Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

ANS) To find the Z score , Z = (X- μ) /σ

Z = (50-45)/8 = 0.625

The probability that the service manager cannot meet his commitment ,

1-stats.norm.cdf(0.625)

Therefore probability = 0.2659

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ =6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

ANS) P(X>44)> P(38<X<44)

A. False.
   Employees older than 44 yrs of age , p(x>44)
   1-stats.norm.cdf(44,loc=38,scale=6)
   p(x>44) = 0.1586
   Employees between 38 to 44 yrs of age , p(38<x<44)
   stats.norm.cdf(44,38,6)-stats.norm.cdf(38,38,6)
   p(38<x<44) = 0.3413
B. Ture.
   Employees under 30 yrs of age
   stats.norm.cdf(30,38,6)
   p(x<30) = 0.0912
   No. of employees attending training program from 400 numbers is
   N*P(x<30) = 36.4844

3. If X_1 ~ $N(\mu, \sigma^2)$ and X_2 ~ $N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

ANS) As we know that if X ~ $N(\mu1, \sigma1^2)$ and Y ~ $N(\mu2, \sigma2^2)$ are two independent random variable then X+Y ~ $N(\mu1+\mu2, \sigma1^2 + \sigma2^2$ and X-Y ~ $N(\mu1-\mu2, \sigma1^2 + \sigma2^2$
Similarly is Z = aX+bY , where X and Y are as defined above , i.e Z is linear combination of X and Y ,then Z ~ $N(a\mu1+b\mu2, a^2\sigma1^2 + b^2\sigma2^2$
Therefore $2X_1$ ~ $N(2\mu, 4\sigma^2)$ and
X1+X2 ~ $N(2\mu, 2\sigma^2)$

4. Let X ~ $N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

ANS) The pro(getting the value between a & b)= 0.99
So, the pro(getting value outside a & b) = 1-0.99 = 0.01
The probability towards left of a = -0.01/2 = 0.05
The probability towards right of b = 0.01/2 = 0.05
By finding the std normal variable ,we need to calculate X :

$$Z = (X-\mu)/\sigma$$
$$Z^*\sigma = (X-\mu)$$
$$X = Z^*\sigma + \mu$$

For a probability of 0.05, z value is -2.57
therefore, -(-2.57)*20+100=151.4
(-2.57)*20+100=48.6

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit_1 ~ $N(5, 3^2)$ and Profit_2 ~ $N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   B. Specify the 5^{th} percentile of profit (in Rupees) for the company
   C. Which of the two divisions has a larger probability of making a loss in a given year?

ANS) Mean profits from two different divisions of company = Mean 1 + Mean 2

Mean = (5+7)*45 = 540

Variance of profits from two different divisions of a company is

SD =(np.sqrt(9+16))*45 = 225

A. Range is Rs (99.00810347848784, 980.9918965215122) in Millions

B. To find 5th percentile , we use the formula X = $Z^{*}\sigma + \mu$

from the z table

X = 540+(-1.645)*225 = 170

C. Probability of Division 1 making loss P(X<0)

stats.norm.cdf(0,5,3) —> 0.047

Probability of Division 2 making loss P(X<0)

stats.norm.cdf(0,7,4) —> 0.040