

A Purple Comet?

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Caption: The Purple Comet is an annual international math competition open to middle schoolers and high schoolers. [1]

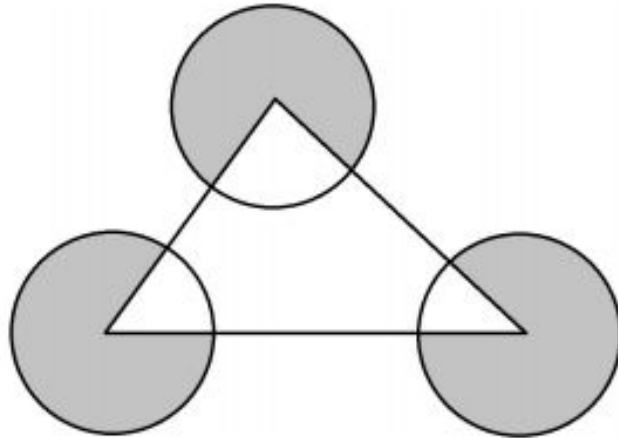
30 problems. 90 minutes. Easy enough, right? That is, until you realize no one got a perfect score (last year, at least). It's the PurpleComet, a math competition with problems ranging from algebra, to number theory, to geometry, and it's open for anyone from 9th to 12th grade.

The PurpleComet is a unique math competition, in that not only does it allow calculators, but encourages the usage of programming, and other offline tools to solve math problems, as well as your five other teammates. I participated in the Purple Comet last year (in my defense, I was an amateur), and scored a 3! That's right, 3 problems right out of 30. Like most competitions, the problems increase in difficulty as one progresses through the contest.

This year's PurpleComet has already begun, with the Freshman teams (coached by Mr. Matthews) kicking it off. I, for one, will participate in the PurpleComet hosted on Friday, April 5th. Let's examine the difficulty of the comet.

Here's a #2 from last year's contest.

A triangle with side lengths 16, 18, and 21 has a circle with radius 6 centered at each vertex. Find n so that the total area inside the three circles but outside of the triangle is $n\pi$.



Caption: Basically, find the area of the shaded region [2]

Last year, one of our teammates, Jin Xie, came up with an brilliant solution, explaining “I realized that the circles were being used as part of the triangle so that the angles of the triangle would be the same as the angles of the circle”. In other words, the sum of the sector angles of all three angles must sum up to 180, since the sum of the sector angles is equal to the sum of the angles in the triangle.

Now, the problem disintegrates into easy steps. We begin by noting that since the sum of the sector angles is 180, we don't have to find the area of each circle separately, but can instead find the area of the semicircle (since a semicircle has an angle measure of 180) with radius 6. This would be $\frac{1}{2}\pi(6)^2 = 18\pi$. Now, we just subtract the area of the white region from the area of all three circles, which turns out to be $3 * \pi(6)^2 - 18\pi = 90\pi$. And huzzah! Our final answer is 90!

Intimidated? Don't be; These solutions are the product of minutes, hours, and even days of deep thought, practice, and hard work. It takes time to solve these problems, so sit back, relax, and enjoy their solutions!

Sources:

[1] [https://purplecomet.org/assets/images/Purple Comet Logo 2013.jpg](https://purplecomet.org/assets/images/Purple%20Comet%20Logo%202013.jpg)

[2] <https://purplecomet.org/views/data/2018HSProblems.pdf>