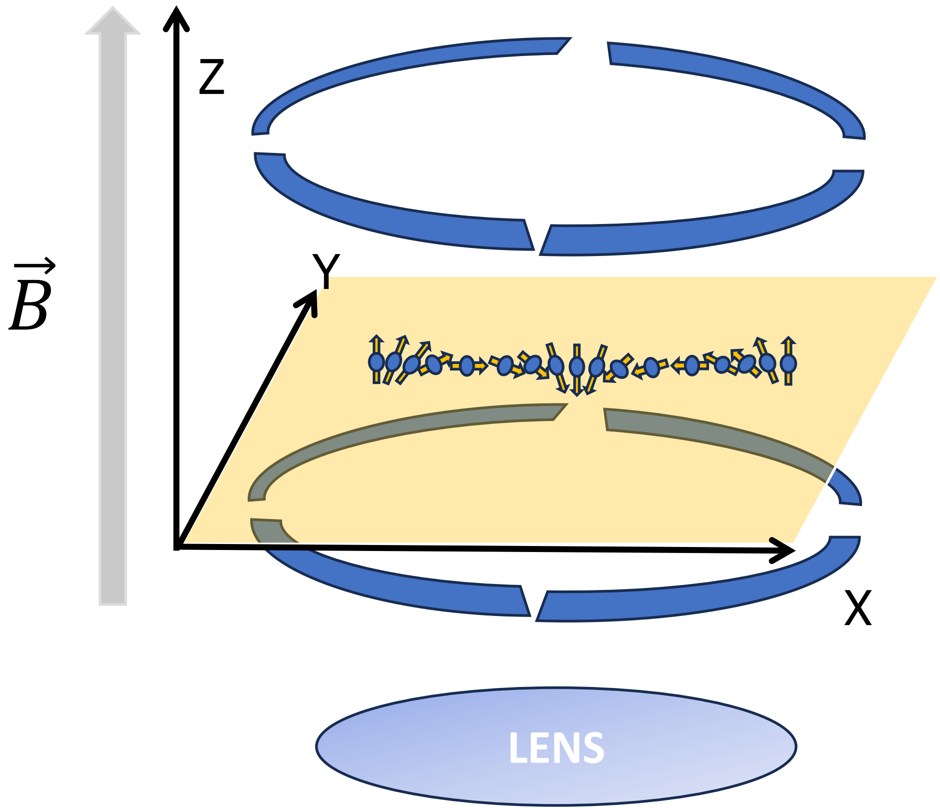
For Neel state ($\lambda = 2a_0$):



$$\Delta\phi = \pi$$

$$\boxed{\tau_{spiral} = \frac{\pi}{\frac{\partial \omega}{\partial x} a_0}}$$

CREATING A FREQUENCY VARIATION WITH POSITION $\left(\frac{\delta\omega}{\delta x}\right)$



Rydberg Hamiltonians in external fields

$$\boxed{H(\vec{E},\vec{B}) = H_o - \hat{d}\cdot\vec{E} \; - \hat{\mu}\cdot\vec{B} + \frac{1}{8m_e}\left|\hat{d}\times\vec{B}\;\right|^2}$$

Electric dipole operator

Magnetic dipole operator

Better experimental control on $\vec{E}$ than on $\vec{B}$

CREATING A FREQUENCY VARIATION WITH POSITION $\left(\frac{\delta\omega}{\delta x}\right)$

For a spiral state:

$$\phi(x) \propto x$$

$$\left|\Psi_{spiral}\right\rangle = \prod_i |\downarrow\rangle_i + e^{\frac{i2\pi}{\lambda}x_i}|\uparrow\rangle_i$$

$$\phi(x) = \omega(x)\tau_{spiral}$$

$$\omega(x) = \frac{\partial\omega(x)}{\partial x}x$$

$$\boxed{\Delta\omega = \frac{\partial\omega(x)}{\partial x}\Delta x = \frac{\partial\omega(\vec{E})}{\partial\left|\vec{E}_{plane}\right|}\frac{\partial\left|\vec{E}_{plane}\right|}{\partial x}\Delta x}$$

ACHIEVING LINEAR VARIATION OF ω

$$\Delta\omega = \frac{\partial\omega(\vec{E})}{\partial|\vec{E}_{plane}|}\frac{\partial|\vec{E}_{plane}|}{\partial x}\Delta x$$

Choosing two energy levels:

- $\frac{\partial\omega}{\partial|\vec{E}_{plane}|} \sim$ constant

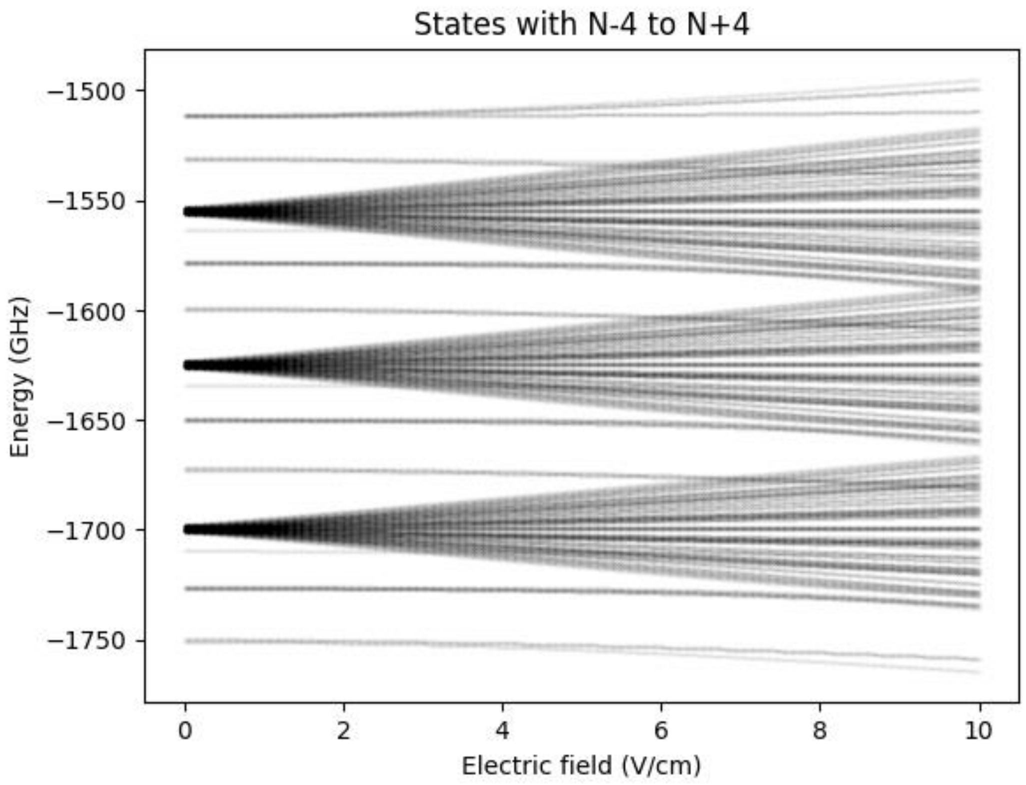
Adjusting electrodes voltages:

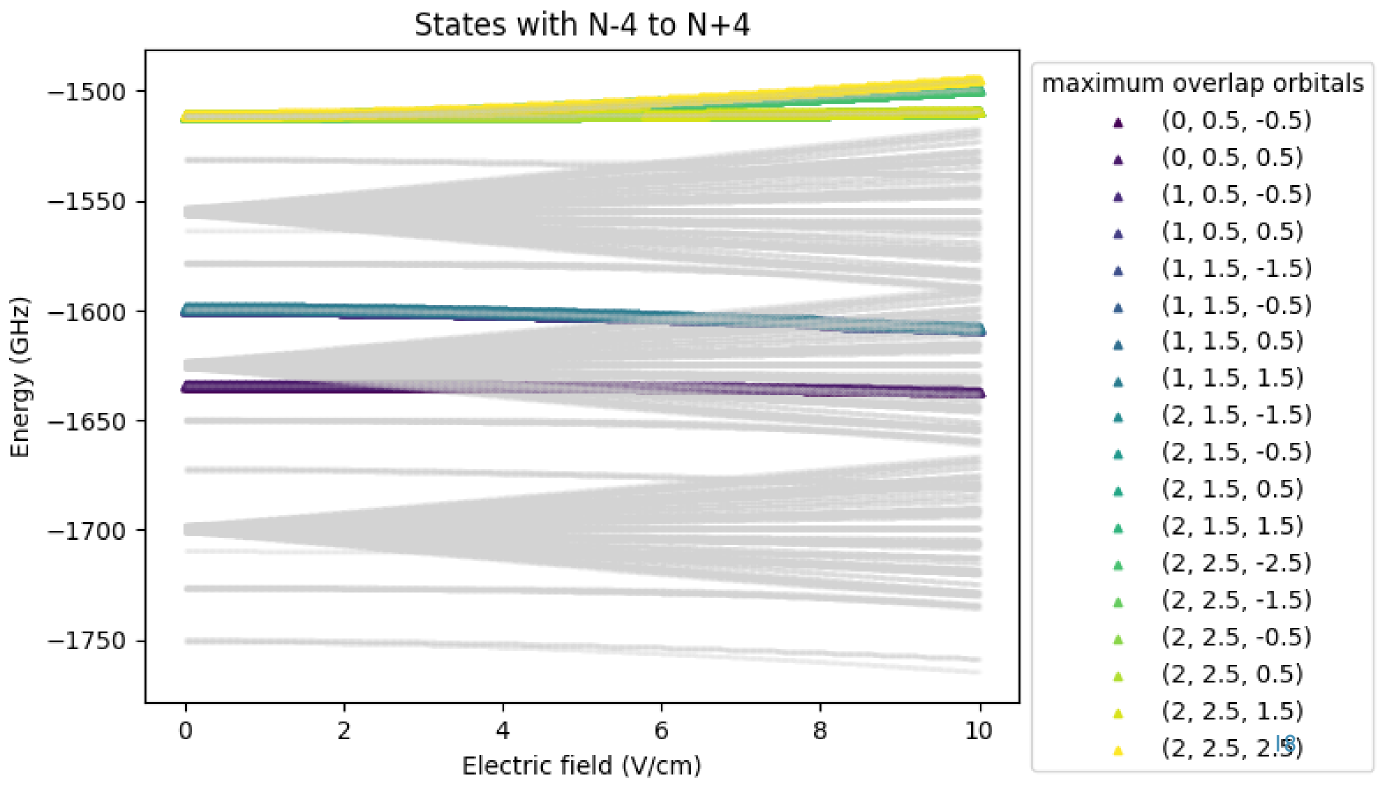
- $\frac{\partial|\vec{E}_{plane}|}{\partial x} \sim$ constant

3.B CHOOSING THE TWO LEVELS

RYDBERG ATOMS IN ELECTRIC FIELD $\left(\frac{\delta\omega}{\delta|\vec{E}|}\right)$

At $\vec{B} = 160\ G\ \hat{z}$ variation of energy levels with $\vec{E}$





CRITERIA FOR CHOOSING ENERGY LEVELS

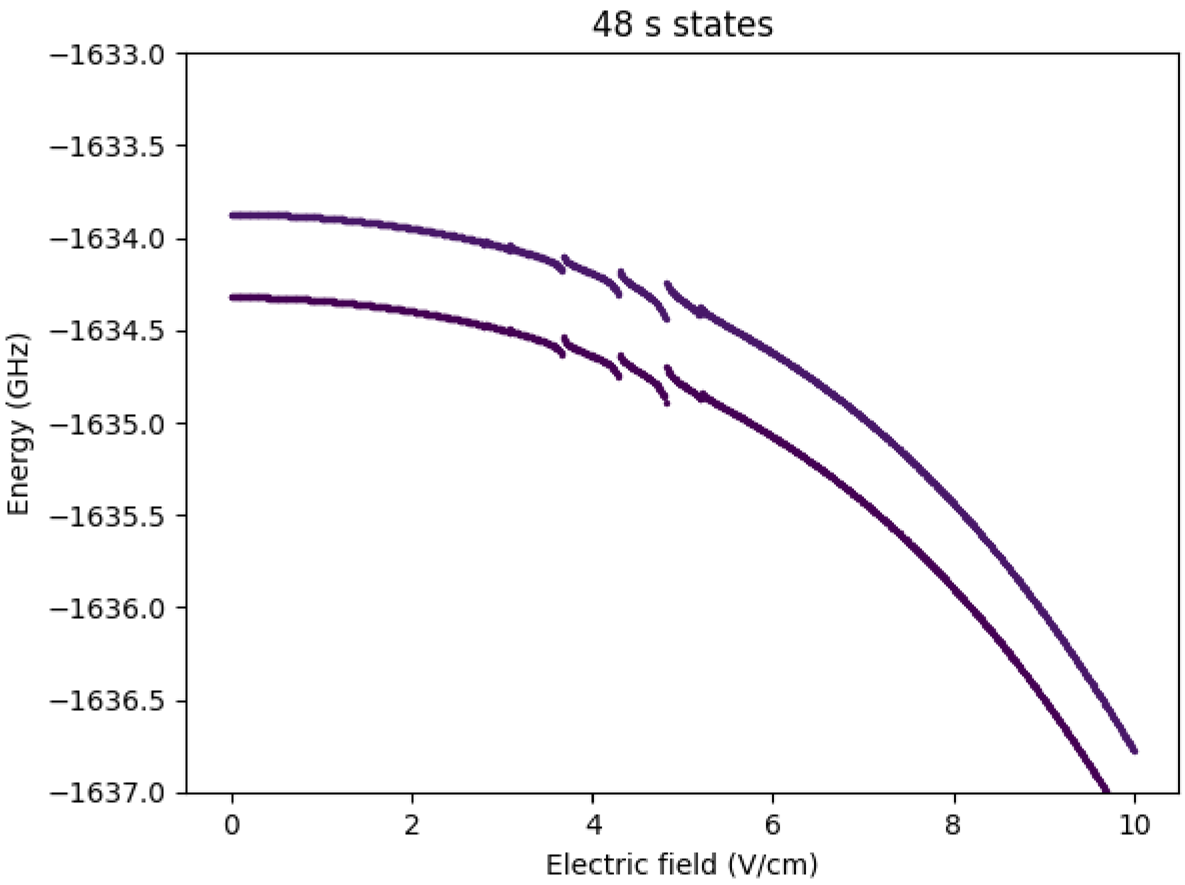
- $\frac{\partial \omega}{\partial |\vec{E}_{plane}|} \sim$ constant for considerable range
- $\tau_{spiral} \ll T_{interaction}$
- $\frac{\partial \omega}{\partial |\vec{E}_{plane}|}$ large enough to get small τ_{spiral}
- Large electric dipole moment

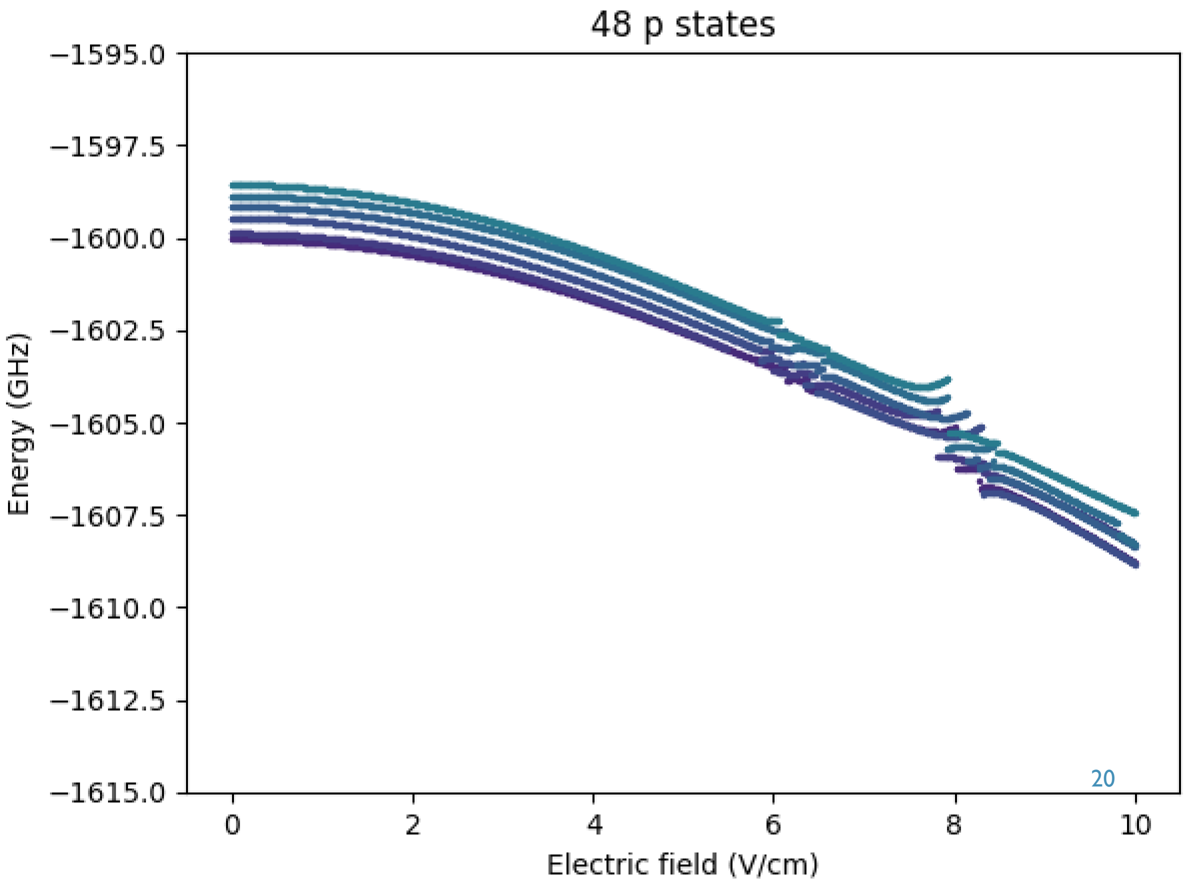
Reference dipole moment : 1.38 GHz (cm/V)

[for $|48\ s\ 0.5\ 0.5\rangle \rightarrow |48\ p\ 1.5\ 0.5\rangle$]

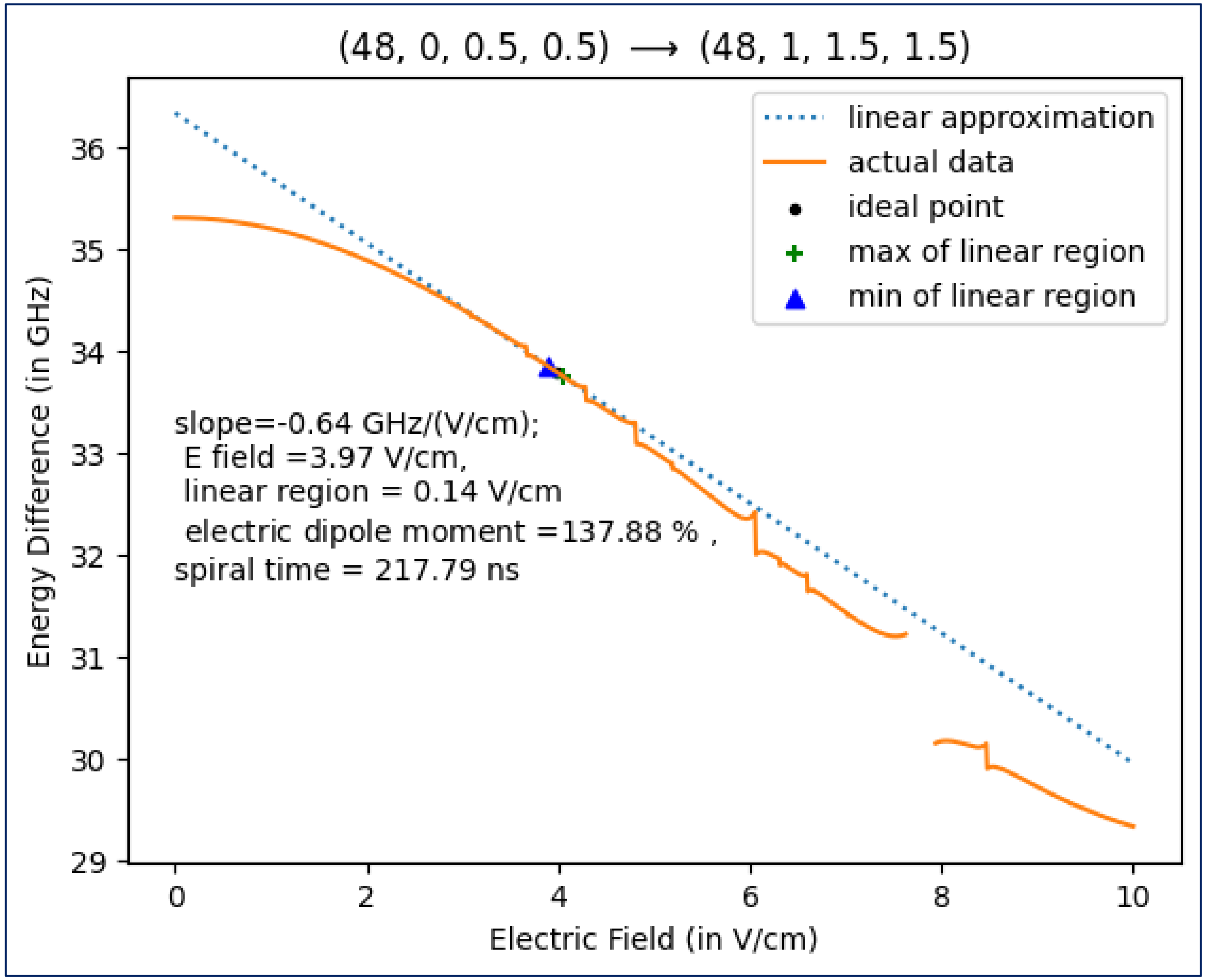
A CLOSER LOOK AT STARK MAPS

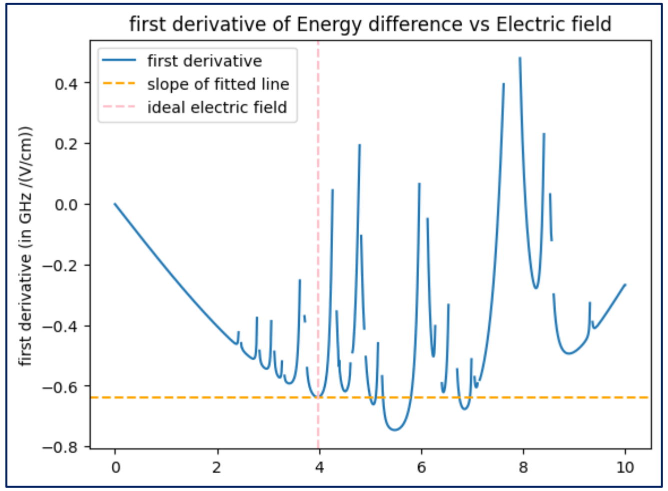
At $\vec{B} = 160\ G\ \hat{z}$ variation of energy levels with $\vec{E}$

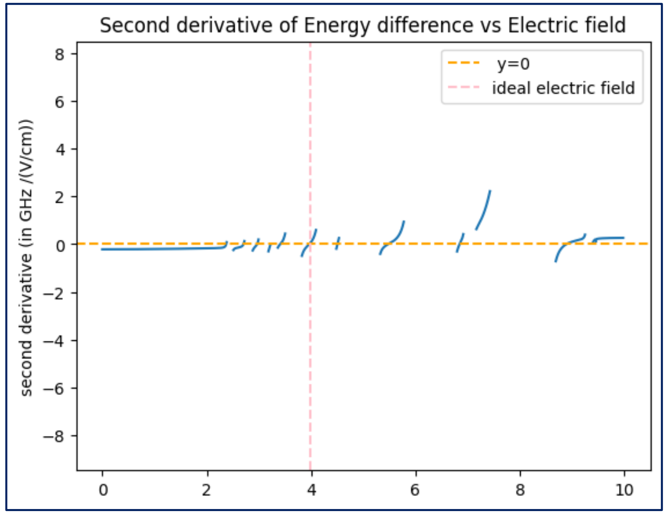




CHOOSING THE TWO LEVEL SYSTEM







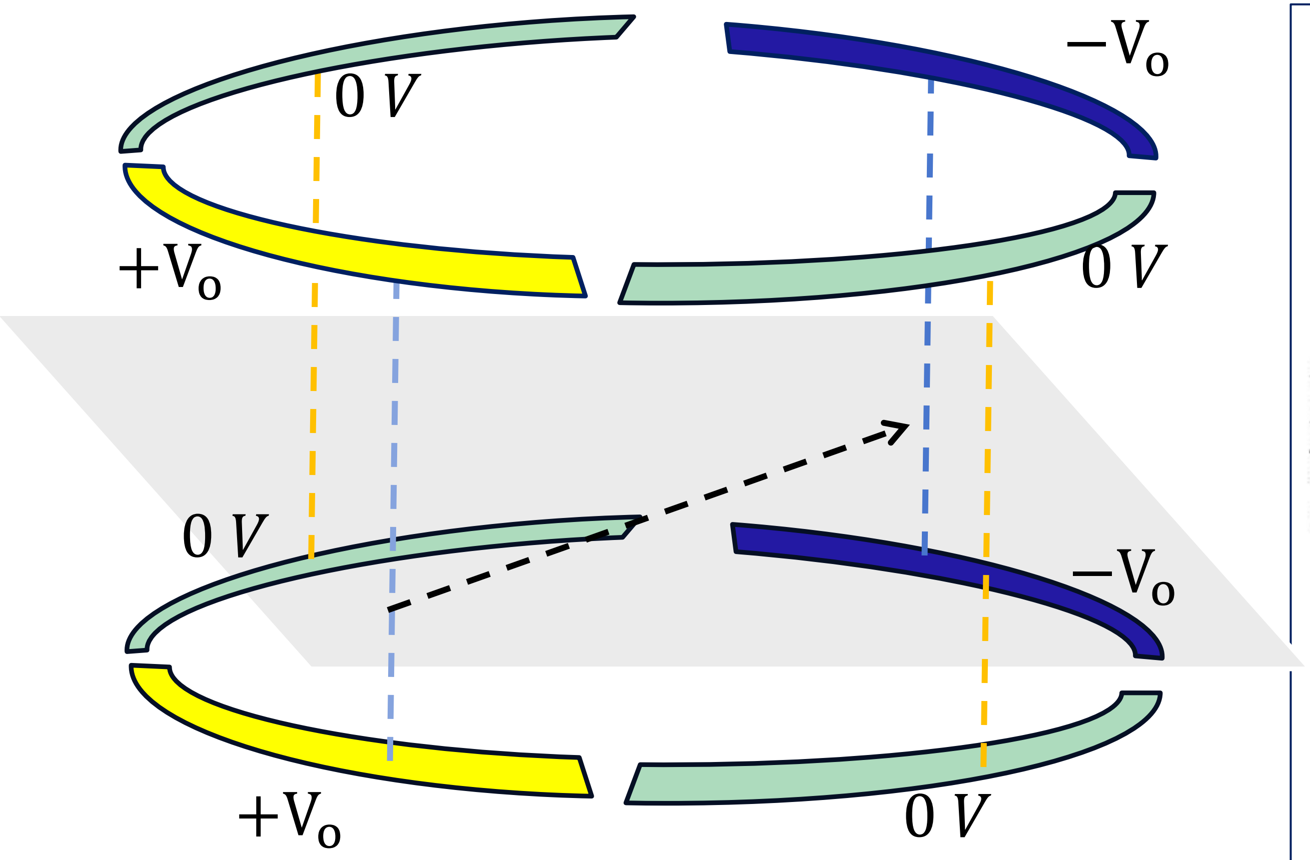
3C. SETTING ELECTRODE VOLTAGES

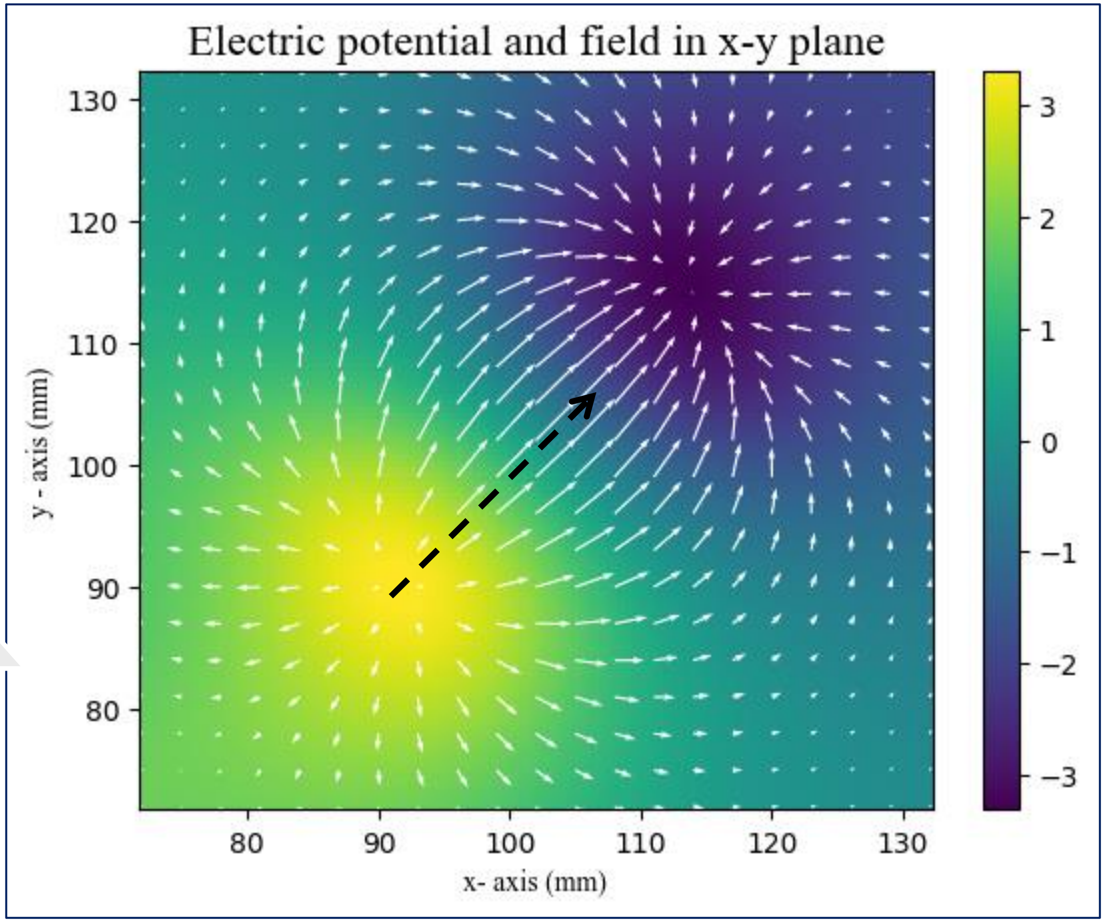
A. Generating offset electric field
B. Generating $\frac{\partial\left|\vec{E}_{plane}\right|}{\partial r}$
C. Combining these two

DESIRED ELECTRIC FIELDS

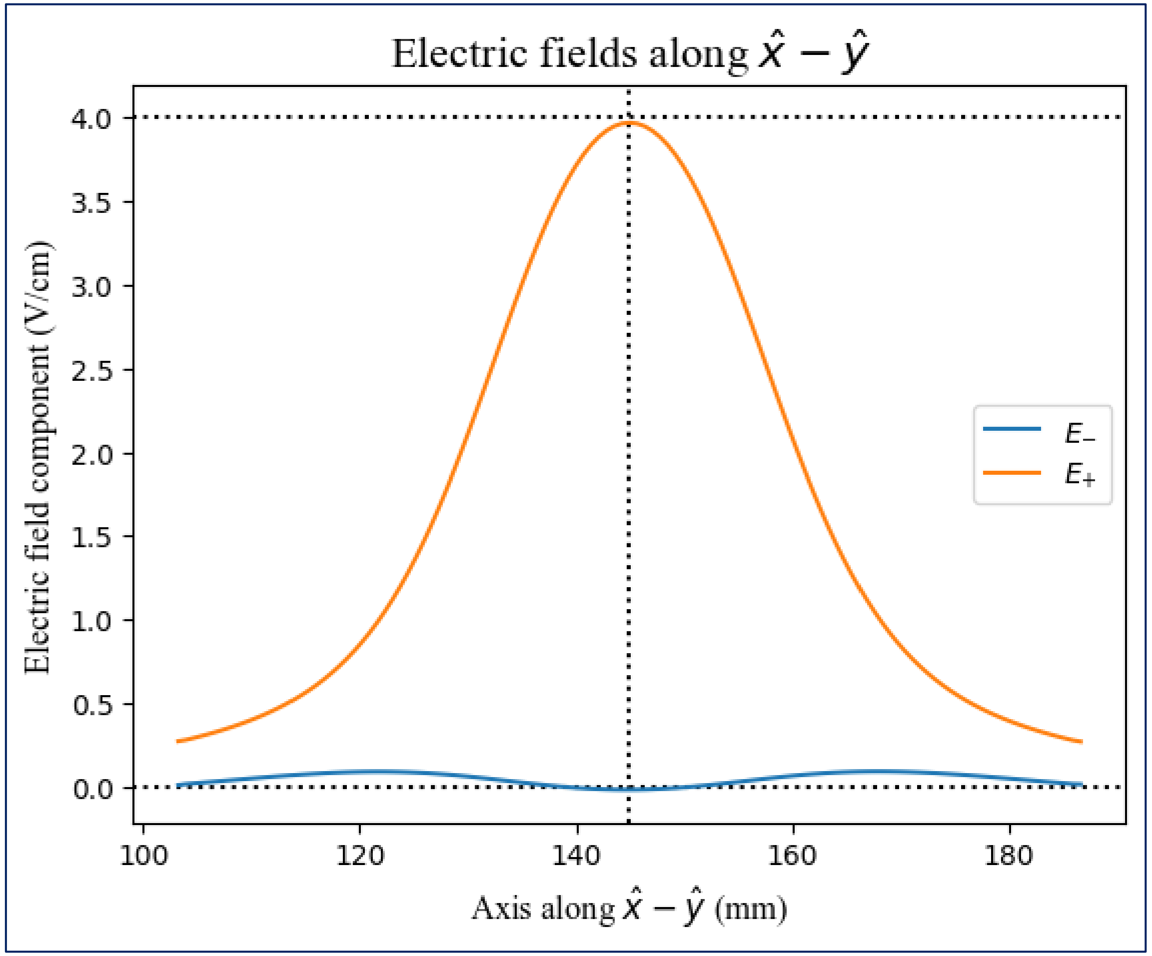
- $\frac{\partial \left| \vec{E}_{plane} \right|}{\partial r} = constant$
- $\frac{\partial \left| \vec{E}_{plane} \right|}{\partial r_{\perp}} \sim 0$
- $\frac{\partial \left| \vec{E}_{plane} \right|}{\partial z} \sim 0$
- Electrode potentials < 40V (to be able to ramp faster)

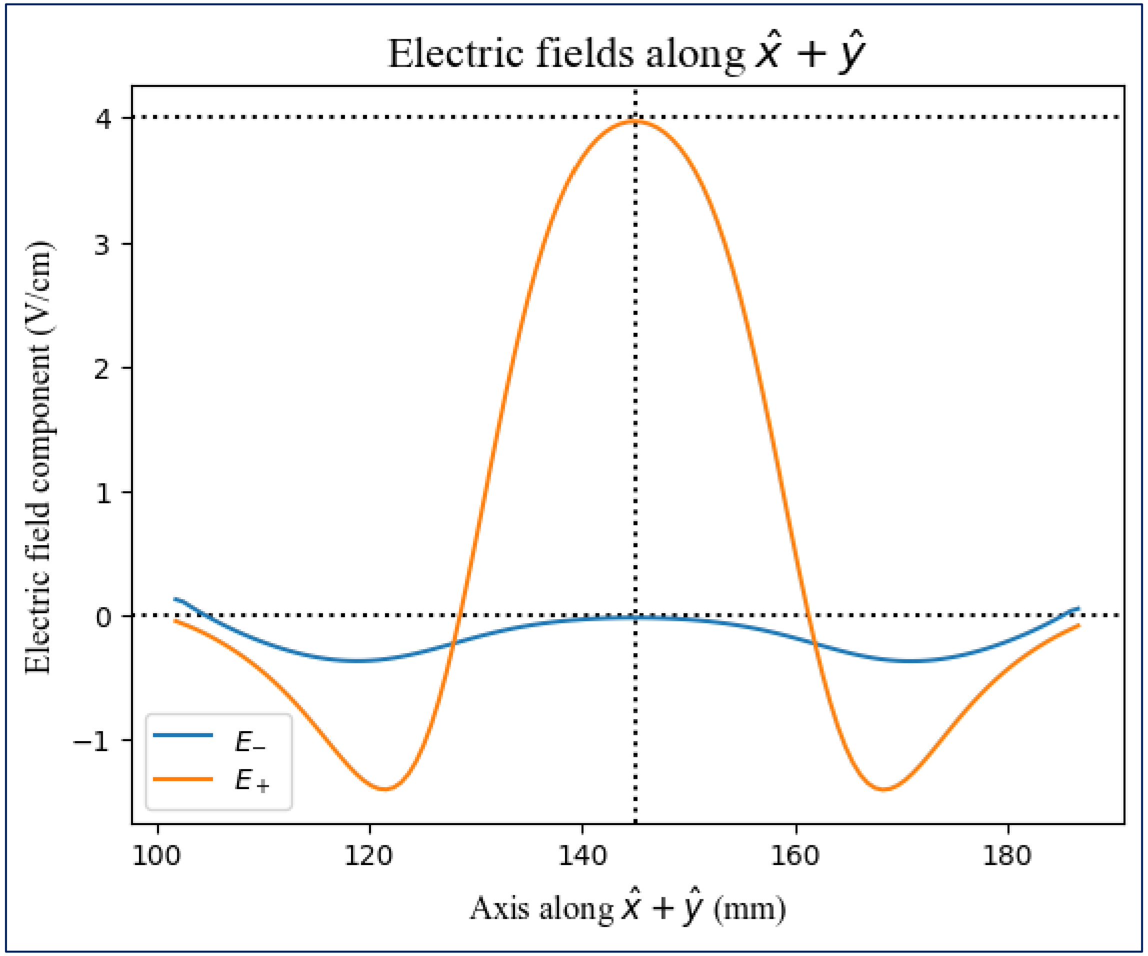
A. GENERATING CONSTANT $|\vec{E}_{plane}|$



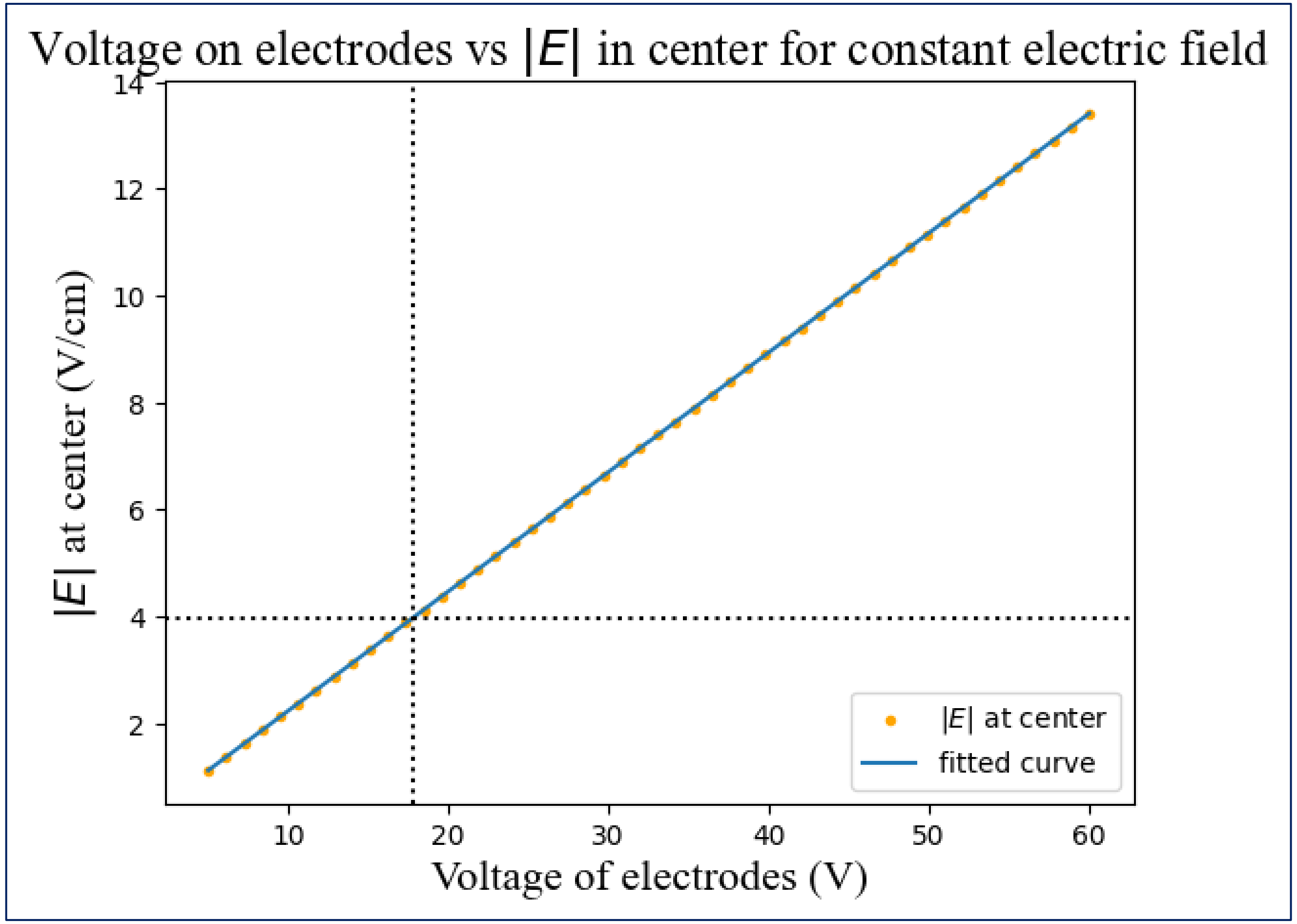


THE OFFSET FIELDS GENERATED

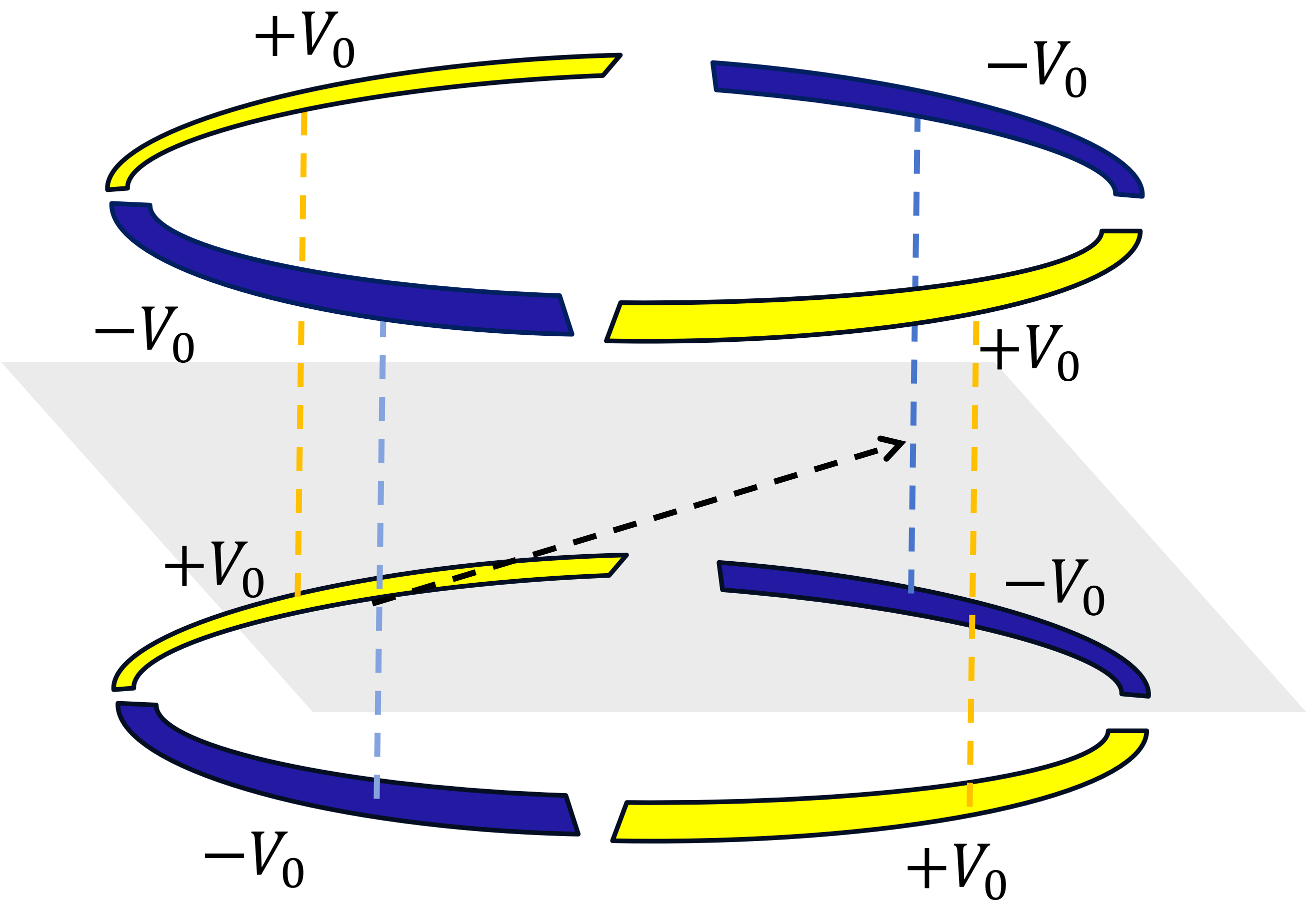


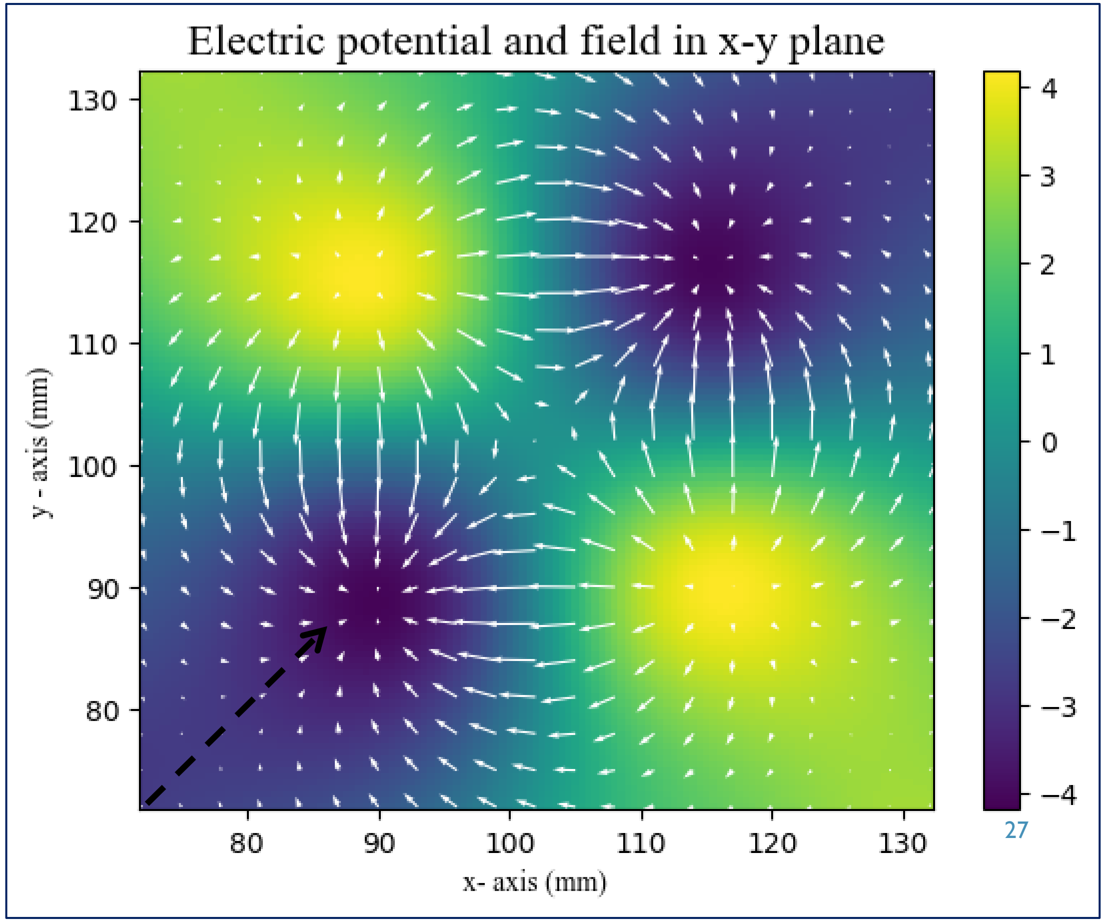


FINALIZING V_o

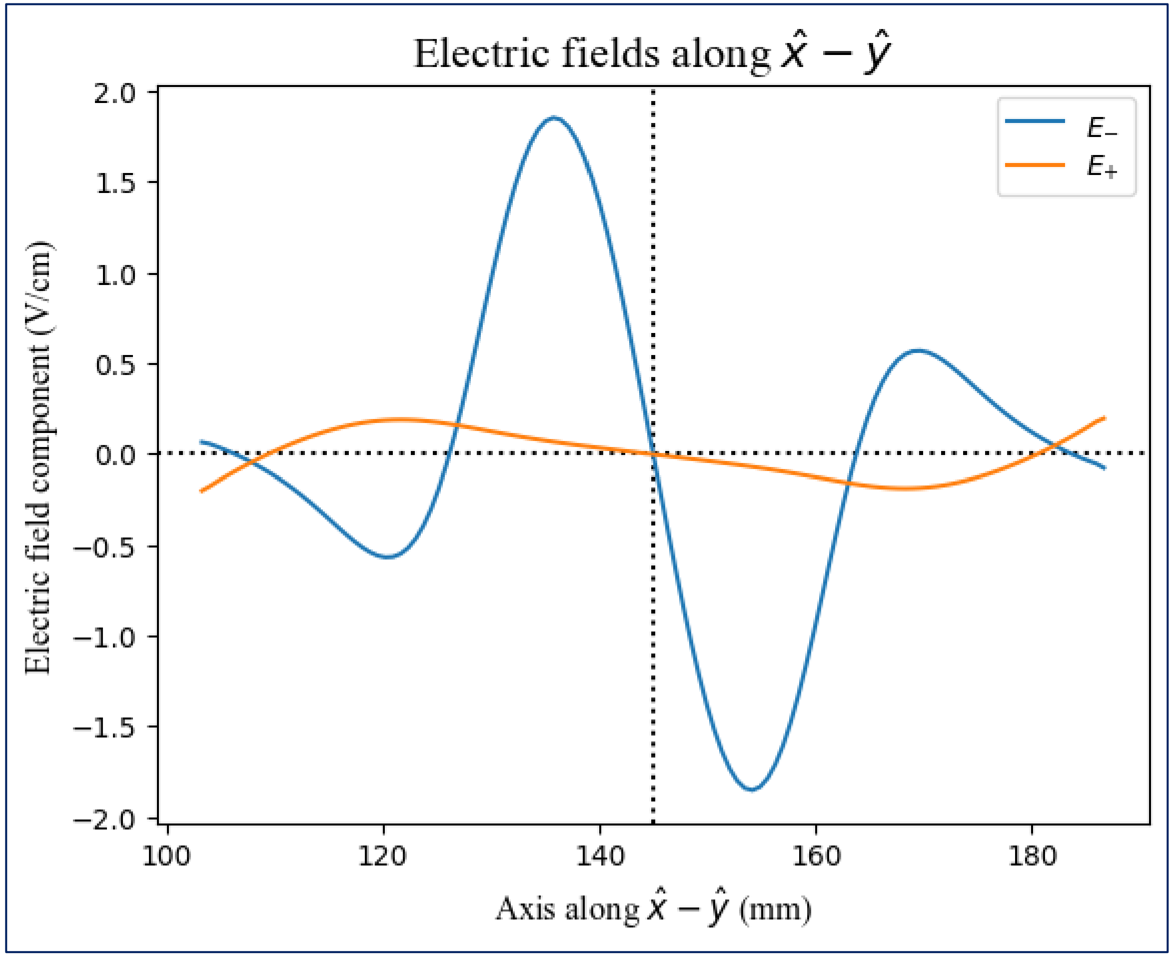


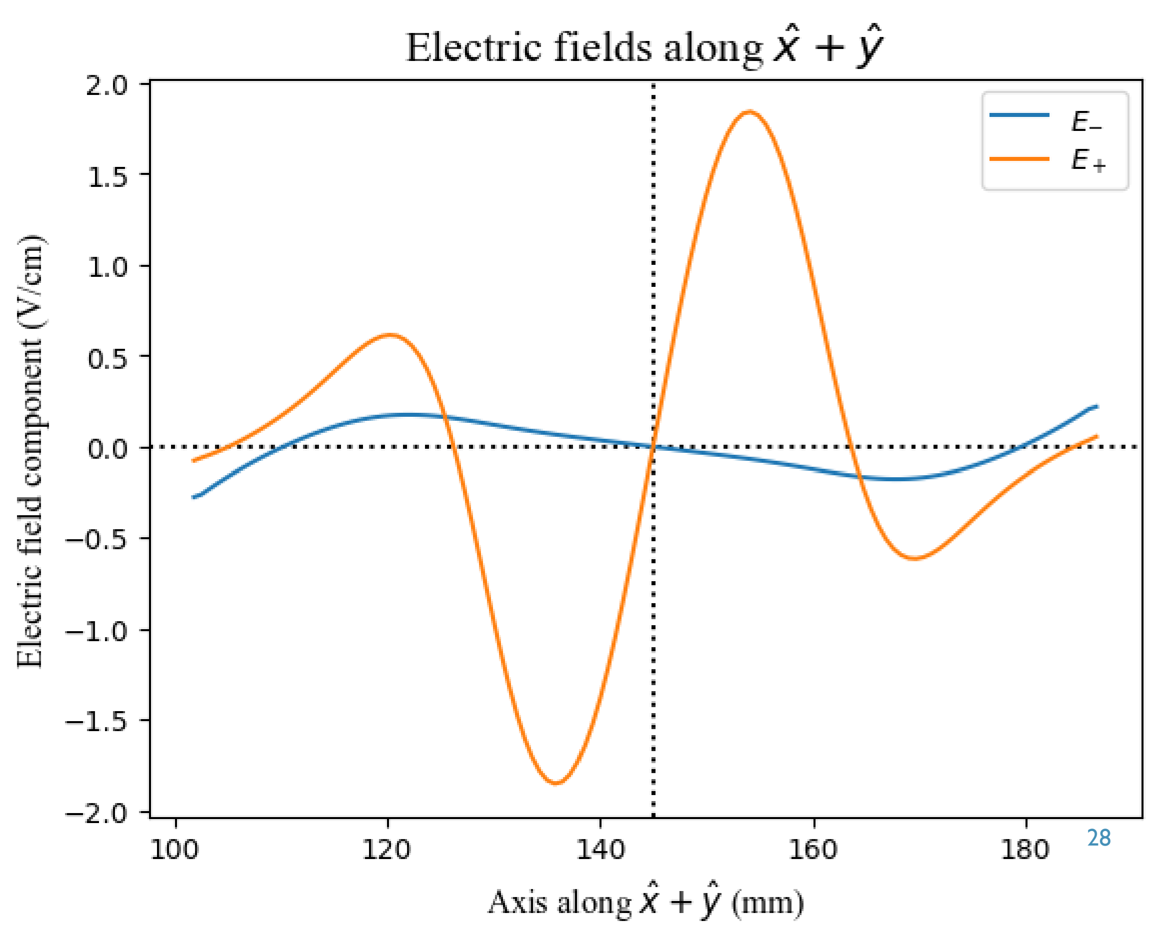
B. GENERATING $\frac{\partial\left|\vec{E}_{plane}\right|}{\partial r}$



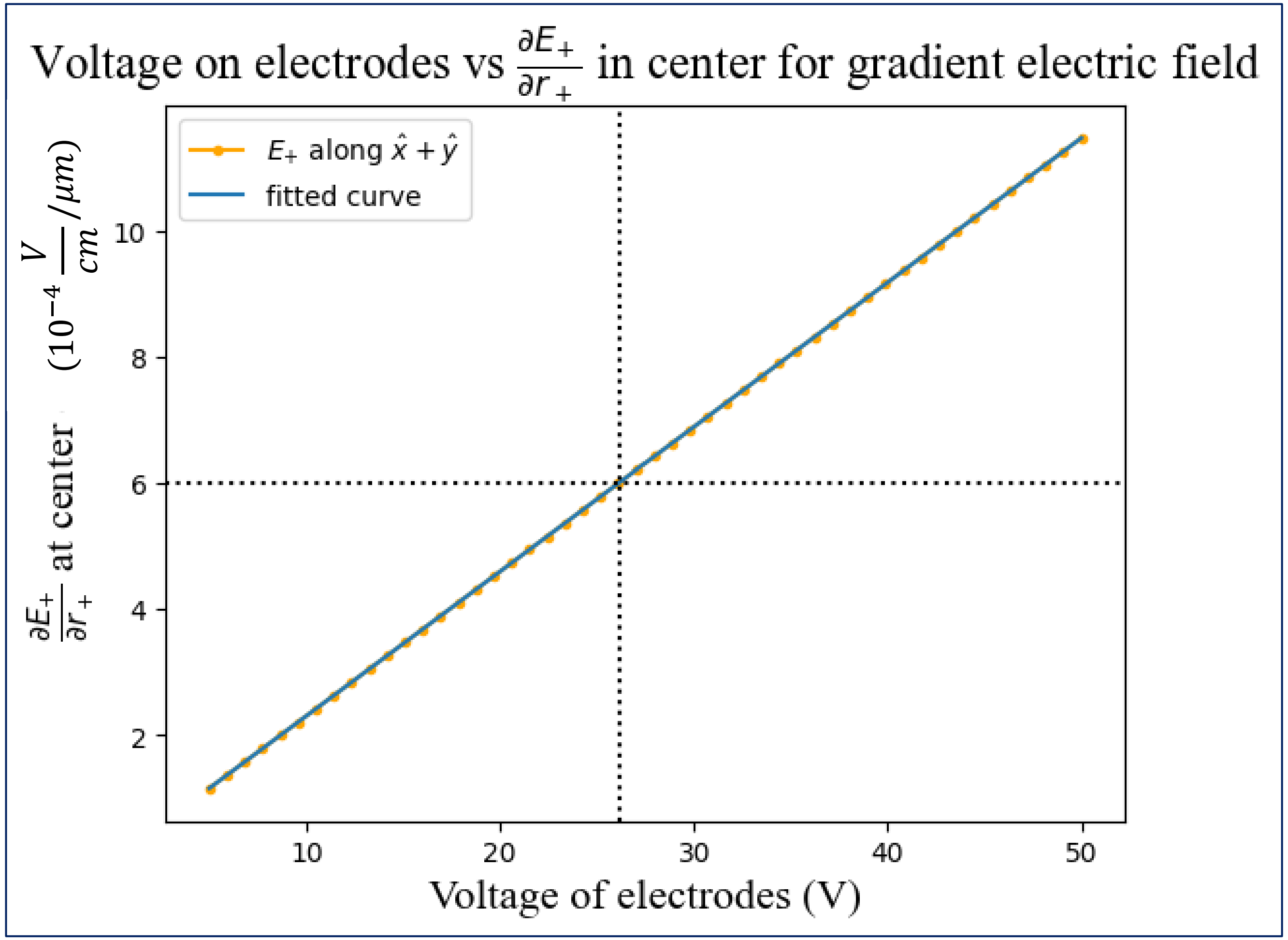


THE GRADIENT FIELDS GENERATED

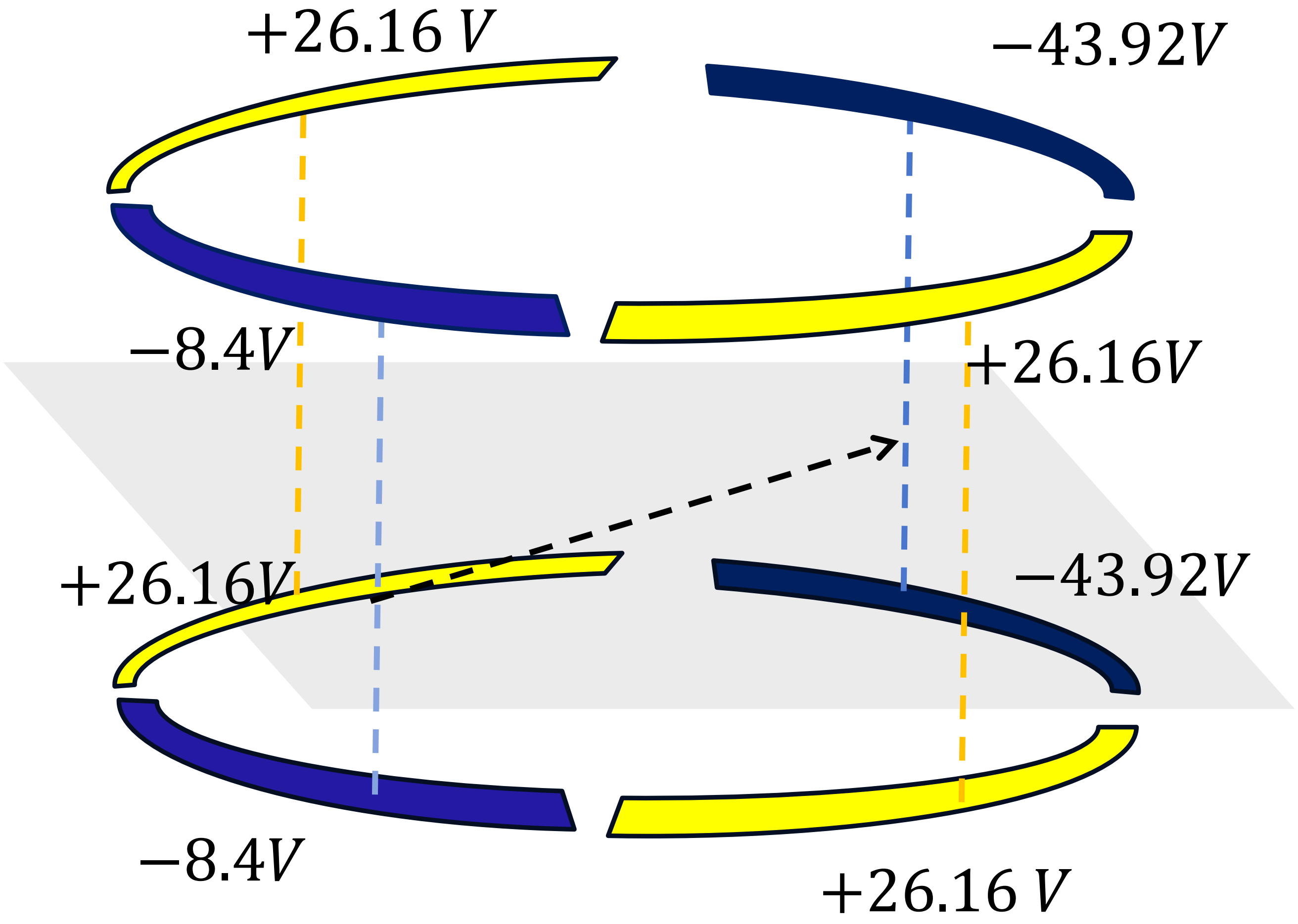


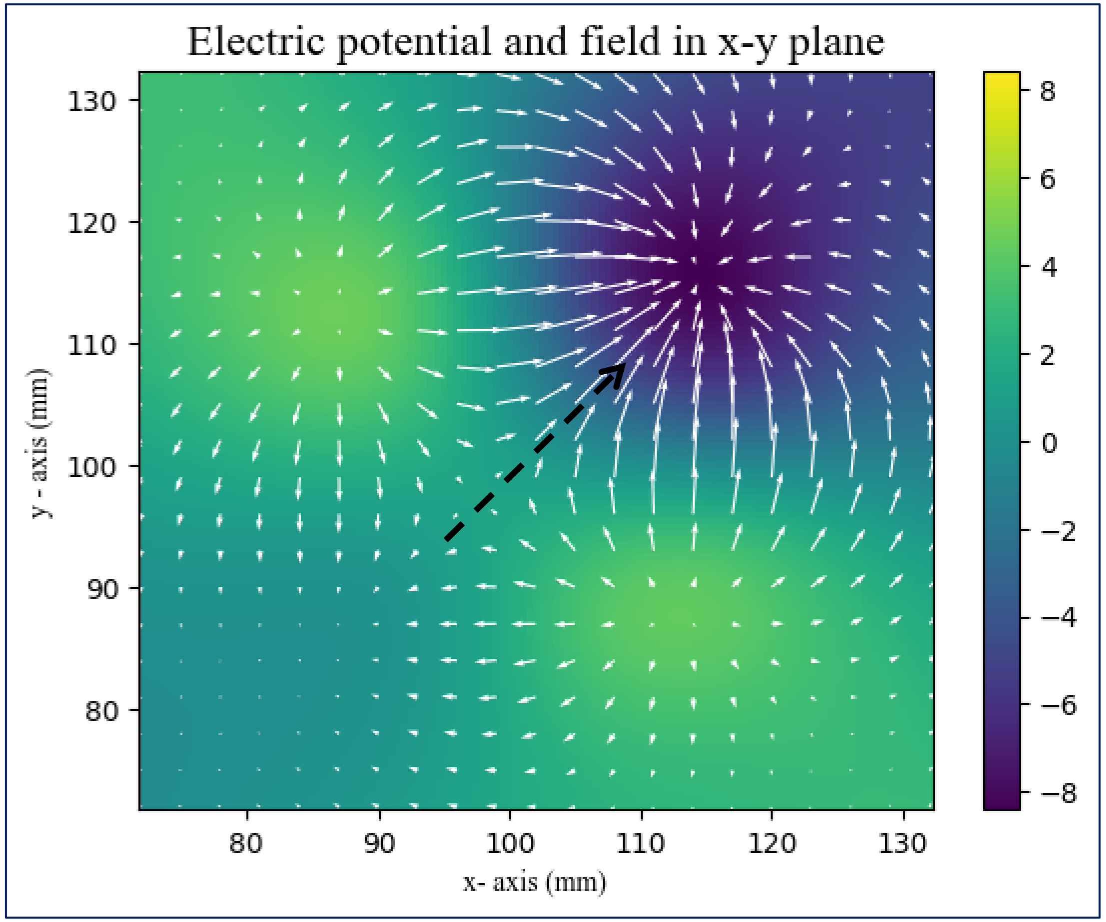


FINALIZING V_o

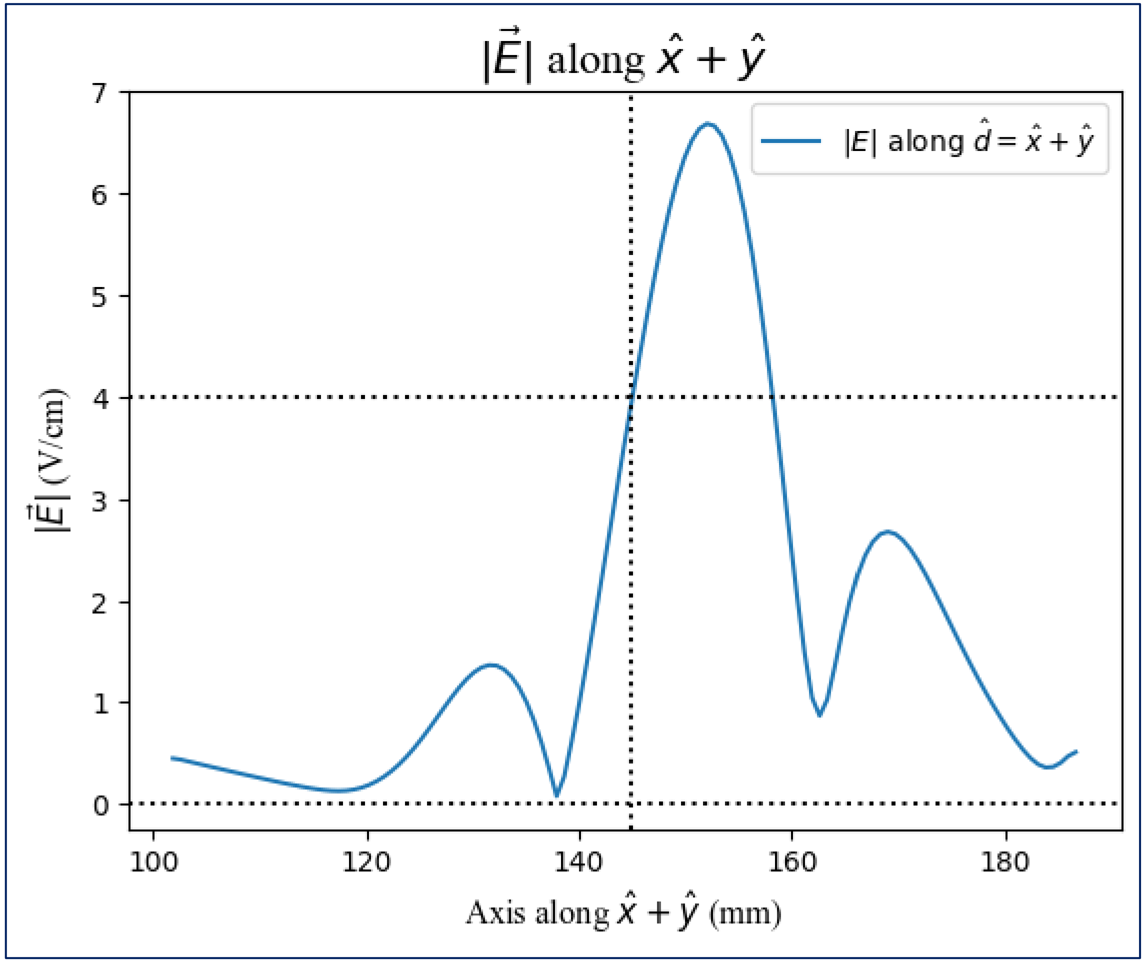


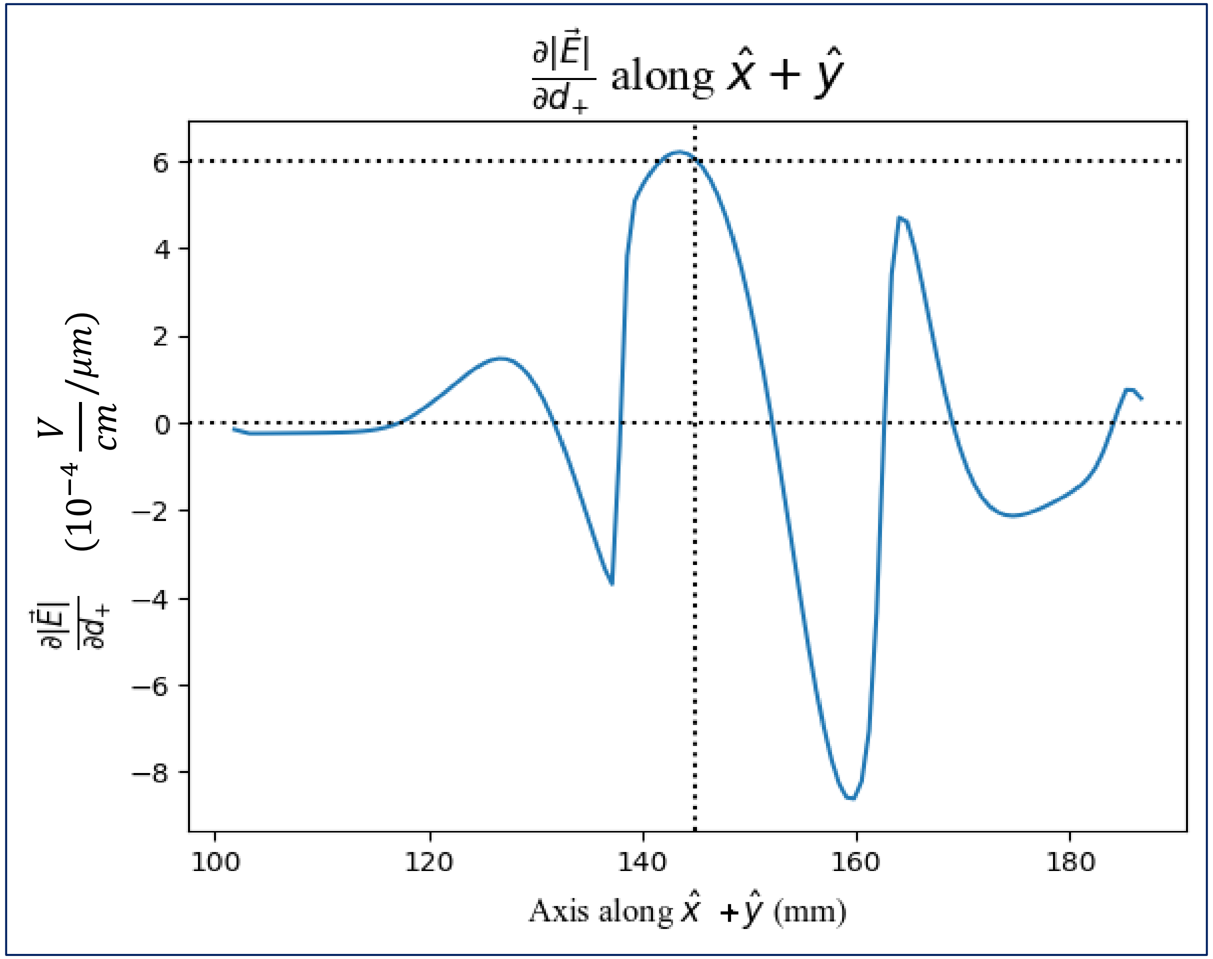
C. GENERATING OFFSET $|\vec{E}_{plane}|$ & $\frac{\partial|\vec{E}_{plane}|}{\partial r}$



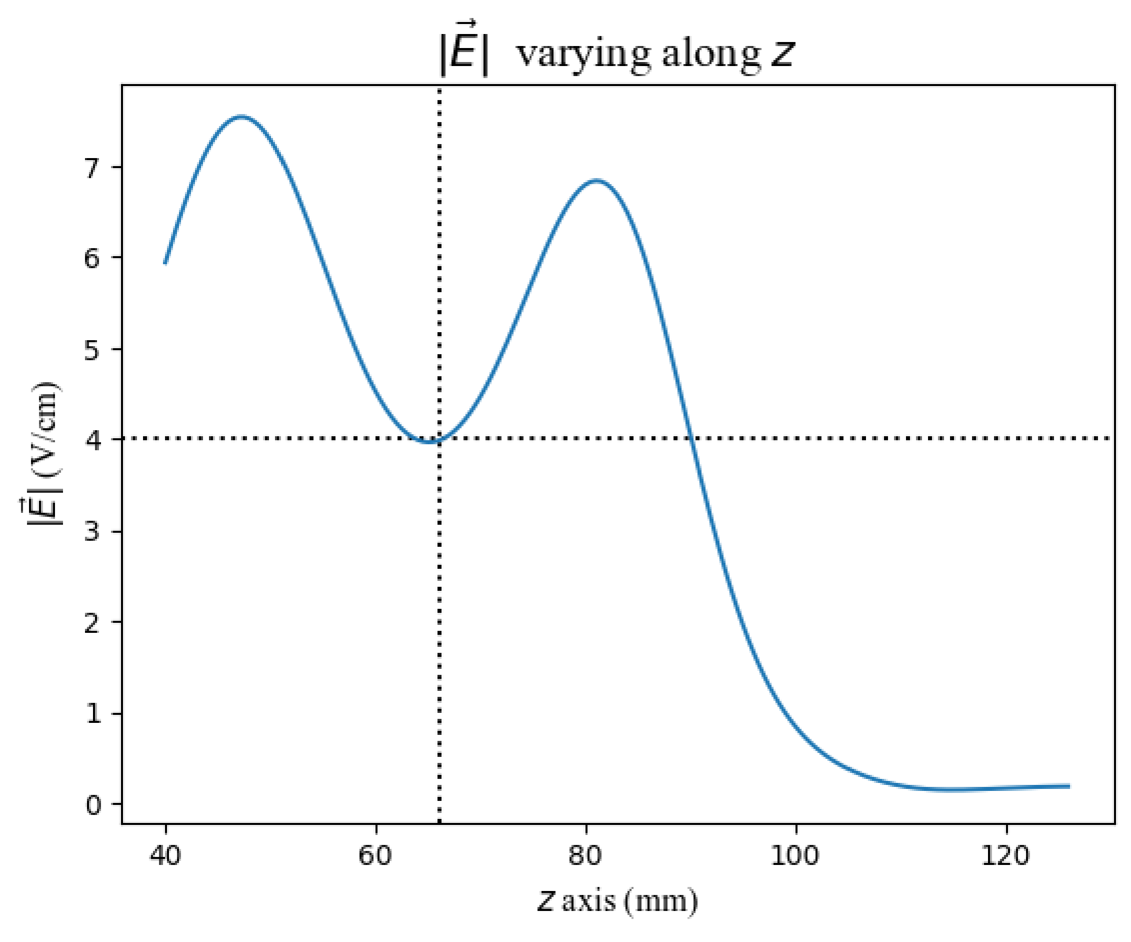


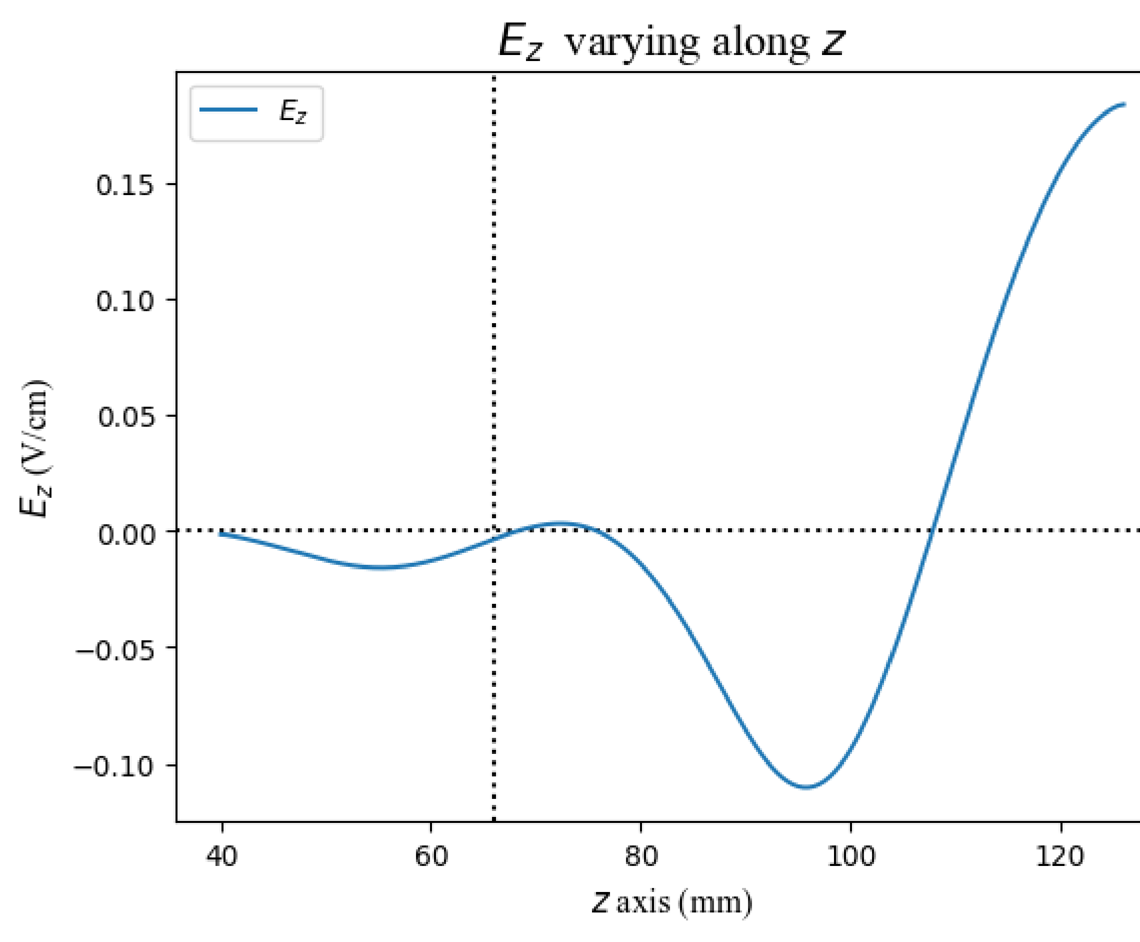
COMBINING THE OFFSET AND GRADIENT



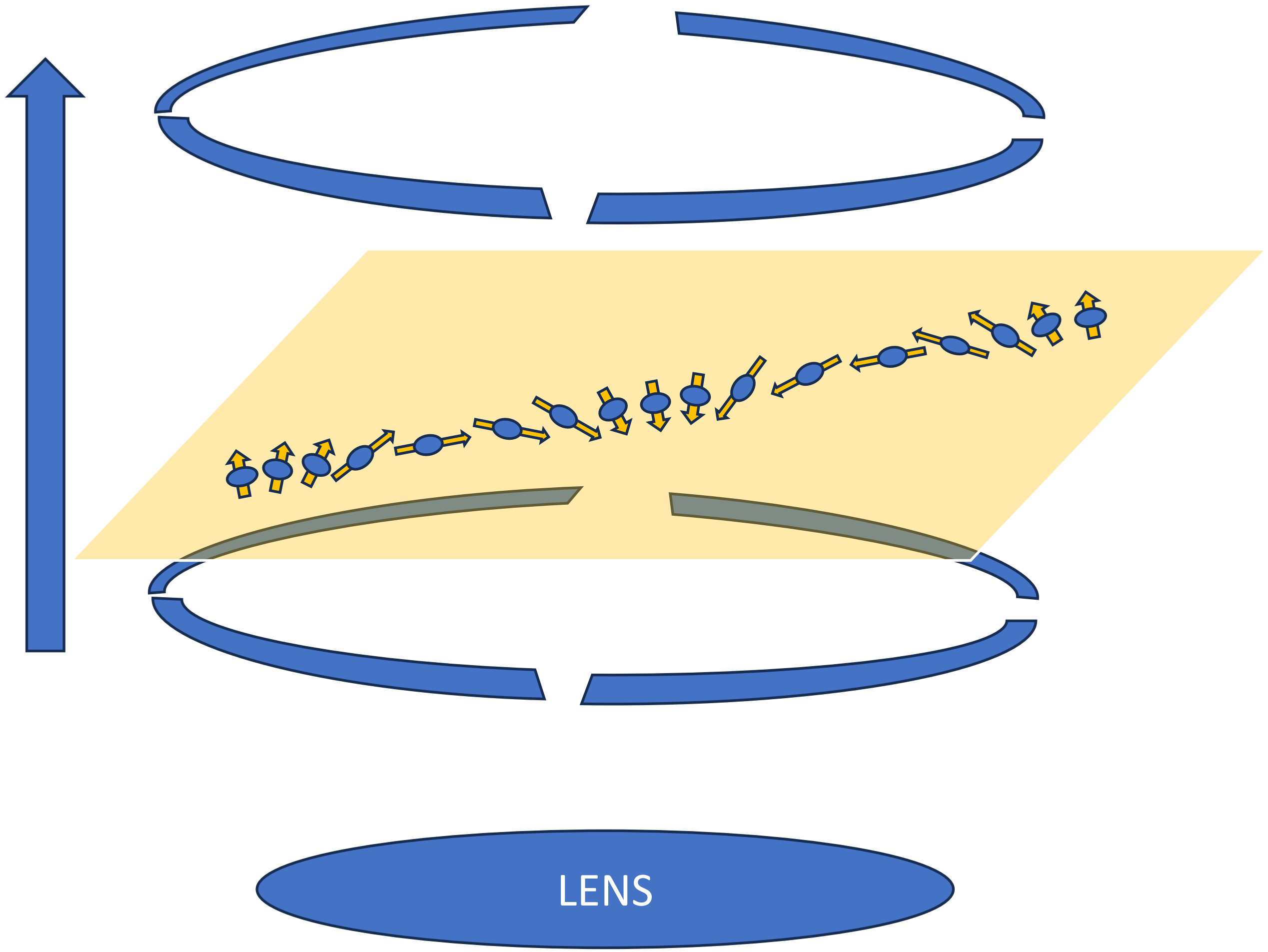


CHARACTERIZING ALONG Z AXIS





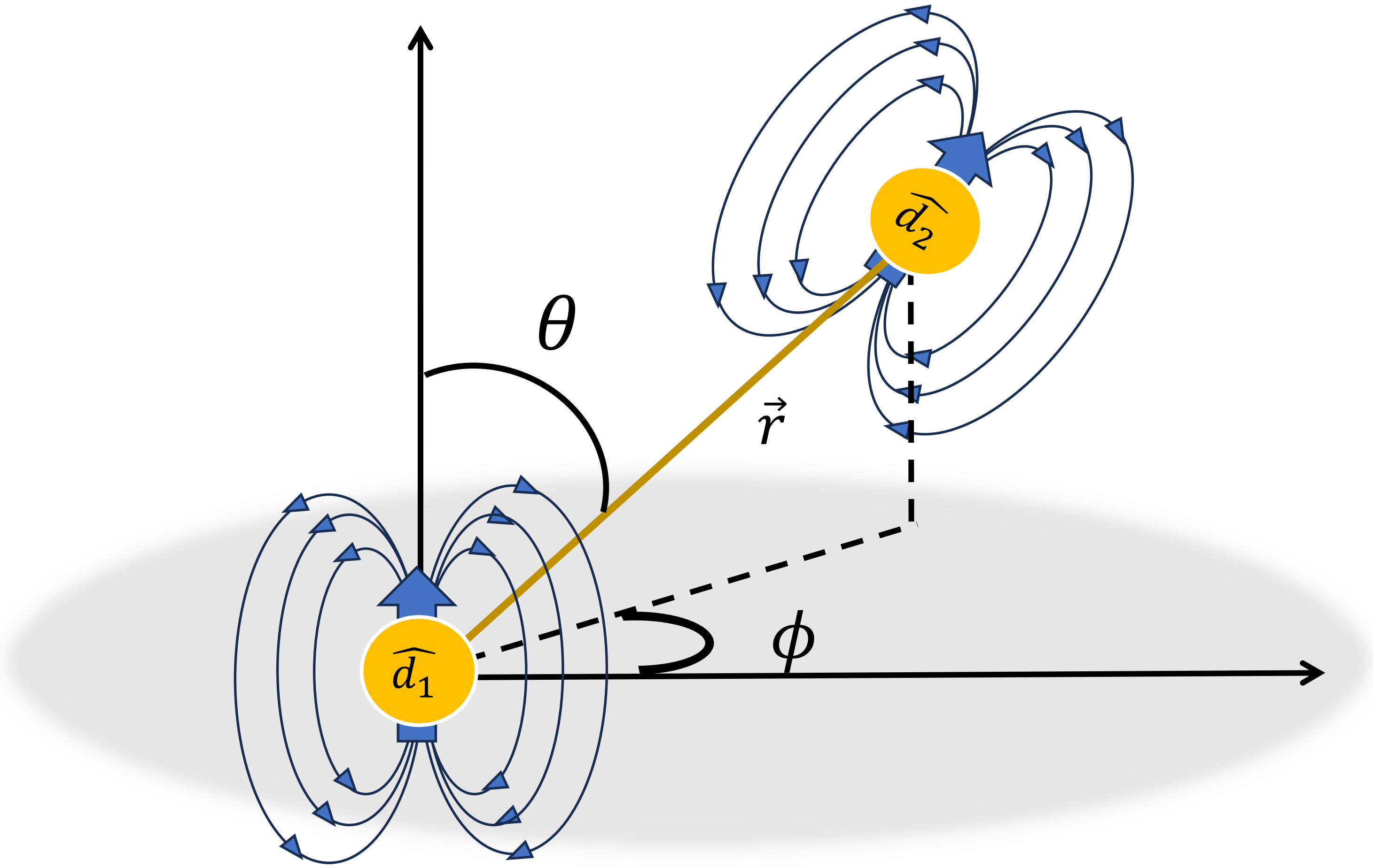
FINAL DIRECTION OF SPIN SPIRAL



4. MAPPING TO A SPIN HAMILTONIAN

INTERACTION HAMILTONIAN FOR TWO ATOMS

To first order, as if dipoles are interacting



$$H_{Total} = H_1 \otimes \mathbb{I} + \mathbb{I} \otimes H_2 + H_{int}$$

$$H_{int} = \frac{1}{4\pi\epsilon_0} \frac{\widehat{d_1} \cdot \widehat{d_2} - 3(\widehat{d_1} \cdot \hat{r})(\widehat{d_2} \cdot \hat{r})}{|\vec{r}|^3}$$

HAMILTONIAN OF THE TWO ATOM SYSTEM

Using $\widehat{d_\pm} = \mp\frac{1}{\sqrt{2}}(\widehat{d_x} \pm i\,\widehat{d_y})$ and $\widehat{d_0} = \widehat{d_z}$

$$\boxed{H_I(\theta,\phi,\vec{r}) = \frac{H_{dd}(\theta,\phi)}{4\pi\epsilon_0|\vec{r}|^3}}$$

$$H_{dd}(\theta,\phi)$$
$$= \frac{1-3\cos^2(\theta)}{2}\left[2\,\widehat{d_{10}}\cdot\widehat{d_{20}} + \widehat{d_{1+}}\cdot\widehat{d_{2-}} + \widehat{d_{1-}}\cdot\widehat{d_{2+}}\right]$$

No azimuthal dependence

$$-\ \frac{3\sin^2(\theta)}{2}\left[\widehat{d_{1+}}\cdot\widehat{d_{2+}}e^{-i\phi} + \widehat{d_{1-}}\cdot\widehat{d_{2-}}e^{i\phi}\right]$$
$$-\ \frac{3\sin^2(\theta)}{2}\left[(\widehat{d_{1+}}\cdot\widehat{d_{20}} + \widehat{d_{10}}\cdot\widehat{d_{2+}})e^{-i\phi} + (\widehat{d_{1-}}\cdot\widehat{d_{20}} + \widehat{d_{10}}\cdot\widehat{d_{2-}})e^{i\phi}\right]$$

Some terms could be ignored for unperturbed states

MAPPING TO A SPIN HAMILTONIAN

- $|\text{Chosen excited state}\rangle = |\uparrow\rangle$ & $|\text{Chosen ground state}\rangle = |\downarrow\rangle$
- Single Atom Hamiltonian $= |\downarrow\rangle\langle\downarrow| + \omega|\uparrow\rangle\langle\uparrow|$
- Find H_{int} in the two spin basis : $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$
- Write H_{int} rotating frame of non-interacting Hamiltonians

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{1,2}^* & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{1,3}^* & a_{2,3}^* & a_{3,3} & a_{3,4} \\ a_{1,4}^* & a_{2,4}^* & a_{3,4}^* & a_{4,4} \end{pmatrix} \xrightarrow[\text{Rotating Wave Approximation}]{e^{i(H_1 \otimes \mathbb{I} + \mathbb{I} \otimes H_2)t}} \begin{pmatrix} a_{1,1} & 0 & 0 & 0 \\ 0 & a_{2,2} & a_{2,3} & 0 \\ 0 & a_{2,3}^* & a_{3,3} & 0 \\ 0 & 0 & 0 & a_{4,4} \end{pmatrix}$$

MAPPING TO A SPIN HAMILTONIAN

$$H_{int} = \frac{1}{4\pi\epsilon_0|\vec{r}|^3} \times \begin{pmatrix} E_{\uparrow\uparrow} & 0 & 0 & 0 \\ 0 & E_{\downarrow\uparrow} & J/2 & 0 \\ 0 & J/2 & E_{\downarrow\uparrow} & 0 \\ 0 & 0 & 0 & E_{\downarrow\downarrow} \end{pmatrix}$$

$$\equiv J^{\perp}\left(S_1^x \otimes S_2^x + S_1^y \otimes S_2^y\right) + J^{\parallel}\left(S_1^Z \otimes S_2^Z\right) + h_z\left(S_1^z \otimes \mathbb{I} + \mathbb{I} \otimes S_2^z\right) + V\mathbb{I}$$

$$J^{\parallel} = E_{\downarrow\downarrow} + E_{\uparrow\uparrow} - 2E_{\downarrow\uparrow}$$

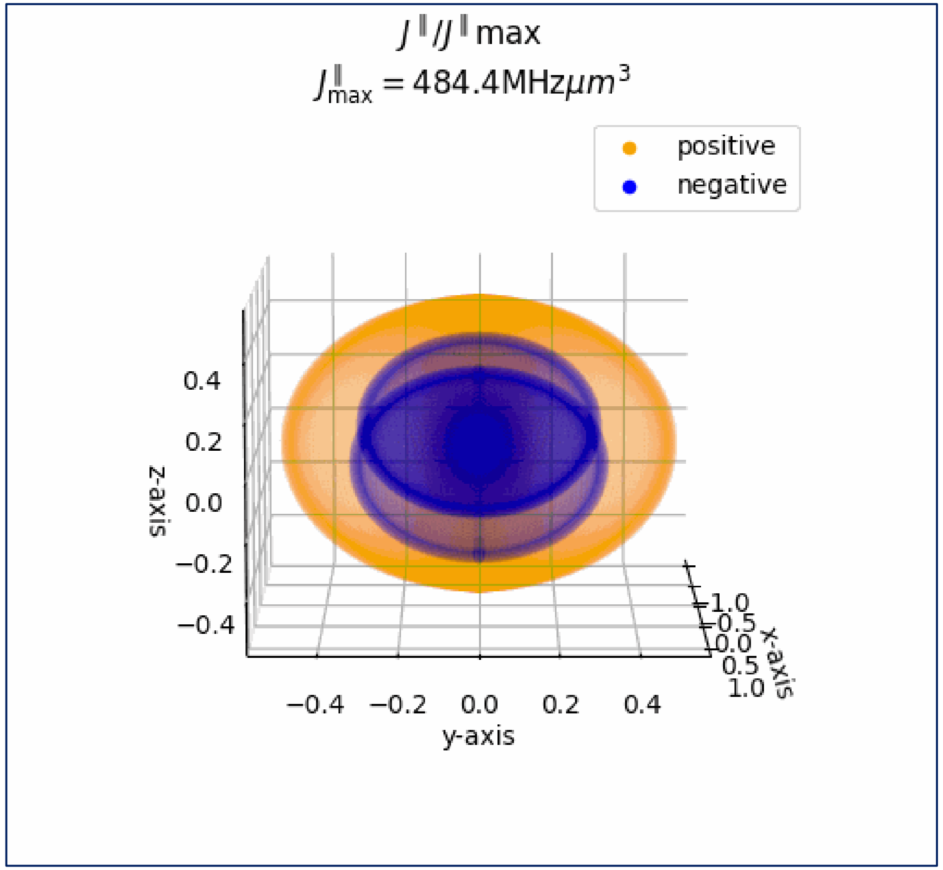
$$J^{\perp} = 2J$$

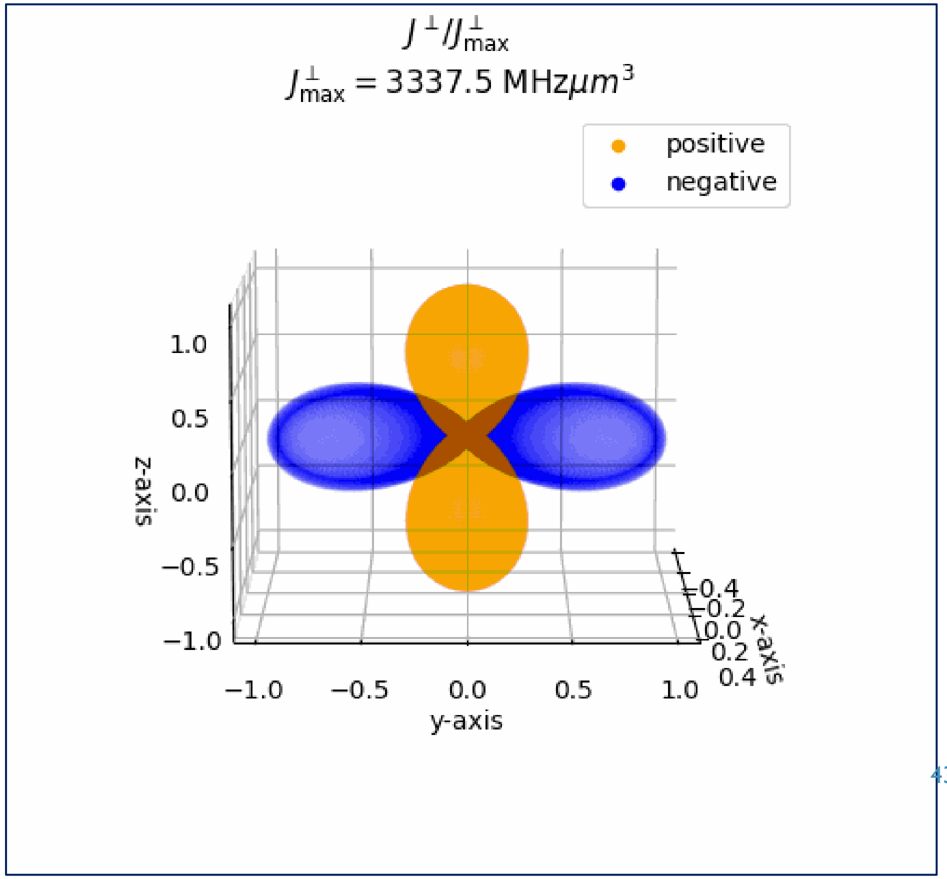
$$h_z = \frac{E_{\downarrow\downarrow} - E_{\uparrow\uparrow}}{2}$$

$$V = \frac{E_{\uparrow\uparrow} + E_{\downarrow\downarrow} + 2E_{\uparrow\downarrow}}{4}$$

ANGULAR DEPENDENCE OF $J^{\parallel}$ AND $J^{\perp}$

For $|48\ s\ ; J = 0.5\ ; m_J = 0.5\ \rangle \rightarrow |48\ p\ ; J = 1.5; m_J = 1.5\ \rangle$

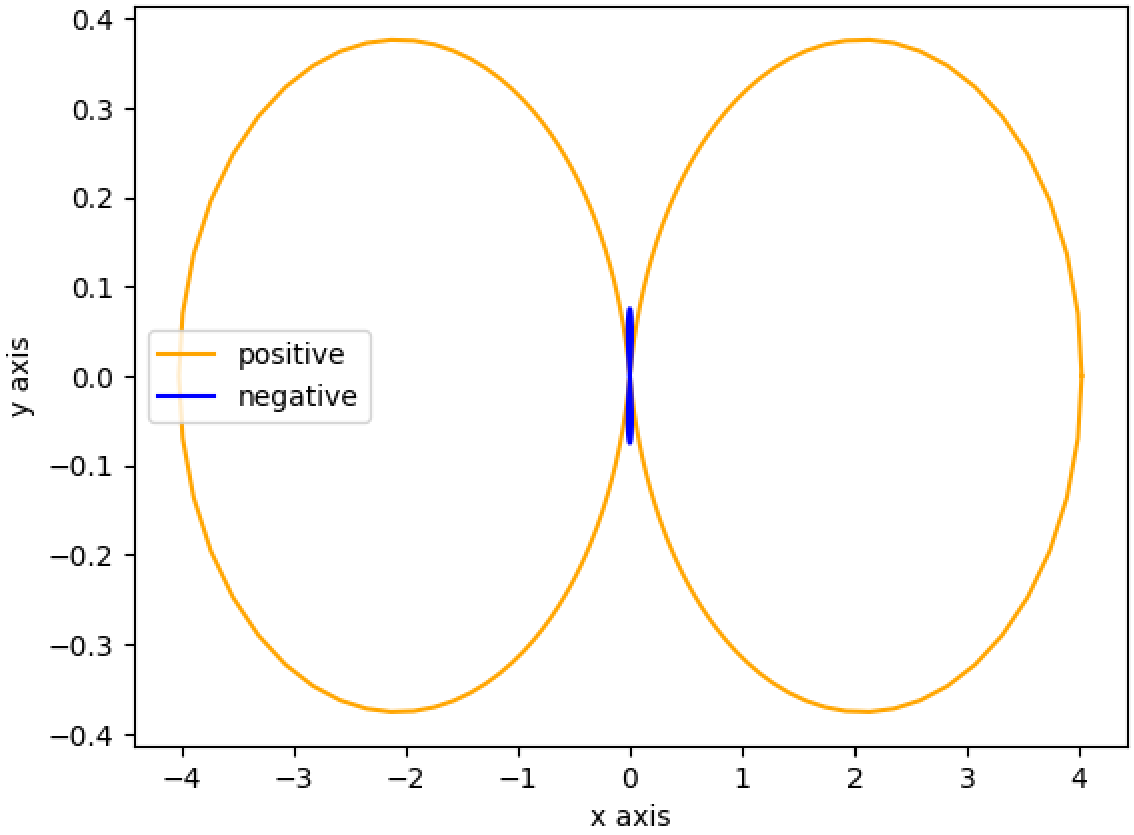


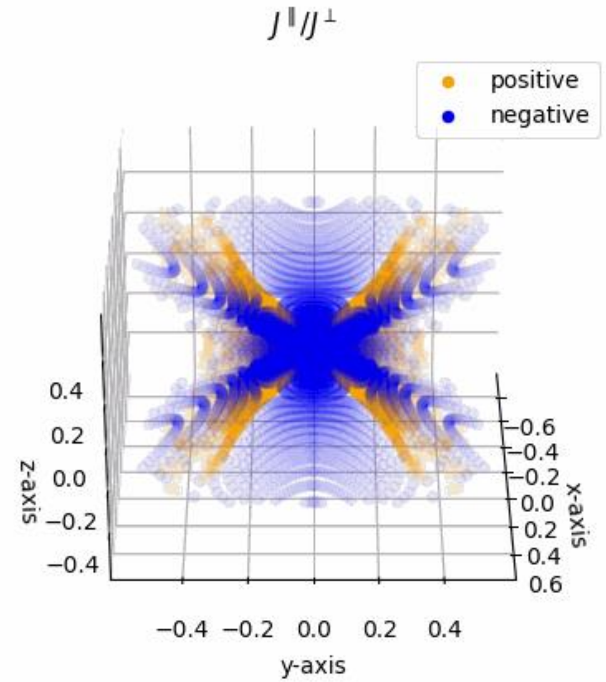


ANGULAR DEPENDENCE OF $J^{\parallel} / J^{\perp}$

For $|48\,s\,; J = 0.5\,; m_J = 0.5\,\rangle \rightarrow |48\,p\,; J = 1.5; m_J = 1.5\,\rangle$

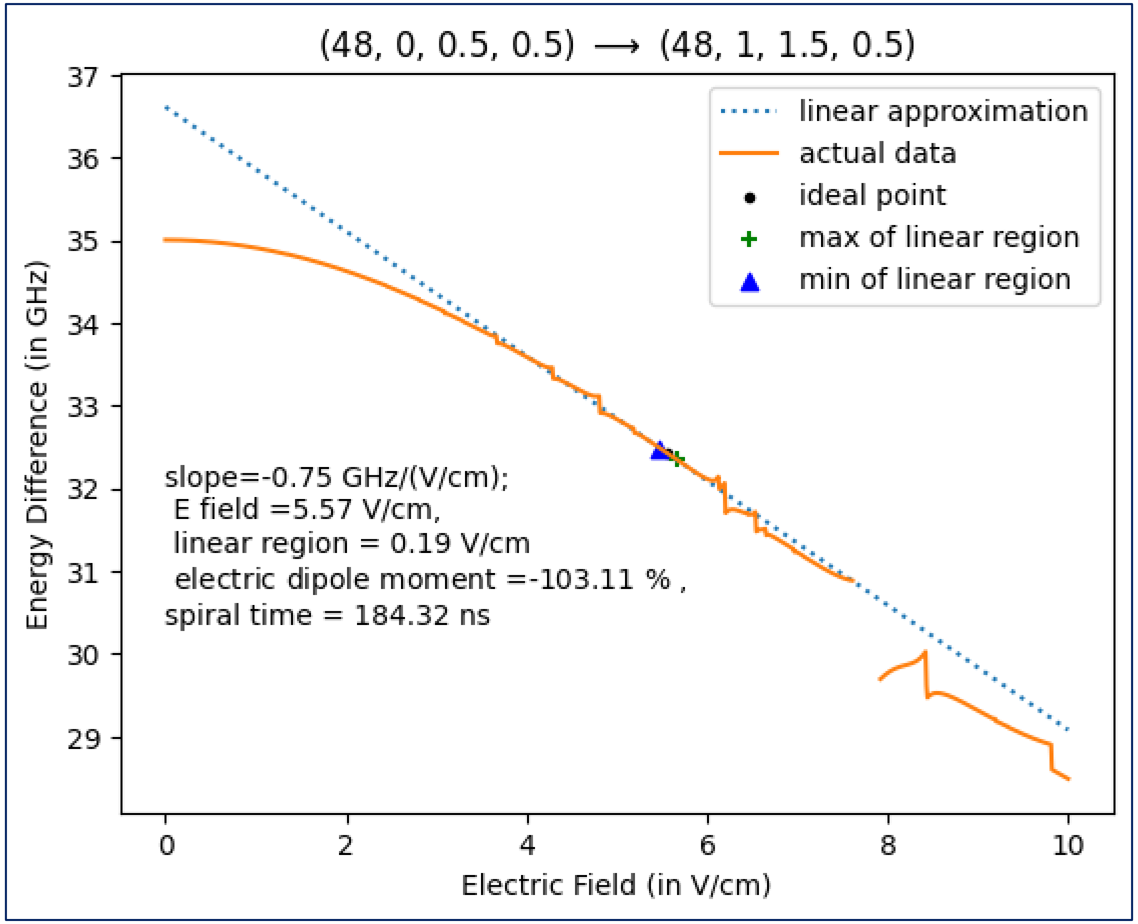
$J^{\parallel}/\,J^{\perp}$ in X-Y plane

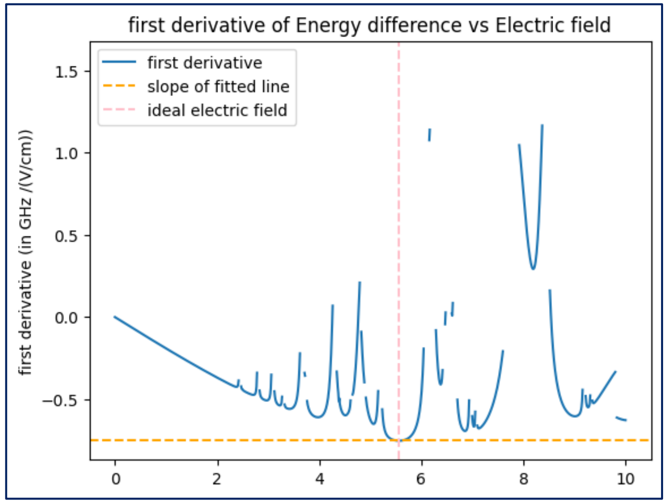


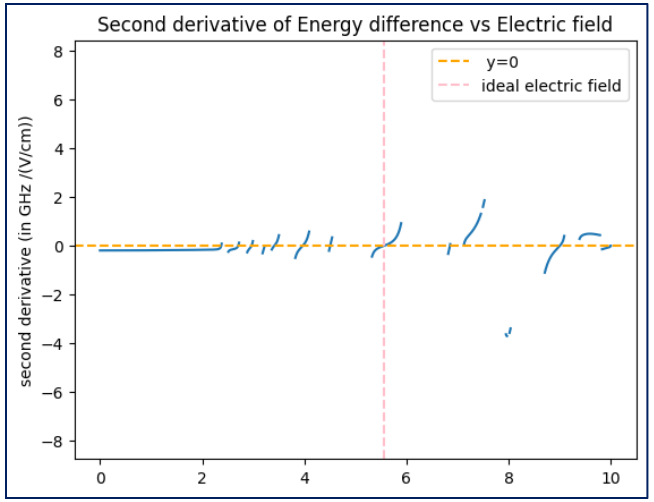


ANOTHER TRANSITION EXAMPLE

For $|48\, s\, ; J = 0.5\, ; m_J = 0.5\, \rangle \rightarrow |48\, p\, ; J = 1.5 ; m_J = 0.5\, \rangle$

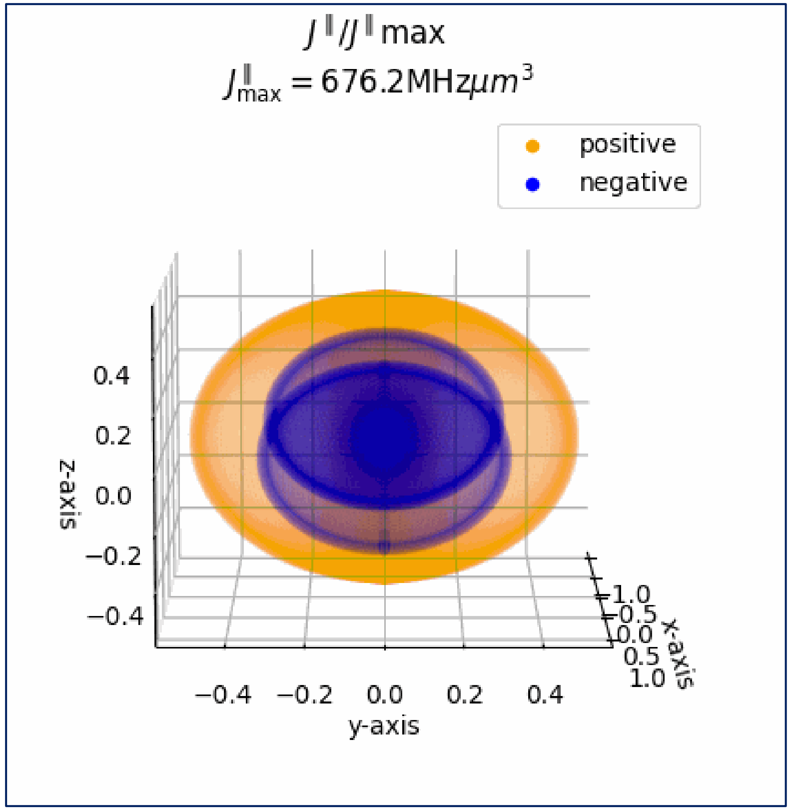


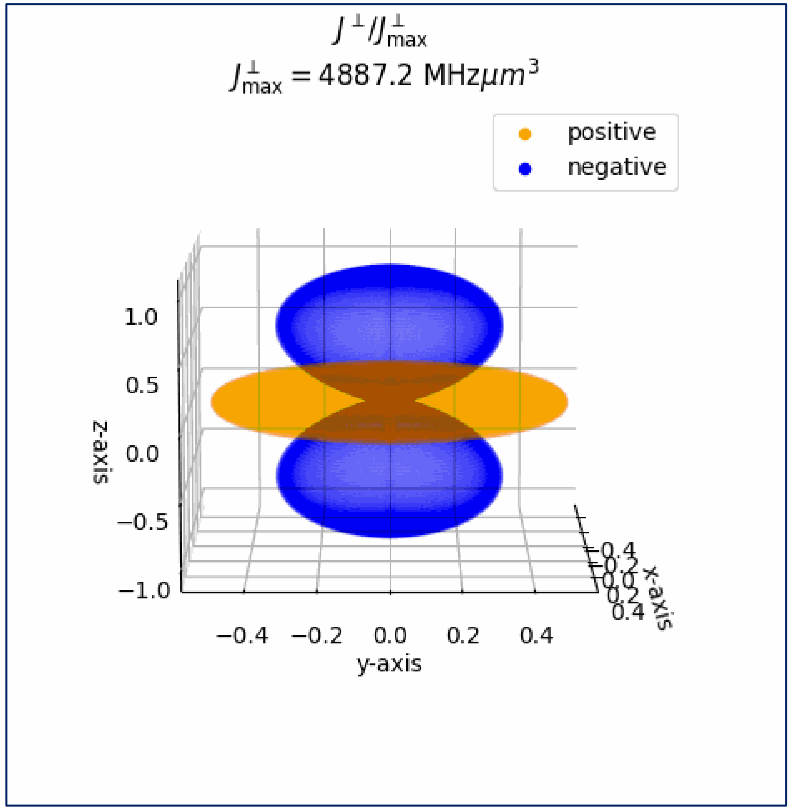




ANGULAR DEPENDENCE OF $J^{\parallel}$ AND $J^{\perp}$

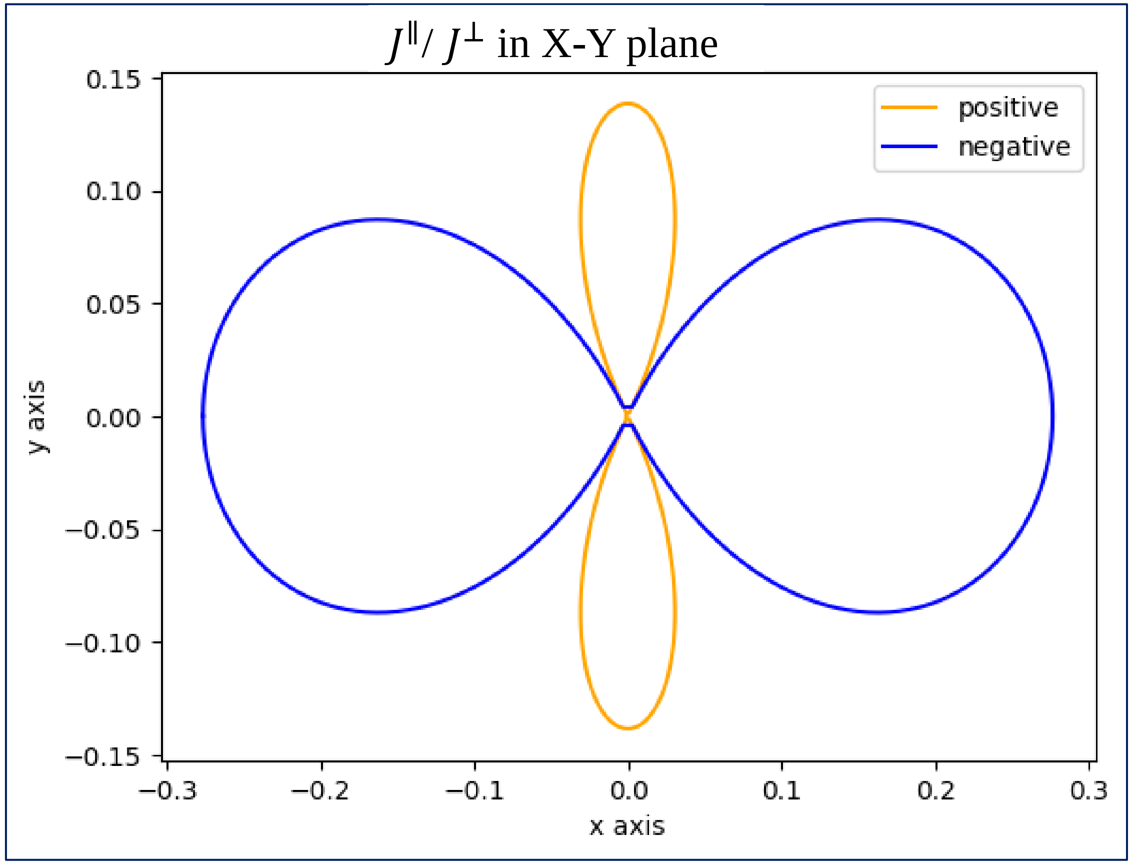
For $|48\, s\, ; J = 0.5\, ; m_J = 0.5\, \rangle \rightarrow |48\, p\, ; J = 1.5; m_J = 0.5\, \rangle$

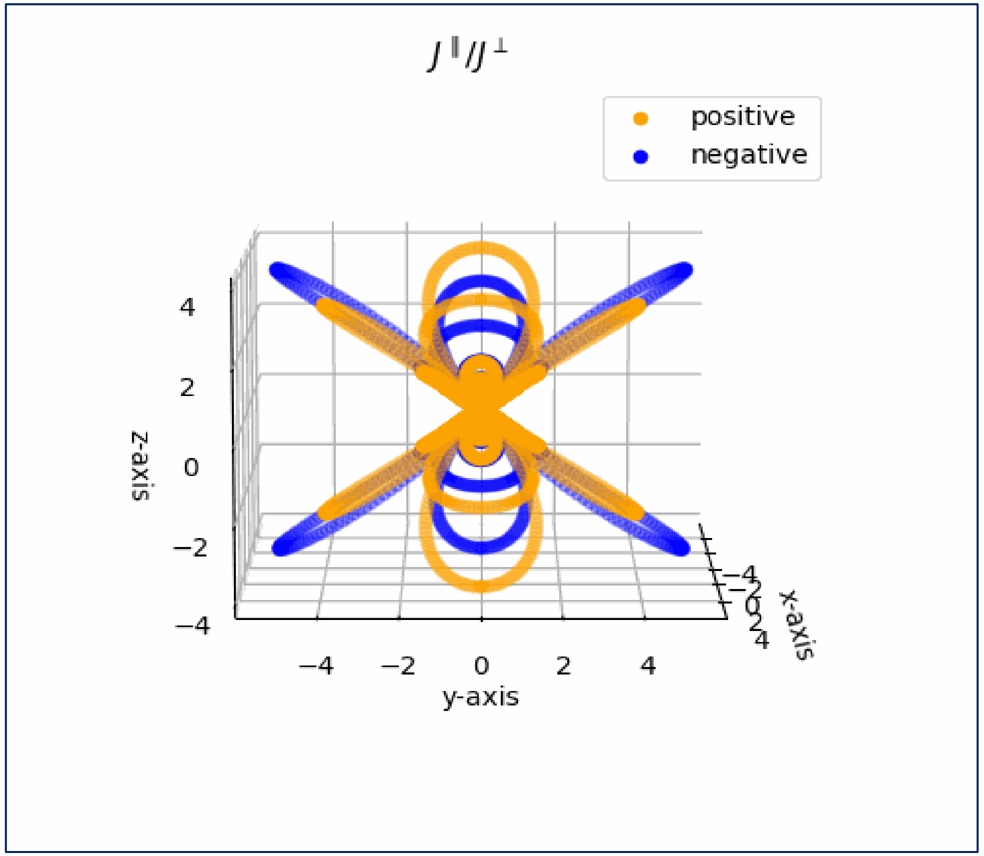




ANGULAR DEPENDENCE OF $J^{\parallel}/J^{\perp}$

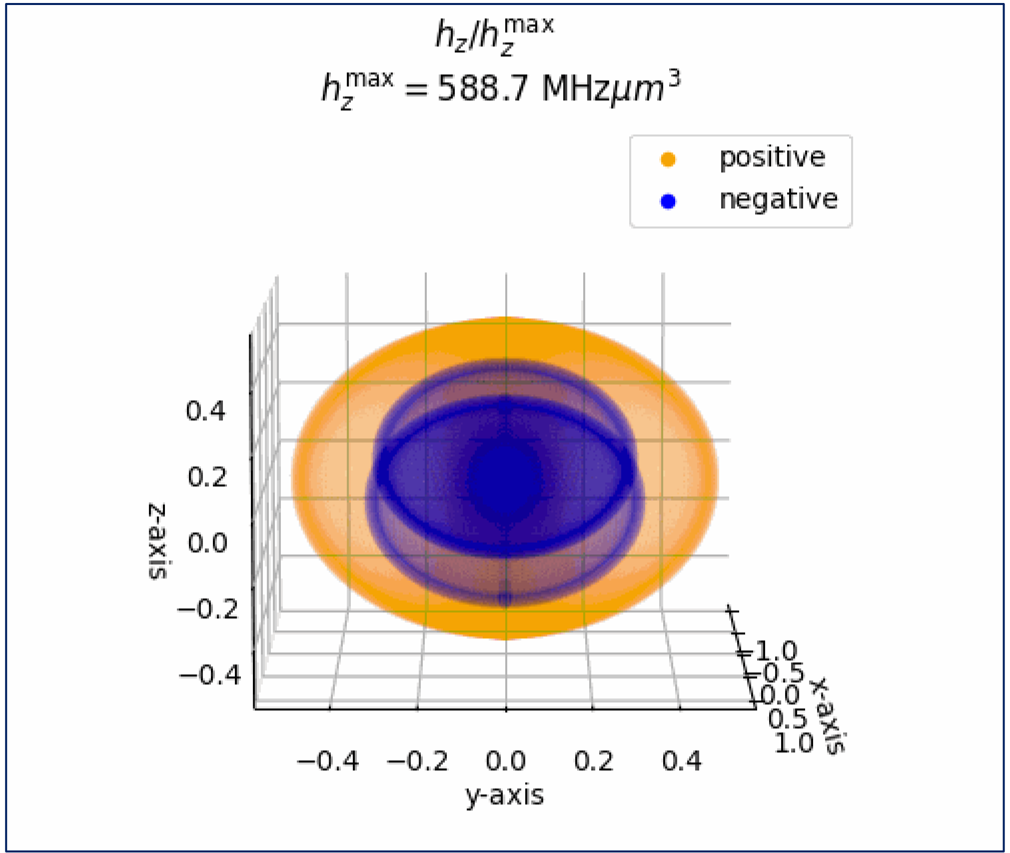
For $|48\,s\,;J=0.5\,;m_J=0.5\,\rangle \rightarrow |48\,p\,;J=1.5;m_J=0.5\,\rangle$

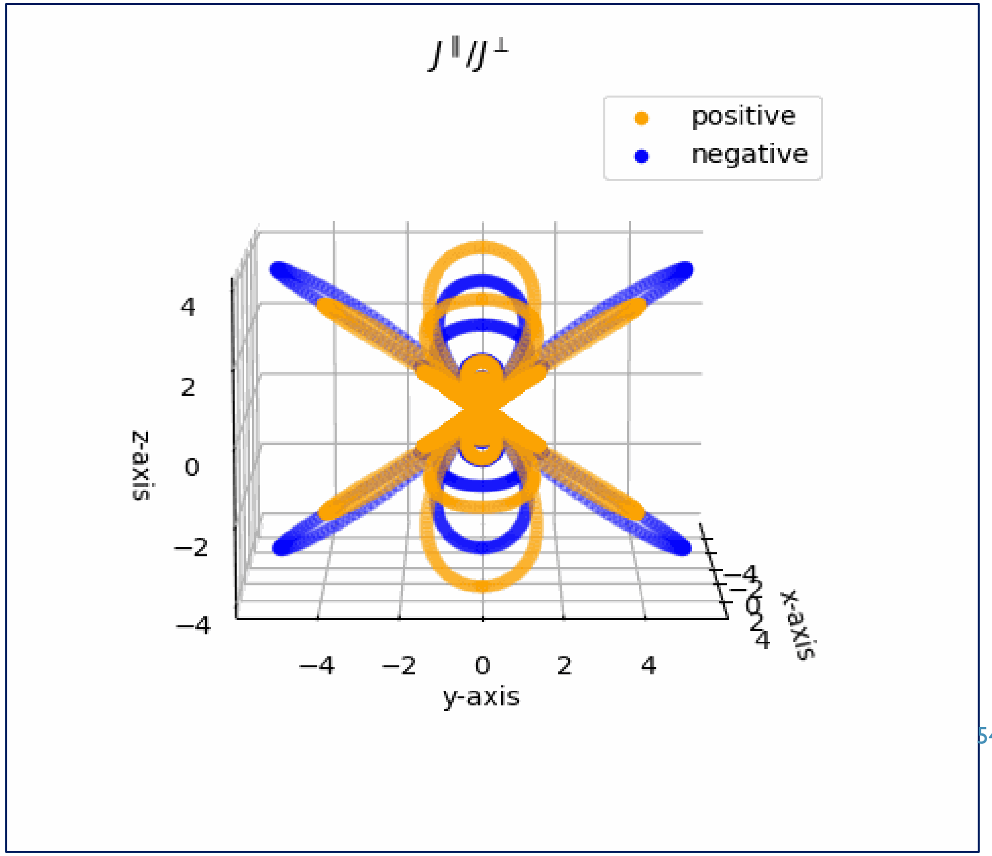




ANGULAR DEPENDENCE OF h_z AND $J^{\parallel}/J^{\perp}$

For $|48\, s\, ; J = 0.5\, ; m_J = 0.5\, \rangle \rightarrow |48\, p\, ; J = 1.5; m_J = 0.5\, \rangle$





OUTLOOK

- Can implement different Hamiltonians for different levels
- Analyze changes with principal quantum number
- Analyze effect of change in direction of fields on Spin model
- Look for Forster resonances

THANK YOU !

For making me a part of the Rydberg Team!