EFormer: An Effective Edge-based Transformer for Vehicle Routing Problems

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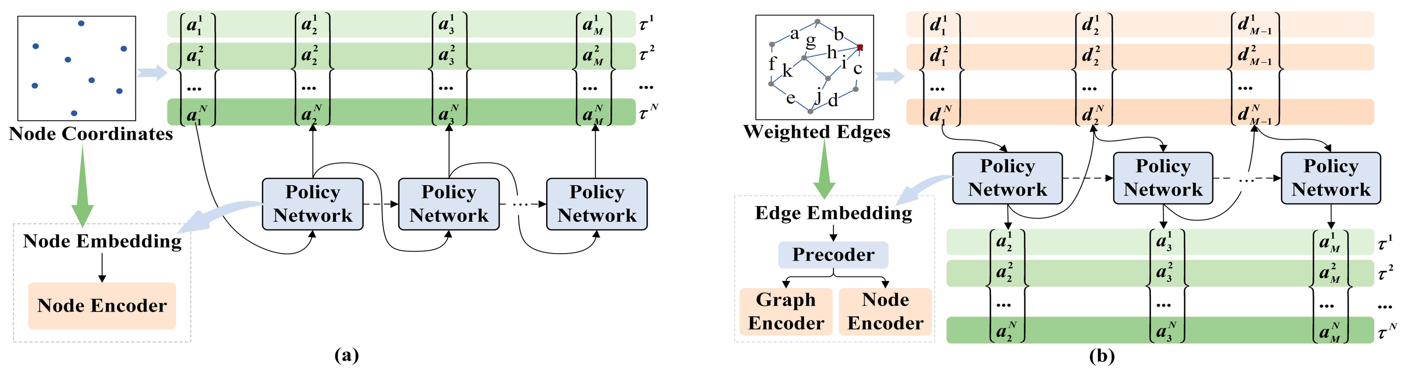
Introduction

Vehicle Routing Problem (VRP)

- ☐ A fundamental NP-hard combinatorial optimization problem(COP) with broad applications.
- ☐ Remains highly challenging due to inherent computational complexity.

Motivation

- ☐ Recently, neural heuristics with deep RL show strong efficiency and generalization.
- ☐ Most existing methods rely heavily on node coordinates, limiting robustness.
- ☐ In practice, edge information (distances, costs) is more available than precise coordinates.
- ☐ Coordinate-based methods struggle when inputs deviate from ideal conditions.



- ☐ **(a) Node-based methods:** Most neural heuristics capture the structure of routing problems by treating the coordinates of instances as node features.
- ☐ **(b) Edge-based method (EFormer):** Our work focuses on edge-based methods and proposes a novel neural network model architecture that improves the solution quality.

Experiments

☐ Generalization to TSPLIB and CVRPLIB.

Method	TSPLIB1-100 Len.	Gap(%)	TSPLIB101-300 Len.	Gap(%)	TSPLIB301-500 Len.	Gap(%)
OPT	19499.583	0.000	40129.375	0.000	43694.666	0.000
GCN-Greedy	25371.528	30.242	63125.529	55.716	81143.342	83.080
GCN-BS	23906.973	22.631	58213.359	46.951	76546.164	72.946
GCN-BS*	21195.267	10.824	55853.973	39.104	72865.950	64.648
MatNet(×1)	20076.205	3.880	46876.768	17.868	63544.470	46.192
MatNet(×8)	19761.313	1.786	45638.627	14.479	62427.744	42.206
EFormer(×1)	19741.575	1.698	45396.807	13.599	61415.142	36.562
EFormer(×8)	19662.845	**0.837**	43360.592	**8.805**	56111.793	**26.924**

Method	CVRPLIB1-100 Len.	Gap(%)	CVRPLIB101-300 Len.	Gap(%)	CVRPLIB301-500 Len.	Gap(%)
OPT	925.800	0.000	33184.483	0.000	82903.857	0.000
GCN-Greedy	1233.255	34.732	44684.202	45.069	138044.859	63.597
GCN-BS	1167.058	27.909	42773.044	38.466	120532.679	49.664
GCN-BS*	1087.529	18.369	41403.524	34.012	114962.535	42.366
MatNet(×1)	1098.828	21.017	38946.522	20.351	103278.318	24.309
MatNet(×8)	1072.623	14.901	38218.037	17.657	98912.043	19.460
EFormer(×1)	1107.504	18.766	38477.709	12.485	98010.813	21.023
EFormer(×8)	1019.698	**9.885**	36968.746	**9.784**	94612.575	**15.325**

☐ Generalization to larger-scale instances.

Method	TSP100 Len.	Gap(%)	Time(m)	TSP200 Len.	Gap(%)	Time(m)	TSP300 Len.	Gap(%)	Time(m)	TSP500 Len.	Gap	Time(m)
Concorde	7.731	0.000	8.42	10.704	0.000	16.01	12.934	0.000	23.02	16.522	0.000	35.12
GCN-Greedy	8.457	9.919	0.07	12.324	15.141	0.28	16.270	25.786	0.85	26.141	58.226	2.87
GCN-BS	8.111	7.506	0.08	12.133	13.351	0.29	15.964	23.424	0.85	25.686	55.468	2.87
GCN-BS*	7.903	2.232	0.64	11.980	11.927	2.29	15.841	22.474	5.33	25.364	53.518	23.69
MatNet(×1)	7.786	0.711	0.04	11.395	6.459	0.16	14.930	15.431	0.72	22.511	36.253	2.12
MatNet(×8)	7.747	0.213	0.08	11.282	5.408	0.35	14.766	14.159	1.08	22.199	34.364	3.89
EFormer(×1)	7.754	0.302	0.04	11.328	5.837	0.25	15.077	16.563	1.02	21.281	28.807	2.12
EFormer(×8)	7.739	**0.113**	0.11	11.136	**4.043**	0.42	14.508	**12.170**	1.86	20.397	**23.457**	5.68

Method	CVRP100 Len.	Gap(%)	Time(m)	CVRP200 Len.	Gap(%)	Time(m)	CVRP300 Len.	Gap(%)	Time(m)	CVRP500 Len.	Gap(%)	Time(m)
LKH3	15.594	0.000	1.11h	20.173	0.000	2.5h	26.028	0.000	3.6h	37.229	0.000	5.8h
GCN-Greedy	16.860	8.112	0.10	22.606	12.063	0.28	33.889	30.202	1.33	49.550	33.096	5.64
GCN-BS	16.442	5.438	0.07	22.282	10.454	0.28	33.213	27.605	1.87	48.923	31.410	3.51
GCN-BS*	16.128	3.427	1.45	22.066	9.387	1.85	32.802	26.025	4.17	48.311	29.767	9.01
MatNet(×1)	16.220	4.017	0.05	21.982	8.971	0.14	31.069	19.365	0.34	46.908	25.999	1.95
MatNet(×8)	16.075	3.086	0.09	21.715	7.648	0.34	30.448	16.982	0.94	46.107	23.845	3.59
EFormer(×1)	15.809	1.381	0.06	20.831	3.266	0.16	28.211	8.387	0.43	42.708	14.716	2.13
EFormer(×8)	15.730	**0.876**	0.10	20.665	**2.443**	0.47	27.723	**6.511**	1.27	41.647	**11.865**	5.44

☐ Generalization to different distributions.

Method	TSP20 Len.	Gap(%)	Time(m)	TSP50 Len.	Gap(%)	Time(m)	TSP100 Len.	Gap(%)	Time(m)
Distribution				Explosion					
Concorde	3.559	0.000	3.43	4.620	0.000	21.53	5.830	0.000	66.11
GCN Greedy	3.673	3.180	0.46	4.931	6.610	1.24	6.491	11.340	7.88
GCN-BS	3.591	0.898	0.44	4.686	1.43	1.21	6.388	9.564	8.48
GCN-BS*	3.560	0.008	34.85	4.645	0.546	41.29	6.247	7.144	78.05
MatNet(×1)	3.561	0.044	0.11	4.643	0.495	0.13	5.981	2.587	0.52
MatNet(×8)	3.560	0.001	0.33	4.626	0.121	1.91	5.915	1.450	5.28
EFormer(×1)	3.560	0.022	0.04	4.630	0.225	0.26	5.905	1.293	1.26
EFormer(×8)	3.559	**0.000**	0.24	4.623	**0.058**	1.34	5.864	**0.586**	6.21
Distribution				Grid					
Concorde	3.845	0.000	3.43	5.969	0.000	21.43	7.790	0.000	66.23
GCN Greedy	3.957	2.912	0.46	5.970	4.915	1.02	8.708	11.781	4.09
GCN-BS	3.876	0.816	0.44	5.732	0.745	1.18	8.322	6.826	4.31
GCN-BS*	3.845	0.004	34.29	5.700	0.167	34.14	8.171	4.892	61.15
MatNet(×1)	3.847	0.041	0.11	5.709	0.336	0.13	7.917	1.633	0.52
MatNet(×8)	3.845	0.001	0.34	5.695	0.085	1.24	7.853	0.805	6.12
EFormer(×1)	3.846	0.017	0.04	5.700	0.167	0.26	7.815	0.325	1.26
EFormer(×8)	3.845	**0.000**	0.24	5.693	**0.055**	1.34	7.799	**0.110**	6.17
Distribution				Implosion					
Concorde	3.785	0.000	2.48	5.600	0.000	21.64	7.610	0.000	66.45
GCN Greedy	3.899	3.024	0.47	5.881	5.02	1.15	8.636	13.476	4.75
GCN-BS	3.814	0.771	0.43	5.645	0.797	1.21	8.285	8.866	5.59
GCN-BS*	3.785	0.004	34.61	5.610	0.176	35.46	8.137	6.919	60.45
MatNet(×1)	3.786	0.457	0.11	5.620	0.351	0.13	7.740	1.709	0.52
MatNet(×8)	3.785	0.006	0.34	5.605	0.092	1.91	7.675	0.850	4.03
EFormer(×1)	3.785	0.019	0.04	5.609	0.167	0.26	7.636	0.335	1.26
EFormer(×8)	3.785	**0.001**	0.23	5.603	**0.056**	1.31	7.619	**0.122**	6.32

Method	CVRP20 Len.	Gap(%)	Time(m)	CVRP50 Len.	Gap(%)	Time(m)	CVRP100 Len.	Gap(%)	Time(m)
Distribution				Explosion					
LKH3	5.748	0.000	2.35h	8.740	0.000	8.52h	12.302	0.000	13.46h
GCN Greedy	5.874	2.191	0.24	9.414	7.714	1.64	13.394	8.713	4.57
GCN-BS	5.836	1.537	0.24	9.258	5.927	1.95	13.176	6.949	5.78
GCN-BS*	5.795	0.826	20.85	8.972	2.651	45.46	12.988	5.425	65.51
MatNet(×1)	5.806	1.020	0.11	9.131	4.470	0.23	12.933	4.979	0.88
MatNet(×8)	5.780	0.568	0.52	8.985	2.803	1.13	12.762	3.585	4.70
EFormer(×1)	5.783	0.616	0.05	8.834	1.078	0.25	12.535	1.749	0.98
EFormer(×8)	5.758	**0.183**	0.28	8.792	**0.595**	1.69	12.458	**1.123**	6.86
Distribution				Grid					
LKH3	6.147	0.000	2.15h	10.400	0.000	8.52h	15.640	0.000	13.46h
GCN Greedy	6.432	4.633	0.49	10.915	4.953	1.73	16.432	5.065	3.93
GCN-BS	6.370	3.627	0.38	10.723	3.104	1.82	16.257	3.947	4.18
GCN-BS*	6.313	2.700	21.18	10.657	2.466	44.49	16.121	3.074	73.07
MatNet(×1)	6.208	0.978	0.15	10.832	4.153	0.34	16.282	4.104	0.52
MatNet(×8)	6.181	0.538	0.71	10.678	2.677	1.68	16.115	3.036	4.65
EFormer(×1)	6.185	0.605	0.05	10.502	0.984	0.25	15.842	1.291	0.98
EFormer(×8)	6.159	**0.180**	0.30	10.459	**0.569**	1.65	15.773	**0.853**	6.86
Distribution				Implosion					
LKH3	6.061	0.000	2.15h	10.260	0.000	8.52h	15.440	0.000	13.46h
GCN Greedy	6.247	3.072	0.24	10.755	4.824	1.47	16.297	5.549	4.46
GCN-BS	6.184	2.037	0.24	10.676	4.056	1.52	16.023	3.776	4.63
GCN-BS*	6.133	1.184	20.84	10.547	2.793	45.09	15.928	3.162	65.87
MatNet(×1)	6.121	0.992	0.09	10.691	4.196	0.13	16.080	4.148	0.78
MatNet(×8)	6.095	0.557	0.51	10.540	2.732	1.18	15.917	3.089	4.74
EFormer(×1)	6.098	0.610	0.06	10.362	0.993	0.25	15.644	1.323	0.98
EFormer(×8)	6.071	**0.176**	0.32	10.319	**0.575**	1.72	15.574	**0.871**	6.86

- ☐ **Synthetic:** Uniform, Explosion, Grid, Implosion.
- ☐ **Real-world benchmarks:** TSPLib and CVRPLib.
- ☐ **Baselines:** GCN-BS, MatNet, GREAT.

Method	TSP20 Len.	Gap(%)	Time(m)	TSP50 Len.	Gap(%)	Time(m)	TSP100 Len.	Gap(%)	Time(m)
Concorde	3.831	0.000	4.43	5.692	0.000	23.53	7.763	0.000	66.45
LKH3	3.831	0.000	2.78	5.692	0.000	17.21	7.763	0.000	49.56
OR-Tools	3.864	0.864	1.16	5.851	2.795	10.75	8.057	3.782	39.05
GCN-Greedy	3.948	3.078	0.32	5.968	4.856	1.34	8.537	9.966	4.09
GCN-BS	3.862	0.825	0.81	5.732	0.712	4.33	8.170	5.234	4.31
GCN-BS*	3.831	0.004	21.41	5.700	0.141	35.46	7.955	2.477	63.05
MatNet(×1)	3.832	0.044	0.11	5.709	0.303	0.13	7.836	0.940	0.52
MatNet(×8)	3.831	0.002	0.22	5.694	0.050	1.24	7.795	0.410	5.28
MatNet(×128)	3.831	0.000	5.71	5.692	0.013	16.47	7.776	0.170	60.11
GREAT(×1)#	-	-	-	-	-	-	7.850	1.210	2.00
GREAT(×8)#	-	-	-	-	-	-	7.820	0.810	18.00
EFormer(×1)	3.831	0.018	0.04	5.699	0.130	0.26	7.788	0.324	1.22
EFormer(×8)	3.831	0.000	0.15	5.692	0.011	1.34	7.772	0.115	6.81
EFormer(×128)	3.831	**0.000**	4.72	5.692	**0.001**	25.81	7.767	**0.045**	66.55

Method	CVRP20 Len.	Gap(%)	Time(m)	CVRP50 Len.	Gap(%)	Time(m)	CVRP100 Len.	Gap(%)	Time(m)
LKH3	6.117	0.000	2.15h	10.347	0.000	8.52h	15.647	0.000	13.46h
HGS	6.112	-0.079	1.48h	10.347	-0.001	4.67h	15.584	-0.401	6.54h
OR-Tools	6.414	4.863	2.37	11.219	8.430	19.35	17.172	9.749	2.61h
GCN-Greedy	6.471	5.794	0.27	11.130	7.567	2.05	16.948	8.314	5.24
GCN-BS	6.284	2.740	0.26	10.786	4.248	2.11	16.487	5.371	5.45
GCN-BS*	6.192	1.232	20.78	10.636	2.796	38.36	16.243	3.811	78.78
MatNet(×1)	6.172	0.907	0.11	10.787	4.253	0.21	16.280	4.401	1.02
MatNet(×8)	6.146	0.469	0.58	10.635	2.787	1.23	16.117	3.356	4.70
MatNet(×128)	6.131	0.229	9.93	10.538	1.847	17.93	15.989	2.530	66.05
EFormer(×1)	6.147	0.490	0.04	10.457	1.067	0.25	15.844	1.259	0.98
EFormer(×8)	6.123	0.098	0.28	10.414	0.650	1.69	15.776	0.830	6.86
EFormer(×128)	6.116	**-0.017**	14.84	10.393	**0.447**	24.47	15.735	**0.563**	85.65

- ☐ EFormer outperforms all edge-based methods under greedy (×1) and augmentation (×8, ×128).
- ☐ ×8 augmentation already surpasses MatNet ×128.

Generalization

- Robust across distributions, larger-scale instances, and real-world benchmarks.
- Demonstrates strong generalization even with edge-only inputs.

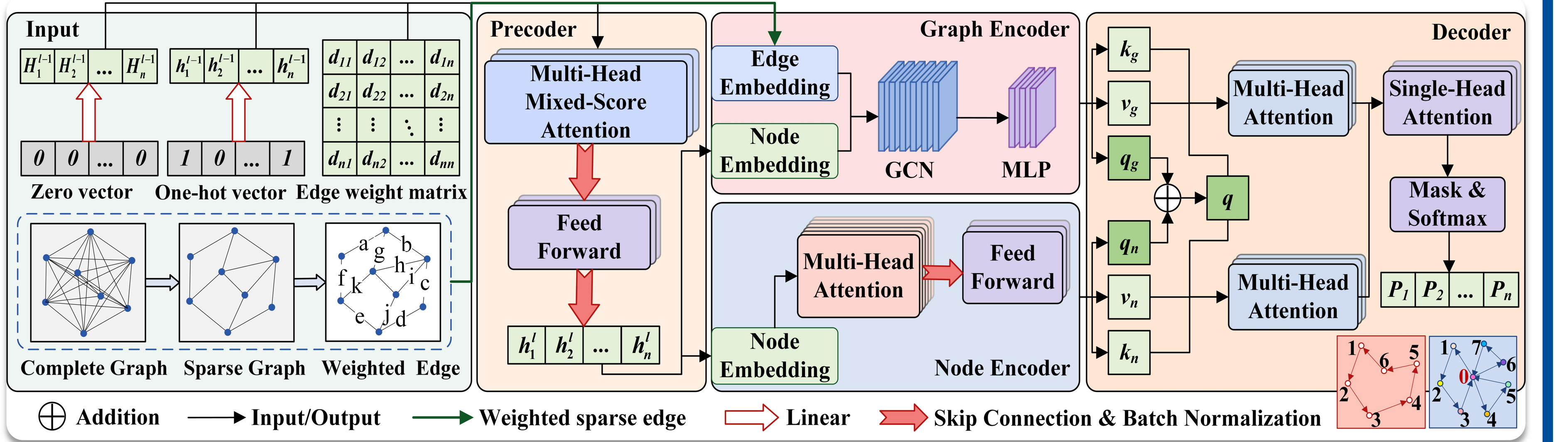
Method: EFormer

EFormer Architecture

☐ **Precoder:**
- Edge → temporary nodes via mixed-score attention.
- Enables stochastic instance augmentation.

☐ **Parallel encoders:**
- Graph Encoder with residual GCNs and MLP for sparse edge embeddings.
- Node Encoder with attention for temporary nodes.

☐ **Decoder:**
- Parallel context embedding and multi-query integration.
- Masked Softmax for autoregressive path construction.

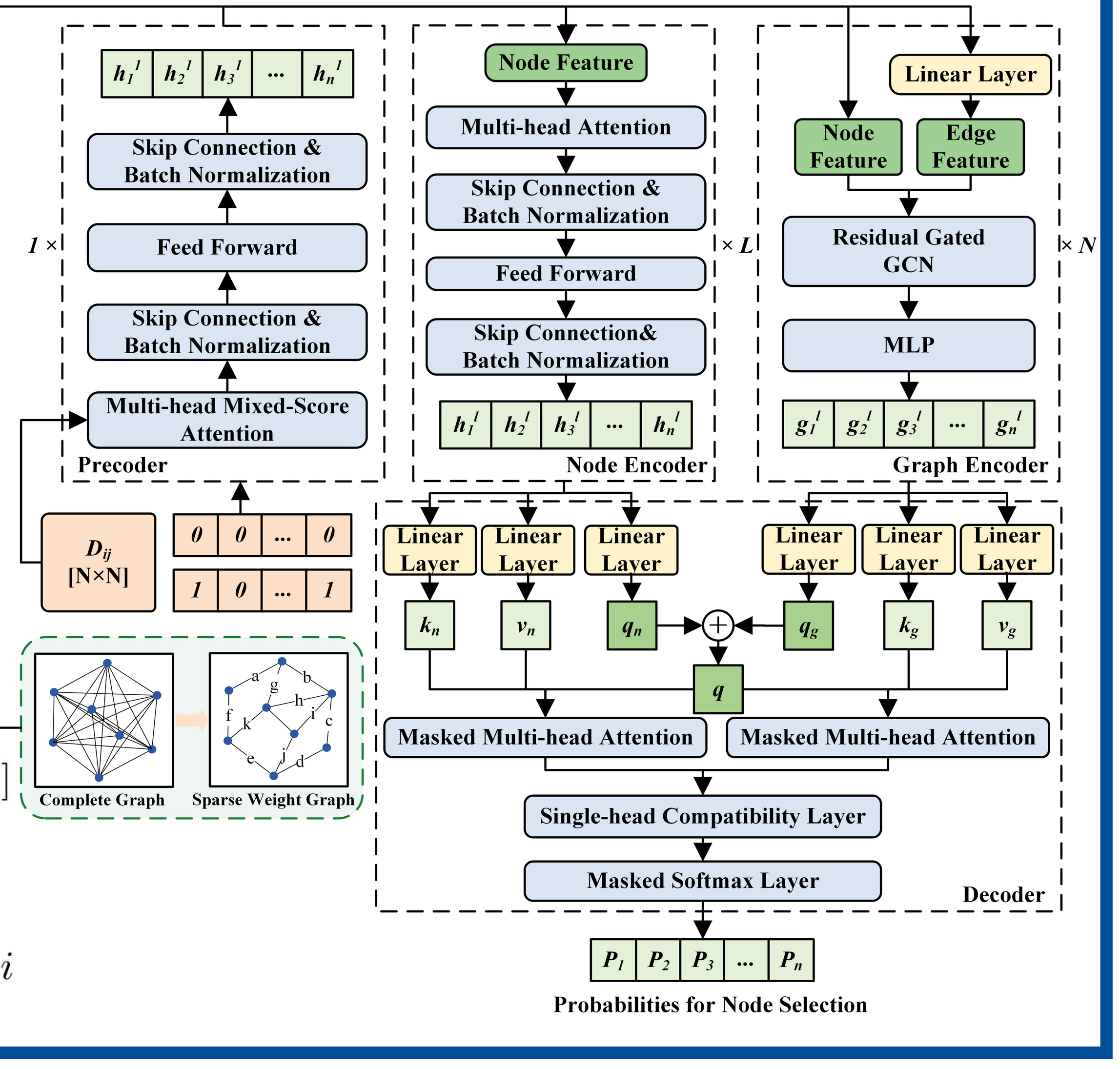


☐ **Precoder: Instance Augmentation**
- Re-encodes each instance into multiple stochastic variants.
- Supports flexible ×N augmentation (vs. POMO's fixed ×8).

☐ **Graph & Node Encoder: Neither Can Be Omitted**
- Graph Encoder uses GCN layers and Node Encoder uses attention layers.
- Complementary modules—GCNs capture rich edge features, while Node Encoder with attention provides efficient node embeddings; both are indispensable.

Trainer

- EFormer integrates into autoregressive solvers.
- Uses **REINFORCE** algorithm with shared baseline.
- Samples n trajectories per batch to compute rewards.
- Applies gradient ascent with score clipping and normalization for stable training.



☐ **Gradient approximation:**

$$\nabla_\theta \mathcal{L}(\theta) \approx \frac{1}{n}\sum_{i=1}^{n}[(f(\tau^i) - b^i(s))\nabla \log p_\theta(\tau^i|s)]$$

☐ **Share baseline:**

$$b^i(s) = b_{\text{shared}}(s) = \frac{1}{n}\sum_{i=1}^{n} f(\tau^i), \text{ for all } i$$

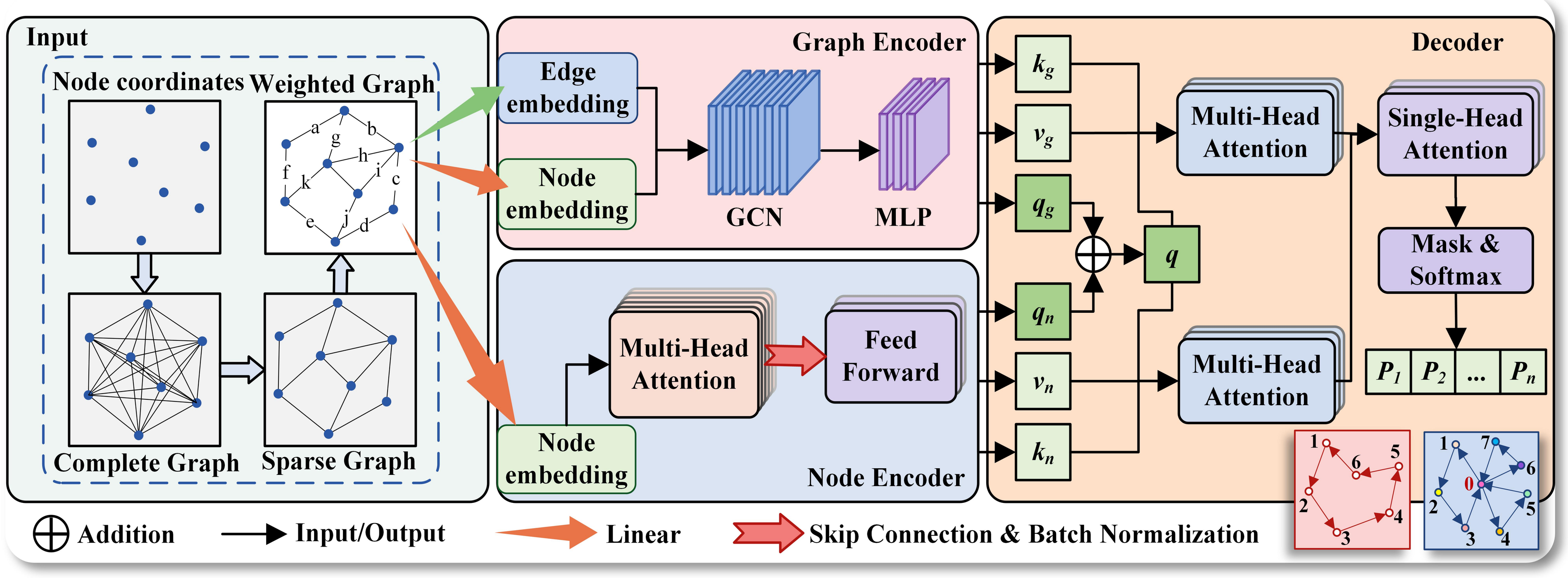
Variant: EFormer-node

Variant: EFormer-node

- ☐ Scalable variant using node coordinates as input.
- ☐ Retains EFormer core; Precoder omitted.

Architecture

- ☐ Graph Encoder, Node Encoder, Decoder.
- ☐ Directly inputs node coords & edge weights → higher efficiency & adaptability.



Method	TSP20 Len.	Gap(%)	Time(m)	TSP50 Len.	Gap(%)	Time(m)	TSP100 Len.	Gap(%)	Time(m)
Concorde	3.831	0.000	4.43	5.692	0.000	23.53	7.763	0.000	66.45
LKH3	3.831	0.000	2.78	5.692	0.000	17.21	7.763	0.001	49.56
OR-Tools	3.864	0.864	1.16	5.851	2.795	10.75	8.057	3.782	39.05
GCN Greedy	3.856	0.650	0.25	5.901	3.678	1.21	8.413	8.373	6.25
GCN-BS	3.835	0.128	0.81	5.710	0.317	4.62	7.931	2.155	17.73
GCN-BS*	3.831	0.000	21.24	5.694	0.041	37.63	7.869	1.368	58.34
POMO(×1)	3.832	0.026	0.08	5.698	0.119	0.24	7.796	0.364	1.03
POMO(×8)	3.831	0.001	0.12	5.693	0.024	0.45	7.775	0.142	2.11
Pointerformer(×1)	3.833	0.067	0.09	5.714	0.391	0.19	7.859	1.229	0.67
Pointerformer(×8)	3.831	0.001	0.18	5.693	0.026	0.52	7.776	0.163	1.51
LEHD Greedy	3.867	0.961	0.14	5.721	0.519	0.24	7.808	0.577	1.02
LEHD RRC10	3.834	0.091	0.35	5.697	0.095	1.10	7.775	0.145	2.12
LEHD RRC50	3.831	0.004	0.62	5.692	0.010	2.45	7.766	0.033	7.93
EFormer(×1)	3.831	0.018	0.04	5.699	0.130	0.26	7.788	0.324	1.22
EFormer(×8)	3.831	0.000	0.15	5.692	0.011	1.34	7.772	0.115	6.81
EFormer(×128)	3.831	**0.000**	4.72	5.692	**0.000**	25.81	7.767	**0.045**	66.55
EFormer-node(×1)	3.831	0.008	0.04	5.695	0.055	0.24	7.772	0.113	1.20
EFormer-node(×8)	3.831	**0.000**	0.18	5.692	**0.005**	1.59	7.765	**0.027**	6.73

Method	CVRP20 Len.	Gap(%)	Time(m)	CVRP50 Len.	Gap(%)	Time(m)	CVRP100 Len.	Gap(%)	Time(m)
LKH3	6.117	0.000	2.15h	10.347	0.000	8.52h	15.647	0.000	13.46h
HGS	6.112	-0.079	1.48h	10.347	-0.001	4.67h	15.584	-0.401	6.54h
OR-Tools	6.414	4.863	2.37	11.219	8.430	19.35	17.172	9.749	2.61h
POMO(×1)	6.160	0.698	0.06	10.533	1.799	0.18	15.837	1.216	0.64
POMO(×8)	6.132	0.254	0.20	10.437	0.875	0.48	15.754	0.689	2.11
LEHD Greedy	6.462	5.647	0.07	10.872	5.075	0.18	16.217	3.648	0.55
LEHD RRC10	6.222	1.718	0.31	10.586	2.307	0.69	15.885	1.529	2.33
LEHD RRC50	6.140	0.375	1.57	10.430	0.803	2.95	15.729	0.524	9.10
EFormer(×1)	6.147	0.490	0.04	10.457	1.067	0.25	15.844	1.259	0.98
EFormer(×8)	6.123	0.098	0.28	10.414	0.650	1.69	15.776	0.830	6.86
EFormer(×128)	6.116	**-0.017**	14.84	10.393	**0.447**	24.47	15.735	**0.563**	85.65
EFormer-node(×1)	6.141	0.388	0.06	10.455	1.045	0.22	15.795	0.950	0.96
EFormer-node(×8)	6.124	**0.114**	0.27	10.411	**0.625**	1.54	15.725	**0.498**	6.84

High Performance

- ☐ Edge-based EFormer also exceeds some node-based methods.
- ☐ Outperforms LEHD RRC50 at equal inference time.
- ☐ EFormer-node surpasses all neural baselines under greedy and augmented inference.

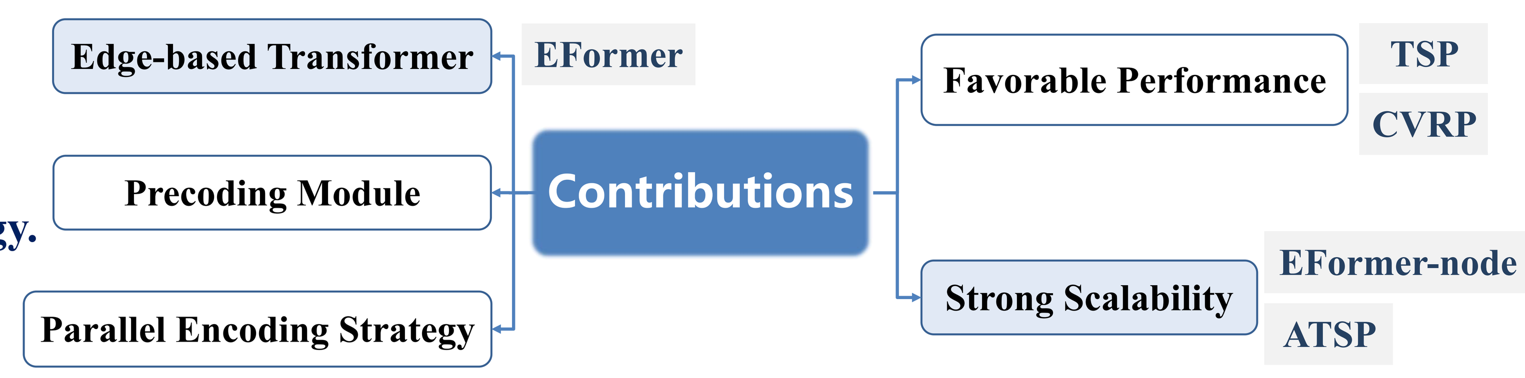
Framework Flexibility

- ☐ EFormer extends seamlessly to EFormer-node and ATSP, showing strong adaptability, efficiency, and robustness.

Conclusion

Contributions

- ☐ **Edge-based Transformer.**
- ☐ **Precoding module.**
- ☐ **Parallel Encoding Strategy.**
- ☐ **Favorable Performance.**
- ☐ **Strong Scalability.**



- ☐ **Looking ahead,** a promising direction is to develop a unified learning-based lightweight framework that can operate effectively on both edge and node.

Ads & Links

- For more information or questions, please contact us.
- Email: mengdian@mail.dlut.edu.cn
- Links of WeChat, Paper, and Code.