# Wilcoxon's Test

## Introduction

The advantage of Wilcoxon's signed rank test over normal sign test is that 'Sign Test' considers only the signs of the deviations, $X_i - \tilde{\mu}$ and not their magnitudes, whereas, Wilcoxon's signed rank test considers sign as well as magnitude of deviations $X_i - \tilde{\mu}$.

## Wilcoxon's Test for One Sample

Wilcoxon's signed rank test assumes that the population of interest is continuous and symmetric. Under this assumption mean and median are approximately same, so null alternative of interest is either $H_0: \mu = \mu_0$(mean) or $H_0: \tilde{\mu} = \tilde{\mu}_0$ (median).

For test procedure, a sample $X_1$, $X_2$, ……………. $X_n$ of size n is taken from population.

Then deviations from mean for each sample observations, i.e., $d_i = X_i - \tilde{\mu}_0$ is computed for i = 1, 2, ………… n. and their absolute values, i.e., $|d_i|$, are noted..

Then ranks are given to these absolute values $|d_i|$ in ascending order and these ranks are given the signs of the corresponding deviations.

If $d_i = X_i - \tilde{\mu}_0 = 0$ for any observation, then such observations are discarded and no ranks are assigned to such deviations.

Then sum of positive ranks and negative ranks are computed. Let $R^+$ denote the sum of positive ranks and $R^-$ denote the sum of negative ranks.

Let $R = \min(R^+, R^-)$

For specified value of α and n, the critical value $R_\alpha^*$ is obtained from 'Wilcoxon's Signed Ranks Test' table. (Given below)

Finally H0 is rejected at α level of significance, if (a) $R \leq R_\alpha^*$ (for two-tailed test) (b) $R^+ \leq R_\alpha^*$ (for left tailed test) (c) $R^- \leq R_\alpha^*$ (for right tailed test)

## Wilcoxon's Test for Paired Observations

In this test two continuous populations are considered. It is not necessary to assume that they are symmetric. Let $\mu_1$ and $\mu_2$ be means of the two populations. Wilcoxon's signed rank test for paired observations is used to test the null hypothesis that the two populations have identical means, i.e.,

$$H_0 : \mu_1 = \mu_2$$

against appropriate alternative hypothesis.

For the test procedure two samples, one from each population, are taken randomly. Both these samples are considered to have same size, n.

Then differences of each pair of sample observations are computed assigning a + sign if first sample observation is greater than second and assigning a – sign otherwise.

If any pair of sample observations are identical, then such pair is discarded and test is based on remaining non-identical observations only.

Then the ranks are given to these differences in ascending order of their absolute values. If two or more differences are tied (or same) then average rank is assigned as the common rank.

Then to each rank sign of their difference is given.

Then the sum of positive ranks and negative ranks are computer. Let $R^+$ denote the sum of positive ranks and $R^-$ denote the sum of negative ranks.

Let $R = \min(R^+, R^-)$

Remaining procedure is same as that of individual observation.

## Wilcoxon' Signed Rank Test Table

| | alpha values | | | | | | |
|---|---|---|---|---|---|---|---|
| n | 0.001 | 0.005 | 0.01 | 0.025 | 0.05 | 0.10 | 0.20 |
| 5 | -- | -- | -- | -- | -- | 0 | 2 |
| 6 | -- | -- | -- | -- | 0 | 2 | 3 |
| 7 | -- | -- | -- | 0 | 2 | 3 | 5 |
| 8 | -- | -- | 0 | 2 | 3 | 5 | 8 |
| 9 | -- | 0 | 1 | 3 | 5 | 8 | 10 |
| 10 | -- | 1 | 3 | 5 | 8 | 10 | 14 |
| 11 | 0 | 3 | 5 | 8 | 10 | 13 | 17 |
| 12 | 1 | 5 | 7 | 10 | 13 | 17 | 21 |
| 13 | 2 | 7 | 9 | 13 | 17 | 21 | 26 |
| 14 | 4 | 9 | 12 | 17 | 21 | 25 | 31 |
| 15 | 6 | 12 | 15 | 20 | 25 | 30 | 36 |
| 16 | 8 | 15 | 19 | 25 | 29 | 35 | 42 |
| 17 | 11 | 19 | 23 | 29 | 34 | 41 | 48 |
| 18 | 14 | 23 | 27 | 34 | 40 | 47 | 55 |
| 19 | 18 | 27 | 32 | 39 | 46 | 53 | 62 |
| 20 | 21 | 32 | 37 | 45 | 52 | 60 | 69 |
| 21 | 25 | 37 | 42 | 51 | 58 | 67 | 77 |
| 22 | 30 | 42 | 48 | 57 | 65 | 75 | 86 |
| 23 | 35 | 48 | 54 | 64 | 73 | 83 | 94 |
| 24 | 40 | 54 | 61 | 72 | 81 | 91 | 104 |
| 25 | 45 | 60 | 68 | 79 | 89 | 100 | 113 |
| 26 | 51 | 67 | 75 | 87 | 98 | 110 | 124 |
| 27 | 57 | 74 | 83 | 96 | 107 | 119 | 134 |

| | alpha values | | | | | | |
|---|---|---|---|---|---|---|---|
| n | 0.001 | 0.005 | 0.01 | 0.025 | 0.05 | 0.10 | 0.20 |
| 28 | 64 | 82 | 91 | 105 | 116 | 130 | 145 |
| 29 | 71 | 90 | 100 | 114 | 126 | 140 | 157 |
| 30 | 78 | 98 | 109 | 124 | 137 | 151 | 169 |
| 31 | 86 | 107 | 118 | 134 | 147 | 163 | 181 |
| 32 | 94 | 116 | 128 | 144 | 159 | 175 | 194 |
| 33 | 102 | 126 | 138 | 155 | 170 | 187 | 207 |
| 34 | 111 | 136 | 148 | 167 | 182 | 200 | 221 |
| 35 | 120 | 146 | 159 | 178 | 195 | 213 | 235 |
| 36 | 130 | 157 | 171 | 191 | 208 | 227 | 250 |
| 37 | 140 | 168 | 182 | 203 | 221 | 241 | 265 |
| 38 | 150 | 180 | 194 | 216 | 235 | 256 | 281 |
| 39 | 161 | 192 | 207 | 230 | 249 | 271 | 297 |
| 40 | 172 | 204 | 220 | 244 | 264 | 286 | 313 |
| 41 | 183 | 217 | 233 | 258 | 279 | 302 | 330 |
| 42 | 195 | 230 | 247 | 273 | 294 | 319 | 348 |
| 43 | 207 | 244 | 261 | 288 | 310 | 336 | 365 |
| 44 | 220 | 258 | 276 | 303 | 327 | 353 | 384 |
| 45 | 233 | 272 | 291 | 319 | 343 | 371 | 402 |
| 46 | 246 | 287 | 307 | 336 | 361 | 389 | 422 |
| 47 | 260 | 302 | 322 | 353 | 378 | 407 | 441 |
| 48 | 274 | 318 | 339 | 370 | 396 | 426 | 462 |
| 49 | 289 | 334 | 355 | 388 | 415 | 446 | 482 |
| 50 | 304 | 350 | 373 | 406 | 434 | 466 | 503 |

## Solved Problem

### #.1

A study in which a rocket motor is formed by binding an igniter propellant and a sustainer propelled inside a metal housing. Results of testing 20 randomly selected motors for the shear strength of the bond between the two propellant types is given below (*in working table of answer*):

.............

.............

Use Wilcoxon's signed rank test to observe whether shear strength is 2000 psi.

Solution-

Here,

$$H_0: \mu = 2000 \qquad H_1: \mu \neq 2000$$

Working table-

| Observation # | Shear Strength (X) | Difference (X-2000) | Signed Rank |
|---|---|---|---|
| 1 | 2158.70 | 158.70 | +3 |
| 2 | 1678.15 | -321.85 | -14 |
| 3 | 2316.00 | 316.00 | +13 |
| 4 | 2061.30 | 61.30 | +2 |
| 5 | 2207.50 | 207.50 | +6 |
| 6 | 1708.30 | -291.70 | -12 |
| 7 | 1784.70 | -215.30 | -7 |
| 8 | 2575.10 | 575.10 | +19 |
| 9 | 2357.90 | 357.90 | +16 |
| 10 | 2256.70 | 256.70 | +11 |
| 11 | 2165.20 | 165.20 | +4 |
| 12 | 2399.50 | 399.50 | +17 |
| 13 | 1779.80 | -220.20 | -8 |
| 14 | 2336.75 | 336.75 | +15 |
| 15 | 1765.30 | -234.70 | -9 |
| 16 | 2053.30 | 53.50 | +1 |
| 17 | 2414.40 | 414.40 | +18 |
| 18 | 2200.50 | 200.50 | +5 |
| 19 | 2654.20 | 654.20 | +20 |
| 20 | 1753.70 | -246.30 | -10 |

Here, $sum\ of\ +ve\ ranks\ (R^{+}) = 150$

$sum\ of\ -ve\ ranks\ (R^{-}) = 60$

$R = \min(R^{+}, R^{-}) = 60$

From Wilcoxon's signed rank critical value table, for $n = 20, R^{*}_{0.05} = 52$

Since it is not true that $R \leq R^{*}_{0.05}\ so\ H_0$ is not rejected.

## #.2

An automobile engineer is investigating two different types of metering devices for an electronic fuel injection system to determine if they differ in their fuel mileage performance. The system is installed on 12 different cars and a test is run with each metering system on each car. The observations are as follows-

| | |
|---|---|
| Car # | 1 2 3 ......... |
| Meter1 | ............. |
| Meter2 | ........... |

Use Wilcoxon's signed rank test to determine if the two metering devices differ in their fule mileage performance.

Solution:

Let $\mu_1\ and\ \mu_2$ denote mean fuel mileage of two metering devices.

Here, $H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 \neq \mu_2$

Working table-

| Car # | Meter1 | Meter2 | Difference | Rank of Diff. |
|---|---|---|---|---|
| 1 | 17.6 | 16.8 | 0.8 | 8 |
| 2 | 19.4 | 20.2 | -0.6 | -5 |
| 3 | 19.5 | 18.2 | 1.3 | 12 |
| 4 | 17.1 | 16.4 | 0.7 | 6.5 |
| 5 | 15.3 | 16.0 | -0.7 | -6.5 |
| 6 | 15.9 | 15.4 | 0.5 | 4 |
| 7 | 16.3 | 16.5 | -0.2 | -1 |
| 8 | 18.4 | 18.0 | 0.4 | 3 |
| 9 | 17.3 | 16.4 | 0.9 | 9 |
| 10 | 19.1 | 20.1 | -1.0 | -10 |
| 11 | 17.8 | 16.7 | 1.1 | 11 |
| 12 | 18.2 | 17.9 | 0.3 | 2 |

Here, $sum\ of + ve\ ranks\ (R^+) = 55.5$

$sum\ of - ve\ ranks\ (R^-) = 22.5$

$R = \min(R^+, R^-) = 22.5$

From Wilcoxon's signed rank critical value table, for $n = 12, R^*_{0.05} = 13$

Since it is not true that $R \leq R^*_{0.05}\ so\ H_0$ is not rejected.

## Mann-Whitney's U-Test

## Introduction

It is a non-parametric test procedure that is used to test the equality of mean of two continuous populations the form of distribution of which may or may not be known.

Let $X_1$ and $X_2$ be two independent continuous populations with means $\mu_1$ and $\mu_2$.

The null and alternative hypothesis underlying the test are:

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 < \mu_2 \text{ or } \mu_1 > \mu_2 \ \ or\ \mu_1 \neq \mu_2$$

For the test procedure random samples of size $n_1$ is taken from population $X_1$ and of size $n_2$ is taken from $X_2$.

Then ranks are assigned to each $n_1 + n_2$ total observations regarding both as one single sample in an ascending order of their magnitudes.

If two or more observations are tied (or identical) then mean of the ranks that would have been assigned they are different is assigned as their common rank.

After assigning ranks to each sample observation, the sum of ranks for sample observations from $X_1$ and fro $X_2$ are computed.

Let R1 be the sum of ranks in the first sample and R2 be the sum of ranks in second sample.

If $R_1 < R_2$ then we compute the value of following U-statistic

$$U = R_1 - \frac{n_1(n_1+1)}{2}$$

and if $R_2 < R_1$, the following U-statistic is computed:

$$U = R_2 - \frac{n_2(n_2+1)}{2}$$

The U-statistic is observed to be normally distributed with mean and variance given by

$$E(U) = \mu_U = \frac{n_1 n_2}{2}$$

$$V(U) = \sigma_U^2 = \frac{n_1 n_2}{n(n^3-1)} . \frac{n^3 - n}{12}$$

If there are ties within sample, then still $Z_0$ is used as test statistic. However, if there are ties across samples with the $i^{th}$ ran repeated $t_i$ times, then $V(U)$ is corrected as

$$V(U) = \sigma_U^2 = \frac{n_1 n_2}{n(n^3-1)} . \left(\frac{n^3 - n}{12} - \sum \frac{t_i^3 - t_i}{12}\right)$$

Now,

$$\frac{U - E(U)}{\sqrt{V(U)}} \sim N(0,1)$$

Let,

$$Z_0 = \frac{U - E(U)}{\sqrt{V(U)}} = \frac{U - \frac{n_1 \times n_2}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}}$$

Finally, the null hypothesis is rejected at $\alpha$ level of significance, if (a) for two tailed test $|Z_0| \geq Z_{\alpha/2}$ and (b) for one tailed test $|Z_0| \geq Z_\alpha$.

## Solved Problem

### #.1

A farmer wishes to determine whether there is a difference in yields between two different varieties of wheat I and wheat II. The following data shows the productions of wheat per unit area using the two varieties. Can the farmer conclude at 0.01 level of significance that a difference exists by using Mann-Whitney's U-test.

| | |
|---|---|
| Wheat I | ....................... |
| Wheat II | ...................... |

Solution-

H0 ............... H1: .................

Working Table-

| Wheat I | Rank | Wheat II | Rank |
|---|---|---|---|
| 15.9 | 9 | 16.4 | 12.5 |
| 15.3 | 5 | 16.8 | 15 |
| 16.4 | 12.5 | 17.1 | 17 |
| 14.9 | 3 | 16.9 | 16 |
| 15.3 | 5 | 18.0 | 19 |
| 16.0 | 10.5 | 15.6 | 8 |
| 14.6 | 2 | 18.1 | 20 |
| 15.3 | 5 | 17.2 | 18 |
| 14.5 | 1 | 15.4 | 7 |
| 16.6 | 14 | | |
| 16.0 | 10.5 | | |
| | $R_1 = 77.5$ | | $R_2 = 132.5$ |

Here, $n_1 = 11 \quad n_2 = 9 \quad n = 11 + 9 = 20$

Since $R_1 < R_2$, so test statistic is

$$U = R_1 - \frac{n_1(n_1+1)}{2} = 77.5 - \frac{11(11+1)}{2} = 11.5$$

Since samples are large we compute

$$E(U) = \frac{n_1.n_2}{2} = \frac{11 \times 9}{2} = 49.5$$

Since there is 1 tie of rank across two groups, so correction factor is

$\sum(t^3 - t) = 2^3 - 2 = 6$

$$V(U) = \frac{n_1 n_2}{n(n-1)}\left(\frac{n^3 - n}{12} - \frac{\sum(t_i^3 - t_i)}{12}\right)$$

$$= \frac{11 \times 9}{20(20-1)}\left(\frac{20^3 - 20}{12} - \frac{6}{12}\right) = 0.26(665 - 0.5) = 172.77$$

The test statistic is

$$Z_0 = \frac{U - E(U)}{\sqrt{V(U)}} = \frac{11.5 - 49.5}{\sqrt{172.77}} = -2.891$$

Rejected.

### Practice Problem

Comparing two kinds of emergency flares, a consumer testing service obtained the following burning times (rounded to the nearest tenth of a minute):

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Brand C | 19.5 | 21.5 | 15.3 | 17.4 | 16.8 | 16.6 | 20.3 | 22.5 | 21.3 | 23.4 | 19.7 | 21 |
| Brand D | 16.5 | 15.8 | 24.7 | 10.2 | 13.5 | 15.9 | 15.7 | 14 | 12.1 | 17.4 | 15.6 | 15.8 |

Use Mann-Whitney U-test at 0.01 level of significance to check whether it is reasonable to say that the burning rates of two kinds of flares are identical.

Solution-

$$H_0: populations\ are\ identical.$$

$$H_1: populations\ are\ not\ identical.$$

Working Table-

| Brand C | Rank | Brand D | Rank |
|---|---|---|---|
| 19.5 | 16 | 16.5 | 11 |
| 21.5 | 21 | 15.8 | 8.5 |
| 15.3 | 5 | 24.7 | 24 |
| 17.4 | 14.5 | 10.2 | 1 |
| 16.8 | 13 | 13.5 | 3 |
| 16.6 | 12 | 15.9 | 10 |
| 20.3 | 18 | 15.7 | 7 |
| 22.5 | 22 | 14.0 | 4 |
| 21.3 | 20 | 12.1 | 2 |
| 23.4 | 23 | 17.4 | 14.5 |
| 19.7 | 17 | 15.6 | 6 |
| 21.0 | 19 | 15.8 | 8.5 |
| | $R_1 = 200.5$ | | $R_2 = 99.5$ |

Here,

$$n_1 = n_2 = 12, \qquad n = 12 + 12 = 24$$

Since $R_2 < R_1$, so test statistic is

$$U = R_2 - \frac{n_2(n_2+1)}{2} = 99.5 - \frac{12(12+1)}{2} = 21.5$$

Since samples are large we compute

$$E(U) = \frac{n_1 . n_2}{2} = \frac{12 \times 12}{2} = 72$$

Since there is 1 tie of rank across two groups, so

$$V(U) = \frac{n_1 n_2}{n(n-1)}\left(\frac{n^3 - n}{12} - \frac{\sum(t_i^3 - t_i)}{12}\right)$$

$$= \frac{12 \times 12}{24(24-1)}\left(\frac{24^3 - 24}{12} - \frac{2^3 - 2}{12}\right) = 0.2680(1150 - 0.5) = 299.789$$

The test statistic is

$$Z_0 = \frac{U - E(U)}{\sqrt{V(U)}} = \frac{21.5 - 72}{\sqrt{299.789}} = -2.91$$

Rejected.

# Kruskal-Wallis H- Test

## Introduction

- It is non-parametric test procedure that is used to test equality of means of several populations the form of distribution of which may or may not be known.
- It is generalization of U-test in the sense that in U-test equality of mean of two populations are considered whereas, in H-test equality of means of several populations are considered.
- Let $X_1$, $X_2$ …………$X_k$ be k independent and continuous populations with means $\mu_1$, $\mu_2$ ………….$\mu_k$.
- The objective of the H-test is to test

  $$H_0: \mu_1 = \mu_2 = \cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots .. = \mu_k$$

  Or,

  $$H_0: All\ k\ populations\ are\ identical.$$

  against the alternative

  $$H_1: k\ populations\ are\ not\ all\ identical.$$

- For the test procedure k samples of sizes $n_1$, $n_2$, ……….. , $n_k$ are taken from each population.
- Then ranks are assigned to entire sample observations jointly.
- Let $R_1$, $R_2$, ……………. , $R_k$ be sum of ranks of observations in $1^{st}$, $2^{nd}$, ………., $k^{th}$ populations.
- Also let $n = \sum_{i=1}^{k} n_i$
- Now, if the null hypothesis is true it is observed that the statistic

$$H = \frac{12}{n(n+1)}\sum_{i=1}^{k}\left(\frac{R_i^2}{n_i}\right) - 3(n+1)$$

is distributed as $\chi 2$ distribution with k-1 degrees of freedom.

If $i^{th}$ rank is repeated ti times either within or between groups then a correction factor of

$$1 - \frac{\sum_i (t_i^3 - t_i)}{n^3 - n}$$

is introduced and corrected test statistic is

$$H = \frac{\frac{12}{n(n+1)}\sum_{i=1}^{k}\left(\frac{R_i^2}{n_i}\right) - 3(n+1)}{1 - \frac{\sum(t_i^3 - t_i)}{n^3 - n}}$$

Finally, the null hypothesis is rejected at $\alpha$ level of significance, if $H \geq \chi^2_{k-1}$.

*(Chi-square table is given below)*

**Chi-square Distribution Table**

| d.f. | .995 | .99 | .975 | .95 | .9 | .1 | .05 | .025 | .01 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 2.71 | 3.84 | 5.02 | 6.63 |
| 2 | 0.01 | 0.02 | 0.05 | 0.10 | 0.21 | 4.61 | 5.99 | 7.38 | 9.21 |
| 3 | 0.07 | 0.11 | 0.22 | 0.35 | 0.58 | 6.25 | 7.81 | 9.35 | 11.34 |
| 4 | 0.21 | 0.30 | 0.48 | 0.71 | 1.06 | 7.78 | 9.49 | 11.14 | 13.28 |
| 5 | 0.41 | 0.55 | 0.83 | 1.15 | 1.61 | 9.24 | 11.07 | 12.83 | 15.09 |
| 6 | 0.68 | 0.87 | 1.24 | 1.64 | 2.20 | 10.64 | 12.59 | 14.45 | 16.81 |
| 7 | 0.99 | 1.24 | 1.69 | 2.17 | 2.83 | 12.02 | 14.07 | 16.01 | 18.48 |
| 8 | 1.34 | 1.65 | 2.18 | 2.73 | 3.49 | 13.36 | 15.51 | 17.53 | 20.09 |
| 9 | 1.73 | 2.09 | 2.70 | 3.33 | 4.17 | 14.68 | 16.92 | 19.02 | 21.67 |
| 10 | 2.16 | 2.56 | 3.25 | 3.94 | 4.87 | 15.99 | 18.31 | 20.48 | 23.21 |
| 11 | 2.60 | 3.05 | 3.82 | 4.57 | 5.58 | 17.28 | 19.68 | 21.92 | 24.72 |
| 12 | 3.07 | 3.57 | 4.40 | 5.23 | 6.30 | 18.55 | 21.03 | 23.34 | 26.22 |
| 13 | 3.57 | 4.11 | 5.01 | 5.89 | 7.04 | 19.81 | 22.36 | 24.74 | 27.69 |
| 14 | 4.07 | 4.66 | 5.63 | 6.57 | 7.79 | 21.06 | 23.68 | 26.12 | 29.14 |
| 15 | 4.60 | 5.23 | 6.26 | 7.26 | 8.55 | 22.31 | 25.00 | 27.49 | 30.58 |
| 16 | 5.14 | 5.81 | 6.91 | 7.96 | 9.31 | 23.54 | 26.30 | 28.85 | 32.00 |
| 17 | 5.70 | 6.41 | 7.56 | 8.67 | 10.09 | 24.77 | 27.59 | 30.19 | 33.41 |
| 18 | 6.26 | 7.01 | 8.23 | 9.39 | 10.86 | 25.99 | 28.87 | 31.53 | 34.81 |
| 19 | 6.84 | 7.63 | 8.91 | 10.12 | 11.65 | 27.20 | 30.14 | 32.85 | 36.19 |
| 20 | 7.43 | 8.26 | 9.59 | 10.85 | 12.44 | 28.41 | 31.41 | 34.17 | 37.57 |
| 22 | 8.64 | 9.54 | 10.98 | 12.34 | 14.04 | 30.81 | 33.92 | 36.78 | 40.29 |
| 24 | 9.89 | 10.86 | 12.40 | 13.85 | 15.66 | 33.20 | 36.42 | 39.36 | 42.98 |
| 26 | 11.16 | 12.20 | 13.84 | 15.38 | 17.29 | 35.56 | 38.89 | 41.92 | 45.64 |
| 28 | 12.46 | 13.56 | 15.31 | 16.93 | 18.94 | 37.92 | 41.34 | 44.46 | 48.28 |
| 30 | 13.79 | 14.95 | 16.79 | 18.49 | 20.60 | 40.26 | 43.77 | 46.98 | 50.89 |
| 32 | 15.13 | 16.36 | 18.29 | 20.07 | 22.27 | 42.58 | 46.19 | 49.48 | 53.49 |
| 34 | 16.50 | 17.79 | 19.81 | 21.66 | 23.95 | 44.90 | 48.60 | 51.97 | 56.06 |
| 38 | 19.29 | 20.69 | 22.88 | 24.88 | 27.34 | 49.51 | 53.38 | 56.90 | 61.16 |

## Solved Problem

### #.1

Following are the number of printing mistakes counted on pages selected at random from three Sunday editions of a newspaper:

| | | | | | | |
|---|---|---|---|---|---|---|
| April 11 | 4 | 10 | 2 | 6 | 4 | 12 |
| April 18 | 8 | 5 | 13 | 8 | 8 | 10 |
| April 25 | 7 | 9 | 11 | 2 | 4 | 12 |

Use Kruskal-Wallis H-test at 0.05 level of significance to test that the samples come from identical populations.

Solution-

$$H_0: populations\ are\ identical$$

$$H_1: populations\ are\ not\ identical$$

Rank Table-

Here, $n_1 = n_2 = n_3 = 6 \quad n = 6 + 6 + 6 = 18$

| | | | | | | | $R_i$ | $R_i^2/n_i$ |
|---|---|---|---|---|---|---|---|---|
| Apr. 11 | 4 | 13.5 | 1.5 | 6 | 4 | 16.5 | 46.5 | 360.375 |
| Apr. 18 | 10 | 6 | 18 | 10 | 10 | 13.5 | 67.5 | 759.375 |
| Apr. 25 | 8 | 12 | 15 | 1.5 | 4 | 16.5 | 57 | 541.5 |
| | | | | | | | | 1661.25 |

Here, rank 1.5 occurs 2 times, rank 4 occurs 3 times, rank 10 occurs 3 times, rank 13.5 occurs 2 times and rank 16.5 occurs 2 times, so

$$\sum\left(t_i^3 - t_i\right) = (2^3 - 2) + (3^3 - 2) + (3^3 - 3) + (2^3 - 2) + (2^3 - 2) = 48$$

The test statistic is

$$H = \frac{\frac{12}{n(n+1)}\sum_{i=1}^{k}\left(\frac{R_i^2}{n_i}\right) - 3(n+1)}{1 - \frac{\sum(t_i^3 - t_i)}{n^3 - n}}$$

$$= \frac{\frac{12}{18(18+1)} \times 1661.25 - 3(18+1)}{1 - \frac{48}{18^3 - 18}} = \cdots \ldots ..$$

Critical value is $\chi^2_{0.05,3-1} = 5.991$

Not rejected.

## Practice Problem

An experiment was designed to compare three preventive methods against corrosion yielded the following maximum depths (in thousands of an inch) in pieces of wires subjected to the respective treatments:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Method I | 77 | 54 | 67 | 74 | 71 | 66 | |
| Method II | 60 | 41 | 59 | 65 | 62 | 64 | 52 |
| Method III | 49 | 52 | 69 | 47 | 56 | | |

Use the Kruskal-Wallis H-test at 0.05 level of significance to test the hypothesis that the three samples come from identical populations.

Solution-

H0: the three samples come from identical populations

H1: the three samples do not come from identical populations

Here, $n_1 = 6, n_2 = 7, n_3 = 5 \qquad n = 6 + 7 + 5 = 18$

Rank Table

| | | | | | | | | $R_i$ | $R_i^2/n_i$ |
|---|---|---|---|---|---|---|---|---|---|
| MethodI | 18 | 6 | 14 | 17 | 16 | 13 | | 84 | 1176 |
| MethodII | 9 | 1 | 8 | 12 | 10 | 11 | 4.5 | 55.5 | 440.04 |
| MethodIII | 3 | 4.5 | 15 | 2 | 7 | | | 31.5 | 198.45 |
| | | | | | | | | | 1814.49 |

Here, rank 4.5 occurs 2 times so

$$\sum\left(t_i^3 - t_i\right) = (2^3 - 2) = 6$$

The test statistic is

$$H = \frac{\frac{12}{n(n+1)}\sum_{i=1}^{k}\left(\frac{R_i^2}{n_i}\right) - 3(n+1)}{1 - \frac{\sum(t_i^3 - t_i)}{n^3 - n}}$$

$$= \frac{\frac{12}{18(18+1)} \times 1814.49 - 3(18+1)}{1 - \frac{6}{18^3 - 18}} = \frac{6.667}{0.99} = 6.67$$

Critical value is $\chi^2_{0.05,3-1} = 5.991$

Not rejected.