MATH303-22S2: Assignment 2

Instructions

- This assignment is due at **5pm on Monday 17 October**.
- It is to be submitted, as a single pdf file, to the dropbox folder on the Learn page.
- This assignment is worth **20%** of your final grade.
- You may work by yourself or with one other person. If you hand in a joint assignment, you will each be given the same mark.
- You must write your name (and your partner's name if you are working as a pair) on your submission.
- Your assignment does not need to be typed, but you should expect to lose marks for poorly presented working and/or incomplete working.

Questions

1. Consider the LP:

$$\begin{aligned} \text{maximize} \quad & x_1 + x_2 \\ \text{subject to} \quad & -x_1 + x_2 \leq 10 \\ & x_1 + x_2 \geq 5 \\ & 2x_1 + x_2 \leq 40 \\ & x_1,\ x_2 \geq 0. \end{aligned}$$

   (a) Re-write this problem in standard form.
   (b) Using one artificial variable, state the Phase I problem for this LP.
   (c) Solve the Phase I problem (using the Simplex Method) to get an initial basic feasible solution.
   (d) Using your initial bfs from part (c), solve the LP using the Simplex Method.

2. Using one artificial variable, determine whether there are any vectors satisfying

$$\begin{aligned} 2x_1 + x_2 &\geq 70 \\ x_1 + x_2 &\leq 40 \\ x_1 + 3x_2 &\geq 90 \\ x_1,\ x_2 &\geq 0. \end{aligned}$$

3. The 100m sprint is held at the Olympic Games. The winning times each Olympics are

Year (Y)	**Time (T)**
1948	10.3
1952	10.4
1956	10.5
1960	10.2
1964	10.0
1968	9.95
1972	10.14
1976	10.06
1980	10.25
1984	9.99

Year (Y)	**Time (T)**
1988	9.92
1992	9.96
1996	9.84
2000	9.87
2004	9.85
2008	9.69
2012	9.63
2016	9.81
2020	9.80

Before you start: You should use a computer to help with this question; do not attempt to do the computations by hand. (Hint: The Week 6 lecture notes, page 7, and your answer to Question 5 from Assignment 1, might be useful here).

Suppose you had already set up a system of equations for this data using a polynomial of degree 1 (a linear fit). i.e., you had already formed $A\mathbf{x} = \mathbf{b}$, where each equation was of the form

$$T = c_0 + c_1 Y.$$

You had solved the least squares system using the QR factorization, and that had given you the best fit in terms of the 2-norm (i.e., the least squares solution was the one that minimized $\|A\mathbf{x} - \mathbf{b}\|_2^2$), and you found it to be $\mathbf{x} = \begin{bmatrix} 28.1690 \\ -0.0092 \end{bmatrix}$. But now you were worried that there were outliers in the data set, and you wanted to re-do the computation, but minimizing the 1-norm instead. So, now, you want to solve

$$\min_{\mathbf{x}} \|A\mathbf{x} - \mathbf{b}\|_1.$$

This problem can be reformulated as an LP:

$$\begin{array}{ll} \min\limits_{\mathbf{x},\mathbf{t}} & \mathbf{1}^T\mathbf{t} \\ \text{subject to} & A\mathbf{x} - \mathbf{t} \le \mathbf{b} \\ & -A\mathbf{x} - \mathbf{t} \le -\mathbf{b} \\ & \mathbf{t} \ge 0 \end{array}$$

(a) Re-write this problem in standard form. (Note that there should not be any non-negativity constraint for the '$\mathbf{x}$').

(b) Solve this problem, for example, using Matlab's in-built linprog function.

(c) On a single set of axes, plot the data, and the linear (degree 1) fit for you found in part (b). (Note that you are only interested in the '$\mathbf{x}$' part of the solution here.)

(d) Using your linear fit from (c), predict the winning time for the Paris 2024 Olympic games.