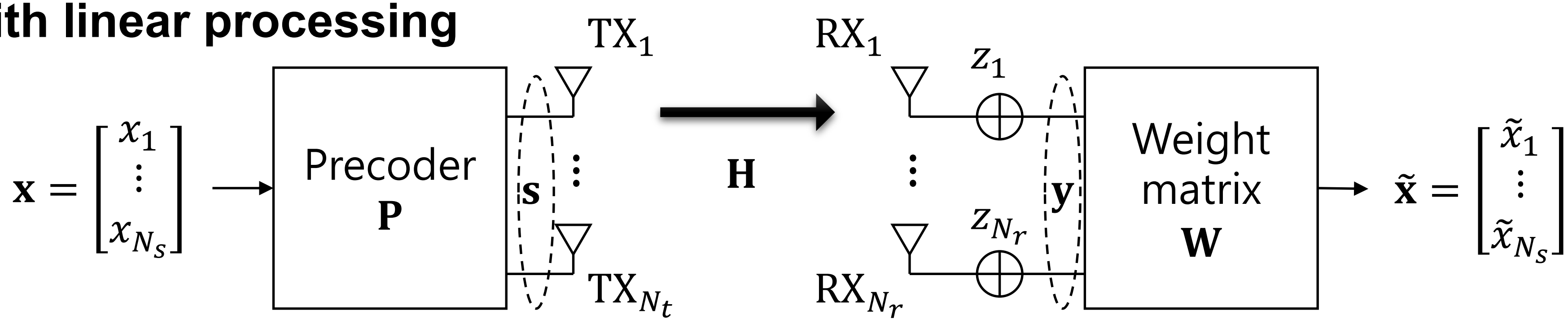
Assignment 07

- **MIMO with linear processing**



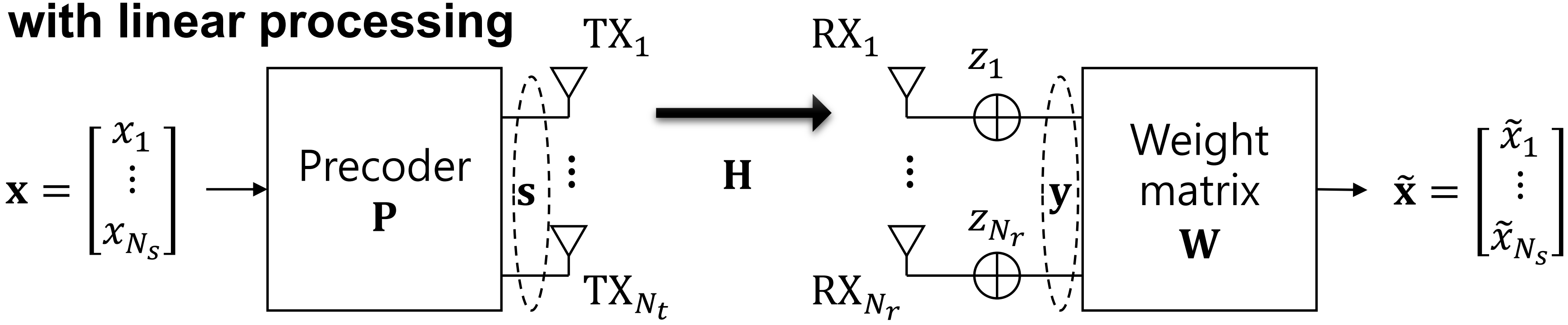
  - ① MIMO with linear processing

  - MIMO without CSIT
    - ② RxMF
    - ③ RxZF
    - ④ RxMMSE

  - MIMO with CSIT
    - ⑤ Joint SVD
  - ⑥ Numerical results

Assignment 07

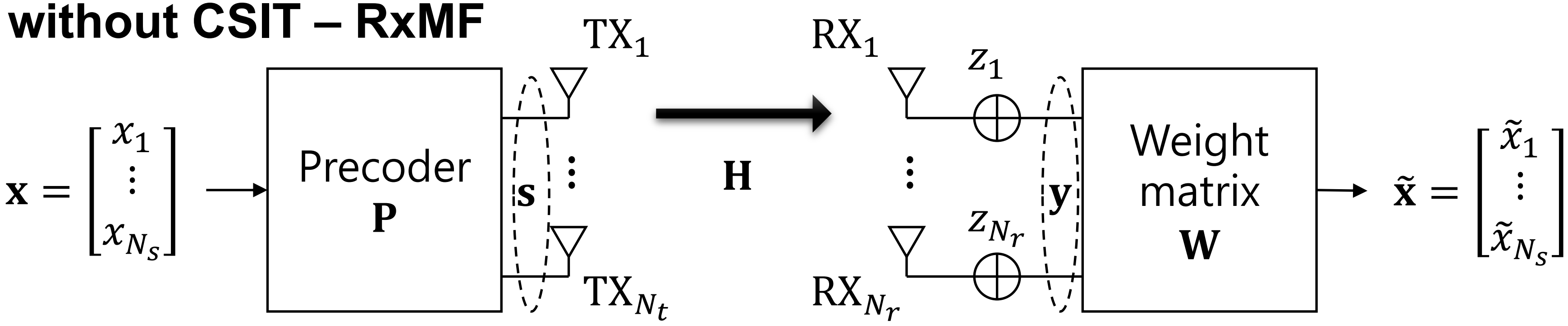
- ① **MIMO with linear processing**



  - Let $x_i, \forall i = 1, \dots, N_s$ and $z_i, \forall i = 1, \dots, N_r$ follow zero-mean complex Gaussian distributions.
  - Define $\mathbf{R_x} \triangleq \mathrm{E}[\mathbf{x}\mathbf{x}^H] = \mathbf{I}_{N_s}$, $\mathbf{R_z} \triangleq \mathrm{E}[\mathbf{z}\mathbf{z}^H]$, $E_s \triangleq \mathrm{E}[|\mathbf{s}|^2]$, $N_s \triangleq r = \mathrm{rank}(\mathbf{H}) = \min(N_t, N_r)$.
  - (1) Specify the dimensions of $\mathbf{P}, \mathbf{H}, \mathbf{W}$.
  - (2) Write an expression of the post-processed signal $\tilde{\mathbf{x}}$ in terms of $\mathbf{W}, \mathbf{H}, \mathbf{P}, \mathbf{x}, \mathbf{z}$.
  - (3) Show that $\mathrm{E}[|\mathbf{s}|^2] = \|\mathbf{P}\|_F^2 = E_s$.
  - (4) Show that the capacity is expressed by $C = \log_2\left|\mathbf{I}_{N_s} + (\mathbf{W}\mathbf{R_z}\mathbf{W}^H)^{-1}\mathbf{W}\mathbf{H}\mathbf{P}\mathbf{P}^H\mathbf{H}^H\mathbf{W}^H\right|$.

Assignment 07

- ② **MIMO without CSIT – RxMF**



  - We maximize the correlation between $\mathbf{x}$ and $\tilde{\mathbf{x}}$.
    - Correlation = Cosine ∝ Inner product
    - “Matched” = “Parallel”
  - At the same time, we want to minimize the noise portion of $\tilde{\mathbf{x}}$.
  - Thus, we solve the following problem:

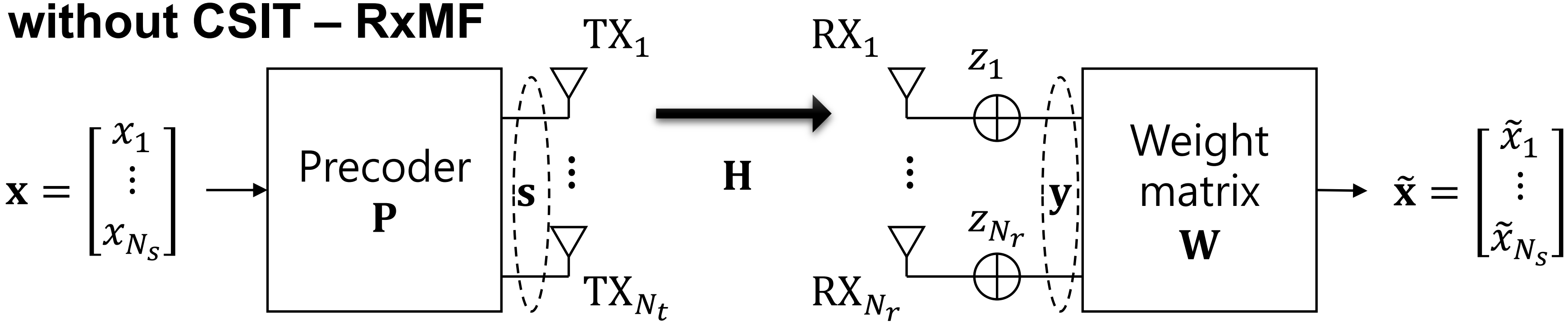
$$\boxed{\max_{\mathbf{W}} \frac{\mathrm{E}[\mathbf{x}^H \tilde{\mathbf{x}}]}{\mathrm{E}[|\mathbf{W}\mathbf{z}|^2]}}$$

  - Since the objective function is a ratio, we may fix the denominator to some $\delta > 0$ without altering the optimal value.

$$\max_{\mathbf{W}} \mathrm{E}[\mathbf{x}^H \tilde{\mathbf{x}}]$$
$$\text{s.t. } \mathrm{E}[|\mathbf{W}\mathbf{z}|^2] = \delta$$

Assignment 07

- ② **MIMO without CSIT – RxMF**



- (1) Show that $\mathrm{E}[\mathbf{x}^H\tilde{\mathbf{x}}] = \mathrm{tr}(\mathbf{WHP})$ and $\mathrm{E}[|\mathbf{Wz}|^2] = \mathrm{tr}(\mathbf{WR_zW}^H)$.

- Using the method of Lagrangian multipliers,

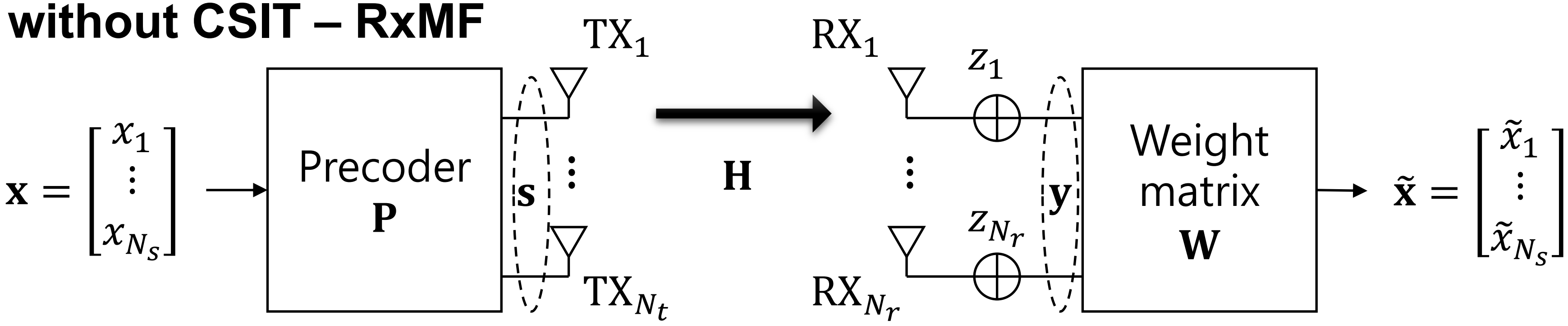
$$\boxed{\begin{aligned} &\max_{\mathbf{W}} \mathrm{tr}(\mathbf{WHP}) \\ &\text{s.t. } \mathrm{tr}(\mathbf{WR_zW}^H) = \delta \end{aligned}}$$

- The optimal solution is one of the stationary points with $\frac{\partial g(\mathbf{W},\lambda)}{\partial \mathbf{W}} = 0, \frac{\partial g(\mathbf{W},\lambda)}{\partial \lambda} = 0$.

$$\boxed{\max_{\mathbf{W}} g(\mathbf{W},\lambda) \triangleq \mathrm{tr}(\mathbf{WHP}) + \lambda(\mathrm{tr}(\mathbf{WR_zW}^H) - \delta)}$$

Assignment 07

- ② **MIMO without CSIT – RxMF**

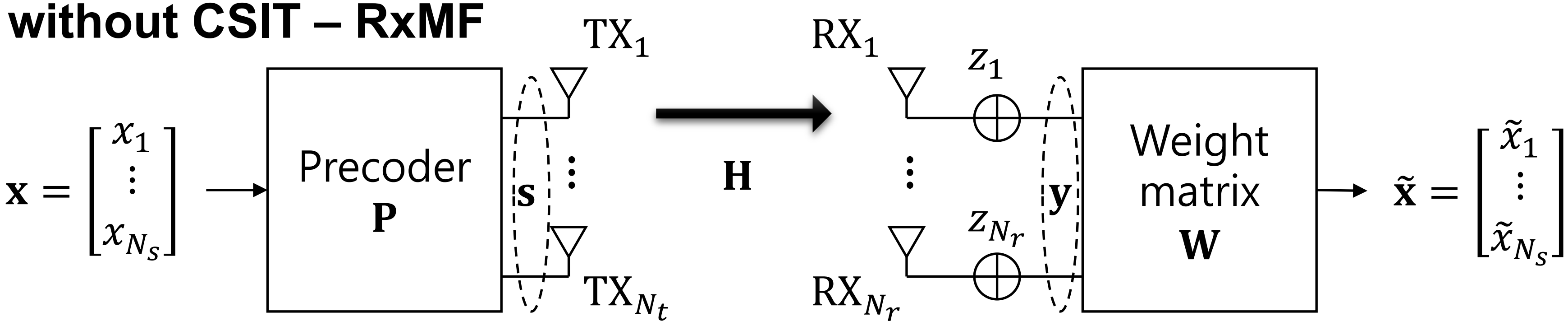


- (2) Show that $\frac{\partial g(\mathbf{W},\lambda)}{\partial \mathbf{W}} = (\mathbf{HP})^T + \lambda \mathbf{W}^* \mathbf{R}_\mathbf{z}^T$ and $\frac{\partial g(\mathbf{W},\lambda)}{\partial \lambda} = \mathrm{tr}(\mathbf{W}\mathbf{R}_\mathbf{z}\mathbf{W}^H) - \delta$.
- (3) Letting $\frac{\partial g(\mathbf{W},\lambda)}{\partial \mathbf{W}} = \mathbf{0}$, show that $\mathbf{W}_{\mathrm{opt}} = -\frac{1}{\lambda}\mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-1}$.
- (4) Letting $\frac{\partial g(\mathbf{W},\lambda)}{\partial \lambda} = 0$, show that $\lambda_{\mathrm{opt}} = \pm\sqrt{\frac{1}{\delta}\mathrm{tr}(\mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-1}\mathbf{HP})}$.

- Since $\delta > 0$ can be freely chosen, λ_{opt} can also be freely chosen.
- For simplicity, we let $\delta = \mathrm{tr}(\mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-1}\mathbf{HP})$ and $\lambda_{\mathrm{opt}} = -1$ such that $\mathbf{W}_{\mathrm{opt}} = \mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-1}$.
- (5) Show that $C = \log_2\left|\mathbf{I}_{N_s} + \mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-H}\mathbf{HP}\right|$.

Assignment 07

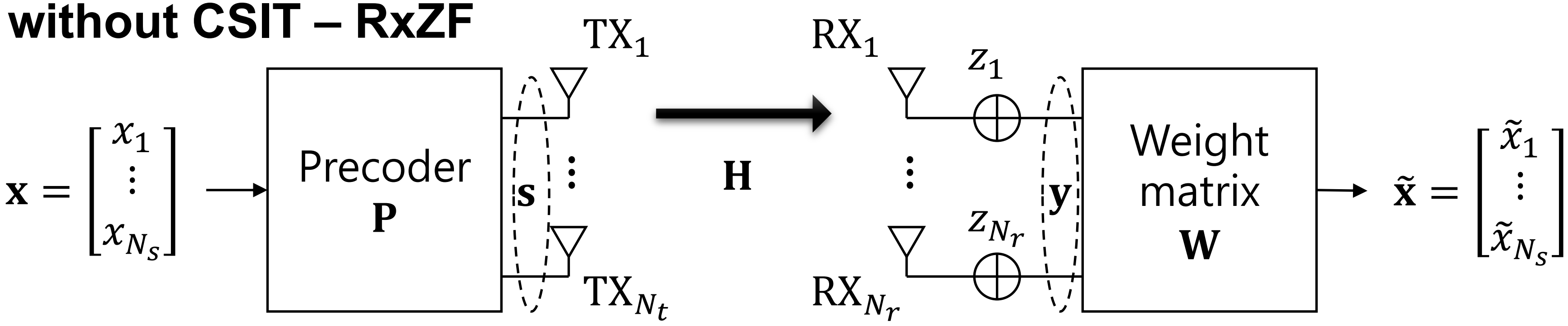
- ② **MIMO without CSIT – RxMF**



  - (6) For AWGN, i.e., $\mathbf{R_z} = \sigma_z^2 \mathbf{I}_{N_r}$, show that $C = \log_2 \left| \mathbf{I}_{N_s} + \frac{1}{\sigma_z^2} \mathbf{P}^H \mathbf{H}^H \mathbf{H} \mathbf{P} \right|$.

Assignment 07

- ③ **MIMO without CSIT – RxZF**



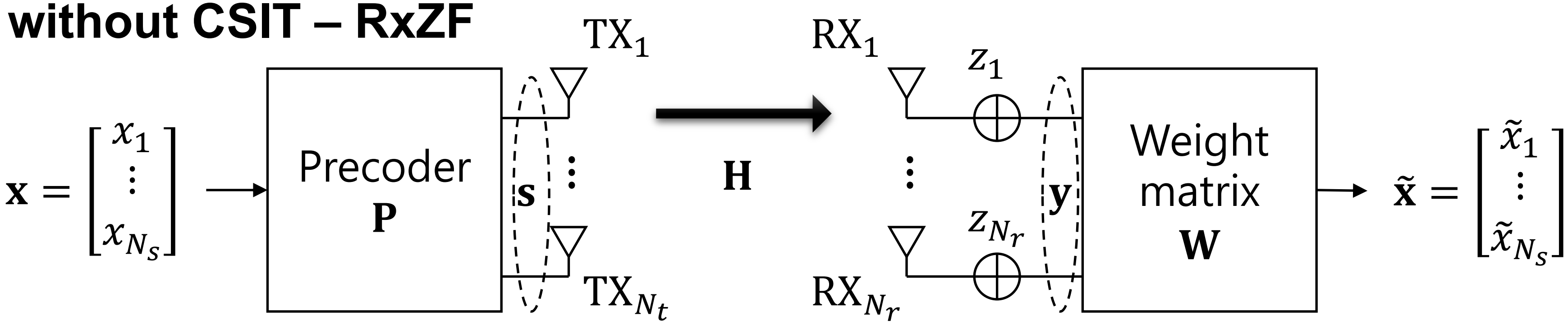
  - We suppress the inter-symbol interference in $\tilde{\mathbf{x}}$ by diagonalizing the symbol portion such that $\mathbf{WHPx} = \mathbf{x}$. ("Zero-forcing" the interference)
    - That is, $\mathbf{WHP} = \mathbf{I}_{N_s}$
  - At the same time, we want to minimize the noise portion of $\tilde{\mathbf{x}}$.
  - Thus, we solve the following problem:

$$\boxed{\begin{aligned} &\min_{\mathbf{W}} \mathrm{E}[|\mathbf{Wz}|^2] \\ &\text{s.t. } \mathbf{WHP} = \mathbf{I}_{N_s} \end{aligned}}$$

  - We assume $\mathrm{rank}(\mathbf{P}) = N_s$.

Assignment 07

- ③ **MIMO without CSIT – RxZF**



- From $\mathrm{E}[|\mathbf{Wz}|^2] = \mathrm{tr}(\mathbf{W}\mathbf{R}_\mathbf{z}\mathbf{W}^H)$ and using standard basis vectors $\mathbf{e}_i$'s in which the i-th element is 1 and the other elements are 0,

$$\boxed{\begin{aligned} &\min_{\mathbf{W}} \mathrm{tr}(\mathbf{W}\mathbf{R}_\mathbf{z}\mathbf{W}^H) \\ &\text{s.t. } \mathbf{e}_i^H\mathbf{WHP}\mathbf{e}_i = 1, \forall i = 1, \dots, N_s \\ &\qquad \mathbf{e}_i^H\mathbf{WHP}\mathbf{e}_j = 0, \forall i \neq j \end{aligned}}$$

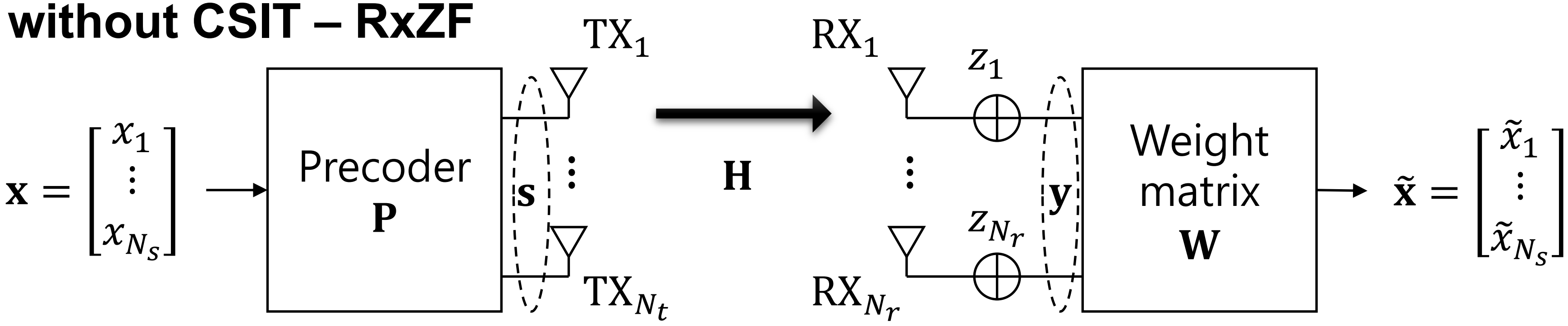
- Using the method of Lagrangian multipliers,

$$\boxed{\max_{\mathbf{W}} g\left(\mathbf{W}, \{\lambda_i\}, \{\rho_{ij}\}\right) \triangleq \mathrm{tr}(\mathbf{WHP}) + \sum_{i=1}^{N_s} \lambda_i\left(\mathbf{e}_i^H\mathbf{WHP}\mathbf{e}_i - 1\right) + \sum_{i\neq j} \rho_{ij}\mathbf{e}_i^H\mathbf{WHP}\mathbf{e}_j}$$

- The optimal solution is one of the stationary points with $\frac{\partial g(\mathbf{W},\{\lambda_i\},\{\rho_{ij}\})}{\partial \mathbf{W}} = \mathbf{0}$, $\frac{\partial g(\mathbf{W},\{\lambda_i\},\{\rho_{ij}\})}{\partial \lambda_i} = 0, \forall i = 1, \dots, N_s$, $\frac{\partial g(\mathbf{W},\{\lambda_i\},\{\rho_{ij}\})}{\partial \rho_{ij}} = 0, \forall i \neq j$.

Assignment 07

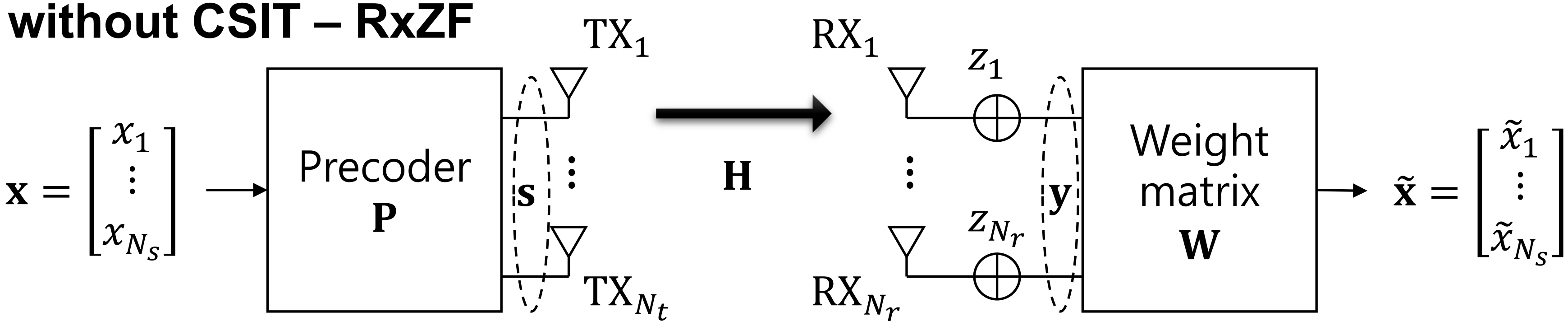
- ③ **MIMO without CSIT – RxZF**



- (1) Show that $\frac{\partial g(\mathbf{W},\{\lambda_i\},\{\rho_{ij}\})}{\partial \mathbf{W}} = \mathbf{W}^*\mathbf{R}_\mathbf{z}^T + \left(\mathbf{HP}\sum_{i=1}^{N_s}\lambda_i\mathbf{e}_i\mathbf{e}_i^H\right)^T + \left(\mathbf{HP}\sum_{i\neq j}\rho_{ij}\mathbf{e}_j\mathbf{e}_i^H\right)^T$,
- (2) Show that $\frac{\partial g(\mathbf{W},\{\lambda_i\},\{\rho_{ij}\})}{\partial \lambda_i} = \mathbf{e}_i^H\mathbf{WHP}\mathbf{e}_i - 1, \forall i = 1, \dots, N_s$ and $\frac{\partial g(\mathbf{W},\{\lambda_i\},\{\rho_{ij}\})}{\partial \rho_{ij}} = \mathbf{e}_i^H\mathbf{WHP}\mathbf{e}_j, \forall i \neq j$
- (3) Letting $\frac{\partial g(\mathbf{W},\{\lambda_i\},\{\rho_{ij}\})}{\partial \mathbf{W}} = \mathbf{0}$, show that $\mathbf{W}_{\mathrm{opt}} = -\left(\sum_{i=1}^{N_s}\lambda_i\mathbf{e}_i\mathbf{e}_i^H + \sum_{i\neq j}\rho_{ij}\mathbf{e}_j\mathbf{e}_i^H\right)^H\mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-1}$.
- (4) Letting $\frac{\partial g(\mathbf{W},\{\lambda_i\},\{\rho_{ij}\})}{\partial \lambda_i} = 0, \forall i = 1, \dots, N_s$ and $\frac{\partial g(\mathbf{W},\{\lambda_i\},\{\rho_{ij}\})}{\partial \rho_{ij}} = 0, \forall i \neq j$, show that $\mathbf{W}_{\mathrm{opt}}\mathbf{HP} = \mathbf{I}_{N_s}$.
- (5) Show that $\mathbf{W}_{\mathrm{opt}} = (\mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-1}\mathbf{HP})^{-1}\mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-1}$.

Assignment 07

- ③ **MIMO without CSIT – RxZF**

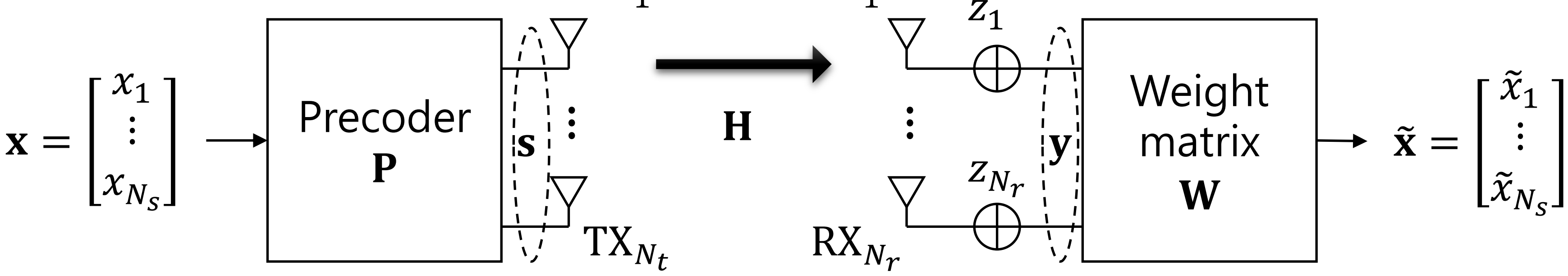


  - (6) Show that $C = \log_2\left|\mathbf{I}_{N_s} + \mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-H}\mathbf{H}\mathbf{P}\right|$.

  - (7) For AWGN, i.e., $\mathbf{R}_\mathbf{z} = \sigma_z^2\mathbf{I}_{N_r}$, show that $C = \log_2\left|\mathbf{I}_{N_s} + \frac{1}{\sigma_z^2}\mathbf{P}^H\mathbf{H}^H\mathbf{H}\mathbf{P}\right|$.

Assignment 07

- ④ **MIMO without CSIT – RxMMSE**



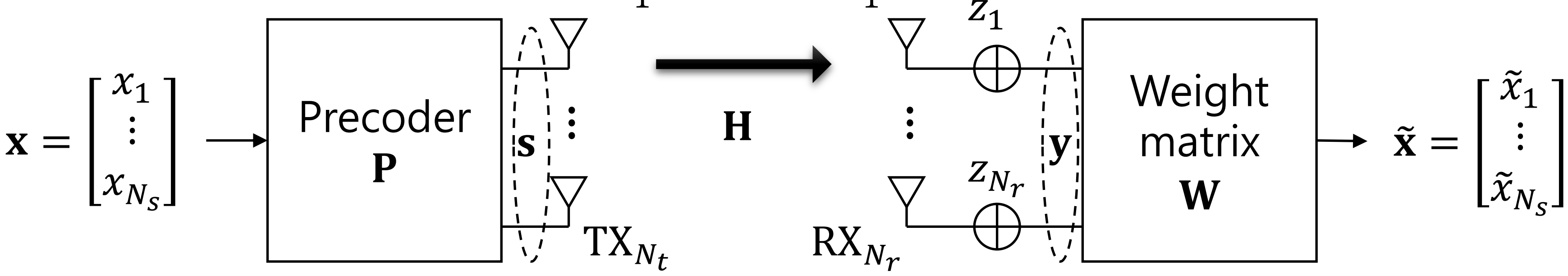
  - We minimize the mean square error (MSE) between $\mathbf{x}$ and $\tilde{\mathbf{x}}$
  - Thus, we solve the following problem:

$$\boxed{\min_{\mathbf{W}} \mathrm{E}[|\mathbf{x} - \tilde{\mathbf{x}}|^2]}$$

  - (1) Show that $\mathrm{E}[|\mathbf{x} - \tilde{\mathbf{x}}|^2] = \mathrm{tr}\left(\mathbf{I}_{N_s} - \mathbf{P}^H\mathbf{H}^H\mathbf{W}^H - \mathbf{WHP} + \mathbf{W}(\mathbf{HPP}^H\mathbf{H}^H + \mathbf{R}_\mathbf{z})\mathbf{W}^H\right)$.

Assignment 07

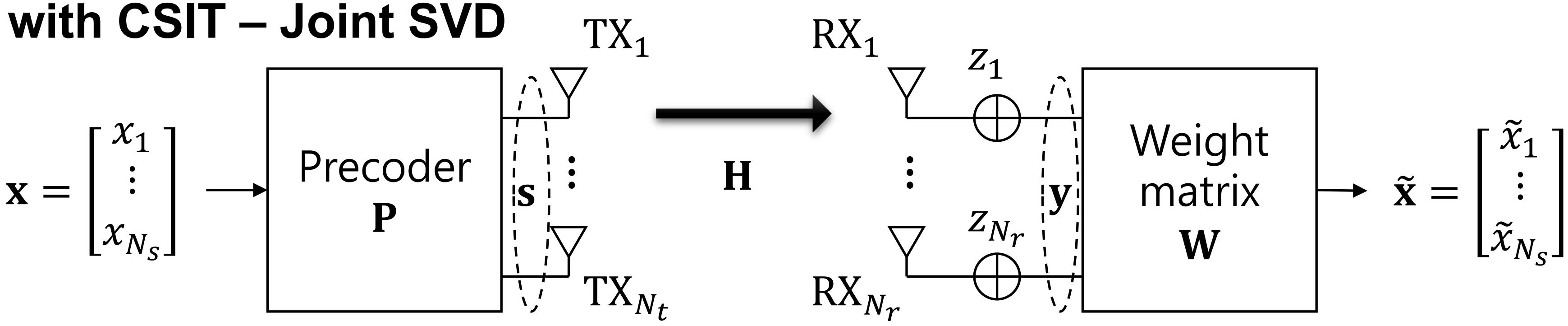
- ④ **MIMO without CSIT – RxMMSE**



  - Since this is an unconstrained convex problem, the optimal solution is a stationary point with $\frac{\partial \mathrm{E}\left[|\mathbf{x}-\tilde{\mathbf{x}}|^2\right]}{\partial \mathbf{W}} = \mathbf{0}$.
  - (2) Show that $\frac{\partial \mathrm{E}\left[|\mathbf{x}-\tilde{\mathbf{x}}|^2\right]}{\partial \mathbf{W}} = -(\mathbf{HP})^T + \mathbf{W}^*(\mathbf{HPP}^H\mathbf{H}^H + \mathbf{R}_\mathbf{z})^T$.
  - (3) Letting $\frac{\partial \mathrm{E}\left[|\mathbf{x}-\tilde{\mathbf{x}}|^2\right]}{\partial \mathbf{W}} = \mathbf{0}$, show that $\mathbf{W}_{\mathrm{opt}} = \mathbf{P}^H\mathbf{H}^H(\mathbf{HPP}^H\mathbf{H}^H + \mathbf{R}_\mathbf{z})^{-1}$.
  - (4) Using the Woodbury matrix identity, also show that $\mathbf{W}_{\mathrm{opt}} = \left(\mathbf{I}_{N_s} + \mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-1}\mathbf{HP}\right)^{-1}\mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-1}$
  - (5) If $\mathrm{rank}(\mathbf{P}) = N_s$, using $|\mathbf{I} + \mathbf{AB}| = |\mathbf{I} + \mathbf{BA}|$, show that $C = \log_2\left|\mathbf{I}_{N_s} + \mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-H}\mathbf{HP}\right|$.
  - (6) If $\mathrm{rank}(\mathbf{P}) = N_s$, for AWGN, i.e., $\mathbf{R}_\mathbf{z} = \sigma_z^2\mathbf{I}_{N_r}$, show that $C = \log_2\left|\mathbf{I}_{N_s} + \frac{1}{\sigma_z^2}\mathbf{P}^H\mathbf{H}^H\mathbf{HP}\right|$.

Assignment 07

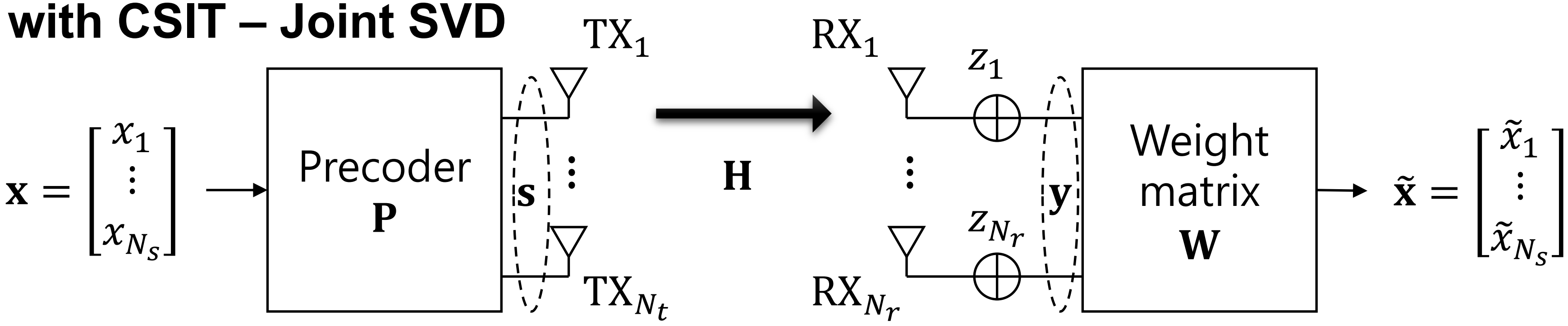
- ⑤ **MIMO with CSIT – Joint SVD**



- By SVD, $\mathbf{H} = \mathbf{U\Sigma V}^H = [\mathbf{U}_1 \quad \mathbf{U}_0] \begin{bmatrix} \mathbf{\Sigma}_1 & \\ & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1^H \\ \mathbf{V}_0^H \end{bmatrix} = \mathbf{U}_1 \mathbf{\Sigma}_1 \mathbf{V}_1^H$, where $\mathbf{U}_1$ and $\mathbf{V}_1$ are singular vector matrices corresponding to non-zero singular values in $\mathbf{\Sigma}_1$.
- For joint SVD, we set $\mathbf{P} = \mathbf{V}_1 \mathbf{Q}_1^{1/2}$ for positive diagonal matrix $\mathbf{Q}_1 \in M_{N_s \times N_s}(\mathbb{R})$ and $\mathbf{W} = \mathbf{U}_1^H$.
- (1) Specify the dimensions of $\mathbf{U}_1, \mathbf{U}_0, \mathbf{\Sigma}_1, \mathbf{V}_1, \mathbf{V}_0$.
- (2) Show that $\tilde{\mathbf{x}} = \mathbf{\Sigma}_1 \mathbf{x} + \mathbf{U}_1^H \mathbf{z}$.
- (3) Show that $\mathrm{E}[|\mathbf{s}|^2] = \mathrm{tr}(\mathbf{Q}_1) = E_s$.

Assignment 07

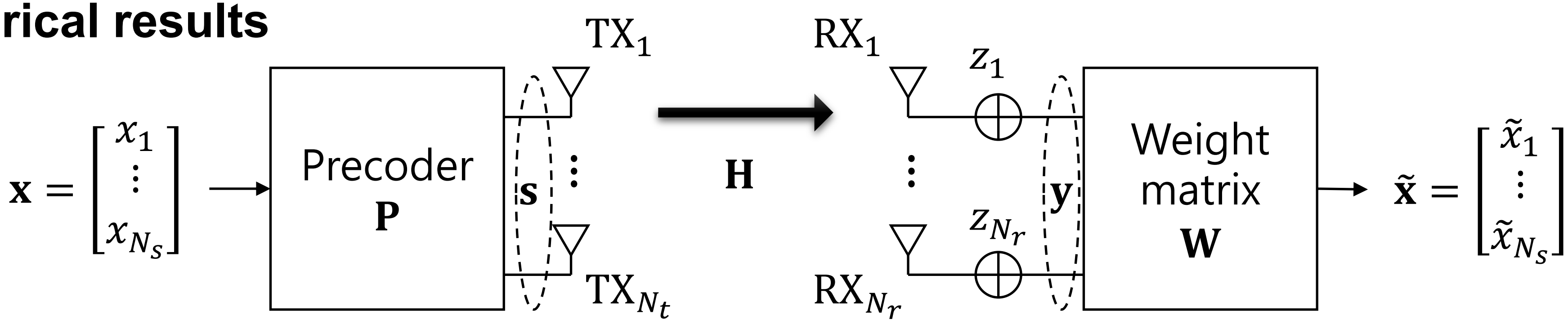
- ⑤ **MIMO with CSIT – Joint SVD**



  - (4) For AWGN, i.e., $\mathbf{R}_\mathbf{z} = \sigma_z^2 \mathbf{I}_{N_r}$, show that $C = \sum_{i=1}^{N_s} \log_2\left(1 + \frac{1}{\sigma_z^2}\sigma_{\mathbf{H},i}^2 q_i\right)$, i.e., achieving the MIMO capacity when q_i's are optimized.
  - (5) Letting $\mathbf{P} = \mathbf{V}_1\mathbf{Q}_1^{1/2}$ for RxMF, RxZF, RxMMSE, show that C becomes equivalent to the case with CSIT, i.e., achieving the MIMO capacity when q_i's are optimized.

Assignment 07

- ⑥ **Numerical results**



- In the absence of the CSIT, one possible design for the precoder is $\mathbf{P} = \sqrt{\frac{E_s}{N_s}} \begin{bmatrix} \mathbf{I}_{N_s} \\ \mathbf{0}_{(N_t - N_s) \times N_s} \end{bmatrix}$ such that the average transmit energy E_s is equally distributed to the symbols and by assigning x_i's to TX_i's in order. ($N_t - N_s$ antennas are unused.)
- (1) Show that $\mathrm{E}[|\mathbf{s}|^2] = E_s$.

Assignment 07

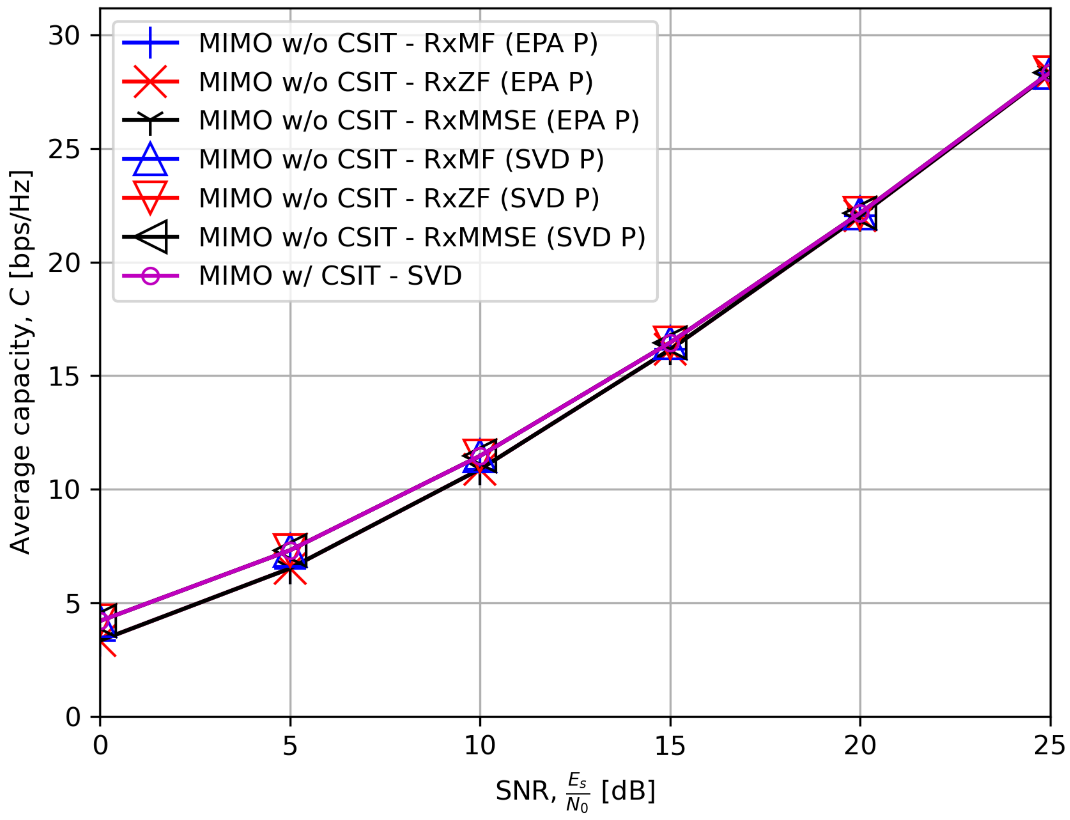
- ⑥ **Numerical results**
  - Setups
    - Unit average transmit energy, i.e., $E_s = 1$
    - AWGN, i.e., $\mathbf{R}_\mathbf{z} = \sigma_z^2 \mathbf{I}_{N_r}$
    - Rayleigh fading channel matrix, i.e., $h_{ij} \sim CN(0,1)$
  - Curve 1: MIMO without CSIT
    - For $\mathbf{P} = \sqrt{\frac{E_s}{N_s}} \begin{bmatrix} \mathbf{I}_{N_s} \\ \mathbf{0}_{(N_t - N_s) \times N_s} \end{bmatrix}$,
    - Curve 1-1: MIMO without CSIT – RxMF
      - $C = \log_2 \left| \mathbf{I}_{N_s} + (\mathbf{W}\mathbf{R}_\mathbf{z}\mathbf{W}^H)^{-1} \mathbf{W}\mathbf{H}\mathbf{P}\mathbf{P}^H\mathbf{H}^H\mathbf{W}^H \right|$ with $\mathbf{W} = \mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-1}$
    - Curve 1-2: MIMO without CSIT – RxZF
      - $C = \log_2 \left| \mathbf{I}_{N_s} + (\mathbf{W}\mathbf{R}_\mathbf{z}\mathbf{W}^H)^{-1} \mathbf{W}\mathbf{H}\mathbf{P}\mathbf{P}^H\mathbf{H}^H\mathbf{W}^H \right|$ with $\mathbf{W} = (\mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-1}\mathbf{H}\mathbf{P})^{-1}\mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-1}$
    - Curve 1-3: MIMO without CSIT – RxMMSE
      - $C = \log_2 \left| \mathbf{I}_{N_s} + (\mathbf{W}\mathbf{R}_\mathbf{z}\mathbf{W}^H)^{-1} \mathbf{W}\mathbf{H}\mathbf{P}\mathbf{P}^H\mathbf{H}^H\mathbf{W}^H \right|$ with $\mathbf{W} = \left(\mathbf{I}_{N_s} + \mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-1}\mathbf{H}\mathbf{P}\right)^{-1}\mathbf{P}^H\mathbf{H}^H\mathbf{R}_\mathbf{z}^{-1}$

Assignment 07

- ⑥ **Numerical results**
  - Setups
    - Unit average transmit energy, i.e., $E_s = 1$
    - AWGN, i.e., $\mathbf{R_z} = \sigma_z^2 \mathbf{I}_{N_r}$
    - Rayleigh fading channel matrix, i.e., $h_{ij} \sim CN(0,1)$
  - Curve 1: MIMO without CSIT
    - For $\mathbf{P} = \mathbf{V}_1[:, 0: N_{s,\mathrm{opt}}]\mathbf{Q}_{1,\mathrm{opt}}^{1/2}[0: N_{s,\mathrm{opt}}, 0: N_{s,\mathrm{opt}}]$, where $N_{s,\mathrm{opt}}$ is the number of non-zero $q_{i,\mathrm{opt}}$'s in $\mathbf{Q}_{\mathrm{opt}}$
    - Curve 1-4: MIMO without CSIT – RxMF
      - $C = \log_2 \left| \mathbf{I}_{N_{s,\mathrm{opt}}} + (\mathbf{W}\mathbf{R_z}\mathbf{W}^H)^{-1}\mathbf{W}\mathbf{H}\mathbf{P}\mathbf{P}^H\mathbf{H}^H\mathbf{W}^H \right|$ with $\mathbf{W} = \mathbf{P}^H\mathbf{H}^H\mathbf{R_z}^{-1}$
    - Curve 1-5: MIMO without CSIT – RxZF
      - $C = \log_2 \left| \mathbf{I}_{N_{s,\mathrm{opt}}} + (\mathbf{W}\mathbf{R_z}\mathbf{W}^H)^{-1}\mathbf{W}\mathbf{H}\mathbf{P}\mathbf{P}^H\mathbf{H}^H\mathbf{W}^H \right|$ with $\mathbf{W} = (\mathbf{P}^H\mathbf{H}^H\mathbf{R_z}^{-1}\mathbf{H}\mathbf{P})^{-1}\mathbf{P}^H\mathbf{H}^H\mathbf{R_z}^{-1}$
    - Curve 1-6: MIMO without CSIT – RxMMSE
      - $C = \log_2 \left| \mathbf{I}_{N_{s,\mathrm{opt}}} + (\mathbf{W}\mathbf{R_z}\mathbf{W}^H)^{-1}\mathbf{W}\mathbf{H}\mathbf{P}\mathbf{P}^H\mathbf{H}^H\mathbf{W}^H \right|$ with $\mathbf{W} = \left(\mathbf{I}_{N_{s,\mathrm{opt}}} + \mathbf{P}^H\mathbf{H}^H\mathbf{R_z}^{-1}\mathbf{H}\mathbf{P}\right)^{-1}\mathbf{P}^H\mathbf{H}^H\mathbf{R_z}^{-1}$
  - Curve 3: MIMO with CSIT – Joint SVD
    - $C = \log_2 \left| \mathbf{I}_{N_s} + (\mathbf{W}\mathbf{R_z}\mathbf{W}^H)^{-1}\mathbf{W}\mathbf{H}\mathbf{P}\mathbf{P}^H\mathbf{H}^H\mathbf{W}^H \right|$ with $\mathbf{P} = \mathbf{V}_1\mathbf{Q}_{1,\mathrm{opt}}^{1/2}$ and $\mathbf{W} = \mathbf{U}_1^H$

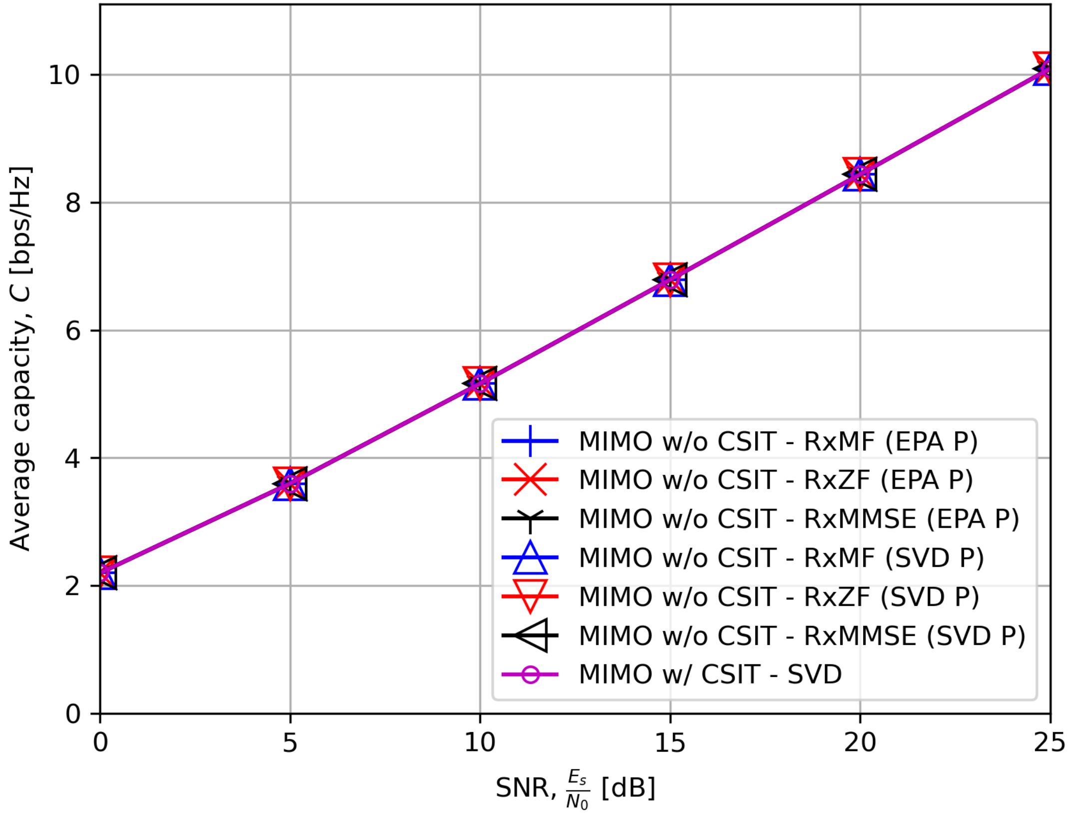
Assignment 07

- ⑥ **Numerical results**
  - For $(N_t, N_r) = (4,4)$ (Fig. 4.6 of "Introduction to Space-Time Wireless Communications")
  - 1000 samples



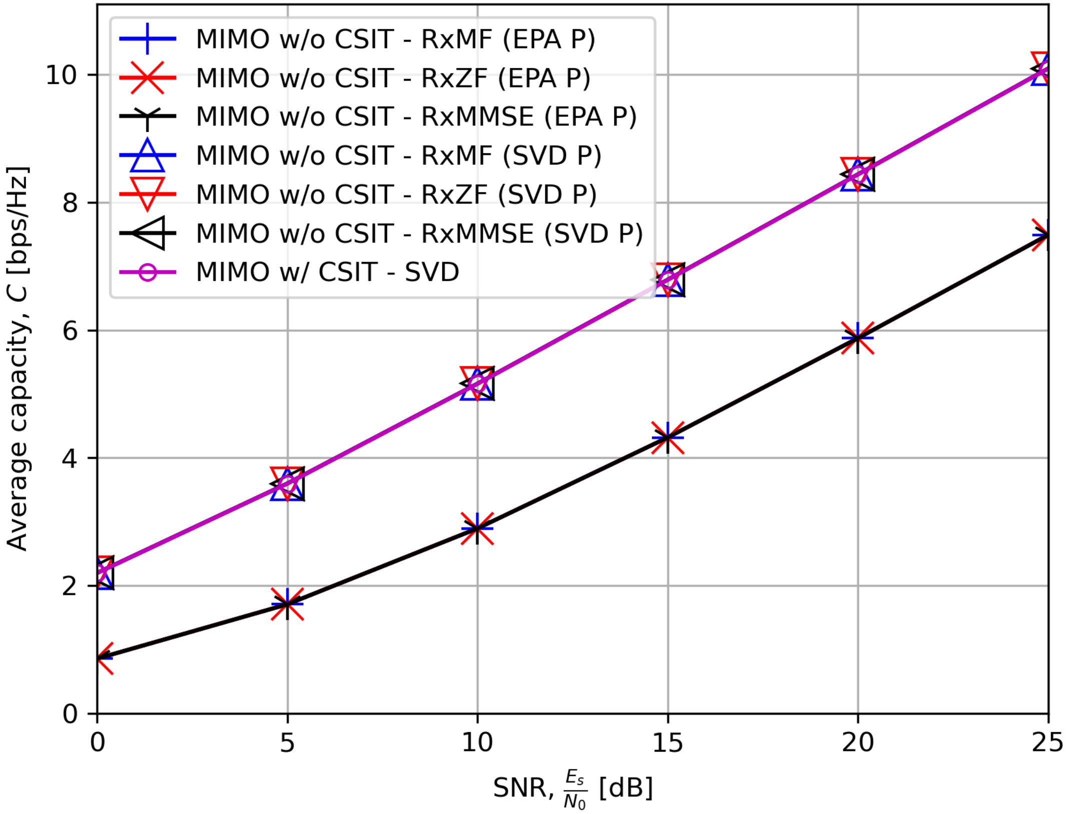
Assignment 07

- ⑥ **Numerical results**
  - SIMO wih $(N_t, N_r) = (1{,}4)$
  - 1000 samples



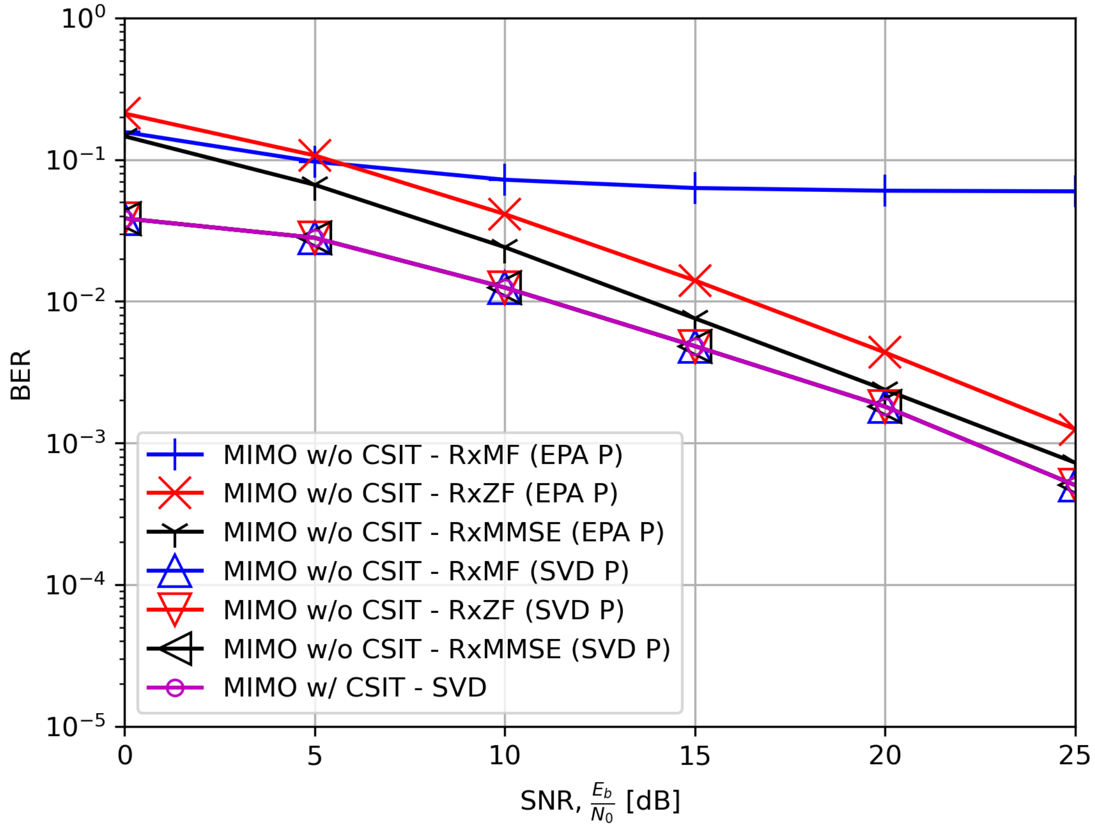
Assignment 07

- ⑥ **Numerical results**
  - MISO with $(N_t, N_r) = (4,1)$
  - 1000 samples



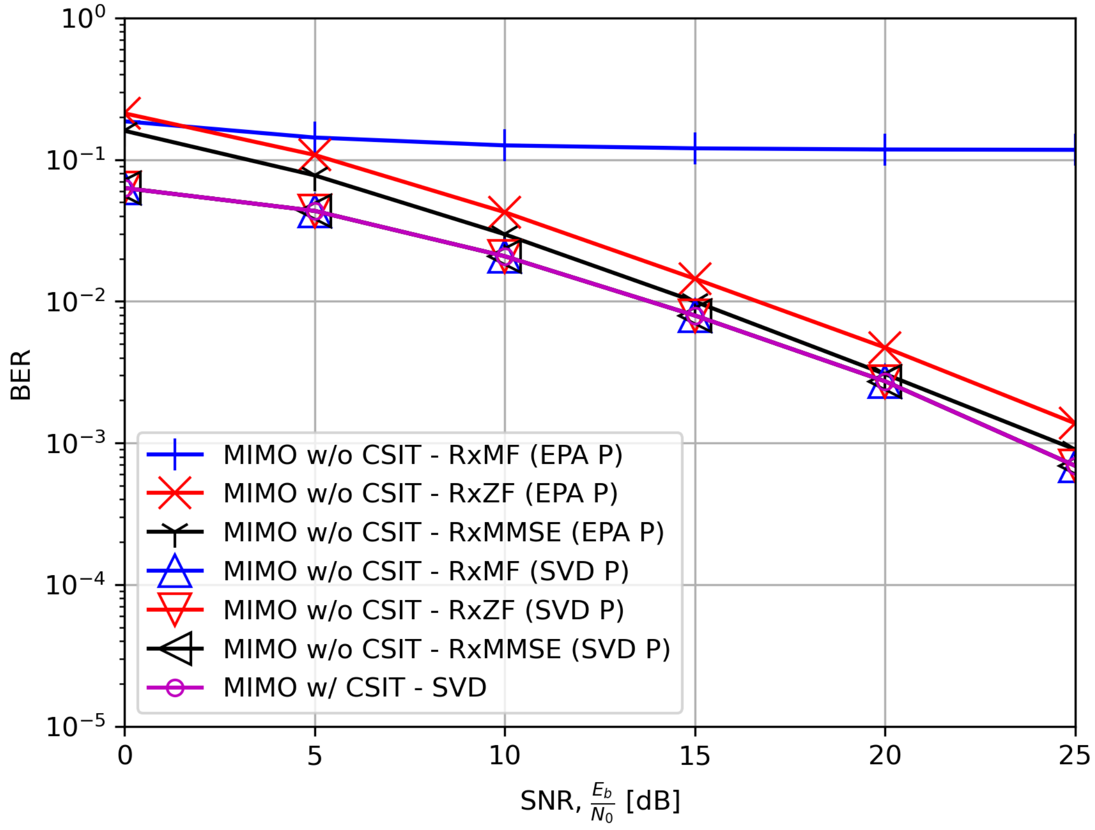
Assignment 07

- ⑥ **Numerical results**
  - BPSK for $(N_t, N_r) = (2{,}2)$ (Fig. 2-3 of “Extending the capacity of next generation … OFDM”)
  - 20000 samples



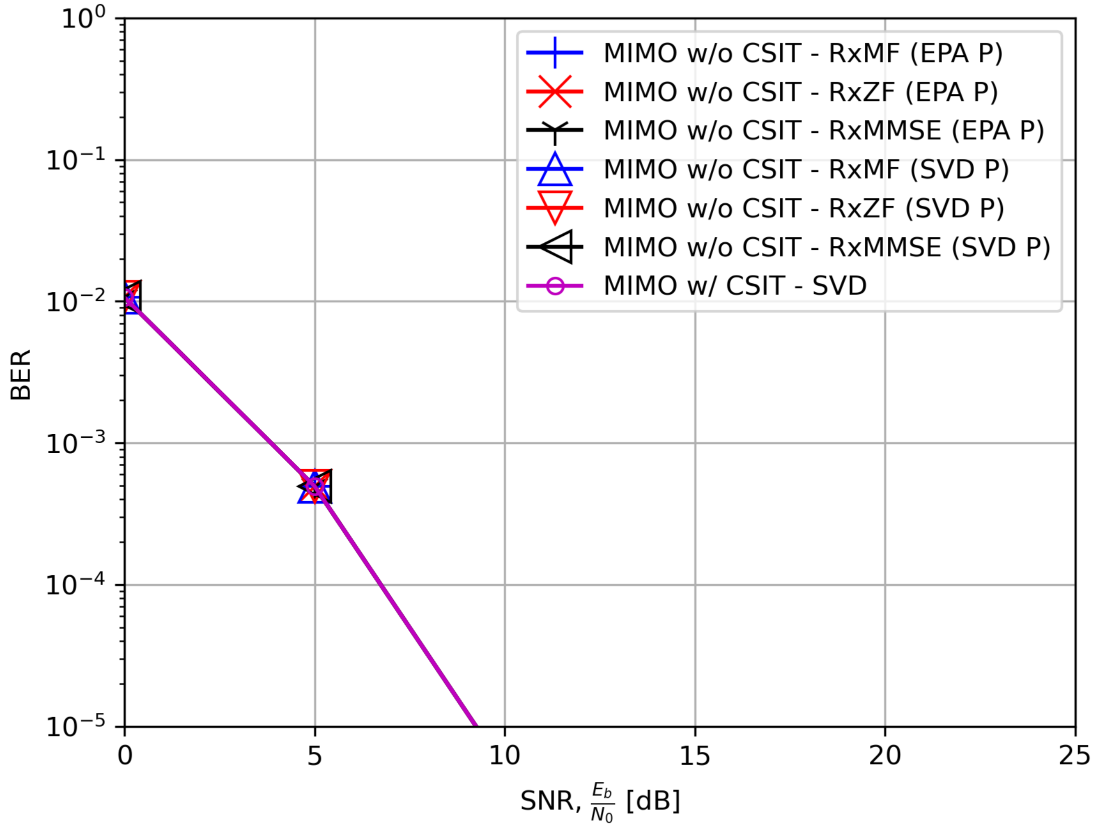
Assignment 07

- ⑥ **Numerical results**
  - QPSK for $(N_t, N_r) = (2,2)$ (Fig. 2-4 of "Extending the capacity of next generation … OFDM")
  - 20000 samples



Assignment 07

- ⑥ **Numerical results**
  - SIMO QPSK for $(N_t, N_r) = (1,4)$
  - 100000 sample



Assignment 07

- ⑥ **Numerical results**
  - MISO QPSK for $(N_t, N_r) = (4,1)$
  - 100000 sample