

## SCHOOL

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Cédric Bonnafé (leave on Thursday)

Titre : Representations of rational Cherednik algebras at  $t=0$

Resume : To any finite reflection group  $W$  acting on a vector space  $V$  is associated its rational Cherednik algebra  $H$  at  $t=0$ , which is a deformation of the semidirect product of  $W$  with the algebra of polynomial functions on  $V \times V^*$ .

We will study the interplay between the representation theory of  $H$  and the geometry of the spectrum of the center  $Z$  of  $H$ . As an application, we will compute the equivariant cohomology of a symplectic resolution of  $(V \times V^*)/W$ , if it exists, and show some application to the character theory of the symmetric group.

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Piotr Przytycki (not on Friday the 2<sup>nd</sup>)

title: The isomorphism problem for Coxeter groups

abstract: When are two Coxeter groups isomorphic? We discuss a conjectural answer proposed by Bernhard Muehlherr. We outline the solution in "twist-rigid" case that is joint work with Pierre-Emmanuel Caprace.

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Vic Reiner (for school)

TITLE:

"Reflection group invariant theory and generatingfunctionology"

ABSTRACT:

We will discuss some enumerative consequences of the beautiful invariant theory of complex reflection groups, including

- Shephard-Todd/Chevalley Theorem,
- Solomon's Theorem,
- exponents, coexponents,
- q-Catalan numbers,

calling upon representation theory and commutative algebra facts where needed.

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Nathan Reading (not on the 29 or 30<sup>th</sup>)

Lattice homomorphisms between weak orders and between Cambrian lattices

Given a finite Coxeter group  $W$ , the automorphisms of the weak order on  $W$  are exactly the diagram automorphisms (the symmetries of the Coxeter diagram of  $W$ ). In this talk, we consider a larger class of maps: surjective lattice homomorphisms between the weak orders on different finite Coxeter groups. Just as for automorphisms, everything is essentially determined by acting on Coxeter diagrams. Specifically, a surjective homomorphism exists from  $W$  to  $W'$  if and only if the diagram for  $W'$  can be obtained from the diagram for  $W$  by deleting vertices and/or decreasing edge labels (and thus possibly erasing edges). Once such an operation on diagrams is decided, the homomorphism is completely determined by its restriction to rank-2 standard parabolic subgroups. Furthermore,  $W'$  is a very simply-described lattice quotient of  $W$ . (The analogy is that we would call  $R'$  a simply-described quotient of a ring  $R$  if  $R' = R/I$  for  $I$  generated by a few low-degree polynomials.)

Our classification of surjective lattice homomorphisms between finite Coxeter groups leads easily to an even nicer classification of surjective lattice homomorphisms between Cambrian lattices in terms of "oriented diagram homomorphisms." If there is time, I'll mention some applications to cluster algebras, namely refinement relations among  $g$ -vector fans and surprising ring homomorphisms between cluster algebras.

## Conference

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**Sara Billey** (University of Washington)

Title: Reduced words and a formula of Macdonald

Abstract:

Macdonald gave a remarkable formula connecting a weighted sum of reduced words for a permutation with the number of terms in a Schubert polynomial. We will review some of the fascinating results on the set of reduced words in order to put our main results in context. Then we will discuss a new bijective proof of Macdonald's formula based on Little's bumping algorithm.

We will also discuss some generalizations of this formula based on work of Fomin, Kirillov, Stanley and Wachs. This project extends earlier work by Benjamin Young on a Markov

process for reduced words of the longest permutation.

This is joint work with Ben Young and Alexander Holroyd.

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**Cesar Ceballos** (Universität Wien), talk on Monday or Tuesday

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**Maria Chlouveraki** (UVSQ / Université Paris-Saclay) Monday or Tuesday)

Title: Combinatorics of multipartitions and families of characters

Abstract: Families of characters were defined for Weyl groups by Lusztig for the determination of the families of unipotent characters of finite reductive groups. Lusztig's definition is inductive and uses his famous  $a$ -function. There are two alternative definitions for families of characters: one that uses Kazhdan-Lusztig cells, and one that uses the blocks of the Iwahori-Hecke algebra over a special ring known as the "Rouquier ring". The former generalises to finite Coxeter groups (under certain assumptions), the latter to all complex reflection groups. Another approach has been recently suggested which would also allow the definition of cells and families for all complex reflection groups: via the rational Cherednik algebras. In this talk, we will discuss the description of the families of characters for the complex reflection groups of the infinite series  $G(l,p,n)$  using combinatorics of multipartitions, and their connection with the families of characters of Weyl groups of type B.

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**Patrick Dehornoy** (Université de Caen) (not on Friday)

Multifraction reduction in Artin-Tits groups

Artin-Tits groups are the braid groups associated with Coxeter groups, and their word problem remains unsolved in the general case. Multifraction reduction is a new rewrite system providing for certain Artin-Tits groups ("FC type") a normal form that extends Ore's fractional decomposition. In the general case, reduction need not give a unique normal form, but massive experiments and partial results support the conjecture that it still solves the word problem. The decidability of the method relies on results by Dyer and Hohlweg about low elements in the underlying Coxeter group, and it is reasonable to think that the main open question ("semiconvergence of reduction") is directly connected with combinatorial properties of the Coxeter group.

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**Anna Felikson** (Durham University)

Hyperbolic Coxeter polytopes.

Abstract:

Hyperbolic Coxeter polytopes are fundamental domains of discrete actions of reflection groups in hyperbolic spaces. Although the classification problem for spherical and Euclidean Coxeter polytopes was solved already in 1934 by H.S.M.Coxeter, similar question in hyperbolic case turned out to be very difficult.

I will overview old and new results concerning classification of hyperbolic Coxeter polytopes and will discuss a range of methods which proved to be useful for different problems. In particular, the methods developed for this classification problem were successfully applied to obtain the classification of quivers of finite mutation type.

Some of the approaches mentioned in the talk are based on joint works with Pavel Tumarkin.

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**Patricia Hersh** (North Carolina State University ) (talk not on the 9th)

Title: Shelling Bruhat order and the proof of a conjecture of Thomas Lam

Abstract: Matthew Dyer's proof that reflection orders give rise to lexicographic shellings for Bruhat order relied on the fact that the number of so-called ascending chains in a closed interval in Bruhat order is the leading coefficient in a polynomial closely related to the Kazhdan-Lusztig polynomial. We give a new, purely combinatorial proof that reflection orders yield shellings for Bruhat order based on a seemingly new characterization of cover relations in Bruhat order. In related work, we also prove a conjecture of Thomas Lam that face posets of stratified spaces of response matrices of electrical networks are lexicographically shellable. We will discuss this work, parts of which are joint work with Rick Kenyon.

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**Kasia Jankiewicz** (McGill University)

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**Martina Lanini** (Università degli Studi di Roma "Tor Vergata")

Title: "Combinatorial Fock space and representations of quantum groups at roots of unity"

Abstract: The classical Fock space arises in the context of mathematical physics, where one would like to describe the behaviour of certain configurations with an unknown number of identical, non-interacting particles. By work of Leclerc-Thibon, it has a realisation in terms of the affine Hecke algebra of type A and it controls the representation theory of the corresponding quantum group at a root of unity. In a joint work with Arun Ram and Paul Sobaje, we produce a generalisation of the  $q$ -Fock space to all Lie types. This gadget can also be realised in terms of affine Hecke algebras and captures the decomposition numbers of standard modules for representations of quantum groups at roots of unity."

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**Timothée Marquis** (UCL)

Title: Conjugacy classes in Coxeter groups

Abstract: A Coxeter group can be thought of as the automorphism group of some geometric object, called its Coxeter complex. This is a simplicial complex, which can be further equipped with a non-positively curved metric (i.e. a CAT(0)-metric). This provides a very powerful tool to study Coxeter groups. We will illustrate this by describing the conjugacy classes in (some finite-index subgroup of) any Coxeter group.

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**Jon McCammond** (UC Santa Barbara)

Title: The intrinsic geometry of the dual braid complex

Abstract: The braid groups have a standard presentation and a dual presentation and there are classifying spaces associated to each. The classifying space for the standard presentation is an example of a Salvetti complex and the one for the dual presentation is what I am

calling the dual braid complex. The dual braid complex is a simplicial complex with a piecewise euclidean metric that has many nice properties --- including being CAT(0) (this is known to be true in low dimensions and strongly conjectured in all dimensions) --- but it is extremely large. In this talk I will introduce a drastically simplified cell structure on the dual braid group derived from the intrinsic geometry of the dual braid complex and discuss the facts that become visible in this less cluttered context.

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**Karola Meszaros** (Cornell University) (n'est la que le 5,6,7 part le 8)

From generalized permutahedra to Grothendieck polynomials via flow polytopes

We prove that the Grothendieck polynomial  $\mathfrak{G}_{1\pi}(\mathbf{x})$ , for permutations  $1\pi$ , where  $\pi$  is dominant, is a weighted lattice point enumerator of its Newton polytope, with all weights nonzero. We also show that the Newton polytopes of the homogeneous components of  $\mathfrak{G}_{1\pi}(\mathbf{x})$  are generalized permutahedra. Moreover, the Schubert polynomial  $\mathfrak{S}_{1\pi}(\mathbf{x})$ , for permutations  $1\pi$ , where  $\pi$  is dominant, equals the lattice point enumerator of a generalized permutahedron. These results imply recent conjectures of Monical, Tokcan and Yong regarding the supports of Schubert and Grothendieck polynomials for permutations  $1\pi$ , where  $\pi$  is dominant. We connect Grothendieck polynomials and generalized permutahedra via a family of dissections of flow polytopes obtained from the subdivision algebra. We naturally label each simplex in a dissection by a sequence, called a left-degree sequence, and show that the left-degree sequences arising from simplices of a fixed dimension in our dissections of flow polytopes are exactly the integer points of generalized permutahedra. We also connect left-degree sequences and Grothendieck polynomials, thereby revealing the beautiful relation

between generalized permutahedra and Grothendieck polynomials. The talk is based on joint papers with Laura Escobar and Avery St. Dizier.

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**Jean Michel** (Université Denis Diderot - Paris 7)

Cyclotomic root systems ( joint work with Michel Broué and Ruth Corran)

Abstract: We generalize the definition and properties of root systems to complex reflection groups - roots become rank one projective modules over the ring of integers of a number field  $k$ . In the irreducible case, we provide a classification of root systems over the field of definition  $k$  of the reflection representation. In the case of spetsial reflection groups, we generalize as well the definition and properties of bad primes.

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**Yuya Mizuno** (Nagoya University)

Title: A connection between the module category over a path algebra and the Coxeter group.

Abstract: A path algebra, which is defined by a quiver, is one of the most fundamental classes of algebras.

For this class, Oppermann-Reiten-Thomas established a bijection between certain subcategories over the algebra and the elements of the Coxeter group of the corresponding graph.

Then, this result allows a representation-theoretical interpretation of the Coxeter group.

In this talk, we explain a close relationship between path algebras, preprojective algebras and the Coxeter group.

Moreover, we discuss the interplay of the theory of  $c$ -sortable elements and torsion pairs.

Our result is a joint work with Hugh Thomas.

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**Piotr Przytycki** (McGill University), (not on Friday the 2<sup>nd</sup>)

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**Nathan Reading** (North Carolina State University)

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**Vic Reiner** (University of Minnesota)

NOT ON Monday or Tuesday)

Title: Invariant derivations and differential forms for reflection groups

Abstract:

(joint work with Anne Shepler, [arxiv.org/abs/1612.01031](https://arxiv.org/abs/1612.01031))

Classical invariant theory of a complex reflection group  $W$  highlights three beautiful structures:

- the  $W$ -invariant polynomials constitute a polynomial algebra, over which
- the  $W$ -invariant differential forms with polynomial coefficients constitute an exterior algebra, and
- the relative invariants of any  $W$ -representation constitute a free module.

When  $W$  is a duality (or well-generated) group, we give an explicit description of the isotypic component within the differential forms of the irreducible reflection representation. This resolves a combinatorial conjecture motivated by  $W$ -Catalan combinatorics, and also relates to a certain thread of results in Lie theory.

We establish this result in a case-free fashion, by examining the space of  $W$ -invariant differential derivations; these are derivations whose coefficients are not just polynomials, but differential forms with polynomial coefficients. . When  $W$  is a duality group, we show that the space of invariant differential derivations is free as a module over the exterior subalgebra of  $W$ -invariant forms generated by all but the top-degree exterior generator. (The basic invariant of highest degree is omitted.)

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**Vivien Ripoll** (Universität Wien)

Title:

Some geometric properties of limit roots of Coxeter groups

Abstract: Let  $W$  be an infinite Coxeter group, and consider the root system constructed from its geometric representation. In the past few years, together with Dyer, Hohlweg, Labbé, Préaux, we investigated the accumulation set of the directions of roots, which is a very pretty subset of the projective space, called the set of limit roots of  $W$ . Recently I have been working on some conjectures on the geometry of this limit set, and I will share some partial results. One of them



involves a description of "parabolic slices" of limit roots (the intersection of the limit set with a vector subspace associated to a parabolic subgroup). I also explore the conjecture stating that the set of limit roots is equal to the intersection of its convex hull with the isotropic cone. As a byproduct of the computations involved in this quest, I obtain an interesting disjunction property about the set of dihedral limit roots.

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**Salvatore Stella** (Università degli studi di Roma "La Sapienza")

Title: Polytopal realizations of finite type  $\mathbf{g}$ -vector fans

Abstract:

To any cluster algebra with principal coefficients  $\mathcal{A}$ , and by extension to any other cluster algebra, it is associated a simplicial fan that encodes the underlying combinatorial structure: the  $\mathbf{g}$ -vector fan.

When  $\mathcal{A}$  is of finite type, this fan has a natural interpretation in terms of a (not necessarily finite) root system. Moreover it is complete, leading to wonder whether it is the normal fan to a polytope or not.

A positive answer to this question, whenever the initial seed of  $\mathcal{A}$  is acyclic, was obtained by Hohlweg, Lange, and Thomas deleting inequalities in the facet description of the associated permutahedron.

We extend the polytopality result to all the other possible initial seeds.

Our construction though, while producing the same generalized associahedra in the acyclic cases, does not preserve the connection with Coxeter combinatorics beyond type  $A_n$ .

This seminar is based on a joint work with C. Hohlweg and V. Pilaud.

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**Christian Stump** (Freie Universität Berlin)

Title: From Shi arrangements to reflection multi-arrangements

Abstract: The module of derivations of a multi-arrangements of hyperplanes was introduced by Ziegler in the 1980's. Maybe the most

important example is the Coxeter arrangement with constant multiplicities resolved by Terao in 2002 by showing that these are free with exponents given uniformly in terms of Coxeter numbers and invariant degrees. For finite Weyl groups, this result is closely related to the combinatorics of Catalan and Shi arrangements.

After a detailed description of this relationship, I plan to present a generalization to all complex reflection group, using the most general notion of Coxeter numbers and an operator on irreducible components introduced by Malle in the context of the associated cyclotomic Hecke algebra, opening a possible way to generalize certain notions of Catalan and Shi arrangements beyond Weyl types.

This is joint work with Torsten Hoge, Toshiyuki Mano and Gerhard Roehrl.

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**Jacek Swiatkowski** (Wroclaw University)

"Coxeter groups with the  $n$ -dimensional Sierpinski compacta as boundaries".

**ABSTRACT:**

The  $n$ -dimensional Sierpinski compacta are some natural higher dimensional analogs of the Sierpinski curve. For any positive integer  $n$  we will show that the right angled Coxeter systems from some vast family have the property that the visual (i.e.  $CAT(0)$ ) boundary of the associated Coxeter-Davis complex is homeomorphic to the  $n$ -dimensional Sierpinski compactum. We will also present some aspects of the proof, in which we confront all the difficulties involved in dealing with boundaries of spaces that are metrically singular, not necessarily hyperbolic (in the sense of Gromov), and whose expected boundaries have no convenient characterization in terms of a list of topological properties.

## Schedule School

	Monday 29	Tuesday 30	Wednesday 31	Thursday 1	Friday 2
<b>9:30</b>	<b>Coffee</b>				
<b>10h-12h</b>	Introduction to Coxeter Groups & Reflection groups	Piotr Przytycki	Piotr Przytycki	Vic Reiner	Vic Reiner
<b>14h-16h</b>	Introduction to Coxeter Groups & Reflection groups	Cédric Bonnafé	Cédric Bonnafé	Nathan Reading	Nathan Reading
<b>16h</b>	<b>Coffee</b>				
<b>18:30</b>		<b>School Dinner</b>			

## Schedule Workshop

	Monday 5	Tuesday 6	Wednesday 7	Thursday 8	Friday 9
<b>9H30</b>	<b>Coffee</b>				
<b>10h00-11h00</b>	Nathan Reading	Patricia Hersh	Patrick Dehornoy	Piotr Przytycki	Sara Billey
<b>11h00-12h00</b>	Salvatore Stella	Vivien Ripoll	Jon McCammond	Timothée Marquis	Vic Reiner
<b>15h30</b>	<b>Coffee</b>				
<b>16h00-17h00</b>	Cesar Ceballos	Maria Chlouveraki	Martina Lanini	Anna Felikson	<b>14h00:</b> Christian Stump
<b>17h00-18h00</b>	Karola Mészáros	Yuya Mizuno	Kasia Jankiewicz	Jacek Swiatkowski	<b>15h00:</b> Jean Michel
<b>18h</b>	<b>Reception</b>				