FUNDAMENTALS OF MACHINE LEARNING
AA 2022-2023
Prova Intermedia (FACSIMILE)

3 Novembre, 2022

Istruzioni: Niente libri, niente appunti, niente dispositivi elettronici, e niente carta per appunti. Usare matita o penna di qualsiasi colore. Usare lo spazio fornito per le risposte.
Instructions: No books, no notes, no electronic devices, and no scratch paper. Use pen or pencil. Use the space provided for your answers.
This exam has 5 questions, for a total of 100 points and 10 bonus points.

Nome: ______________________________

Matricola: ______________________________

1. **Multiple Choice**: Select the correct answer from the list of choices.
   (a) [5 points] True or False: Using the kernel trick, we can get non-linear decision boundaries using algorithms designed originally for linear models. ◯ True ◯ False
   (b) [5 points] True or False: A zero-mean Gaussian Prior (i.e. $p(\mathbf{w}) = \mathcal{N}(\mathbf{0}, \sigma I)$) on model parameters in a MAP estimate corresponds to adding an L2 regularization term (e.g. $||\mathbf{w}||_2$) to the loss in an MLE estimate. ◯ True ◯ False
   (c) [5 points] True or False: In the Primal Form of the SVM, increasing hyperparameter C will decrease the complexity of the resulting classifier. ◯ True ◯ False
   (d) [5 points] True or False: The Maximum a Priori and Maximum likelihood solution for linear regression are always equivalent. ◯ True ◯ False
   (e) [5 points] If a hard-margin support vector machine tries to minimize $||\mathbf{w}||_2$ subject to $y_n(\mathbf{w}^T\mathbf{x}_n + b) \geq 2$, what will be the size of the margin?
   ◯ $\frac{1}{||\mathbf{w}||}$ ◯ $\frac{2}{||\mathbf{w}||}$ ◯ $\frac{1}{2||\mathbf{w}||}$ ◯ $\frac{1}{4||\mathbf{w}||}$
   (f) [5 points] The posterior distribution of B given A is:
   - ◯ $P(B \mid A) = \frac{P(A|B)P(A)}{P(B)}$
   - ◯ $P(B \mid A) = \frac{P(A,B)P(B)}{P(A)}$
   - ◯ $P(B \mid A) = \frac{P(A|B)P(B)}{P(A)}$
   - ◯ $P(B \mid A) = \frac{P(A|B)P(B)}{P(A,B)}$

   (g) [5 points] Let $\mathbf{w}^*$ be the solution obtained using unregularized least-squares regression. What solution will you obtain is you scale all input features by a factor of c before solving?
   ◯ $c\mathbf{w}^*$ ◯ $c^2\mathbf{w}^*$ ◯ $\frac{1}{c^2}\mathbf{w}^*$ ◯ $\frac{1}{c}\mathbf{w}^*$

Total Question 1: 35

2. Multiple Answer Select ALL correct choices: there may be more than one correct choice, but there is always at least one correct choice.
   (a) [5 points] What are support vectors?
   - ◯ The examples $\mathbf{x}_n$ from the training set required to compute the decision function $f(\mathbf{x})$ in an SVM.
   - ◯ The class means.
   - ◯ The training samples farthest from the decision boundary.
   - ◯ The training samples $\mathbf{x}_n$ that are on the margin (i.e. $y_n f(\mathbf{x}_n) = 1$).

(b) [5 points] Which of the following are true about the relationship between the MAP and MLE estimators for linear regression?

- ◯ They are equal if $p(\mathbf{w}) = 1$
- ◯ They are equal if $p(\mathbf{w}) = \mathcal{N}(\mathbf{0}, \sigma)$ for very small σ.
- ◯ They are never equal.
- ◯ They are equal in the limit of infinite training samples.

(c) [5 points] You train a linear classifier on 10,000 training points and discover that the training accuracy is only 67%. Which of the following, done in isolation, has a good chance of improving your training accuracy?

- ◯ Add novel features.
- ◯ Train on more data.
- ◯ Train on less data
- ◯ Regularize the model

(d) [5 points] In a soft-margin support vector machine, if we increase C, which of the following are likely to happen?

- ◯ The margin will grow wider.
- ◯ Most nonzero slack variables will decrease.
- ◯ $||\mathbf{w}||_2$ will grow larger.
- ◯ There will be more points inside the margin.

(e) [5 points] Which of the following are reasons why you might adjust your model in ways that increase the bias?

- ◯ You observe high training error and high validation error.
- ◯ You have few data points.
- ◯ You observe low training error and high validation error.
- ◯ Your data are not linearly separable.

(f) [5 points] Which of the following are true of polynomial regression (i.e. least squares regression with polynomial basis mapping)?

- ◯ If we increase the degree of polynomial, we increase variance.
- ◯ The regression function is nonlinear in the model parameters.
- ◯ The regression function is linear in the original input variables.
- ◯ If we increase the degree of polynomial, we decrease bias.

(g) [5 points] Which of the following classifiers can be used on non linearly separable datasets?

- ◯ The hard margin SVM.
- ◯ Logistic regression.
- ◯ The linear generative Bayes classifiers.
- ◯ Fisher's Linear Discriminant.

Total Question 2: 35

3. [15 points] Assume the class conditional distributions for a two-class classification problem are $p(\mathbf{x} \mid \mathcal{C}_1) = \mathcal{N}(\boldsymbol{\mu}_1, \beta^{-1}I)$ and $p(\mathbf{x} \mid \mathcal{C}_2) = \mathcal{N}(\boldsymbol{\mu}_2, \beta^{-1}I)$. Show that the optimal decision boundary is *linear*, i.e. that it can be written as $H = \{\mathbf{x} \mid \mathbf{w}^T\mathbf{x} + b = 0\}$ for some $\mathbf{w}$ and b.

   Hint: Remember that points $\mathbf{x}$ on the optimal decision boundary will satisfy $p(\mathbf{x} \mid \mathcal{C}_1) = p(\mathbf{x} \mid \mathcal{C}_1)$, and that the formula for the multivariate Gaussian density is:

$$\mathcal{N}(\mathbf{x} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{D/2}|\boldsymbol{\Sigma}|^{1/2}} \exp\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\}$$

4. [15 points] Assume we have a training set of only two points (one from each class):

$$\mathcal{D} = \{([0,0], -1), ([2,0], +1)\}$$

   Solve for the optimal hard margin primal SVM parameters $\mathbf{w}$ and b for this dataset.

5. [10 points (bonus)] Write the primal SVM objective as an empirical risk minimization problem over a linearly separable dataset $\mathcal{D} = \{(\mathbf{x}_n, y_n)\}_{n=1}^N$. Show that any $\mathbf{w}$ and b minimizing this empirical risk is also a solution to the standard hard-margin SVM objective with constraints.