Knowledge

→ knowledge is a set of facts about domain that can be used to solve problems in that domain.

classification of knowledge

1) classification-based knowledge
   - Ability to classify information.

2) Decision-oriented knowledge
   - choosing the best option.

3) Descriptive knowledge
   - state of some world.

4) Procedural knowledge
   - How to do something

5) Reasoning knowledge
   - what conclusion is valid in what situation.

6) Assimilative knowledge
   - what its impact is ?

Procedural knowledge vs Declarative knowledge

Procedural knowledge	Declarative knowledge
1. knowledge about "how to do something ?".	1. knowledge about "that something is true or false.
2. Process oriented.	2. Data oriented.
3 Less general	3. More general than PK
4. Eg. How to cook vegetable ?	4. Eg. The 1st step in cooking a vegetable is chopping it.

Knowledge Representation

→ Knowledge representation is a method used to encode the knowledge about the world in intelligent system.
→ Knowledge representation is concerned with converting informal knowledge into computer understandable form.

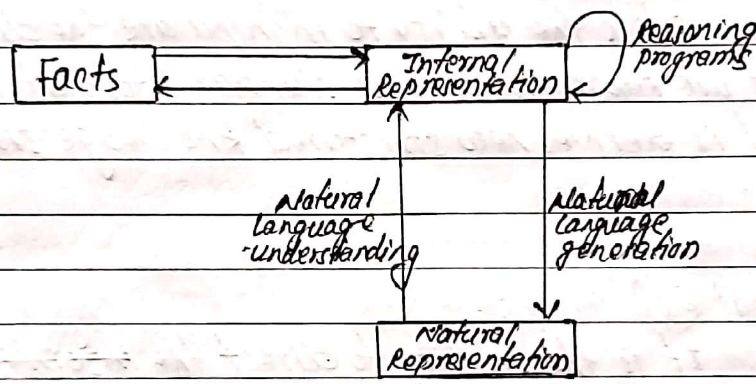


Fig: Mapping betn facts and representation

Issues in Knowledge Representation

1. How can the problem be represented?
2. What is the nature of knowledge and how do we represent it?
3. What specific knowledge about the world is required?
4. At what level should knowledge be represented?
5. How can relevant part be accessed when they are needed?
6. How can the knowledge be debugged, maintained and improved?
7. Are there any important relationships that exists among attributes of objects?
8. How to find the important attribute from the bank of information about an objects or world?
9. Should the scheme be declarative or procedural?

Properties of knowledge Representation system

1. Representational Adequacy:
It is the ability to represent all kinds of knowledge that are needed in a particular domain.

2. Inferential Adequacy:
It is the ability to manipulate the different facts that are represented in a standard format in such a way that it derives new structured knowledge from an old one.

3. Inferential Efficiency:
It is the ability to direct the ~~information~~ inferential mechanism into the most productive directions by storing appropriate guides.

4. Acquisitional Efficiency:
It is the ability to acquire new knowledge from the environment in an efficient manner.

Rule Based knowledge Representation System (RBS)

A rule-base system (or Production system) is a knowledge based system in which the knowledge is stored as an IF-THEN rule. A rule provides some description of how to solve a problem. Any rule consists of two parts: the IF part, called the antecedent (premise or Condition) and the THEN part called the Consequent (Conclusion or action).

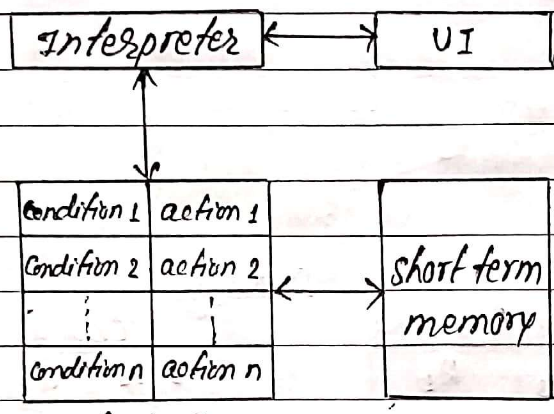
The syntax structure is:

IF <premise> THEN <Action>

E.g.

1. IF it is raining THEN open the umbrella.
2. If the 'traffic light' is green THEN go.

Architecture of production system



Rule Base

Components of RBS:

1. short term memory:
   - Contains description of the current state.

2. Rule Base:
   - It contains rules; each rule is a step in a problem solving Process. Rule is a set of Condition-action pairs.

3. Interpreter:
   - the processing engine which carries out reasoning on the rules and derives an answer.

4. User Interface (UI):
   - through which input and output signals are received and sent.

Examples:

1. Sorting of string

Problem: sorting a string of letters a, b, c.

Short term memory: cbaca (Input state)

Goal state: aabcc

Production set:

R_1 : ba → ab

R_2 : ca → ac

R_3 : cb → bc

Iteration	Memory	Conflict set	Rule fired
0	cbaca	R_1, R_2, R_3	R_1
1	cabca	R_2	R_2
2	acbca	R_2, R_3	R_2
3	acbac	R_1, R_3	R_1
4	acabc	R_2	R_2
5	aacbc	R_3	R_3
6	aabcc		

State space:

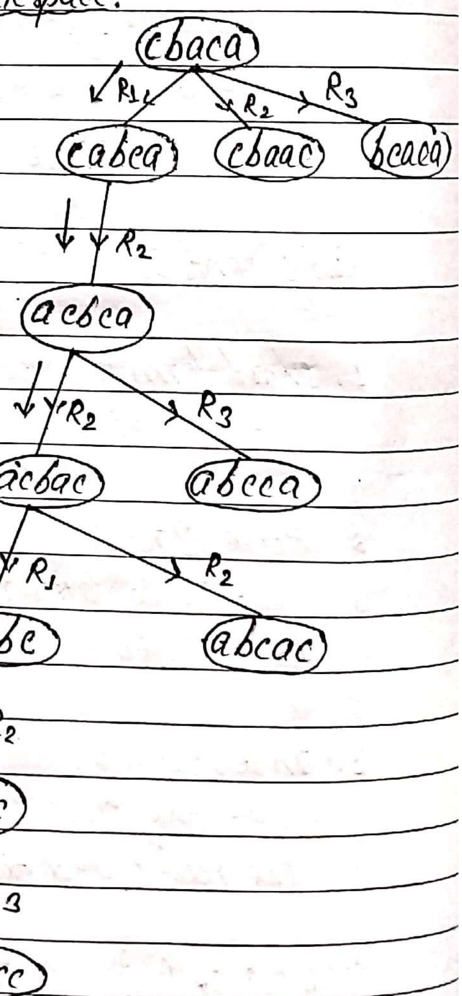


fig: state space for sorting of string.

2. Water Jug Problem

We are given two jugs, a 4-liter one and a 3-liter one, a pump which has unlimited water which can be used to fill the jug and the ground on which water may be poured. Neither jug has any measuring markers on it.

Problem: How can we get exactly $n = \{0, 1, 2, 3, 4\}$ liter of water into one of two jugs?

We will represent the state of the problem as (x, y) where x represents the amount of water in the 4-liter jug and y represent the amount of water in the 3-liter jug.

Note: $0 \leq x \leq 4$ & $0 \leq y \leq 3$

The rule base contains the following production rules:

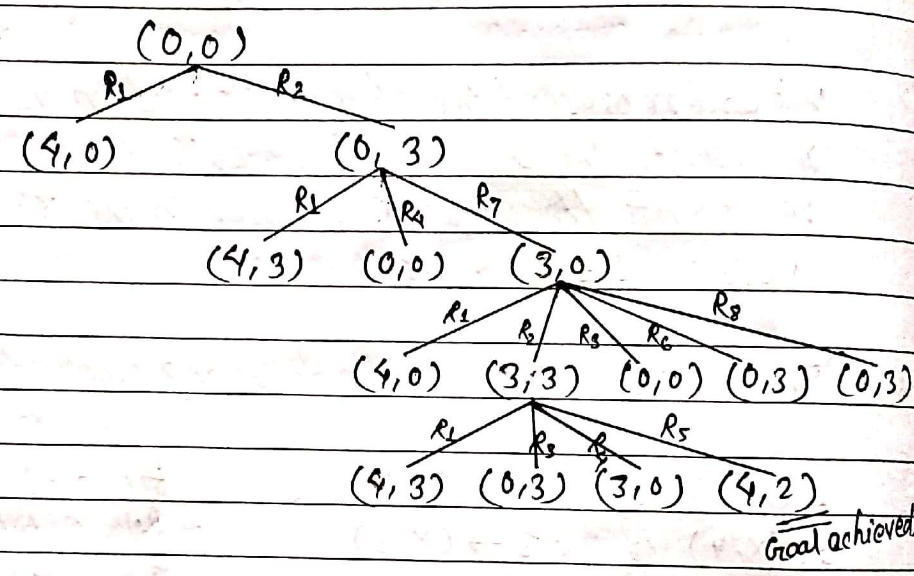
1. (x, y) if $x < 4 \rightarrow (4, y)$ fill 4-lit jug
2. (x, y) if $y < 3 \rightarrow (x, 3)$ fill 3-lit jug
3. (x, y) if $x > 0 \rightarrow (0, y)$ Empty 4-lit jug on ground
4. (x, y) if $y > 0 \rightarrow (x, 0)$ Empty 3-lit jug on ground
5. (x, y) if $x+y \geq 4, x < 4, y > 0 \rightarrow (4, y-(4-x))$ Pour water from 3-lit jug to fill 4-lit jug.
6. (x, y) if $x+y \geq 3, x > 0, y < 3 \rightarrow (x-(3-y), 3)$ Pour water from 4-lit jug to fill 3-lit jug
7. (x, y) if $x+y \leq 4, y > 0 \rightarrow (x+y, 0)$ Pour all of water from 3-lit jug into 4-lit jug
8. (x, y) if $x+y \leq 3, x > 0 \rightarrow (0, x+y)$ Pour all of water from 4-lit jug into 3-lit jug.

Consider: initial state = $(0, 0)$

goal state = $(4, 2)$ or $(n, 2)$

Iteration	Memory	conflict set	Rule fired
0	(0,0)	1, 2	2
1	(0,3)	1, 4, 7	7
2	(3,0)	1, 2, 3, 6, 8	2
3	(3,3)	1, 3, 4, 5	5
4	(4,2)		

State space:



TU. **Q.** Consider the following a production system characterized by

- Initial short term memory: C_5, C_1, C_3
- Production rules:

1. C_1 & $C_2 \rightarrow C_4$
2. $C_3 \rightarrow C_2$
3. C_1 & $C_3 \rightarrow C_6$
4. $C_4 \rightarrow C_6$
5. $C_5 \rightarrow C_1$

Show a possible sequence of two recognize-act cycles. Which will be the new content of the short term memory after those two cycles?

classmate
Date
Page

Sol^n

Iteration	Memory	Conflict set	Rule fired
0	C_5, C_1, C_3	2, 3, 5	2
1	C_5, C_1, C_2	1, 5	1
2	C_5, C_4		

The new content of the short term memory after two cycles = C_5, C_4.

Object Based knowledge Representation system

- Frames
- semantic Network
- scripts
- Conceptual Dependencies
- OAV triplets

Frames

A frame is a data structure containing typical knowledge about concept or objects. A frame represents knowledge about real world things (or entities).

Each frame contains frame name and slots. slots contains attributes defining frame object and values associated with the attributes. Values may be:

- a default values
- an inherited value from a higher frame
- a procedure
- a specific value

When the slots of a frame are all filled, the frame is said to be instantiated. Empty frames are sometimes called object prototypes.

E.g.

Employee

Name	Ram
Address	ktm
Salary	15,000/-
Tax	15% of salary
Gender	M
Department	

Department

DID	001
DName	HR
DLocation	Lalitpur

Semantic Network

Semantic networks can
- show natural relationship between objects / concepts.
- be used to represent declarative/descriptive knowledge.

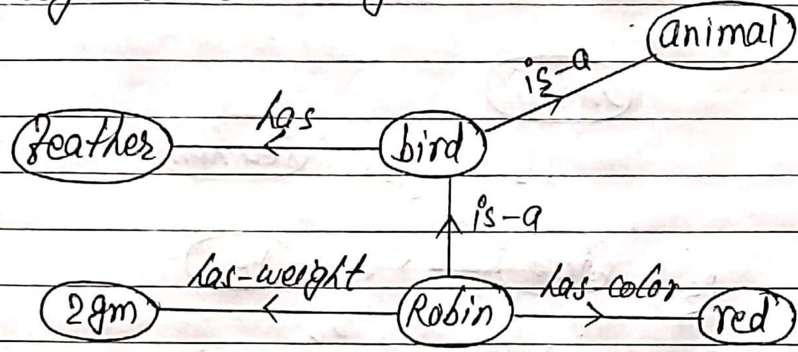
Knowledge is represented as a collection of concepts; represented by nodes. Thus, semantic networks are constructed using nodes (vertices) linked by directional lines called arcs (edges).

A node represents:
- physical object
- concept
- event
- attributes
- attribute values etc.

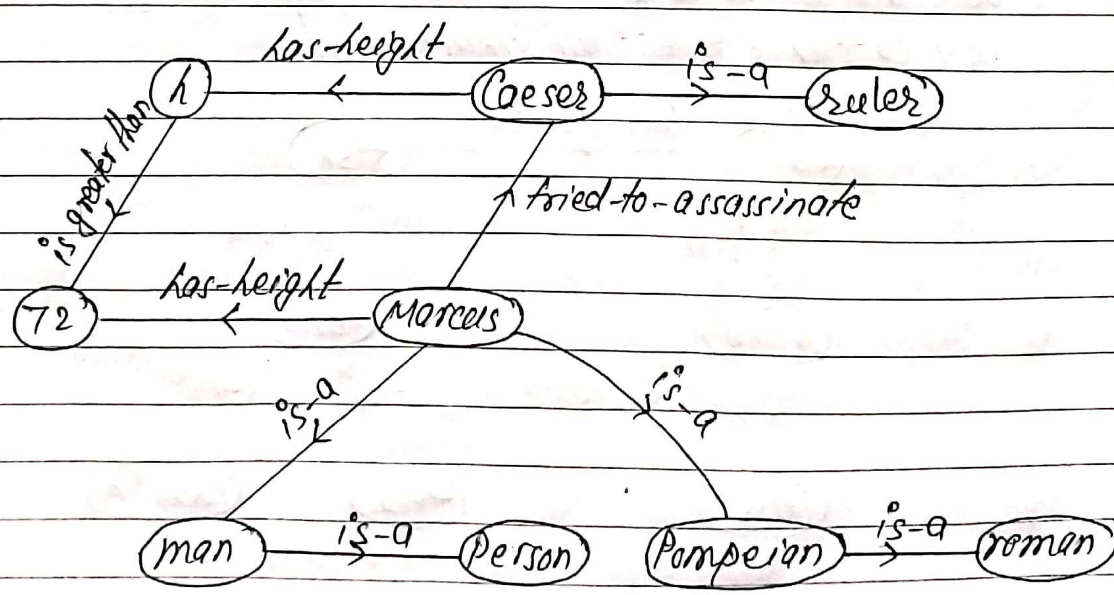
An arc represents relationship betⁿ nodes. Relationship types are:
- Is-a : represent class / instance relationships
- Has-a : identify property relationship.

E.g:

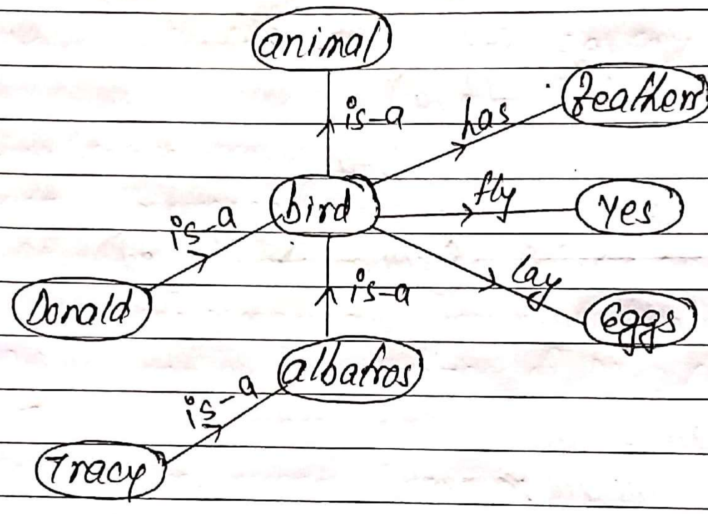
1. All birds are animal.
Birds have feathers.
Robin is a bird.
Robin is red in color.
Weight of Robin is 2gm.



2. All men are person.
All pompeians are roman.
Marcus, is a men, who is pompeian.
Caeser is ruler.
Marcus tried to assassinate caeser.
Marcus has height 72.
Height of marcus is less than the height of caeser.

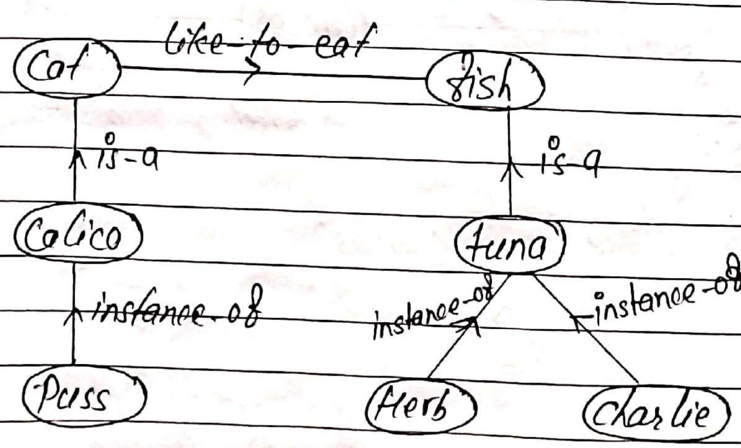


(3) Birds are animals.
Birds have feathers, fly and lay eggs.
Albatros is a bird.
Donald is a bird.
Tracy is an albatros.

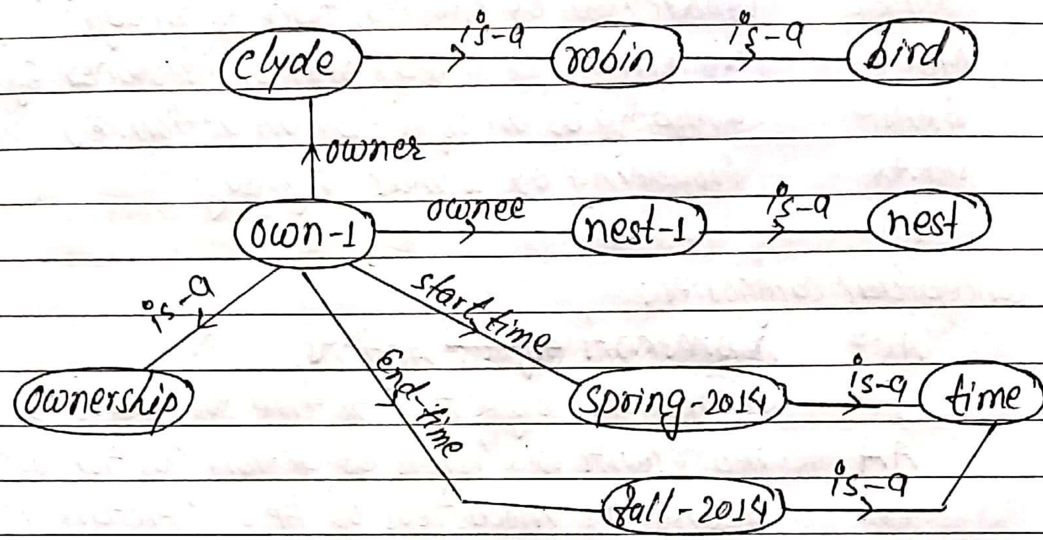


(4) Puss is a calico.
Herb is a tuna.
Charlie is a tuna.
All tunas are fishes.
All calicos are cats
All cats like to eat all kinds of fishes.

{is/instance-of}



5) Robin is bird
clyde is a Robin
clyde owns a nest from spring 2014 to fall 2014.



Conceptual Dependencies

Conceptual dependency (CD) is a theory of how to represent the knowledge about events contained in natural language sentences in such a way that
- facilitates for drawing inference from the sentence.
- the representation (CD) is independent of the language in which the sentences were originally stated.

Various symbols are used to encode knowledge as:
- → represents direction of dependency.
- ⇔ indicates two way link betⁿ actor and action
- P indicates past tense.
- O indicates object case relation
- R indicates recipient case relation.
- F indicates future tense.

CD primitives:

ATRANS	transfer of an abstract relationship. E.g. give, took
PTRANS	transfer of the physical location of an object. e.g. go
PROPEL	Application of physical force to an object E.g. push, pull
MOVE	Movement of a body part by its owner. E.g. kick
GRASP	Grasping of an object by an action. E.g. throw
SPEAK	Producing of sounds E.g. say
⋮	

Conceptual categories:

ACT	represents ~~object~~ Actions.
PP	represents objects (Picture procedures)
AA	represents modifiers of action (action aiders)
PA	represents modifiers of PP's (Picture aiders)
T	represents time
LOC	represents Locations.

E.g.

① John ran

$\text{John} \overset{P}{\Longleftrightarrow} \text{PTRANS}$

② John Pushed door.

$\text{John} \overset{P}{\Longleftrightarrow} \text{PROPEL} \overset{O}{\longleftarrow} \text{Door}$

③ John will run.

$\text{John} \overset{F}{\Longleftrightarrow} \text{PTRANS}$

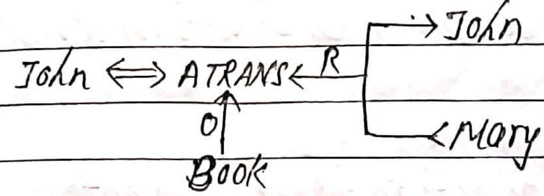
④ Bird is singing.

$\text{Bird} \Longleftrightarrow \text{SPEAK}$

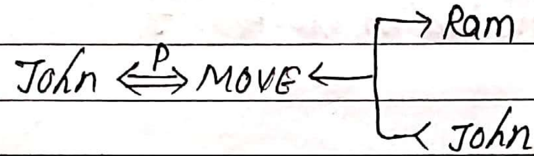
⑤ John Pushed the bike.

$\text{John} \overset{P}{\Longleftrightarrow} \text{PROPEL} \overset{O}{\longleftarrow} \text{bike}$

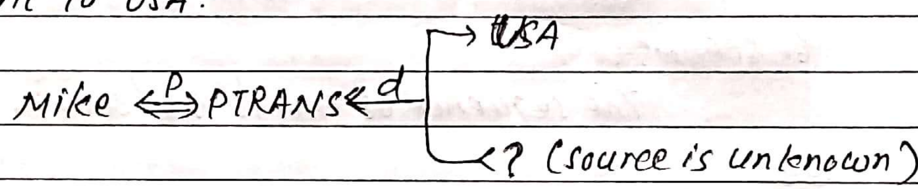
(6) John took the book from Mary



(7) John kicked Ram



(8) Mike went to USA.



Script

A script is a structured representation describing a stereotyped sequence of events in a particular context.

→ It consists of set of slots containing default values along with information about the types of values.

→ Script represent from entry condition to end condition.

Components of script.

1. Entry Condition:

· Must be satisfied before events in the script can occur.

2. Results :
Conditions that will be true after events in script occur.

3. Props :
Slots representing objects involved in events

4. Roles :
Persons involved in the events.

5. Track :
specific variation on more general pattern in the script.

6. scenes :
The sequence of events that occur.

E.g:
script for having launch/dinner at Resturant

Script : Resturant	scene 1 : Entering
Track : Coffee shop	customer PTRANS customer
Props : Tables, menu	customer ATTEND eyes to table
food, money	custome MOVE to sitting position
Roles : customer,	scene 2 : Ordering
waiter, cook,	customer PTRANS menu to customer
Cashier, owner.	customer PTRANS waiter to table
Entry conditions :	scene 3 : Eating
• Customer in hungry	Cook ATRANS food to waiter.
• customer has money	waiter ATRANS food to customer.
Results :	Customer INGEST food.
• customer is not hungry	scene 4 : Exiting
• customer has less money	customer PTRANS waiter to table.
• Owner has more money.	waiter ATRANS bill to customer.

Knowledge Representation using Logic

Logic is defined as a formal language for expressing knowledge and ways of reasoning.

→ Logic makes statement about the world which are true or false.
→ Logic is concise, unambiguous, context insensitive, expressive & effective for inferences.
→ It has syntax, semantics, and proof theory.

Syntax:

It defines the formal structure of sentences. E.g. $x+y=4$ is a well formed sentence where $x2y+$ is not.

Semantic:

It defines the meaning of sentences. It determines the truth w.r.t to models. E.g. $x+y=4$ is true in the world where $x=2$ & $y=2$ but false in the world where $x=1$ & $y=1$.

Proof theory:

set of rules for generating new sentences that are necessarily true given that the old sentences are true.

Types of Logic

→ Propositional logic
→ First-order logic (first-order predicate logic)

*Entailment:

Entailment means that one thing follows from other.

$KB \models \alpha$

knowledge base KB entails sentence α if and only if α is true in all worlds where KB is true.

Given knowledge base KB containing a statement α then KB is model for α i.e. $M(\alpha) = KB$

E.g.

$KB = \{p \to q, p\}$

Then $KB \models \{p \to q, p, q\}$

$M(q) = KB$, $M(p \to q) = KB$, $M(p) = KB$

* **Reasoning:**

Reasoning is the act of deriving a conclusion from certain premise using given methodology.

* **Inference:**

Inference is a process by which new sentences are derived from existing sentences in KB. For E.g. Modus tollens is a role of inference which derives new knowledge.

$$\frac{\begin{array}{c} p \to q \\ p \end{array}}{\therefore q}$$

* **Soundness:**

The inference algorithm is sound if it derives only sentences that are entailed by KB.

i is sound if whenever $KB \vdash_i \alpha$ it is also true that $KB \models \alpha$.

* **Completeness:**

The inference algorithm is complete if it can derive any sentence that is entailed by KB.

i is complete if whenever $KB \models \alpha$ it is also true that $KB \vdash_i \alpha$

$\{KB \vdash_i \alpha \Rightarrow i$ derives α from KB$\}$

Propositional Logic

A propositional logic is a declarative sentence which can be either true or false but not both or either.

- In propositional logic, there are atomic sentences and complex sentences.
- Atomic sentences consist of single proposition symbol. Each such symbol stands for proposition that can be true or false.
- The complex sentences are constructed from simpler sentences (atomic sentences) using logical connectives.

A set of connectives:

¬ (Negation) ∨ (disjunction) ⇔ (Bidirectional)
∧ (Conjunction) ⇒ (Implication)

Truth Table

P	Q	¬P	P∧Q	P∨Q	P⇒Q	P⇔Q
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Grammar / syntax for propositional logic

Sentence → automic sentence / Complex sentence
Automic sentence → True / False / symbol
symbol → P / q / r / ...
Complex sentence → ¬ sentence / sentence OP sentence
OP → ∧ / ∨ / → / ↔

E.g: $p \rightarrow$ It is raining.
$q \rightarrow$ It is cloudly
If it is raining then it is cloudy. $p \rightarrow q$

* Validity

A sentence is valid if it is true in all models.
For E.g. $p \vee \neg p$, $p \Rightarrow p$
- valid sentences are also known as tautologies.

* Satisfiability

~~A sentence is valid if it is true in all models.~~
~~For E.g $p \vee q$, $p \rightarrow q$, $p \wedge q$~~

- A sentence is satisfiable if it is true in some model. For E.g. $p \vee q$, $p \rightarrow q$, $p \wedge q$
- A sentence is unsatisfiable if it is true in no models. for E.g. $p \wedge \neg p$

Q. State whether the following sentences are valid, unsatisfiable, or neither.

a. smoke $\Rightarrow$ smoke
b. smoke $\Rightarrow$ fire
c. (smoke $\Rightarrow$ fire) $\Rightarrow$ ($\neg$smoke $\Rightarrow$ $\neg$ fire)
d. smoke $\vee$ fire $\vee$ $\neg$ fire.

Soln

a) Truth table for smoke $\Rightarrow$ smoke

smoke	smoke	smoke $\Rightarrow$ smoke
T	T	T
F	F	T

Here (smoke ⇒ smoke) is true for all models. Hence it is a valid sentence.

b) smoke ⇒ fire

smoke	fire	smoke ⇒ fire
T	T	T
T	F	F
F	T	T
F	F	T

It is neither valid nor unsatisfiable i.e. it is satisfiable.

c) (smoke ⇒ fire) ⇒ (¬smoke ⇒ ¬fire)

smoke	fire	¬smoke	¬fire	smoke ⇒ fire	¬smoke ⇒ fire	(smoke ⇒ fire) ⇒ (¬smoke ⇒ ¬fire)
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

It is neither valid nor unsatisfiable but is satisfiable.

d) smoke ∨ fire ∨ ¬fire

smoke	fire	¬fire	smoke ∨ fire	smoke ∨ fire ∨ ¬fire
T	T	F	T	T
T	F	T	T	T
F	T	F	T	T
F	F	T	F	T

Here, the given sentence is true for all model hence it is valid.

* Resolution Rule / Unit resolution

Given any disjunction of literals (propositions / predicate) and a literal

$P_1 \vee P_2 \vee P_3 \vee \dots \vee P_n$ & q

such that for any P_i, P_i is negation of q, then

$$\frac{P_1 \vee P_2 \vee P_3 \vee \dots \vee P_{i-1} \vee P_i \vee P_{i+1} \vee \dots \vee P_n,\ q}{P_1 \vee P_2 \vee P_3 \vee \dots \vee P_{i-1} \vee P_{i+1} \vee \dots \vee P_n}$$

E.g.

$$\frac{p \vee q \vee r,\ \neg p}{q \vee r}$$

* Generalized Resolution Rule

Given any two disjunction of literals say $P_1 \vee P_2 \vee P_3 \vee \dots \vee P_n$ & $q_1 \vee q_2 \vee \dots \vee q_m$ such that any P_i is negation of q_j.

$$\frac{P_1 \vee P_2 \vee \dots \vee P_{i-1} \vee P_i \vee P_{i+1} \vee \dots \vee P_n,\ q_1 \vee q_2 \vee \dots \vee q_{j-1} \vee q_j \vee q_{j+1} \vee \dots \vee q_m}{P_1 \vee P_2 \vee \dots \vee P_{i-1} \vee P_{i+1} \vee \dots \vee P_n \vee q_1 \vee q_2 \vee \dots \vee q_{j-1} \vee q_{j+1} \vee \dots \vee q_m}$$

E.g.

1. $$\frac{p \vee q \vee r,\ p \vee \neg q \vee \neg r}{p}$$
2. $$\frac{p \vee q \vee r,\ \neg r \vee s}{p \vee q \vee s}$$

* Conjuctive Normal Form (CNF)

A sentence that is expressed as a conjunction of disjunction of literals is said to be in conjuctive normal form (CNF)

E.g.

→ $(p \vee q) \wedge (p \vee r)$

→ p, q, $p \wedge q$, $p \wedge q \wedge r$

CNF Conversion Algorithm

1. Eliminate $\Leftrightarrow$
   Rewriting $P \Leftrightarrow q$ as $(P \Rightarrow q) \wedge (q \Rightarrow P)$
2. Eliminate $\Rightarrow$
   Rewriting $P \Rightarrow q$ as $\neg P \vee q$
3. Use De-Morgan's law to push $\neg$ inwards
   Rewriting $\neg(P \vee q)$ as $\neg P \wedge \neg q$
   Rewriting $\neg(P \wedge q)$ as $\neg P \vee \neg q$
4. Eliminate double negation
   Rewrite $\neg(\neg P)$ as P
5. Use the distribution laws to get CNF
   $P \vee (q \wedge r) \approx (P \vee q) \wedge (P \vee r)$
6. Use
   - $P \wedge (q \wedge r)$ as $P \wedge q \wedge r$
   - $(P \vee q) \vee r$ as $P \vee q \vee r$

Disjunctive Normal Form (DNF)

A sentence that is expressed as a disjunction of conjunction of literals is said to be in DNF

E.g.

$(P \wedge q) \vee (P \wedge r)$

$(P \vee q) \wedge r \approx (P \wedge r) \vee (q \wedge r)$

DNF Conversion Algorithm

- Same as CNF conversion algorithm except the step 5. All steps are same & step 5 changes to
5. Use the distribution laws to get DNF
   $P \wedge (q \vee r) \approx (P \wedge q) \vee (P \wedge r)$

Q. Convert to CNF
$b \leftrightarrow (a \vee c)$

Soln

not necessary in exam.

$b \leftrightarrow (a \vee c)$
$= (b \rightarrow (a \vee c)) \wedge ((a \vee c) \rightarrow b)$ [by using $p \leftrightarrow q \approx (p \rightarrow q) \wedge (q \rightarrow p)$]
$= (\neg b \vee (a \vee c)) \wedge (\neg (a \vee c) \vee b)$ [by using $p \rightarrow q \approx \neg p \vee q$]
$= (\neg b \vee a \vee c) \wedge (\neg (a \vee c) \vee b)$ [By using associative law]
$= (\neg b \vee a \vee c) \wedge ((\neg a \vee \neg c) \vee b)$ [By using demorgans law]
$= (\neg b \vee a \vee c) \wedge ((\neg a \vee b) \wedge (\neg c \vee b))$ [By using distributive law]
$= (\neg b \vee a \vee c) \wedge (\neg a \vee b) \wedge (\neg c \vee b)$ [By using associative law]

$\therefore$ Knowledge Base (KB) in CNF:

1. $\neg b \vee a \vee c$
2. $\neg a \vee b$
3. $\neg c \vee b$

Resolution Algorithm

1. Convert KB into CNF.
2. Add negation of conclusion to be inferred into KB
3. Apply resolution rule to resulting clauses.
4. The process continues until:
   - Repetition of statement or
   - Empty clause is not found.

Q. "We will be happy. If we will be happy then sun will shine. The sun will shine if and only if either it will rain or it will not be cloudy. It will be cloudy but I will not be happy".
Convert the above sentence into proposition.

Solⁿ

let

$p \rightarrow$ we will be happy.

$q \rightarrow$ sun will shine.

$r \rightarrow$ It will rain.

$s \rightarrow$ It will be cloudy.

The KB is

1. p
2. $p \rightarrow q$
3. $q \leftrightarrow (r \vee \neg s)$
4. $s \wedge \neg p$

Q. Given fact

$$(b \leftrightarrow (a \vee c)) \wedge \neg b$$

Infer a.

Solⁿ

First convert to CNF

$(b \leftrightarrow (a \vee c)) \wedge \neg b$

$= [(b \rightarrow (a \vee c)) \wedge ((a \vee c) \rightarrow b)] \wedge \neg b$

$= [(\neg b \vee a \vee c) \wedge (\neg(a \vee c) \vee b)] \wedge \neg b$

$= [(\neg b \vee a \vee c) \wedge ((\neg a \wedge \neg c) \vee b)] \wedge \neg b$

$= (\neg b \vee a \vee c) \wedge (\neg a \vee b) \wedge (\neg c \vee b) \wedge \neg b$

The KB in CNF

1. $\neg b \vee a \vee c$
2. $\neg a \vee b$
3. $\neg c \vee b$
4. $\neg b$
5. $\neg a$ (negation of conclusion)

Now using resolution in 1 & 2

$$\frac{\neg b \vee a \vee c, \neg a \vee b}{c}$$

Since c is new fact not in KB so add it to KB.

6. c

Using resolution in 6 & 3

$$\frac{c, \neg c \vee b}{b}$$

Since b is not in KB so add it to KB

7. b

Using resolution in 4 & 7

$$\frac{\neg b, b}{\phi}$$

Hence 'a' can be inferred

T.U. 068

Q. Consider the KB:
" If it is hot and humid, then it is raining. If it is humid, then it is hot. It is humid". Is it raining?

Soln

a) let
- $p \rightarrow$ It is hot.
- $q \rightarrow$ It is humid.
- $r \rightarrow$ It is raining.

b) The KB in proposition,
1. $(p \wedge q) \Rightarrow r$
2. $q \Rightarrow p$
3. q

c) r to show. so add it's negation into KB

Converting KB into CNF

1. $(p \wedge q) \Rightarrow r \approx \neg(p \wedge q) \vee r \approx \neg p \vee \neg q \vee r$
2. $q \Rightarrow p \approx \neg q \vee p$
3. q
4. $\neg r$ (Negation of conclusion)

The KB in CNF

1. $\neg p \vee \neg q \vee r$
2. $\neg q \vee p$
3. q
4. $\neg r$

Using resolution in 2 & 3

$$\frac{\neg q \vee p,\ q}{p}$$

Since p is new fact not in KB so add it to KB

5. p

Using resolution in 1 & 5

$$\frac{\neg p \vee \neg q \vee r,\ p}{\neg q \vee r}$$

Since $\neg q \vee r$ is not in KB so add it to KB.

6. $\neg q \vee r$

Using resolution in 6 & 3

$$\frac{\neg q \vee r,\ q}{r}$$

Since r is not in KB so add it to KB

7. r

Using resolution in 4 & 7

$$\frac{\neg r,\ r}{\phi}$$

Hence, it is raining.

Q. Consider the following sentence:

$$[(F \Rightarrow P) \vee (D \Rightarrow P)] \Rightarrow [(F \wedge D) \Rightarrow P]$$

a. Convert the right hand and left hand sides of main implication into CNF.

b. Prove that validity of sentence using resolution.

Soln

a) Right hand side of main implication is

$$(F \Rightarrow P) \vee (D \Rightarrow P)$$

Converting to CNF,

$= (\neg F \vee P) \vee (\neg D \vee P)$

$= \neg F \vee P \vee \neg D \vee P$

$= \neg F \vee P \vee \neg D$

Again, left hand side of main implication is

$$(F \wedge D) \Rightarrow P$$

Converting to CNF,

$(F \wedge D) \Rightarrow P$

$= \neg (F \wedge D) \vee P$

$= (\neg F \vee \neg D) \vee P$

$= \neg F \vee \neg D \vee P$

b) Now taking negation of given sentence,

$$\neg[[(F \Rightarrow P) \vee (D \Rightarrow P)] \Rightarrow [(F \wedge D) \Rightarrow P]]$$

Converting it into CNF,

$= \neg [(\neg F \vee P \vee \neg D) \Rightarrow (\neg F \vee \neg D \vee P)]$

$= \neg [\neg(\neg F \vee P \vee \neg D) \vee (\neg F \vee \neg D \vee P)]$

$= \neg [(F \wedge \neg P \wedge D) \vee (\neg F \vee \neg D \vee P)]$

$= \neg (F \wedge \neg P \wedge D) \wedge \neg(\neg F \vee \neg D \vee P)$

$= (\neg F \vee P \vee \neg D) \wedge (F \wedge D \wedge \neg P)$

$= (\neg F \vee P \vee \neg D) \wedge F \wedge D \wedge \neg P$

The KB in CNF

1. $\neg F \vee P \vee \neg D$
2. F
3. D
4. $\neg P$

Using resolution in ① & ② we get

$$\frac{\neg F \vee P \vee \neg D,\ F}{P \vee \neg D}$$

5. $P \vee \neg D$

Using resolution in ③ & ⑤

$$\frac{D,\ P \vee \neg D}{P}$$

6. P

Using resolution on ④ & ⑥

$$\frac{\neg P,\ P}{\phi}$$

Hence, the given sentence is valid.

Q. It is raining today. If it is raining today then we will go for swimming. We will go for swimming if and only if it will be holiday tomorrow. It will be holiday tomorrow but the course will not be completed on time.

Now infer if it is not raining today then either we will go for swimming or course will not be completed on time.

Soln

let, $P \rightarrow$ It is raining today.

$q \rightarrow$ we will go for swimming

$r \rightarrow$ It will be holiday tomorrow.

$s \rightarrow$ The course will be completed on time.

The KB is

1. p
2. $p \to q \approx \neg p \vee q$
3. $q \leftrightarrow r \approx (q \to r) \wedge (r \to q) \approx (\neg q \vee r) \wedge (\neg r \vee q)$
4. $r \wedge \neg s$

The KB in CNF

1. p
2. $\neg p \vee q$
3. $\neg q \vee r$
4. $\neg r \vee q$
5. r
6. $\neg s$

Conclusion to be inferred

$p \to (q \vee \neg s) \approx \neg p \vee q \vee \neg s$

Add negation of conclusion to KB

$\neg(\neg p \vee q \vee \neg s) \approx p \wedge \neg q \wedge s$

7. $\neg q$
8. s

Using resolution on 6 & 8

$$\frac{\neg s,\ s}{\phi}$$

Hence, the given sentence can be inferred.

Forward chaining (data-driven)

→ When we have some data and we make a decision based on this data then the process is called as forward chaining.
→ Forward chaining starts with the available data and uses inference rules to extract more data until a goal is reached.

E.g.

① While diagnosing a patient the doctor first check the symptoms and medical condition of the body such as temperature, blood pressure, pulse, blood etc. After that, the patient symptoms are analysed and compared against the predetermined symptoms. Then the doctor is able to provide the medicines according to the symptoms of the patient.

② "If it is raining then we will take umbrella". Here "it is raining" is data and "we will take umbrella" is a decision. It was already known that it is raining that is why we are going to take umbrella. This is forward chaining.

Advantages

→ Runs greater when a problem naturally begins by collecting data and searching for information that can be collected from it to be used in future steps.
→ Forward chaining has the capability of providing a lot of data from the available ~~few~~ initial data or facts.
→ Forward chaining ~~has f~~ is a very popular technique for implementation to expert system, and system using production rules in the knowledge base. For expert system that needs interruption, control, monitoring and planning, the forward chaining is the best choice.

Backward chaining (goal-driven)

→ When we have a decision and based on the decision if we fetch the initial data that support goal then the process is called as backward chaining.
→ Backward chaining starts with a goal and then searches back through inference rules to find the facts that support the goal.

E.g.
"If it is raining then we will take umbrella." Here we have our possible conclusion "we will take umbrella". If we are taking umbrella then it can be stated that "it is raining." Here based on conclusion we guessed that the data can be "it is raining". This process is called backward chaining.

Advantages

→ The system that uses backward chaining tries to set goals in order which they arrive in the knowledge base.
→ The search in backward chaining is directed.
→ While searching, the backward chaining considers those parts of knowledge base which are directly related to the considered problem or backward chaining never performs unnecessary inferences.
→ Backward chaining is an excellent tool for specific type of problem such as diagnosing & debugging.

KB = {
$p \rightarrow q$
$l \wedge m \rightarrow p$
$b \wedge l \rightarrow m$
$a \wedge p \rightarrow l$
$a \wedge b \rightarrow l$
a
b
} Infer q.

Forward chaining

Given a and b are true.
By $a \wedge b \rightarrow l$, l is true. Being b true and l true, $b \wedge l \rightarrow m$ leads m to be true.
Being l true and m true, $l \wedge m \rightarrow p$ leads p to be true.
p is true and $p \rightarrow q$ means q is true.

Backward chaining

We start from q.
For q to be true p should be true; since $p \rightarrow q$
p is true when l and m are true as $l \wedge m \rightarrow p$
l is true when a & b are true as $a \wedge b \rightarrow l$
since a & b are given facts which are true. so l is true.
m is true since $b \wedge l \rightarrow m$ as b & l both are true.
Hence q is true.

Predicate Logic / First order predicate logic (FOPL)

Predicate logic / first-order logic is symbolized reasoning in which each sentences or statement is broken down into a subject and a predicate. The predicate modifies or defines the properties of the subject.

A sentence in first-order logic is written in the form P_x or $P(x)$, where P is the predicate and x is the subject, represented as variable.

Syntax:

Sentence → Automic sentence / sentence connective sentence / Quantifier variable sentence / ¬ sentence

Automic sentence → predicate (term) / term

term → function (term) / constant / variable

Connective → $\land$ / $\lor$ / $\rightarrow$ / $\leftrightarrow$

Quantifier → $\forall$ / $\exists$

Constant → x_1 / x_2 / Ram / ...

variable → x / y / z / a / b / ...

function → height-of / boy / man / ..

predicate → greater-than / younger-than / ...

The first order logic assumes the world contains

- object
- Relation
- function

* **Quantifier**

↳ allows us to express properties of collection of objects instead of enumerating object by name.

Two quantifier:

universal : "for all" $\forall$

Existential : "there exists" $\exists$

All are talent

$\forall x$ talent(x)

↓

talent(Ram)

talent(shyam)

→ $\forall x\ P$ is true in a model m iff P is true for all x in the model.

→ $\exists x\ P$ is true in a model m iff P is true for at least one x in the model.

* **Universal Instantiation (UI)**

- Replace variable by ground term (term without variables).

E.g.

$\forall x$ Pompeian(x) ⇒ Roman(x)

Pompeian(marcus)

Roman(caser)

It's UI is

Pompeian(marcus) ⇒ Roman(caser)

Pompeian(marcus)

Roman(caser)

classmate
Date
Page

* Existential Instantiation (EI) / Skolemization

Skolemization is the process of replacement of existential quantified variable with skolem constant that is not in KB and deletion of the respective quantifier.

E.g.

$\exists x\, P(x)$

After EI

$P(C_1)$ where C_1 is skolem constant.

* Unification and Lifting

→ A unifier of two atomic formulae is a substitution of terms for variables that makes them identical.

Unifier of $P(x, f(a), z)$ and $P(z, z, u)$:

$\{x/f(a),\ z/f(a),\ u/f(a)\}$

x knows y ⟶ knows (x, y)

making alike

x is lifted with value Jane

p	q	unifier (θ)
1. knows(John, x)	knows(John, Jane)	{x/ Jane}
2. knows(John, x)	knows(y, Ram)	{x/Ram, y/John}
3. knows(John, x)	knows(y, mother(y))	{y/John, x/mother(y)} or {y/John, x/mother(John)}
4. knows(John, x)	knows(x, Ram)	fail
5. knows(John, x)	knows(y, z)	{y/John, x/z} or {y/John, x/John, z/John}

→ unification no. (4) is failed due to overlap of variables, x cannot take the values of John and Ram at the same time.

→ Can be unified by renaming variable

{ x/Ram , y/John }

knows (John, x)   knows (y, Ram)

Standardizing apart

Most generic unifier
↳ variable लाई variable ले replace

Most specific unifier
↳ variable लाई constant ले replace

let C_1 and C_2 be two clauses. If C_1 and C_2 have no variables in common, they are said to be standardized apart. standardized apart eliminate overlap of variables to avoid clashes by renaming variable.

Q. Attempt to unify the following pair of expressions. Either show their most general unifiers, most specific unifier or explain why they will not unify.

a. P(x, y) and P(a, z)

b. P(x, x) and P(a, b)

c. ancestor(x, y) and ancestor (bill, father(bill))

Soln

a. P(x, y) and P(a, z)
→ The most general unifier is {a/x, y/z}

b. P(x, x) and P(a, b)
→ fails to unify since both a & b cannot be substituted for x.

c. ancestor(x, y) and ancestor(bill, father(bill))
→ The most general unifier is {bill/x, father(bill)/y}.

FOPL Examples

Q. Convert following sentences to FOPL.
If every helper is busy then there is a job in the queue.
A job is in queue but the helper is not busy.
Every helpers are teased by someone.

Soln

The predicate in given KB are
H(x) → x is helper.
B(x) → x is busy.
J(x) → x is job.
Q(x) → x is in queue.
T(x, y) → x is teased by y.

The facts in FOPL

1. $\forall x \exists y [(H(x) \land B(x)) \Rightarrow (J(y) \land Q(y))]$
2. $\exists y \exists x [(J(y) \land Q(y)) \land (H(x) \land \neg B(x))]$
3. $\forall x \exists y (H(x) \Rightarrow T(x, y))$

Q. "A person born in Nepal, each of whose parents is a Nepali citizen by birth, is a Nepali citizen by birth. A person born outside Nepal, one of whose parent is a Nepali citizen by birth, is a Nepali citizen by decent. Several developed countries have dual citizenship provision, but Nepal doesn't have that provision". Represent the above sentence in first order logic and explain each step.

Sol^n

The predicate in given KB are,

Person(x) → x is a person.

Born(x,y) → Person x born in country y.

Parent(x,y) → x is parent of y.

citizen(x,c,r) → x is a citizen of country c for reason r.

DC(x) → x is developed country.

CP(x) → x have dual citizenship provision.

The facts in FOPL,

1. $\forall x \forall y [((person(x) \land Born(x, Nepal) \land Parent(y,x)) \Rightarrow citizen(y, Nepal, Birth)) \Rightarrow citizen(x, Nepal, Birth)]$
2. $\forall x \exists y [((person(x) \land \neg Born(x, Nepal) \land Parent(y,x)) \land citizen(y, Nepal, Birth)) \Rightarrow citizen(x, Nepal, Decent)]$
3. $\exists x [(DC(x) \land CP(x)) \land \neg CP(Nepal)]$

Q. Convert into FOPL.

John likes all kinds of food

Apples are food.

chicken is food.

Anything anyone eats and isn't killed by is food.

Bills eats peanuts and is still alive.

Sue eats everything Bill eats.

Sol^n

The predicate in KB are

Food(x) → x is food.

likes(x,y) → x likes y.

Eats(x,y) → x eats y.

killed-by(x,y) → x killed y.

Alive(x) → x is still alive

The facts in FOPL,

1. $\forall x\,[\,Food(x) \Rightarrow likes(John, x)\,]$
2. $Food(Apples)$
3. $Food(chicken)$
4. $\forall x \forall y\,[\,(Eats(y,x) \wedge \neg killed\text{-}by(y,x)) \Rightarrow Food(x)\,]$
5. $Eats(Bill, peanuts) \wedge Alive(Bill)$
6. $\forall x\,[\,Eats(sue, x) \Rightarrow Eats(Bill, x)\,]$

Q. "A deductive system is sound if any formula that can be derived in the system is logically valid. Conversely, a deductive system is complete if every logically valid formula is derivable. All of the system discussed in this article are both sound and complete. They also share the property that it is possible to effectively verify that a purportedly valid deduction is actually a deduction; such deduction systems are called effective". Represent the above sentences in first-order logic and explain each step.

Sol^n

The predicate in KB are,

System(x) → x is a system.

Deductive(x) → x is deductive.

Sound(x) → x is sound.

F(x) → x is formula.
Derived_in (x, y) → x is derived in y.
Derivable (x) → x is derivable
LV (x) → x is logically valid.
Discussed-in (x, y) → x is discussed in y.
Complete (x) → x is complete
SP (x, y) → x share the property of y.
EV (x) → x is effictively verify.
Pd (x) → x is purportedly valid.
Effective (x) → x is effective.

The facts in FOPL;

1. $\forall x \forall y [(F(x) \wedge Derived\text{-}in(x,y) \wedge LV(x)) \Rightarrow (system(y) \wedge Deductive(y) \wedge sound(y))]$
2. $\forall x \forall y [(LV(x) \wedge F(x) \wedge Derivable(x)) \Rightarrow (system(y) \wedge Deductive(y) \wedge complete(y))]$
3. $\forall y [(system(y) \wedge Discussed\text{-}in(y, article)) \Rightarrow (sound(y) \wedge Complete(y))]$
4. $\forall x \exists y [(Pd(x) \wedge system(x) \wedge sp(x,y) \wedge EV(x) \wedge Deductive(x)) \Rightarrow Effective(x)]$

Q. Rabin likes only easy courses. Science courses are hard. All courses in the CSIT are easy. CSC 101 is a CSIT course.
a. Translate the sentence into predicate logic.
b. Convert your sentences into clausal normal form (CNF).

Soln

The predicate in given KB are;

$Easy(x) \rightarrow x$ is easy course.
$likes(x, y) \rightarrow x$ likes y.
$science(x) \rightarrow x$ is science course.
$CSIT(x) \rightarrow x$ is course in CSIT

a) The facts in FOPL,

1. $\forall x\,(Easy(x) \Rightarrow likes(Rabin, x))$
2. $\forall x\,(science(x) \Rightarrow \neg Easy(x))$
3. $\forall x\,(CSIT(x) \Rightarrow Easy(x))$
4. $CSIT(CSC101)$

b) Converting to CNF;

1. $\forall x\,(Easy(x) \Rightarrow likes(Rabin, x))$
   $= \forall x\,(\neg Easy(x) \vee likes(Rabin, x))$
   $= \neg Easy(x) \vee likes(Rabin, x)$

2. $\forall x\,(science(x) \Rightarrow \neg Easy(x))$
   $= \forall x\,(\neg science(x) \vee \neg Easy(x))$
   $= \neg science(x) \vee \neg Easy(x)$

3. $\forall x\,(CSIT(x) \Rightarrow Easy(x))$
   $= \forall x\,(\neg CSIT(x) \vee Easy(x))$
   $= \neg CSIT(x) \vee Easy(x)$

4. $CSIT(CSC101)$

$\therefore$ KB in CNF are,
1. $\neg Easy(x) \vee likes(Rabin, x)$
2. $\neg science(x) \vee \neg Easy(x)$
3. $\neg CSIT(x) \vee Easy(x)$
4. $CSIT(CSC101)$

Q. Consider the following statements.
All cats like fish, cats eat everything they like, and ziggy is a cat.
a) Translate the sentence into FOPL.
b) convert the sentence into CNF.
c) Answer using FOPL, if ziggy eats fish?

Sol^n

The predicate in given KB are;
cat(x) → x is cat.
likes (x, y) → x likes y.
eats (x, y) → x eats y.

a) The sentence into FOPL,
1. $\forall x\, [\, cat(x) \Rightarrow likes(x, fish)]$
2. $\forall x \forall y\, [\, (cat(x) \wedge likes(x,y)) \Rightarrow eats(x,y))$
3. $Cat(ziggy)$

b) Converting to CNF,
1. $\forall x\, [\, cat(x) \Rightarrow likes(x, fish)]$
$= \forall x\, [\, \neg cat(x) \vee likes(x, fish)]$
$= \neg cat(x) \vee likes(x, fish)$

2. $\forall x \forall y\, [\, (cat(x) \wedge likes(x,y)) \Rightarrow eats(x,y)]$
$= \forall x \forall y\, [\, \neg(cat(x) \wedge likes(x,y)) \vee eats(x,y)]$
$= \neg cat(x) \vee \neg likes(x,y) \vee eats(x,y)$

3. $Cat(ziggy)$

CNF in KB are,
1. $\neg cat(x) \vee likes(x, fish)$
2. $\neg cat(x) \vee \neg likes(x,y) \vee eats(x,y)$

3. Cat(ziggy)

c) To infer,

eats(ziggy, fish)

so add negation of conclusion into KB.

4. $\neg$eats(ziggy, fish)

Now, using resolution in ① & ② by lifting {y/fish}

$\neg$cat(x) $\vee$ likes(x, fish) , $\neg$cat(x) $\vee$ $\neg$likes(x, fish) $\vee$ eats(x, fish)

$\neg$cat(x) $\vee$ eats(x, fish)

since $\neg$cat(x) $\vee$ eats(x, fish) is not in KB so add it to KB.

5. $\neg$cat(x) $\vee$ eats(x, fish)

Using resolution in ③ & ⑤ by lifting {x/ziggy}

cat(ziggy) , $\neg$cat(ziggy) $\vee$ eats(ziggy, fish)

eats(ziggy, fish)

since eats(ziggy, fish) is not in KB so add it to KB.

6. eats(ziggy, fish)

Using resolution in ④ & ⑥

$\neg$eats(ziggy, fish), eats(ziggy, fish)

ϕ

∴ Hence, ziggy eats fish.

Q. All students are smart. All students are talent. Infer "All smarts are talent".

Sol^n

The predicate in KB are;

$stu(x) \rightarrow x$ is student.

$sm(x) \rightarrow x$ is smart.

$tl(x) \rightarrow x$ is talent.

Now the KB in FOPL,

1. $\forall x\ (stu(x) \Rightarrow sm(x))$
2. $\forall y\ (stu(y) \Rightarrow tl(y))$

The KB in CNF

1. $\neg stu(x) \vee sm(x)$
2. $\neg stu(y) \vee tl(y)$

To infer: $\forall z\ (sm(z) \Rightarrow tl(z))$

Add its negation by converting into CNF into KB

$\neg(sm(z) \Rightarrow tl(z))$

$= \neg(\neg sm(z) \vee tl(z))$

$= sm(z) \wedge \neg tl(z)$

3. $sm(z)$
4. $\neg tl(z)$

Using resolution in ② & ④ by lifting $\{y/z\}$

$$\frac{\neg stu(z) \vee tl(z),\ \neg tl(z)}{\neg stu(z)}$$

Since it is a new clause not in KB so add it to KB.

5. $\neg stu(z)$

Since, resolution rule is no more applicable on the clause in KB hence the clause "All smarts are talent" cannot be inferred.

Q. Translate the following sentence into first order logic.

i) "Everyone's DNA is unique and is derived from their parents' DNA".

Sol^n

The predicate in given KB are,

$DNA(x) \rightarrow$ x's DNA.

$Unique(x, y) \rightarrow$ x and y are unique.

$DerivedFrom(x, y) \rightarrow$ x is derived from y.

$Parent(x, y) \rightarrow$ x is parent of y.

The facts in FOPL,

$$\forall x \forall y \forall z [unique(DNA(x), DNA(y)) \Rightarrow (DNA(x) \land DNA(y) \land Parent(y, x) \land DerivedFrom(y, z) \land Parent(z, y))]$$

---

ii) "No dog bites a child of its owner"

$Dog(x) \rightarrow$ x is dog

$Child(x, y) \rightarrow$ x is child of y.

$Bites(x, y) \rightarrow$ x bites y.

$Owner(x, y) \rightarrow$ x is owner of y.

$$\forall x \forall y \forall z [(Dog(x) \land Owner(y, x) \land child(z, y)) \Rightarrow \neg Bites(x, z)]$$

---

iii) "Every gardner likes the sun".

Gardner(x) → x is gardner.
Likes (x, y) → x likes y.

$\forall x \, (Gardner(x) \Rightarrow Likes(x, sun))$

iv) "All purple mushrooms are poisonous"

Purple(x) → x is purple.
Mushroom(x) → x is mushroom.
Poisonous(x) → x is poisonous.

$\forall x \, [(Mushroom(x) \wedge Purple(x)) \Rightarrow Poisonous(x)]$

v) "No two adjacent countries have the same color".

Country(x) → x is country.
Adj(x, y) → x and y are adjacent
Distinct(x, y) → x and y are distinct.
Color(x) → x is color.

$\forall x \forall y \, [(Country(x) \wedge Country(y) \wedge Adj(x, y)) \Rightarrow Distinct(Color(Country(x)), Color(Country(y)))]$

vi) "There are exactly two purple mushroom".

$\exists x \exists y \forall z \, [(Mushroom(x) \wedge Purple(x) \wedge Mushroom(y) \wedge Purple(y) \wedge \neg(x = y) \wedge (Mushroom(z) \wedge Purple(z))) \Rightarrow ((x = z) \vee (y = z))]$

vii) "You can fool some of the people all of the time".

Person(x) → x is person.
Time(t) → t is time
Can-fool(x,t) → Person x can fool at time t.

$\exists x \forall t [(person(x) \wedge Time(t)) \Rightarrow Can\text{-}fool(x,t)]$

viii) "You can fool all of the people some of the time".

$\forall x \exists t [person(x) \Rightarrow (Time(t) \wedge Can\text{-}fool(x,t))]$

Q. Convert the following statement into FOPL & then into CNF.
All oversmart persons are stupid.
children's of all stupid persons are naughty.
Ram is children of Hari.
Hari is oversmart.

Soln

The predicate in given KB are;
Person(x) → x is a person.
oversmart(x) → x is oversmart
stupid(x) → x is stupid.
children(x,y) → x is children of y.
Naughty(x) → x is naughty.

The facts in FOPL,

1. $\forall x [(person(x) \land oversmart(x)) \Rightarrow stupid(x)]$
2. $\forall x \forall y [(children(x,y) \land stupid(y) \land person(y)) \Rightarrow naughty(x)]$
3. children (Ram, Hari)
4. oversmart (Hari)

Converting to CNF,

1. $\forall x [(person(x) \land oversmart(x)) \Rightarrow stupid(x)]$
$= \forall x [\neg(person(x) \land oversmart(x)) \lor stupid(x)]$
$= \neg Person(x) \lor \neg oversmart(x) \lor stupid(x)$

2. $\forall x \forall y [(children(x,y) \land stupid(y) \land person(y)) \Rightarrow naughty(x)]$
$= \forall x \forall y [\neg(children(x,y) \land stupid(y) \land person(y)) \lor naughty(x)]$
$= \neg children(x,y) \lor \neg stupid(y) \lor \neg person(y) \lor naughty(x)$

3. children (Ram, Hari)
4. oversmart (Hari)

∴ The KB in CNF,
1. $\neg Person(x) \lor \neg oversmart(x) \lor stupid(x)$
2. $\neg children(x,y) \lor \neg stupid(y) \lor \neg person(y) \lor naughty(x)$
3. children (Ram, Hari)
4. oversmart (Hari)

Q. Anyone passing his history exams and winning the lottery is happy. But anyone who studies or is lucky can pass all his exams. John did not study but John is lucky. Anyone who is lucky wins the lottery.
Now, use resolution to infer "John is happy".

Sol^n

The predicate in KB are

Pass(x, y) → x passes his y exam.
Win(x, y) → x wins y.
Happy(x) → x is happy.
study(x) → x study.
Lucky(x) → x is lucky.

The facts in FOPL,

1. $\forall x\,[(pass(x, History) \land win(x, lottery)) \Rightarrow Happy(x)]$
2. $\forall x \forall y\,[(study(x) \lor lucky(x)) \Rightarrow Pass(x, y)]$
3. $\neg study(John) \land lucky(John)$
4. $\forall x\,[lucky(x) \Rightarrow wins(x, lottery)]$

Converting to CNF,

1. $\forall x\,[(Pass(x, History) \land win(x, lottery)) \Rightarrow Happy(x)]$
$= \forall x\,[\neg(Pass(x, history) \land win(x, lottery)) \lor Happy(x)]$
$= \neg Pass(x, history) \lor \neg win(x, lottery) \lor Happy(x)$

2. $\forall x \forall y\,[(study(x) \lor lucky(x)) \Rightarrow Pass(x, y)]$
$= \forall x \forall y\,[\neg(study(x) \lor lucky(x)) \lor Pass(x, y)]$
$= \forall x \forall y\,[(\neg study(x) \land \neg lucky(x)) \lor Pass(x, y)]$
$= (\neg study(x) \lor Pass(x, y)) \land (\neg lucky(x) \lor Pass(x, y))$

3. $\neg study(John) \land lucky(John)$

4. $\forall x\,[Lucky(x) \Rightarrow win(x, lottery)]$
$= \forall x\,[\neg Lucky(x) \lor win(x, lottery)]$
$= \neg lucky(x) \lor win(x, lottery)$

CNF in KB are

1. ¬Pass (x, History) ∨ ¬win (x, lottery) ∨ Happy(x)
2. ¬study(x) ∨ Pass (x, y)
3. ¬lucky(x) ∨ Pass (x, y)
4. ¬study (John)
5. Lucky (John)
6. ¬lucky (x) ∨ win(x, lottery)

To infer
Happy (John)
so add it's negation into KB.

7. ¬Happy (John)

Now, using resolution in 1 & 7 by lifting {x/John}

¬Pass (John, History) ∨ ¬win(John, lottery) ∨ Happy (John), ¬Happy(John)

¬Pass (John, History) ∨ ¬win(John, lottery)

since it isn't in KB so add it to KB

8. ¬Pass (John, History) ∨ ¬win (John, lottery)

using resolution in 6 & 8 by lifting {x/John}.

¬lucky(John) ∨ win(John, lottery), ¬Pass(John, History) ∨ ¬win (John, lottery)

¬Pass (John, History) ∨ ¬lucky(John)

since it isn't in KB so add it to KB.

9. ¬Pass (John, History) ∨ ¬lucky(John)

using resolution in ⑤ & ⑨

$$\frac{lucky(John),\ \neg Pass(John, History) \vee \neg lucky(John)}{\neg Pass(John, History)}$$

It is not in KB so add it to KB.

10. ¬Pass(John, History)

using resolution in ③ & ⑩ by lifting {x/John, y/History}

$$\frac{\neg lucky(John)\ \cancel{History} \vee Pass(John, History),\ \neg Pass(John, History)}{\neg lucky(John)}$$

It is not in KB so add it to KB.

11. ¬lucky(John)

using resolution in ⑤ & ⑪

$$\frac{lucky(John),\ \neg lucky(John)}{\phi}$$

Hence, John is happy.

Q. Convert into FOPL.

1. Everyone likes someone.
2. All the student who visited science museums are not CSIT student
3. Nabina likes all fruits that are rich vitamin A.
4. Asmita likes all the movies that Manju likes.

Uncertain knowledge
→ Probability
→ Probabilistic logic / Probabilistic reasoning

- → Probabilistic reasoning is using logic and probability to handle uncertain situations.
- → The aim of a probabilistic logic / probabilistic reasoning is to combine the capacity of probability theory to handle uncertainity with the capacity of deductive logic to exploit structure of formal argument.
- → In situations where "the relevant world is random" or appears to be random because of poor representation" or "not random but our program can not access large database", probabilistic reasoning is to be applied.
- → One has to apply probabilistic reasoning in deciding about the next card to play in a game of cards or in diagnosing the illness from the symptoms. These are random world.
- → Uncertainities can arise from an inability to predict outcomes due to unreliable, vague, incomplete or inconsistent knowledge.

Probability
→ The chance that something will happen.
→ real number in the range 0 to 1.

- → $P(A) = 0$ indicates total uncertainity i.e. there is no chance that a particular event A will occur.
- → $P(A) = 1$ indicates total certainity i.e. event A is certain to occur.

* Random variables

- variables that would occur in KB.
- These values might represent the possible outcomes of one experiment or potential values of a quantity whose value is concertain.

For e.g.

The possible outcomes for one fair coin toss can be described using the following random variables:

$$X = \begin{cases} \text{head,} \\ \text{tail} \end{cases}$$

Random variable's domain

- Boolean $\rightarrow$ $x = \{T, F\}$
- Discrete $\rightarrow$ Distinct values that a random variable have. $x = \{d_1, d_2, d_3, d_4\}$
- Continuous $\rightarrow$ $x = \{1, 2, 3, \ldots \infty\}$

- Types of probability

1. Prior probability / unconditional probability

The prior/unconditional probability $P(a)$ is the probability ~~P(a)~~ of a to be true.

E.g.

$P(a) = 0.7$

- probability of a to be true $= 0.7$

$P(\text{weather} = \text{sunny}) = 0.72$

$P(\text{weather} = \text{rain}) = 0.1$

2. Posterior probability / conditional probability

The posterior / Conditional probability $P(a/b)$ is the probability of a to be true given the condition b.

e.g.

$P(cavity / toothache) = 0.8$ means if a patient have toothache and no other information is yet available, the the probability of patient's having the cavity is 0.8.

$$\begin{cases} P(a/b) = \dfrac{P(a \wedge b)}{P(b)} \;;\; P(b) \neq 0 \\ P(a \wedge b) = P(a/b) \cdot P(b) = P(b/a) \cdot P(a) \end{cases}$$

Q. In a group of 100 sports car buyers, 40 bought alarm system, 30 purchased bucket seats, and 20 purchased an alarm system and bucket seats. If a car buyer chosen at random bought an alarm system, what is the probability they also bought bucket seats?

Soln

let us define,

A = purchasing alarm system , B = purchasing bucket seats

Then,

$P(A) = 40/100 = 0.4$

$P(B) = 30/100 = 0.3$

$P(A \wedge B) = 20/100 = 0.2$

$$P(B/A) = \frac{P(A \wedge B)}{P(A)} = \frac{0.2}{0.4} = 0.5$$

$\therefore$ The probability that a buyer bought bucket seats given that they purchased an alarm system is 50%.

Probability Distribution

A probability distribution is a table or a mathematical function that provides the probabilities of occurrence of different possible outcomes in an experiment.

→ Probability distribution gives values for all possible assignments.

E.g.
If P(weather=sunny) = 0.72, P(weather=rain) = 0.1, P(weather=cloudy) = 0.08 & P(weather=snow) = 0.1 then

$$P(weather) = (0.72, 0.1, 0.08, 0.1)$$

If two coins are tossed simultaneously, then the probability distribution of getting head!

→ If two coins are being tossed, there can be either 0 heads, 1 head or 2 heads.

Sample space S = {HH, HT, TH, TT}

The prob. distribution of getting heads can be shown as:

X	0	1	2
p(x)	1/4	1/4	1/4

If a coin is tossed, the probability distribution of getting tails.

X	0	1
p(x)	1/2	1/2

→ {If a coin is tossed, either 0 tail or 1 tail can be obtained}

Joint Probability Distribution

Joint probability is a statistical measure that calculates the likelihood of two events occurring together and at the same point in time.
Joint probability is the probability of event Y occurring at the same time that event X occurs.
For E.g.
If we have two random variables: weather and cavity with set of domain for weather = {rainy, sunny, cloudy, snow} and cavity = {True, false} the P(weather, cavity) = 4×2 matrix of values

weather =	sunny	rainy	cloudy	snow
cavity = true	0.144	0.02	0.016	0.02
cavity = false	0.576	0.08	0.064	0.08

Inference using full joint probability distribution

We use the full joint distribution as the knowledge base from which answers to all questions may be derived. The probability of a proposition is equal to the sum of the probabilities of the atomic events in which it holds.

$$P(a) = \sum P(e_i)$$

Therefore, given a full joint distribution that specifies the probabilities of all the atomic events, one can compute the probability of any proposition.

E.g.
The full joint distribution is the following 2×2×2 table.

	toothache		¬toothache	
	Gum Problem	¬Gum Problem	Gum Problem	¬Gum Problem
Cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

$P(\text{cavity or toothache}) = 0.108 + 0.012 + 0.072 + 0.08 + 0.016 + 0.064$
$= 0.28$

Marginalization or summing out

Distribution over Y can be obtained by summing out all the other variables from any joint distribution containing Y. This process is called marginalization.

$P(Y) = \sum P(Y, z)$

↳ No. of variables where Y seems to be true.

e.g.
From above table

$P(\text{cavity}) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2$
$P(\neg GP) = 0.012 + 0.064 + 0.008 + 0.576 = 0.66$
$P(\neg\text{toothache}) = 0.072 + 0.008 + 0.144 + 0.576 = 0.8$
$P(\text{cavity}, \neg\text{toothache}) = 0.072 + 0.008 = 0.08$

$P(Y) = \sum P(Y, z)$
$P(Y, z) = P(Y/z) \cdot P(z)$
Therefore, for any set of variables Y & Z:
$P(Y) = \sum P(Y/z) \cdot P(z)$
↳ This rule is the Conditioning rule.

Calculating conditional probability,

$$P(\neg cavity / toothache) = \frac{P(\neg cavity \wedge toothache)}{P(toothache)}$$

$$= \frac{(0.016 + 0.064)}{(0.108 + 0.012 + 0.016 + 0.064)}$$

$$= 0.4$$

$$P(cavity / toothache) = \frac{P(cavity \wedge toothache)}{P(toothache)}$$

$$= \frac{(0.108 + 0.012)}{(0.108 + 0.012 + 0.016 + 0.064)}$$

$$= 0.6$$

Independence

A and B are independent iff

$$P(A/B) = P(A) \text{ or } P(B/A) = P(B) \text{ or } P(A,B) = P(A).P(B)$$

e.g.

$P(toothache, Gum\ problem, cavity, weather)$

$= P(toothache, Gum\ problem, cavity).P(weather)$

Here, weather is independent of other three variables.

Imp

Baye's Theorem / Rule

Baye's theorem is a way to apply conditional probability for prediction. Conditional probability is the probability of an event happening, given that it has some relationship to one or more other events.
Mathematically, Baye's theorem is defined as:

$$P(b/a) = \frac{P(a/b) * P(b)}{P(a)}$$

Proof:

We know that,

$$P(b/a) = \frac{P(b \wedge a)}{P(a)}$$

$$P(b \wedge a) = P(b/a) * P(a) \text{ ——— (i)}$$

Similarly,

$$P(a/b) = \frac{P(a \wedge b)}{P(b)}$$

$$P(a \wedge b) = P(a/b) * P(b) \text{ ——— (ii)}$$

from eq^n (i) & (ii)

$$P(b/a) * P(a) = P(a/b) * P(b) \qquad [P(a \wedge b) \approx P(b \wedge a)]$$

$$P(b/a) = \frac{P(a/b) * P(b)}{P(a)}$$

→ Bayes theorem provides a way to revise existing predictions or theories (update probabilities) given new or additional evidence. This, in turn, makes the predictions more accurate.

Applications of Bayes Theorem

1. Medical science :
Baye's rule is used for predicting a particular disease based on the symptoms and physical condition of the patient.

2. Weather forcasting:
Baye's rule is a powerful algorithm for predictive modeling weather forcast.

3. Robotics :
Baye's rule is used to calculate the probability of a robot's next steps given the steps the robot has already executed.

4. Finance :
Baye's theorem can be used to rate the risk of lending money to potential borrowers.

Q. A doctor knows that the disease meningitis causes the patient to have a stiff neck 50% of the time. The doctor also knows that the probability that a patient has meningitis is 1/50,000, and the probability that any patient has a stiff neck is 1/20. Now find the probability that a patient with stiff neck has meningitis.

Sol^n

let s be the proposition that the patient has a stiff neck and m be the proposition that the patient has meningitis.

Here, we are given

$P(s/m) = 0.5$

$P(m) = 1/50,000$

$P(s) = 1/20$

$P(m/s) = ?$

Now using Baye's rule

$$P(m/s) = \frac{P(s/m) * P(m)}{P(s)} = \frac{0.5 * 1/50,000}{1/20} = 0.0002$$

Hence, the probability that a patient with a stiff neck has meningitis is 0.0002.

Q. 1% of women at age forty who participates in routine screening have breast cancer 80% of women with breast cancer will get positive mammographies. 9.6% of women without breast cancer will also get positive mammographies. A women in this age group has a positive mammography in a routine screening. What is the probability that she actually has breast cancer?

Soln

Let, B be the proposition that women has breast cancer.

B' be the proposition that women without breast cancer.

+ be the proposition that women getting positive mammographies.

Here, we are given

$P(B) = 0.01$

$P(+/B) = 0.8$

$P(+/B') = 0.096$

$P(B/+) = ?$

We have,

$$P(B/+) = \frac{P(+/B) * P(B)}{P(+)}$$

Here $P(B)$, $P(+/B)$ & $P(+/B')$ are known. $P(+)$ is needed to find $P(B/+)$.

$$P(+) = P(+/B) * P(B) + P(+/B') * P(B')$$
$$= (0.8 * 0.01) + (0.096 * 0.99)$$
$$= 0.1030$$

$$\therefore P(B/+) = \frac{0.8 * 0.01}{0.1030} = 0.07767$$

T.U. 074

Q. Consider in Nepal, 51% of adults are males and rests are females, Consider one adult is randomly selected for a survey of drinking alcohol. It is found that 15% of males drink alcohol where as 2% of female drink alcohol. Now find the probability that the selected adult is male.

Soln

let M be the adult males.
F be the adult females.
A be the adult who is drinking alcohol.

Here, we are given,

$P(M) = 0.51$

$P(F) = 0.49$

$P(A/M) = 0.15$

$P(A/F) = 0.02$

$P(M/A) = ?$

Now,

$P(A) = P(M) * P(A/M) + P(F) * P(A/F)$
$= 0.51 * 0.15 + 0.49 * 0.02$
$= 0.0863$

By using Baye's rule

$$P(M/A) = \frac{P(A/M) * P(M)}{P(A)}$$
$$= \frac{0.15 * 0.51}{0.0863}$$
$$= 0.8864$$

Q. Two different suppliers, A and B, provid a manufacturer with the same part. All supplies of this part are kept in a large bin. In the past, 5% of the parts supplied by A and 9% of the parts supplied by B have been defective. A supplies four times as many parts as B. suppose you reach into the ~~been~~ bin and select a part, and find it is non-defective. what is the prob. that it was supplied by A?

Soln

let,

A is the parts supplied by A.
B is the parts supplied by B.
D is the non-defective parts.

Given,

$P(D/A) = 0.095 \quad (1 - 0.05)$

$P(D/B) = 0.91$ $(1-0.09)$
$P(A) = 0.8$
$P(B) = 0.2$
$P(A/D) = ?$

Now,
$$P(D) = P(D/A) * P(A) + P(D/B) * P(B)$$
$$= 0.95 * 0.8 + 0.91 * 0.2$$
$$= 0.942$$

By using Baye's rule

$$P(A/D) = \frac{P(D/A) * P(A)}{P(D)} = \frac{0.95 * 0.8}{0.942} = 0.8068$$

Q. Manju is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 5 days each year. Unfortunately, the weatherman has predicated rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts 10% of the time. What is the probability that it will rain on the day of Manju's wedding?

Sol^n

Let,

Event A_1 → It rains on Manju's wedding.
Event A_2 → It does not rain on Manju's wedding.
Event B → The weatherman predicts rain.

Given,

$P(A_1) = \frac{5}{365} = 0.0136985 \approx 0.014$

$P(A_2) = \frac{360}{365} = 0.9863015 \approx 0.986$

$P(B/A_1) = 0.9$

$P(B/A_2) = 0.1$

$P(A_1/B) = ?$

Now,

$P(B) = P(A_1) * P(B/A_1) + P(A_2) * P(B/A_2)$

$= 0.014 * 0.9 + 0.986 * 0.1$

$= 0.1112$

Using Baye's rule

$P(A_1/B) = \frac{P(A_1) * P(B/A_1)}{P(B)} = \frac{0.014 * 0.9}{0.112} = 0.111$

∴ Prob. of rain on the day of Manju's wedding, given a forecast of rain by the weatherman is 0.111.

1.0 068 **Q3.** After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease, and the test is 99 % accurate (i.e. the probability of testing positive given that you have the disease is 0.99, as the probability of testing negative if you don't have disease). The good news is that is a rare disease, striking only one in 10,000 people?

a) Why it is good news that the disease is rare?

b) What are the chances that you actually have the disease?

Sol

Sol^n

let, T^+ : test is positive , T^- : test is negative.
D : has disease $\bar{D}$: doesn't have disease

Here, we are given,

$P(T^+/D) = 0.99$

$P(T^-/D) = 0.99$

$P(D) = \frac{1}{10,000} = 0.0001$

a) As probability of having disease is very small i.e. 0.0001, so it is good news that the disease is rare.

b) we can infer,

$P(\bar{D}) = 1 - 0.0001 = 0.9999$

$P(T^+/\bar{D}) = 1 - 0.99 = 0.01$

$P(D/T^+) = ?$

Now,

$$P(T^+) = P(T^+/D) * P(D) + P(T^+/\bar{D}) * P(\bar{D})$$
$$= 0.99 * 0.0001 + 0.01 * 0.9999$$
$$= 0.010098$$

Now, using Baye's rule

$$P(D/T^+) = \frac{P(T^+/D) * P(D)}{P(T^+)}$$
$$= \frac{0.99 * 0.0001}{0.010098}$$
$$= 0.09804$$

V.Imp.

Bayesian Network (Belief N/w / causal nets / Baye's N/w)

Bayesian networks are a type of probabilistic graphical model that uses Bayesian inference for probability computations.

- → Bayesian network is a probabilistic graphical model that represents a set of random variables and their conditional dependencies via a directed acyclic graph.
- → Nodes in the graph represent the random variables and the directed edges between nodes represent conditional dependencies.
- → The edge exists between nodes iff there exists conditional probability i.e. a link from x to y means y is dependent of x.
- → Each nodes are labelled with probability.

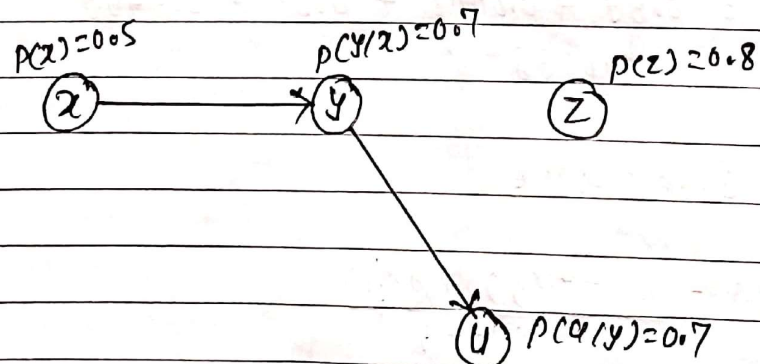
For e.g.

$P(x) = 0.5$

$P(y/x) = 0.7$

$P(z) = 0.8$

$P(u/y) = 0.47$



Bayesian Graph.

Inference with Bayesian Network:

Using a Bayesian network to compute probabilities is called inference in Bayesian network.
The first task is to compute the posterior probability distribution for the query variable x, given some assignment of values e to the set of evidence variable $E = E_1, \dots, E_n$ and the hidden variables are $Y = Y_1, \dots Y_n$.

From the full joint probability distribution we can answer the query $P(x/e)$ by computing

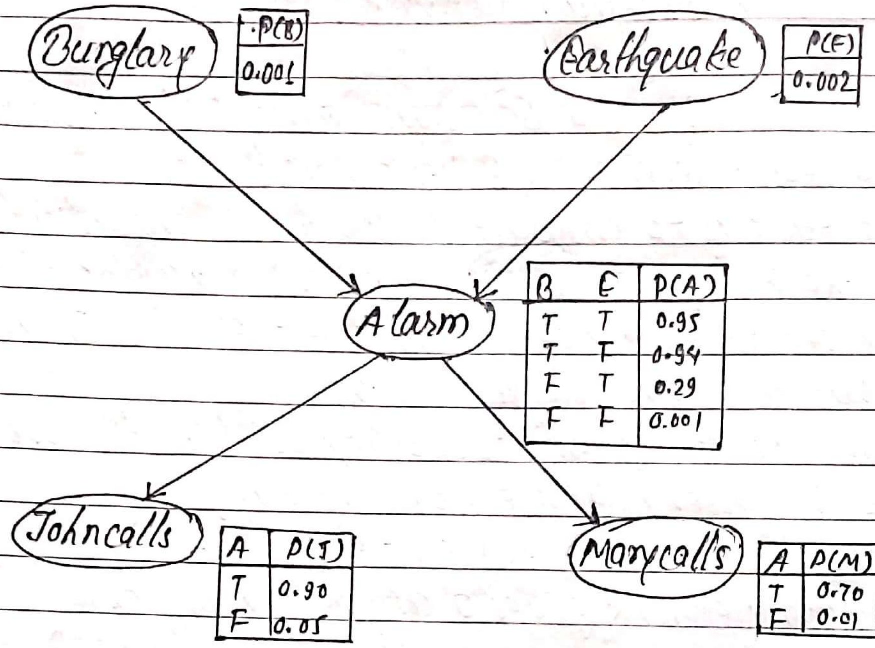
$$P(x/e) = \alpha P(x, e) = \alpha \sum_y (x, e, y)$$

A Bayesian network gives a complete representation of the full joint distribution, specifically, the terms $P(x, e, y)$ can be written as products of conditional probabilities from the network.

Therefore, a query can be answered using a Bayesian network by computing sums of products of conditional probabilities from the network.

* Bayesian N/w e.g.

You have a burglar alarm installed in your home. It is fairly reliable at detecting a burglary, but also responds on occasion to minor earthquakes. You also have two neighbors, John and Mary, who have promised to call you at work when they hear the alarm. John always calls when he hears the alarm, but sometimes confuses the telephone ringing with the alarm and calls then, too. Mary, on the other hand, likes rather loud music and sometimes misses the alarm altogether.



P(B)
0.001

P(E)
0.002

B	E	P(A)
T	T	0.95
T	F	0.94
F	T	0.29
F	F	0.001

A	P(J)
T	0.90
F	0.05

A	P(M)
T	0.70
F	0.01

Q. What are conceptual graphs? Represents the following statements into conceptual graph.
"King Ram marry sita, the daughter of king Janak".

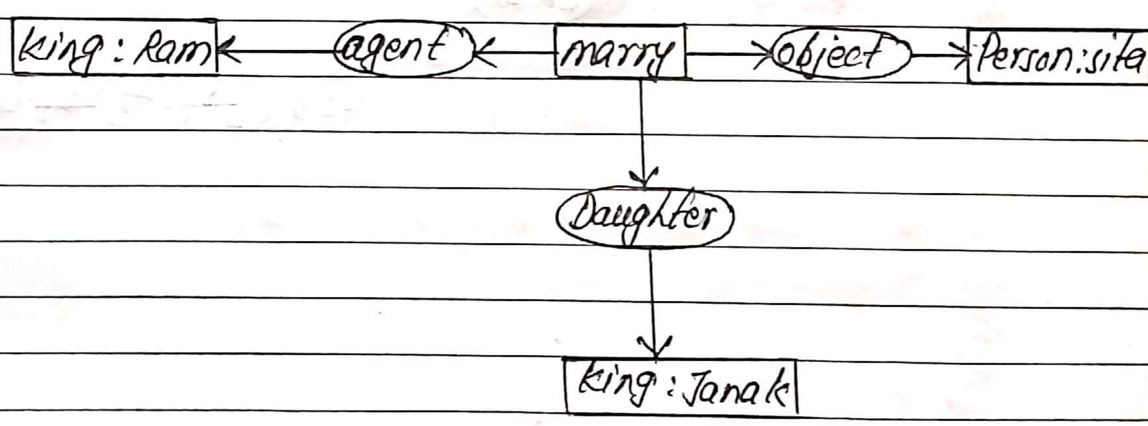
Soln

Conceptual graph is a graph representation for logic based on the semantic networks and the existential graphs of charles sanders Peirce.

A Conceptual graph consists of concept nodes and relation nodes.

- The concept node represents entities, attributes, states and events.
- The relation node show how the concepts are interrelated.

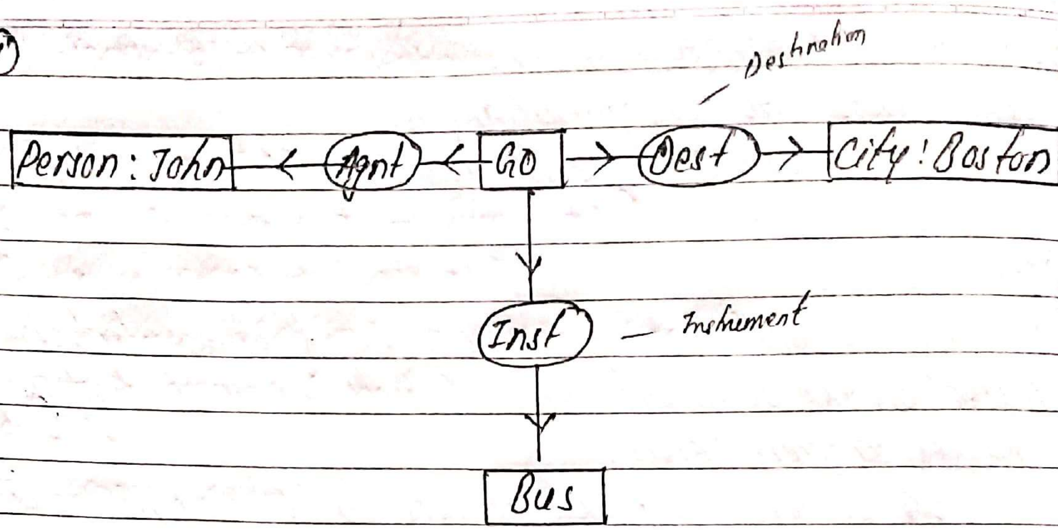
Conceptual graph for given sentence:



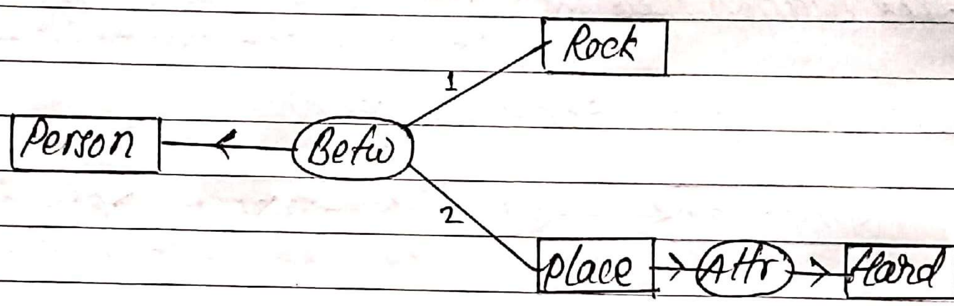
* Conceptual graphs e.g.

(i) John is going to Boston by bus.
(ii) A person is betn a rock and a hard place.
(iii) A cat is on a mat.

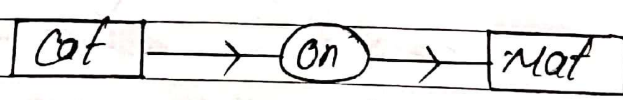
(1)


Destination
Person : John
Agnt
GO
Dest
City : Boston
Inst
Instrument
Bus

(2)


Rock
1
Person
Betw
2
place
Attr
Hard

(3)

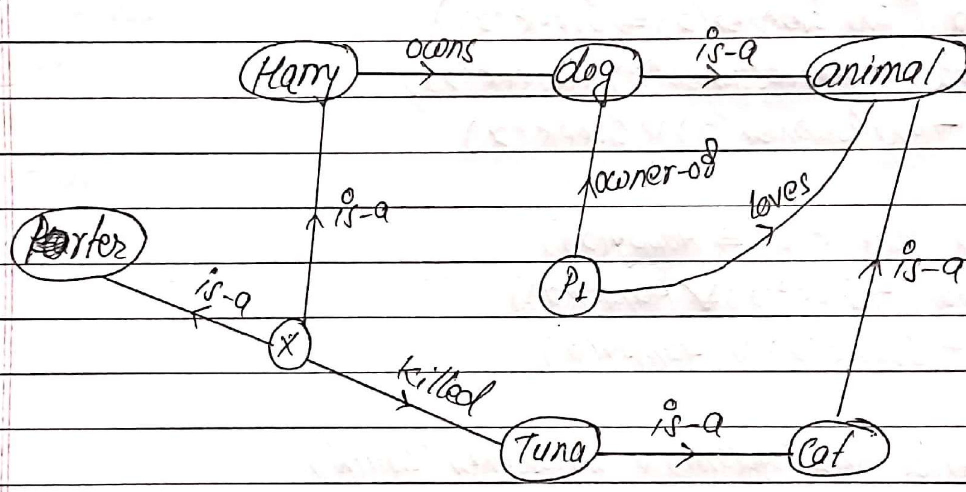

Cat
On
Mat

Some Q & A

Q. Represent following knowledge using semantics network.

Harry owns a dog.
Every dog owner loves animal.
Dog is an animal.
Either Harry or Porter killed Tuna.
Tuna is a cat.
All cats are animal.

Sol^n



Q. Prove the following argument is valid using resolution.
1. All philosophers are Greek.
2. All Greeks are happy.
3. Either Adam or smith is a philosopher.
4. Smith is not a philosopher.
5. Therefore, Adam is happy.

Sol^n
The predicate in KB are;
Philosopher(x) → x is philosopher.
Greek(x) → x is Greek.

Happy(x) → x is happy.

The facts in FOPL

1. $\forall x\,(philosopher(x) \Rightarrow Greek(x))$
2. $\forall x\,(Greek(x) \Rightarrow Happy(x))$
3. $philosopher(Adam) \vee philosopher(smith)$
4. $\neg philosopher(smith)$

To infer:

5. $Happy(Adam)$

Converting to CNF.

1. $\forall x\,(philosopher(x) \Rightarrow Greek(x))$
   $= \forall x\,(\neg Philosopher(x) \vee Greek(x))$
   $= \neg Philosopher(x) \vee Greek(x)$

2. $\forall x\,(Greek(x) \Rightarrow Happy(x))$
   $= \forall x\,(\neg Greek(x) \vee Happy(x))$
   $= \neg Greek(x) \vee Happy(x)$

3. $philosopher(Adam) \vee philosopher(smith)$
4. $\neg philosopher(smith)$

The KB in CNF

1. $\neg Philosopher(x) \vee Greek(x)$
2. $\neg Greek(x) \vee Happy(x)$
3. $philosopher(Adam) \vee philosopher(smith)$
4. $\neg Philosopher(smith)$
   Add negation of conclusion into KB.
5. $\neg Happy(Adam)$

Using resolution in (1) & (2)

$$\frac{\neg Philosopher(x) \vee Greek(x),\ \neg Greek(x) \vee Happy(x)}{\neg Philosopher(x) \vee Happy(x)}$$

It is a new clause not in KB. so add it to KB.

6. $\neg philosopher(x) \vee Happy(x)$

Using resolution in (5) & (6) by lifting $\{x/Adam\}$

$$\frac{\neg Happy(Adam),\ \neg philosopher(Adam) \vee Happy(Adam)}{\neg philosopher(Adam)}$$

It is a new clause not in KB. so add it to KB

7. $\neg philosopher(Adam)$

Using resolution in (3) & (7)

$$\frac{philosopher(Adam) \vee philosopher(smith),\ \neg philosopher(Adam)}{philosopher(smith)}$$

~~using~~ It is a new clause not in KB. so add it to KB.

8. $philosopher(smith)$

using resolution in (4) & (8)

$$\frac{\neg philosopher(smith),\ philosopher(smith)}{\phi}$$

Therefore, the argument "Adam is happy" is valid.