$$\cancel{V}_m(t) = \frac{\pi}{4}\int_0^{L_m} [D(\xi,t)]^2 d\xi \tag{1}$$

$$\cancel{V}_C(t) = \cancel{V}_R + \cancel{V}_m(t) - \cancel{V}_{max} \tag{2}$$

$$F = \frac{\partial}{\partial t}\int_{C\cancel{V}} V(x,t)\rho d\cancel{V} + \int_{CS} V(x,t)\rho V(x,t) dA \tag{3}$$

$$\frac{p_m}{\rho} + \frac{V_m^2}{2} + gz_m = \frac{p_j}{\rho} + \frac{V_j^2}{2} + gz_j + \int_{s_o}^{s_j} \frac{dV(s,t)}{dt} ds \tag{4}$$

$$\begin{aligned} T_{f(h)} &= \frac{1}{2}C_x\rho V^2 S && (5)\\ D &= \frac{1}{2}C_D\rho_w S_w U^2 \\ T_{f(V)} &= \frac{1}{2}C_z\rho V^2 S && (6)\end{aligned}$$

$$\begin{cases} D = \frac{1}{2}C_D\rho_w S_w U^2 & \text{(7a)}\\ L = \frac{1}{2}C_L\rho_w S_w U^2 & \text{(7b)}\end{cases}$$

$$\eta_f = \frac{2V}{V+V_j} = \frac{V}{V+\frac{\Delta V}{2}} \tag{8}$$

$$\begin{aligned} T_{j(h)} &= T_j\cos\alpha && (9)\\ T_{j(v)} &= T_J\sin\alpha && (10)\end{aligned}$$

$$\begin{aligned} &F_r + D + R_A = T_{j(h)} + T_{f(h)} && \text{horizontal direction} && (11)\\ &B = T_{j(h)} + T_{f(h)} + L && \text{vertical direction} && (12)\end{aligned}$$

$$C_{Df} = \begin{cases} \frac{1.328}{Re_L^{1/5}} & Re_L \le 5\times 10^5 \\ \frac{0.0735}{Re_L^{1/5}} - \frac{1742}{Re_L} & 5\times 10^5 \le Re_L \le 10^7 \\ \frac{0.455}{(\log Re_L)^{2.58}} - \frac{1742}{Re_L} & 10^7 \le Re_L \le 10^9 \end{cases} \tag{13}$$

$$\bar{T} = \rho\frac{\pi^3}{16}L^2D^2f^2 \tag{14}$$

$$ma = \bar{T}\left(1 - e^{-\lambda t}\right) - \frac{1}{2}C_d\rho A V^2 \tag{15}$$

$$\frac{dV}{dt} = \frac{\bar{T}\left(1 - e^{-\lambda t}\right) - \frac{1}{2} C_d \rho A V^2}{m} \tag{16}$$

$$\begin{cases} m\ddot{x_1} & = 2kx_1 - kx_2 \\ m\ddot{x_2} & = -kx_1 + 2kx_2 - kx_3 \\ m\ddot{x_3} & = -kx_2 + 2kx_3 - kx_4 \\ \cdots & \\ m\ddot{x_8} & = -kx_7 + 2kx_8 - kx_9 \\ m\ddot{x_9} & = -kx_8 + 2kx_9 - kx_{10} \\ m\ddot{x_{10}} & = -kx_9 + 2kx_{10} \end{cases} \tag{17}$$

$$\mathbf{M} \begin{bmatrix} \ddot{x_1} \\ \ddot{x_2} \\ \ddot{x_3} \\ \vdots \\ \ddot{x_8} \\ \ddot{x_9} \\ \ddot{x_{10}} \end{bmatrix} = \mathbf{K_1} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_8 \\ x_9 \\ x_{10} \end{bmatrix} \tag{18}$$

$$\left(\mathbf{K_1} - \omega^2 \mathbf{M_1}\right) = \begin{bmatrix} 2k - \omega^2 m & -k & 0 & \cdots & 0 & 0 & 0 \\ -k & 2k - \omega^2 m & -k & \cdots & 0 & 0 & 0 \\ -k & -k & 2k - \omega^2 m & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 2k - \omega^2 m & -k & 0 \\ 0 & 0 & 0 & \cdots & -k & 2k - \omega^2 m & -k \\ 0 & 0 & 0 & \cdots & 0 & -k & 2k - \omega^2 m \end{bmatrix} \tag{19}$$

$$\mathbf{M}\ddot{\mathbf{x}} = \begin{bmatrix} 2k & -k & 0 & \cdots & 0 & 0 & 0 \\ -k & 2k & -k & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -k & 2k & -k \\ 0 & 0 & 0 & \cdots & 0 & -k & k \end{bmatrix} \mathbf{X} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} f_1 \tag{20}$$

$$\sin\theta \approx \theta \qquad \cos\theta \approx 1 \tag{21}$$

$$\begin{cases} m_1 l_1^2 \ddot{\theta}_1 = -m_1 g l_1 \theta_1 + m_2 g l_1 \left(\theta_2 - \theta_1\right) \\ m_2 l_1 l_2 \ddot{\theta}_1 + m_2 l_2^2 \ddot{\theta}_2 = -m_2 g l_2 \theta_2 \end{cases} \tag{22}$$

$$\theta_1(0) = -0.57\text{rad} \qquad \ddot{\theta}_1(0) = 0\text{rad}/s \tag{23}$$

$$\theta_2(0) = 0\text{rad} \qquad \ddot{\theta}_2 = 0\text{rad}/s \tag{24}$$

$$
\begin{aligned}
\frac{V_m}{V_{in}}(s) &= \frac{1}{m}\frac{p_7}{V_{in}}(s) = \frac{|\mathbf{A}_2\left(\mathbf{V}\right)|}{|\mathbf{A}|} \\
&= \frac{\left(-k_a\left(m + m m_a s\right) + \left(m + \left(ms + b_a\right) m_a\right)\right) s}{m(k + (b + ms)s)\left(s - k_a\right) + m m_a\left(b_a s^2 + \left(k + (b + ms)s\right)\left(s - k_a\right)\right) s}
\end{aligned}
\tag{25}
$$