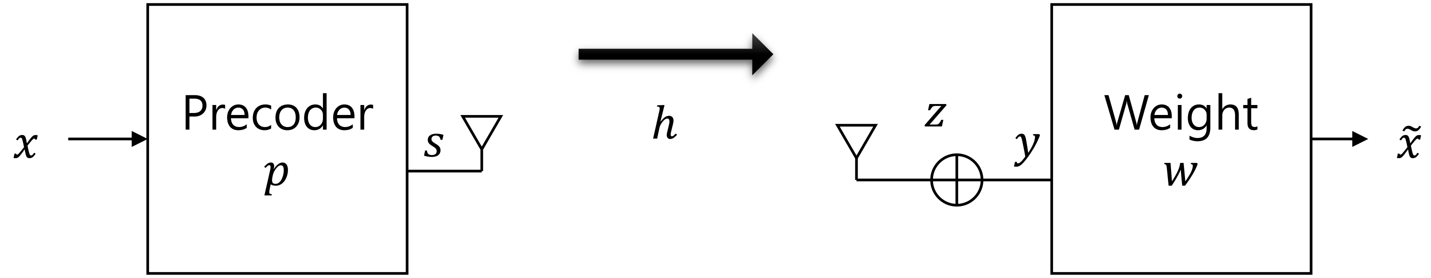
Assignment 05

- **SIMO / MISO combining schemes**
  - ① SISO
  - SIMO
    - ② Selection combining (SC)
    - ③ Equal gain combining (EGC)
    - ④ Maximum ratio combining (MRC)
  - MISO without CSIT
    - ⑤ Equal power allocation (EPA)
    - ⑥ Alamouti
  - MISO with CSIT
    - ⑦ Maximum ratio transmission (MRT)
  - ⑧ Numerical results

Assignment 05

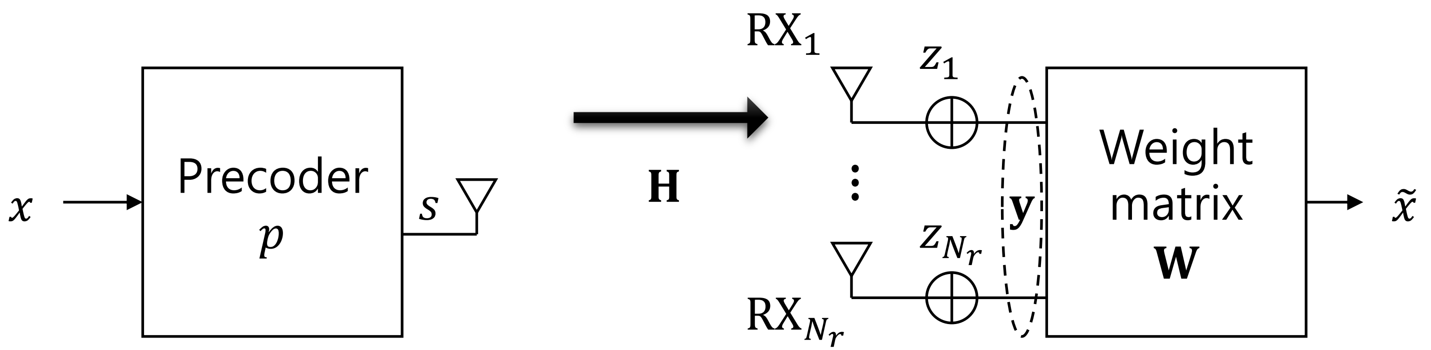
- ① **SISO**



- Assume $x \sim CN(0,1)$ and $z \sim CN(0, \sigma_z^2)$.
- (1) Write an expression of the recovered signal $\tilde{x}$ in terms of w, h, p, x, z
- (2) Show that the average transmit energy is $E_s = |p|^2$.
- (3) For the weight $w = \frac{1}{hp}$, show that the estimated symbol is $\tilde{x} = x + \frac{z}{h\sqrt{E_s}}$.

Assignment 05

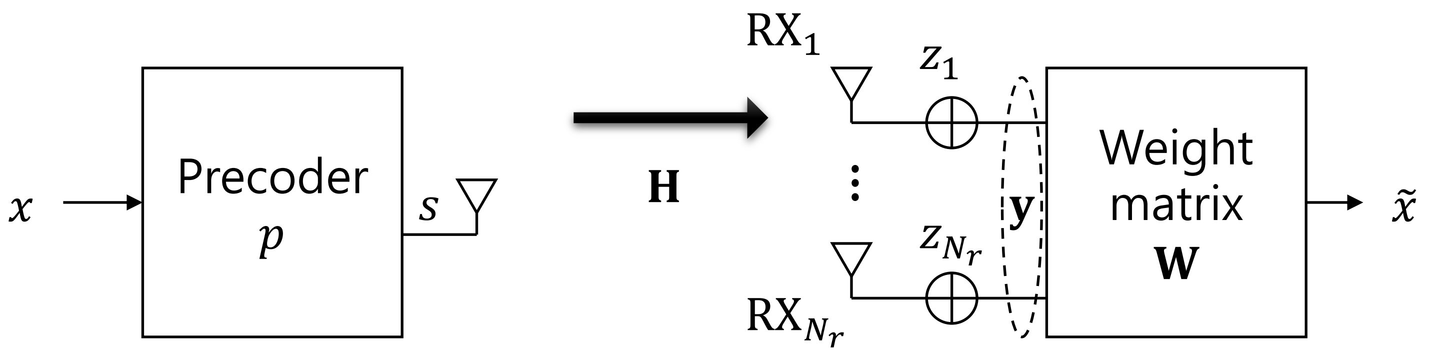
- ② **SIMO – SC**



- Let $x \sim CN(0,1)$ and $z_i, \forall i = 1, \dots, N_r$ follow zero-mean complex Gaussian distributions.
- Define $\mathbf{R_z} \triangleq \mathrm{E}[\mathbf{z}\mathbf{z}^H]$, $E_s \triangleq \mathrm{E}[|s|^2]$.
- (1) Specify the dimension of $\mathbf{H}, \mathbf{W}$.
- (2) Write an expression of the post-processed signal $\tilde{x}$ in terms of $\mathbf{W}, \mathbf{H}, p, x, \mathbf{z}$.
- (3) Show that $\mathrm{E}[|s|^2] = |p|^2 = E_s$.

Assignment 05

- ② **SIMO – SC**

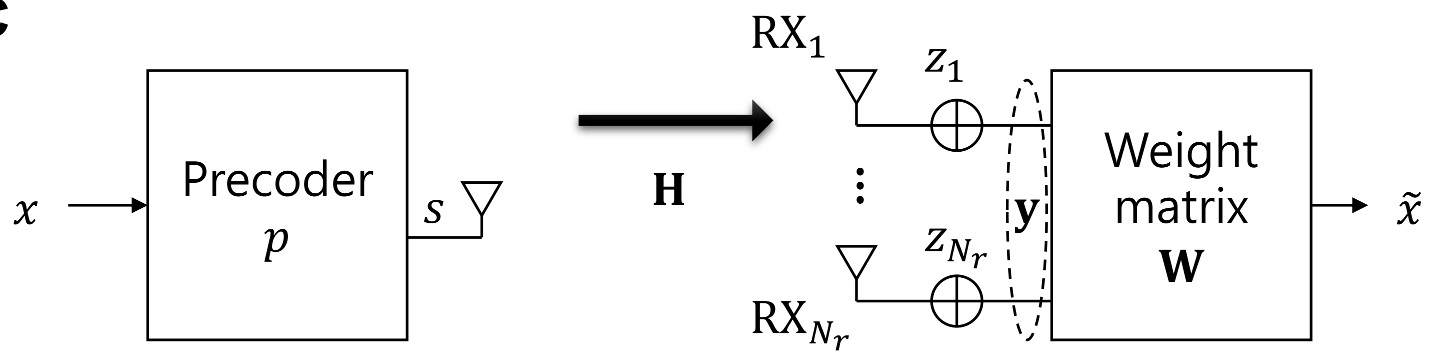


- (4) For AWGN, i.e., $\mathbf{R_z} = \sigma_z^2 \mathbf{I}_{N_r}$, show that the SNR of the i-th receive antenna is $\gamma_i = \frac{\mathrm{E}\left[\left|\mathbf{H}_{(i,1)} p x\right|^2\right]}{\mathrm{E}[|z_i|^2]} = \frac{\left|\mathbf{H}_{(i,1)}\right|^2 E_s}{\sigma_z^2}$

- In SC, antenna $k \triangleq \arg\max_i \gamma_i$ is selected for decoding.
- Although simple, SC does not make full use of the receive antennas.
- (5) For $\mathbf{W} = \frac{1}{\mathbf{H}_{(k,1)} p} \mathbf{e}_k^T$ where $\mathbf{e}_k$ is a standard basis vector in which the k-th element is 1 and the other elements are 0, show that the estimated symbol is $\tilde{x} = x + \frac{z_k}{\mathbf{H}_{(k,1)} \sqrt{E_s}}$.

Assignment 05

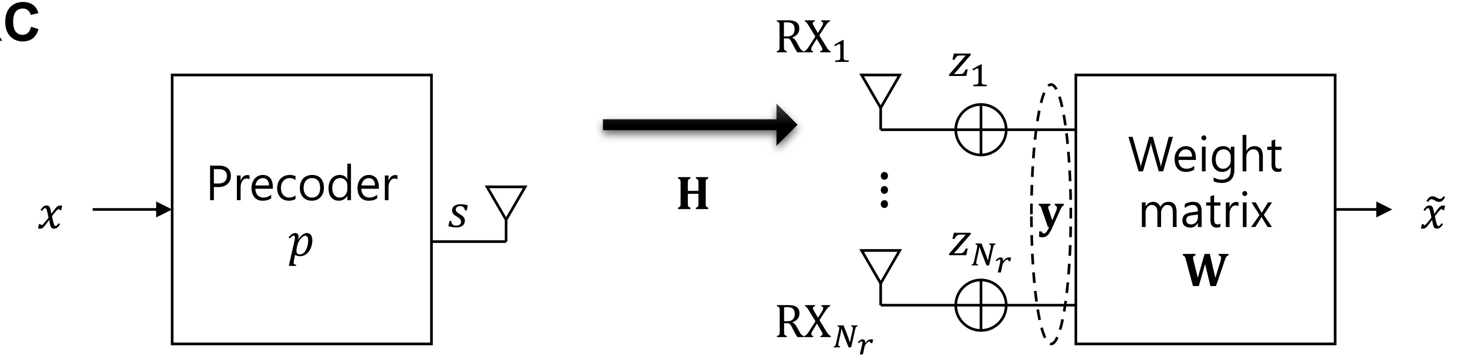
- ③ **SIMO – EGC**



- In EGC, the output of the i-th receive antenna is phase-compensated, and all the outputs are combined for decoding.
- Although making full use of the receive antennas, EGC is not optimal.
- (1) For $\mathbf{W} = \frac{1}{\sum_{i=1}^{N_r} |\mathbf{H}_{(i,1)}|} \left[\frac{1}{\angle \mathbf{H}_{(1,1)} p} \quad \cdots \quad \frac{1}{\angle \mathbf{H}_{(N_r,1)} p} \right]$ where $\angle \mathbf{H}_{(i,1)} = \arctan\left(\frac{\mathrm{Im}\{\mathbf{H}_{(i,1)}\}}{\mathrm{Re}\{\mathbf{H}_{(i,1)}\}} \right)$ is the phase of $\mathbf{H}_{(i,1)}$, show that the estimated symbol is $\tilde{x} = x + \frac{1}{\sum_{i=1}^{N_r} |\mathbf{H}_{(i,1)}|} \sum_{i=1}^{N_r} \left(\frac{z_i}{\angle \mathbf{H}_{(i,1)} \sqrt{E_s}} \right)$.

Assignment 05

- ④ **SIMO – MRC**

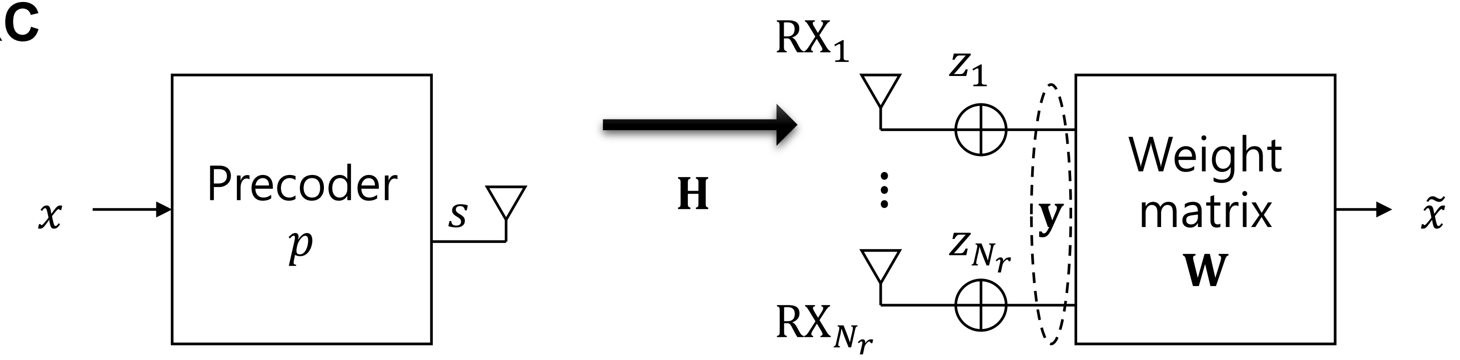


- In MRC, the output of the i-th receive antenna is phase-compensated and weighted proportional to the channel gain.
- That is, branches with strong signal are further amplified, while weak signals are attenuated.
- (1) For AWGN, i.e., $\mathbf{R}_\mathbf{z} = \sigma_z^2 \mathbf{I}_{N_r}$, show that the combined SNR $\gamma_\Sigma \triangleq \frac{\mathrm{E}[|\mathbf{WH}px|^2]}{\mathrm{E}[|\mathbf{Wz}|^2]} = \frac{E_s}{\sigma_z^2} \frac{|\mathbf{H}^H \mathbf{W}^H|^2}{|\mathbf{W}^H|^2}$

- (2) Using the Cauchy-Schwartz inequality, find the optimal $\mathbf{W}$ that maximizes γ_Σ and show that the maximum SNR is $\gamma_\Sigma = \frac{E_s}{\sigma_z^2} |\mathbf{H}|^2$.

Assignment 05

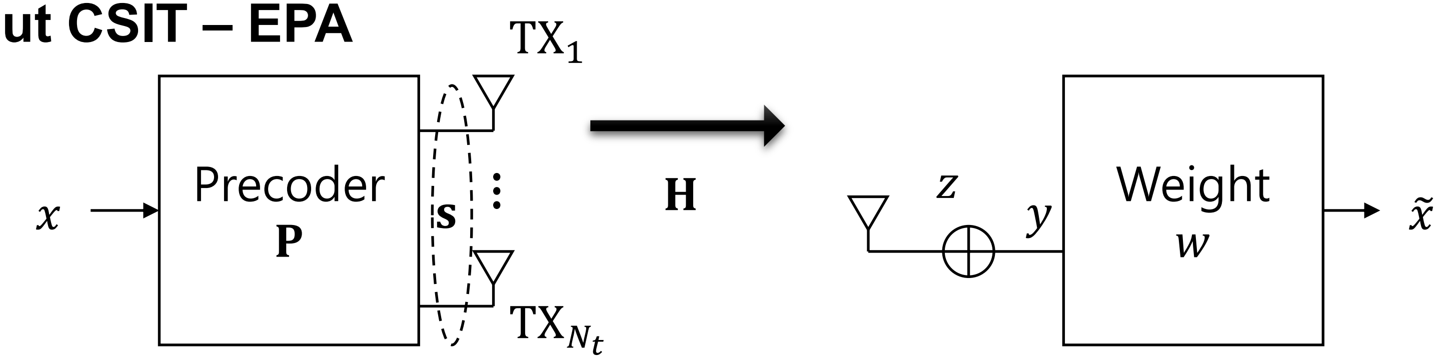
- ④ **SIMO – MRC**



  - Although optimal, MRC additionally needs amplifiers and attenuators besides phase-shifters, yielding a higher implementation cost over EGC
  - (3) For $\mathbf{W} = \frac{1}{|\mathbf{H}^H|^2 p}\mathbf{H}^H$, show that the estimated symbol is $\tilde{x} = x + \frac{1}{|\mathbf{H}^H|^2\sqrt{E_s}}\mathbf{H}^H\mathbf{z}$.

Assignment 05

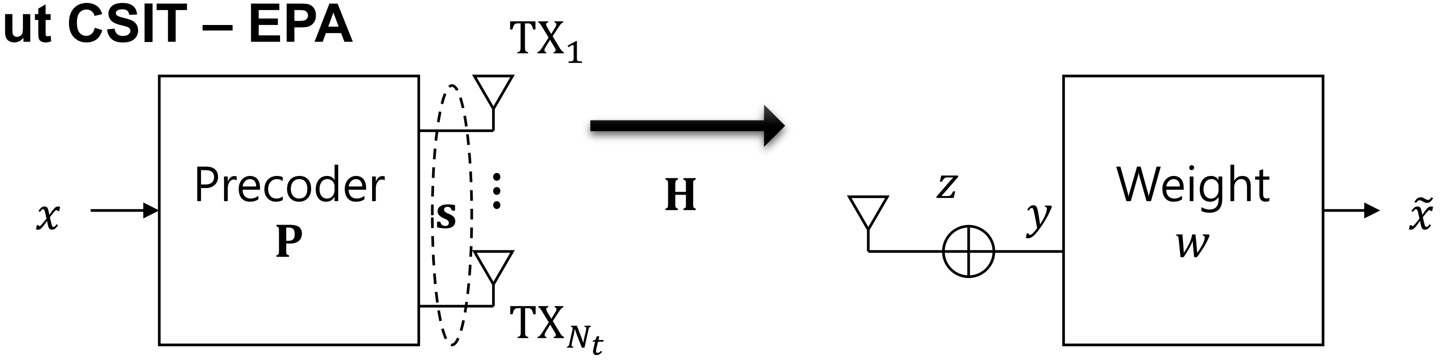
- ⑤ **MISO without CSIT – EPA**



  - Let $x \sim CN(0,1)$ and $z \sim CN(0, \sigma_z^2)$.
  - (1) Specify the dimension of $\mathbf{P}, \mathbf{H}$.
  - (2) Write an expression of the post-processed signal $\tilde{x}$ in terms of $w, \mathbf{H}, \mathbf{P}, x, z$.
  - (3) Show that the average transmit energy is $E_s = |\mathbf{P}|^2$.

Assignment 05

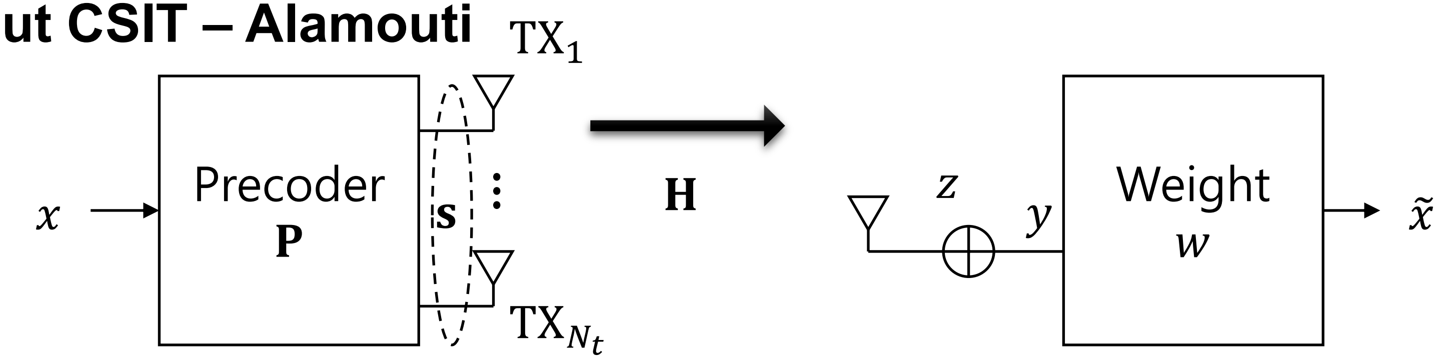
- ⑤ **MISO without CSIT – EPA**



  - We design $\mathbf{s}$ such that the average transmit energy E_s is equally distributed to the antennas, i.e., $\mathbf{P} = \sqrt{\frac{E_s}{N_t}}\mathbf{1}_{N_t}$. ("Equal power allocation")
  - (4) For $w = \frac{1}{\mathbf{HP}}$, show that the estimated symbol is $\tilde{x} = x + \frac{1}{\sqrt{\frac{E_s}{N_t}}\sum_{i=1}^{N_t}\mathbf{H}_{(1,i)}} z.$

Assignment 05

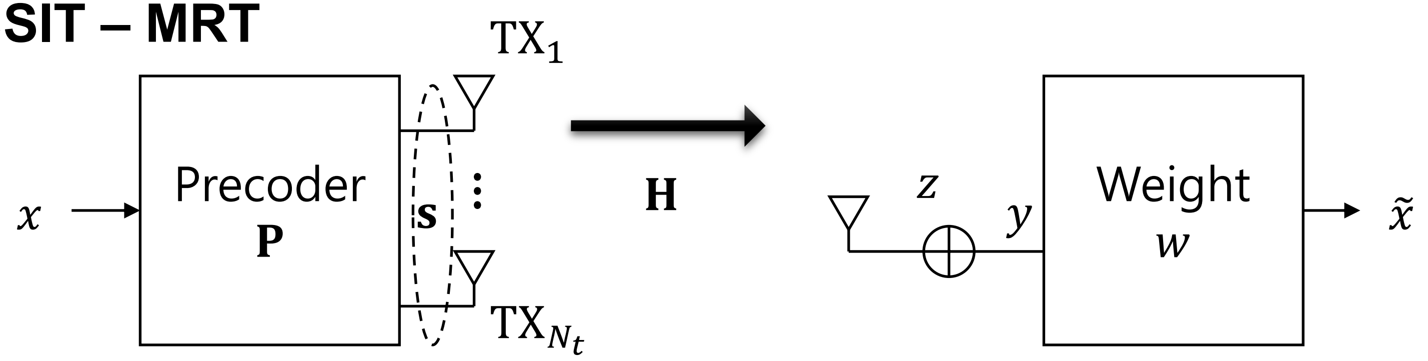
- ⑥ **MISO without CSIT – Alamouti**



  - (Refer to the paper)

Assignment 05

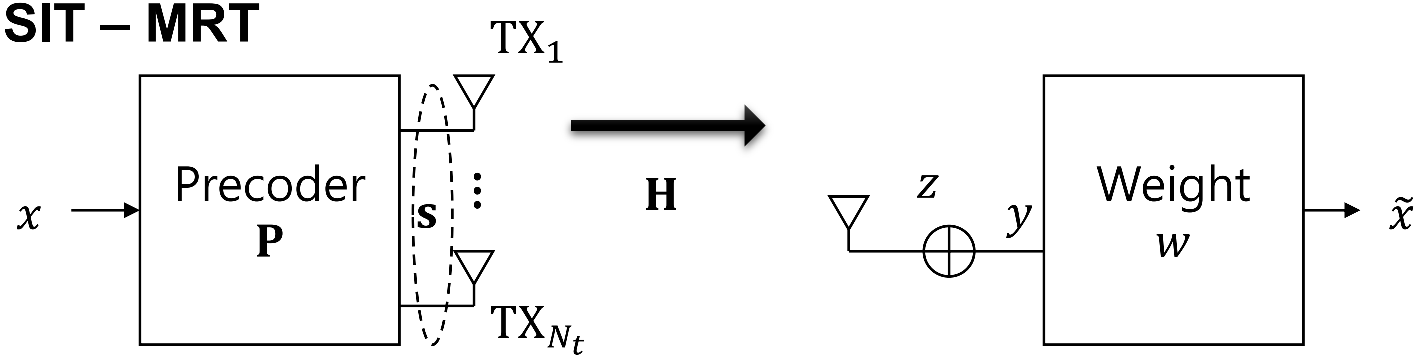
- ⑦ **MISO with CSIT – MRT**



- Similar to MRC, the the i-th transmit antenna is phase-compensated and weighted proportional to the channel gain.
- That is, branches with strong signal are further amplified, while weak signals are attenuated.
- (1) For AWGN, i.e., $\mathbf{R}_\mathbf{z} = \sigma_z^2 \mathbf{I}_{N_r}$, show that the receive SNR $\gamma = \frac{\mathrm{E}\left[|w\mathbf{HP}x|^2\right]}{\mathrm{E}[|wz|^2]} = \frac{|\mathbf{HP}|^2}{\sigma_z^2}$.
- (2) Using the Cauchy-Schwartz inequality, find the optimal $\mathbf{P}$ that maximizes γ with $E_s = |\mathbf{P}|^2$ and show that the maximum SNR is $\gamma = \frac{|\mathbf{H}^H|^2 E_s}{\sigma_z^2}$.

Assignment 05

- ⑦ **MISO with CSIT – MRT**



  - (3) For $w = \frac{1}{\mathbf{HP}}$, show that the estimated symbol is $\tilde{x} = x + \frac{1}{\sqrt{\frac{E_S}{N_t}}\sum_{i=1}^{N_t}\mathbf{H}_{(1,i)}} z.$

Assignment 05

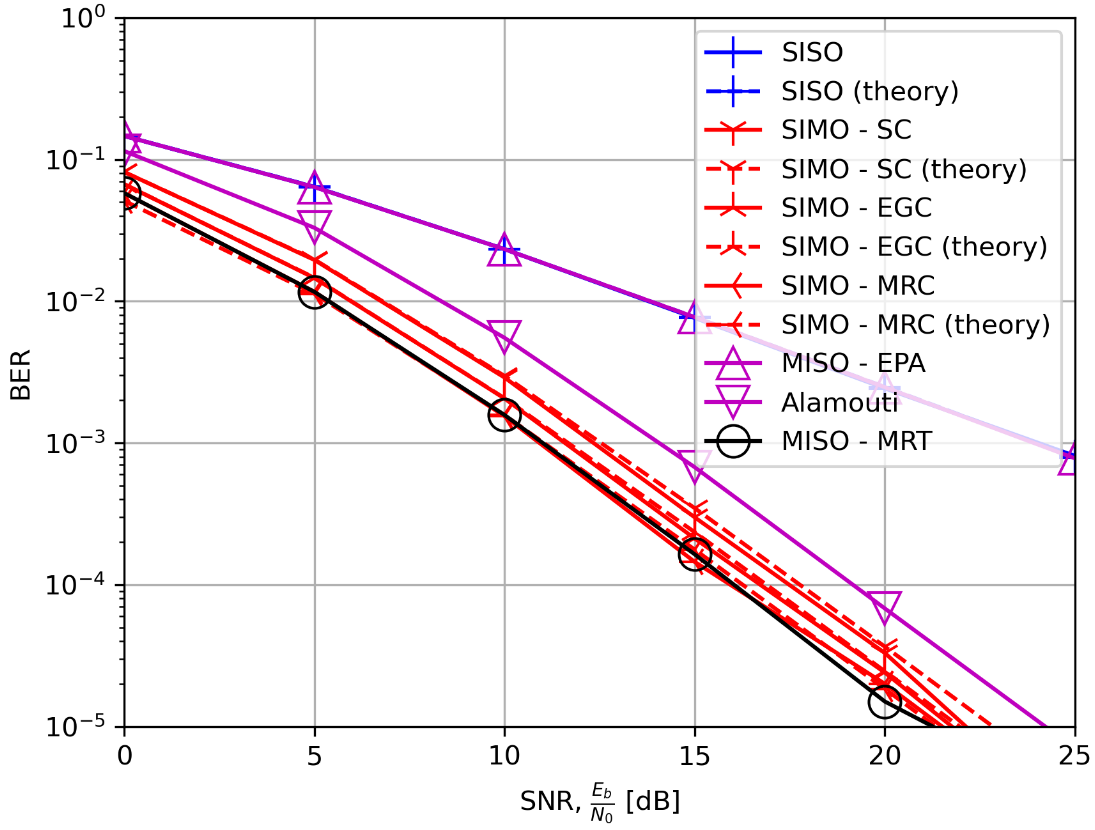
- ⑧ **Numerical results**
  - Setups
    - Unit average transmit energy, i.e., $E_s = 1$
    - AWGN, i.e., $\mathbf{R_z} = \sigma_z^2 \mathbf{I}_{N_r}$
    - Rayleigh fading channel matrix, i.e., $h_{ij} \sim CN(0,1)$
  - Curve 1: SISO $(N_t = 1, N_r = 1)$
  - Curve 2: SIMO $(N_t = 1, N_r = 2)$
    - Curve 2-1: SIMO – SC
    - Curve 2-1: SIMO – EGC
    - Curve 2-1: SIMO – MRC
  - Curve 3: MISO $(N_t = 2, N_r = 1)$
    - Curve 3-1: MISO – EPA
    - Curve 3-2: MISO – Alamouti
    - Curve 3-1: MISO – MRT

Assignment 05

- ⑧ **Numerical results**
  - Setups
    - Unit average transmit energy, i.e., $E_s = 1$
    - AWGN, i.e., $\mathbf{R}_\mathbf{z} = \sigma_z^2 \mathbf{I}_{N_r}$
    - Rayleigh fading channel matrix, i.e., $h_{ij} \sim CN(0,1)$
  - Curve 4: SIMO $(N_t = 1, N_r = 4)$
    - Curve 4-1: SIMO – SC
    - Curve 4-1: SIMO – EGC
    - Curve 4-1: SIMO – MRC
  - Curve 5: MISO $(N_t = 4, N_r = 1)$
    - Curve 5-1: MISO – EPA
    - Curve 5-2: MISO – Alamouti $(N_t = 2, N_r = 2)$
    - Curve 5-1: MISO – MRT

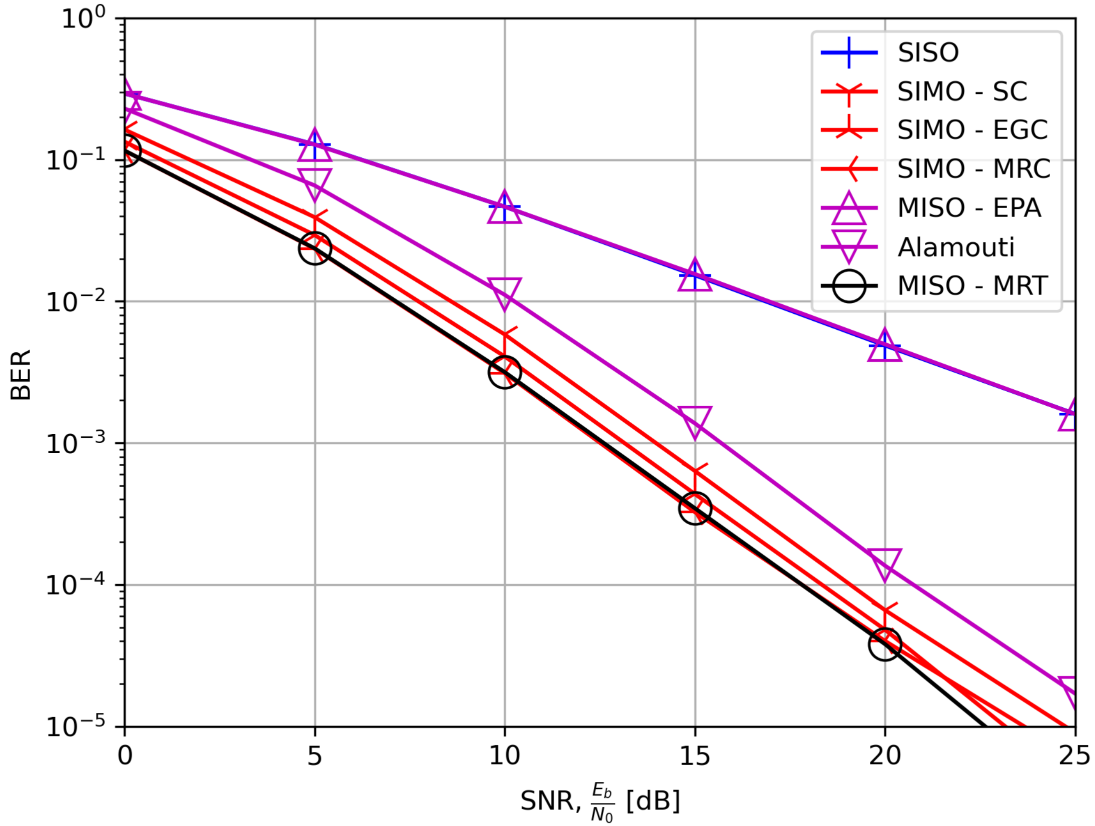
Assignment 05

- ⑧ **Numerical results**
  - BPSK with 2 antennas (Fig. 7, 8, 9, 16 of "Critical and Important … MIMO Technology")
  - 10^6 samples



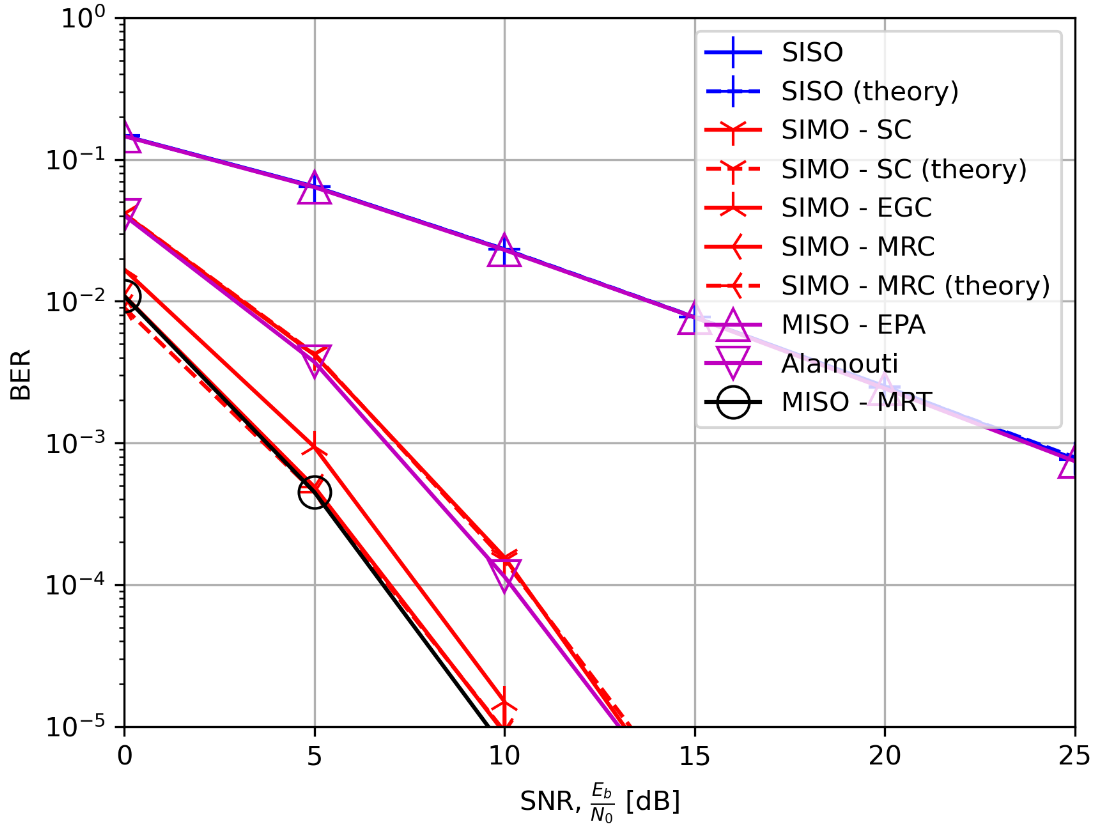
Assignment 05

- ⑧ **Numerical results**
  - QPSK with 2 antennas
  - 10^6 samples



Assignment 05

- ⑧ **Numerical results**
  - BPSK with 4 antennas (Fig. 7, 8, 9, 16 of “Critical and Important … MIMO Technology”)
  - 10^6 samples



Assignment 05

- ⑧ **Numerical results**
  - QPSK with 4 antennas
  - 10^6 samples