Computations

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Contents

1 Blast wave Computations

$$f = \frac{1}{\exp\{\beta(u_\tau m_T cosh(y-\eta) - u_r p_T cos(\phi_p - \phi))\} + a} \tag{1}$$

$$\frac{d^3N}{d^2p_T dy} = \frac{1}{(2\pi)^3}\int_0^R rdr \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} \tau d\eta m_T cosh(y-\eta) f \tag{2}$$

The above integral can be simplified to be written in the form

$$\frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy} = A m_T \int_0^1 rdr K_1(\frac{m_T u_\tau}{T_{fo}}) I_0(\frac{p_T u_r}{T_{fo}}) \tag{3}$$

if we take f to the Maxwell Juttner distribution. Here A is some normalisation constant.
We now define the surface velocity

$$\beta_T = \frac{u_r}{u_\tau} \tag{4}$$

He we use the model

$$u_r = u_0 \frac{r}{R}$$

where R is the maximum freeze out radius. Let

$$\frac{r}{R} = \xi$$

where

$$0 < \xi < 1$$

. We can now write the expression for transverse velocity

$$\beta_T = \frac{u_0 \xi}{\sqrt{1 + (u_0\xi)^2}}. \tag{5}$$

The maximum surface velocity is given when $\xi = 1$

$$\beta_s = \frac{u_0}{\sqrt{1 + (u_0)^2}} \tag{6}$$

We will have to find the average value of β_T [4]

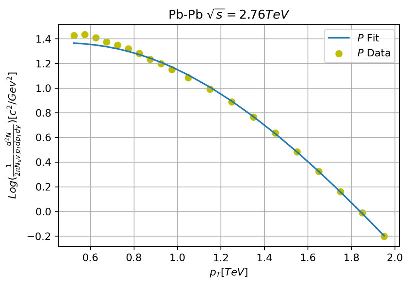
$$\langle \beta_T \rangle = 2\int_0^1 \frac{u_0\xi}{\sqrt{1+(u_0\xi)^2}} \xi d\xi \tag{7}$$

where the factor of 2 comes from dividing by the term

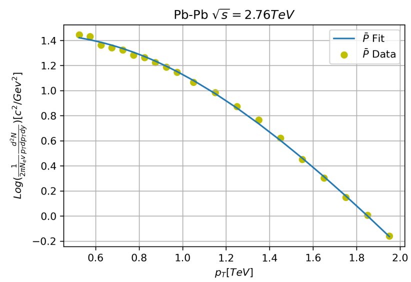
$$\int_0^1 \xi d\xi = \frac{1}{2}$$

The average value can then be computed to be given by the function

$$\langle \beta_T \rangle = 2u_0 \left(\frac{\sqrt{1+u_0}}{u_0} - \frac{\sinh \sqrt{u_0}}{u_0^{3/2}} \right) \tag{8}$$

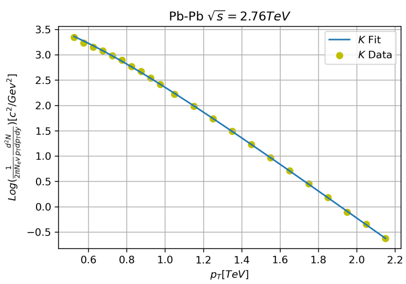


(a) P

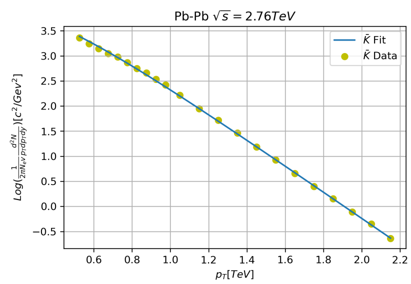


(b) $\bar{P}$

Figure 1: Data fit for proton and anti-proton



(a) K



(b) $\bar{K}$

Figure 2: Data fit for Kon and anti-Kon

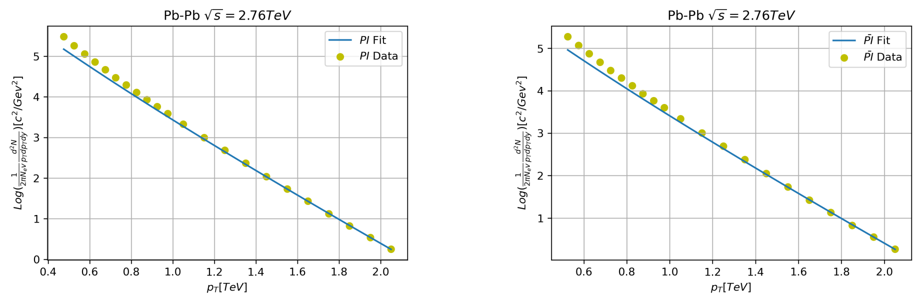
2 Blast wave Fit Results

We have taken the data set from [1]. The Blast wave model fit was used to extract the radial flow velocity with freeze-out temperature fixed at 0.13 GeV for Pb-Pb collision at $\sqrt{s_{NN}} = 2.76$ TeV. The values for various rapidity are given in the tables below.

The corresponding fit values is given in the table

Particle	$\langle\beta_T\rangle$	β_{max}	χ^2_{red}
P	0.542	0.768	19.3
$\bar{P}$	0.538	0.765	16.6
PI	0.546	0.772	60.0
$\bar{PI}$	0.547	0.773	60.7
K	0.554	0.779	8.88
$\bar{K}$	0.551	0.776	8.87

For 20-30 % centrality [2] we have the results



(a) PI (b) $\bar{P}I$

Figure 3: Data fit for pion and anti-pion

2.1 Elliptic Flow

The elliptic flow is computed in the Blast Wave model as

$$v_n(p_T) = \frac{\int_0^{2\pi} d\phi cos\,(n\phi)\, \frac{d^3N}{p_T dp_T d\phi dy}}{\int_0^{2\pi} d\phi \frac{d^3N}{p_T dp_T d\phi dy}} \tag{9}$$

$$v_2(p_T) = \frac{1}{N(p_T, y)} \frac{A}{(2\pi)^3} \int_0^{2\pi} d\phi cos\,(n\phi) \int_0^R rdr \int_{-\infty}^{\infty} \tau d\eta m_T cosh(y-\eta) e^{-\beta(u_\tau m_T cosh(y-\eta) - u_r p_T cos(\phi_p - \phi))} \tag{10}$$

The η integral can be done to give a Modified Bessel function of the second kind. We have re-scaled the value of R and absorbed it into the normalisation.

$$v_2(p_T) = \frac{1}{N(p_T, y)} \frac{A\tau}{(2\pi)^3} \int_0^{2\pi} d\phi \int_0^1 rdr K_1(\beta u_\tau m_T) \int_0^{2\pi} d\phi_p cos\,(2\phi_p)\, e^{\beta(u_r p_T cos(\phi_p - \phi))} \tag{11}$$

By making a change of variables

$$\phi_p - \phi = \psi$$

we have,

$$v_2(p_T) = \frac{1}{N(p_T, y)} \frac{A\tau}{(2\pi)^3} \int_0^{2\pi} d\phi \int_0^1 rdr K_1(\beta u_\tau m_T) \int_0^{2\pi} d\psi cos\,(2(\psi + \phi))\, e^{\beta(u_r p_T cos(\psi))} \tag{12}$$

Using

$$cos\,(n(\psi + \phi)) = cos\,(2\psi)\, cos\,(2\phi) - sin\,(2\psi)\, sin\,(2\phi)$$

we have two integrals

$$cos\,(2\phi)\int_0^{2\pi} d\psi cos\,(2\psi)\,e^{\beta u_r p_T cos(\psi)} = 2\pi I_2(\beta u_r p_T)$$

and

$$sin\,(2\phi)\int_0^{2\pi} d\psi sin\,(2\psi)\,e^{\beta u_r p_T cos(\psi)} = 0$$

. The $sin\,(2\psi)$ integral is zero as the exponential term is even in ψ. This reduces our final integral to be

$$v_2(p_T) = \frac{1}{N(p_T,y)}\frac{A\tau 2\pi}{(2\pi)^3}\int_0^{2\pi} d\phi cos\,(2\phi)\int_0^1 rdrK_1(\beta u_\tau m_T)I_2(\beta u_r p_T) \quad (13)$$

Now,

$$N(p_T,y) = \frac{A\tau 2\pi}{(2\pi)^3}\int_0^{2\pi} d\phi \int_0^1 rdrK_1(\beta u_\tau m_T)I_0(\beta u_r p_T) \quad (14)$$

This gives,

$$v_2(p_T) = \frac{\int_0^{2\pi} d\phi cos\,(2\phi)\int_0^1 rdrK_1(\beta u_\tau m_T)I_2(\beta u_r p_T)}{\int_0^{2\pi} d\phi \int_0^1 rdrK_1(\beta u_\tau m_T)I_0(\beta u_r p_T)} \quad (15)$$

We took v_2 data from [3]

References

[1] Betty Abelev et al. Pion, Kaon, and Proton Production in Central Pb–Pb Collisions at $\sqrt{s_{NN}} = 2.76$ TeV. *Phys. Rev. Lett.*, 109:252301, 2012.

[2] Betty Abelev et al. Centrality dependence of π, K, p production in Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV. *Phys. Rev. C*, 88:044910, 2013.

[3] Betty Bezverkhny Abelev et al. Elliptic flow of identified hadrons in Pb-Pb collisions at $\sqrt{s_{\mathrm{NN}}} = 2.76$ TeV. *JHEP*, 06:190, 2015.

[4] K. Adcox et al. Single identified hadron spectra from $\sqrt{s_{NN}} = 130$-GeV Au+Au collisions. *Phys. Rev. C*, 69:024904, 2004.