Quantitative Modeling Assignment
Module 9 – Goal Programming

1. The Research and Development Division of the Emax Corporation has developed three new products. A decision now needs to be made on which mix of these products should be produced. Management wants primary consideration given to three factors: total profit, stability in the workforce, and achieving an increase in the company's earnings next year from the $75 million achieved this year. In particular, using the units given in the following table, they want to

   Maximize Z = P - 6C - 3D, where

   P = total (discounted) profit over the life of the new products,

   C = change (in either direction) in the current level of employment,

   D = decrease (if any) in next year's earnings from the current year's level.

The amount of any increase in earnings does not enter into Z, because management is concerned primarily with just achieving some increase to keep the stockholders happy. (It has mixed feelings about a large increase that then would be difficult to surpass in subsequent years.)

The impact of each of the new products (per unit rate of production) on each of these factors is shown in the following table:

	Unit Contribution				
	Product:				
Factor	**1**	**2**	**3**	**Goal**	**Units**
Total profit	20	15	25	Maximize	Millions of dollars
Employment level	6	4	5	= 50	Hundreds of employees
Earnings next year	8	7	5	≥ 75	Millions of dollars

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Questions

1) Define y_1^+ and y_1^-, respectively, as the amount over (if any) and the amount under (if any) the employment level goal. Define y_2^+ and y_2^- in the same way for the goal regarding earnings next year. Define x_1, x_2, and x_3 as the production rates of Products 1, 2, and 3, respectively. With these definitions, use the goal programming technique to express y_1^+, y_1^-, y_2^+ and y_2^- algebraically in terms of x_1, x_2, and x_3. Also express P in terms of x_1, x_2, and x_3.

Solution:

Variables:

y1 = 6 x1 + 4 x2 + 5 x3
y2 = 8 x1 + 7 x2 + 5 x3

y1 and y2 can be positive or negative

$$y_i = y_i^+ - y_i^- \quad \forall i = 1, 2, 3$$

$$y_i^+ = \begin{cases} y_i & \text{if } y_i \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$y_i^- = \begin{cases} |y_i| & \text{if } y_i \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

Formulation:

Maximize Z = P - 6C - 3D

6 x1 + 4 x2 + 5 x3 = 50
8 x1 + 7 x2 + 5 x3 >= 75

($Y1^+$ - $Y1^-$) = 6 x1 + 4 x2 + 5 x3 - 50
($Y2^+$ - $Y2^-$) = 8 x1 + 7 x2 + 5 x3 – 75

6 x1 + 4 x2 + 5 x3 - $Y1^+$ - $Y1^-$ = 50
8 x1 + 7 x2 + 5 x3 - $Y2^+$ - $Y2^-$ >= 75

- We are coding $Y1^+$ - $Y1^-$ and $Y2^+$ - $Y2^-$ as y1m, y1p, y2m, y2p

❖ The LP file will look like this formulation:

<u>For first case:</u>

Max: 20 x1 + 15 x2 + 25 x3 - 6 y1p - 6 y1m - 3 y2m
s.t
6 x1 + 4 x2 + 5 x3 + y1m - y1p = 50
8 x1 + 7 x2 + 5 x3 + y2m - y2p = 75

<u>For the Streamlined case :</u>

Max: 20 x1 + 15 x2 + 25 x3 - 1000 y1p - 6 y1m - 3 y2m
s.t
6 x1 + 4 x2 + 5 x3 + y1m - y1p = 50
8 x1 + 7 x2 + 5 x3 + y2m - y2p = 75