Non-Interacting Quantum Scalar Field Theory in the case of a Fixed Background Causal Set

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Abstract

Quantum fields typically yield problematic divergences at arbitrarily short length scales (or high energies). The discreteness of causal sets automatically regulates such phenomena since, for example, wavelengths shorter than the discreteness scale cannot exist.In this context of a non-interacting quantum scalar field theory living on a fixed background causal set, one can use the retarded Green function as a solution to the inhomogeneous wave equation in the conventional continuum theory.By invoking the use of this function we can set up and study properties of vacuum states and entanglement entropy in the Quantum field. We can also observe analogues of these Green functions in the causal set. The aim of the project is to gather computational evidence for a proposed relation between Causal Set Theory and Entanglement theory, using Hypergraphs and a framework. Regardless of the positive preliminary results, due to restrictions in time and computational capabilities, it is not possible to come to a final conclusion regarding the question posed. It is recommended that these tools be applied to larger causal sets.

Introduction

The purpose of the causal set theory approach to a quantum scalar field is to arrive at a theory of quantum gravity. The Causal Set overlays a new concept on the physical world to allow for the creative freedoms to make profound observations. The motivation for use of the Causal set can be seen in the spacetime causal order from General Relativity and the path integral from quantum theory.The discreteness of spacetime at the Plank scale considers discreteness, causal order and the path integral [1]. From these principles the discreteness and spacetime causal order can be combined to model the foundations of spacetime as a causal set. The structure of causal sets and the relationship between a causal set and a continuum spacetime are key intricacies that must be considered in investigations as sensitive as that which encompasses the topic of this discussion. For the sake of allowing the discreteness concept to hold,one must ignore unphysical scales smaller than the Plank scale- avoiding anomalous and non-consistent observations[2]. Non-locality will be heavily leveraged (from the result of the combination of discreteness and Lorentz invariance) to validate the use of Causal Set theory as an approach to Quantum Gravity and Quantum Scalar Fields in General. The partial order on a causal set represents a fundamental relation while local finiteness indicates discreteness. It follows naturally that finite volume regions in the continuum contain only a finite number of causal set elements.Such discreteness does not violate the continuum approximation of local Lorentz invariance[3]. First we must consider the 2 dimensional case and work upwards. So far the features of the 2d theory have been studied in detail.Considering 3 dimensions as consisting of 2 spatial and 1 temporal dimension, the discussion will consist of the logical progression, ending at the 3 dimensional investigation. The spectrum of a certain key operator in 2d exhibits an inverse decay of 1/k where k can be understood as a wavenumber. Each nth eigenfunction of this operator has an eigenvalue proportional to 1/n (where the wavenumber k is proportional to n). In 3d it is known that the decay goes as $1/\sqrt{n}$. A deep understanding of these results would support entanglement entropy calculations in 3 and higher dimensions.[4]. All of the Figures will be created using the conditions: the Number of sprinkled elements is 4000, the length of each side of the Minkowski Diamond (in both 2 and 3 dimensions) is 1 unit and the density equals the number of sprinkled elements over the volume of the associated diamond.

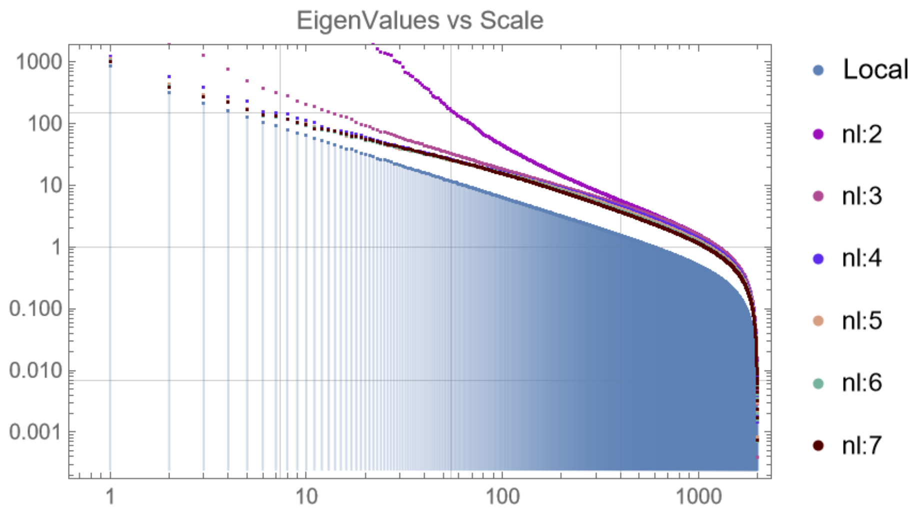

Fig. 1.— The Non-Local spectra distribution in relation to locality scales, across the discreteness range. The colours have been randomized to allow for distinction. The Local spectra has been included as a point of reference. The model has been modified to include discreteness length scale=2 to discrete length scale =7 ; the simulated data shows a clear convergence to some plot that appears to be a vertically shifted Local Spectra.

Theory

To construct a theory of quantum gravity we need to abandon the continuous nature of space . Building up from continuum theory we can approach quantum gravity using Causal sets. Using black hole thermodynamics we can get hints about quantum gravity and its nature due to infinities that arise. Such erroneous calculations could be avoided if discreteness was used as a model. Infinities in quantum gravity and singularities in general relativity are reasons why discreteness could simplify our physical picture.

Using it as a starting point to building a new quantum theory of gravity, one can draw the property of causality from the concept of a causal set in order to draw causal relations that mimic the natural interpretation of cause and effect. This eradicates the need to assume topology as with the normal picture of space time (and also eradicates the need for a differential structure and metric). As causal sets may encode this information about the causal relations in a metric one can come to the conclusion that it may be better to begin with a discrete space time in order to build on the foundational theories of quantum gravity. This is assuming the causal relations of the elements (points) of this spacetime can be proven for certain. This acts as an explanation as to the starting point of the causal set approach to quantum gravity.

Observations

One may consider the trend to be that as the discreteness scale increases, the tendency towards a middling value becomes more apparent (particularly in the 2-dimensional case. For the sake of identifying the rate of convergence and looking at the most important parts of the graph it was obvious that we had to slice the data from the linear part of Figure 1. We took logarithms of the X axis and Y axis. Considering the data was a large data, trends could be identified. From this, I can tell that,as a quantum field deviates from the assumed location of its initial disturbance, the scale and effect of each prevailing interaction becomes decreased. My aim is to figure out the speed of this decrease and how it changes in relation to the size of the causal set that the field is being modelled on.

Using such mathematics as the retarded green theorem and using such operators such as the a Scaled d'Albertian, one can use the data from the initial list plots of the causal set in a Minkowski 2 dimensional diamond to create a causal matrix that will be operated upon by these factors that have been created in conjunction with Sorkin- Johnston's eigenvalue research from the last decade[5]. As this is the three dimensional case, the values we get, although complex, cannot form a full picture of reality due to the difficulties in translating such things as three dimensions or more to a suitable visualization.

Rate of convergence

The speed of convergence was calculated by finding the difference between the list log plots values and the values of each individual nonlocality scale in turn. I did this by indexing the log of each point in the data sets, finding the difference and dividing it by the relative magnitude in order to find a proper estimation of convergence rate.

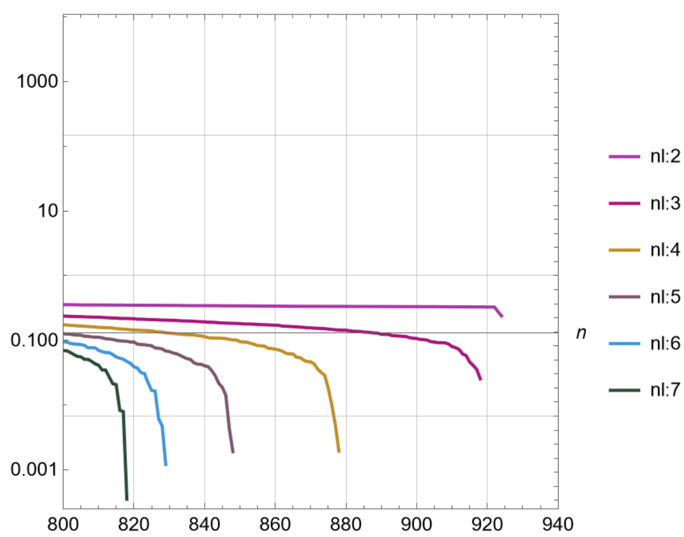

Fig. 2.— The relative rates of convergence of Non-Local spectra onto the Local spectra. The trend shows the speed of convergence increases as you increase the non locality scale.The X axis is the non locality scale and the Y axis is the rate of convergence (found by taking the difference in Eigenvalues compared to the local scale and dividing that by the sum of the same values).

Example Code for this Operation in Mathematica:

```
ListLogPlot[ReverseSort[(Log[Take[Select[EEnl, #> 0 &],
 listlimiter]] -Log[Select[EE, #> 0 &]])/(Log[Select[EE, # > 0 &]] +
 Log[Take[Select[EEnl, #> 0 &], listlimiter]])]
Where listlimiter = -Min[Length[Select[EE, #> 0 &]],
Length[Select[EE, #> 0 &]]];
```

The rates of convergence increase dramatically, from 10^{-4} to 10^{0}. To explain this I must call upon calculations by Yasamin Yazdi on entanglement entropy. According to these calculations and research on Causal sets, Scalar fields can be defined on a causal set by taking their value at each point on the set. The Pauli-Jordan operator and the Wightman Function can then be used and through them entanglement entropy can be highlighted from these specific Scalar fields. Considering the above figure, I would expect the larger nl value to get further away from locality. Theories with larger nl values tend to have smoother features on larger scales. Figure 2 is closer to what the local theory looks like. This is the current trend mathematically, however further investigation should be carried out to verify the insight.

Spectra visualisations in 3 dimensions

In this use case when looking at the 3 dimensional situation we find that the Eigenvalues seem to span a broad spectrum of power scales.

This acts as evidence that in the three dimensional case, the divergences become more distinguished. The non-local plots eventually cross above the local.To explain this, when the discreteness length scale is set to 1, then it means that each element is roughly 1 unit away from the nearest related one. Setting the discreteness length scale to 2 would mean that the nonlocality affects elements not just nearest to each other but also one more further element. Similarly, for the value to be equal to 3 it means that the effect reaches out even further, to the third element away, etc. The crossing over is due to the breakdown of the properties of the quantum field at these short distances, resulting in large scale fluctuations in field values.

Eigenvector visualisations in 2 dimensions

When visualising the eigenfunctions, one must take into account the real parts and imaginary parts of the eigenvectors separately. This is because they each show amplitudes of the functions at certain points, with varying levels of noise present. When creating these plots, I had to choose a plot that would represent a large number of waves in this inhomogeneous wave equation analogue so I used the nonlocality scale of 6.

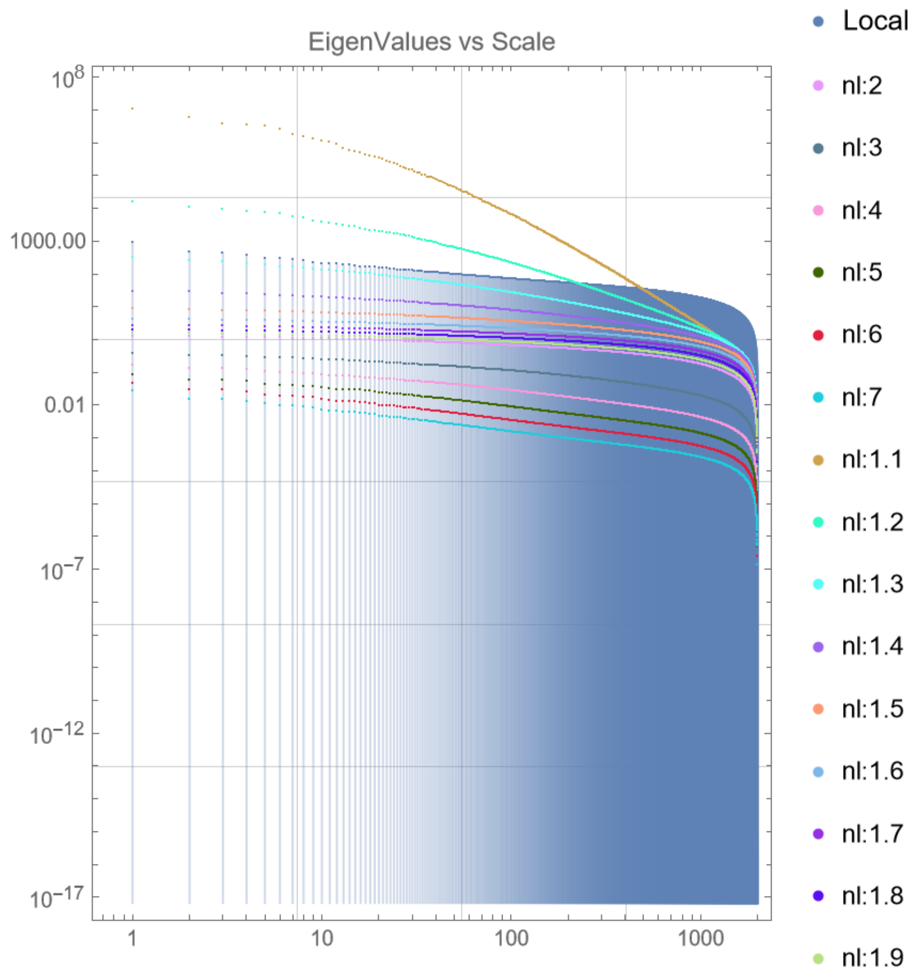

Fig. 3.— The Non-Local spectra distribution in relation to locality scales, across the discreteness range. Once again, colours have been randomized to allow for distinction. The Local spectra has been included as a point of reference, with a blue graph filling being used for effect. The model includes non locality=1.1 to non locality=7 ; the data shows a convergence in the non-linear region.

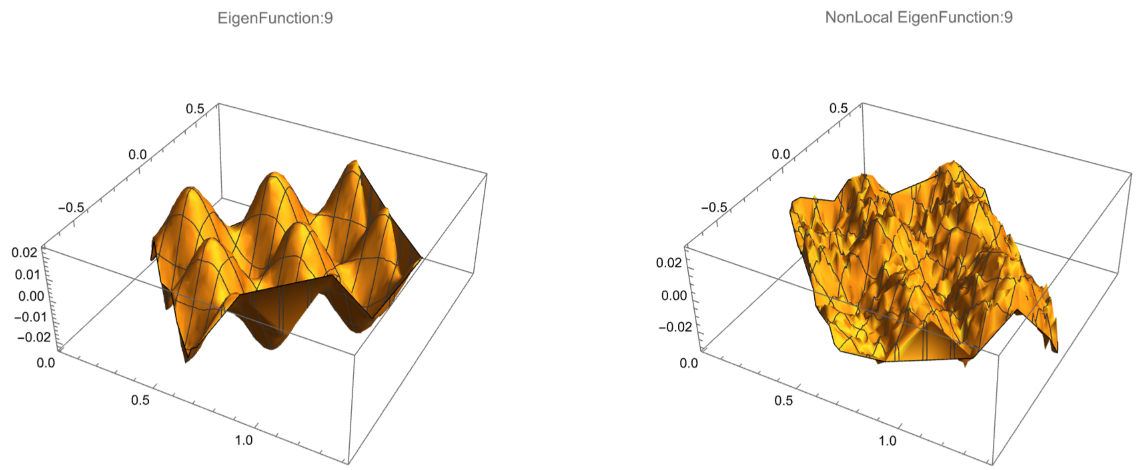

Fig. 4.— This figure shows a clear and smooth wave that models the interactions of the quantum scalar field at a set point with the causal data separated by discrete elements from the centre of a Scalar Field. I chose the 9th EigenFunction.

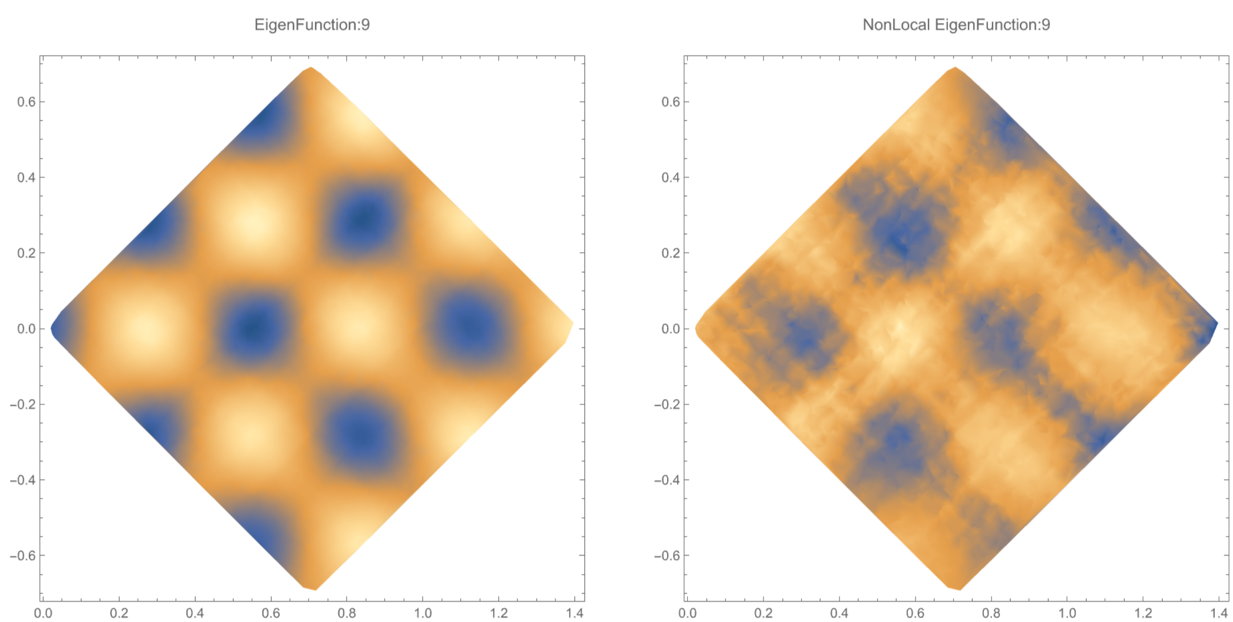

Fig. 5.— The noise is significantly more apparent in this comparison, as a ListDensityPlot. The aberrations are due to the conversion of the EigenFunction into a set of discrete points.

Comparing all Eigenvector visualisations in 2 dimensions

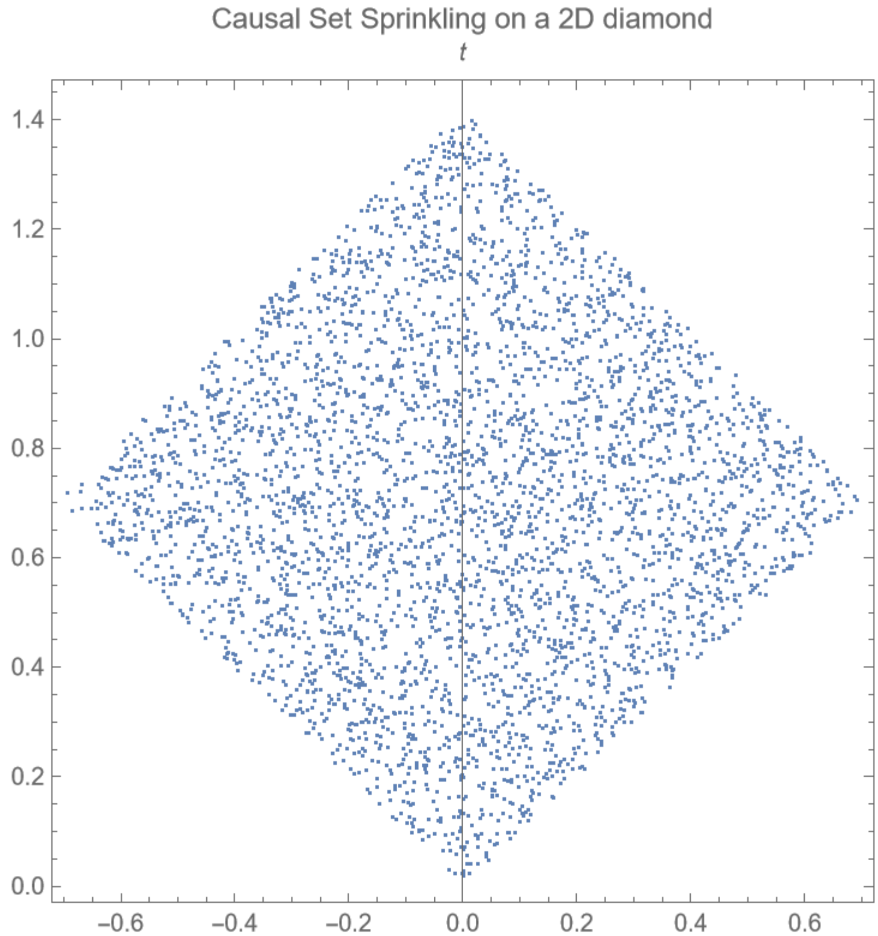

Fig. 6.— This figure presents the Causal set sprinkling of a Minkowski diamond in two dimensions as a Listplot

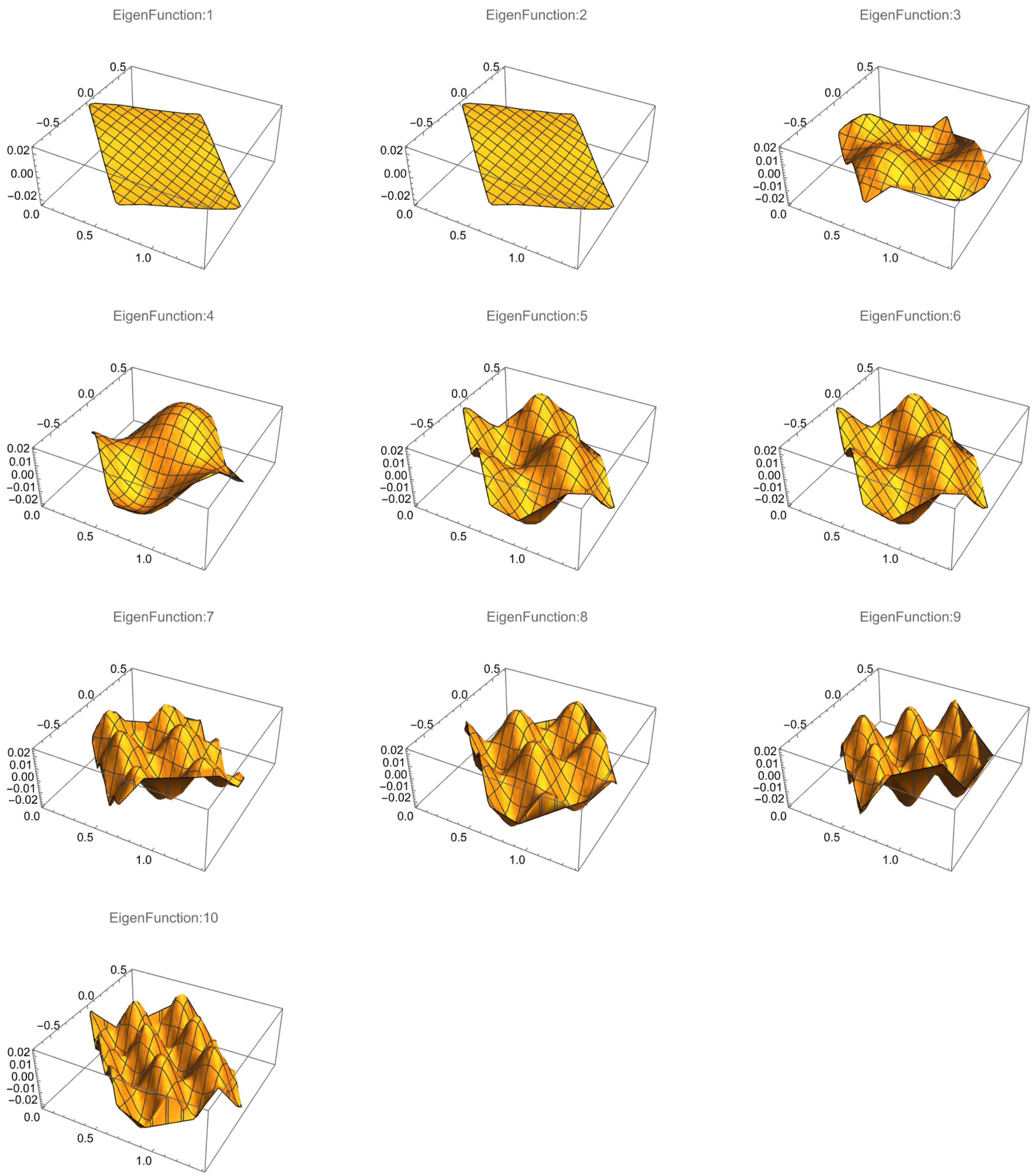

Fig. 7.— This figure shows a clear and smooth wave that models the interactions of the quantum scalar field at a set point with the causal data separated by discrete elements from the Scalar Field.- Local (ListPlots)

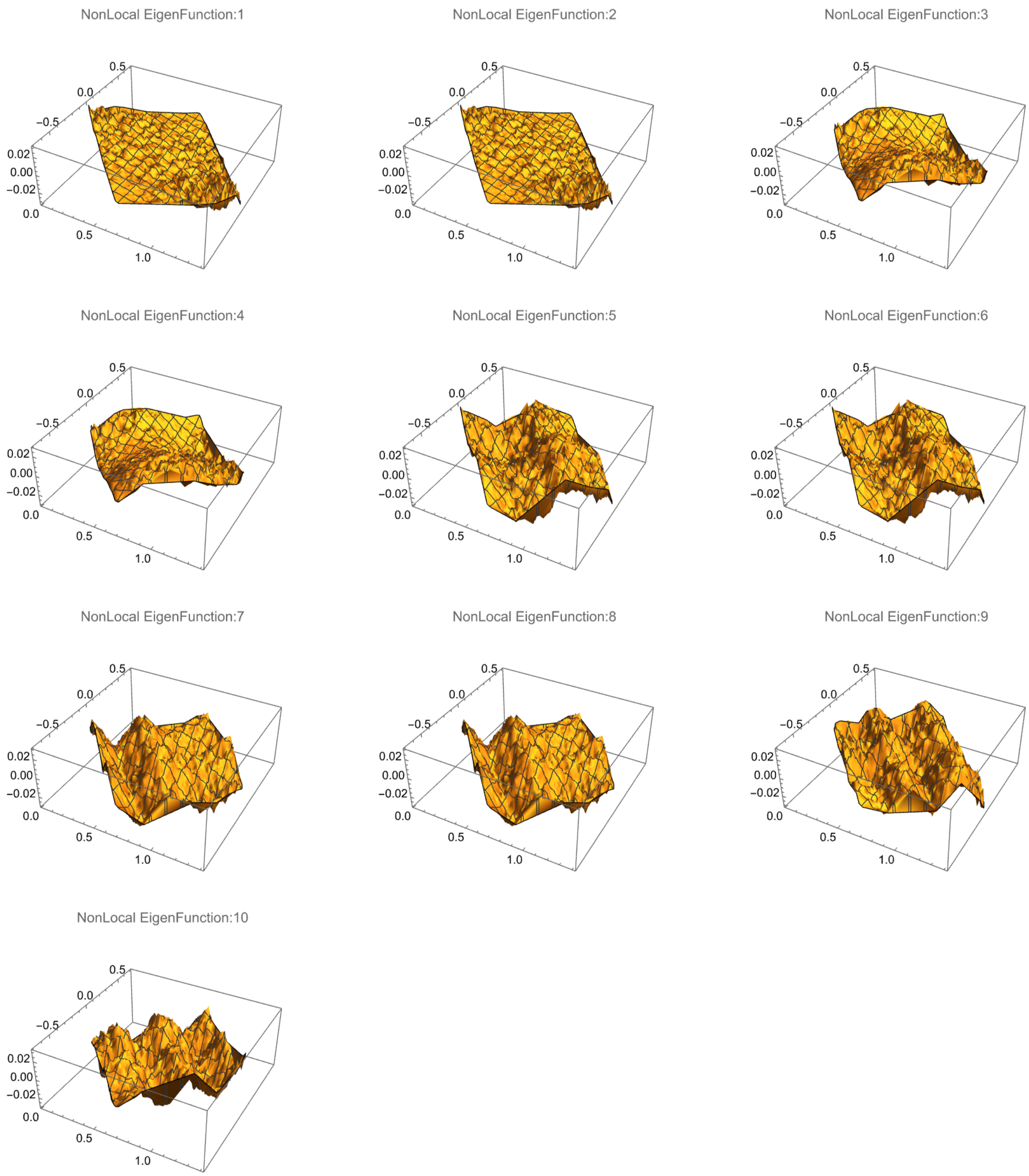

Fig. 8.— This figure shows a clear and smooth wave that models the interactions of the quantum scalar field at a set point with the causal data separated by discrete elements from the Scalar Field.- Non Local (ListPlots)

Comparing all Eigenvector visualisations in 3 dimensions

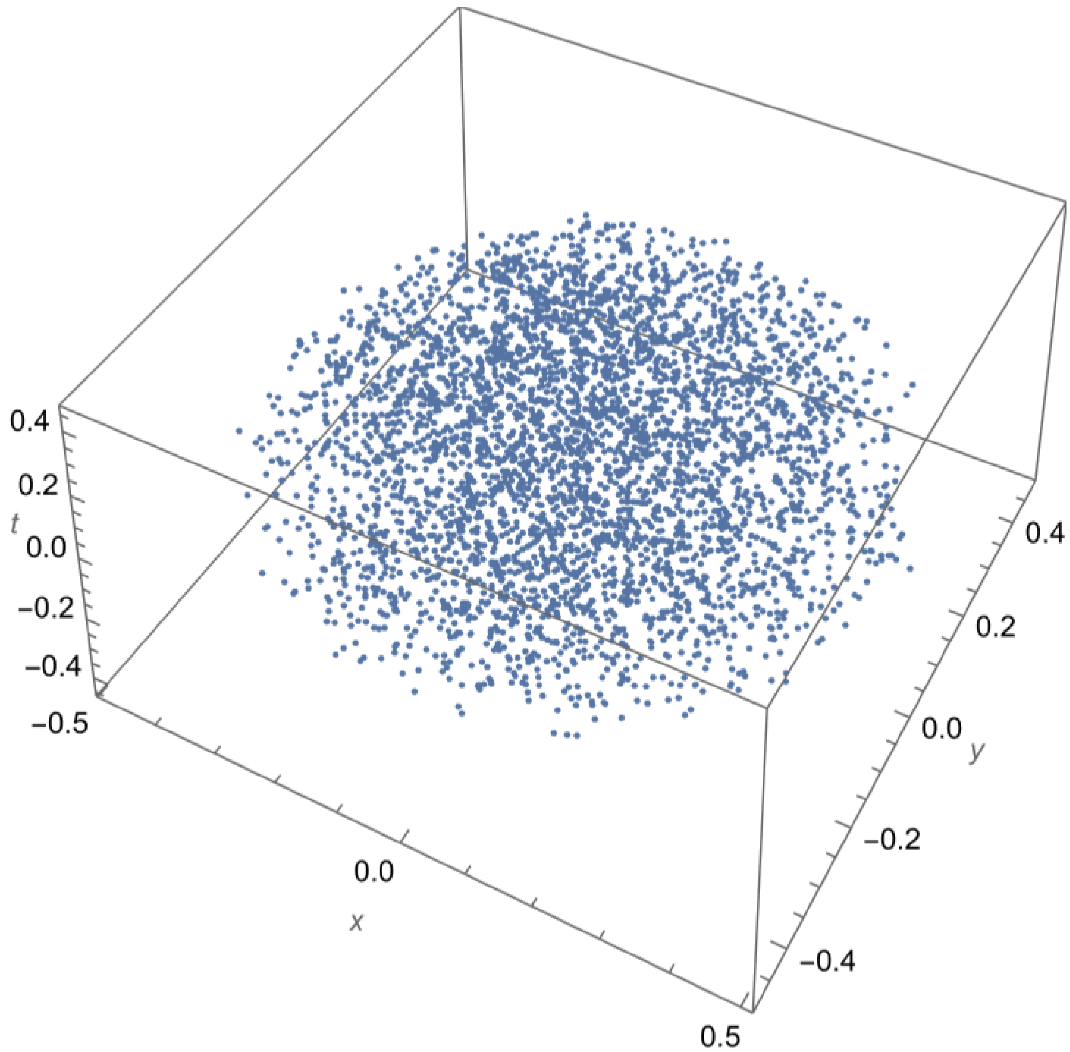

Fig. 9.— This figure presents the Causal set sprinkling of a minkowski diamond in three dimensions as a Listplot.

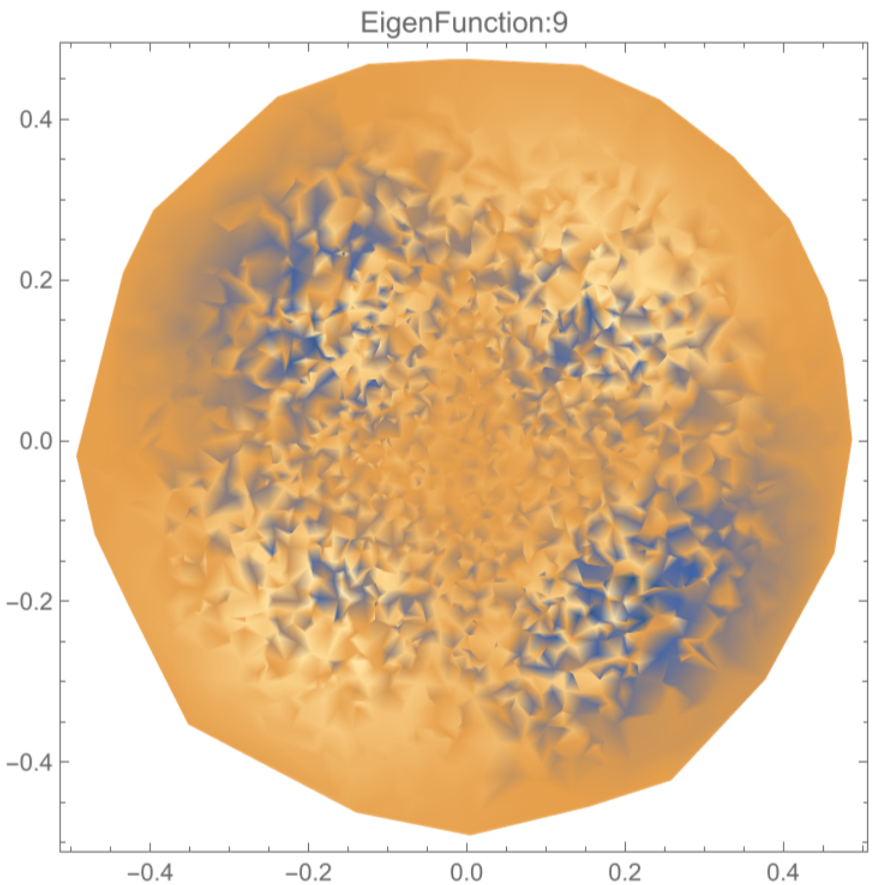

Fig. 10.— This figure is the ListDensityPlot of the 9th EigenFunction in 3 dimensions.

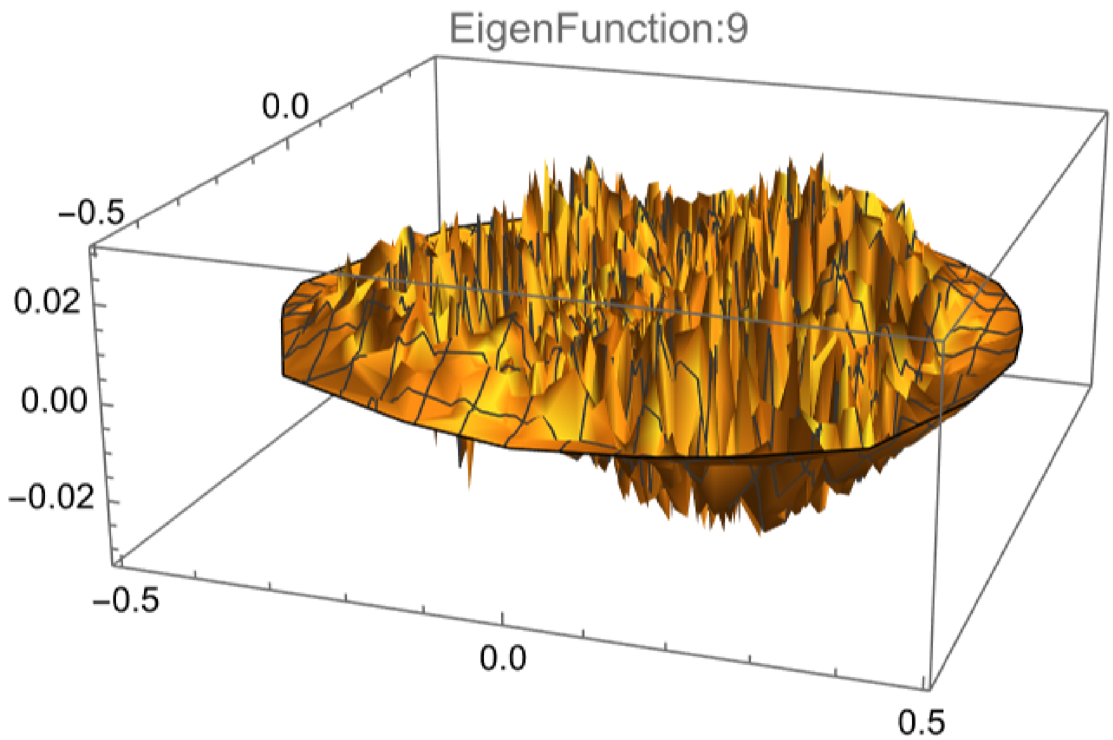

Fig. 11.— This figure is the ListPlot of the 9th EigenFunction in 3 dimensions.

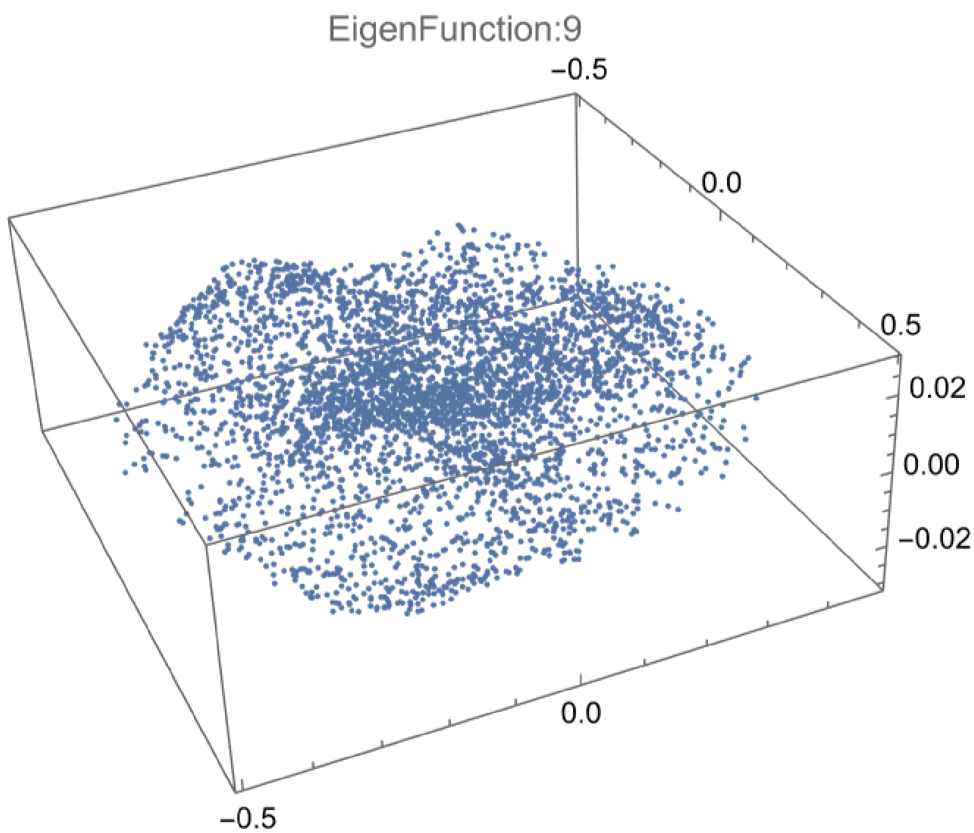

Fig. 12.— This figure is the ListPointPlot of the 9th EigenFunction in 3 dimensions.

The Wave number Interpretation

Considering the analytic expressions for the eigenfunctions in the 2D local theory (found in equations 4.3 and 4.4 in [6]), the eigenvalues (lambda) are represented by the relationship L/K. The k in this expression is a wavenumber. In the causal set, the density of elements sets the maximum k value. In the 3D and the nonlocal theory, we are yet to explore how the eigenfunctions and eigenvalues may relate to wavenumber. Typically, analytic expressions don't exist in these cases, so they must be deduced entirely from the causal set.

Conclusion

The work has presented some encouraging results. However, it does not yet provide enough evidence as to the relation between Causal Sets and small scale divergences in quantum scalar fields in relation to their entanglement entropies. The developed code can be applied to a larger Causal Set, which would provide greater validation on the topic. It would be interesting to see if the Causal matrix could be developed in fewer steps. If a relation between the two approaches can be established, the result could be groundbreaking..

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