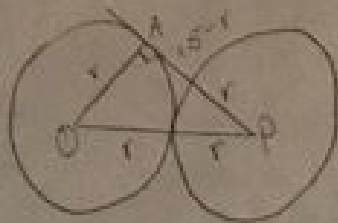


Chapter 10

343.



$$(15)^2 + (r)^2 = (22.5)^2$$

$$225 + r^2 = 506.25$$

$$225 = 31^2$$

$$75 = r^2$$

$$\sqrt{75} = \sqrt{75} = 1$$

$$\boxed{5\sqrt{3} = r}$$

344.



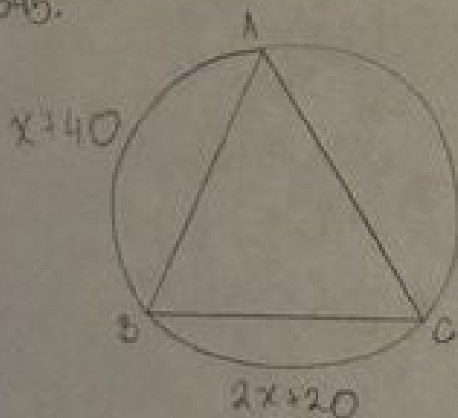
$$AC = \sqrt{4^2 + 6^2}$$

$$= \sqrt{16 + 36}$$

$$= \sqrt{52}$$

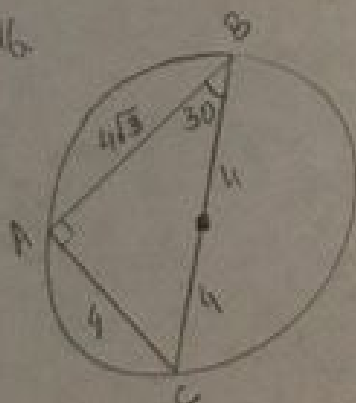
$$\boxed{AC = 5.2}$$

345.



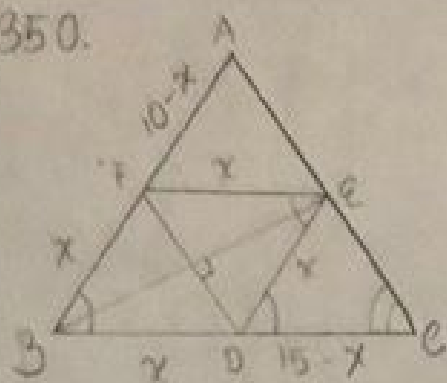
$$\left. \begin{array}{l} x + 40 + 2x - 20 + 2x + 20 = 360 \\ 5x = 320 \\ \boxed{x = 64} \end{array} \right\}$$

346.



$$\frac{1}{2} \cdot 4 \cdot 4\sqrt{3} = \boxed{8\sqrt{3}}$$

350.


 $\triangle CDE \sim \triangle CBA$ by AA

$$\frac{CD}{CB} = \frac{DE}{BA}$$

$$\frac{15-x}{15} = \frac{x}{10}$$

$$10(15-x) = 15x$$

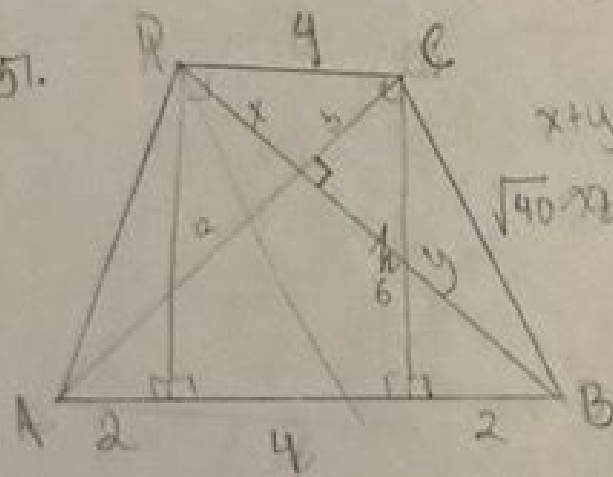
$$2(15-x) = 3x$$

$$30 - 2x = 3x$$

$$30 = 5x$$

$$\boxed{6=x}$$

351.



$$x+y = a+b$$

$$\sqrt{40} \cdot h \Rightarrow x+y = 16$$

$$a+b = 8$$

$$\frac{1}{2} \cdot b \cdot y = \frac{1}{2} \cdot b \cdot x + \frac{1}{2} \cdot a \cdot y + \frac{1}{2} \cdot a \cdot x$$

$$= \frac{1}{2} \cdot (a+b)(x+y)$$

$$= \frac{1}{2} (x+y)^2$$

$$2\left(\frac{1}{2} \cdot 2 \cdot h\right) + 4h = \frac{1}{2} (x+y)^2 = 6h$$

$$2h + 4h = \frac{1}{2} (x+y)^2$$

$$a^2 + x^2 = b^2 + y^2$$

$$\sqrt{(\sqrt{40})^2 - 4} = \sqrt{36} = 6$$

$$= \boxed{36}$$

$$x^2 + y^2 + a^2 + b^2 = 80$$

$$(x+y)(a+b)$$

$$a^2 + b^2 + y^2 = 80$$

$$b^2 + y^2 = 40$$

$$= \left(\frac{1}{2} \cdot 2 \cdot y\right) 2 + 4y$$

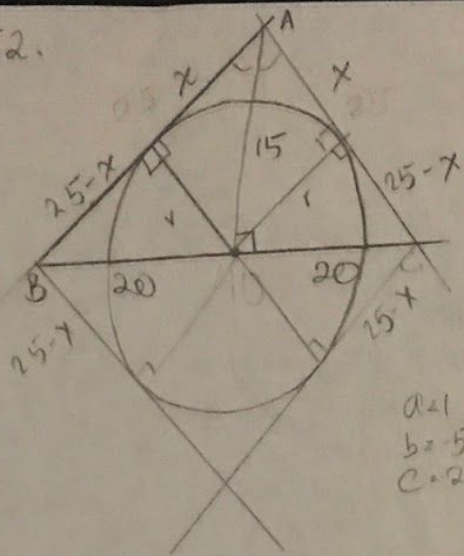
$$= \frac{1}{2} \cdot x^2 + 2xy + y^2 = 2y + 4y$$

$$x^2 + 2xy + y^2 = 4y + 4y$$

$$x^2 + 2xy + y^2 - 12y = 0$$

$$x^2 - 12y = 0$$

352.



$$(25-x)^2 + r^2 = 20^2$$

$$25^2 - 2(25 \cdot x) + x^2 + r^2 = 400$$

$$625 - 50x + x^2 + r^2 = 400$$

$$x^2 + r^2 = 50x - 225$$

$$r^2 = 50x - 225$$

$$r^2 =$$

$$\begin{aligned} a &= 1 \\ b &= 50 \\ c &= 225 \end{aligned}$$

$$\frac{50 \pm \sqrt{2500 - 4(-2250)}}{2}$$

$$\frac{50 \pm \sqrt{2500}}{2}$$

$$\frac{50 \pm \sqrt{2500 + 10000}}{2}$$

$$\begin{array}{r} 12500 \\ 100 \quad 125 \\ 5 \quad 20 \quad 5 \\ 54 \end{array}$$

$$\frac{1}{2} \cdot 20 \cdot 15 = \frac{1}{2} \cdot r \cdot 25$$

$$20 \cdot 15 = 25r$$

$$300 = 25r$$

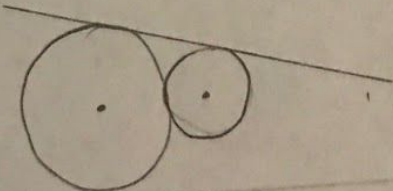
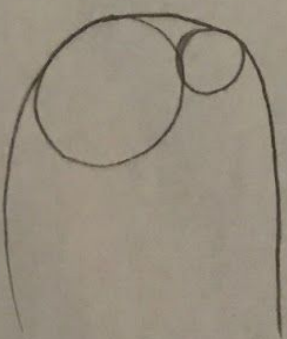
$$12 = r$$

$$\Rightarrow 172\pi$$

$$\begin{aligned} 25 - 20 &= x \\ 5 &= 4 - 3 \end{aligned}$$

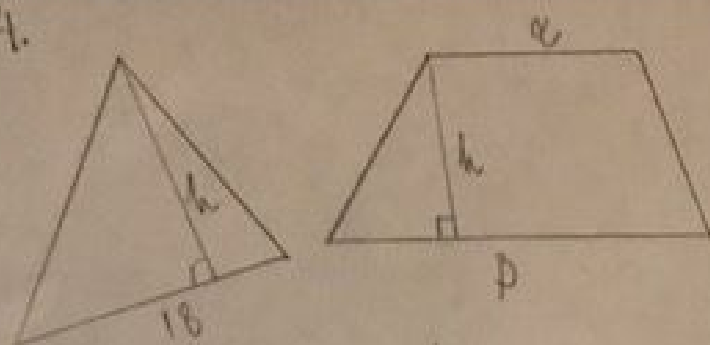
$$\pi r^2 \cdot A_0 \rightarrow \frac{\pi r^2}{2} \cdot A_0 + \frac{\pi (12)^2}{2} = \frac{144\pi}{2}$$

353.



$$\begin{aligned} \sqrt{3^2 + 6^2} &= \sqrt{9 + 36} \\ &= \sqrt{45} \\ &= \sqrt{9} \sqrt{5} \\ &= 3\sqrt{5} \end{aligned}$$

354.

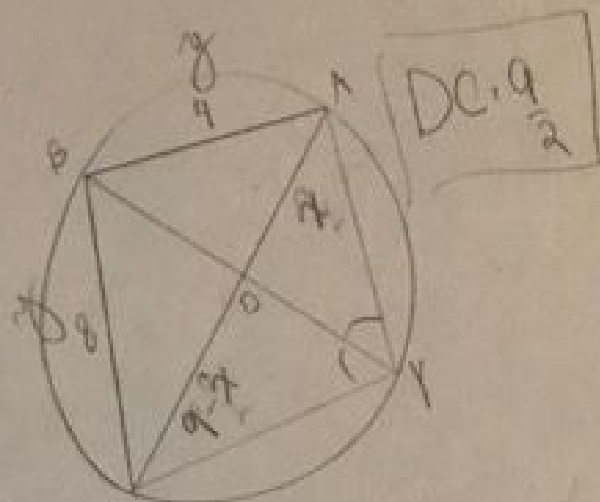


$$\frac{1}{2} \cdot 18 \cdot h = \frac{1}{2} (a+b) \cdot h$$

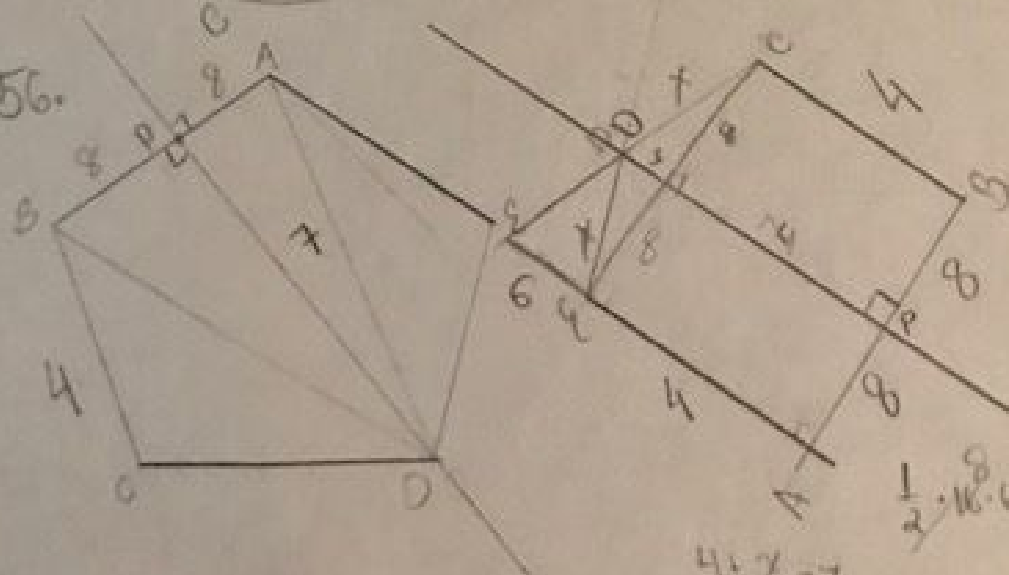
$$18h = (a+b)h$$

$$18 = a+b$$

355.



356.



$$\frac{1}{2} \cdot 16 \cdot 6 + 16 \cdot 4$$

$$48 + 64$$

$$112$$

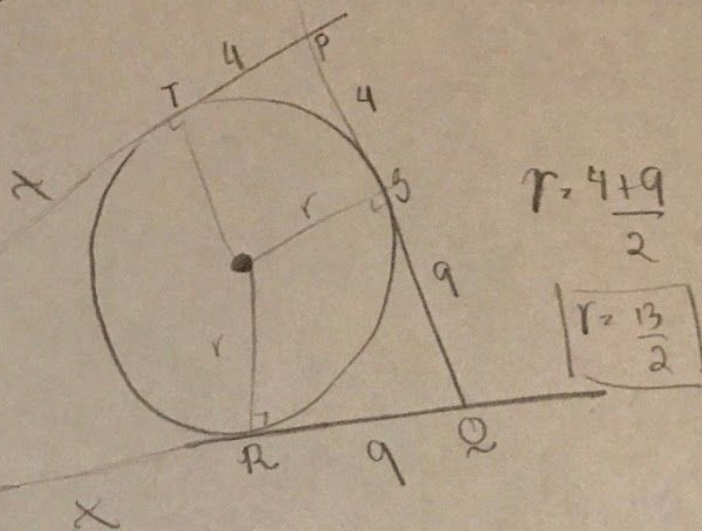
$$4 \cdot x = 7$$

$$\frac{4 \cdot x}{2} = 7$$

$$4 \cdot x = 14$$

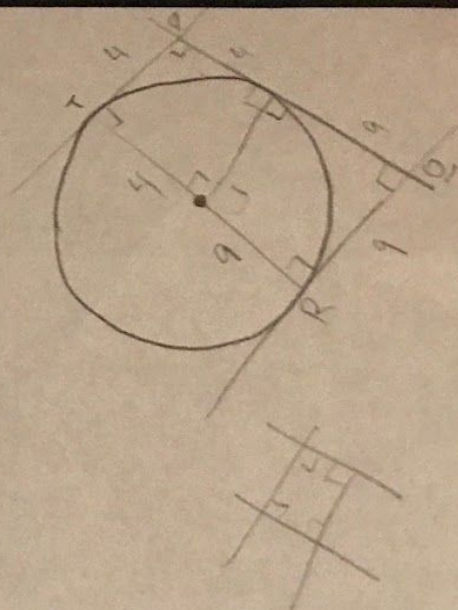
$$x = 10$$

357.

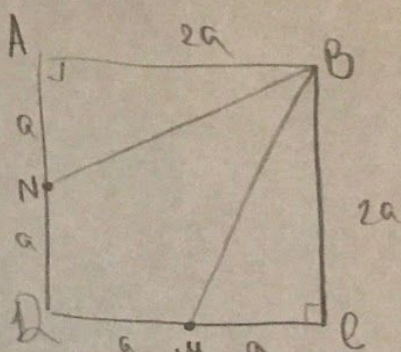


$$r = \frac{4+9}{2}$$

$$r = \frac{13}{2}$$



358.



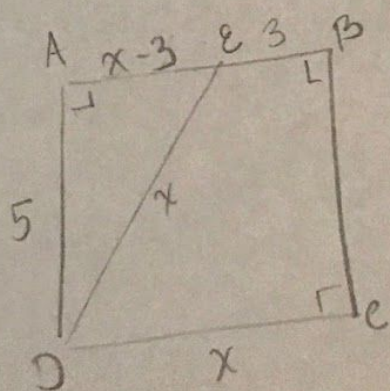
$$\frac{BM \cdot DN}{AB \cdot CD} = \frac{a + a + \sqrt{(a^2 + (2a)^2)} + \sqrt{a^2 + (2a)^2}}{4(2a)}$$

$$= \frac{2a + 2(\sqrt{5a^2})}{8a} = \frac{2a + 2(a\sqrt{5})}{8a}$$

$$= \frac{2a + 2a\sqrt{5}}{8a} = \frac{2 + 2\sqrt{5}}{8}$$

$$= \frac{1 + \sqrt{5}}{4}$$

359.



$$25 + (x-3)^2 = x$$

$$25 = x - (x^2 - 6x + 9)$$

$$25 = x - x^2 + 6x - 9$$

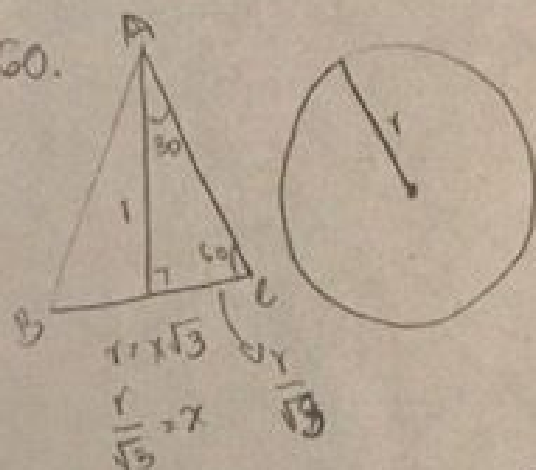
$$0 = -x^2 + 7x - 34$$

$$0 = x^2 - 7x + 34$$

$$x = \frac{7 \pm \sqrt{49 - 4(34)}}{2}$$

359. $25 + (x-3)^2 = x^2$
 $25 + x^2 - 6x + 9 = x^2$
 $25 - 6x + 9 = 0$
 $-6x = -34$
 $x = \frac{34}{6} = \boxed{\frac{17}{3}}$

360.

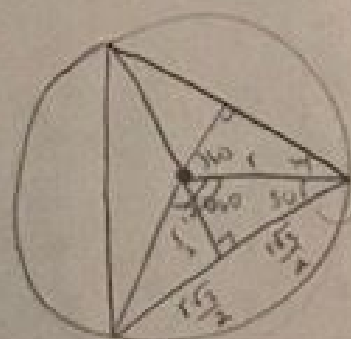


$30-60-90$
 $x - x\sqrt{3} - 2x$

$S \cdot 2x = \frac{2r}{\sqrt{3}} \cdot [\Delta ABC] = \frac{8\sqrt{3}}{4} \cdot \left(\frac{2r}{\sqrt{3}}\right)^2 \sqrt{3} = \frac{4r^2 \sqrt{3}}{3}$

~~$\frac{4r^2 \sqrt{3}}{3} = \frac{4r^2 \sqrt{3}}{12} = \frac{r^2 \sqrt{3}}{3}$~~

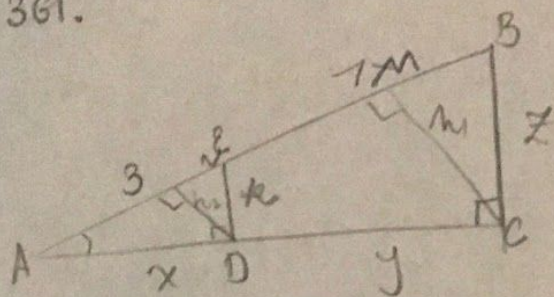
$\frac{r}{\sqrt{3}} + \frac{r}{\sqrt{3}} + \frac{2r}{\sqrt{3}} + \frac{2r}{\sqrt{3}}$
 $= \frac{6r}{\sqrt{3}} = \Delta ABC$



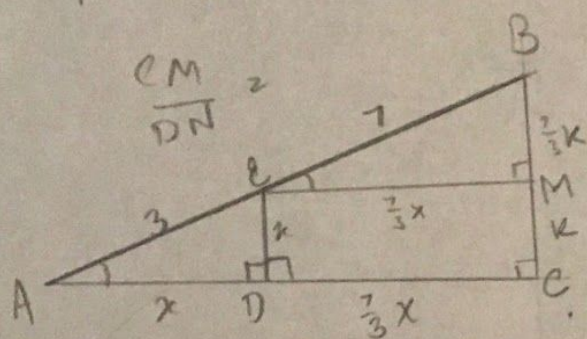
$73 \left(\frac{r\sqrt{3}}{2} \right) = \frac{3r\sqrt{3}}{2}$

$\frac{\frac{6r}{\sqrt{3}} \cdot 2\sqrt{3}}{\frac{3r\sqrt{3}}{2} \cdot 2\sqrt{3}} = \frac{6r \cdot 2}{9r}$
 $\frac{12r}{9r} = \boxed{\frac{4}{3}}$

361.



$$\triangle CMA \sim \triangle DNA$$



$$\begin{aligned} &= \frac{1}{2} xk - \frac{100}{18} xk \\ &= xk \left(\frac{1}{2} - \frac{100}{18} \right) \end{aligned}$$

$$[\triangle ADE] = \frac{1}{2} xk$$

$$[\triangle ABC] - [\triangle ADE] = \left(\frac{1}{2} \cdot \frac{10}{3} x \cdot \frac{10}{3} k \right) - \left(\frac{1}{2} xk \right)$$

$$= \left(\frac{100}{18} xk \right) - \left(\frac{1}{2} xk \right)$$

$$= \left(\frac{100}{18} xk \right) - \left(\frac{9}{18} xk \right)$$

$$= \frac{91}{18} xk$$

$$\therefore \frac{[\triangle ADE]}{[\triangle ABC] - [\triangle ADE]} = \frac{\frac{1}{2} xk}{\frac{91}{18} xk} = \frac{\frac{1}{2} \cdot 18}{91} = \boxed{\frac{9}{91}}$$

$$\frac{[\triangle ADE]}{[\triangle ABC] - [\triangle ADE]}$$

$$\triangle AED \sim \triangle ABC \text{ by AA}$$

$$\frac{[\triangle ADE]}{[\triangle ABC]} = \frac{3m_1 h_1}{10m_2 h_2}$$

$$\frac{1}{2} \cdot 10 \cdot h_1 = \left(\frac{1}{2} \cdot 3 \cdot h_2 \right)$$

$$5h_1 = \frac{3}{2} h_2$$

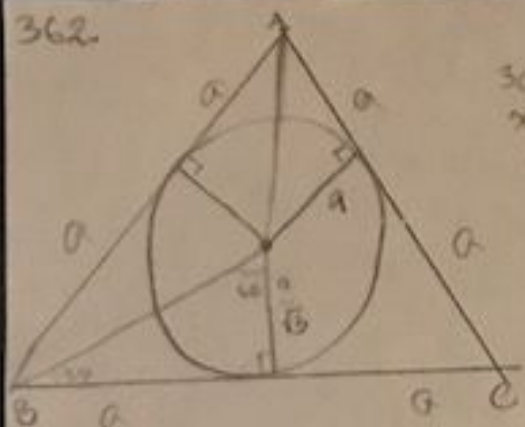
$$5h_1 = \frac{3}{2} \left(\frac{10}{3} h_1 \right)$$

$$\triangle ADE \sim \triangle EBM \text{ by AA}$$

$$\therefore [\triangle ADE] = \frac{1}{2} \cdot xk$$

$$\begin{aligned} [\triangle ABC] - [\triangle ADE] &= \frac{1}{2} \cdot xk \\ &= \left(\frac{1}{2} \cdot \frac{10}{3} x \cdot \frac{10}{3} k \right) \end{aligned}$$

362.



$$30-60-90$$

$$x - x\sqrt{3} - 2x$$

$$x\sqrt{3} = a$$

$$x = \frac{a}{\sqrt{3}}$$

$$\frac{a}{\sqrt{3}} = 9$$

$$a = 9\sqrt{3}$$

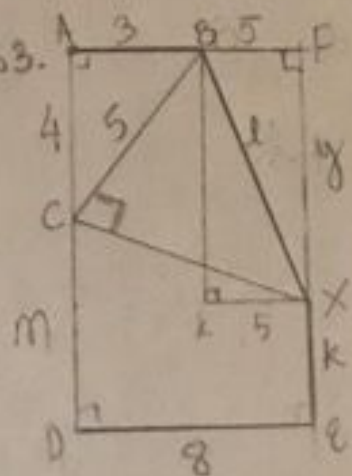
$$[\Delta ABE] = \frac{9\sqrt{3}}{4}$$

$$[\Delta ABE] = \frac{(9\sqrt{3})^2\sqrt{3}}{4}$$

$$[\Delta ABE] = \frac{81 \cdot 3\sqrt{3}}{4}$$

$$[\Delta ABE] = \frac{243\sqrt{3}}{4}$$

363.



$$A(0,0)$$

$$B(3,0)$$

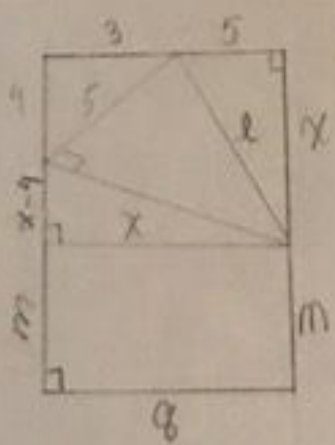
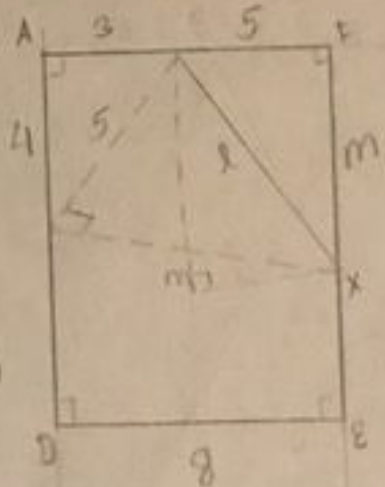
$$C(8,0)$$

$$D(0,m)$$

$$E(8,4+m)$$

$$X(8,4+m)$$

$$Y(3,-(4+m))$$



$$(x+m) \cdot 8 = \left(\frac{8+x}{2}\right)x + \left(\frac{8+x}{2}\right)$$

$$\therefore x = 8$$

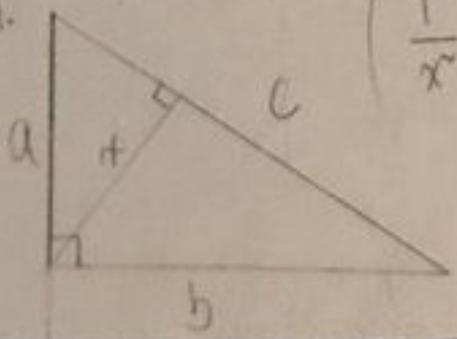
$$5^2 + 8^2 = l^2$$

$$25 + 64 = l^2$$

$$89 = l^2$$

$$l = \sqrt{89}$$

364.



$$\left(\frac{1}{x} = \frac{1}{a} + \frac{1}{b}\right) a^2 b^2 x^2$$

$$a^2 b^2 = a^2 x + b^2 x$$

$$a^2 b^2 = x^2 (a^2 + b^2)$$

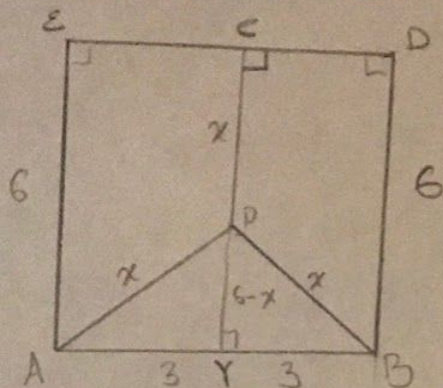
$$a^2 b^2 = x^2 c^2$$

$$ab = xc$$

$$\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$$

$$\frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$

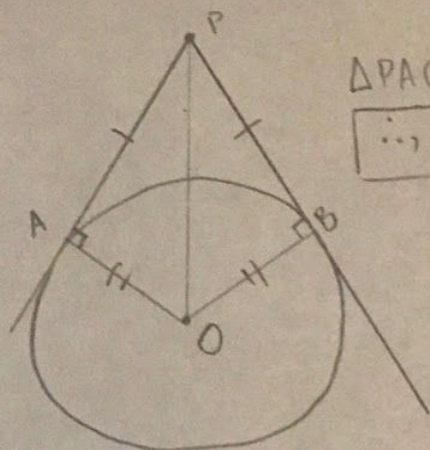
365.



$$\begin{aligned} (3)^2 + (6-x)^2 &= x^2 & (6-x)(6-x) \\ 9 + 36 - 12x + x^2 &= x^2 & 36 - 12x + x^2 \\ 45 - 12x &= 0 \\ 45 &= 12x \\ 15 &= 4x \\ \frac{15}{4} &= x \end{aligned} \quad \left| \begin{aligned} [\Delta APB] &= \frac{1}{2} \cdot PY \cdot AB \\ &= \frac{1}{2} \cdot 6 \cdot (6 - \frac{15}{4}) \\ &= 3(6 - \frac{15}{4}) \\ &= 18 - \frac{45}{4} \end{aligned} \right.$$

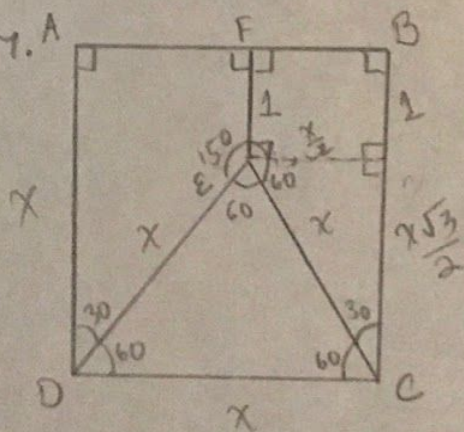
$$\frac{72}{27} = \frac{72}{4} - \frac{45}{4} = \boxed{\frac{27}{4}}$$

366.

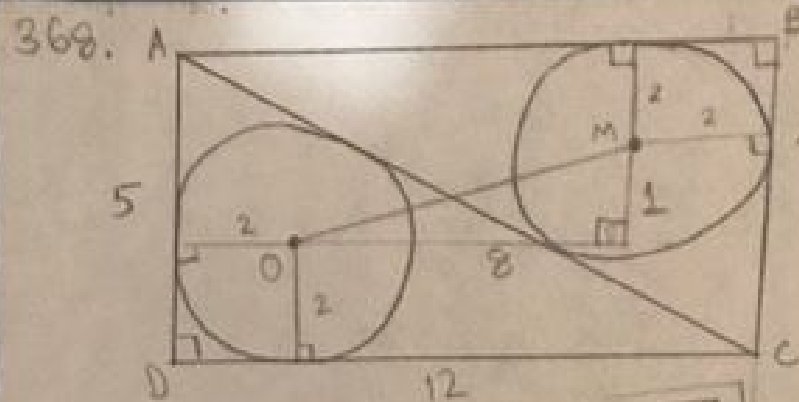


$\Delta PAO \cong \Delta PBO$ by HL \cong thm.
 $\therefore \angle POB \cong \angle POA = \frac{1}{2} \angle AOB$ by CPCTC

367.



$$\begin{aligned} 360 - (90 + 90 + 30) &= 360 - 210 = 150 \\ \frac{x}{2} = 1 &\Rightarrow x = 2 \\ \frac{x\sqrt{3}}{2} &= \frac{2\sqrt{3}}{2} = \sqrt{3} \end{aligned} \quad \left. \vphantom{\begin{aligned} 360 - (90 + 90 + 30) &= 360 - 210 = 150 \\ \frac{x}{2} = 1 &\Rightarrow x = 2 \\ \frac{x\sqrt{3}}{2} &= \frac{2\sqrt{3}}{2} = \sqrt{3} \end{aligned}} \right\} DC = BC = \boxed{2 + \sqrt{3}}$$

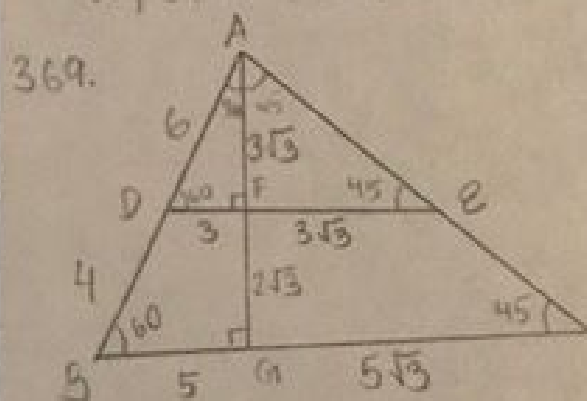


$$[\triangle ACD] = \frac{1}{2} \cdot 12 \cdot 5 = 30$$

$$A = 15 \quad \left| \begin{array}{l} 9 = \frac{5+12+13}{2} \\ 30 = 15r \\ 2 = r \end{array} \right. \quad \left| \begin{array}{l} 5 = \frac{30}{2} = 15 \\ 12 - 4 = 8 \\ 5 - 4 = 1 \end{array} \right.$$

$$\therefore OM^2 = 8^2 + 1^2 \Rightarrow OM = \sqrt{65}$$

$$\begin{array}{r} 675 \\ 27 \\ \hline 48 \end{array}$$



$$[FECG] = [\triangle AGC] - [\triangle AFE]$$

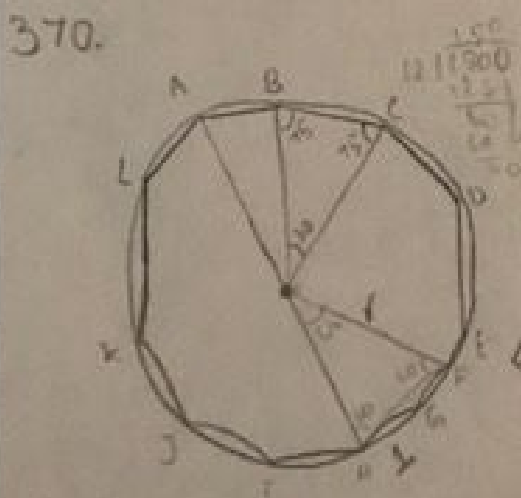
$$[FECG] = \left(\frac{1}{2} \cdot 5\sqrt{3} \cdot 5\sqrt{3} \right) - \left(\frac{1}{2} \cdot 3\sqrt{3} \cdot 3\sqrt{3} \right)$$

$$[FECG] = \left(\frac{1}{2} \cdot (5\sqrt{3})^2 \right) - \left(\frac{1}{2} \cdot (3\sqrt{3})^2 \right)$$

$$[FECG] = \left(\frac{1}{2} \cdot 25 \cdot 3 \right) - \left(\frac{1}{2} \cdot 9 \cdot 3 \right)$$

$$[FECG] = \left(\frac{1}{2} \cdot 75 \right) - \left(\frac{1}{2} \cdot 27 \right)$$

$$[FECG] = \frac{1}{2} (75 - 27) = \frac{1}{2} (48) = \underline{24}$$



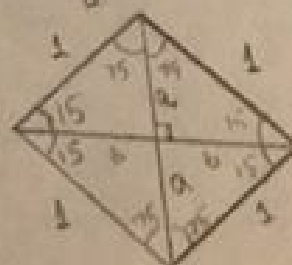
$$\frac{180(12-2)}{12} = \frac{180 \cdot 10}{12} = \frac{1800}{12} = 150$$

$$\frac{360}{12} = 30$$

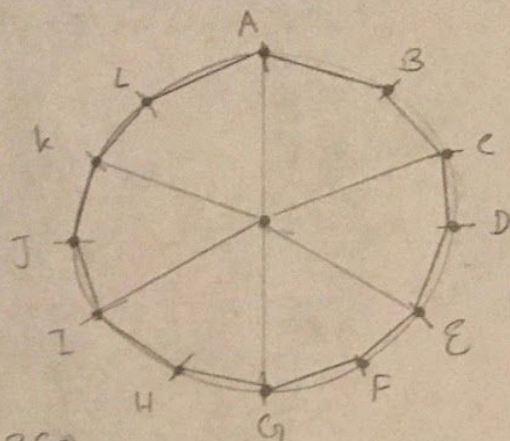
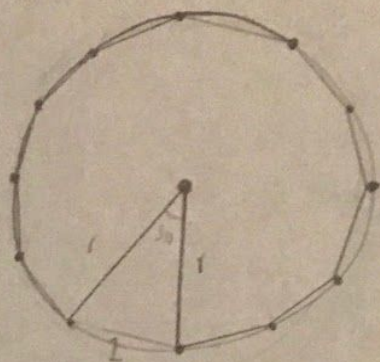
$$\frac{180-30}{2} = \frac{150}{2} = 75$$

1. Find area of sector.
2. Sum of area of all sectors.
3. $\frac{A}{a} = \frac{r}{r}$

$$[AB \dots L] = 6 \left(\frac{r^2 \sqrt{3}}{4} \right) + \pi r^2$$



370.



$$\frac{nlr}{2} = [AB \dots L]$$

$$\frac{360}{12} = 30^\circ$$

$$nlr = 2[AB \dots L]$$

$$r = \frac{2[AB \dots L]}{nl} = \frac{2[AB \dots L]}{12} = \frac{[AB \dots L]}{6}$$

$$12 \left(\sqrt{3(s-a)(s-b)(s-c)} \right) \Rightarrow S = \frac{1+1+1}{2} = \frac{2+1}{2}$$

$$12 \left(\sqrt{\frac{2r+1}{2} \left(\frac{2r+1}{2} - 1 \right) \left(\frac{2r+1}{2} - 1 \right)} \right) = \frac{12 \cdot 1 \cdot r}{2}$$

$$24 \sqrt{\frac{2r+1}{2} \left(\frac{2r+1}{2} - 1 \right) \left(\frac{2r+1}{2} - 1 \right)} = 12r$$

$$2 \sqrt{\frac{2r+1}{2} \left(\frac{2r+1}{2} - 1 \right) \left(\frac{2r+1}{2} - 1 \right)} = r$$

$$2 \sqrt{\frac{2r+1}{2} \left(\frac{2r+1}{2} - \frac{2r}{2} \right) \left(\frac{2r+1}{2} - \frac{2}{2} \right)} = r$$

$$\left(2 \sqrt{\frac{2r+1}{2} \left(\frac{1}{2} \right) \left(\frac{2r-1}{2} \right)} = r \right)^2$$

$$4 \left(\frac{(2r+1) \cdot 2 \cdot (2r-1)}{2^3} \right) = r^2$$

$$4 \left(\frac{(2r+1)(2r-1) \cdot 2}{8} \right) = r^2 \quad 2 \left(\frac{1}{\sqrt{2}} \right) = d$$

$$\frac{4r^2 - 1}{2} = r^2$$

$$4r^2 - 1 = 2r^2$$

$$-1 = -2r^2$$

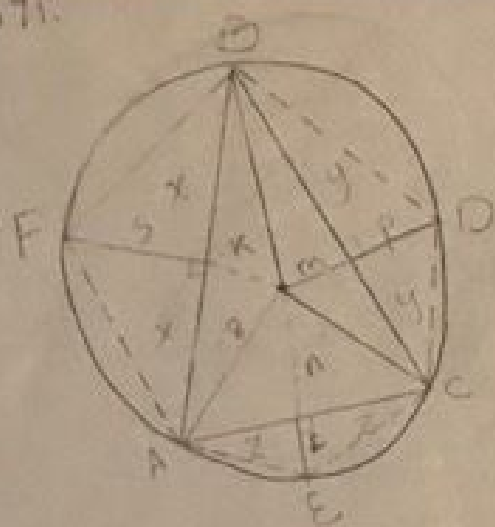
$$1 = 2r^2$$

$$\frac{1}{2} = r^2 \Rightarrow \sqrt{\frac{1}{2}} = r$$

$$\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = d$$

$$\boxed{\sqrt{2} = d}$$

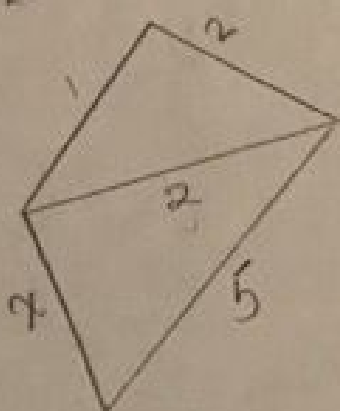
371.



$$\begin{aligned}
 AB + BC + AC + 2(x + y + z) &= 35 \\
 [AECDBF] &= [\triangle ABC] + [\triangle FAB] \\
 &+ [\triangle BDC] + [\triangle ACE] \\
 &= \frac{1}{2} \cdot AB \cdot k + \frac{1}{2} \cdot AC \cdot n + \frac{1}{2} \cdot BC \cdot m \\
 &+ \frac{1}{2} \cdot AB \cdot s + \frac{1}{2} \cdot AC \cdot t + \frac{1}{2} \cdot BC \cdot p \\
 &= \frac{1}{2} AB(k + s) + \frac{1}{2} AC(n + t) + \frac{1}{2} BC(m + p)
 \end{aligned}$$

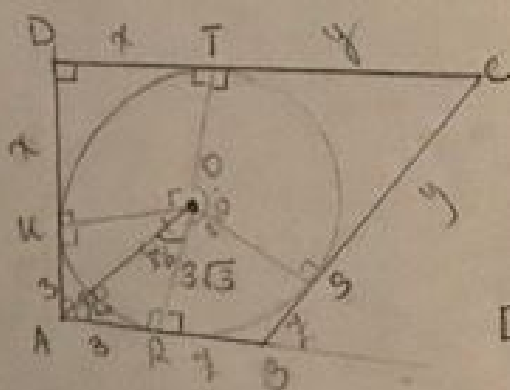
$$= \frac{1}{2} AB \cdot 8 + \frac{1}{2} AC \cdot 8 + \frac{1}{2} BC \cdot 8 = 4(35) = \boxed{140}$$

372.



$$\begin{array}{l|l}
 |2-1| < y < 2+1 & |5-2| < x < 5+2 \\
 1 < y < 3 & 3 < x < 7 \\
 y = 2 & x = 4, 5, 6 \\
 \hline
 & \boxed{15}
 \end{array}$$

373

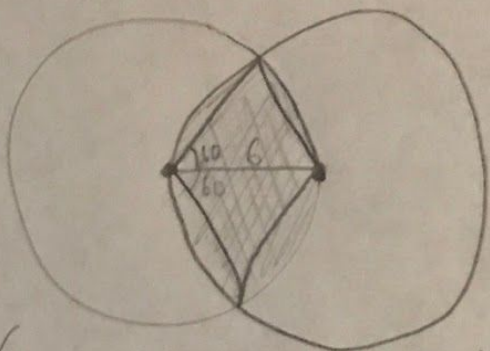


$$\begin{array}{r}
 360 \\
 -210 \\
 \hline
 150 \\
 -90 \\
 \hline
 60
 \end{array}$$

$$\begin{aligned}
 360 - (60 + 40 + 90) \\
 = 360 - (190) \\
 = 360 - 190 \\
 = 170
 \end{aligned}$$

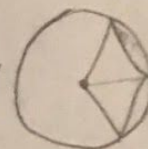
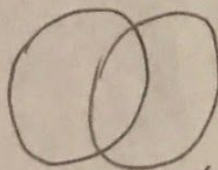
$$\begin{aligned}
 [OO] \cdot \pi r^2 &= (3\sqrt{3})^2 \pi \\
 &= \boxed{27\pi}
 \end{aligned}$$

374.



$$2\left(\frac{6\sqrt{3}}{4}\right) = \frac{6\sqrt{3}}{2} = \frac{36\sqrt{3}}{2} = 18\sqrt{3}$$

$$\begin{array}{r} 672 \\ 18 \\ \hline 54 \end{array}$$

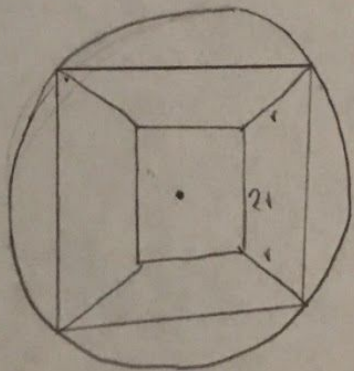
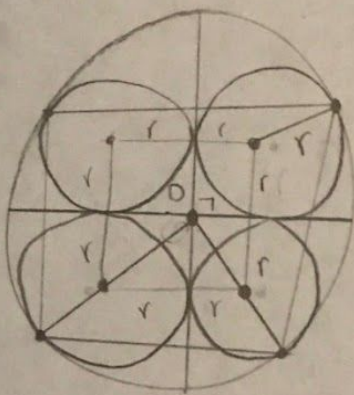
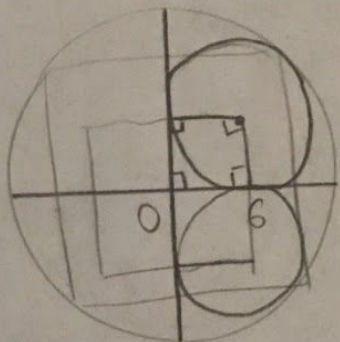


$$\left(\left(\frac{120}{360} \cdot \pi r^2\right) - (18\sqrt{3})\right) 4 + 18\sqrt{3} = \left(\left(\frac{\pi \cdot 36}{3}\right) - 18\sqrt{3}\right) 4$$

$$= (12\pi - 18\sqrt{3}) 4 + 18\sqrt{3} = 48\pi - 72\sqrt{3} + 18\sqrt{3}$$

$$= \boxed{48\pi - 54\sqrt{3}}$$

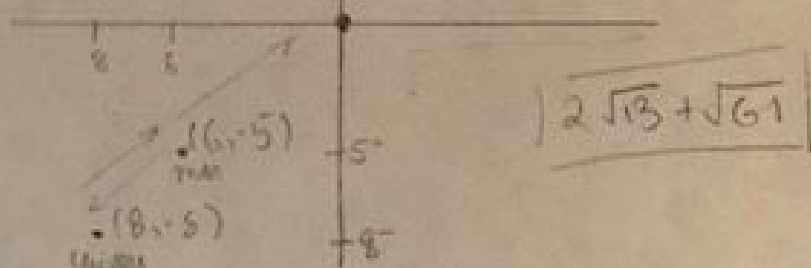
375.



$$2r\sqrt{2} + r + r = 2r\sqrt{2} + 2r = 12$$

$$r(2\sqrt{2} + 2) = 12$$

$$r = \frac{12}{2\sqrt{2} + 2} = \boxed{\frac{6}{1+\sqrt{2}}}$$

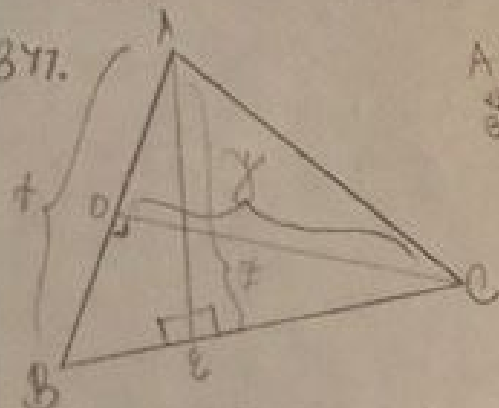
$$\frac{\sqrt{(-8+5)^2 + (-6-6)^2}}{\sqrt{13}} = \frac{\sqrt{(6)^2 + (-12)^2}}{\sqrt{61}}$$


AE, AB, CD are known

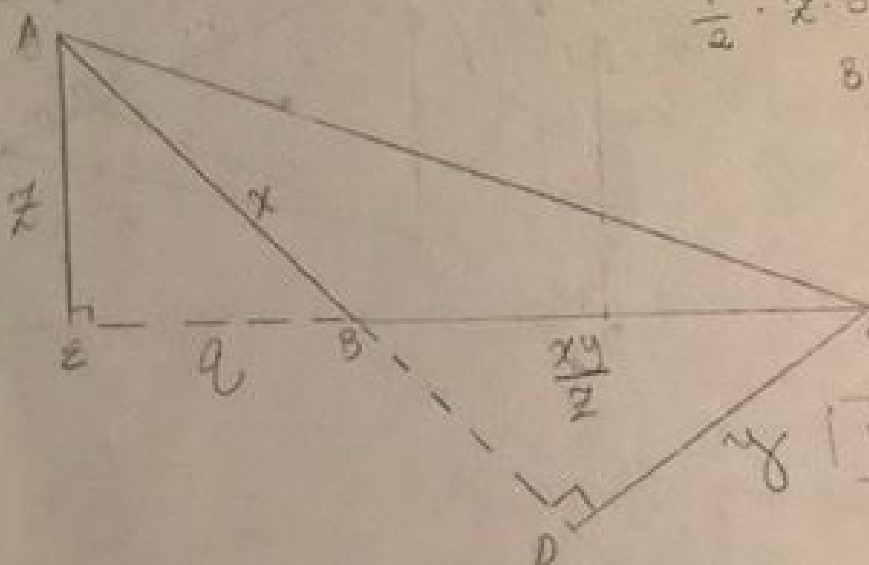
$\triangle BEC$ is also known, BC is also known,
 EC is also known

$$\frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot BC \cdot AE$$

$$\frac{DB^2 = CB^2 - CD^2}{DB = \sqrt{CB^2 - CD^2}} \quad \therefore \text{if 2 CB are acute}$$



10) LG is obtuse



$$\frac{1}{2} \cdot 2 \cdot 30 = \frac{1}{2} \cdot 2 \cdot 8$$

86-2-2-3

50. 23

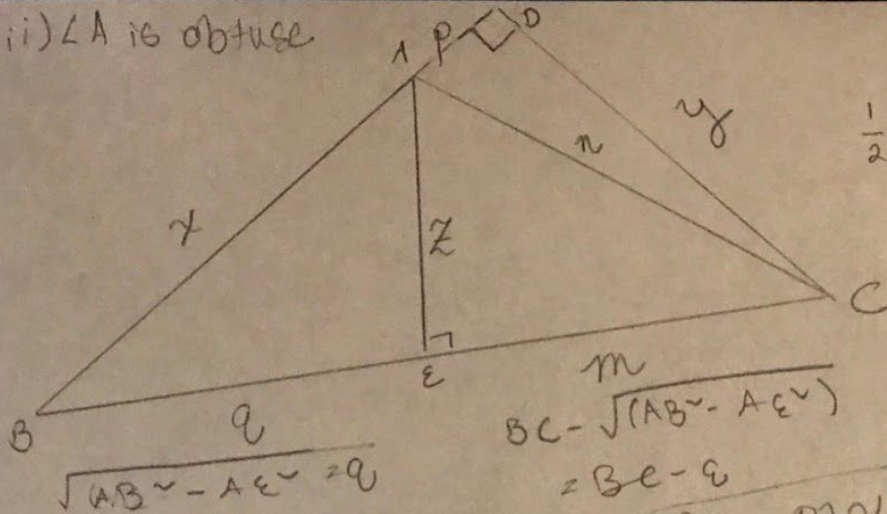
22

$$b) \sim: Be^{\sim} - Dc^{\sim}$$

$$30 \cdot \sqrt{\left(\frac{xy}{2}\right)^2 - (y^2)^2}$$

$$BD = \sqrt{\left(\frac{r_2}{2} - y\right)^2}$$

ii) $\angle A$ is obtuse



$$\frac{1}{2} \cdot y \cdot x = \frac{1}{2} \cdot z \cdot BC$$

$$\sqrt{z^2 + (BC - \sqrt{(AB)^2 - (AE)^2})^2} = n$$

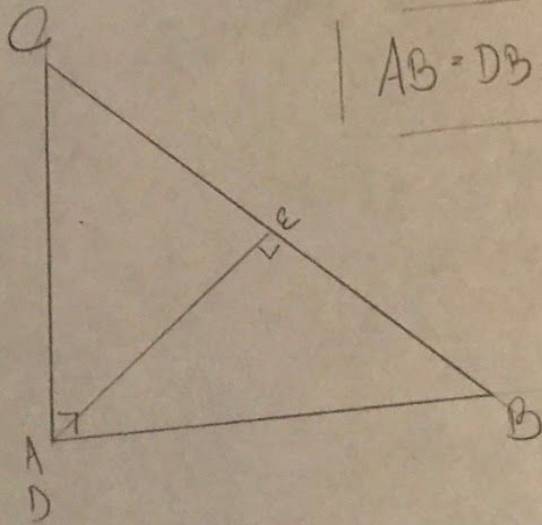
$$\sqrt{z^2 + (BC - e)^2} = n$$

$$\sqrt{n^2 - y^2} = AD$$

$$BC - \sqrt{(AB)^2 - (AE)^2} = BC - e$$

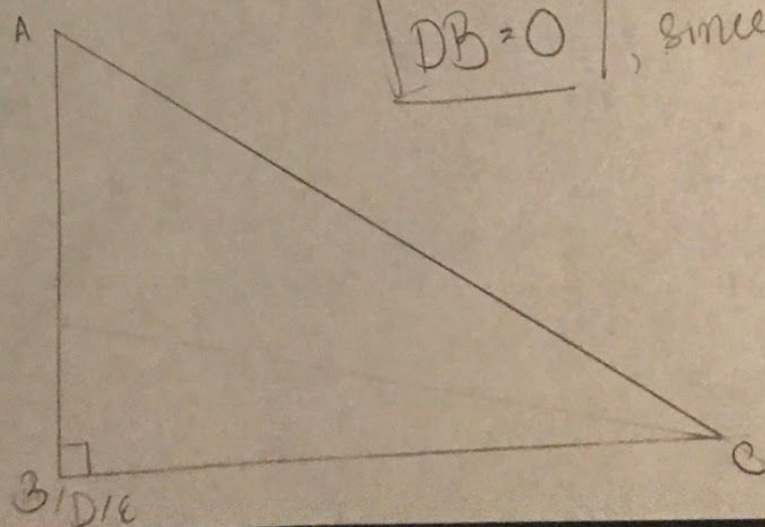
$$DB = p + x$$

iii) $\angle A = 90^\circ$



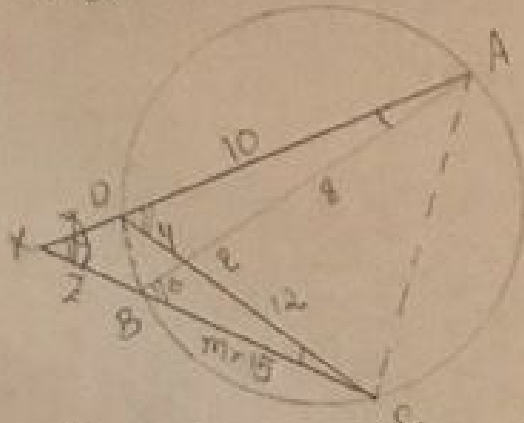
$$AB = DB, \therefore DB \text{ is known as a given}$$

iv) $\angle B = 90^\circ$



$$DB = 0, \text{ since } B = D = E$$

378.



$$AE \cdot EB = DE \cdot EC$$

$$8 \cdot 6 = 12 \cdot x$$

$$48 = 12x$$

$$4 = x$$

$\triangle DEC \sim \triangle BDA$

$$\frac{AD}{DE} = \frac{BE}{EC}$$

$$\frac{10}{4} = \frac{6}{12}$$

$$2 \cdot 16 = 3 \cdot 14$$

$$16x = 14y$$

$\triangle BEC \sim \triangle DEA$

$$\frac{BC}{DE} = \frac{EC}{EA}$$

$$\frac{m}{10} = \frac{6}{4}$$

$$8x = 7y$$

$$x = \frac{7}{8}y$$

$$x \cdot D \cdot xA = xB \cdot xC$$

$$y \cdot (10+y) = 2(x+m)$$

$$4m = 60 \Rightarrow m = 15$$

$$10y + y = x + 15x$$

$$10y + y = \left(\frac{7}{8}y\right) + 15\left(\frac{7}{8}y\right)$$

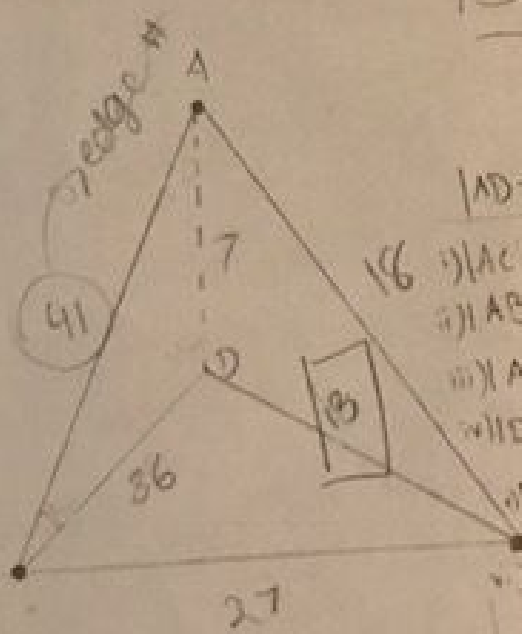
$$y + 10y = \frac{49}{64}y + \frac{105}{8}y$$

$$y + 10 = \frac{49}{64}y + \frac{105}{8}$$

$$\frac{64-49}{64}y = \frac{105-80}{8} \Rightarrow y = \frac{105-80}{8} \cdot \frac{64}{64-49} = \frac{25}{8} \cdot 64 = \frac{200}{15} = \frac{40}{3}$$

$$Ax = AD + DX = 10 + \frac{40}{3} = \frac{70}{3}$$

379.



7, 13, 18, 27, 36

$$|AD - DB| < 41 < AD + DB$$

$$|AC - DC| < AD < AC + DC$$

$$|AB - BD| < AD < AB + BD$$

$$|AB - AD| < BD < AB + AD$$

$$|DC - BC| < BD < DC + BC$$

$$|AC - AD| < DC < AC + AD$$

$$|BC - BD| < DC < BC + BD$$

$$(AD, DB) \neq$$

$$(7, 13) \cdot 6$$

$$(7, 18) \cdot 11$$

$$(7, 27) \cdot 20$$

$$(13, 18) \cdot 5$$

$$(13, 27) \cdot 9$$

$$(7, 36) \cdot 29$$

$$(13, 36) \cdot 23$$

$$(27, 36) \cdot 9$$

$$(18, 27) \cdot 9$$

$$(18, 36) \cdot 18$$

$$i) 6 < 7 < 43$$

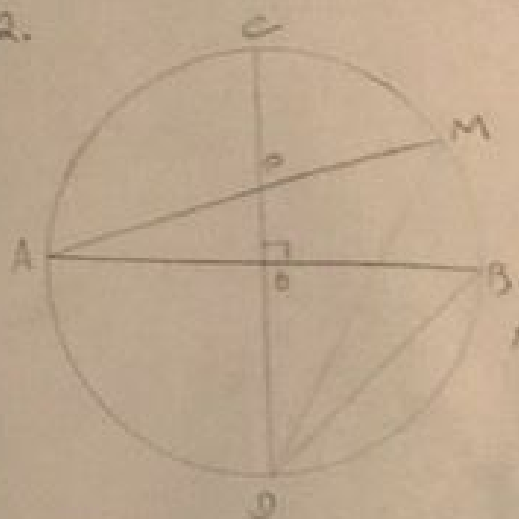
$$ii) 34 < 7 < 48$$

$$v) 11 < 7 < 25$$

$$vi) 9 < 7 < 63$$

$$iii) 14 < 7 < 40$$

382.



$$\begin{aligned}
 AP \cdot AM &= AP(AP + PM) \quad \text{--- } AO \cdot OB \\
 AP \cdot AM &\stackrel{?}{=} AO^2 & \text{--- } i) AO \cdot AB &= AO \cdot AO + AO^2 \\
 AP^2 + AP \cdot PM & & \text{--- } ii) CP \cdot CD &= CP(CP + PD) \\
 & & \text{--- } iii) CP \cdot CD &= CO^2 + CP \cdot PD \\
 AO^2 &= OP^2 + CP \cdot PD & \text{--- } iv) CO \cdot OD &= (CO + OP) \cdot OP \\
 & & &= OP^2 + OP \cdot CP \\
 AO^2 + OP^2 + CP \cdot PD &= AO^2 \\
 CP^2 + CP \cdot PD &= AO^2 - AO^2 \\
 OP^2 + CP \cdot PD &= AO^2 (AO - 1) \\
 OP^2 + CP(OP + OD) & \\
 OP^2 + OP \cdot CP + OD \cdot CP &
 \end{aligned}$$

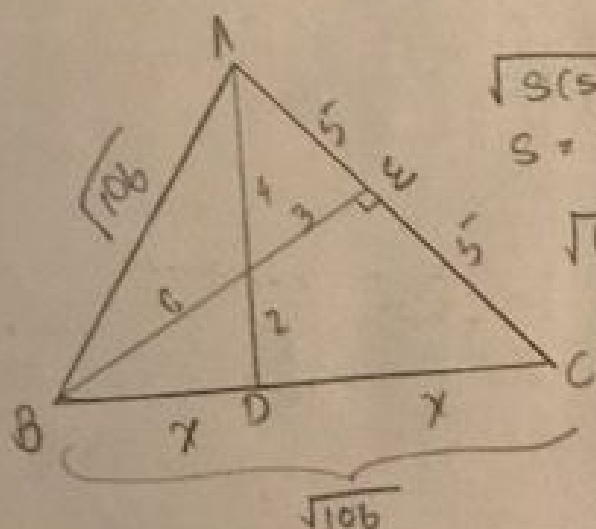
$$OP(OP + CP) + CP \cdot OD \rightarrow OP \cdot CO + OD \cdot CP$$

$$AO \cdot OB = CO \cdot OD = OD(CP + OP) = OD \cdot CP + OP \cdot OD$$

$$AP \cdot PM = CP \cdot PD = CP(OD + OP) = OD \cdot CP + OP \cdot CP$$

$$AO \cdot AB = AO(AO + OB) = AO^2 + AO \cdot OB = AO^2 + OD \cdot CP + OP \cdot OD + OD \cdot CP + OP \cdot CP$$

383.



$$\sqrt{s(s-a)(s-b)(s-c)} = [\Delta ABC]$$

$$s = \frac{2\sqrt{106} + 10}{2} = \sqrt{106} + 5$$

$$\begin{aligned}
 &\sqrt{(\sqrt{106} + 5)(\sqrt{106} + 5 - 10)} \\
 &(\sqrt{106} + 5 - \sqrt{106})(\sqrt{106} + 5 - \sqrt{106}) \\
 &= \sqrt{(\sqrt{106} + 5)(\sqrt{106} - 5)(5)}
 \end{aligned}$$

$$= \sqrt{(\sqrt{106} + 5)(\sqrt{106} - 5)(5)}$$

$$= \sqrt{((\sqrt{106})^2 - 5^2) \cdot 25}$$

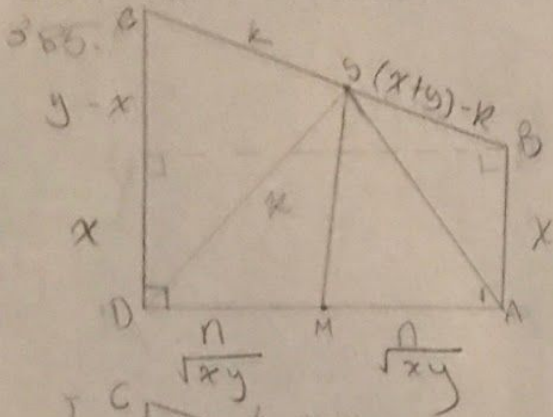
$$= \sqrt{(106 - 25) \cdot 25}$$

$$= \sqrt{81 \cdot 25}$$

$$= \sqrt{2025}$$

$$= \boxed{45}$$

384. Shown in blackboard



$$(y-x)^2 + (2n)^2 = (x+y)^2$$

$$y^2 - 2xy + x^2 + 4n^2 = x^2 + 2xy + y^2$$

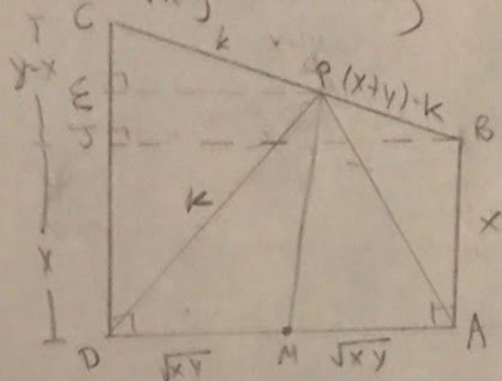
$$4n^2 = 4xy$$

$$n^2 = xy$$

show $SM = n$, or

$$SD^2 + SA^2 = 4xy = 4n^2$$

$$(2\sqrt{xy})^2 = 4xy$$



$$(k)^2 - \left(\frac{y}{2}\right)^2 = PE^2$$

$$k^2 - \frac{y^2}{4} = PE^2$$

$$CP = k$$

$$JB = 2\sqrt{xy}$$

$$CB = x+y$$

$$PE = \sqrt{k^2 - \frac{y^2}{4}}$$

$$CJ = \frac{y}{2}$$

$$(y-x = \frac{y}{2})^2$$

$$2y - 2x = y$$

$$-2x = -y$$

$$2x = y$$

$\triangle CEP \sim \triangle CJB$ by AA

$$\frac{CE}{CJ} = \frac{PE}{BJ} \Rightarrow CE \cdot 2\sqrt{xy} = \left(\sqrt{k^2 - \frac{y^2}{4}}\right) \left(\frac{y}{2}\right)$$

$$CE = \frac{\left(\sqrt{\frac{4k^2 - y^2}{4}}\right) \left(\frac{y}{2}\right)}{2\sqrt{xy}} = \frac{\left(\frac{\sqrt{4k^2 - (2x)^2}}{2} \cdot \frac{y}{2}\right)}{2\sqrt{x} \cdot 2x} = \frac{y\sqrt{4k^2 - 4x^2}}{4 \cdot 2\sqrt{2x} \cdot 4}$$

$$CE = \frac{y\sqrt{4k^2 - 4x^2}}{8x\sqrt{2}}$$

$$\frac{CP}{CB} = \frac{PE}{BJ} \Rightarrow \frac{k}{x+y} = \frac{PE}{2x\sqrt{2}}$$

$$2xk\sqrt{2} = (x+y)PE$$

$$2xk\sqrt{2} = (x+2x)PE$$

$$2xk\sqrt{2} = 3xPE$$

$$2k\sqrt{2} = 3 \cdot \sqrt{4(k^2 - x^2)}$$

$$(2k\sqrt{2} = 3 \cdot 2\sqrt{k^2 - x^2})^2 \Rightarrow 4k^2 \cdot 2 = 6(k^2 - x^2)$$

$$(2k\sqrt{2})^2 = (3\sqrt{4k^2 - 4x^2})^2$$

$$4k^2 \cdot 2 = 9(4k^2 - 4x^2)$$

$$8k^2 = 36k^2 - 36x^2$$

$$-28k^2 = -36x^2$$

$$28k^2 = 36x^2$$

$$k\sqrt{28} = 6x$$

$$2k\sqrt{7} = 6x$$

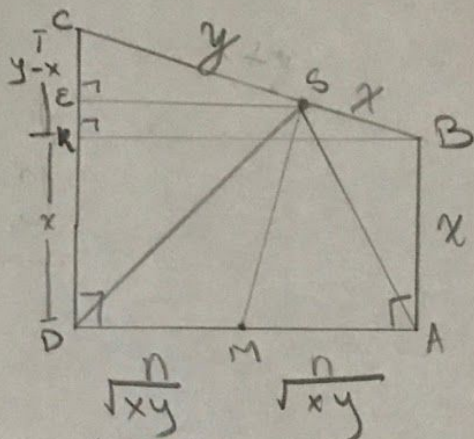
$$8k^2 = 6k^2 - 6x^2$$

$$2k^2 = 6x^2$$

$$k^2 = 3x^2$$

NOT PLausible

385.


 $\triangle CES \sim \triangle KCB$ by AA

$$\frac{CS}{CB} = \frac{ES}{KB} \Rightarrow CS \cdot KB = CB \cdot ES$$

$$y(2\sqrt{xy}) = (y+x)(\sqrt{2yx-x^2})$$

$$(2y\sqrt{xy} = x+y(\sqrt{2yx-x^2}))^2$$

$$(2y)^2 \cdot (\sqrt{xy})^2 = (x+y)^2 \cdot (\sqrt{2yx-x^2})^2 \Rightarrow 4y^2 \cdot xy = (x^2+2xy+y^2)(2yx-x^2)$$

$$4y^3x = 2yx \cdot x^2 - x^4 + 2xy \cdot 2xy - x^2 \cdot 2xy + 2y^3x - x^2y^2$$

$$4y^3x = 2yx^3 - x^4 + 4x^2y^2 - 2x^3y + 2y^3x - x^2y^2$$

$$4xy^3 = 2x^3y - x^4 + 4x^2y^2 - 2x^3y + 2xy^3 - x^2y^2$$

$$2xy^3 = 3x^2y^2 - x^4$$

$$2y^3 = 3xy^2 - x^3$$

$$2y^3 - 3xy^2 = -x^3$$

$$3xy^2 - 2y^3 = x^3$$

$$y^2(3x-2y) = x^3$$

$$2y^3 = x(3y^2 - x^2)$$

$$2y^3 = x(3y+x)(3y-x)$$

$$-x^4 + 3x^2y^2 - 2xy^3 = 0$$

$$x^4 - 3x^2y^2 + 2xy^3 = 0$$

$$\frac{CE}{CK} = \frac{CS}{CB} \Rightarrow CE \cdot CB = CS \cdot CK$$

$$CE = \frac{CS \cdot CK}{CB} = \frac{y \cdot (y-x)}{y+x} = \frac{y^2 - yx}{y+x}$$

$$CK^2 + BK^2 = BE^2$$

$$(y-x)^2 + (2n)^2 = (y+x)^2$$

$$(2n)^2 = (y+x)^2 - (y-x)^2$$

$$4n^2 = y^2 + 2yx + x^2 - (y^2 - 2yx + x^2)$$

$$4n^2 = y^2 + 2yx + x^2 - y^2 + 2yx - x^2$$

$$4n^2 = 4yx$$

$$n^2 = xy \Rightarrow n = \sqrt{xy}$$

$$SE^2 + CE^2 = ES^2$$

$$SE^2 = ES^2 - CE^2$$

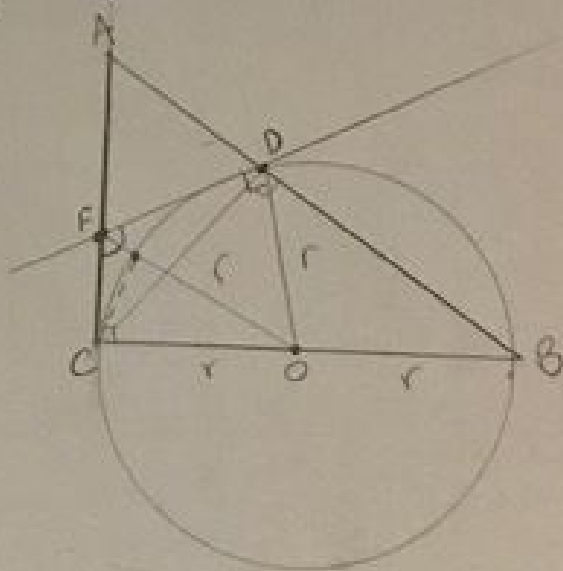
$$SE^2 = y^2 - (y-x)^2$$

$$SE = \sqrt{y^2 - (y^2 - 2yx + x^2)}$$

$$SE = \sqrt{y^2 - y^2 + 2yx - x^2}$$

$$SE = \sqrt{2yx - x^2}$$

286-

 $\angle COF \cong m$ $\triangle FOC \cong \triangle FOD$ by HL $\therefore \angle OFC = \angle OFD$ $\therefore \text{COF \& LDOF by CPLTC}$
$$LCBD = \frac{1}{2} \widehat{CD} \Rightarrow \frac{1}{2} \cdot 2 \cdot LCOP = \frac{1}{2} \cdot 2m$$
$$\therefore \angle COF = \angle OBC$$
$$\therefore \triangle GOC \sim \triangle ABE \text{ by AA}$$

$\angle ACB \neq \angle ACD$

2007 LDBE

$$\frac{OC}{BC} = \frac{FC}{AC} \Rightarrow \frac{r}{21} = \frac{FC}{AC}$$

ii) $DF = FC = AE$

$$\therefore FC = AF$$

i) $\frac{FC}{AC} = \frac{1}{2} \Rightarrow \therefore, \overline{DF}$ bisects \overline{AC} .

iii) $\triangle ABC \sim \triangle DBC, \therefore \angle A = \angle BCD$

$$iv) \text{MLCFD} = \frac{DDE - DFC}{2}$$

iv) $\Delta COF \neq \Delta FOD$ by $\Delta H_1 \neq$

$$\therefore, LCTO = LOFO \text{ by edetc}$$
$$\triangle FGC \sim \triangle ABC \text{ by } AA \sim$$
 $\therefore \text{LCF-O}^{\text{red}} \text{LA}$
$$\therefore, 2\text{LEFO} \Rightarrow \text{LEFO}$$

$$\therefore, LCPD = 2 \text{ mL A}$$

$$MLA = \frac{BC}{2}$$

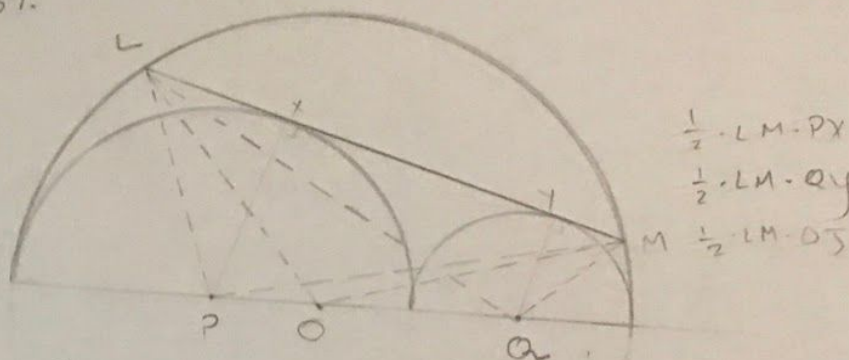
$$m\angle CFD = \widehat{DB} + \widehat{BC} - \widehat{DC}$$

$$\frac{\widehat{DB}}{0.2} = \frac{\widehat{DFC} + \widehat{BCE}}{2} = 2 \left(\frac{\widehat{BCE}}{2} \right)$$

$$\widehat{DB} - \widehat{DFC} + \widehat{BC} = \widehat{BC}$$

$$\overline{DB} = \sqrt{2} \overline{FC} = \overline{BC}$$

387.

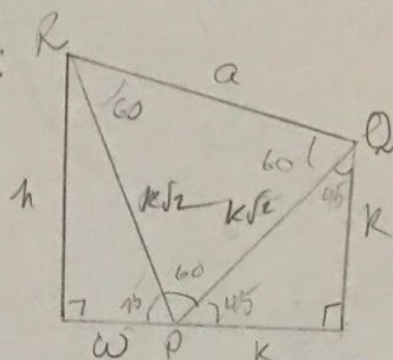


$$\frac{1}{2} \cdot LM \cdot PX$$

$$\frac{1}{2} \cdot LM \cdot QY$$

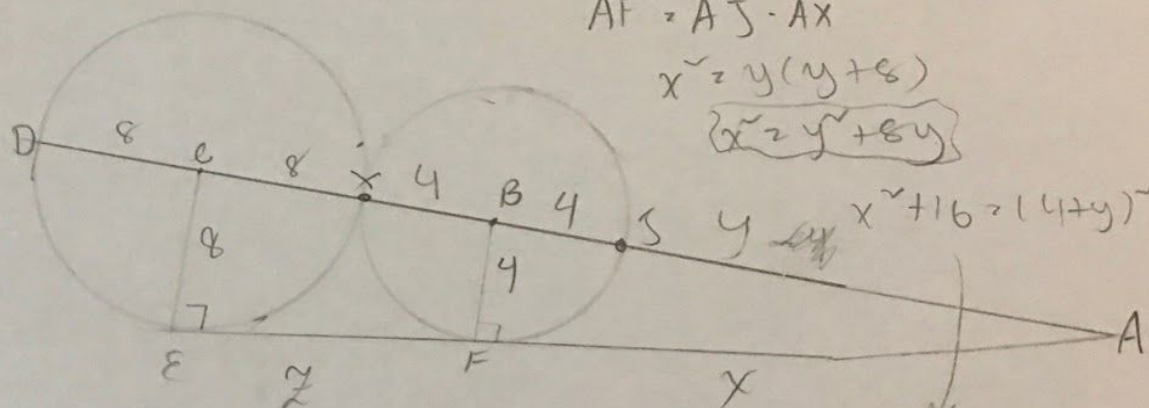
$$M \frac{1}{2} \cdot LM \cdot OJ$$

388.



$$W = \sqrt{h^2 - 2k^2}$$

389



$$AF^2 = AX \cdot AX$$

$$x^2 = y(y+8)$$

$$\{x^2 = y^2 + 8y\}$$

$$x^2 + 16 = (4+y)^2$$

$$x^2 = y(y+8) \Rightarrow x^2 = y^2 + 8y$$

$$x^2 + 16 = y^2$$

$$x^2 = x^2 + 16 + 4y$$

$$-16 = 4y$$

$$x^2 + 16 = 16 + 8y + y^2$$

$$x^2 = 8y + y^2$$

$$y^2 + 8y = 0$$

$$y^2 + 8y + x(2x+2) = y^2 + 32y + 192$$

$$2(2x+2) = 24y + 192$$

$$2\sqrt{y^2 + 8y} \cdot 2 + x^2 = 24y + 192$$

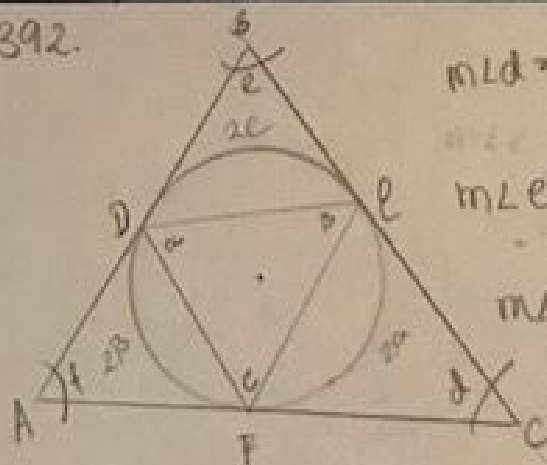
$$AE^2 = AX \cdot AD$$

$$(x+2)^2 = (y+8)(y+24)$$

$$x^2 + 2x + 2 = y^2 + 32y + 192$$

$$\frac{34}{102}$$

392.



$$m + d = \frac{(2\beta + 2\alpha) - 2a}{2} \Rightarrow 2m + d = (2\beta + 2\alpha) - 2a$$

$$2m + d - (2\beta + 2\alpha) = -2a$$

$$m + e = \frac{(2\beta + 2\alpha) - 2c}{2} \Rightarrow 2m + e = (2\beta + 2\alpha) - 2c$$

$$2m + e - (2\beta + 2\alpha) = -2c$$

$$m + f = \frac{(2\alpha + 2\epsilon) - 2b}{2} \Rightarrow 2m + f = (2\alpha + 2\epsilon) - 2b$$

$$2m + f - (2\alpha + 2\epsilon) = -2b$$

$$(2\beta + 2\epsilon) - 2m + d = 2a \Rightarrow a = \beta + \epsilon - m + d$$

$$(2\beta + 2\alpha) - 2m + e = 2c \Rightarrow c = \beta + \alpha - m + e$$

$$(2\alpha + 2\epsilon) - 2m + f = 2b \Rightarrow b = \alpha + \epsilon - m + f$$

$$a + b = 2\epsilon + \beta + \alpha - m + d - m + f$$

$$2c = m + d + m + f$$

$$c = \frac{m + d + m + f}{2}$$

2

$$a + c = 2\beta + \alpha + \epsilon - m + d - m + e$$

$$2b = m + d + m + e$$

$$b = \frac{m + d + m + e}{2}$$

2

$$\beta + \epsilon = 2a + \beta + \epsilon - m + e - m + f$$

$$2a = m + e + m + f$$

$$a = \frac{m + e + m + f}{2}$$

2

$$m + d + m + f < 180$$

$$m + d + m + e < 180$$

$$m + e + m + f < 180$$

$$a < \frac{180}{2}$$

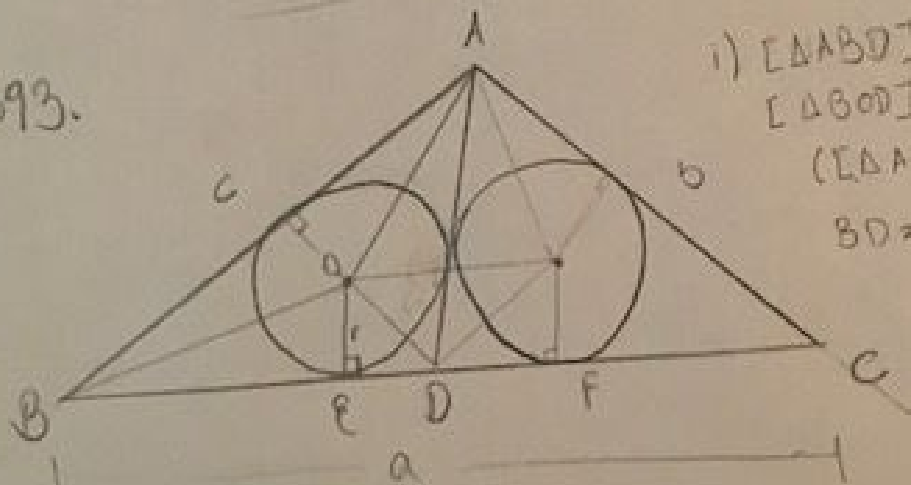
$$b < \frac{180}{2}$$

$$c < \frac{180}{2}$$

$\therefore \triangle DEF$ is acute



393.



$$1) [\triangle ABD] = rs$$

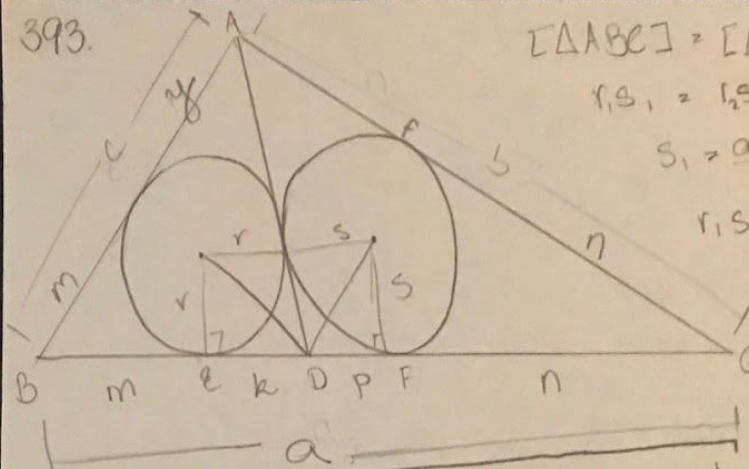
$$[\triangle BOD] = [\triangle ABC] -$$

$$([\triangle AOB] + [\triangle AOD])$$

$$BD = \frac{[\triangle ABC] - ([\triangle AOB] + [\triangle AOD])}{r}$$

r

393.



$$[\triangle ABC] = [\triangle ADC] + [\triangle ADB]$$

$$r_1 S_1 = r_2 S_2 + r_3 S_3$$

$$S_1 = \frac{a+b+c}{2}$$

$$r_1 S_1 = \sqrt{\left(\frac{a+b+c}{2}\right) \left(\frac{a+b+c}{2} - a\right) \left(\frac{a+b+c}{2} - b\right) \left(\frac{a+b+c}{2} - c\right)}$$

$$\text{let } AD = q$$

$$S_2 = \frac{b+n+p+q}{2}$$

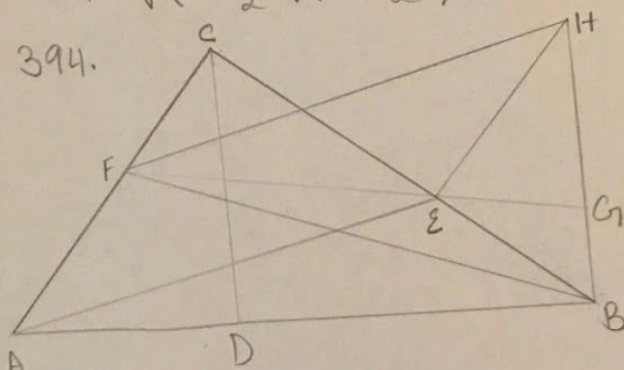
$$S_3 = \frac{c+m+k+q}{2}$$

$$r_2 S_2 = \sqrt{\left(\frac{b+n+p+q}{2}\right) \left(\frac{b+n+p+q}{2} - b\right) \left(\frac{b+n+p+q}{2} - q\right) \left(\frac{b+n+p+q}{2} - (n+p)\right)}$$

$$r_3 S_3 = \sqrt{\left(\frac{c+m+k+q}{2}\right) \left(\frac{c+m+k+q}{2} - c\right) \left(\frac{c+m+k+q}{2} - q\right) \left(\frac{c+m+k+q}{2} - (m+k)\right)}$$

$$r_1 S_1 = \sqrt{\left(\frac{a+b+c}{2}\right) \left(\frac{a+b+c}{2} - a\right) \left(\frac{a+b+c}{2} - b\right) \left(\frac{a+b+c}{2} - c\right)}$$

394.



$$EC = EB \quad FH = AE$$

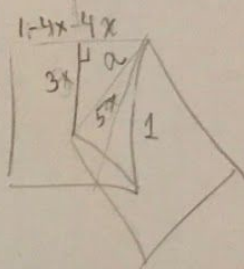
$$AF = CE \quad FH \parallel AE$$

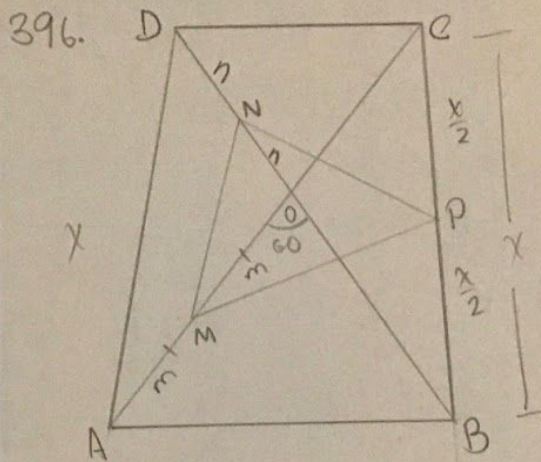
$$AD = BD$$

AEHF is a rhombus, b/c
 $FH \parallel AE$ & $FH = AE$.

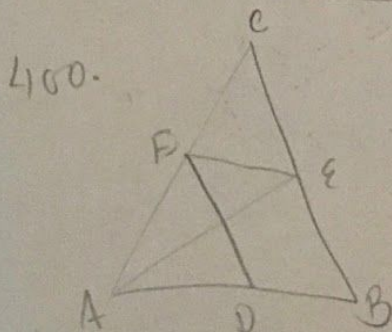
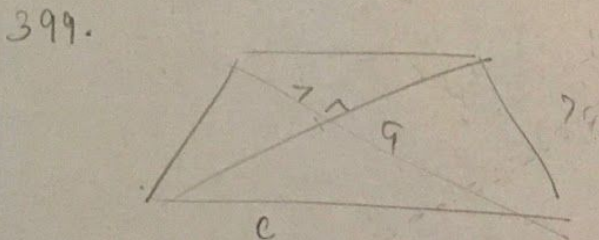
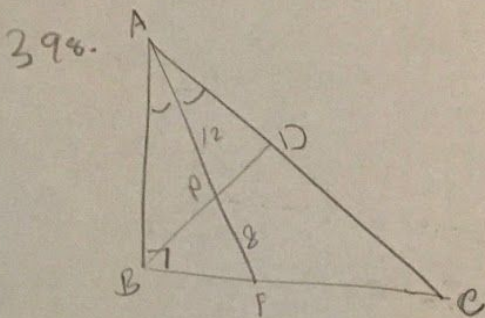
$$BH = DE$$

395.





397. $\frac{10}{2} = \frac{x}{5}$ $\therefore x = 25??$



401.

