MATH303-22S2 - *Assignment 2*

`Due Date:` **Monday 17th October 5pm**

`Submission by:` Richard Hodges, Henry Hastings

`Student IDs:` 11139318, 35556669

Question 1

a)

$$\begin{aligned} \underset{\underline{x}}{\text{minimise}} \quad & -x_1 + x_2 \\ \text{subject to} \quad & x_1 - x_2 + x_3 && = 10 \\ & -x_1 - x_2 + x_4 && = -5 \\ & 2x_1 + x_2 + x_5 && = 40 \\ & \underline{x} \geq 0 \end{aligned}$$

Where $x_3,\ x_4,\ x_5$ are slack variables.

$$A = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \underline{b} = \begin{bmatrix} 10 \\ -5 \\ 40 \end{bmatrix}$$

---

b) Let $B = \{3, 4, 5\}$ as $A_B = I \Rightarrow \exists\ A_B^{-1} = I$

$$\underline{x}_B = A_B^{-1}\underline{b} = \begin{bmatrix} 10 \\ -5 \\ 40 \end{bmatrix}$$

Thus x_4 leaves B as this corresponds to the most negative element of $\underline{x}_B$

We can replace x_4 in B with a singular artificial variable as seen in part 8.1 of the lecture notes. The artificial variable, $\underline{a}_6$, which is adjoined to the matrix A, is obtained in the following way:

$$\underline{a}_6 = \underline{b} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \\ 39 \end{bmatrix}$$

From part 8.1 of the lecture notes we see that $B^{\#} = \{3,\ 5,\ 6\}$ is a B.F.S of the following L.P:

$$[A \,|\, a_6] \begin{bmatrix} \vec{x} \\ x_6 \end{bmatrix} = \underline{b}$$

This represents the constraints of our phase 1 problem, as outlined in part 8.1 of the lecture notes. The phase 1 problem requires us to minimise x_6 given this constraint, removing it from B which yields a B.F.S of our original L.P.

Phase 1 problem:

$$\begin{aligned} \underset{\underline{x}}{\text{minimise}} \quad & x_6 \\ \text{subject to:} \quad & [A \,|\, a_6] \begin{bmatrix} \vec{x} \\ x_6 \end{bmatrix} = \underline{b} \\ & x_6,\ \underline{x} \geq 0 \end{aligned}$$

---

c) We solve this L.P using the simplex method as described in Algorithm 1 of the week 7 notes.

$B = \{3,\ 5,\ 6\}$ by **b)** thus $N = \{1,\ 2,\ 4\}$

$$\Rightarrow A_B = \begin{bmatrix} 1 & 0 & 9 \\ 0 & 0 & -5 \\ 0 & 1 & 39 \end{bmatrix} \Rightarrow A_B^{-1} = \begin{bmatrix} 1 & 1.8 & 0 \\ 0 & 7.2 & 1 \\ 0 & -0.2 & 0 \end{bmatrix}, \quad A_N = \begin{bmatrix} -1 & 1 & 0 \\ -1 & -1 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$\underset{\sim}{c}_N = \underset{\sim}{0}, \quad \underset{\sim}{c}_B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We define $\underset{\sim}{r} = \underset{\sim}{c}_N - A_N^T A_B^{-T} \underset{\sim}{c}_B$

$$\Rightarrow \underset{\sim}{r} = \vec{0} - \begin{bmatrix} -1 & -1 & 2 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1.8 & 7.2 & -0.2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.2 \\ -0.2 \\ 0.2 \end{bmatrix}$$

Choose x_1 to enter as it has a lower index of the variables corresponding to the negative values of $\underset{\sim}{r}$, using Bland's anti-cycling rules.

$$\hat{\underset{\sim}{b}} = A_B^{-1} \underset{\sim}{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \underset{\sim}{d} = -A_B^{-1} \underset{\sim}{a}_1 = \begin{bmatrix} 2.8 \\ 5 \\ -0.2 \end{bmatrix}$$

x_6 must be the leaving variable as it represents the only negative value in $\underset{\sim}{d}$.

$$\Rightarrow B = \{1,\ 3,\ 5\}, \quad N = \{2,\ 4,\ 6\}$$

$$\Rightarrow A_B = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix} \Rightarrow A_B^{-1} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \quad A_N = \begin{bmatrix} 1 & 0 & 9 \\ -1 & 1 & -5 \\ 1 & 0 & 39 \end{bmatrix}$$

$$\underset{\sim}{c}_N = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \underset{\sim}{c}_B = \underset{\sim}{0}$$

$$\Rightarrow \underset{\sim}{r} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - A_N^T A_B^{-T} \underset{\sim}{0} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

As $\underset{\sim}{r} \geq 0$, $B = \{1,\ 3,\ 5\}$ is an optimal basis for the phase 1 problem and thus a B.F.S. for the original L.P.

---

d) We solve this L.P using the simplex method as described in Algorithm 1 of the week 7 notes

$$\Rightarrow B = \{1,\ 3,\ 5\}, \quad N = \{2,\ 4\}$$

$$\Rightarrow A_B = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 0 \\ 2 & 0 & 1 \end{bmatrix} \quad A_B^{-1} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \quad A_N = \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\underset{\sim}{c}_N = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \quad \underset{\sim}{c}_B = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} \Rightarrow \underset{\sim}{r} &= \begin{bmatrix} -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} -1 \\ 0 \end{bmatrix} - \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ -1 \end{bmatrix} \end{aligned}$$

So x_4 is the entering variable

$$\hat{\underset{\sim}{b}} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ -5 \\ 40 \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \\ 30 \end{bmatrix}, \quad \underset{\sim}{d} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \underset{\sim}{a}_4 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$$

Giving us x_5 as the leaving variable, as this corresponds to the only negative entry in $\underset{\sim}{d}$

$$\Rightarrow B = \{1,\,3,\,4\}, \quad N = \{2,\,5\}$$

$$\Rightarrow A_B = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & 0 & 0 \end{bmatrix} \quad A_B^{-1} = \begin{bmatrix} 0 & 0 & 0.5 \\ 1 & 0 & 0.5 \\ 0 & 1 & 0.5 \end{bmatrix} \quad A_N = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\underline{c}_N = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, \quad \underline{c}_B = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} \Rightarrow \underline{r} &= \begin{bmatrix} -1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} -1 \\ 0 \end{bmatrix} - \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \end{aligned}$$

So x_2 is the entering variable

$$\hat{\underline{b}} = A_B^{-1}\underline{b} = \begin{bmatrix} 0 & 0 & 0.5 \\ 1 & 0 & 0.5 \\ 0 & 1 & 0.5 \end{bmatrix} \begin{bmatrix} 10 \\ -5 \\ 40 \end{bmatrix} = \begin{bmatrix} 20 \\ 30 \\ 15 \end{bmatrix}$$

$$\underline{d} = A_B^{-1}\underline{a}_2 = \begin{bmatrix} 0 & 0 & 0.5 \\ 1 & 0 & 0.5 \\ 0 & 1 & 0.5 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.5 \\ -1.5 \\ 0.5 \end{bmatrix}$$

$$\frac{-\hat{b}_1}{d_1} = 40 \qquad \frac{-\hat{b}_2}{d_2} = 20$$

As $\frac{-\hat{b}_2}{d_2} < \frac{-\hat{b}_1}{d_1}$ we get x_3 as the leaving variable, which corresponds to $\hat{b}_2$ and d_2.

$$\Rightarrow B = \{1,\,2,\,4\}, \quad N = \{3,\,5\}$$

$$\Rightarrow A_B = \begin{bmatrix} -1 & 1 & 0 \\ -1 & -1 & 1 \\ 2 & 1 & 0 \end{bmatrix} \Rightarrow \quad A_B^{-1} = \begin{bmatrix} \frac{-1}{3} & 0 & \frac{1}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 1 & \frac{2}{3} \end{bmatrix}, \quad A_N = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\underline{c}_N = \underline{0}, \quad \underline{c}_B = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$$

$$\begin{aligned} \Rightarrow \underline{r} &= \underline{0} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{-1}{3} & \frac{2}{3} & \frac{1}{3} \\ 0 & 0 & 1 \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} \end{aligned}$$

As $\underline{r} \geq \underline{0}$ we have an optimal basis

$$\underline{x}_B^* = A_B^{-1}\underline{b} = \begin{bmatrix} 10 \\ 20 \\ 25 \end{bmatrix} \quad \underline{x}_N^* = \underline{0}$$

$$\underline{x}^* = \begin{bmatrix} 10 \\ 20 \\ 0 \\ 25 \\ 0 \end{bmatrix}$$

This gives minimal value of

$$-x_1 - x_2 = -30$$

and a maximal value of

$$x_1 + x_2 = 30$$

The solution ot the L.P. is

$x_1 = 10,\ x_2 = 20;\ x_1 + x_2 = 30$

All inverses for this question were found using Symbolab.

Question 2

In standard form our constraints are:

$$\begin{aligned}
-2x_1 - x_2 + x_3 &= -70 \\
x_1 + x_2 + x_4 &= 40 \\
-x_1 - 3x_2 + x_5 &= -90 \\
\underline{x} &\geq 0
\end{aligned}$$

Where $x_3,\ x_4,\ x_5$ are slack variables.

$$\Rightarrow A = \begin{bmatrix} -2 & -1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ -1 & -3 & 0 & 0 & 1 \end{bmatrix} \qquad \underline{b} = \begin{bmatrix} -70 \\ 40 \\ -90 \end{bmatrix}$$

Let $B = \{3,\ 4,\ 5\}$ as $\exists\ A_B^{-1}$ as $A_B = I_3$

$$\underline{x}_B = \begin{bmatrix} -70 \\ 40 \\ -90 \end{bmatrix} = \underline{b}$$

We can replace x_5 in B as it corresponds to the most negative entry of $\underline{x}_B$. We replace it with a singular artificial variable as seen in part 8.1 of the lecture notes. The artificial variable, x_6, which is adjoined to the matrix A, is obtained in the following way:

$$\underline{a}_6 = \underline{b} - \underline{a}_3 - \underline{a}_4 + \underline{a}_5 = \begin{bmatrix} -71 \\ 39 \\ -90 \end{bmatrix}$$

This yields a phase 1 problem with $B = \{3,\ 4,\ 6\}$:

$$\begin{aligned}
\underset{\underline{x}}{\text{minimise}} \quad & x_6 \\
\text{subject to} \quad & [A \,|\, \underline{a}_6] \begin{bmatrix} \underline{x} \\ x_6 \end{bmatrix} = \underline{b} \\
& \underline{x},\ x_6 \geq \underline{0}
\end{aligned}$$

We solve this L.P using the simplex method as described in Algorithm 1 of the week 7 notes

$$B = \{3,\ 4,\ 6\} \quad N = \{1,\ 2,\ 5\}$$

$$A_B = \begin{bmatrix} 1 & 0 & -71 \\ 0 & 1 & 39 \\ 0 & 0 & -90 \end{bmatrix} \quad A_B^{-1} = \begin{bmatrix} 1 & 0 & \frac{-71}{90} \\ 0 & 1 & \frac{39}{90} \\ 0 & 0 & \frac{-1}{90} \end{bmatrix} \quad A_N = \begin{bmatrix} -2 & -1 & 0 \\ 1 & 1 & 0 \\ -1 & -3 & 1 \end{bmatrix}$$

$$\underline{c}_N = \underline{0}, \quad \underline{c}_B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{aligned}
\underline{r} = \underline{0} - &\begin{bmatrix} -2 & -1 & 0 \\ 1 & 1 & 0 \\ -1 & -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{-71}{90} & \frac{39}{90} & \frac{-1}{90} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\
= &\begin{bmatrix} \frac{-1}{90} \\ \frac{-3}{90} \\ \frac{1}{90} \end{bmatrix}
\end{aligned}$$

So x_2 is the entering variable.

$$\underline{\hat{b}} = A_B^{-1}\underline{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \underline{d} = -A_B^{-1}\underline{a}_2 = \begin{bmatrix} \frac{-123}{90} \\ \frac{27}{90} \\ \frac{3}{90} \end{bmatrix}$$

$$\frac{-\hat{b}_1}{d_1} = \frac{90}{123} \quad \frac{-\hat{b}_3}{d_3} = \frac{90}{3}$$

So x_3 is the leaving variable, as $\frac{-\hat{b}_1}{d_1} < \frac{-\hat{b}_3}{d_3}$ and x_3 corresponds to $\hat{b}_1$ and d_1.

$$\Rightarrow B = \{2,\,4,\,6\} \quad N = \{1,\,3,\,5\}$$

$$\Rightarrow A_B = \begin{bmatrix} -1 & 0 & -71 \\ 1 & 1 & 39 \\ -3 & 0 & -90 \end{bmatrix} \Rightarrow \quad A_B^{-1} = \begin{bmatrix} \frac{30}{41} & 0 & \frac{-71}{90} \\ \frac{9}{41} & 1 & \frac{39}{90} \\ \frac{-1}{41} & 0 & \frac{-1}{90} \end{bmatrix}, \quad A_N = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\underline{c}_N = \underline{0}, \quad \underline{c}_B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\underline{r} = \underline{0} - \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{30}{41} & \frac{9}{41} & \frac{-1}{41} \\ 0 & 1 & 0 \\ \frac{-71}{90} & \frac{39}{90} & \frac{-1}{90} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{-5}{123} \\ \frac{1}{41} \\ \frac{-1}{123} \end{bmatrix}$$

Therefore x_1 is the entering variable.

$$\underline{\hat{b}} = A_B^{-1}\underline{b} = \begin{bmatrix} \frac{30}{41} \\ \frac{50}{41} \\ \frac{40}{41} \end{bmatrix} \quad \underline{d} = -A_B^{-1}\underline{a}_2 = \begin{bmatrix} \frac{109}{123} \\ \frac{-37}{123} \\ \frac{-5}{123} \end{bmatrix}$$

$$\frac{-\hat{b}_2}{d_2} = \frac{150}{37} \quad \frac{-\hat{b}_3}{d_3} = \frac{120}{5}$$

So x_4 is the leaving variable, as $\frac{-\hat{b}_2}{d_2} < \frac{-\hat{b}_3}{d_3}$ and x_4 corresponds to $\hat{b}_2$ and d_2

$$\Rightarrow B = \{1,\,2,\,6\} \quad N = \{3,\,4,\,5\}$$

$$\Rightarrow A_B = \begin{bmatrix} -2 & -1 & -71 \\ 1 & 1 & 39 \\ -1 & -3 & -90 \end{bmatrix} \Rightarrow \quad A_B^{-1} = \begin{bmatrix} \frac{27}{37} & \frac{123}{37} & \frac{32}{37} \\ \frac{51}{37} & \frac{109}{37} & \frac{7}{37} \\ \frac{-2}{37} & \frac{-5}{37} & \frac{-1}{37} \end{bmatrix}, \quad A_N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\underline{c}_N = \underline{0}, \quad \underline{c}_B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\underline{r} = \underline{0} - \begin{bmatrix} \frac{27}{37} & \frac{51}{37} & \frac{-2}{37} \\ \frac{123}{37} & \frac{109}{37} & \frac{-5}{37} \\ \frac{32}{37} & \frac{7}{37} & \frac{-1}{37} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{2}{37} \\ \frac{5}{37} \\ \frac{1}{37} \end{bmatrix} \geq \underline{0}$$

Thus, the optimal basis is $B = \{2,\,4,\,6\}$

The optimal basis still contains the artificial variable. This shows us that there is no basic feasible solution to our set of constraints without the artificial variable. Thus, this shows there are no vectors satisfying our constraints.

All inverses for this question were found using Symbolab.

Question 3

a)

$$A = \begin{bmatrix} 1948 & 1 \\ \vdots & \vdots \\ 2020 & 1 \end{bmatrix} \quad \underline{b} = \begin{bmatrix} 10.3 \\ \vdots \\ 9.80 \end{bmatrix} \underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \underline{t} = \begin{bmatrix} t_1 \\ \vdots \\ t_{19} \end{bmatrix}$$

Using this we can rewrite our constraints in the form

$$\left[\begin{array}{c|c} A & -I_{19} \\ -A & -I_{19} \end{array}\right] \begin{bmatrix} \underline{x} \\ \underline{t} \end{bmatrix} \leq \begin{bmatrix} \underline{b} \\ -\underline{b} \end{bmatrix}$$

Thus we add 38 slack variables using I_{38}, naming the variables $s_1, \ldots, s_{38}$ which form the vector $\underline{s}$. This results in the standard form LP constraint.

$$\left[\begin{array}{c|c|c} A & -I_{19} & I_{38} \\ -A & -I_{19} & \end{array}\right] \begin{bmatrix} \underline{x} \\ \underline{t} \\ \underline{s} \end{bmatrix} \begin{bmatrix} \underline{b} \\ -\underline{b} \end{bmatrix}$$

The LP in standard form is:

$$\begin{aligned} \underset{\underline{x}}{\text{minimise}} \quad & \underline{1}^T \underline{t} \\ \text{subject to} \quad & \left[\begin{array}{c|c|c} A & -I_{19} & I_{38} \\ -A & -I_{19} & \end{array}\right] \begin{bmatrix} \underline{x} \\ \underline{t} \\ \underline{s} \end{bmatrix} = \begin{bmatrix} \underline{b} \\ -\underline{b} \end{bmatrix} \\ & \underline{t},\ \underline{s} \geq \underline{0} \end{aligned}$$

---

b) We let:

$$X = \left[\begin{array}{c|c} A & -I_{19} \\ -A & -I_{19} \end{array}\right]$$

$$\underline{c} = \begin{bmatrix} \underline{b} \\ -\underline{b} \end{bmatrix}$$

We define our objective function through the vector $\underline{f}$

$$\underline{f} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

Such that $\underline{f}$ is 21×1.

As $\underline{1}^T \underline{t} = \underline{0}^T \underline{x} + \underline{1}^T \underline{t}$

The function linprog$(f,\ X,\ c)$ will minimise f, such that $X\underline{x}^* \leq \underline{c}$, where $\underline{x}^*$ is the vector of all our constraints.

This is equivalent to:

$$\left[\begin{array}{c|c} A & -I_{19} \\ -A & -I_{19} \end{array}\right] \begin{bmatrix} \underline{x} \\ \underline{t} \end{bmatrix} \leq \begin{bmatrix} \underline{b} \\ -\underline{b} \end{bmatrix}$$

Hence, using linprog$(f,\ X,\ c)$ will solve our L.P.

The solution to this LP using the linprog function. which satisfies the non-negativity constraints of $\underline{t}$ and $\underline{s}$ is:

$$\begin{bmatrix} -0.0083 \\ 26.4085 \\ 0 \\ 0.1331 \\ 0.2662 \\ 0.0008 \\ 0.1677 \\ 0.1846 \\ 0.0385 \\ 0.0085 \\ 0.2146 \\ 0.0123 \\ 0.0492 \\ 0.0238 \\ 0.0631 \\ 0 \\ 0.0131 \\ 0.1138 \\ 0.1408 \\ 0.0723 \\ 0.0954 \end{bmatrix}$$

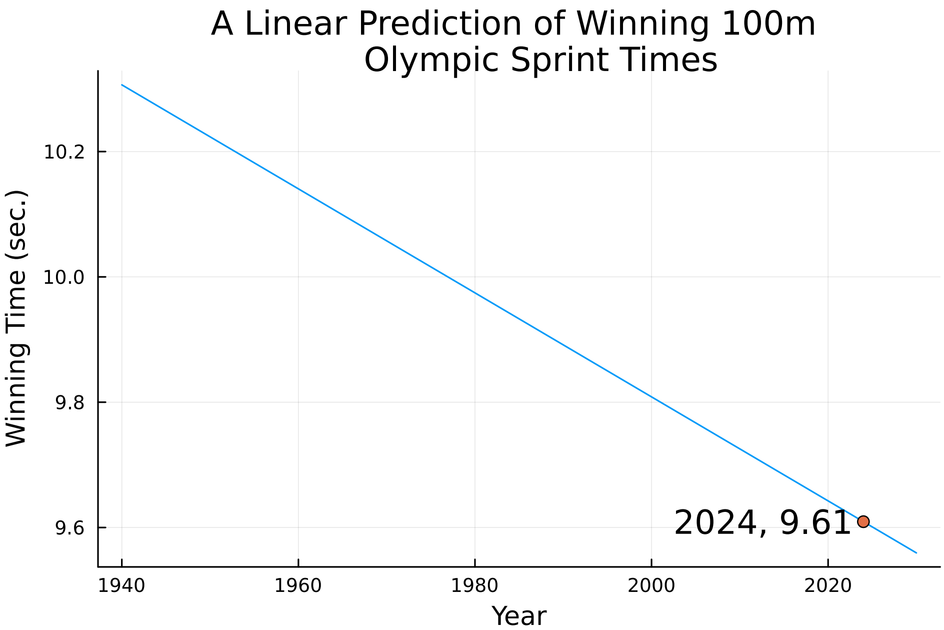
---

c) Given our solution to the LP we find:

$$x_1 = -0.0083 \quad x_2 = 26.4085$$

This gives a linear approximation of

$$T = 26.4085 - 0.0083x$$



`Figure 1. - A linear prediction of winning 100m olympic sprint times.`

---

d) Using the linear approximation for the line of best fit

$$T = 26.4085 - 0.0083x$$

and inputting the year $x = 2024$ yields

$$T = 26.4085 - 0.0083 \times 2024 = 9.6093$$

Our linear approximation predicts a winning 100m spring time of 9.6093 seconds at the 2024 Olympic Games.