Practice Problems on 'Confidence Interval'

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Test of Hypothesis on Mean (μ) of a $N(\mu, \sigma^2)$ Population

Case : I When population variance (σ^2) is known

#.1 **The manufacturer of a 75 Watt light bulb claims that average life of its bulb is more than 1000 hours. It is known that the life of 75 Watt light bulbs is normally distributed with a standard deviation of 25 hours. A random sample of 20 number of 75 Watt bulbs is found to have average life of 1014 hours. Test the claim of the manufacturer at 5% level of significance.**

Solution-

Let X be random variable denoting life of 75 W light bulbs.

It is given that $X \sim N(\mu, \sigma^2)$, where $\sigma = 25$.

Here the null and alternative hypotheses are-

$$H_0 : \mu = 1000$$

$$H_1 : \mu > 1000$$

Since the population s.d. is known, so the test statistic is

$$Z_0 = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

Here,

sample mean $(\bar{X}) = 1014$

sample size (n) = 20

Hence,

$$Z_0 = \frac{1014 - 1000}{25/\sqrt{20}} = \frac{14}{5.59} = 2.5$$

It is right-tailed test (since alternative hypothesis is of > form), so critical value is $Z_{0.05} = 1.64$.

Since $Z_0 > 1.64$, so the null hypothesis is rejected.

#.2 **A manufacturer produces piston rings for an automobile engine. It is known that ring diameter is normally distributed with a standard deviation of 0.01 mm. A random sample of 15 rings has a mean diameter of 74.036 mm. Test the hypothesis that the true mean diameter of piston ring is 74.035. Use α = 0.01.**

Solution-

Let X denote diameter of piston rings.

It is given that $X \sim N(\mu, \sigma^2)$, where $\sigma = 0.01$.

Here, the null and alternative hypotheses are :

$$H_0: \mu = 74.035$$

$$H_1: \mu \neq 74.035$$

Here, population standard deviation is given, so the test statistic is

$$Z_0 = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

Here,

sample size (n) = 15

sample mean $(\bar{X}) = 74.036$

Hence,

$$Z_0 = \frac{74.036 - 74.035}{0.01/\sqrt{15}} = \frac{0.001}{0.01/\sqrt{15}} = 0.38$$

It is two-tailed test (since the alternative hypothesis is of < > form), so critical values are

$$Z_{\frac{0.01}{2}} = Z_{0.005} = 2.58 \ and - Z_{\frac{0.01}{2}} = -Z_{0.005} = -2.58$$

Since it is not true that $Z_0 > 2.58$ and it is also not true that $Z_0 < -2.58$, so the null hypothesis is not rejected.

#.3 The average commission charged by full-service brokerage firms on a sale of common is $144, with a standard deviation of $52. Joel Freelander has taken a random sample of 121 trades by his clients and determined that they paid an average commission of $151. At a 0.10 level significance, can Joel conclude that his client's commissions are higher that the industry average? [Hints: Z_0 = 1.48, not rejected]

#.4 Do middle-aged male executive have different averaged blood pressure than the general population? The National Center for Health Statistics reports that the mean systolic blood pressure for males of 35 to 44 years of age is 128 and the standard deviation in this population is 15. The medical director of a company looks at the medical records of 72 company executives in this age group and finds that the mean systolic blood pressure in this sample is

$\bar{x} = 126.7$. Is this evidence that executive blood pressure differ from the national average? [Hints: Z_0 = -0.74, not rejected]

Case – II When population variance (σ^2) is unknown

Sub-case (a) When sample size is large (i.e, n >30)

#.1 An industrial engineer collected data on the labor time required to produce an order of automobile mufflers using a heavy stamping machine. The data on times (hours) for n = 52 orders of different parts has mean of 1.865 hours with a variance of 1.5623 (hours)^2. Conduct a test of hypothesis with the intent of showing that the mean labor time is less than 1.955 hours.

Solution-

Let μ denote actual mean labor time required to produce an order.

Here, the null and alternative hypotheses are:

$$H_0: \mu = 1.955$$

$$H_1: \mu < 1.955$$

Here, population variance is not given and sample is large, so test statistic is

$$Z_0 = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

Here,

sample size (n) = 52

sample mean $(\bar{X}) = 1.865$

sample variance $(s^2) = 1.5623$

Hence,

$$Z_0 = \frac{1.865 - 1.955}{\sqrt{1.5623}/\sqrt{52}} = -\frac{0.090}{1.73} = -0.52$$

It is left tailed test (since alternative hyp. is of < form), so critical value is $-Z_\alpha = -Z_{0.05} = -1.64$

Since it is not true that $Z_0 < -1.64$, so the null hypothesis is not rejected.

#.2 The manufacturer of large liquid crystal display (LCD) is difficult. Some defects are minor and can be removed; others are unremovable. The number of unremovable defects for each of n = 45 displays

1	0	5	3	0	7	6	0	0	4	6	8
5	0	9	1	0	8	6	0	3	2	0	0
0	6	0	10	0	6	0	0	1	0	0	0
0	1	5	1	0	5	0	0	2			

has mean $\bar{x}$ = 2.667 and s = 3.057 unremovable defects. Conduct a test of hypothesis with the intent of showing that the mean number of unremovable defects is less than 3.6. Take α = 0.025. [Ans. Calc. Z = -2.05, reject]

Sub-case (b) When sample is small, i.e., when n<=30

#.1 Data on number of work hours lost per day on a construction project due to weather related conditions over 11 working days are recorded as

8.8 8.8 12.5 12.2 5.4 13.3 12.8 6.9 9.1 2.2 14.7

Assuming that the work hours are normally distributed, is there any evidence to conclude at 1% level of significance that the mean number of work hours lost per day is smaller than 11 hours?

Solution-

Let μ denote actual mean number of work hours lost.

Here, the null and alternative hypotheses are:

$$H_0: \mu = 11$$

$$H_1: \mu < 11$$

Here, population variance is not known and sample is small, so the test statistic is

$$t_0 = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

<u>Calculation of sample mean and sample variance-</u>

X	x2
8.8	77.44
8.8	77.44
12.5	156.25
12.2	148.84
5.4	29.16
13.3	176.89
12.8	163.84
6.9	47.61
9.1	82.81
2.2	4.84
14.7	216.09
106.7	1181.21

We have

$$\bar{X} = \frac{1}{n}\sum X = \frac{1}{11} \times 106.7 = 9.7$$

Next,

$$S^2 = \frac{1}{n-1}\sum (X - \bar{X})^2 = \frac{n}{n-1}\left(\frac{1}{n}\sum (X - \bar{X})^2\right)$$

$$= \frac{n}{n-1}\left(\frac{1}{n}\sum X^2 - \bar{X}^2\right)$$

$$= \frac{11}{11-1}\left(\frac{1}{11} \times 1181.21 - 9.7^2\right)$$

$$= \frac{11}{10} \times 13.293 = 14.622$$

So, $S = \sqrt{14.622} = 3.8239$

Hence,

$$t_0 = \frac{9.7 - 11}{\frac{3.8239}{\sqrt{11}}} = -\frac{1.3}{1.153} = -1.13$$

The critical value is $-t_{\alpha,n-1} = -t_{0.01,11-1} = -t_{0.01,10} = -2.76$

Since it is not true that $t_0 < -2.76$, so the null hypothesis is not rejected.

#.2 If a sample of 25 observations reveals a sample mean of 52 and a sample variance of 4.2, test the hypothesis that the population mean is 65, against the alternative that it is some other value. Use the 0.01 level of significance. [t_0 = -31.72, critical value is $t_{0.005,24}$ = 2.797, reject]

#.3 A real-state agent took a random sample of 12 homes in a prestigious suburb of Chicago and found the average appraised market value to be $ 780,000, with a standard deviation of $ 49,000. Test the hypothesis that for all homes in the area, the mean appraised value is $ 825,000, against the alternative that it is less than $ 825,000. Use 0.05 level of significance. [t_0 = -3.18, critical value is $t_{0.05,11}$ = 1.796, reject]