Inferential Statistics **First semester, 2021**

Exercises on Point estimation

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Example 4.1 *Let $X > 0$ be the number of defective items in a shipment of a very large number of items and suppose that $X \sim F_\theta$, with p.d.f. $f(x;\theta) = c_\theta \theta^x$, $x = 1, 2, \ldots,$, with $\theta \in (0,1)$ the unknown parameter and $c_\theta > 0$ a real depending on θ. Let $X_1, \ldots, X_n$ be an i.i.d random sample with $X_i \sim F_\theta$.*

(a) *Formulate the statistical model and write the likelihood and the log-likelihood function*

(b) *Is there a sufficient statistic for θ ?*

(c) *Find a method of moments estimator and compute its asymptotic distribution; what can you say about its asymptotic bias?*

(d) *Given the observed sample $x_1 = 8, x_2 = 2, x_3 = 3, x_4 = 1$, compute the MLE and state a probability distribution for this estimator.*

(e) *Answer to (d), when the observed sample is $x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 8$. Why the answer is as in (d)?*

Solution.

Example 4.2 *Let $X_i \overset{\text{iid}}{\sim} \mathrm{N}(\mu, \sigma^2)$, $i = 1, \ldots, n$. Suppose that μ is known and σ^2 is unknown. Get the MLE of $\psi = \log \sigma$ and determine its distribution, where $\sigma = \sqrt{\sigma^2}$*

Solution.

Example 4.3 *Let* $X_i \overset{\text{iid}}{\sim} \mathrm{N}(\mu, \sigma^2)$, $i = 1, \ldots, n$. *Suppose that* μ *and* σ^2 *are unknown.*

(a) *Get the MLE of* $\psi = \sigma/\mu$ *and determine its distribution.*

(b) *Suppose that an observed sample of size* 10 *produced* $\overline{x} = 1.5$ *and* $n^{-1}\sum_{i=1}^{n}(x_i - \overline{x})^2 = 3$, *approximate the probability that the MLE is greater than* $1 + \psi$.

Solution.

Example 4.4 *Let* $X_i \overset{\text{iid}}{\sim} \mathrm{N}(\theta, \theta^2)$, $i = 1, \ldots, n$, *with* $\theta > 0$ *unknown.*

(a) *Find a sufficient statistic for* θ

(b) *Compute the MLE of* θ

Solution.

Example 4.5 *Let* $X_i \overset{\text{iid}}{\sim} \mathrm{Unif}(0, \theta)$, $i = 1, \ldots, n$, *with* θ *unknown. Find the distribution of the MLE of* θ. *What is its limiting distribution as* $n \to \infty$?

Solution.