Efficiency of RBD over CRD

The efficiency of an experiment carried in RBD with 'a' treatments and 'n' number of replicates in each block is given by

$$Efficiency\ (\eta) = 1 + \frac{(n-1)(MS_{Bl} - MS_E)}{(na-1)MS_E}$$

Solved Problem

From following ANOVA table of RBD, determine its efficiency with respect to CRD-

S.V.	S.S.	d.f.	M.S.
Treatments	750	5	150
Blocks	180	3	60
Error	200	15	13.33
Total	1130	23	---

Here,

$$a = 6$$

$$n = 4$$

The d.f. of treatment is 5. We know d.f. of treatment = a – 1, so 5 = a – 1 and a = 6

We need to study only those form of RBD in which there are only one observation for each treatment belonging to each block. So number of blocks (b) = number of replicates (n). Here it is given that degrees of freedom of blocks is 3. Now degrees of freedom of blocks = b – 1, so, 3 = b – 1 and b = 4. Hence number of replicates n = 4.

Also, $MS_{Tr} = 150, MS_{Bl} = 60\ and\ ME_E = 13.33$

So, efficiency of RBD over CRD is

$$Efficiency\ (\eta) = 1 + \frac{(n-1)(MS_{Bl} - MS_E)}{(na-1)MS_E}$$

$$= 1 + \frac{(4-1)(60-13.33)}{(4\times 6-1)\times 13.33} = 1.456 = 145.6\%$$

So, RBD is 45.6% more efficient than CRD.

Practice problem-

From following ANOVA table of RBD, determine its efficiency with respect to CRD-

S.V.	S.S.	d.f.	M.S.
Treatments	15.66	3	5.22
Blocks	21.55	5	4.31
Error	12.3	15	0.82
Total	49.51	23	---

Ans. 12.75% more efficient.

Efficiency of LSD over CRD

Efficiency of LSD (having m treatments and so m rows and m columns) with respect to CRD is given by

$$Efficiency\ (\eta) = \frac{(m-1)MS_E + MS_R + MS_C}{(m+1)MS_E}$$

Solved problem

For following ANOVA table of $4 \times 4\ LSD$, determine its efficiency with respect to CRD-

S.V.	S.S.	d.f.	M.S.
Rows	2.133	3	0.711
Columns	2.203	3	0.734
Treatments	10.633	3	3.554
Error	7.059	6	1.177
Total	22.058	15	---

Solution- Here $number\ of\ rows = number\ of\ columns = number\ of\ treatments = m = 4$.

Also, $MS_R = 0.711, MS_C = 0.734, MS_{Tr} = 3.554$

Now efficiency is given by

$$Efficiency\ (\eta) = \frac{(m-1)MS_E + MS_R + MS_C}{(m+1)MS_E}$$

$$= \frac{(4-1)1.177 + 0.711 + 0.734}{(4+1) \times 1.177} = 0.845 = 84.5\%$$

Hence LSD is $(100 - 84.5)\% = 15.5\%$ less efficient than CRD.

Efficiency of LSD over RBD

LSD consists of two way blocking (one into rows and another into columns) and there is only one type blocking in RBD.

So, to consider efficiency of LSD over RBD, we need to consider whether we need to keep row-wise blocking by removing column-wise blocking or vice-versa.

Case-I – When row is taken as block

$$Efficiency\ (\eta) = \frac{(m-1)MS_E + MS_C}{(m+1)MS_E}$$

Case-II – When column is taken as block

$$Efficiency\ (\eta) = \frac{(m-1)MS_E + MS_R}{(m+1)MS_E}$$

Solved problem

For following ANOVA table of $4 \times 4\ LSD$, determine its efficiency with respect to RBD when (a) row is taken as block (b) column is taken as block-

S.V.	S.S.	d.f.	M.S.
Rows	2.133	3	0.711
Columns	2.203	3	0.734
Treatments	10.633	3	3.554
Error	7.059	6	1.177
Total	22.058	15	---

Solution – (*asking to solve yourself*)

Estimation of a Missing Value in RBD

As shown in problem below, in RBD if one of the observations, say x, is missing, then it can be estimated by

$$x = \frac{aT' + bB' - G'}{(a-1)(b-1)}$$

where,

$a = number\ of\ treatments \qquad b = number\ of\ blocks$

B' = sum of all known values in block containing missing value

T' = sum of all known values in treatment containing missing value

G' = grand sum of all knows values

In missing plot technique after estimating unknown (missing) value by using above expression, it is substituted for the missing value and ANOVA is carried out. However, one degree of freedom is decreased in total sum of squares (SS_T) which causes decrease in one degree of freedom in SS_E.

Hence to get better result, an adjustment factor k (given below) is subtracted from SS_{Tr}, so that adjusted sum of square of treatments is $SS_{Tr}(Adj.) = SS_{Tr} - k$, where

$$k = \frac{(B' + aT' - G')^2}{a(a-1)(b-1)^2}$$

Solved problem

The table below represents the yield of 3 varieties in 4 block experiment for which one observation is missing. Estimate the missing value and analyze the data.

A 18.1	B ??	A 15.2	C 13.2
C 16.0	A 12.1	B 17.5	A 16.6
B 16.3	C 13.4	C 16.3	B 18.1

Solution-

Estimation of missing value, (say x).

Working table-

Treatments	Blocks				Total
	I	II	III	IV	
A	18.1	12.1	15.2	16.6	62
B	16.3	x	17.5	18.1	51.9+x
C	16.0	13.4	16.3	13.2	58.9
Total	50.4	25.5+x	49.0	47.9	172.8+x

Here, $T' = 51.9$

$B' = 25.5$

$G' = 172.8$

$a = 3$

$b = 4$

The estimated missing value is

$$x = \frac{aT' + bB' - G'}{(a-1)(b-1)} = \frac{3 \times 51.9 + 4 \times 25.5 - 172.8}{(3-1)(4-1)} = 14.15$$

Carrying out ANOVA-

Here-

$$H_{0Tr}: there\ is\ no\ significant\ difference\ among\ treatments$$

$$H_{1Tr}: there\ is\ significant\ difference\ among\ treatments$$

$$H_{0Bl}: there\ is\ no\ significant\ difference\ among\ blocks$$

$$H_{1Bl}: there\ is\ significant\ difference\ among\ blocks$$

Working table-

Treatments	Blocks				Total
	I	II	III	IV	
A	18.1	12.1	15.2	16.6	62
B	16.3	14.15	17.5	18.1	66.05
C	16.0	13.4	16.3	13.2	58.9
Total	50.4	39.65	49.0	47.9	186.95

Here, N = 12

$$C.F. = \frac{1}{12} \times 186.95^2 = 2912.525$$

$$SS_T = (18.1^2 + 12.1^2 + \cdots \ldots \ldots \ldots 47.9^2) - 2912.525 = 2955.88 - 2912.525 = 43.354$$

$$SS_{Tr} = \frac{1}{4}(62^2 + 66.05^2 + 58.9^2) - 2912.525 = 6.428$$

$$SS_{Bl} = \frac{1}{3}(50.4^2 + 39.65^2 + 49.0^2 + 47.9^2) - 2912.525 = 23.372$$

$$Adjustment\ Factor\ (k) = \frac{(B' + aT' - G')^2}{a(a-1)(b-1)^2}$$

$$= \frac{(25.5 + 3 \times 51.9 - 172.8)^2}{3(3-1)(4-1)^2} = 1.306$$

$$SS_{Tr}(Adj.) = SS_{Tr} - k = 6.428 - 1.306 = 5.121$$

$$SS_E = SS_T - SS_{Tr}(Adj.) - SS_{Bl} = 43.354 - 5.121 - 23.372 = 14.861$$

Remaining calculations are presented in ANOVA table below-

S.V.	S.S.	d.f.	M.S.	F
Treatments	5.121	2	2.56	0.861
Blocks	23.372	3	7.79	2.62
Error	14.861	5 (=10-5)	2.972	
Total	43.354	10 (=12-1-1)		

$$It\ is\ not\ true\ that\ F_{Tr}(0.861) > F_{0.05,2,5}(5.79), so\ H_{0Tr}\ is\ not\ rejected.$$

$$It\ is\ not\ true\ that\ F_{Bl}(2.62) > F_{0.05,3,5}(5.41), so\ H_{0Bl}\ is\ not\ rejected.$$

Practice Problem

The table given below shows yields of 3 varieties in 4 blocks experiment for which one observations is missing. Estimate the missing values and then analyze the data.

P 19 Q 26 R 21	R 29 P ?? Q 28	P 23 Q 27 R 22	Q 33 R 26 R 26

Estimation of a Missing Value in LSD

The expression for calculating a missing value in LSD is

$$x = \frac{m(R' + C' + T') - 2G'}{(m-1)(m-2)}$$

where

m = number of rows = number of columns = number of treatments

R' = sum of all known values in row containing missing value

C' = sum of all known values in column containing missing value

T' = sum of all known values in treatment containing missing value

G' = grand sum of all known values

Next, in SS_{Tr} following adjustment factor is subtracted-

$$Adjustment\ Factor\ (k) = \frac{\left((m-1)T' + R' + C' - G'\right)^2}{\left((m-1)(m-2)\right)^2}$$

So adjusted SS_{Tr} is

$$SS_{Tr}(Adj.) = SS_{Tr} - k$$

Solved Problem

Determine the missing value and carry out ANOVA of following design-

D 20.1 C 17.5 A ?? B 25.1	B 19.4 A 10.4 D 18.1 C 30.4	C 30.6 D 21.2 B 24.6 A 10.2	A 7.9 B 19.1 C 25.2 D 28.0

Solution –

Calculation of missing value-

				Total
D 20.1	B 19.4	C 30.6	A 7.9	78
C 17.5	A 10.4	D 21.2	B 19.1	68.2
A ??	D 18.1	B 24.6	C 25.2	R'=67.9
B 25.1	C 30.4	A 10.2	D 28.0	93.7
Total C'=62.7	78.3	86.6	80.2	G'=307.8

Here m = 4

Now missing value is

$$x = \frac{m(R' + C' + T') - 2G'}{(m-1)(m-2)} = \frac{4(62.7 + 67.9 + 28.5) - 2 \times 307.8}{(4-1)(4-2)} = 3.46$$

Analysis of data

It is $4 \times 4\ LSD$.

Hypotheses to test are-

$$H_{0Tr}: there\ is\ no\ significant\ difference\ among\ treatments$$

$$H_{1Tr}: there\ is\ significant\ difference\ among\ treatments$$

$$H_{0R}: there\ is\ no\ significant\ difference\ among\ row\ blocks$$

$$H_{1R}: there\ is\ significant\ difference\ among\ row\ blocks$$

$$H_{0C}: there\ is\ no\ significant\ difference\ among\ column\ blocks$$

$$H_{1C}: there\ is\ significant\ difference\ among\ column\ blocks$$

Working table

				Total
D 20.1 C 17.5 A 3.46 B 25.1	B 19.4 A 10.4 D 18.1 C 30.4	C 30.6 D 21.2 B 24.6 A 10.2	A 7.9 B 19.1 C 25.2 D 28.0	78 68.2 71.36 93.7
Total 66.16	78.3	86.6	80.2	311.26

Here $N = m \times m = 4 \times 4 = 16$

$$C.F. = \frac{1}{16}311.26^2 = 6055.17$$

$$SS_T = (20.1^2 + 19.4^2 + \cdots \ldots + 28.0\text{^}2) - 6055.17 = 7029.79 - 6055.17 = 974.62$$

$$SS_R = \frac{1}{4}(78^2 + 68.2^2 + 71.36^2 + 93.7\text{^}2) - 6055.17 = 96.62$$

$$SS_C = \frac{1}{4}(66.16^2 + 78.3^2 + 86.6^2 + 80.2^2) - 6055.17 = 54.73$$

Calculation of treatment totals-

Treatment A sum = 3.46+10.4+10.2+7.9=31.96

Treatment B sum = 25.1+19.4+24.6+19.1 = 88.2

Treatment C sum = 17.5+30.4+30.6+25.2 = 103.7

Treatment D sum = 20.1+18.1+21.2+28.0 = 87.4

$$SS_{Tr} = \frac{1}{4}(31.96^2 + 88.2^2 + 103.7^2 + 87.4^2) - 6055.17 = 743.11$$

$$Adjustment\ Factor\ (k) = \frac{\left((m-1)T' + R' + C' - G'\right)^2}{\left((m-1)(m-2)\right)^2}$$

$$= \frac{\left((4-1) \times 28.5 + 67.9 + 62.7 - 307.8\right)^2}{\left((4-1)(4-2)\right)^2} = 233.58$$

$$SS_{Tr}(Adj.) = SS_{Tr} - k = 743.11 - 233.58 = 509.53$$

$$SS_E = 974.62 - 96.62 - 54.73 - 509.53 = 313.74$$

Remaining calculations are done in ANOVA table below-

S.V.	S.S.	d.f.	M.S.	F
Rows	96.62	3	32.2	0.513
Columns	54.73	3	18.24	0.291
Treatments	509.53	3	169.84	2.707
Error	313.74	5 (=15-3-3-3-1)	62.748	----
Total	974.62	15	---	----

$$Since\ it\ is\ not\ true\ that\ 0.513 > F_{0.05,3,5}(= 5.41), so\ H_{0R}\ is\ not\ rejected.$$

$$Since\ it\ is\ not\ true\ that\ 0.291 > F_{0.05,3,5}(= 5.41), so\ H_{0C}\ is\ not\ rejected.$$

$$Since\ it\ is\ not\ true\ that\ 2.707 > F_{0.05,3,5}(= 5.41), so\ H_{0Tr}\ is\ not\ rejected.$$