Tutorial-4

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① $T(n) = 3T(n/2) + n^2.$

$a = 3$
$b = 2$ , $n^{\log_b a} = n^{\log_2 3}$

Now, $n^{\log_2 3} < n^2$ (Case 3).

∴ according to master's thm : $T(n) = \Theta(n^2)$

② $T(n) = 4T(n/2) + n^2.$

$a = 4$ , $b = 2$

$n^{\log_2 4} = n^2 = f(n)$ (Case 2)

∴ according to master's thm: $T(n) = \Theta(n^2 \log n)$.

③ $T(n) = T(n/2) + 2^n.$

$a = 1$
$b = 2$

$n^{\log_2 1} = n^0 = 1$

Clearly $1 < 2^n$ (Case 3)

∴ according to master's thm : $T(n) = \Theta(2^n)$.

④ $T(n) = 2^n T(n/2) + n^n.$

This can't be solved using Master's thm as a depends on n.

(5) $T(n) = 16T(n/4) + n$

$a = 16, \; b = 4.$

$n^{\log_b a} = n^{\log_4 16} = n^2$

Now, $n^2 > f(n) = n$ (Case 1)

$\therefore$ According to master's thm, $T(n) = \Theta(n^2)$

(6) $T(n) = 2T(n/2) + n\log n.$

$a = 2, \; b = 2$

$n^{\log_b a} = n^{\log_2 2} = n^1 = n$

Now, $n\log^{k} n = f(n) = n\log n.$ (Case 2)

$\therefore$ According to master's thm: $T(n) = \Theta(n\log^2 n).$

(7) $T(n) = 2T(n/2) + n/\log n.$

→ Master's thm is not applicable since $f(n) = n/\log n$ isn't a polynomial fn.

(8) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, \; b = 4$

$n^{\log_4 2} = n^{0.5}$

$n^{0.5} < n^{0.51} = f(n)$ (Case 3)

$\therefore$ according to master's thm : $T(n) = \Theta(n^{0.51})$

(9) $T(n) = 0.5\,T(n/2) + 1/n$

↳ Master's thm is not applicable since $a < 1$.

(10) $T(n) = 16T(n/4) + n!$

↳ $a = 16$.

$b = 4$.

↳ $n!$ can be written as $n^n$. (Polynomial).

$n^{\log_b a} = n^{\log_4 a} = n^2$.

$n^2 < n!$ (Case 3).

$\therefore$ according to master's thm ; $T(n) = \Theta(n^n) = \Theta(n!)$.

(11) $T(n) = 4T(n/2) + \log n$.

$a = 4, b = 2$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > \log n = f(n)$ (case 1)

$\therefore$ according to master's thm; $T(n) = \Theta(n^2)$.

(12) $T(n) = \text{sqrt}(n)\, T(n/2) + \log n$.

↳ Master's thm isn't applicable since sqrt(n) isn't Constant.

(13) $T(n) = 3T(n/2) + n$.

$a = 3,\ b = 2$.

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$n^{1.58} > n$ (1st case).

$\therefore$ According to Master's thm ; $T(n) = \theta(n^{\log_2 3})$.

(14) . $T(n) = 3T(n/3) + \sqrt{n}$.

$a = 3,\ b = 3$

$n^{\log_b a} = n^{\log_3 3} = n^1$

$n > \sqrt{n}$ (Case 1)

$\therefore$ According to Master's thm ; $T(n) = \theta(n)$.

(15) $T(n) = 4T(n/2) + cn$.

$a = 4,\ b = 2$.

$n^{\log_b a} = n^{\log_2 4} = n^2$

$n^2 > cn$

$\therefore$ According to master's thm : $T(n) = \Theta(n^2)$.

(16) $T(n) = 3T(n/4) + n\log n$.

$a = 3$ , $b = 4$.

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$.

$n^{0.79} < n\log n$ (Case 3).

$\therefore$ according to master's thm : $T(n) = \Theta(n\log n)$.

(17) $T(n) = 3T(n/3) + n/2$

$a = 3$, $b = 3$

$n^{\log_b a} = n^{\log_3 3} = n^1$

$\Theta(n) = \Theta(n/2)$ (Case-2).

$\therefore$ according to master's thm ; $T(n) = \Theta(n\log n)$.

(18). $T(n) = 6T(n/3) + n^2\log n$.

$a = 6$ , $b = 3$.

$n^{\log_3 6} = n^2$

$n^2 < n^2\log n$ (Case 3)

$\therefore$ according to master's thm : $T(n) = \Theta(n^2\log n)$

(19) $T(n) = 4T(n/2) + n/\log n$.

$a = 4, \quad b = 2.$

$n^{\log_b a} = n^{\log_2 4} = n^2.$

$n^2 > n/\log n.$ (case 1)

$\therefore$ according to master's thm: $T(n) = \Theta(n^2)$.

(20). $T(n) = 64T(n/8) - n^2 \log n,$

↳ Master's thm is not applicable since $f(n)$ is negative.

(21) $7T(n/3) + n^2$

$a = 7, b = 3$

$n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

$n^{1.7} < n^2$ (case 3)

↳ according to master's thm; $T(n) = \Theta(n^2)$

(22) $T(n) = T(n/2) + n(2 - \cos n)$

↳ Master's thm isn't applicable since regularity condition is violated in case 3.