A Beauty in Mathematics

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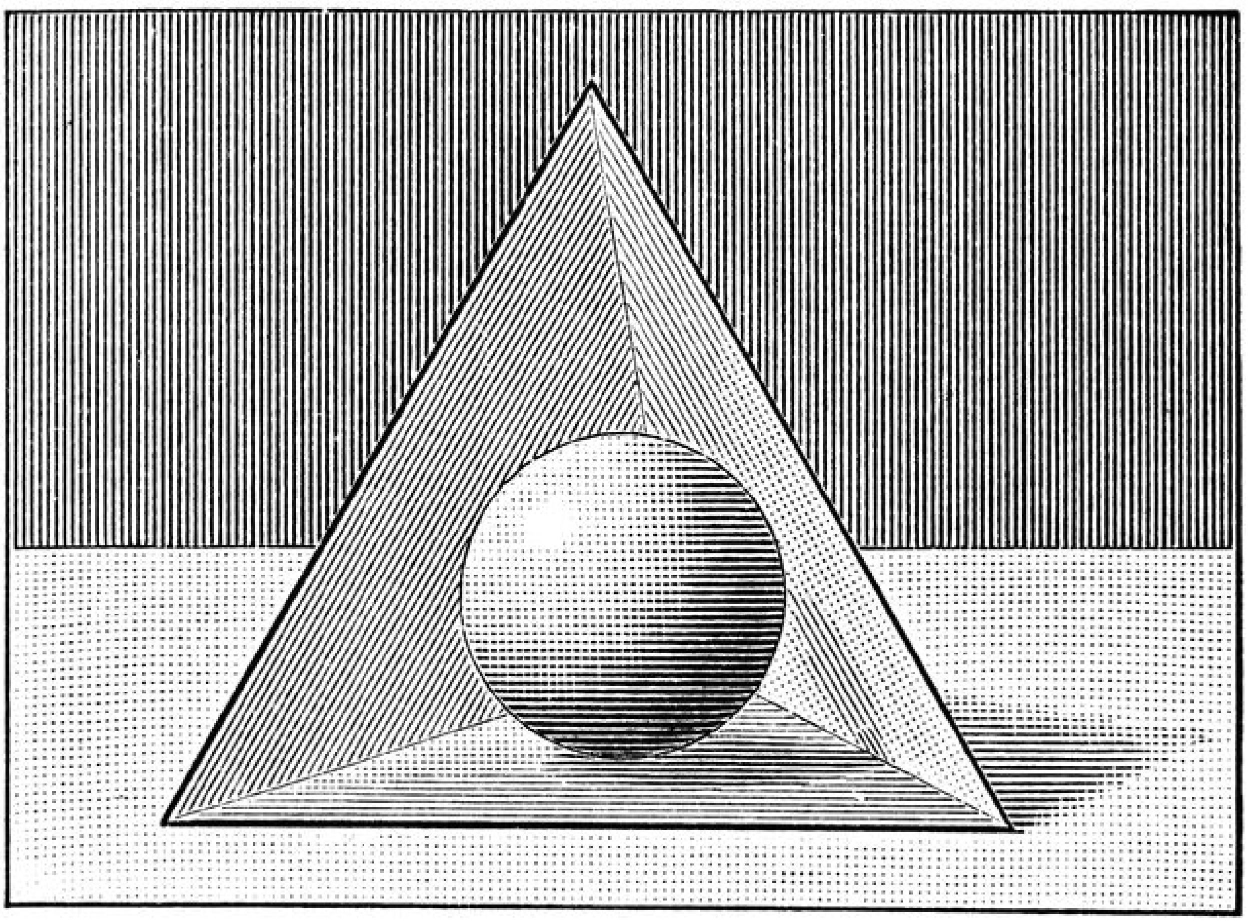
Find the radius given the circumference. Find x if $\sqrt{x} = 16$. Find this. Find that. This is no math. This is common core. This is math absolved of its beauty. Math without math. True mathematics does not ask one to find something right in front of his eyes. No. True mathematics is the journey of solving problems despite hardship; despite difficulty; despite failure. See, creativity does not come at a discount, and when students indoctrinated with the belief that mathematics is the simple action of substitution, meaningless tactics, and a easy steps that lead to a solution are exposed to true problems, their fight-or-flight responses turn on. Many run for their lives, believing such problems to be abnormal or heresy. Others -- the true solvers -- set themselves down to persevere through a good problem and emerge with revelations that change their perspective of the world. See, mathematics is not only a teacher of creativity, ingenuity, and perseverance, but an embodiment of our understanding of the universe. It enables us to describe our world quantitatively and lays the groundwork for the natural sciences.

Brooklyn Tech Math Team coach Mr. Matthews seeks to instill this curiosity for mathematics early on within students. With this goal in mind, he has curated a collection of math problems originating from all over the globe, scaling all levels of difficulty. Each student is be assigned to one problem of their choosing, and tasked with presenting their solution. Jonathan Collard de Beaufort is a freshman in the Math Club, tasked with presenting his solution to "$find\ x\ if\ \sqrt[3]{x\sqrt{x}} = 4$"(Level 2). Jonathan describes the problem as being fairly easy, using Algebra to solve for x by cubing both sides, and then isolating, and thus solving for x.

On the other side of the spectrum, we have a level 5 problem, such as the one I'm tasked with: "A sphere is inscribed in a regular tetrahedron. If the length of an altitude of the tetrahedron is 36, what is the length of a radius of the sphere?". Daunting, yet achievable. Our goal is to try to simplify this problem to a 2D version, and then apply our understanding to the 3D version. We begin by defining a tetrahedron: A 3D figure composed of 4 faces which are equilateral triangles. Now, before we take a look at a sphere inscribed in a tetrahedron, let's look at it's 2D counterpart: a circle inscribed in an equilateral triangle. There can exist only one unique inscribed circle in any triangle, with a specific location, and a specific radius. Realize, in an equilateral triangle, all the points of concurrency of all the cevians (ie, Centroid, Orthocenter, Incenter, etc) of a triangle lie on the same point (otherwise, they are all collinear on the *Euler Line)*.

Furthermore, both the altitude and inradius are perpendicular to the opposite side of the triangle. Thus, we conclude that the inradius of an equilateral triangle lies on it's altitude. But, what portion of the altitude is the inradius? To find out, I used the two area formulas involving the inradius & altitude: $A = \frac{1}{2}bh$ & $A = rs$. b & h represent the base and height of a triangle, whereas r is the inradius, and s is the semiperimeter. After some 30-60-90 triangles, and hardcore geometry, we find that the inradius is in a ratio of 1:3 to the altitude of the equilateral triangle. Now let's track back to 3D. Say we found each of the incenters of each of the 4 faces of the tetrahedron. Now, if we perpendicularly project each of these incenters onto a 3D space, they will dissect the altitude into a ratio of 1:3. Thus, the incenter will be the smaller portion of that ratio, which is ¼ the altitude. Now, we are given the altitude of the tetrahedron was 36, and thus, substituting 36, we get a final result of ¼*36=9. Boom.



CAPTION: A sphere inscribed in a tetrahedron, in 3-dimensions.