

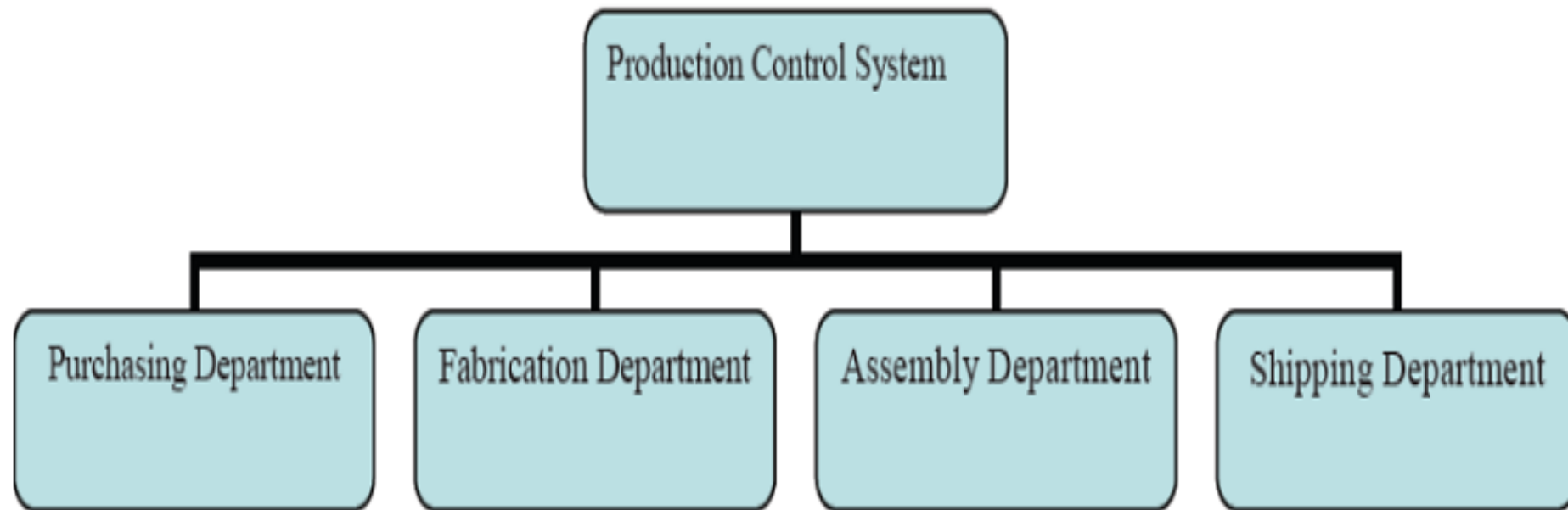
Simulation and modeling

# Introduction to Simulation

- Simulation is the imitation of the operation of a real-world process or system over time.
- Simulation involves the generation of an artificial history of the system, and the observation of that artificial history to draw inferences concerning the operating characteristics of the real system that is represented.
- Simulation is the numerical technique for conducting experiments on digital computer, which involves logical and mathematical relationships that interact to describe the behavior and the structure of a complex real world system over extended period of time.
- The process of designing a model of a real system, implementing the model as a computer program, and conducting experiments with the model for the purpose of understanding the behavior of the system, or evaluating strategies for the operation of the system.

# System Concepts

- A system is defined as a group of objects that are joined together in some regular interaction or interdependence for the accomplishment of some task. For example:  
*Production system for manufacturing automobiles.*
- A system is usually considered as a set of inter-related factors, which are described as entities activities and have properties or attributes.
- Processes that cause system changes are called activities.
- The state of a system is a description of all entities, attributes and the activities at any time.



# Components of system

- An **entity** represents an object that requires explicit definition.
- An entity can be dynamic in that it moves through the system, or it can be static in that it serves other entities.
- In the example, the customer is a dynamic entity, whereas the bank teller is a static entity.
- An entity may have **attributes** that pertain to that entity alone. Thus, attributes should be considered as local values.
- In the example, an attribute of the entity could be the time of arrival.
- Attributes of interest in one investigation may not be of interest in another investigation.

# Components of system

- Thus, if red parts and blue parts are being manufactured, the color could be an attribute.
- Processes that cause system changes are called **Activities**. E.g. Manufacturing process of the department.
- **An event** is defined as an instantaneous occurrence that may change the state of the system.
- In the bank example, events include the arrival of a customer for service at the bank, the beginning of service for a customer, and the completion of a service.
- There are both internal and external events, also called endogenous and exogenous events, respectively.

# Components of system

Example

System	Entities	Attributes	Activities
Traffic	Cars, bus, pedestrian	Speed, model	Driving, walking
Bank	Customer	Balance	Depositing, arrival of costomer, ....
Supermarket	Customers	Shopping list	Checking_out, .....

# Components of system

- An endogenous event in the example is the beginning of service of the customer since that is within the system being simulated.
- An exogenous event is the arrival of a customer for service since that occurrence is outside of the simulation.



# Components of system

- The state of a system is defined to be that collection of variables necessary to describe the system at any time, relative to the objectives of the study.
- In the study of a bank, possible state variables are the number of busy tellers, the number of customers waiting in line or being served, and the arrival time of the next customer.
- So the system state variables are the collection of all information needed to define what is happening within the system to a sufficient level (i.e., to attain the desired output) at a given point in time.

## Example

<i>System</i>	<i>Entities</i>	<i>Attributes</i>	<i>Activities</i>	<i>Events</i>	<i>State Variables</i>
Banking	Customers	Checking account balance	Making deposits	Arrival; departure	Number of busy tellers; number of customers waiting
Rapid rail	Riders	Origination; destination	Traveling	Arrival at station; arrival at destination	Number of riders waiting at each station; number of riders in transit
Production	Machines	Speed; capacity; breakdown rate	Welding; stamping	Breakdown	Status of machines (busy, idle, or down)
Communications	Messages	Length; destination	Transmitting	Arrival at destination	Number waiting to be transmitted
Inventory	Warehouse	Capacity	Withdrawing	Demand	Levels of inventory; backlogged demands

# Components of system

## **Open System/Close System**

- A system with exogenous activities is considered as open system and a system with strict endogenous activities is called a closed system.

# Components of system

## **System Environment**

- The external components which interact with the system and produce necessary changes are said to constitute the system environment.
- In modeling systems, it is necessary to decide on the boundary between the system and its environment.
- This decision may depend on the purpose of the study.
- Example: In a factory system, the factors controlling arrival of orders may be considered to be outside the factory but yet a part of the system environment.
- When, we consider the demand and supply of goods, there is certainly a relationship between the factory output and arrival of orders.
- This relationship is considered as an activity of the system.

# Components of system

## **Endogenous System**

- The term endogenous is used to describe activities and events occurring within a system. Example: Drawing cash in a bank.

## **Exogenous System**

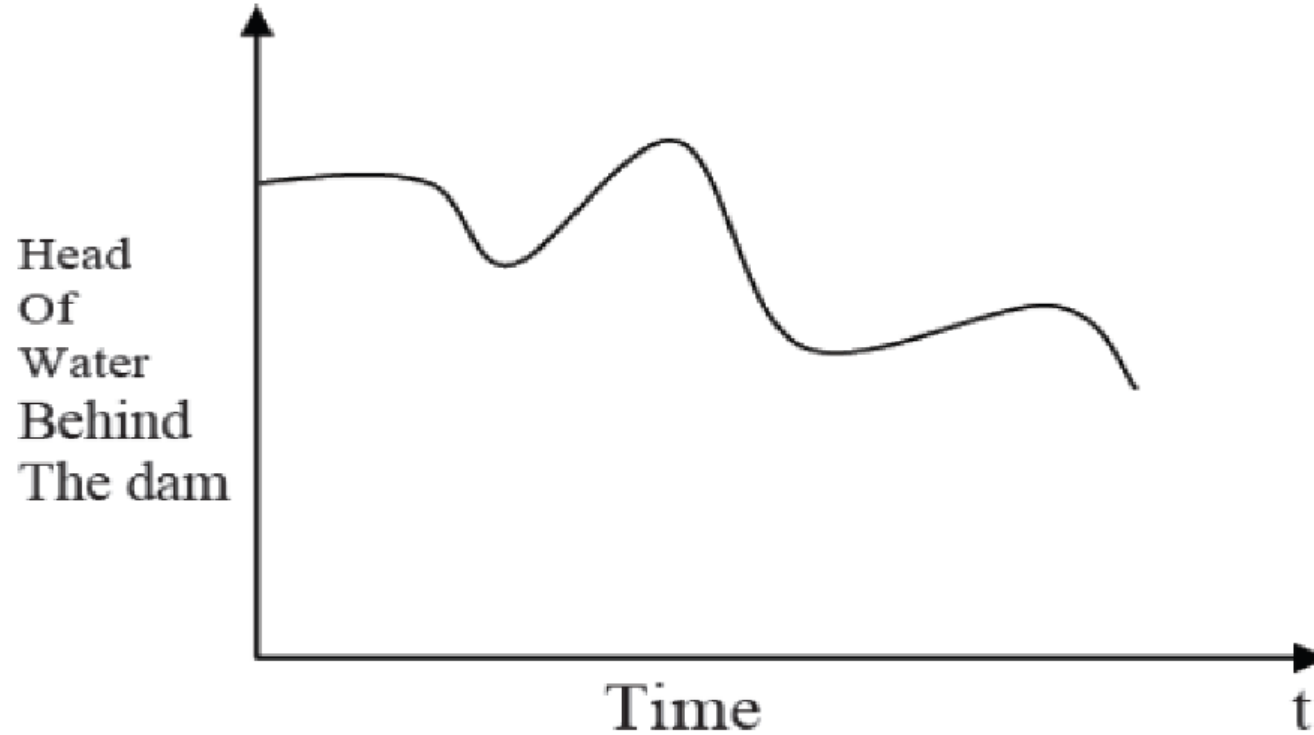
- The term exogenous is used to describe activities and events in the environment that affect the system. Example: Arrival of customers.

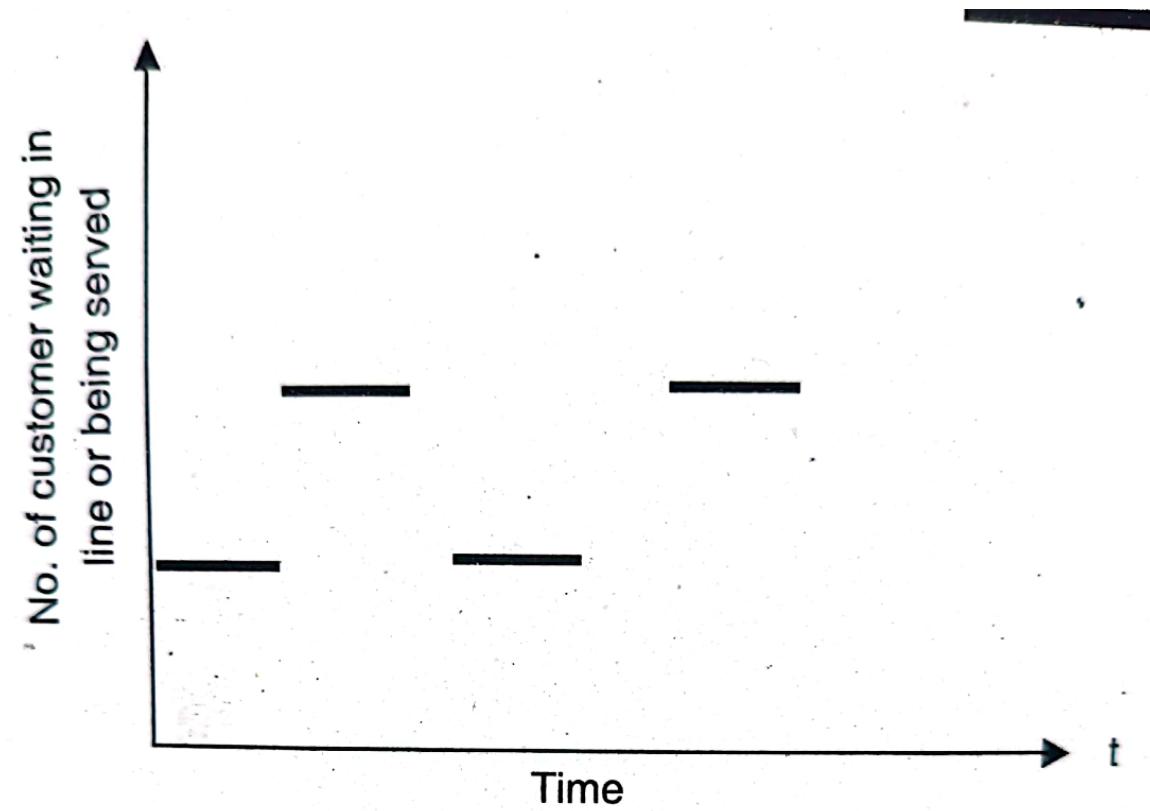
# Components of system

## **Discrete and continuous system**

- Discrete system is one in which the state variables changes only at a discrete set of time. For example: banking system in which no of customers (state variable) changes only when a customer arrives or service provided to customer i.e. customer depart form system.
- Continuous system is one in which the state variables change continuously over time. For example, during winter seasons level of which water decreases gradually and during rainy season level of water increase gradually. The change in water level is continuous.

- The figure below shows the change of water level over time.





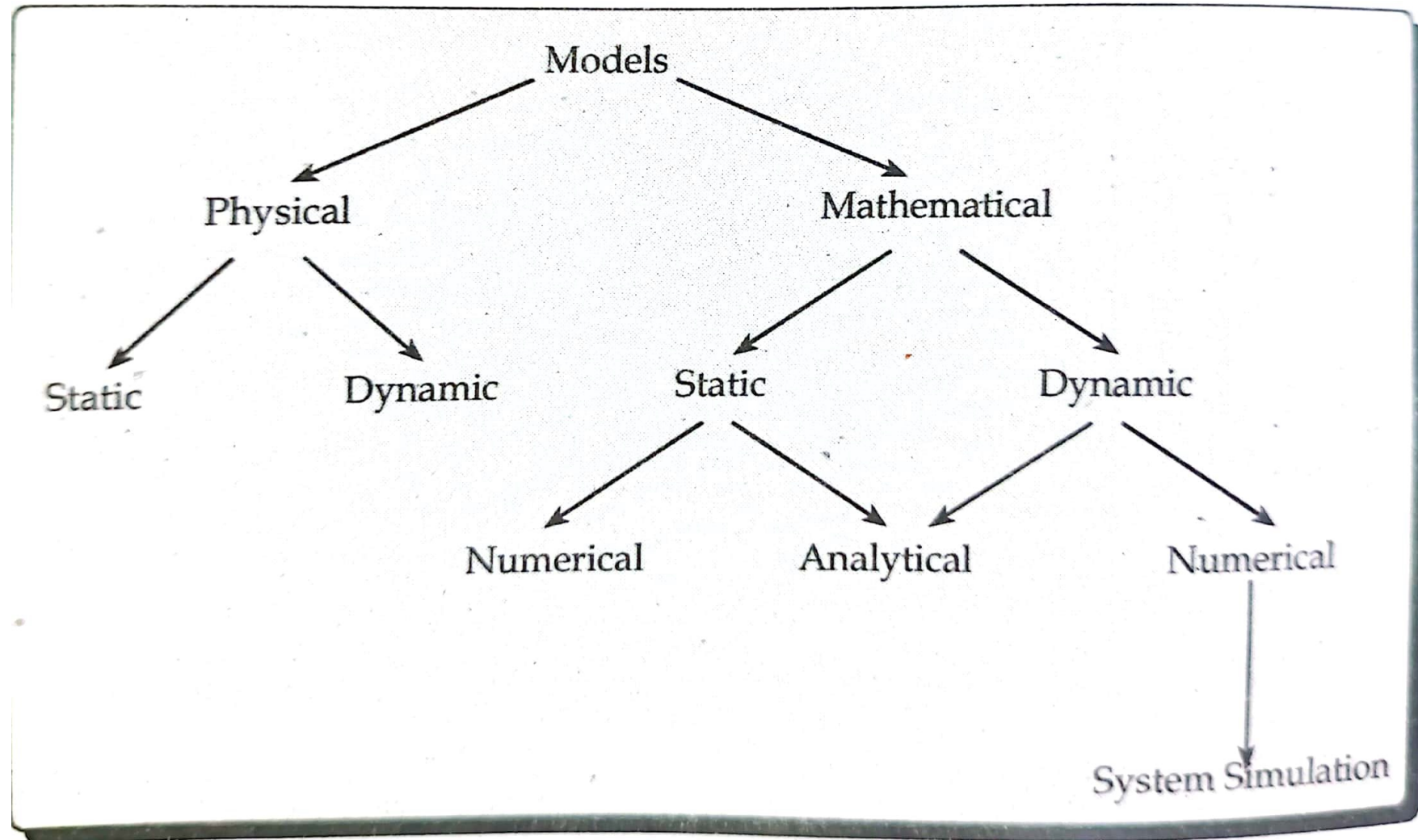
**Figure 1.4: Discrete-system state variable**



# Model of a System

- A model is defined as a representation of a system for the purpose of studying the system
- It is necessary to consider only those aspects of the system that affect the problem under investigation. These aspects are represented in a model, and by definition it is a simplification of the system.

# Types of Models



# Static Physical Model

- The best known examples of physical models are scale models. In ship building, making a scale model provides a simple way of determining the exact measurements of the plates covering the hull, rather than having to produce drawings of complicated 3D shapes.
- Scientists have used models in which spheres represent atoms and rods or specially shaped sheets of metal connect the sphere to represent atomic bonds. Such DNA molecule models are static physical models.

# Dynamic Physical Model

- They rely upon an analogy between the system being studied and some other system of a different nature, the analogy usually depending upon an underlying similarity in the forces governing the behaviour of the systems.

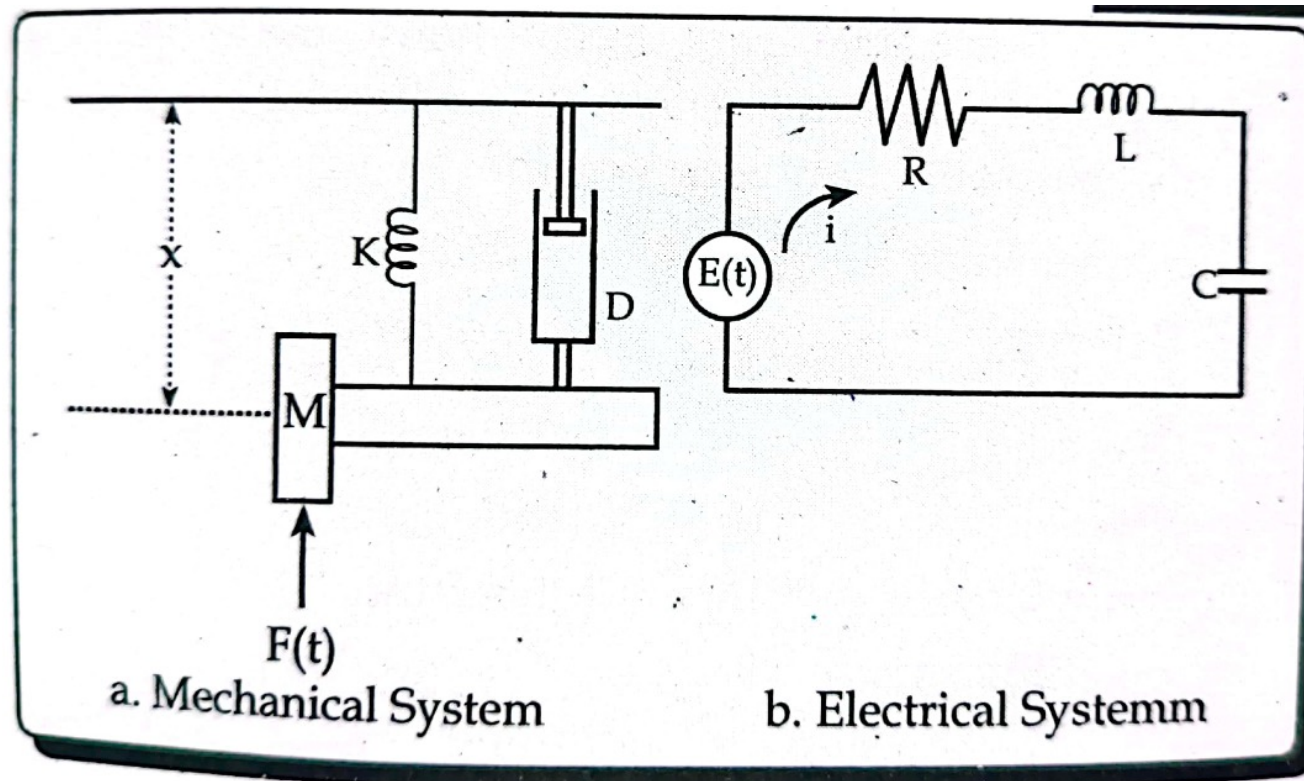


Fig: 1.6: Analogy between mechanical and electrical systems

- Figure a represents a mass that is subject to an applied force  $F(t)$  varying with time, a spring whose force is proportional to its extension or contraction, and a shock absorber that exerts a damping force proportional to the velocity of the mass. The system might, for example, represent the suspension of an automobile wheel when the automobile body is assumed to be immobile in a vertical direction. It can be shown that the motion of the system is described by the following differential equations:

$$M\ddot{x} + D\dot{x} + Kx = KF(t)$$

where  $x$  is the distance moved,

$M$  is the mass...

$K$  is the stiffness of the spring,

$D$  is the damping factor of the shock absorber.

Figure 1.6(b) represents an electrical circuit with an inductance  $L$ , a resistance  $R$ , and a capacitance  $C$ , connected in series with a voltage source that varies in time according to the function  $E(t)$ . If  $q$  is the charge on the capacitance, it can be shown that the behavior of the circuit is governed by the following differential equation:

$$L\ddot{q} + R\dot{q} + \frac{q}{C} = \frac{E(t)}{C}$$



Inspection of these two equations shows that they have exactly the same form and that the following equivalences occur between the quantities in the two systems:

Displacement	$x$	Charge	$q$
Velocity	$\dot{x}$	Current	$I = (\dot{q})$
Force	$F$	Voltage	$E$
Mass	$M$	Inductance	$L$
Damping factor	$D$	Resistance	$R$
Spring stiffness	$K$	$\frac{1}{C}$	$\frac{1}{C}$
		Capacitance	

# Static Mathematical Model

- A static model gives the relationships between the system attributes when the system is in equilibrium. If the point of equilibrium is changed by altering any of the attribute values, the model enables the new values for all the attributes to be derived but does not show the way in which they change to their new values.



Since the relationships have been assumed linear, the complete market model can be written mathematically as follows:

$$Q = a - bP$$

$$S = c + dP$$

$$S = Q$$

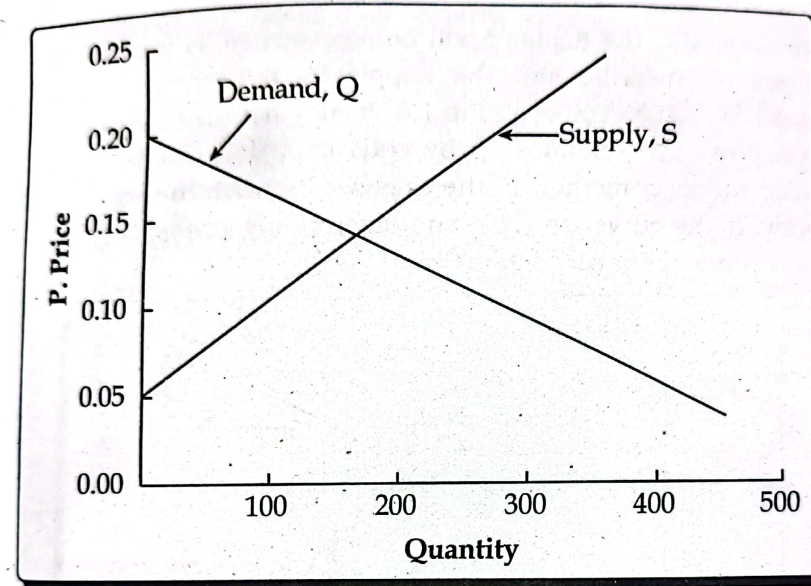


Figure 1.7: Linear Market model

The last equation states the condition for the market to be cleared; it says supply equals demand and, so, determines the price to which the market will settle.

For the model to correspond to normal market conditions in which demand goes down and supply increases as price goes up the coefficients  $b$  and  $d$  need to be positive numbers. For realistic, positive results, the coefficient  $a$  must also be positive. Figure 1.7 has been plotted for the following values of the coefficients:

$$a = 60$$

$$b = 3,000$$

$$c = -100$$

$$d = 2,000$$

The fact that linear relationships have been assumed allows the model to be solved analytically. The equilibrium market price, in fact, is given by the following expression:

$$P = \frac{a - c}{b + d}$$

With the chosen values, the equilibrium price is 0.14, which corresponds to a supply of 180.

More usually, the demand will be represented by a curve that slopes downwards, and the supply by a curve that slopes upwards, as illustrated in Fig 1.8. It may not then be possible to express the relationship by equations that can be solved. Some numeric method is then needed to solve the equations. Drawing the curves to scale and determining graphically where they intersect is one such method.

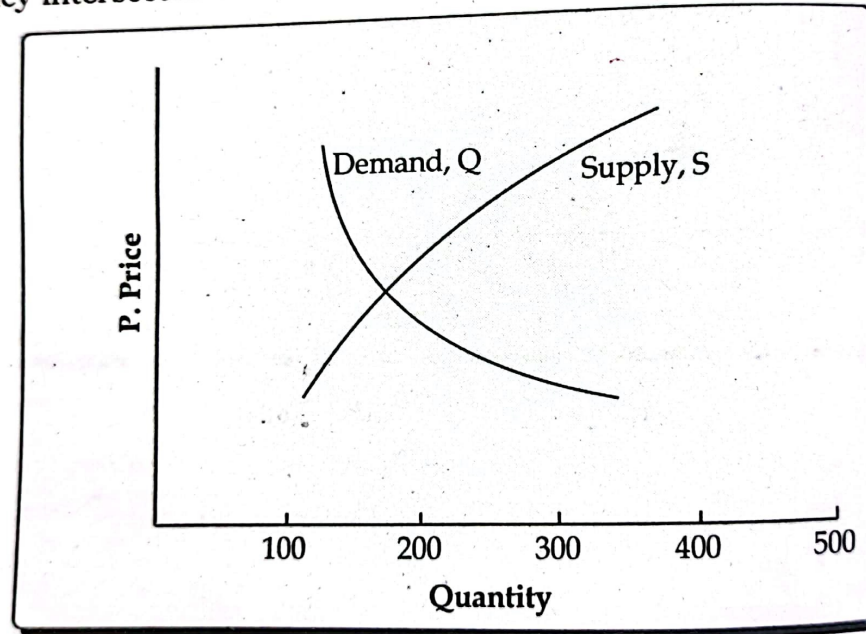


Figure 1.8: Non-linear Market model

# Mathematical Model

- It uses symbolic notation and mathematical equation to represent system.
- The system attributes are represented by variables and the activities are represented by mathematical function. Example:  $f(x) = mx + c$  is a mathematical model of a line.

# Static Model

- Static models can only show the values that the system attributes value does not change over time.
- Example: Scientist has used models in which sphere represents atom, sheet of metal to connect the sphere to represent atomic bonds. Graphs are used to model the various system based on network.
- A map is also a kind of graph. These models are sometimes said to be iconic models and are of kind static physical models.

# Dynamic Model

- Dynamic models follow the changes over time that result from system activities. The mechanical and electrical systems are the example of dynamic system.
- Generally, dynamic models involve the computation of variable value over time and hence they are represented by differential equations.

# Analytical Models:

- In mathematical model, we can differentiate the model on the basis of solution technique used to solve the model.
- Analytical technique means using deductive reasoning of mathematical theory to solve a model. Such models are known as analytical model

# Numerical models

- Numerical models involve applying computational process to solve equations.
- For example: we may solve differential equation numerically when the specific limit of variable is given.
- The analytical methods to produce solution may take situation numerical methods are preferred.

# Deterministic Model

- It contains no random variables.
- They have a known set of inputs which will result in a unique set of outputs. Ex: Arrival of patients to the Dentist at the scheduled appointment time.



# Stochastic Model

- **Stochastic Model** Has one or more random variable as inputs.  
Random inputs leads to random outputs.
- Ex: Simulation of a bank involves random inter-arrival and service times.

# Principles used in Modeling

- Guidelines used in modeling
  - It is not possible provide rule by which models are built. But a number of guidelines can be stated.
  - The different viewpoints from which we can judge whether certain info.

# 1. Block –Building:

- The description of system should be organized as a sequence of blocks.
- It simplifies the interaction between block within system.
- Then it will be easy to describe the whole system in terms of interaction between the block and can be represented graphically as simple block diagram. For example:-the block of factory system. Fig:  
Block diagram of factory system

## 2. Relevance:

- The model should only include relevant information.
- For example, if the factory system study aims to compare the efficiency of different operating rules it is not relevant to consider the mining of employee as an activity.
- Irrelevant information should not include despite of being no harm because it increases the complexity of model and takes more time and effort to solve model.

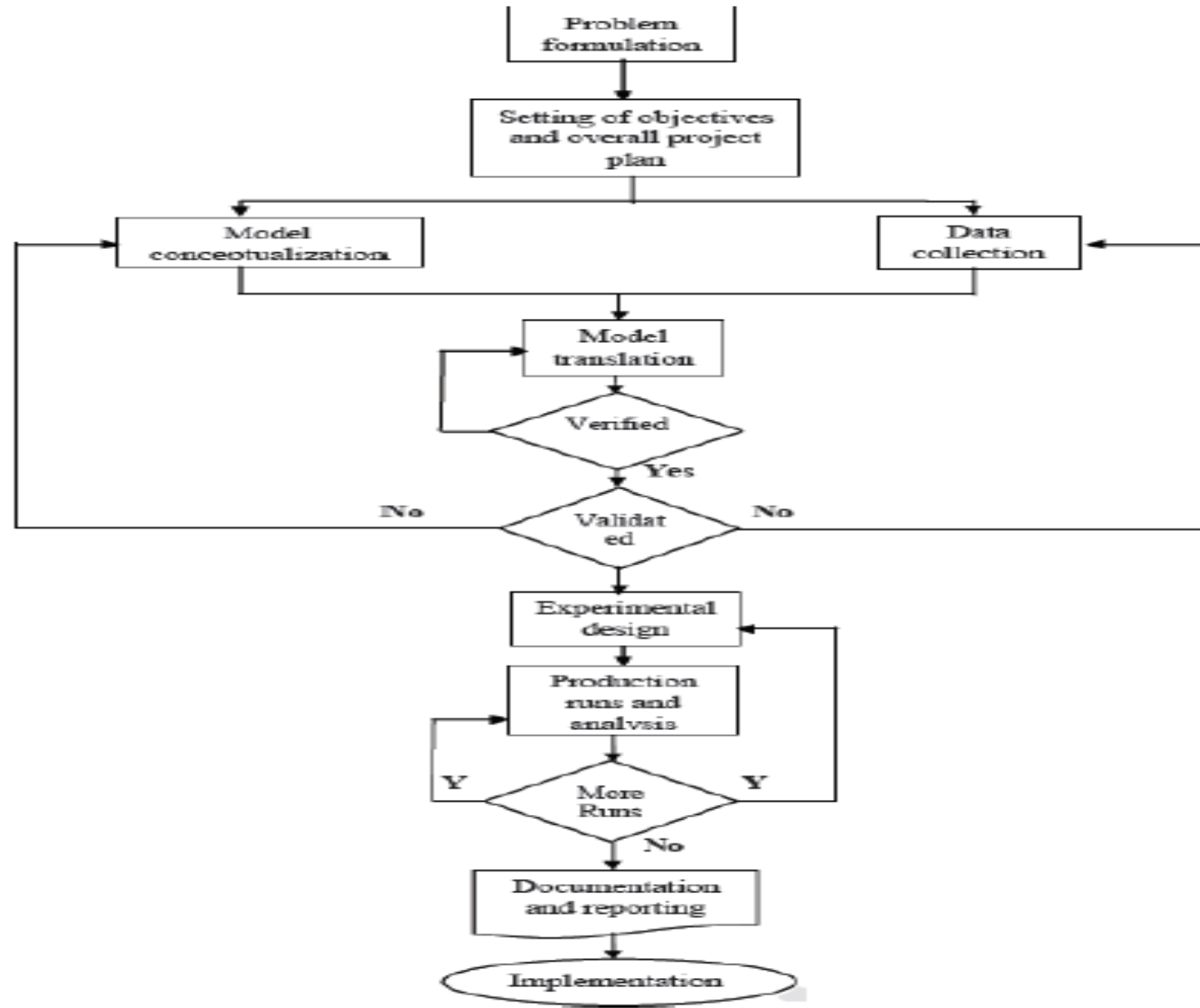
### 3. Accuracy:

- The gathered information should be accurate as well.
- For example in aircraft system the accuracy as movement of the aircraft depends upon the representations of airframe such as a rigid body.

## 4. Aggregation:

- It should be considered that to which numbers of individual entities can be grouped into a block.
- For example in factory system, different department are grouped together handled by production manger.

# Steps on simulation Study



# Problem formulation:

- Every study begins with a study of the problem, provided by policy makers.
- Analyst ensures it is completely understood.
- If it is developed by analyst, policy makers should understand and agree with it.
- *Summary: Clearly state the problem.*



# Setting of objectives and overall project plan:

- The objectives indicate the questions to be answered by simulation.
- At this point, a determination should be made concerning whether simulation is the appropriate methodology or analytical techniques are sufficient.
- Assuming it is appropriate, the overall project plan should include plan for the study in terms of the number of people involved, the cost of the study, and the number of days required to accomplish each phase of the work with the anticipated results
- *Summary: How we should approach the problem?*

# Model conceptualization:

- The construction of a model of a system is probably as much art as science.
- The art of modeling is enhanced by ability to abstract the essential features of a problem such as entities, attributes, events, activities and state variables.
- It is better to start with a simple model and build toward greater complexity.
- Model conceptualization enhances the quality of the resulting model and increases the confidence of the model user in the application of the model.
- *Summary: Establish a reasonable model.*

# Data collection:

- There is a constant interplay between the construction of model and the collection of needed input data.
- It is done in the early stages. Objective types of data are collected.

*Summary: Collect the data necessary to run the simulation.*

# Model translation:

- Real-world systems result in models that require a great deal of information storage and computation.
- It can be programmed by using simulation languages or special purpose simulation software.
- Simulation languages are powerful and flexible.
- *Summary: Convert the model into a programming language.*

# Verification:

- It pertains to the computer program and checking the performance.
- If the input parameters and logical structure are correctly represented, verification is completed.

*Summary: Verify the model by checking if the program works properly.  
Perform debugging and remove logical errors.*

# Validation:

- It is the determination that a model is an accurate representation of the real system.
- It is achieved through calibration of the model.
- The calibration of model is an iterative process of comparing the model to actual system behavior and the discrepancies between the two.
- *Summary: Check if the system accurately represents the real system. Testing is done here.*

# Experimental design:

- An *Experimental Design* is the laying out of a detailed experimental plan in advance of doing the experiment.
- Well chosen experimental designs maximize the amount of "information" that can be obtained for a given amount of experimental effort.
- *Summary: How many runs? For how long? What kind of input variations?*

# Production runs and analysis:

- They are used to estimate measures of performance for the system designs that are being simulated.
- *Summary: Actual running the simulation, collect and analyze the output*



# Repetition:

- Based on the analysis of runs that have been completed, the analyst determines if additional runs are needed and what experimental design those additional experiments should follow.
- *Summary: Repeat the experiments if necessary.*

# Documentation and Reporting:

- Two types of documentation:

- Program documentation:**

- Can be used by the same or different analysts to understand how the program operates.

- Further modification will be easier.

- Model users can change the input parameters for better performance.

- Process documentation:**

- Gives the history of a simulation project.
    - The result of all analysis should be reported clearly and concisely in a final report.
    - This enables to review the final formulation and alternatives, results of the experiments and the recommended solution to the problem.

- *Summary: Document and report the results.*

# Implementation:

- Success depends on the previous steps.
- If the model user has been thoroughly involved and understands the nature of the model and its outputs, likelihood of a vigorous implementation is enhanced.

# Phases of a simulation study

- **Phase 1: Problem Formulation**

This includes problem formulation step.

It is period of discovery/orientation ·

The analyst may have to restart the process if it is not fine-tuned ·

Recalibrations and clarifications may occur in this phase or another phase.

- **Phase 2: Model Building**

This includes model construction , data collection , programming and Validation of the model.

continuing interplay is required among the steps ·

Exclusion of model user results in implications during implementation

# Cont..

- **Phase 3: Running the model**

This includes experimental design, simulation runs and analysis of results.

Conceives a thorough plan for experimenting ·

Discrete-event stochastic is a statistical experiment ·

The output variables are estimates that contain random error and therefore proper statistical analysis is required.

- **Phase 4: Implementation**

This includes documentation and implementation.

Successful implementation depends on the involvement of user and every steps successful completion.

# When is simulation appropriate?

System simulation is the technique of solving problems by the observation of the performance, over time, of a dynamic model of the system.

## **Simulation is appropriate:**

- When we want to try out new policies and decision rules for operating a system, before running the risk of experimenting of the real system.
- When we want to experiment with new situations about which we have little or no information so as to prepare for what may happen.
- When we want to experiment in a low cost, low risk environment.
- When we want to verify analytic solutions or when analytical solutions can't solve the problem.
- When we want to train personnel's since it allows learning without the cost and without disruption of on-the-job learning.

# When is simulation inappropriate?

- Simulation is inappropriate when the problem can be solved using common sense.
- Simulation is inappropriate when the problem can be solved analytically or when it is easier to perform direct experiments.
- Simulation is inappropriate when the cost exceeds savings.
- Simulation is inappropriate when there is not enough time.
- Simulation is inappropriate if managers have unreasonable expectation on the power of simulation.
- If the system behavior is too complex or cannot be defined, simulation is not appropriate.

# Advantages of simulation

- A simulation study can help in understanding how the system operates rather than how individuals think the system operates.
- Managerial and technical decisions can be taken for new policies, operating procedures, etc without disrupting the ongoing operations of the real system.
- Dangerous (atomic reactor, flight simulator, etc) systems can be tested in extreme conditions.
- “what-if” type questions, how and why certain phenomenon occurred can be answered closely.
- Prohibitively high cost of developing an infeasible project can be cut out in the design phase.



# Limitations of the simulation technique

- Model building requires special training.
- Simulation results may be difficult to interpret.
- It can be quite time consuming and expensive.
- Simulation can be used in some cases when an analytical solution is possible or even preferable.

# Application areas of simulation

- There are various application areas of simulation study such as experimentation, operational planning, training, etc.

## **Manufacturing:**

- Simulation can be used to predict the performance of an existing or planned system in manufacturing.
- Some measures of system performance include throughput under average and peak loads, system cycle time ( time to produce one part), utilization of resources, staffing requirements, etc.

# Cont..

## **Urban Planning:**

- Metropolitan planning agencies or urban planners use simulators like “UrbanSim” to evolve cities under various policy decisions.
- They explicitly represent land use, transport, electricity systems, etc.

## **Disaster Preparation:**

- Disaster preparedness simulations involve training on how to handle terrorism attacks, natural disasters, or other life-threatening emergencies.

# Cont..

## **Flight:**

- Flight simulation training devices are used to train pilots on the ground.
- In comparison to training in an actual aircraft, simulation based training allows for the training of maneuvers or situations that may be impractical (or even dangerous) to perform in the aircraft, while keeping the pilot and instructor in a relatively low-risk environment on the ground.
- For example, electrical system failures, hydraulic system failures, etc can be simulated without risk to the pilots or an aircraft.

# Sports:

- In sports, computer simulations are often done to predict the outcome of events and the performance of individual sportsperson.
- Companies like What If Sports and StatFox specialize in using their simulations for not only predicting their game results, but how well an individual player will do too. For this, they use various statistics.

# Cont..

## **Weather:**

- weather conditions by inferring from previous data is one of the real uses of simulation.
- It can be used to forewarn about extreme weather conditions like the path of an active hurricane.

# Hybrid simulation

- For most studies, the system under study is clearly either of continuous or of discrete nature and it is the determining factor in deciding whether to use an analog or a digital computer for system simulation.
- If the system being simulated is an interconnection of continuous and discrete subsystem, then such system simulation is known as hybrid simulation.
- Such hybrid system can be digital computer being linked together.
- Hybrid simulation requires high speed converters to transform signals from analog to digital and vice-versa.

# Real-time simulation

- In real time simulation, actual devices (which are part of a system) are used in conjunction with either digital computer or hybrid computer.
- It provides the simulation of the parts of the system that do not exist or that cannot be easily used in an experiment.
- That is, the basic idea of real-time simulation is “uses the actual part if they are appropriate to use in an experiment otherwise uses the simulation of the parts of the system”.
- Real-time simulation requires computers that can operate in real-time; this means that they must be able to respond immediately to signals sent from the physical devices, and send out signals at specific points in time.
- A well-known example is “simulation to train pilots”. It uses the devices for training pilots by giving them the impression that they are at the controls of an aircraft. It requires real-time simulator of the plane, its control system, the weather and other environmental conditions.



# Assignment

1. What is model? What are the different types of model? Give example for each.
2. Differentiate between dynamic physical models and static physical models with example.
3. Define system modeling and simulation. Describe the difference between static and dynamic physical model with suitable example.
4. Differentiate between discrete and continuous system.
5. Differentiate between analytical models and numerical models.
6. When is simulation appropriate and when it is not?
7. What are the advantages and disadvantages of simulation?
8. Explain different phases of simulation in brief.
9. Explain different phases of simulation study in brief.
10. Define differential equation. What is the role of differential equations in simulation and modeling?