The AMCs

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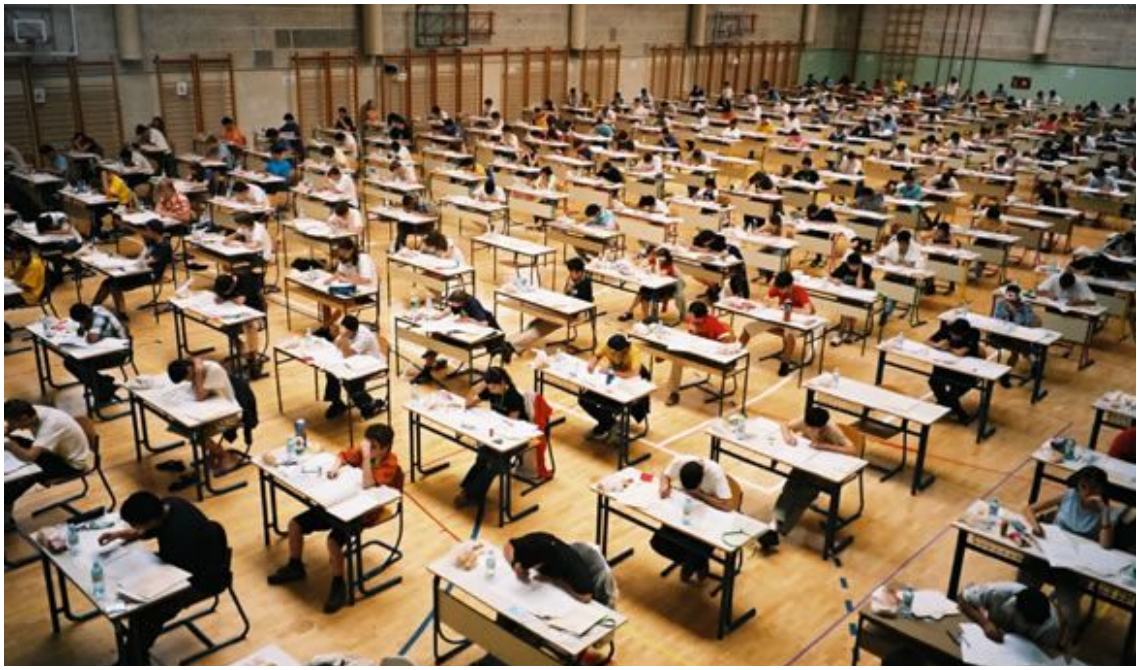


Caption: The International Mathematical Olympiad exam takes place in a different country each year. [1]

It's all headshots. All 25 of 'em. Only 5% pass. Ring any bells? Sound like a Clash of Clans ad? Wrong. It's the AMC: American Mathematical Competition; The first step towards the ultimate challenge: The International Math Olympiad, or IMO.

Hold up. What *is* the IMO? Exactly what it sounds like. It's the olympics. But with math. And the only muscle you'll be using is your brain. Think of it as the boss fight. But there's six levels before it, the first of which is the AMC.

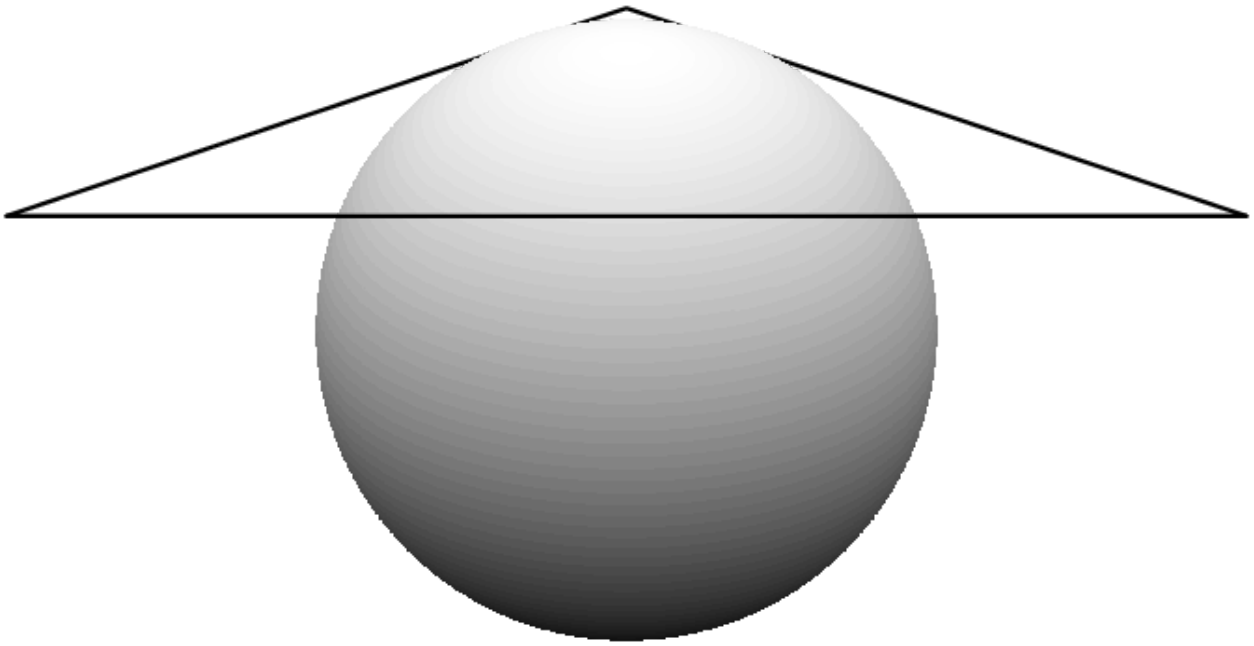
The AMC was given on February 15th, 2019 at Tech. Only two people made it to the next round, both of whom were seniors. It's not exactly the easiest test in the world. Here's some stats. There's four versions of the AMC given every year: the 10A, 10B, 12A, and 12B. The 10 means it's intended for 10th graders and under; same for 12. The average score was 51.66 for the 10A, and 58.42 for the 10B. For the record, I scored 74 on both, which is in the top 25% for the 10A, and top 20% for the 10B. Yep, not exactly something to write home about.

But! This is not why we do math. Math exists for the pain of the masses, and joy of the freaks. Here's a problem that tickled my funny bone:

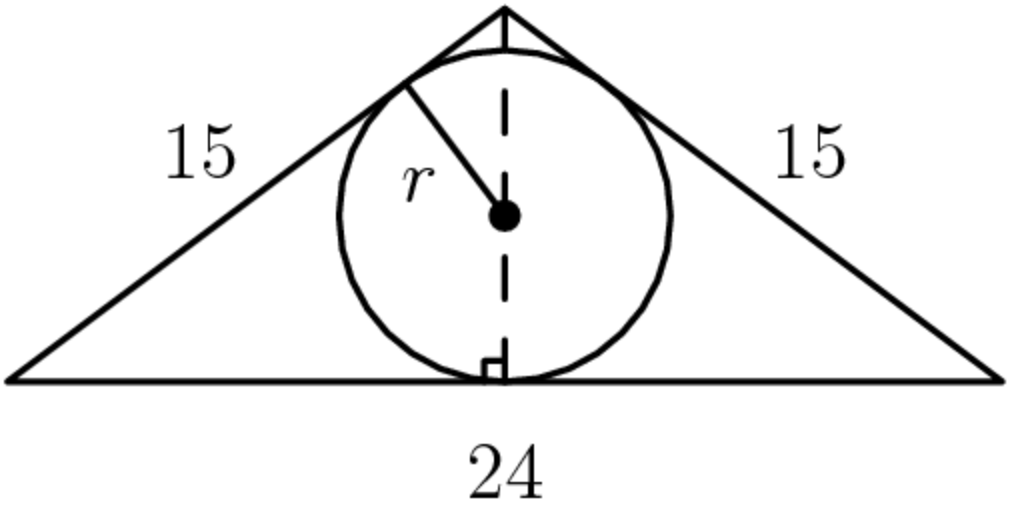
A sphere with center O has radius 6. A triangle with sides of length 15, 15, and 24 is situated in space so that each of its sides is tangent to the sphere. What is the distance between O and the plane determined by the triangle?

Ah, yes. 3D Geometry. How do I put it nicely? Communism is to AP Euro teachers, as 3D geometry is to math competitors. A niche joke? (it only applies to Sophomores taking both AP Euro, and math competitions) Yes. But I couldn't resist.

So, how do you tackle a beast of this nature? Well, as with all geometry problems, draw a diagram! A few rudimentary sketches work. No picasso needed. You should end up with something like:



Notice that we can take the cross-section created by the triangle and the sphere. Can you guess what that'll be? Yup, a circle! And we can use that to our advantage.



Since we're given all three sides, we can find the inradius of the circle using A=rs, where r is the inradius and s is the semiperimeter. The semiperimeter is -- you guess it -- half the perimeter of the triangle. And of course, you can find the area of the triangle trillions of ways; I just used good ol' 1/2bh, since we've got an isosceles triangle. You should end up getting an inradius of 4.

Now comes the tricky part (Wait, that wasn't it?); How can we use the inradius and the center of the sphere to find the distance from the center of the sphere to the triangle? It turns we can actually connect the sphere's center to the incenter (this will be perpendicular), and connect one of the vertices of the triangle to the sphere's center. We end up with a right triangle with a hypotenuse of 6 (the radius of the sphere), and a leg of length 4. We just need to find the third side. A little Pythagorean theorem gives you $2\sqrt{5}$!

And boom! You've just AMC'ed a problem! Already hooked? You can find more problems and solutions on *aops.com*.

Sources:

[1] https://www.maa.org/sites/default/files/images/2006imo.jpg
[2] https://latex.artofproblemsolving.com/4/9/d/49d18095565c52482472bb0f99152e97e368cc5a.png