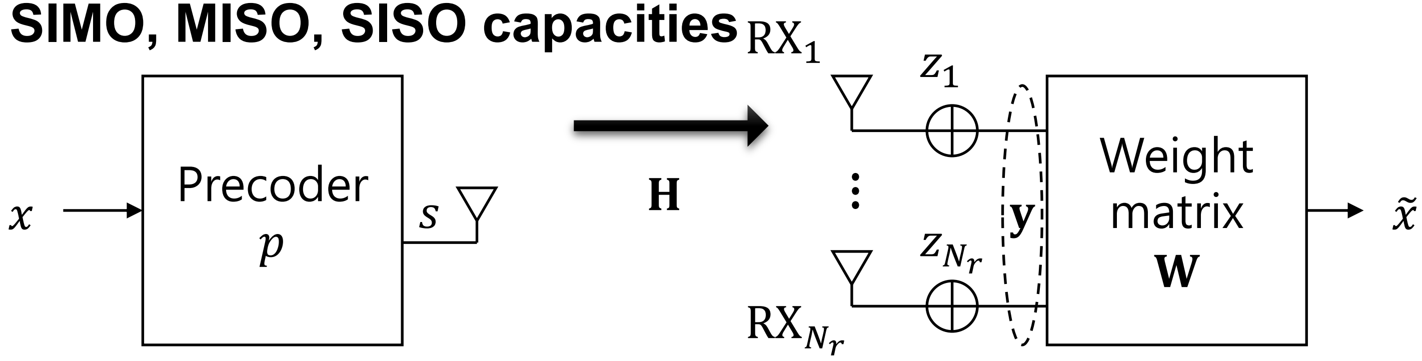
Assignment 08

- **Special cases: SIMO, MISO, SISO capacities**



  - ① SIMO
  - ② MISO without CSIT
  - ③ MISO with CSIT
  - ④ SISO

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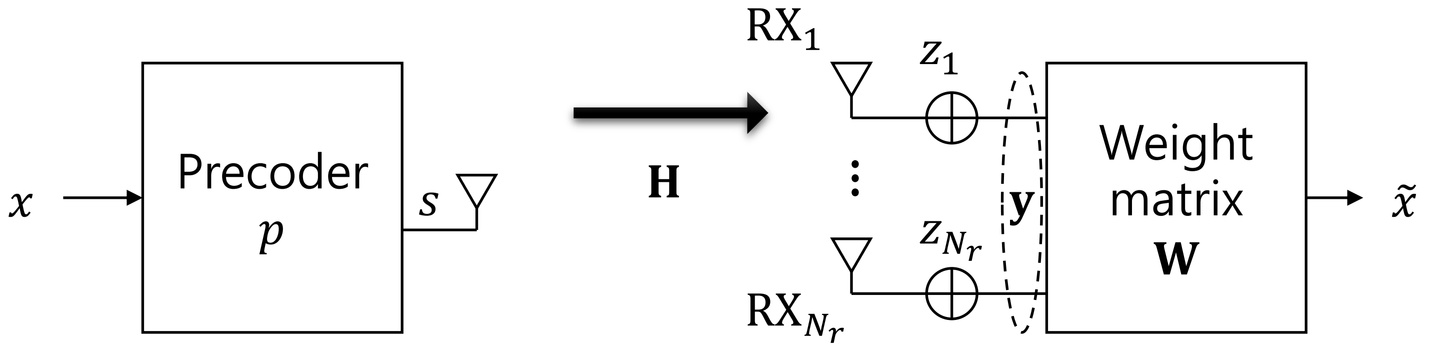
- ① **SIMO**



- Let $x \sim CN(0,1)$ and $z_i, \forall i = 1, \ldots, N_r$ follow zero-mean complex Gaussian distributions.
- Define $\mathbf{R_z} \triangleq \mathrm{E}[\mathbf{z}\mathbf{z}^H]$, $E_s \triangleq \mathrm{E}[|s|^2]$.
- (1) Specify the dimension of $\mathbf{H}, \mathbf{W}$.
- (2) Write an expression of the post-processed signal $\tilde{x}$ in terms of $\mathbf{W}, \mathbf{H}, p, x, \mathbf{z}$.
- (3) Show that $\mathrm{E}[|s|^2] = |p|^2 = E_s$.
- (4) Show that the capacity is expressed by $C = \log_2\left(1 + \frac{E_s \mathbf{W}\mathbf{H}\mathbf{H}^H\mathbf{W}^H}{\mathbf{W}\mathbf{R_z}\mathbf{W}^H}\right)$.

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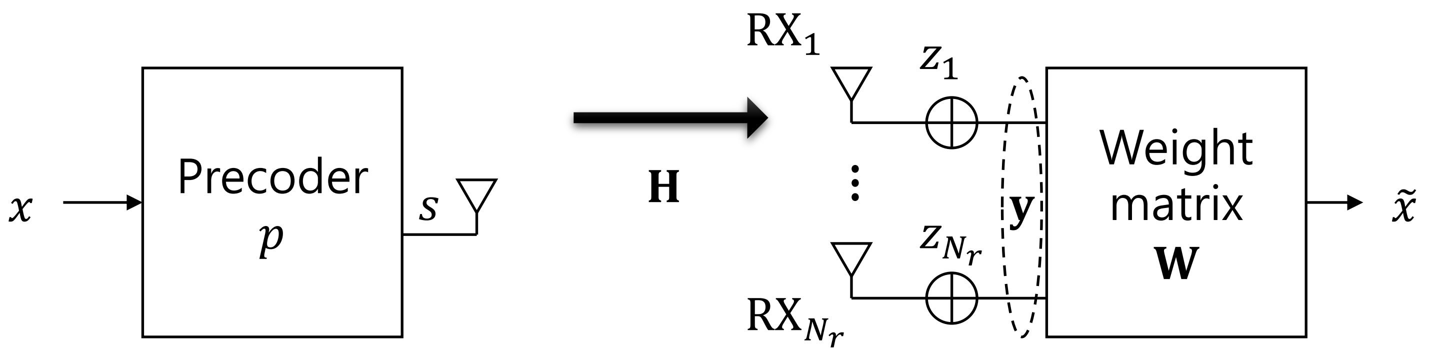
- ① **SIMO**



- (5) For AWGN, i.e., $\mathbf{R}_\mathbf{z} = \sigma_z^2 \mathbf{I}_{N_r}$, show that $C = \log_2\left(1 + \frac{E_s}{\sigma_z^2}\frac{|\mathbf{H}^H\mathbf{W}^H|^2}{|\mathbf{W}^H|^2}\right)$.

- (6) Using the Cauchy-Schwartz inequality, find the optimal $\mathbf{W}$ that maximizes C and show that the maximum capacity is $C = \log_2\left(1 + \frac{E_s}{\sigma_z^2}|\mathbf{H}|^2\right)$.

- (7) Show that MRC achieves the maximum capacity $C = \log_2\left(1 + \frac{E_s}{\sigma_z^2}|\mathbf{H}|^2\right)$.

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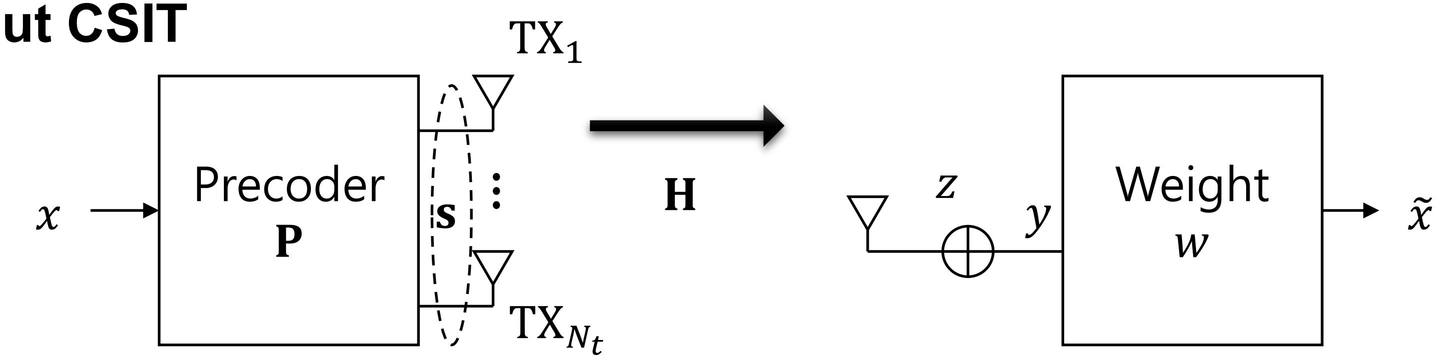
- ② **MISO without CSIT**



- Let $x \sim CN(0,1)$ and $z \sim CN(0, \sigma_z^2)$.
- (1) Specify the dimension of $\mathbf{P}, \mathbf{H}$.
- (2) Write an expression of the post-processed signal $\tilde{x}$ in terms of $w, \mathbf{H}, \mathbf{P}, x, z$.
- (3) Show that the average transmit energy is $E_s = |\mathbf{P}|^2$.
- (4) Show that the capacity is expressed by $C = \log_2\left(1 + \frac{|\mathbf{HP}|^2}{\sigma_z^2}\right)$.

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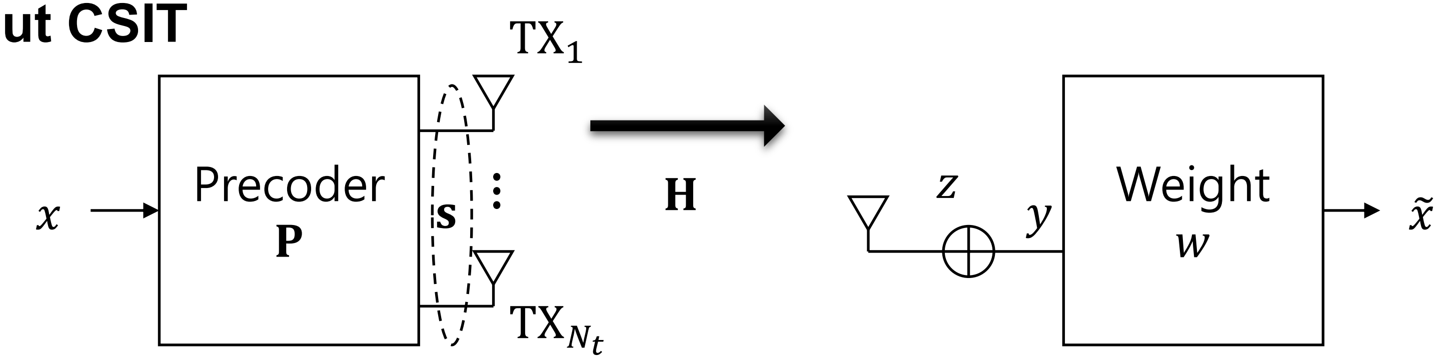
- ② **MISO without CSIT**



  - First approach: We design $\mathbf{s}$ such that the average transmit energy E_s is equally distributed to the antennas, i.e., $\mathbf{P} = \sqrt{\frac{E_s}{N_t}}\mathbf{1}_{N_t}$. ("Equal power allocation")
  - (5) Show that the capacity is $C = \log_2\left(1 + \frac{E_s}{N_t\sigma_z^2}\left|\mathbf{H}\mathbf{1}_{N_t}\right|^2\right)$.

  - Second approach: We design $\mathbf{P} = \sqrt{E_s}\mathbf{e}_1$ such that the average transmit energy E_s is concentrated and by assigning x to TX_1. ($N_t - 1$ antennas are unused.) ("Antenna selection")
  - (6) Show that the capacity is $C = \log_2\left(1 + \frac{E_s}{\sigma_z^2}\left|\mathbf{H}_{(1,1)}\right|^2\right)$.

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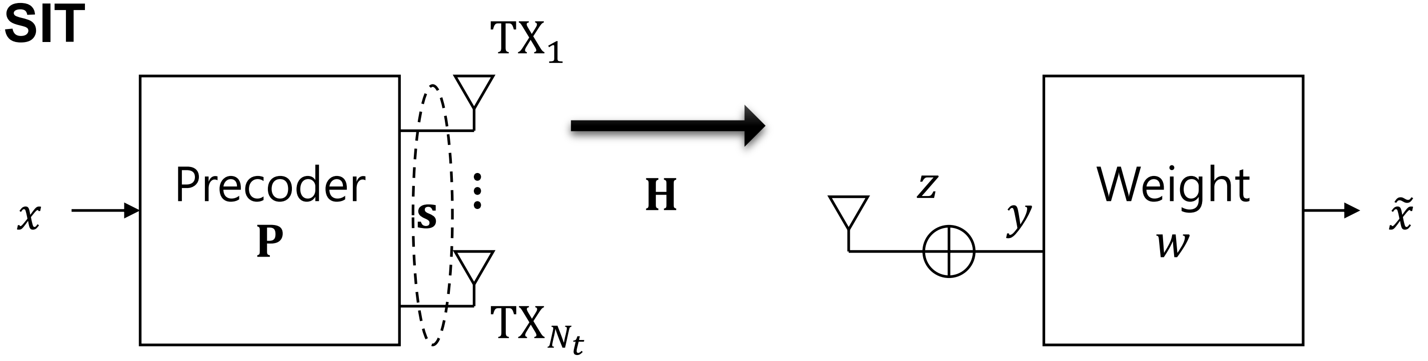
- ③ **MISO with CSIT**

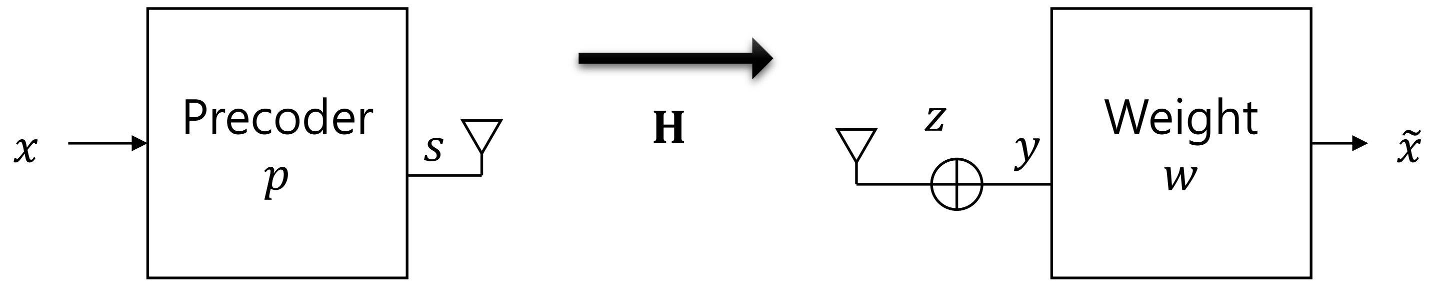


- In this case, we can optimize the precoder $\mathbf{P}$ to achieve the maximum capacity C.
- (1) Using the Cauchy-Schwartz inequality, find the optimal $\mathbf{P}$ that maximizes C with $E_s = |\mathbf{P}|^2$ and show that the maximum capacity is $C = \log_2\left(1 + \frac{|\mathbf{H}^H|^2 E_s}{\sigma_z^2}\right)$.

- (2) Show that MRT achieves the maximum capacity $C = \log_2\left(1 + \frac{|\mathbf{H}^H|^2 E_s}{\sigma_z^2}\right)$.

Assignment 08

- ④ **SISO**



- Assume $x \sim CN(0,1)$ and $z \sim CN(0, \sigma_z^2)$.
- (1) Write an expression of the recovered signal $\tilde{x}$ in terms of w, h, p, x, z
- (2) Show that the average transmit energy is $E_s = |p|^2$.
- (3) Show that the capacity is expressed by $C = \log_2\left(1 + \frac{|h|^2 E_s}{\sigma_z^2}\right)$.