MATH363-22S2 - *Assignment 2*

Language Revitalisation Modeling

`Due Date`: **Monday, 10 October 2022, 5:00 PM**

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All programming is done in Julia and the code and full size images will be available at:

`http://github.com/Reezy-git/363Language`

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using PlutoUI
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Abstract

Around the world there are many languages listed as endangered in particular in New Zealand the use of te reo Māori has seen a significant decline since colonisation and settlement of Europeans. There are a number of reasons why language diversity is considered important such as the language-culture link and enhanced social development [2,9] as such the preservation of existing languages such as te reo Māori should take precedence.

This assignment explores through mathematical modeling the rates at which language death and revival can occur. With the goal of finding whether or not a language will become extinct or flourish in the long term and the required system parameters for this to occur. We use bifurcation analysis of our proposed ODE systems to achieve this.

There exists a level for each language based on the learning and acquisition rates for above which it will be self sustaining and reach a fixed point at which it is used and below which it will reach the fixed point related to extinction. Similarly, there exists a set of learning rates above the threshold given a non-zero starting population of speakers will lead to self sustained growth of a language to the fixed point at which it is used and below which it will reach the fixed point related to extinction.

Policy Implications

When a language is on the trajectory towards extinction additional support to alter the language acquisition rates in the short term may be enough to push the trajectory toward a stable existent future. Once the population reaches it's critical threshold language acquisition is predicted to be self-sustaining therefore the support for revitalisation required is finite. Unless the long term acquisition rates are too low in which case no remedy short of improving these will see a result differing from extinction.

Introduction

With approximately 7000 languages currently spoken around the world [1] and the present prediction that 50-90% of these will be extinct by 2100 [8]. We stand to preserve a significant amount of language and therefore cultural diversity [2] by protecting the already existing 7000 languages. In order to effectively preserve language diversity, we must understand the forces in play and explore potential critical tipping points (in this case we look at bifurcations) within the system. It is hypothesised that once a certain percentage of the population reaches proficiency in the language the revitalisation will become self sustaining resulting in a thriving language and subsequently culture.

As with anything there are different levels of mastery within language acquisition. The Common European Framework of Reference for Languages suggests 3 broad levels of language mastery: Basic, Independent and Proficient [6]. As such, we will break our population into 3 sub populations in relation to their language mastery levels and describe the transition rates between them similar to Barrett-Walker [4]. The model we use is not one of Lokta-Volterra style competition where one language is competing for dominance, as is discussed by Baggs [7] instead considering languages to be non-predatory in nature. We find that the trajectory of a language is either toward extinction or that of a non-zero-use steady state and is entirely dependent on the initial conditions.

We find that for te reo Māori with the estimated learning rates in this assignment, we would expect that even with a population of 100% proficiency the language would die. In 2013 we had a proficient population of merely 10.6% [4] and so if te reo Māori is to survive long term, there needs to be an inrease of the learning rates. No other factor will change the long term outcome, $\beta = \gamma = 0.03$ is simply too low for the language to survive.

Model

In this model we make a few assumptions. Firstly, our overall population, N is static, i.e. birth rates are equal to death rates and total population $N = 1$ this allows us to view the different proficiency levels are a percentage of the total population. We also assume that individuals move from the basic category to the independednt to the proficient category without the ability to lose proficiency or skip a category. Below in figure 1 we see the proposed structural causal model (SCM) for the language acquisition.

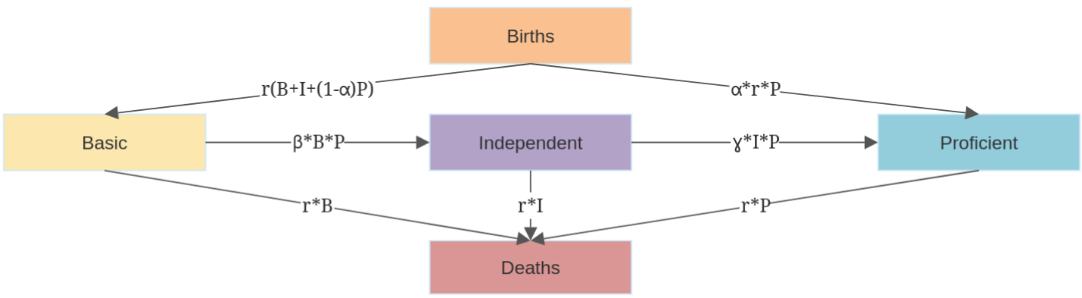

Figure 1 - The SCM represented as a digraph.

We therefore can construct the following models:

For the Basic users, we observe the rate of change is driven by births at a rate of r where all offspring of basic and independent users are assumed to be basic users, and a certain percentage of proficient users baby's are proficient leaving the remainder in the basic category. Basic users can transition to independent users at a learning rate of β dependent on the population of basic and proficient users $B \,\&\, P$ and finally the basic population suffers a death rate of r this gives:

$$\frac{dB}{dt} = r(B + I + (1-\alpha)P) - \beta BP - rB$$

The change in independent users is driven by the transition from basic to independent βBP the transition rate from independent to proficient γIP and of course the death rate. This gives:

$$\frac{dI}{dt} = \beta IP - \gamma IP - rI$$

Finally the change in proficient users is driven by the percentage of proficiency acquired at birth α the transition from independent to proficient γIP and the death rate.

$$\frac{dP}{dt} = r\alpha P + \gamma IP - rP$$

Research in [3-5] suggests that appropriate values for these variables may be near:

$$\alpha = .47 \quad \beta = 0.03 \quad \gamma = 0.03 \quad r = \frac{1}{80}$$

For the simplicity of the model we will assume that $\gamma = \beta$ that is to say that independent learners are half way to fluency.

Since we have fixed our population size to $N = 1$ we can observe that $B = 1 - I - P$ and as such we can reduce our model to the following two equations with the remaining population falling into category B.

$$\frac{dP}{dt} = r\alpha P + \gamma IP - rP \quad , \quad \frac{dI}{dt} = \beta(1 - P - I)P - \gamma IP - rI \tag{1}$$

Results

With a little algebra we can work out the null clines from (1) as:

$$null(I) \Rightarrow \beta P^{*2} + (\beta(I-1) + \gamma I)P^* - rI = 0$$

$$null(P) \Rightarrow I^* = \frac{(1-\alpha)r}{\gamma}$$

There are fixed points when both systems in $(1) = 0$ that is $(I^*, P^*) = (0, 0)$ and when the null clines are equal.

$$\Rightarrow \; (I^*, P^*) = (0, 0) \quad or \quad \left(\frac{(1-\alpha)r}{\gamma}, \frac{-b \pm \sqrt{b^2 - 4\beta c}}{2\beta} \right)$$

Where:

$$b := \beta(\frac{(1-\alpha)r}{\gamma} - 1) + (1-\alpha)r \quad \& \quad c := \frac{r^2(1-\alpha)}{\gamma}$$

We can observe from this a saddle node bifurcation occurring along $b^2 = 4\beta c$ as a result of the square root. i.e. $\beta = \gamma \approx 0.03145$ ignoring the negative solution as it is nonsensical in this context.

A useful visualisation of the model is to construct streamplot (vector field) of the system with a few perturbations of the parameters. In this case we will observe the perturbation of $\beta = \gamma$ the acquisition rates see figure 2.

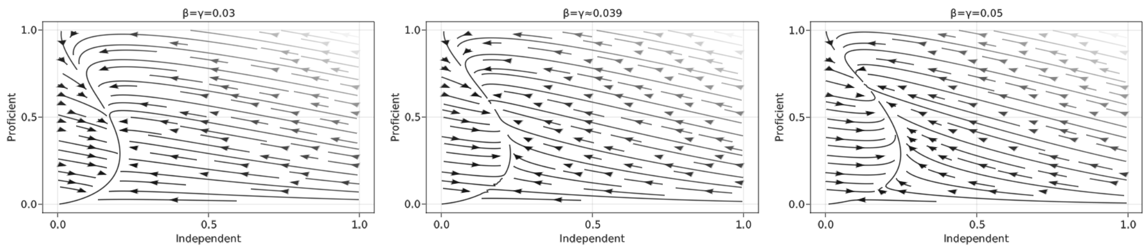


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Figure 2 - Vector field of the ODE system with various parameter levels (NOT phase portraits) We see in 2.1 only one fixed point exists at the origin. In 2.2 and 2.3 two additional fixed points exist a saddle and a stable node.
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Looking at these streamplots we can observe that for $\beta = \gamma = 0.03$ (figure 2.1) there is only one stable fixed point at the origin. But once we increase our acquisition rates a new stable fixed point occurs off the origin and a saddle node point occurs between them. Applying our initial conditions $(B, I, P) = (.774,\ .120,\ .106)$ obtained from [4] for te reo Maori to these parameters we can observe the predicted outcomes for various learning rates see figure 3.

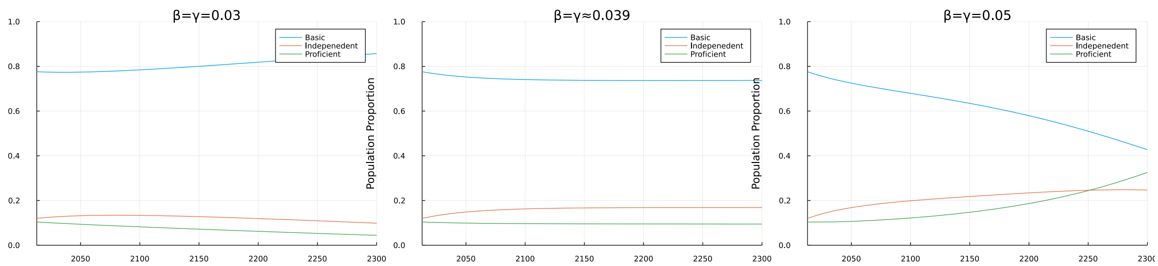


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Figure 3 - The behaviour of the language acquisition model at various acquistion rates. 3.1 shows the decline of the language toward extinction. 3.2 shows at the initial condition provided we are to stay at the saddle node. 3.3 the system is on a trajectory toward a stable node at P≈0.7
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For the parameter values of $\alpha = \beta = 0.03,\ \alpha = 0.47\ r = \frac{1}{80}$ we can see what will happen if $(B, I, P) = (0,\ 0,\ 1)$ over a few thousand years see figure 4. This reflects what has happened as a result of colonialism. However, in the past 100-odd years the decline from 100% usage of te reo Maori among the Māori population has been more dramatic than observed in the system. Reducing to just 10.6% proficient users at this time.

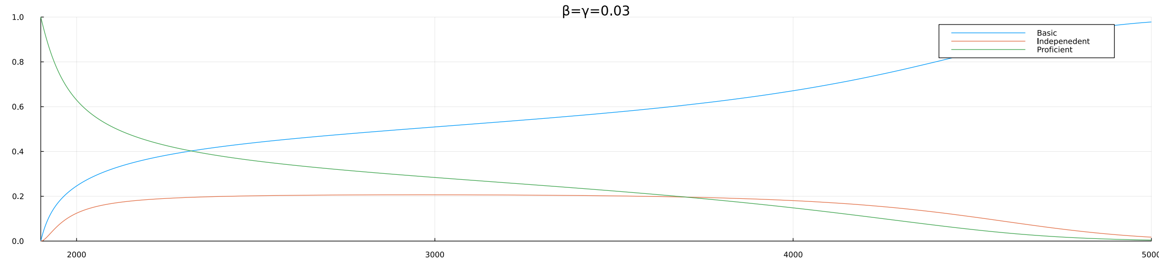

Figure 4 - The behaviour of the language acquisition model at β=γ=0.03 with initial condition (B,I,P)=(0, 0, 1). The death of the language

Discussion

We tested the hypothesis that once a certain percentage of the population reaches proficiency in the language the revitalisation will become self sustaining resulting in a thriving language and culture. Although one should notice that the previous statement fails to account for learning and intergenerational preservation rates, one should also note that in the long term these are essentially fixed by the attitudes of the community. Therefore, if a stable point for the language exists where there is a present population of speakers we need only to influence the rates for a finite period to reach a condition where the system will draw itself to such a fixed point. On the other hand, if the parameters of the system are such that there is no stable fixed point off the origin the language is doomed to extinction. Therefore the attitude of the target community for the endangered language needs to be the target of any revitalisation scheme in order to get the acquisition rates to increase. If this is not done expect the language to continue declining as it did in the past. Figure 4 demonstrates the effect of a low learning rate by demonstrating that even with 100% proficiency initial condition a language can and will still die off.

Consistent with Barrett-Walker [4] this assignment predicts that in the current observed state te reo Māori will continue to decline to extinction. Assuming the 50-90% of existent languages predicted to see extinction in the next hundred years [8] are similar in nature serious grass roots community work needs to be done to alter the learning and acquisition rates for their target populations. Research by Bauer [10] suggests an even more dramatic drop in language usage than is observed in figure 4. Therefore one might suggest that the acquisition rates proposed by Barret-Walker [4] could be too optimistic, at least for the past 100 years since we can read from figure 4. that at 2013 there should have been ~60% Proficients however the data suggests $P = 10.6\%$.

Without intervention we expect te reo Māori to continue to decline if the learning rates are in fact $\beta = \gamma \approx 0.03$ we expect very little change if $\beta = \gamma \approx 0.039$ and growth up to a point of $P \approx 0.7$ if the acquisition rates are $\beta = \gamma \approx 0.05$. Factors that can affect these rates are reported to be: language immersion programs, age of learners [11,12] among others. So if te reo Māori is to be preserved strategies along the lines of increasing availability of immersion programs such as schooling and the targeting of young children are likely to help. In order for a language to survive there needs to exist a non-zero fixed point and as we have seen this existence is reliant on the language's learning rates.

References

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