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Dipartimento di Ingegneria dell'Informazione

Software Engineering for Embedded Systems

Advanced topics on schedulability analysis

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Outline

Introduction

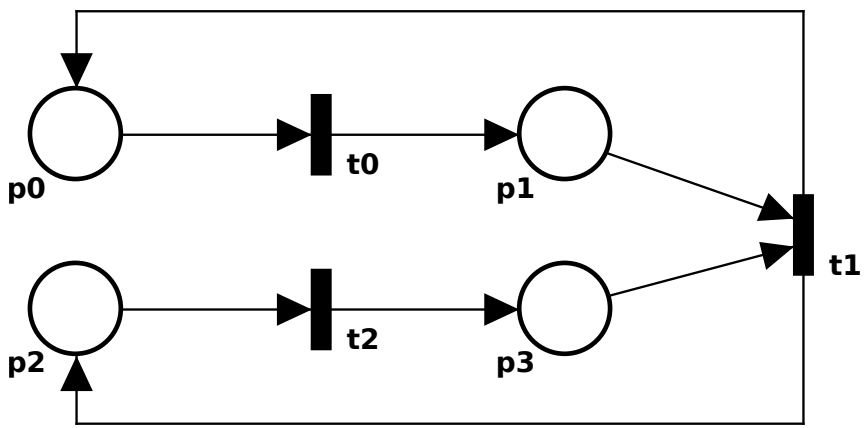
Analytical approaches vs state-space based methods

- Analytical approaches
  - Assume tasks with deterministic Worst Case Execution Time (WCET)
  - Provide exact results for task-sets made of periodic and independent tasks
  - Provide pessimistic results for task-sets including sporadic tasks and inter-task dependencies (e.g., semaphore synchronizations, data-flow precedences)
  - Provide efficient schedulability tests
- State-space based methods
  - Encompass temporal parameters varying within a min-max interval
  - Support modeling and analysis of a wide class of real-time systems (e.g., real-time task-sets, communication protocols, . . .)
  - Provide exact results for any task-set that can be modeled and analyzed
  - Require larger computational complexity than analytical approaches

Petri Nets

PN syntax (1/2)

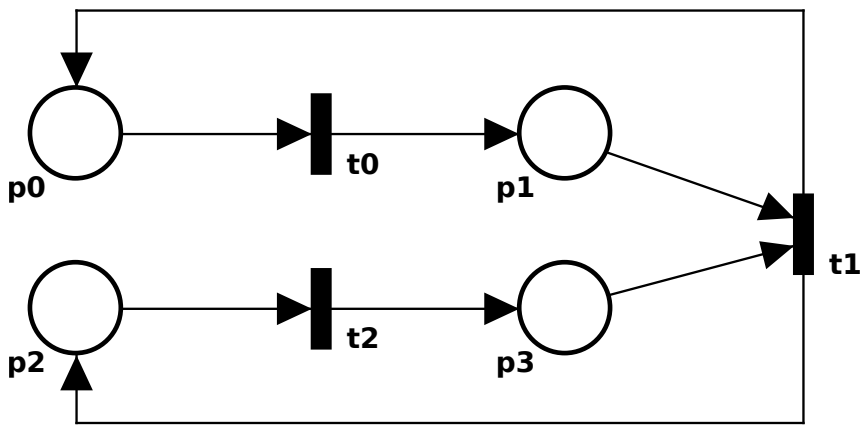
- Petri Nets (PNs) support the representation of **concurrent** systems
- A PN is a tuple $\langle P, T, A^-, A^+ \rangle$ where:
  - P is the set of **places** (circles), T is the set of **transitions** (bars), $P \cap T = \varnothing$
  - $A^- \subseteq P \times T$ is the set of **preconditions** (directed arcs from circles to bars)
  - $A^+ \subseteq T \times P$ is the set of **postconditions** (directed arcs from bars to circles)
- A PN is a directed bipartite graph where:
  - the set of vertices is $P \cup T$
  - the set of arcs is $A^- \cup A^+$



C.A. Petri, "Kommunication mit Automaten," Ph.D. thesis, Technischen Hoschule Darmstadt, 1962.

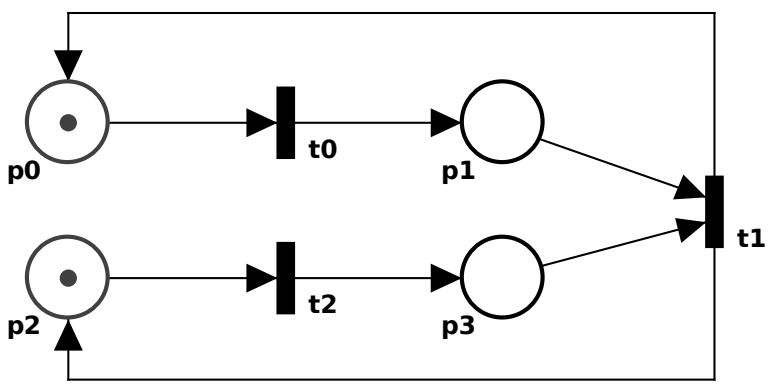
PN syntax (2/2)

- A place p is said to be an **input place** for a transition t if $(p, t) \in A^-$ (i.e., if there is a directed arc from p to t)
  - e.g., `p0` is an input place for `t0`
- A place p is said to be an **output place** for a transition t if $(t, p) \in A^+$ (i.e., if there is a directed arc from t to p)
  - e.g., `p1` is an output place for `t0`



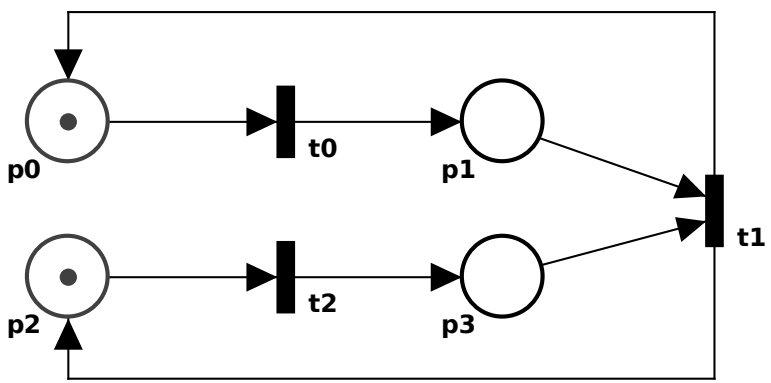
State of a PN

- A **marking** $m : P \to \mathbb{N}$ assigns a natural number of **tokens** to each place
  - Tokens do not have identity
  - e.g., `p0` and `p1` contain one token each
- A transition t is **enabled** by a marking m iff $m(p) > 0 \; \forall \, p \in P \mid (p, t) \in A^-$ (i.e., iff m assigns at least one token to each input place of t)
  - Enabled transition represent **concurrent events**
  - e.g., `t0` and `t2` are enabled, `t1` is not enabled
- The **state** of a PN is a singleton $s = \langle m \rangle$ where m is a marking
  - A transition is enabled in a state $s = \langle m \rangle$ iff it is enabled by marking m
  - An enabled transition eventually fires or is disabled
  - e.g., the current state of the example PN is $\langle \mathtt{p0}, \mathtt{p2} \rangle$



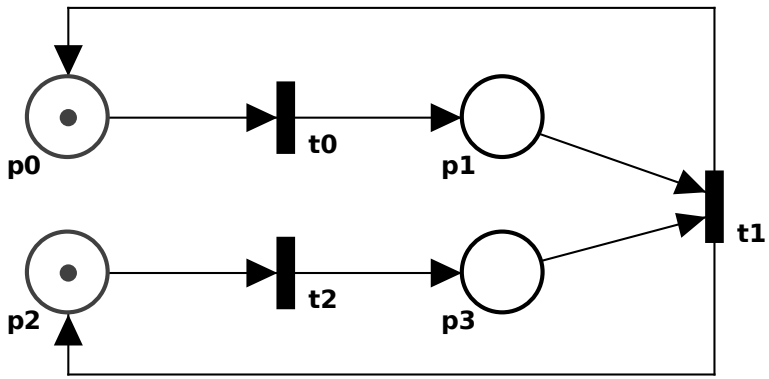
PN semantics (1/2)

- An **execution** of a PN is a path $\omega = s_0 \xrightarrow{\gamma_1} s_1 \xrightarrow{\gamma_2} s_2 \xrightarrow{\gamma_3} \ldots$ such that:
  - $s_0 = \langle m_0 \rangle$ is the initial state and m_0 is the initial marking
  - γ_i is the i-th fired transition
  - $s_i = \langle m_i \rangle$ is the state reached after the firing of γ_i
  - γ_i is selected among the transitions **enabled** in state $s_{i-1} = \langle m_{i-1} \rangle$
  - after the firing of γ_i, the new marking m_i is derived from m_{i-1} by
    1. removing a token from each input place of γ_i
       (i.e., $m_{\text{tmp}} = m_{i-1}(p) - 1 \; \forall \, p \,|\, (p, \gamma_i) \in A^-$)
    2. adding a token to each output place of γ_i
       (i.e., $m_i = m_{\text{tmp}}(p) + 1 \; \forall \, p \,|\, (\gamma_i, p) \in A^+$)
  - ω is a finite or infinite path
- E.g., $\langle \text{p0 p2} \rangle \xrightarrow{\text{t0}} \langle \text{p1 p2} \rangle \xrightarrow{\text{t2}} \langle \text{p1 p3} \rangle \xrightarrow{\text{t1}} \langle \text{p0 p2} \rangle$ is a finite execution

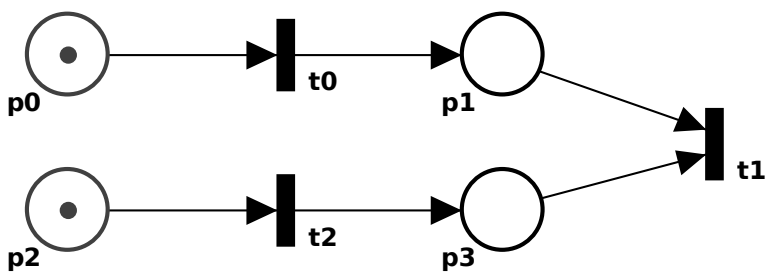


PN semantics (2/2)

- Example 1: a PN that allows infinite executions
  - e.g., $\omega = \langle \mathrm{p0\,p2} \rangle \xrightarrow{\mathrm{t0}} \langle \mathrm{p1\,p2} \rangle \xrightarrow{\mathrm{t2}} \langle \mathrm{p1\,p3} \rangle \xrightarrow{\mathrm{t1}} \langle \mathrm{p0\,p2} \rangle$ is a finite execution
  - e.g., the infinite repetition of ω is an infinite execution

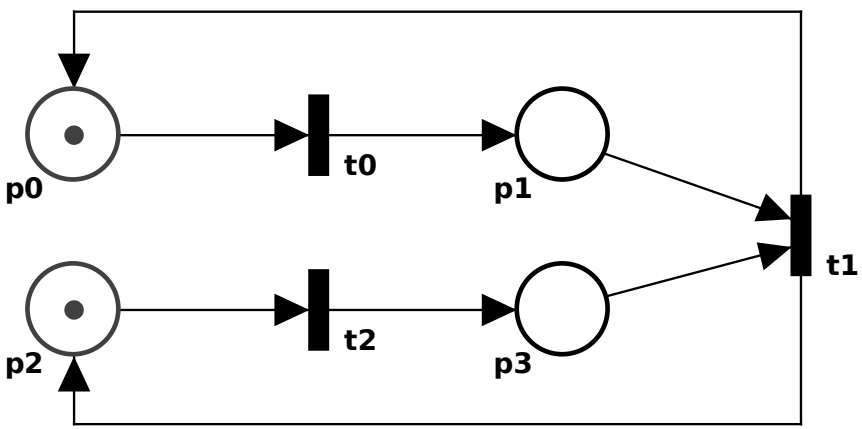


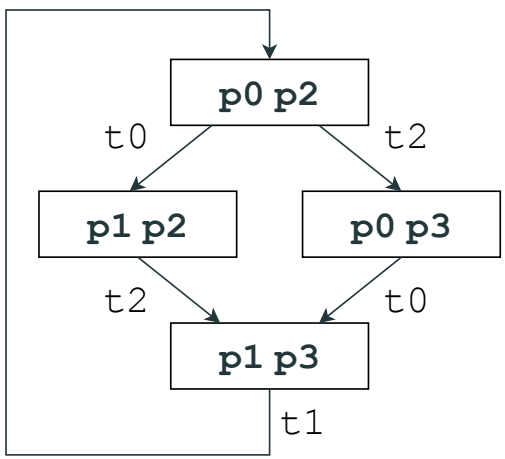
- Example 2: a PN that does not allow infinite executions
  - e.g., $\langle \mathrm{p0\,p2} \rangle \xrightarrow{\mathrm{t0}} \langle \mathrm{p1\,p2} \rangle \xrightarrow{\mathrm{t2}} \langle \mathrm{p1\,p3} \rangle \xrightarrow{\mathrm{t1}} \langle \rangle$
  - e.g., $\langle \mathrm{p0\,p2} \rangle \xrightarrow{\mathrm{t2}} \langle \mathrm{p0\,p3} \rangle \xrightarrow{\mathrm{t0}} \langle \mathrm{p1\,p3} \rangle \xrightarrow{\mathrm{t1}} \langle \rangle$



Reachability graph of a PN

- The set $\mathcal{R}(m_0)$ of markings that can be reached from the initial marking m_0 is the minimum set of markings such that:
  - $m_0 \in \mathcal{R}(m_0)$
  - if $m \in \mathcal{R}(m_0) \wedge \langle m \rangle \xrightarrow{t} \langle m' \rangle$ with $t \in T$ then $m' \in \mathcal{R}(m_0)$
- The **reachability graph** is a directed graph such that:
  - the set of vertices is $\mathcal{R}(m_0)$
  - the set of arcs includes an arc from m_i to m_j iff $\exists\, t \in T \mid \langle m_i \rangle \xrightarrow{t} \langle m_j \rangle$
- A path in the reachability graph identifies an execution of the PN





Enumeration of the reachability graph

Algorithm 1 State-space exploration from the initial marking m_0

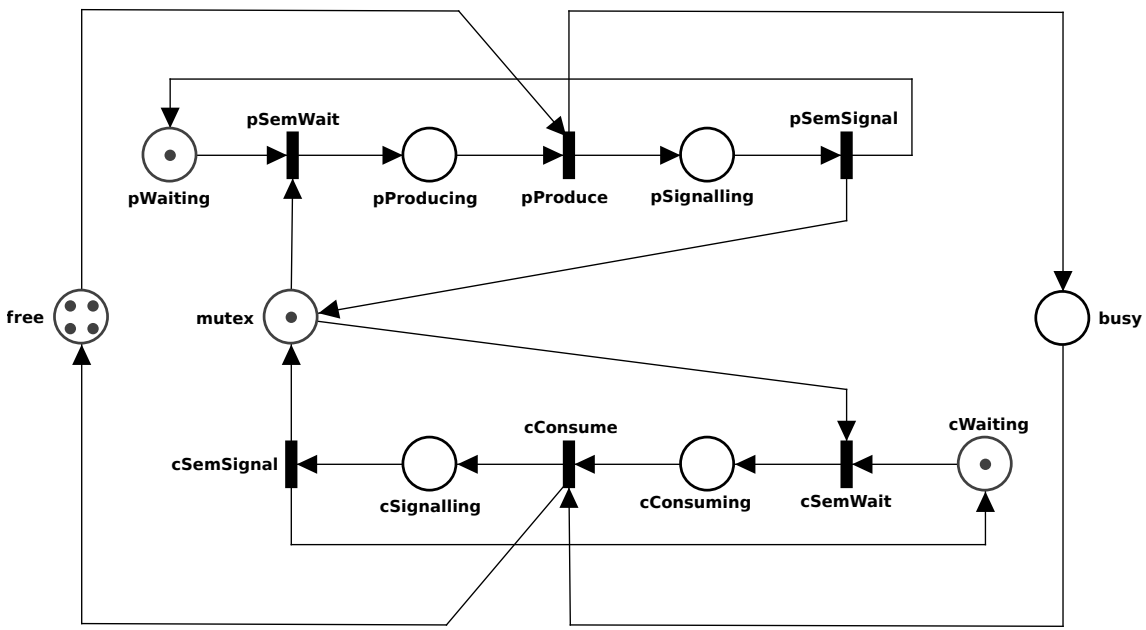
1: $Q \leftarrow \{\cdot \dot{\rightarrow} m_0\}$ // Q is the set of successions to explore
2: $G \leftarrow \varnothing$ // G is the reachability graph
3: **while** $Q \neq \varnothing$ **do**
4: select and remove a succession $m \xrightarrow{t} m'$ from Q
5: **if** IsNewSuccession$(m \xrightarrow{t} m')$ **then**
6: $G = G \cup \{m \xrightarrow{t} m'\}$
7: **for** each $t' \in$ EnabledTransitions(m') **do**
8: $m' \xrightarrow{t'} m'' =$ SuccessionEvaluator(m', t')
9: $Q = Q \cup \{(m, t, m')\}$
10: **end for**
11: **end if**
12: **end while**

The producer/consumer problem (1/4)

- Problem definition
  - A producer produces objects and puts them into a buffer
  - A consumer consumes objects and removes them from the buffer
  - The buffer may have a limited capacity
  - Production and consumption are in mutual exclusion (**safety** requirement)
  - Production and consumption will eventually occur (**liveness** requirement)
- Problem solution
  - Use a binary semaphore to guarantee mutual exclusion

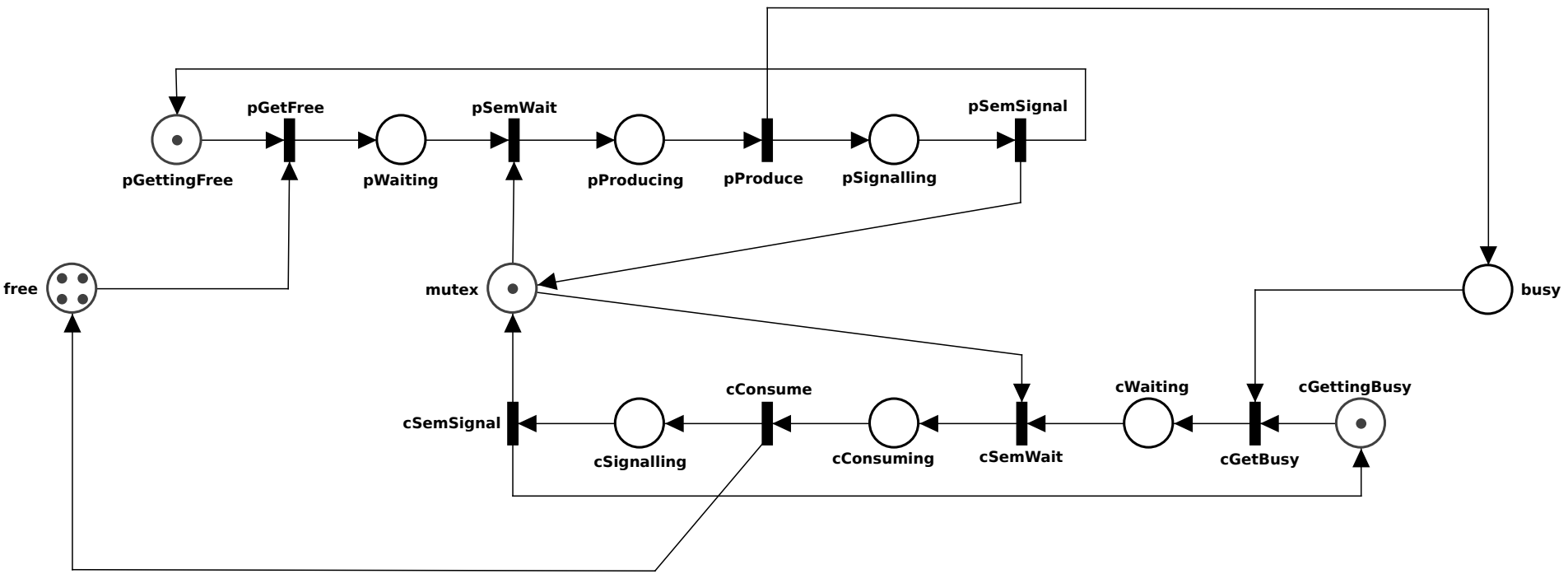
The producer/consumer problem (2/4)

- Place invariants
  - $m(\texttt{pWaiting}) + m(\texttt{pProducing}) + m(\texttt{pSignalling}) = 1 \; \forall \, m \in \mathcal{M}$
  - $m(\texttt{cWaiting}) + m(\texttt{cProducing}) + m(\texttt{cSignalling}) = 1 \; \forall \, m \in \mathcal{M}$
  - $m(\texttt{free}) + m(\texttt{busy}) = 4 \; \forall \, m \in \mathcal{M}$ (4 is the **buffer capacity**)
- The model guarantees mutual exclusion (safety requirement satisfied)
- The model is prone to **deadlock** (liveness requirement not satisfied)
  - No enabled transition in any marking $m \,|\, m(\texttt{pProducing}) = 1 \wedge m(\texttt{busy}) = 4$
  - No enabled transition in any marking $m \,|\, m(\texttt{cConsuming}) = 1 \wedge m(\texttt{free}) = 4$



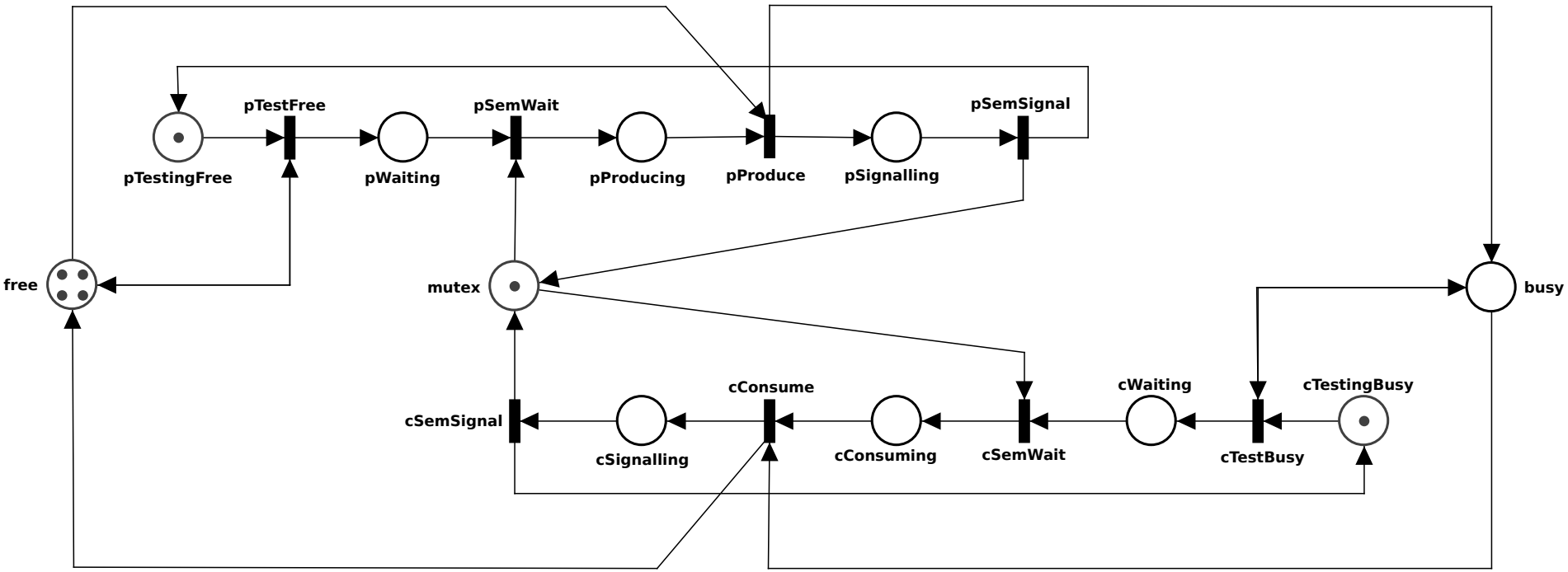
The producer/consumer problem (3/4)

- The model guarantees mutual exclusion (safety requirement satisfied)
- The model prevents deadlock (liveness requirement satisfied)
- The model cannot be translated into real-time code
  - The number of current products could be represented by a shared variable `sv`
  - There may be a context switch (due to the fact that the semaphore is busy) after the producer/consumer has read the variable and before it modifies it



The producer/consumer problem (4/4)

- The model guarantees mutual exclusion (safety requirement satisfied)
- The model prevents deadlock (liveness requirement satisfied)
- The model can be translated into real-time code
  - The shared variable is first tested by the producer/consumer and then modified



Petri nets with inhibitor arcs (1/2)

- PNs with inhibitor arcs have the Turing machine expressivity (PNs do not)
- A PN with inhibitor arcs is a tuple $\langle P, T, A^-, A^+, A^\bullet \rangle$ where:
  - P, T, A^-, A^+ are the elements of a PN
  - $A^\bullet \subseteq P \times T$ is the set of **inhibitor arcs**
    (directed bullet-headed arcs from circles to bars)
- A place p is said to be an **inhibitor place** for a transition t if $(p, t) \in A^\bullet$ (i.e., if there is a bullet-headed arc from p to t)
  - e.g., `reading` is an inhibitor place for transition `writeWait`
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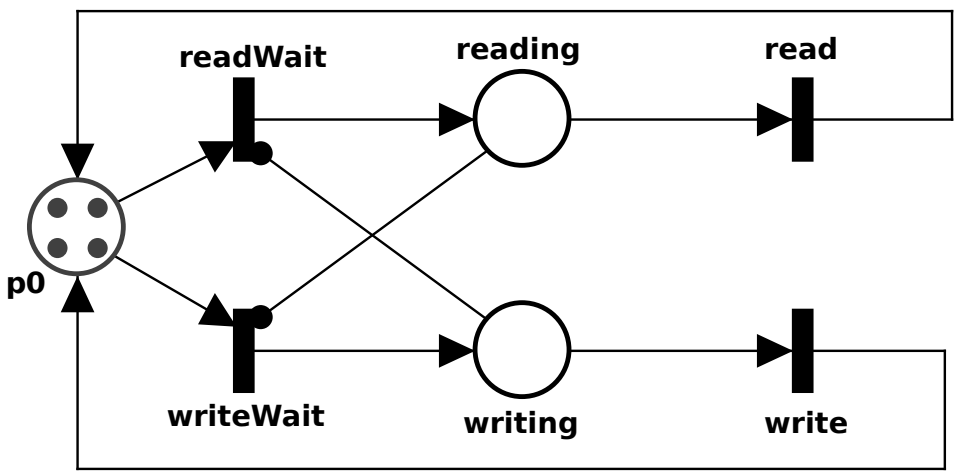
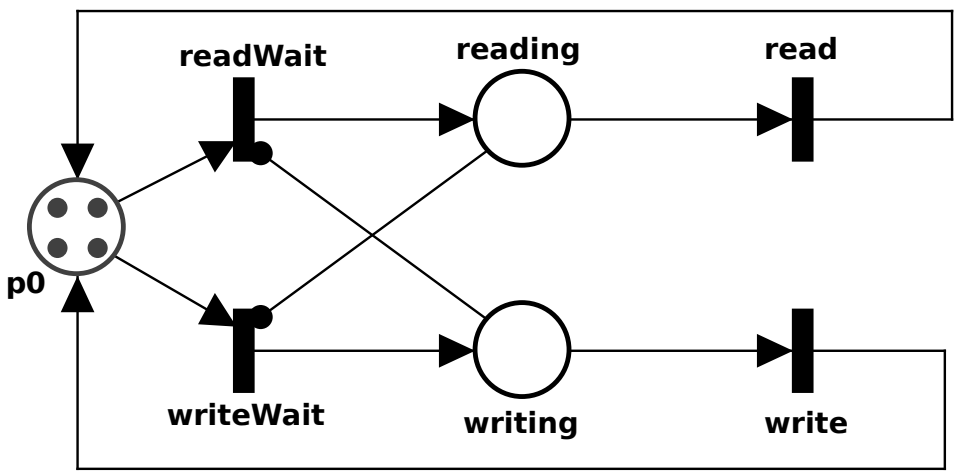


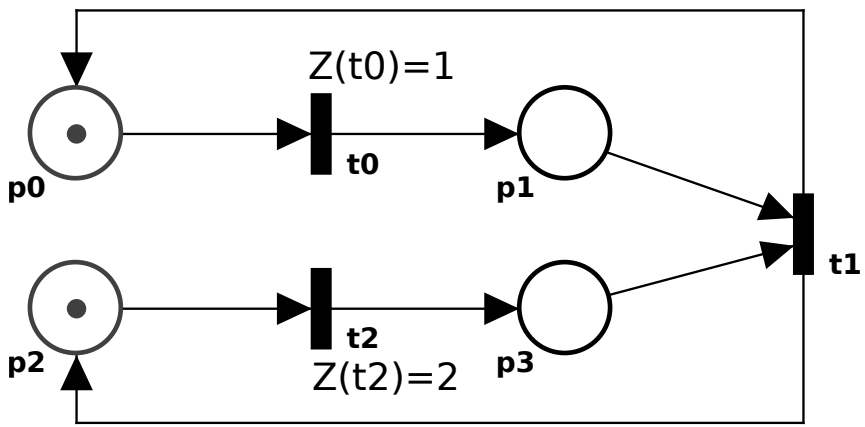
Petri nets with inhibitor arcs (2/2)

- A transition t is **enabled** by a marking m iff:
  - $m(p) > 0 \; \forall \, p \in P \,|\, (p, t) \in A^-$
    (i.e., m assigns at least one token to each input place of t)
  - $m(p) = 0 \; \forall \, p \in P \,|\, (p, t) \in A^\bullet$
    (i.e., m assigns no token to each inhibitor place of t)
- Example: read-write synchronization
  - Transition `readWait` cannot fire if place `writing` contains a token
    (i.e., a process is not allowed to read if another process is writing)
  - Transition `writeWait` cannot fire if place `reading` contains a token
    (i.e., a process is not allowed to write if another process is reading)



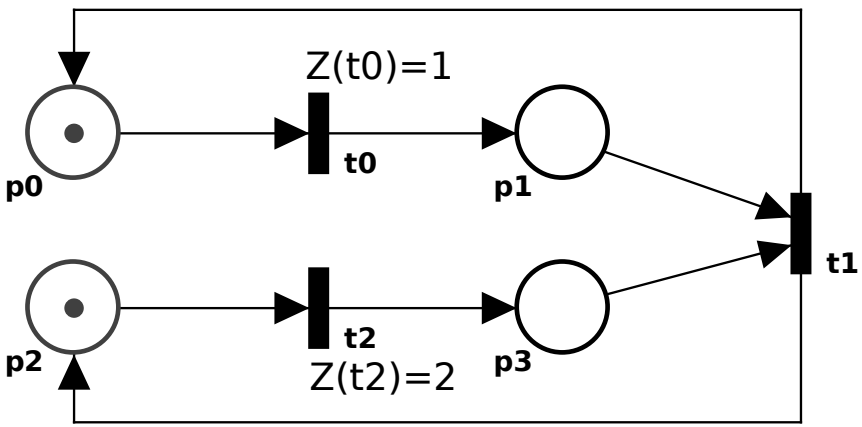
Petri nets with priorities (1/2)

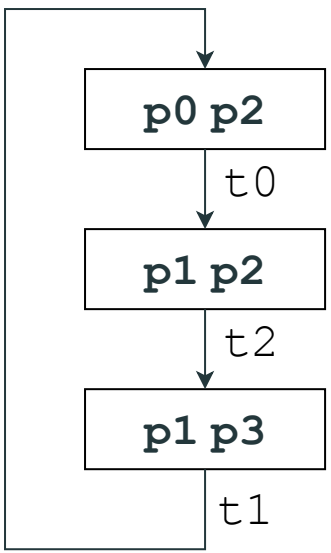
- PNs with priorities have the Turing machine expressivity (PNs do not)
- A PN with priorities is a tuple $\langle P, T, A^-, A^+, Z \rangle$ where:
  - P, T, A^-, A^+ are the elements of a PN
  - $Z : T \to \mathbb{N}$ associates each transition with a **priority** (possibly annotated next to the bar representing the transition)
  - The lowest the priority number is, the largest the priority level is
- Example: `t0` has larger priority level than `t2`



Petri nets with priorities (2/2)

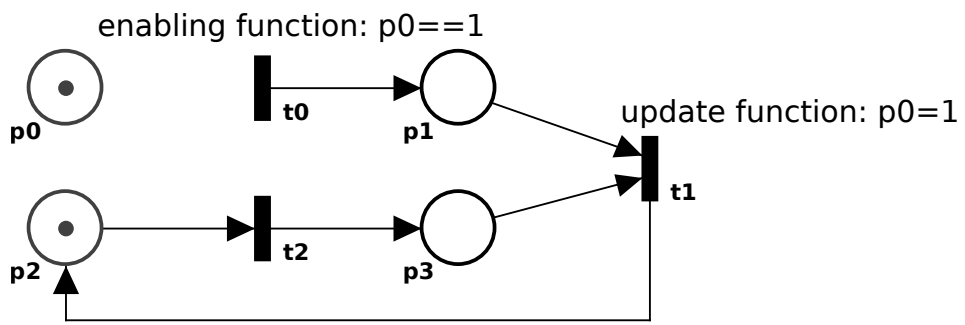
- A transition t is **enabled** by a marking m iff:
  - $m(p) > 0 \; \forall \, p \in P \mid (p, t) \in A^-$
    (i.e., m assigns at least one token to each input place of t)
  - $\forall \, t' \in T \mid t \neq t'$, if $m(p) > 0 \; \forall \, p \in P \mid (p, t') \in A^-$ then $Z(t) < Z(t')$
    (i.e., t has larger priority level than any transition with non-empty input places)
- Example: a single transition is enabled in each reachable marking





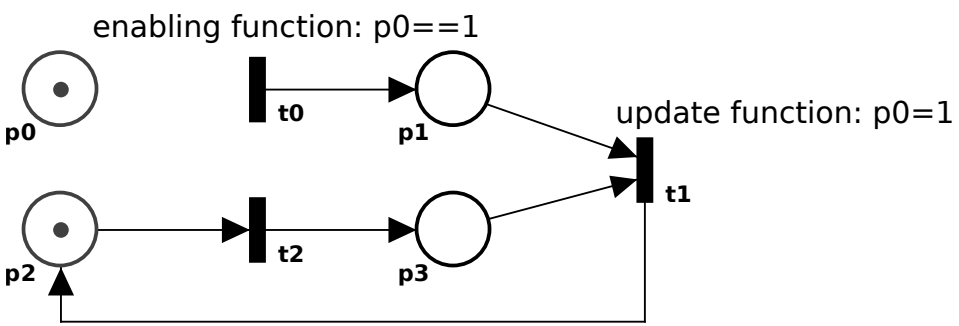
Petri nets with enabling and update functions (1/2)

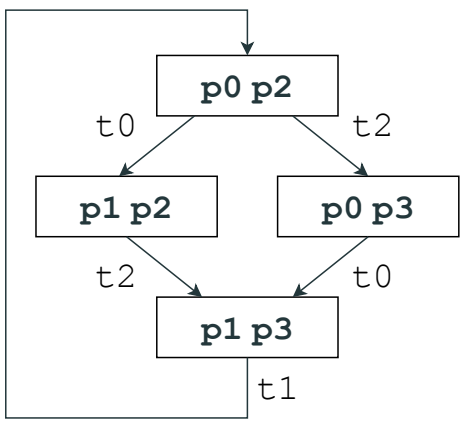
- Improve the modeling convenience wrt PNs with inhibitor arcs
- Do not increase the model expressivity wrt PNs with inhibitor arcs
- A PN with enabling and update functions is a tuple $\langle P, T, A^-, A^+, E, U \rangle$:
  - P, T, A^-, A^+ are the elements of a PN
  - E associates each transition $t \in T$ with an **enabling function** $E(t) : \mathcal{M} \to \{\text{TRUE}, \text{FALSE}\}$, with $\mathcal{M}$ being the set of reachable markings
  - U associates each transition $t \in T$ with an **update function** $U(t) : \mathcal{M} \to \mathcal{M}$
  - $E(t)$ and $U(t)$ may be annotated next to the bar representing the transition
- Example
  - `t0` has an enabling function that evaluates to TRUE if `p0` contains a token
  - `t1` has an update function that assigns to a marking m another marking m' obtained from m by assigning one token to place `p0`



Petri nets with enabling and update functions (2/2)

- A transition t is **enabled** by a marking m iff:
  - $m(p) > 0 \; \forall \, p \in P \,|\, (p, t) \in A^-$
    (i.e., m assigns at least one token to each input place of t)
  - $E(t)(m) = \text{TRUE}$ (i.e., the enabling function of t evaluates to true in m)
- Given a marking m_{i-1} and a transition γ_i enabled by m_{i-1}, marking m_i reached after the firing of γ_i is derived from m_{i-1} by
  1. removing a token from each input place of γ_i
     (i.e., $m_{\text{tmp}} = m_{i-1}(p) - 1 \; \forall \, p \,|\, (p, \gamma_i) \in A^-$)
  2. adding a token to each output place of γ_i
     (i.e., $m_{\text{tmp2}} = m_{\text{tmp}}(p) + 1 \; \forall \, p \,|\, (\gamma_i, p) \in A^+$)
  3. applying the update function of γ_i (i.e., $m_i = U(\gamma_i)(m_{\text{tmp2}})$)

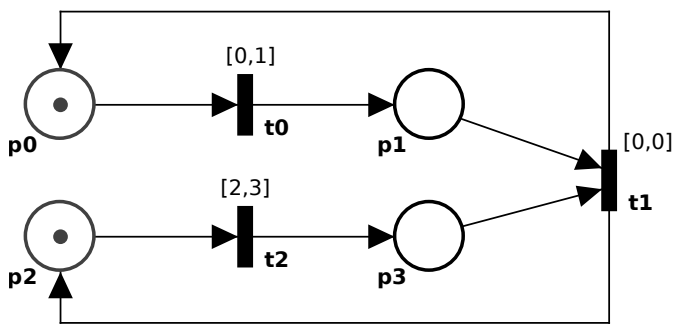




Time Petri Nets

TPN syntax

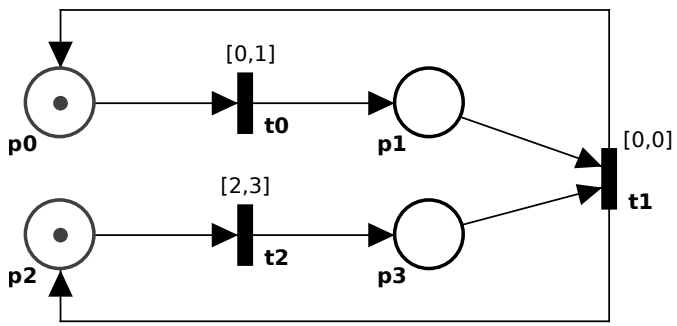
- Time Petri Nets (TPNs) extend PNs (with inhibitor arcs) with **time**
- A TPN is a tuple $\langle P, T, A^-, A^+, A^\cdot, EFT, LFT \rangle$ where:
  - P, T, A^-, A^+, $A^\cdot$ are the elements of a PN with inhibitor arcs
  - EFT associates each transition t with an **earliest firing time** $EFT(t) \in \mathbb{Q}_{\geq 0}$
  - LFT associates each transition t with a **latest firing time** $LFT(t) \in \mathbb{Q}_{\geq 0}$
  - $EFT(t) \leq LFT(t)\ \forall\, t \in T$
- Example
  - `t0` has earliest firing time equal to 0 and latest firing time equal to 1
  - `t1` has earliest firing time equal to 2 and latest firing time equal to 3
  - `t2` has earliest and latest firing time equal to 0



E. Vicario, "Static Analysis and Dynamic Steering of Time-Dependent Systems Using Time Petri Nets," IEEE Transactions on Software Engineering, 2001.

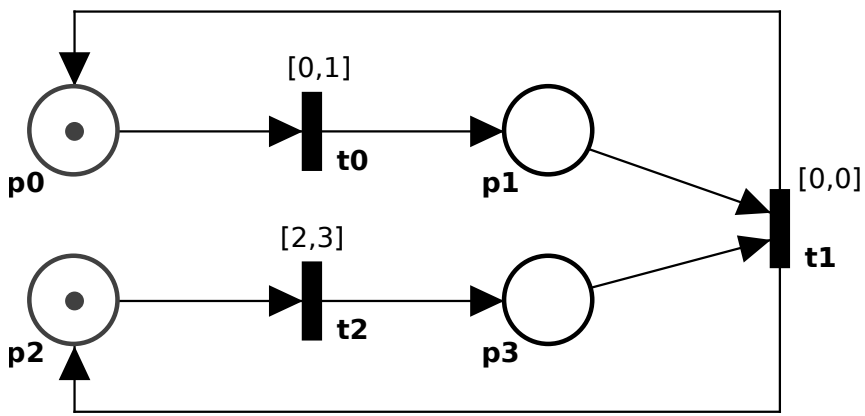
State of a TPN

- The **state** of a TPN is a pair $\langle m, \tau \rangle$ where
  - m is a marking (representing the logical location of the system)
  - τ associates each enabled transition t with a **time to fire** $\tau(t) \in \mathbb{R}_{\geq 0}$
- Example
  - `t0` can be associated with a time to fire $\tau(\texttt{t0}) \in [0, 1]$
  - `t1` can be associated with a time to fire $\tau(\texttt{t1}) \in [2, 3]$



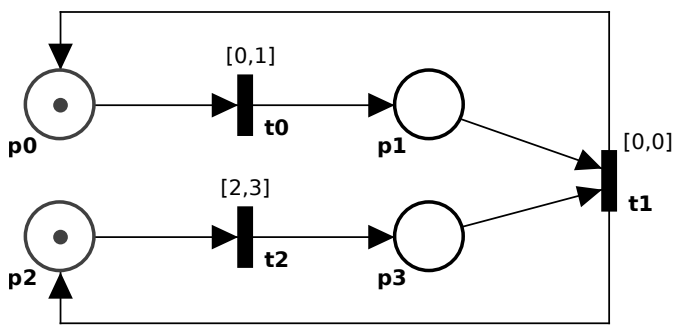
TPN semantics (1/5)

- **Strong** semantics of time constraints
  - A transition t **cannot fire before** it has been enabled with continuity for a time $T_{\min}$ not lower than its earliest firing time (i.e., $T_{\min} \geq EFT(t)$)
  - A transition t **cannot avoid to fire when** it has been enabled with continuity for a time $T_{\max}$ equal to its latest firing time (i.e., $T_{\max} = LFT(t)$, meaning that time cannot advance before t fires or is disabled by another firing)
  - Transition firings occur in zero-time
- **Weak** semantics of time constraints (not implemented by TPNs)
  - A transition that has not fired within its latest firing time could not be able to fire unless it is newly enabled again



TPN semantics (2/5)

- As in PNs with inhibitor arcs, a transition t is **enabled** by a marking m iff:
  - $m(p) > 0 \; \forall \, p \in P \mid (p, t) \in A^-$
    (i.e., m assigns at least one token to each input place of t)
  - $m(p) = 0 \; \forall \, p \in P \mid (p, t) \in A^\bullet$
    (i.e., m assigns no token to each inhibitor place of t)
- A transition t is **firable** in a state $s = \langle m, \tau \rangle$ iff:
  - t is enabled by marking m
  - $\tau(t) \leq \tau(t') \; \forall \, t' \in T \mid t'$ is enabled by m (i.e., the time to fire of t is not larger than the time to fire of any other enabled transition)
- In any possible initial state of the example TPN:
  - `t0` and `t2` are both enabled
  - `t0` is firable while `t2` is not firable

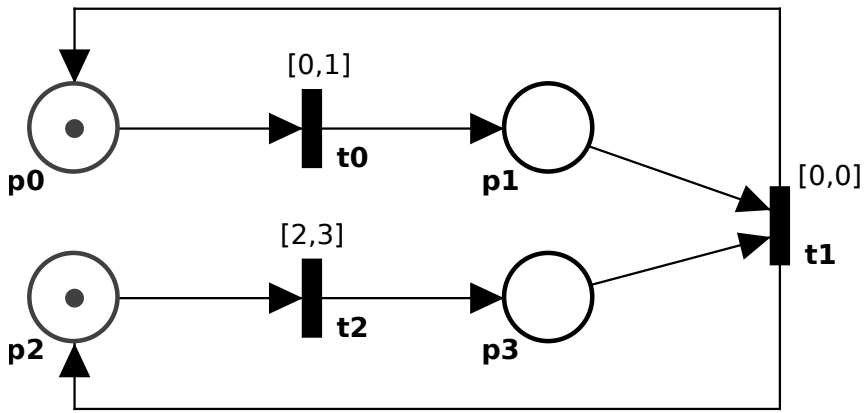


TPN semantics (3/5)

- The firing of a transition t replaces $s_0 = \langle m_0, \tau_0 \rangle$ by $s_1 = \langle m_1, \tau_1 \rangle$
  - As in PNs, m_1 is derived from m_0 by:
    1. removing a token from each input place of t
       (i.e., $m_{\text{tmp}} = m_0(p) - 1 \; \forall \, p \,|\, (p, t) \in A^-$)
    2. adding a token to each output place of t
       (i.e., $m_1 = m_{\text{tmp}}(p) + 1 \; \forall \, p \,|\, (t, p) \in A^+$)
  - A transition t' enabled by m_1 is termed:
    - **persistent** if it is not t and it is enabled both by m_0 and m_{tmp}
    - **newly enabled** if it is not enabled by m_0 or by m_{tmp},
      or if it is the fired transition t and it is enabled by m_1
      (i.e., no multiple enabledness, single server semantics)
  - A transition is termed **disabled** if it is enabled by m_0 but not by m_1
  - τ_1 is derived from τ by:
    1. reducing the time to fire of each **persistent** transition by the value of $\tau_0(t)$
       (i.e., $\tau_1(t') = \tau_0(t') - \tau_0(t) \; \forall \, t' \in T \,|\, t'$ is persistent in m_1)
    2. eliminating the time to fire of each transition **disabled** by the firing of t
    3. sampling the time to fire of each **newly enabled** transition t' in its firing
       interval (i.e., $\tau_1(t') \in [EFT(t'), LFT(t')] \; \forall \, t' \in T \,|\, t'$ is newly enabled in m_1)

TPN semantics (4/5)

- Example TPN
  - t0 is the only firable transition in any initial state with marking p0 p2
  - The firing of t0 in $\langle \mathtt{p0}\,\mathtt{p2}, \tau_0 \rangle$ yields $\langle \mathtt{p1}\,\mathtt{p2}, \tau_1 \rangle$ with $\tau_1(\mathtt{t2}) = \tau_0(\mathtt{t2}) - \tau_0(\mathtt{t0})$

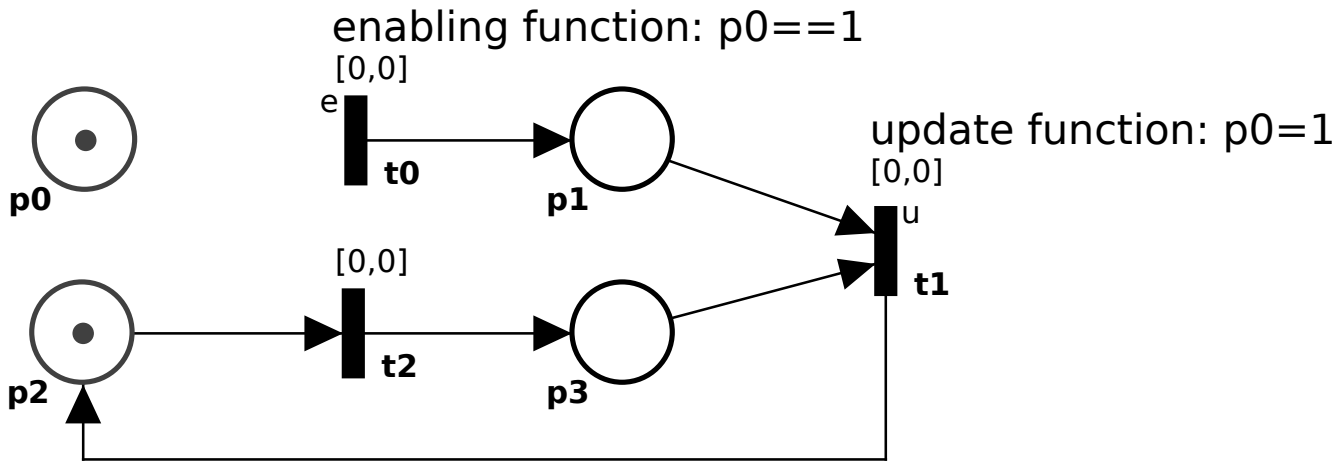


TPN semantics (5/5)

- An **execution** of a TPN is a path $\omega = s_0 \xrightarrow{\gamma_1} s_1 \xrightarrow{\gamma_2} s_2 \xrightarrow{\gamma_3} \dots$ such that:
  - $s_0 = \langle m_0, \tau_0 \rangle$ is the initial state
  - γ_i is the i-th fired transition
  - $s_i = \langle m_i, \tau_i \rangle$ is the state reached after the firing of γ_i
  - γ_i is selected among the transitions **firable** in state $s_{i-1} = \langle m_{i-1}, \tau_{i-1} \rangle$
  - after the firing of γ_i, the new marking m_i is derived from m_{i-1} by
    1. removing a token from each input place of γ_i
       (i.e., $m_{\text{tmp}} = m_{i-1}(p) - 1 \; \forall \, p \,|\, (p, \gamma_i) \in A^-$)
    2. adding a token to each output place of γ_i
       (i.e., $m_i = m_{\text{tmp}}(p) + 1 \; \forall \, p \,|\, (\gamma_i, p) \in A^+$)
  - τ_i is derived from τ_{i-1} by:
    1. reducing the time to fire of each persistent transition by the value of $\tau_{i-1}(\gamma_i)$
       (i.e., $\tau_i(t') = \tau_{i-1}(t') - \tau_{i-1}(\gamma_i) \; \forall \, t' \in T \,|\, t'$ is persistent in m_i)
    2. eliminating the time to fire of each transition disabled by the firing of γ_i
    3. sampling the time to fire of each newly enabled transition in its firing interval
       (i.e., $\tau_i(t') \in [EFT(t), LFT(t)] \; \forall \, t' \in T \,|\, t$ is newly enabled in m_i)
  - ω is a finite or infinite path

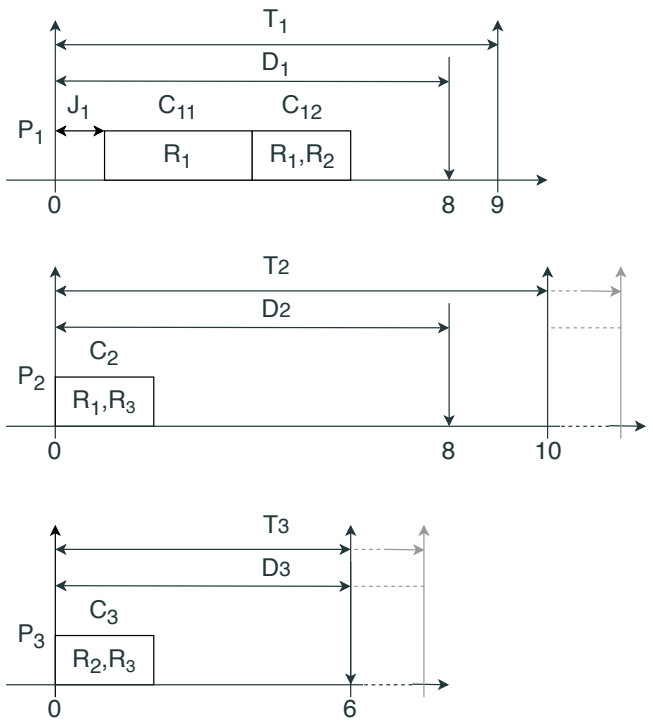
Extension of syntax and semantics of TPNs

- TPNs could be extended with enabling functions, update functions, priorities
  - These features would improve the modeling convenience wrt TPNs
  - These features would not increase the model expressivity wrt TPNs



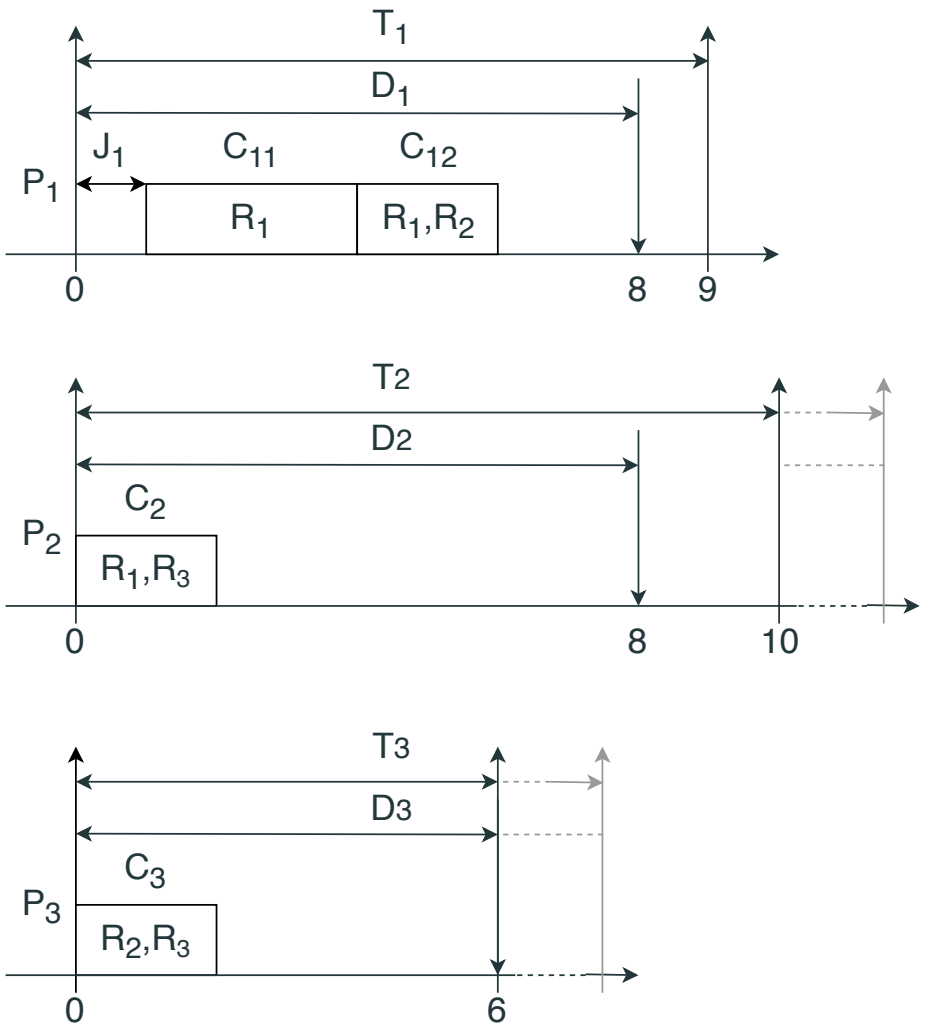
Modeling real-time task-sets with TPNs: an example (1/4)

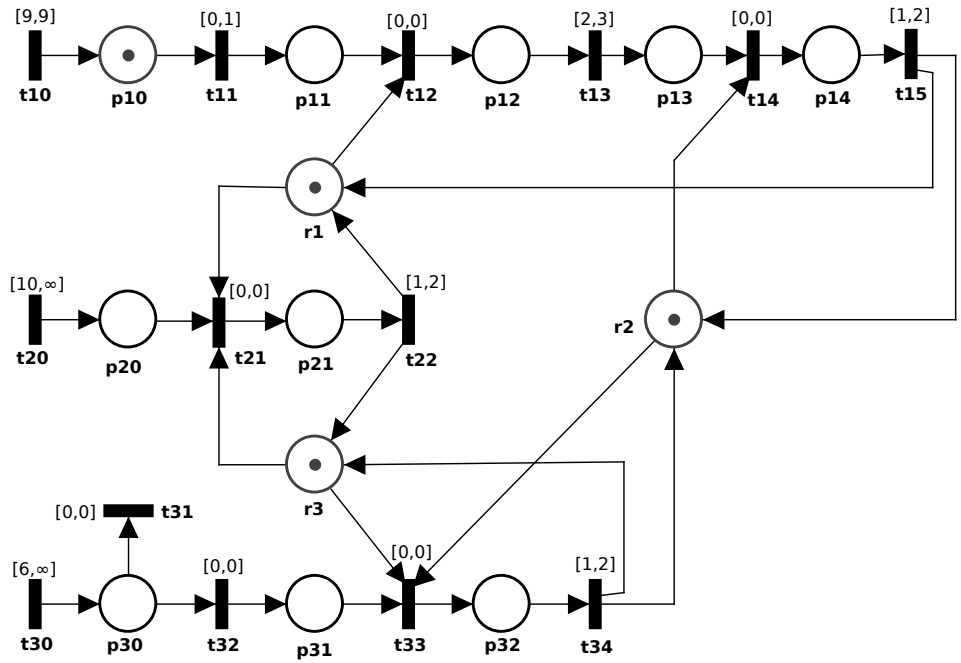
- 3 tasks P_1, P_2, P_3 contending for 3 **non-preemptable** resources R_1, R_2, R_3
  - P_1 and P_2 are hard real-time (mandatory), P_3 is soft real-time (optional)
- P_1 is periodic
  - period $T_1 = 9$
  - relative deadline $D_1 = 8$
  - absolute start time jitter $J_1 \leq 1$
  - computation 1: WCET $C_{11} \in [2, 3]$, uses R_1
  - computation 2: WCET $C_{12} \in [1, 2]$, uses R_1, R_2
- P_2 is sporadic
  - inter-arrival time $T_2 \geq 8$
  - relative deadline $D_2 = 8$
  - computation: WCET $C_{21} \in [1, 2]$, uses R_1, R_3
- P_3 is sporadic
  - inter-arrival time $T_3 \geq 6$
  - relative deadline $D_3 = 6$
  - computation: WCET $C_{31} \in [1, 2]$, uses R_2, R_3



Modeling real-time task-sets with TPNs: an example (2/4)

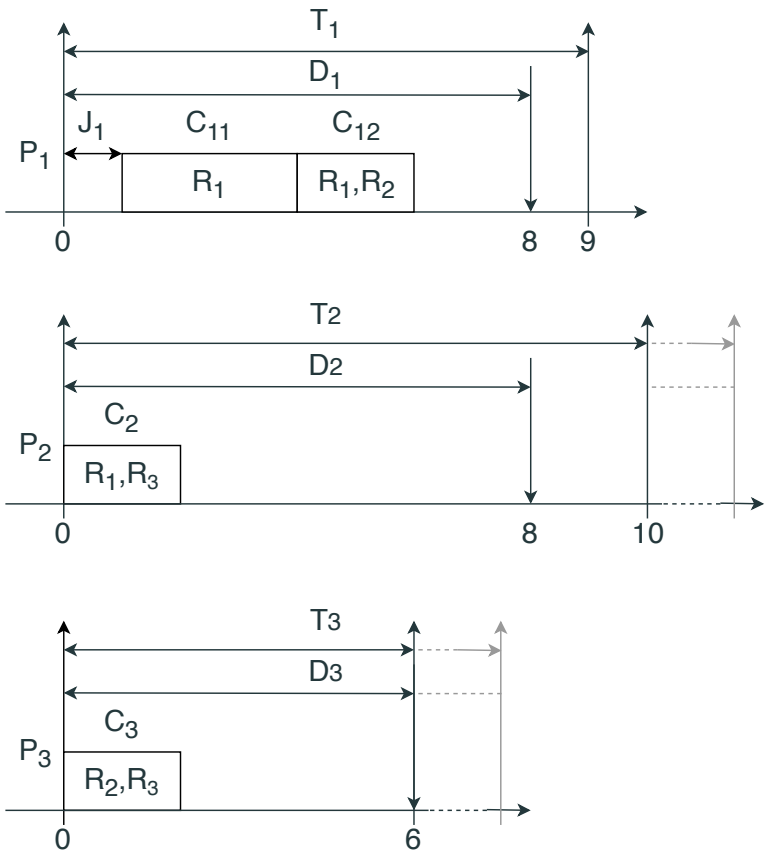
- The TPN could be automatically derived from the timeline

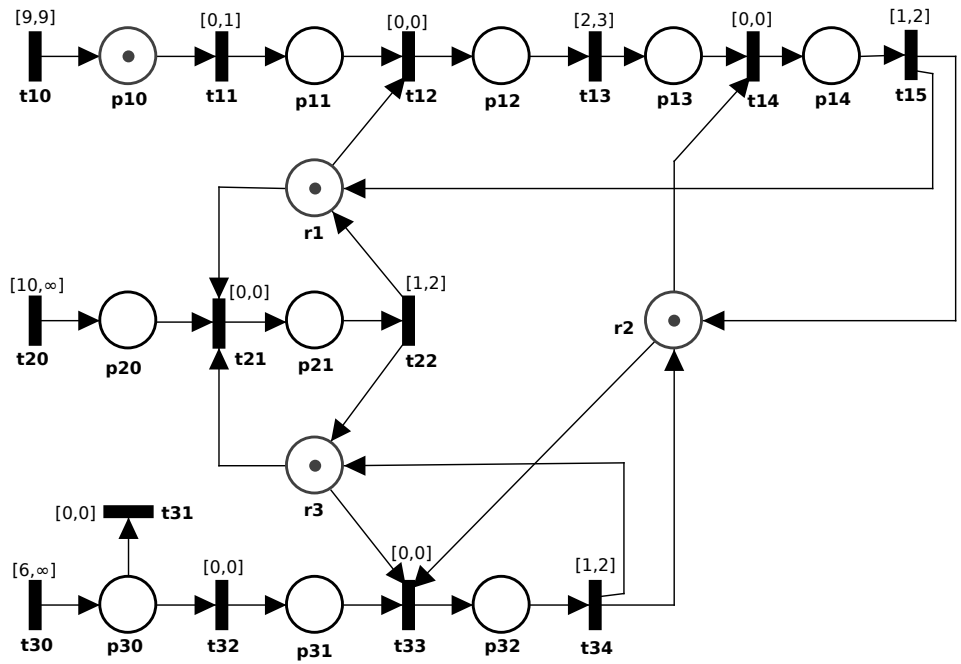




Modeling real-time task-sets with TPNs: an example (3/4)

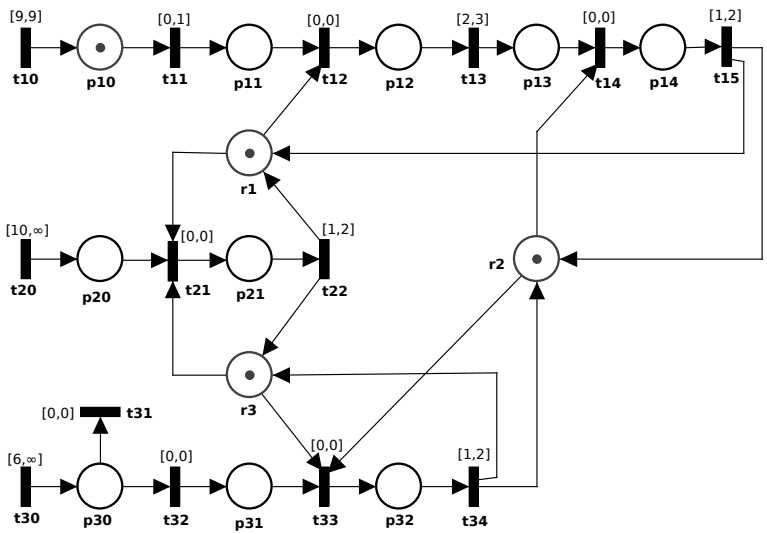
- Remarks
  - The TPN does not capture relative deadlines (which are properties to verify)
  - The model includes two types of non-determinism:
    - in the selection of the times to fire (not under the control of the designer)
    - in the acceptance rejection of the jobs of P_3 (under the control of the designer)





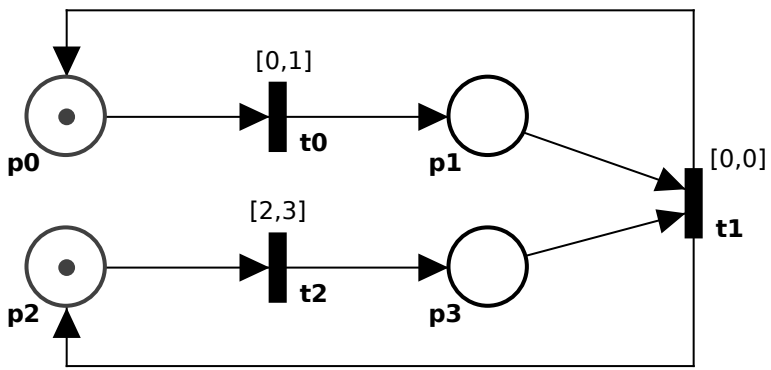
Modeling real-time task-sets with TPNs: an example (4/4)

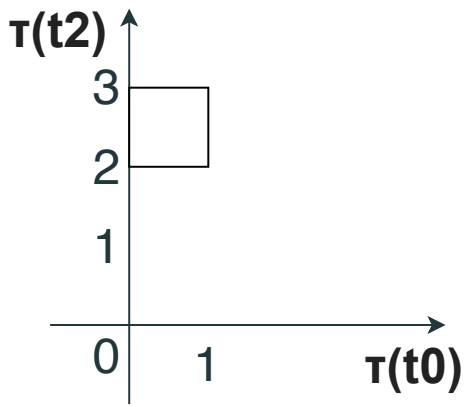
- Which questions could the analysis of the TPN model answer?
  - If P_3 jobs are always rejected, do P_1 and P_2 jobs always meet their deadline?
  - If P_3 jobs are always accepted, do P_1 and P_2 jobs always meet their deadline?
  - Is there an acceptance/rejection strategy guaranteeing deadlines of P_1 and P_2?
- Which questions could the TPN analysis answer in general?
  - **Timed reachability**: which are the feasible orderings among transitions under the constraints imposed by the temporal parameters of the model?
  - **Min/max duration**: what is the minimum and maximum time at which some firing sequence (i.e., sequence of transition firings) can be completed?



TPN analysis: preliminaries (1/3)

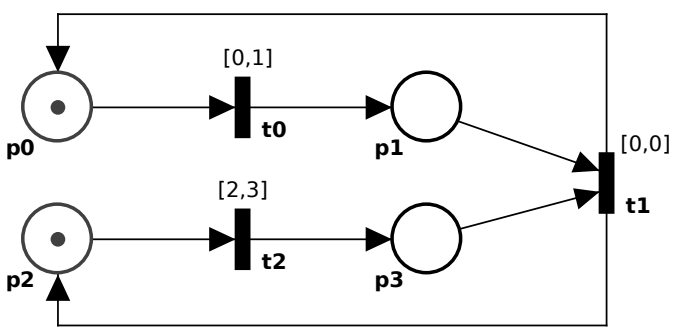
- Times-to-fire take values in dense sets ⇒
  ⇒ the reachability relation between states is not explicitly enumerable ⇒
  ⇒ the state space is covered through **equivalence classes**, each collecting a continuous variety of states (same marking, different times to fire vector)

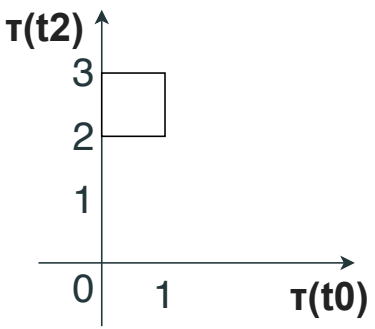




TPN analysis: preliminaries (2/3)

- A **state class** is a pair $S = \langle m, D \rangle$ where:
  - m is a marking (representing the logical location of the system)
  - D is a continuous set of values for the times to fire of the enabled transitions (D is usually termed **firing domain**)
- A state $s = \langle m_s, \vec{\tau_s} \rangle$ is included in a state class $S = \langle m, D \rangle$ if:
  - they have the same marking (i.e., $m_s = m$)
  - the times to fire vector $\vec{\tau_s}$ satisfies the constraints of D (i.e., $\vec{\tau_s} \in D$)
- The form of the constraints of D depend on:
  - the way the times to fire are advanced in the TPN semantics
  - the semantics of the reachability relation to establish on state classes
  - the form of the constraints in the initial state class



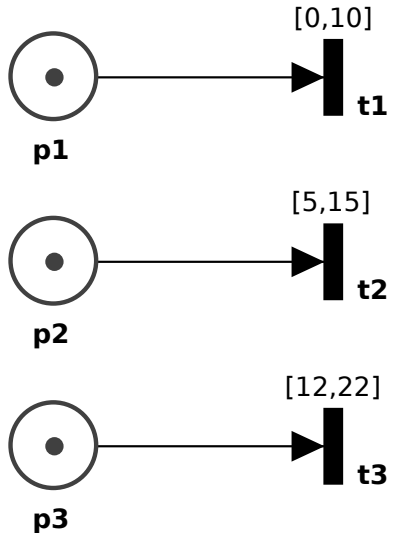


TPN analysis: preliminaries (3/3)

- A state class S' is **reachable** from class S through transition t ($S \xrightarrow{t} S'$) if and only if it contains all and only the states that are reachable from some state collected in S through some feasible firing of t, i.e., S' is reachable from S through t if and only if:
  - $\forall\, s' \in S'\ \exists\, s \in S \mid s \xrightarrow{t} s'$
  - $s \in S \wedge s \xrightarrow{t} s' \Rightarrow s' \in S'$
- Weak reachability relation termed **AE reachability**
  - Does not guarantee that every state $s' \in S'$ is reachable from every state $s \in S$
  - Guarantees that for any state $s' \in S'$ there exists at least one state $s \in S$ and a time τ such that t can be fired from s with firing time τ yielding state s'
- Strong enough to decide reachability properties under timing constraints
  - A state s' is reachable from some state in the initial class S_0 iff s' is contained in some class S which is reachable from S_0
  - A firing sequence ρ is firable iff there exists a class S' reachable from S_0 from which there exists a path reproducing the same events of ρ

TPN analysis: an intuitive introduction (1/5)

- A TPN model with 3 concurrent transitions



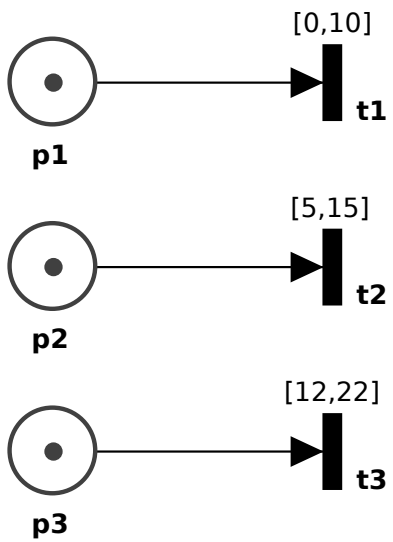
- Times to fire of the enabled transitions initially range within a hyper-rectangle (to simplify notation, the time to fire of transition t_i is denoted as $\tau_i \; \forall \, t_i \in T$)

$$D = \begin{cases} 0 \leq \tau_1 \leq 10 \\ 5 \leq \tau_2 \leq 15 \\ 12 \leq \tau_3 \leq 22 \end{cases}$$

TPN analysis: an intuitive introduction (2/5)

- The assumption that t_2 fires first restricts the firing domain:

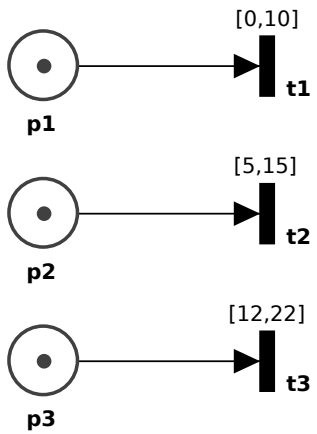
$$D_{t_2} = \begin{cases} D \\ \tau_2 \leq \tau_1 \\ \tau_2 \leq \tau_3 \end{cases} = \begin{cases} 0 \leq \tau_1 \leq 10 \\ 5 \leq \tau_2 \leq 15 \\ 12 \leq \tau_3 \leq 22 \\ \tau_2 - \tau_1 \leq 0 \\ \tau_2 - \tau_3 \leq 0 \end{cases}$$



TPN analysis: an intuitive introduction (3/5)

- Both t_1 and t_3 are persistent so their times to fire are shifted by τ_2:
  - $\tau_1' = \tau_1 - \tau_2 \Rightarrow \tau_1 = \tau_1' + \tau_2$
  - $\tau_3' = \tau_3 - \tau_2 \Rightarrow \tau_3 = \tau_3' + \tau_2$
- By replacing variables, D_{t_2} can be equivalently written as:

$$D_{t_2} = \begin{cases} 0 \leq \tau_1' + \tau_2 \leq 10 & (a) \\ 5 \leq \tau_2 \leq 15 & (b) \\ 12 \leq \tau_3' + \tau_2 \leq 22 & (c) \\ 0 \leq \tau_1' & (d) \\ 0 \leq \tau_3' & (e) \end{cases}$$



TPN analysis: an intuitive introduction (4/5)

- τ_2 is eliminated to obtain a domain D' depending only on the times to fire of the enabled transitions, i.e., $\langle \tau'_1, \tau'_3 \rangle \in D'$ iff $\exists\, \tau_2$ s.t. $\langle \tau'_1, \tau_2, \tau'_3 \rangle \in D_{t_2}$:

$$D'_{t_2} = \begin{cases} -15 \leq \tau'_1 \leq 5 & (a+(-b)) \\ -3 \leq \tau'_3 \leq 17 & (c+(-b)) \\ -22 \leq \tau'_1 - \tau'_3 \leq -2 & (a+(-c)) \\ 0 \leq \tau'_1 & (d) \\ 0 \leq \tau'_3 & (e) \\ \\ \mathbf{5 \leq \tau_2 \leq 15} & \mathbf{(b)} \end{cases}$$

$$D' = \begin{cases} -15 \leq \tau'_1 \leq 5 & (a+(-b)) \\ -3 \leq \tau'_3 \leq 17 & (c+(-b)) \\ -22 \leq \tau'_1 - \tau'_3 \leq -2 & (a+(-c)) \\ 0 \leq \tau'_1 & (d) \\ 0 \leq \tau'_3 & (e) \end{cases}$$

TPN analysis: an intuitive introduction (5/5)

- What is the meaning of the set D'?
  - The successor class contains all and only the state reachable from the initial class: $\forall \langle \tau_1', \tau_2' \rangle \in D' \; \exists \langle \tau_1, \tau_2, \tau_3 \rangle \in D$ such that $\tau_1' = \tau_1 - \tau_2 \wedge \tau_3' = \tau_3 - \tau_2$
  - This is the AE reachability relation!
- What about the form of the constraints of D'?
  - The firing domain D of the initial class is an hyper-rectangle
  - D' is a set of inequalities constraining the difference between times to fire
  - The form of D' is termed **Difference Bounds Matrix (DBM)**
  - Does repetitive derivation yield a general linear convex polyhedron? No!

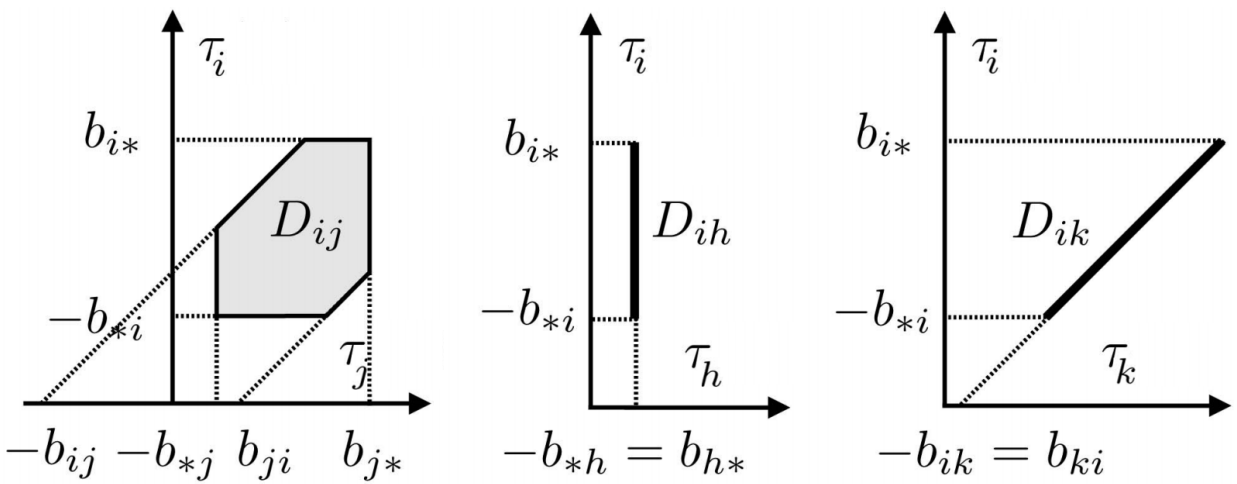
$$D = \begin{cases} 0 \leq \tau_1 \leq 10 \\ 5 \leq \tau_2 \leq 15 \\ 12 \leq \tau_3 \leq 22 \end{cases} \qquad D' = \begin{cases} -15 \leq \tau_1' \leq 5 & (a+(-b)) \\ -3 \leq \tau_3' \leq 17 & (c+(-b)) \\ -22 \leq \tau_1' - \tau_3' \leq -2 & (a+(-c)) \\ 0 \leq \tau_1' & (d) \\ 0 \leq \tau_3' & (e) \end{cases}$$

Difference Bounds Matrix: definition

- A DBM D in the space of a vector of unknown values $\boldsymbol{\tau} = \langle \tau_0, \tau_1, \ldots, \tau_N \rangle$ is a set of linear inequalities constraining the difference between elements of $\boldsymbol{\tau}$:

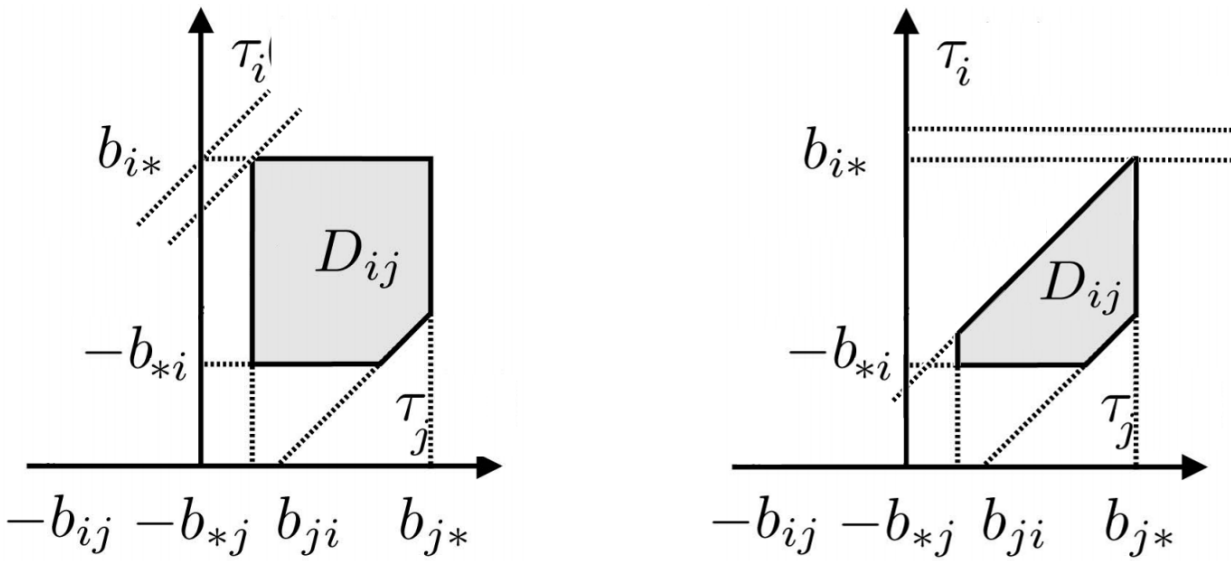
$$D = \begin{cases} \tau_i - \tau_j \leq b_{ij} & \forall\, i, j \in \{0, 1, \ldots, N-1\} \cup \{\star\} \text{ with } i \neq j,\ b_{ij} \in \mathbb{R} \cup \{\infty\} \\ \tau_\star = 0 \end{cases}$$

- $\tau_\star$ is a fictitious variable, also termed **ground**, used to:
  - cast constraints of the form $\tau_i \leq b_{i\star}$ into diagonal constraints $\tau_i - \tau_\star \leq b_{i\star}$
  - cast constraints of the form $\tau_i \geq -b_{\star i}$ into diagonal constraints $\tau_\star - \tau_i \leq b_{\star i}$
- The set of solution of a set of inequalities in DBM form is termed **DBM zone**



Difference Bounds Matrix: normal form (1/2)

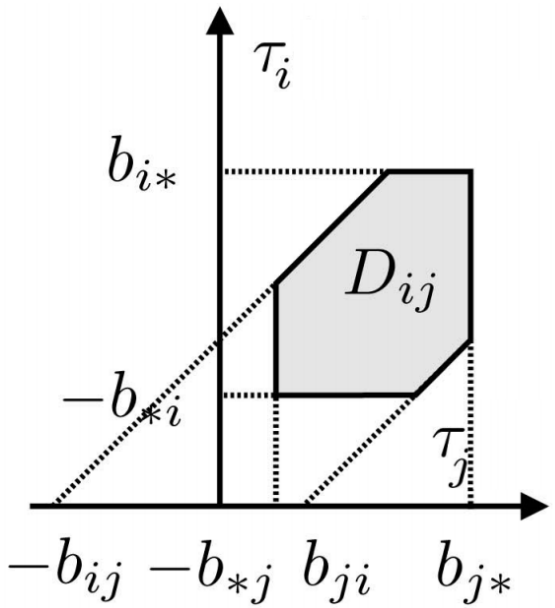
- May different sets of inequalities represent the same set of solutions?
  - Some constraints may be implied by the combination of other constraints
  - The same set of solutions can be represented by different coefficients b_{ij}



- A **normal form** is introduced where each coefficient b_{ij} is maximally tight (i.e., it cannot be reduced without changing the set of solutions)

Difference Bounds Matrix: normal form (2/2)

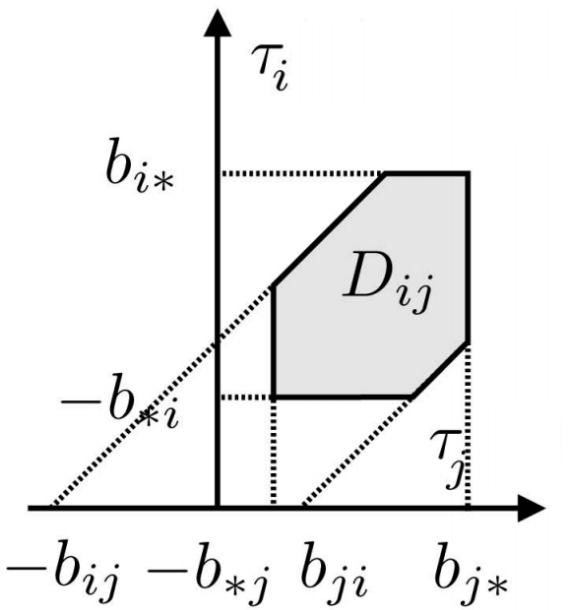
- A DBM D is in **normal form** if b_{ij} is the maximum value of $\tau_i - \tau_j$ that can be attained by any solution of the set $\forall\, i, j \in \{0, 1, \ldots, N-1\} \cup \{\star\} \mid i \neq j$

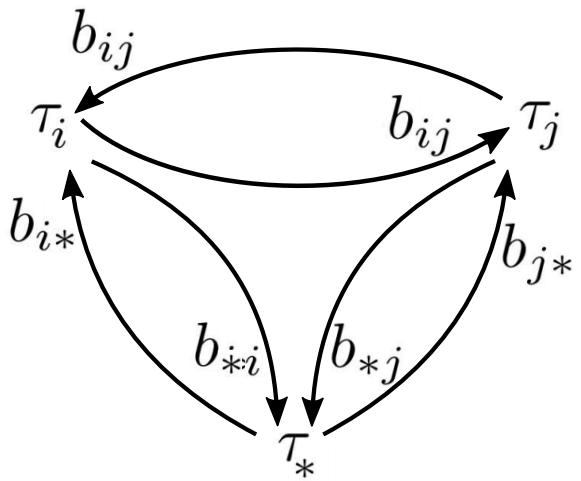


- A non-empty DBM is represented as a **graph** to prove that the normal form:
  - exists and is unique
  - is computed as the solution of an **all shortest path** problem in time $O(N^3)$

Difference Bounds Matrix: graph representation

- The DBM identified by the matrix of coefficients b_{ij} is biunivocally associated with a **complete directed labeled graph** $\langle V, E, b \rangle$ where:
  - the set of vertices V consists of the unknown values τ_i
  - a directed edge exists between any two edges (i.e., $E = V \times V$)
  - the edge from vertex τ_j to vertex τ_i is labeled by b_{ij}

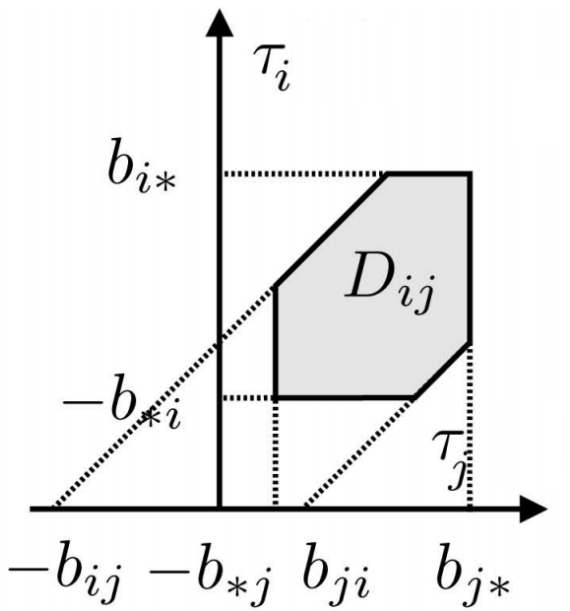


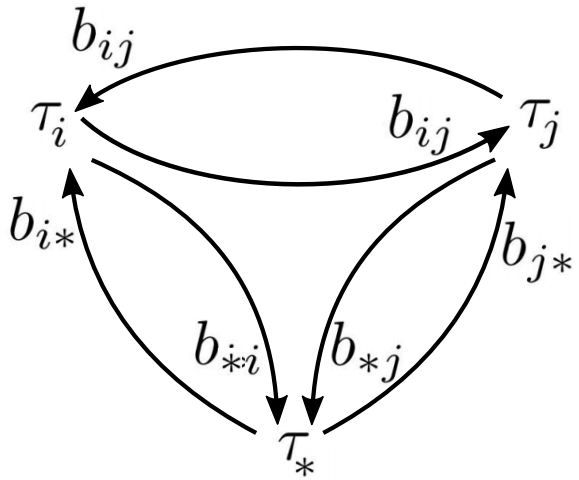


Difference Bounds Matrix: properties (1/4)

Lemma 1

*If a DBM zone is not empty, then the corresponding graph does not include any cycle where the sum of labels on traversed edges is negative (***negative cycle***).*

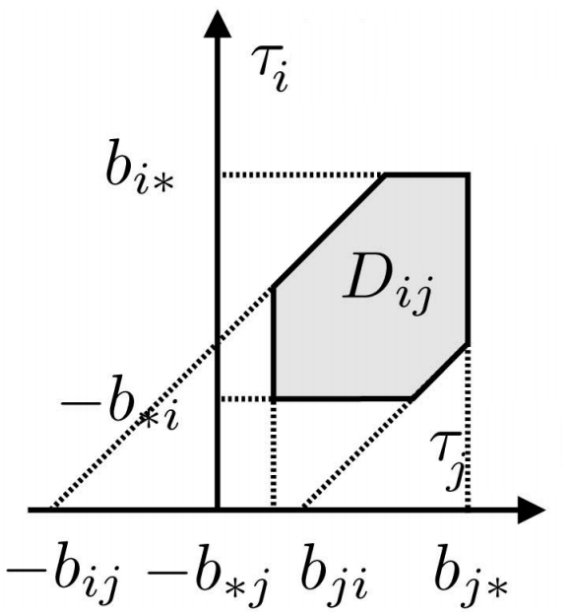


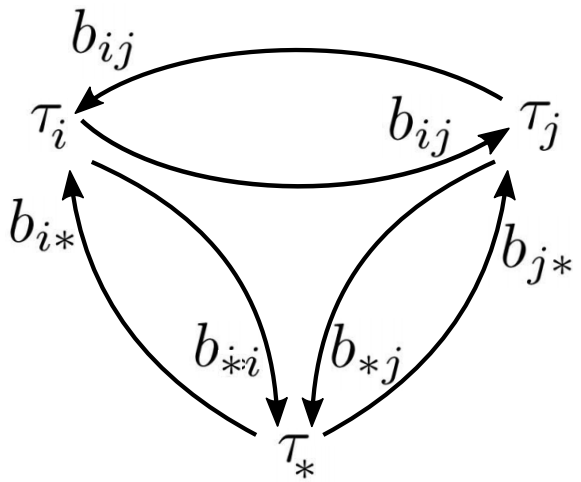


Difference Bounds Matrix: properties (2/4)

Lemma 2

A DBM is in normal form if and only if b_{ij} is the shortest path in the graph from vertex τ_j to vertex τ_i $\forall\, i, j \in \{0, 1, \ldots, N-1\} \cup \{\star\}$ with $i \neq j$.

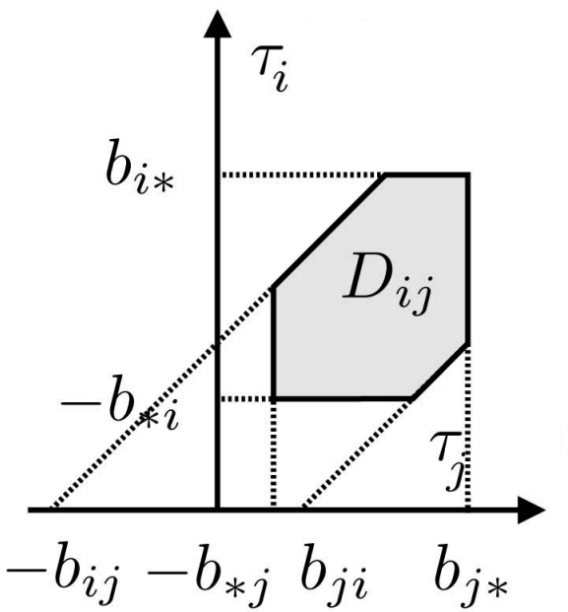


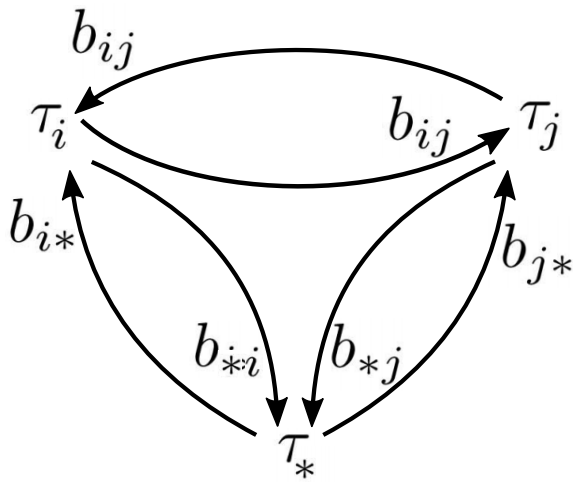


Difference Bounds Matrix: properties (3/4)

Lemma 3

Coefficients b_{ij} are the shortest paths between any two vertices τ_j and τ_i if and only if $b_{ij} \leq b_{ik} + b_{kj}$ $\forall\, i, j, k \in \{0, 1, \ldots, N-1\} \cup \{\star\}$ with $i \neq j$, $i \neq k$, $j \neq k$.

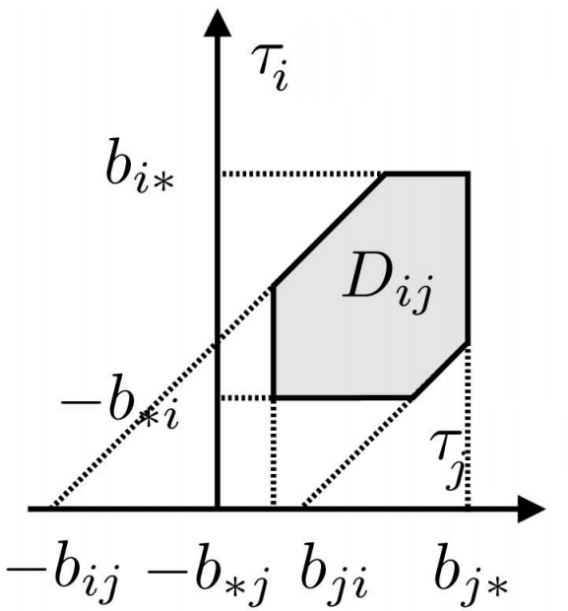


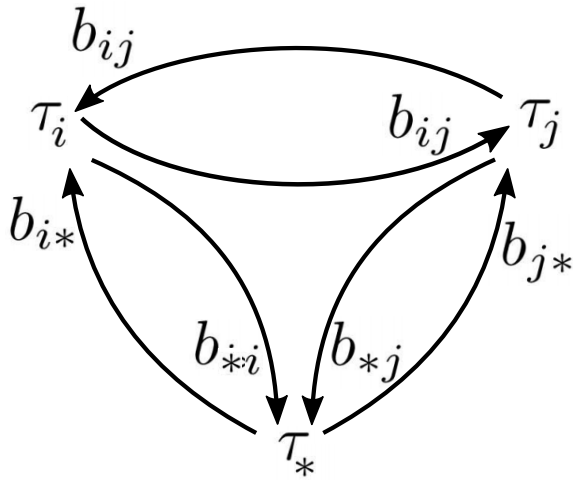


Difference Bounds Matrix: properties (4/4)

Theorem 4

A DBM is in normal form if and only if $b_{ij} \leq b_{ik} + b_{kj}$
$\forall\, i, j, k \in \{0, 1, \ldots, N-1\} \cup \{\star\}$ *with* $i \neq j$, $i \neq k$, $j \neq k$.





Difference Bounds Matrix: how to compute the normal form?

- The normal form can be computed by the **Floyd-Warshall algorithm**
  - N iterations indexed by $k = 0, 1, \ldots, N-1$
  - Iteration k derives b_{ij}^{k+1} defined as the shortest path from τ_j to τ_i that does not visit any intermediate node with index larger than k
    - **Initialization**: b_{ij}^0 is the initial value of coefficient b_{ij}
    - **First iteration** ($k = 0$): b_{ij}^1 is the shortest path from τ_j to τ_i that does not visit any intermediate node with index larger than 0
    - **Last iteration** ($k = N-1$): b_{ij}^N is the shortest path from τ_j to τ_i

Algorithm 2 Floyd-Warshall algorithm

1: **for** each $i, j \in \{0, 1, \ldots, N-1\} \cup \{\star\}$ with $i \neq j$ **do**
2: $\quad b_{ij}^0 = b_{ij}$
3: **end for**
4: **for** each $k = 0, 1, \ldots, N-1$ **do**
5: $\quad$ **for** each $i \in \{0, 1, \ldots, N-1\} \cup \{\star\}$ **do**
6: $\quad\quad$ **for** each $j \in \{0, 1, \ldots, N-1\} \cup \{\star\}$ with $i \neq j$ **do**
7: $\quad\quad\quad b_{ij}^{k+1} = \min\{b_{ij}^k, b_{ik}^k + b_{kj}^k\}$
8: $\quad\quad$ **end for**
9: $\quad$ **end for**
10: **end for**

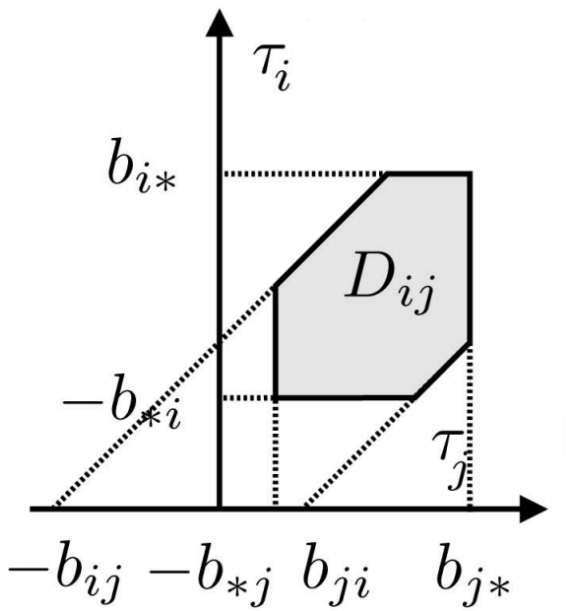
Difference Bounds Matrix: deciding emptiness

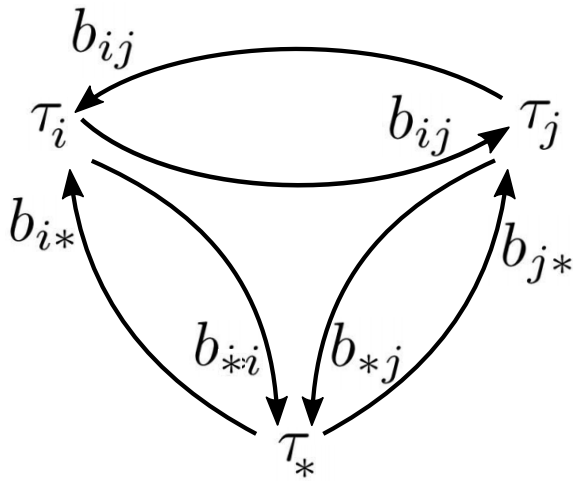
Lemma 5

If the graph corresponding to a DBM does not include any negative cycle, then the DBM zone is not empty.

Lemma 6

A DBM zone is not empty if and only if the corresponding graph does not include any negative cycle.





Difference Bounds Matrix: projection

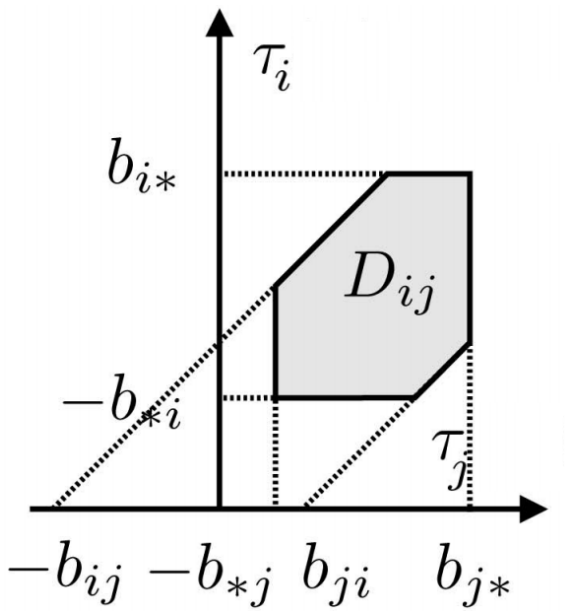
- Given a DBM D in the space of unknown values $\boldsymbol{\tau} = \langle \tau_0, \tau_1, \ldots, \tau_{N-1} \rangle$, the **projection** of D over the space of unknown values $\boldsymbol{\tau}_{\downarrow 0} = \langle \tau_1, \ldots, \tau_{N-1} \rangle$ is $D_{\downarrow 0} := \{ \langle \tau_1, \ldots, \tau_{N-1} \rangle \mid \exists\ \tau_0 \in \mathbb{R} \cup \{\infty\}$ such that $\langle \tau_0, \tau_1, \ldots, \tau_{N-1} \rangle \in D \}$
- Projection is used in the enumeration of the reachability relation between state classes
- Projection is efficiently implemented over a DBM zone in normal form

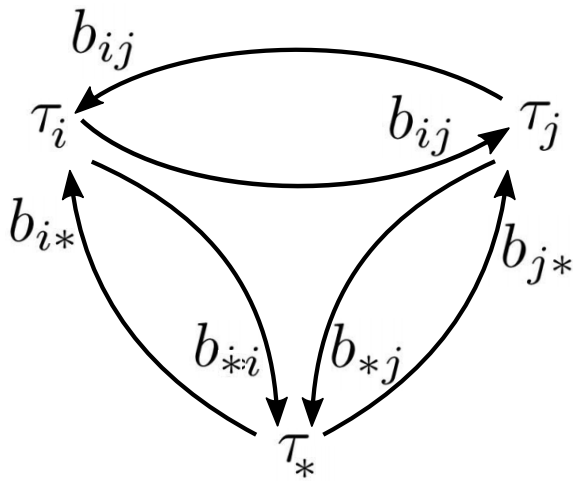
Lemma 7

If a DBM D in the space of unknown values $\boldsymbol{\tau} = \langle \tau_0, \tau_1, \ldots, \tau_{N-1} \rangle$ is in normal form, then the matrix of coefficients of the projection $D_{\downarrow 0}$ is obtained from the matrix of coefficients of D by discarding row 0 and column 0, i.e., the graph of $D_{\downarrow 0}$ is obtained from the graph of D by eliminating node τ_0 (without changing the constraint between any other two nodes).

TPN analysis: initial state class

- We assume that the firing domain D of the initial state class $S = \langle m, D \rangle$ is represented as a set of linear inequalities in DBM normal form, i.e., $D = \{\tau(t_i) - \tau(t_j) \leq b_{ij} \; \forall \, t_i, t_j \in T_e(m) \cup \{t_\star\}$ with $i \neq j$, where:
  - $b_{ij} \in \mathbb{R} \cup \{\infty\}$
  - $T_e(m)$ is the set of transitions enabled by marking m
  - $\tau(t_i)$ is the time to fire of transition $t_i \; \forall \, t_i \in T_e(m)$
  - $t_\star$ is the fictitious event of entrance into the class
  - $\tau(t_\star)$ is the **ground** time at which the class is entered





TPN analysis: successor existence

- A state class $S = \langle m, D \rangle$ with D in DBM normal form has a **successor** through transition t_0 if and only if t_0 is enabled by m and D accepts solutions where $\tau(t_0)$ is not larger than the time to fire of any other enabled transition, i.e., the **restricted firing domain** D_{t_0} has a non-empty set of solutions:

$$D_{t_0} = \begin{cases} \tau(t_i) - \tau(t_j) \leq b_{ij} \\ \tau(t_0) - \tau(t_h) \leq 0 \\ \forall\, t_i, t_j \in T_e(m) \cup \{t_\star\} \text{ with } t_i \neq t_j,\ \forall\, t_h \in T_e(m) \setminus \{t_0\} \end{cases}$$

Lemma 8

*A state class $S = \langle m, D \rangle$ with D in DBM normal form has a **successor** through transition t_0 if and only if $t_0 \in T_e(m)$ and $b_{h0} \geq 0\ \forall\, t_h \in T_e(m)$.*

- Successor detection: linear complexity wrt the number of enabled transitions
- How to compute the firing domain D' of the successor S' of S through t_0?

TPN analysis: successor computation - conditioning

- Condition D to the fact that t_0 fires first:

$$D_{t_0} = \begin{cases} D \\ \tau(t_0) - \tau(t_h) \leq 0 \\ \forall\, t_h \in T_e(m) \setminus \{t_0\} \end{cases} = \begin{cases} \tau(t_i) - \tau(t_j) \leq b_{ij} \\ \tau(t_0) - \tau(t_h) \leq 0 \\ \forall\, t_i, t_j \in T_e(m) \cup \{t_\star\} \,|\, t_i \neq t_j \\ \forall\, t_h \in T_e(m) \setminus \{t_0\} \end{cases}$$

$$= \begin{cases} \tau(t_i) - \tau(t_j) \leq b_{ij} \\ \tau(t_0) - \tau(t_h) \leq \min\{0, b_{0h}\} \\ \forall\, t_i, t_j \in T_e(m) \cup \{t_\star\} \,|\, t_i \neq t_j,\ \forall\, t_h \in T_e(m) \setminus \{t_0\} \end{cases}$$

- Represent D_{t_0} in normal form and let B_{ij} be its coefficients (quadratic complexity wrt the number of enabled transitions):

$$D_{t_0} = \begin{cases} \tau(t_i) - \tau(t_j) \leq B_{ij} \\ \tau(t_0) - \tau(t_i) \leq B_{0i} \\ \forall\, t_i, t_j \in T_e(m) \cup \{t_\star\} \,|\, t_i \neq t_j,\ t_i \neq t_0 \end{cases}$$

TPN analysis: successor computation - time advancement

- Reduce times to fire of persistent transitions in S' by the time elapsed in S, i.e., $\tau'(t_i) = \tau(t_i) - \tau(t_0)\ \forall\, t_i \in T_{\mathrm{e}}(m) \cap T_{\mathrm{p}}(S') \setminus \{t_0\}$ (where $T_{\mathrm{p}}(S')$ is the set of transitions that are persistent in S'):

$$D_{t_0} = \begin{cases} \tau(t_i) - \tau(t_j) \leq B_{ij} \\ \tau(t_i) - \tau(t_0) \leq B_{i0} \\ \tau(t_0) - \tau(t_i) \leq B_{0i} \\ \\ \tau(t_i) - \tau(t_\star) \leq B_{i\star} \\ \tau(t_\star) - \tau(t_i) \leq B_{\star i} \\ \tau(t_0) - \tau(t_\star) \leq B_{0\star} \\ \tau(t_\star) - \tau(t_0) \leq B_{\star 0} \\ \\ \forall\, t_i, t_j \in T_{\mathrm{e}}(m) \cap T_{\mathrm{p}}(S') \cup \{t_\star\} \mid \\ t_i \neq t_j,\ t_i \neq t_0,\ t_i \neq t_\star \end{cases} \qquad D'_{t_0} = \begin{cases} \tau'(t_i) - \tau'(t_j) \leq B_{ij} \\ \tau'(t_i) \leq B_{i0} \\ -\tau'(t_i) \leq B_{0i} \\ \\ \tau'(t_i) + \tau(t_0) \leq B_{i\star} \\ -\tau(t_0) - \tau'(t_i) \leq B_{\star i} \\ \tau(t_0) - \tau(t_i) \leq B_{0i} \\ \tau(t_0) - \tau(t_\star) \leq B_{0\star} \\ \\ \forall\, t_i, t_j \in T_{\mathrm{e}}(m) \cap T_{\mathrm{p}}(S') \cup \{t_\star\} \mid \\ t_i \neq t_j,\ t_i \neq t_0,\ t_j \neq t_\star \end{cases}$$

TPN analysis: successor computation - projection

- Eliminate the time to fire of the fired transition through a projection:

$$D''_{t_0} = \begin{cases} \tau'(t_i) - \tau'(t_j) \leq B_{ij} \\ \tau'(t_i) \leq B_{i0} \\ -\tau'(t_i) \leq B_{0i} \\ \forall\, t_i, t_j \in T_{\mathrm{e}}(m) \cup \{t_\star\} \,|\, t_i \neq t_j,\ t_i \neq t_0 \end{cases}$$

- Let C_{ij} be the normal form coefficients (i.e., $C_{ij} = B_{ij}$, $C_{i\star} = B_{i0}$, $C_{\star i} = B_{0i}$):

$$D''_{t_0} = \begin{cases} \tau'(t_i) - \tau'(t_j) \leq C_{ij} \\ \tau'(t_i) - \tau'(t_\star) \leq C_{i\star} \\ \tau'(t_\star) - \tau'(t_i) \leq C_{\star i} \\ \forall\, t_i, t_j \in T_{\mathrm{e}}(m) \cap T_{\mathrm{p}}(S') \cup \{t_\star\} \,|\, t_i \neq t_j,\ t_i \neq t_\star \end{cases}$$

- Times-to-fire of transitions not enabled by m' can be similarly eliminated

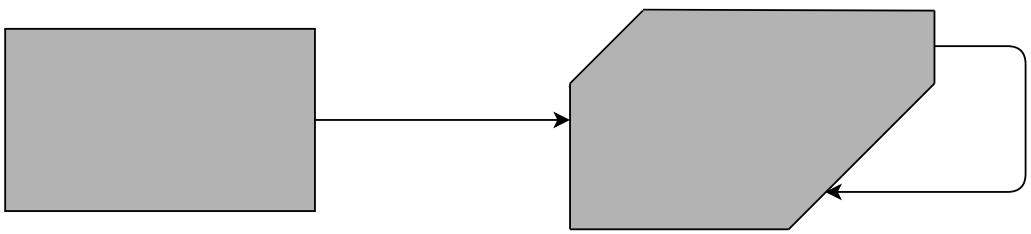
TPN analysis: successor computation - newly enabling

- Add the time to fire of each newly-enabled transition:

$$D' = \begin{cases} D''_{t_0} \\ \tau'(t_j) - \tau'(t_\star) \leq LFT(t_j) \\ \tau'(t_\star) - \tau'(t_j) \leq -EFT(t_j) \\ \forall\, t_j \in T_{\mathrm{n}}(S') \end{cases}$$

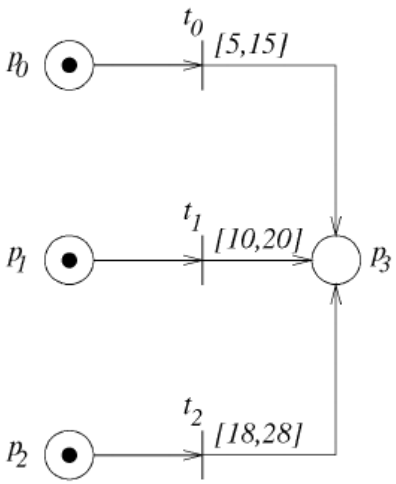
  where $T_{\mathrm{n}}(S')$ is the set of transitions newly-enabled in S'

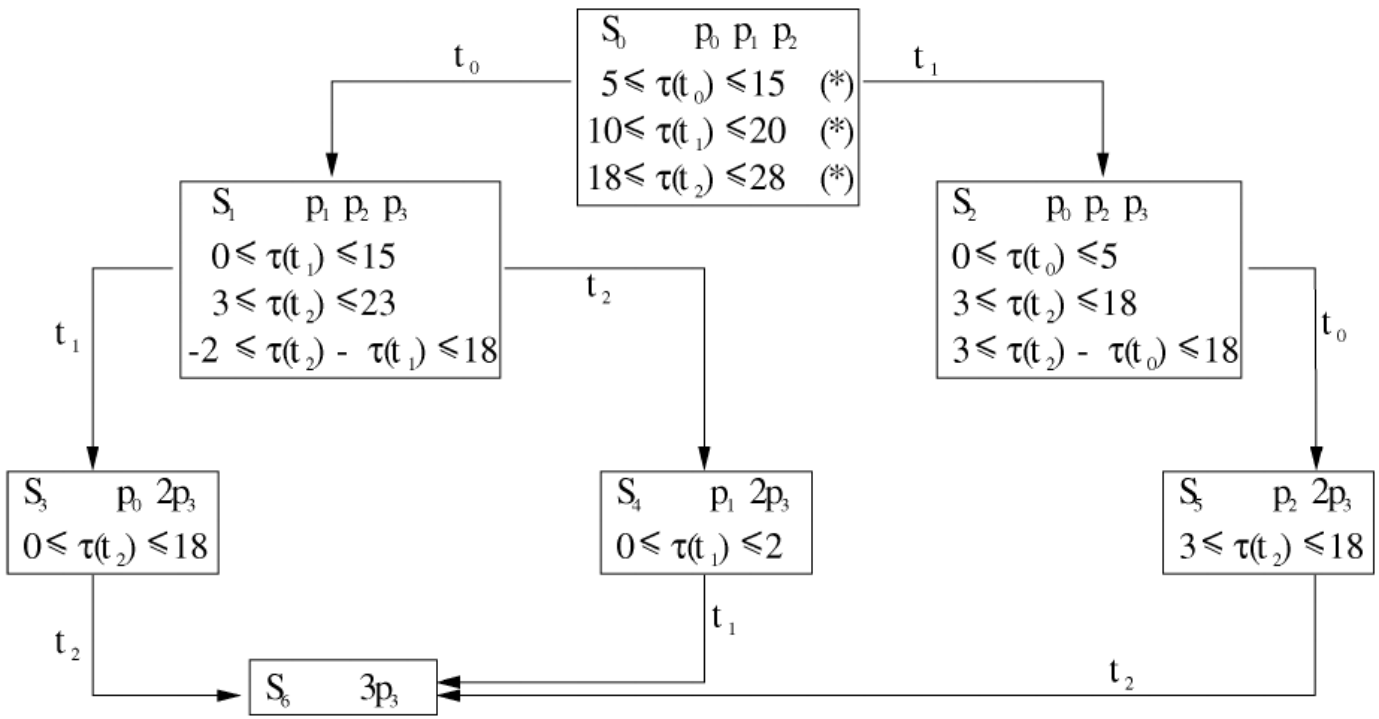
- The firing domain D' of the successor class S' is still in DBM form! $\Rightarrow$
  $\Rightarrow$ The DBM form is closed wrt the steps of successor computation!
  $\Rightarrow$ The reachability relation among state classes can be enumerated!



TPN analysis: state space enumeration

- Enumeration yields the **state class graph**
- A **symbolic run** is a path in the state class graph
  - Represents the dense variety of runs firing a given set of transitions in a given qualitative order with a dense variety of timings between subsequent firings
  - e.g., $S_0 \xrightarrow{t_0} S_1 \xrightarrow{t_2} S_4 \xrightarrow{t_1} S_6$





Domain of timings of a symbolic run

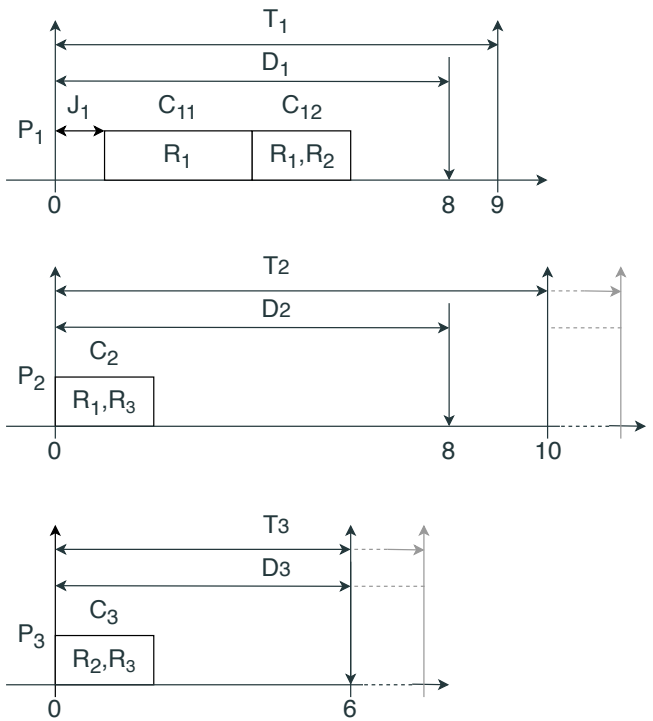
- Consider a symbolic run $\rho = S_0 \overset{t^0_{f(0)}}{\rightarrow} S_1 \overset{t^1_{f(1)}}{\rightarrow} S_2 \dots S_{N-1} \overset{t^{N-1}_{f(N-1)}}{\rightarrow} S_N$
  - $S_0 = \langle m_0, D_0 \rangle$ is the initial state class
  - $S_n = \langle m_n, D_n \rangle$ is the n-th state class visited by $\rho \; \forall \, n \in \{1, \dots, N\}$
  - $f(n)$ is the index of the transition fired in state class $S_n \; \forall \, n \in \{0, 1, \dots, N-1\}$
  - t^k_h is the time to fire of transition t_h in state class S_k
- The domain of timings of ρ is in DBM form:

$$D_\rho = \begin{cases} D_n \\ \tau(t^n_{f(n)}) - \tau(t^n_i) \leq 0 \\ \tau(t^n_{f(n)}) = \tau(t^{n+1}_\star) \\ \forall \, n \in \{0, 1, \dots, N-1\}, \; \forall \, t^n_i \in T_{\mathrm{e}}(m) \end{cases}$$

- Sojourn times in classes are **not independent** $\Rightarrow$ The worst case completion of ρ may not be the sum of the worst case sojourn times in $S_0, \dots, S_{N-1}$
- Computation of D_ρ: complexity $O(N^3)$
  - Note that N is the number of transitions **fired** along ρ
  - The number of transitions **enabled** along ρ can be $>> N$

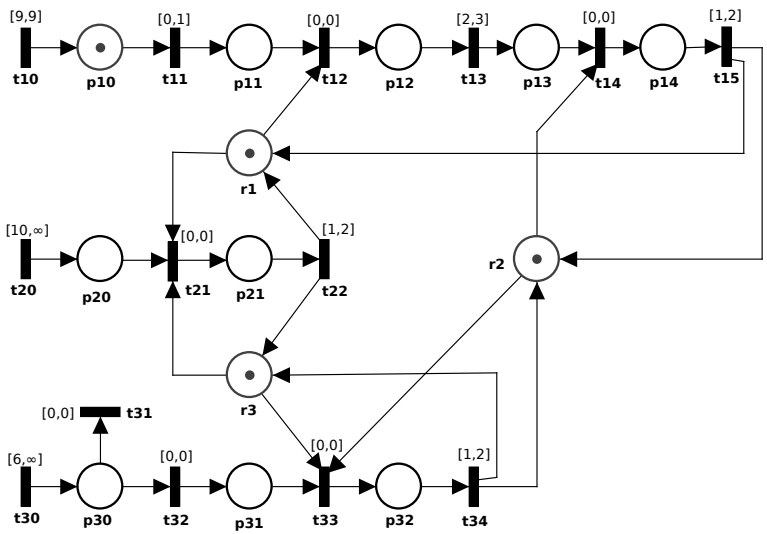
Modeling real-time task-sets with TPNs: an example (1/3)

- 3 tasks P_1, P_2, P_3 contending for 3 non-preemptable resources R_1, R_2, R_3
  - P_1 and P_2 are hard real-time (mandatory), P_3 is soft real-time (optional)
- P_1 is periodic
  - period $T_1 = 9$
  - relative deadline $D_1 = 8$
  - absolute start time jitter $J_1 \leq 1$
  - computation 1: WCET $C_{11} \in [2, 3]$, uses R_1
  - computation 2: WCET $C_{12} \in [1, 2]$, uses R_1, R_2
- P_2 is sporadic
  - inter-arrival time $T_2 \geq 8$
  - relative deadline $D_2 = 8$
  - computation: WCET $C_{21} \in [1, 2]$, uses R_1, R_3
- P_3 is sporadic
  - inter-arrival time $T_3 \geq 6$
  - relative deadline $D_3 = 6$
  - computation: WCET $C_{31} \in [1, 2]$, uses R_2, R_3



Modeling real-time task-sets with TPNs: an example (2/3)

- Which questions could the analysis of the TPN model answer?
  - If P_3 jobs are always rejected, do P_1 and P_2 jobs always meet their deadline?
  - If P_3 jobs are always accepted, do P_1 and P_2 jobs always meet their deadline?
  - Is there an acceptance/rejection strategy guaranteeing deadlines of P_1 and P_2?
- Which questions could the TPN analysis answer in general?
  - **Timed reachability**: which are the feasible orderings among transitions under the constraints imposed by the temporal parameters of the model?
  - **Min/max duration**: what is the minimum and maximum time at which some firing sequence (i.e., sequence of transition firings) can be completed?

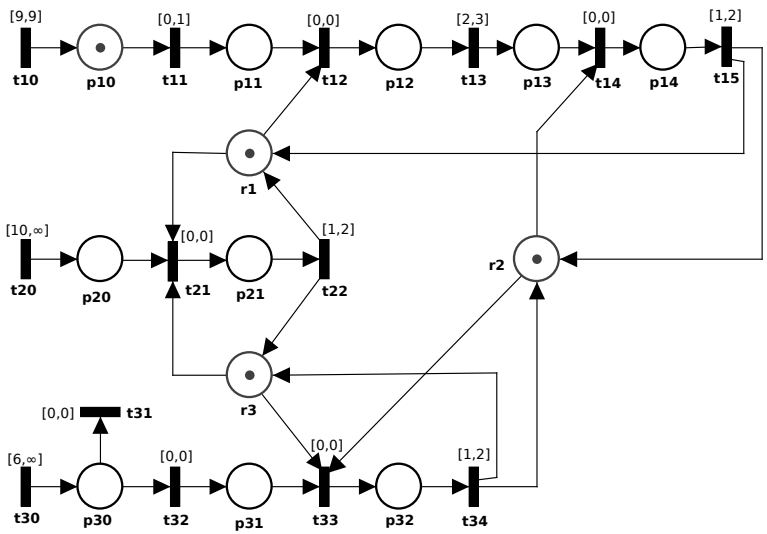


Modeling real-time task-sets with TPNs: an example (3/3)

- Both hard real-time tasks can miss their deadline:

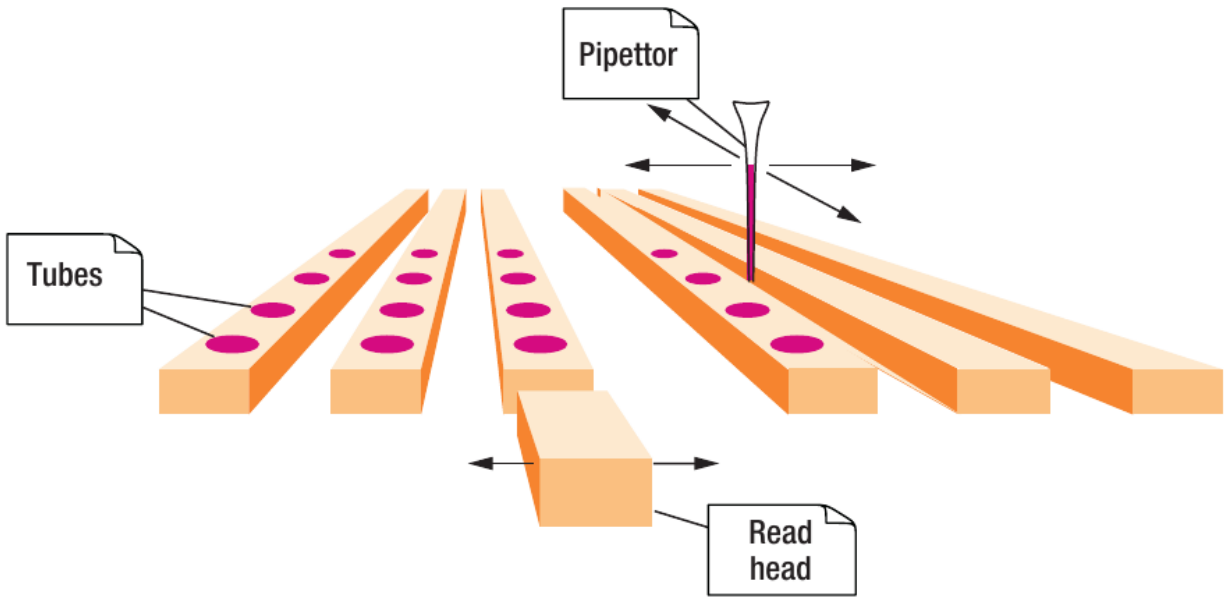
Task	Number of symbolic runs	Possible failures
P_1	3594	103
P_2	1871	352
P_3	1327	0

- How to decide acceptance/rejection of jobs of P_3 at runtime ? Using the **actual** execution timings to determine whether failing symbolic runs are feasible! A job of P_3 is discarded if a job of P_1/P_2 may miss its deadline!



Online scheduling in a biological analysis system (1/5)

- An electromechanical system for immunoenzymatic analysis by BioMérieux
  - Each slot comprises several test tubes
  - A pipettor and a read head can act simultaneously on test tubes

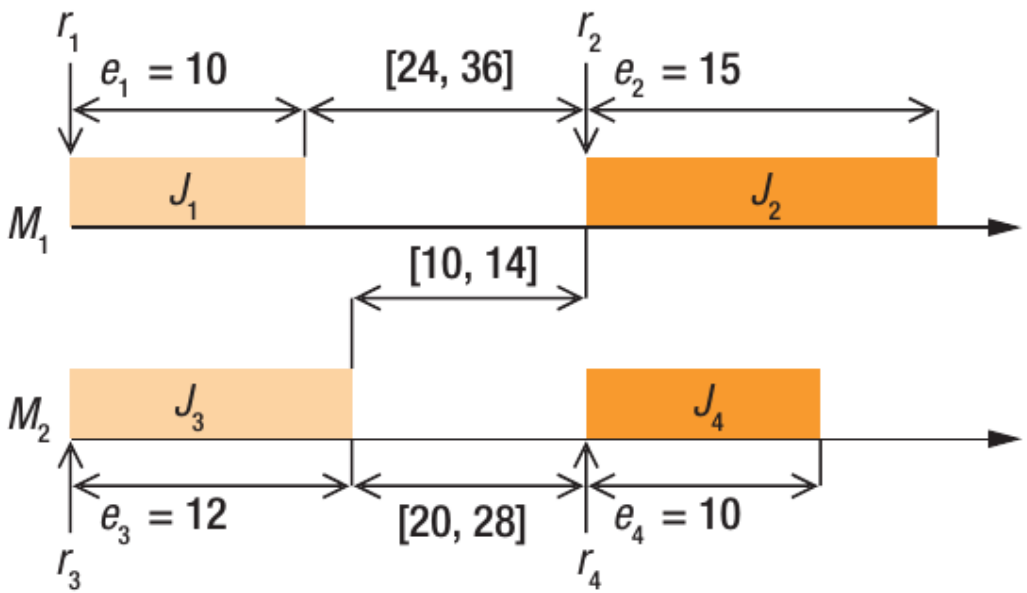


L. Ridi, J. Torrini, E. Vicario, "Developing a Scheduler with Difference-Bound Matrices and the Floyd-Warshall Algorithm," IEEE Software, 2012.

E. Vicario, L. Ridi, A. Carignano, J. Torrini, "Job scheduler for electromechanical system for biological analyses," Patent WO2013098088A1, 2013.

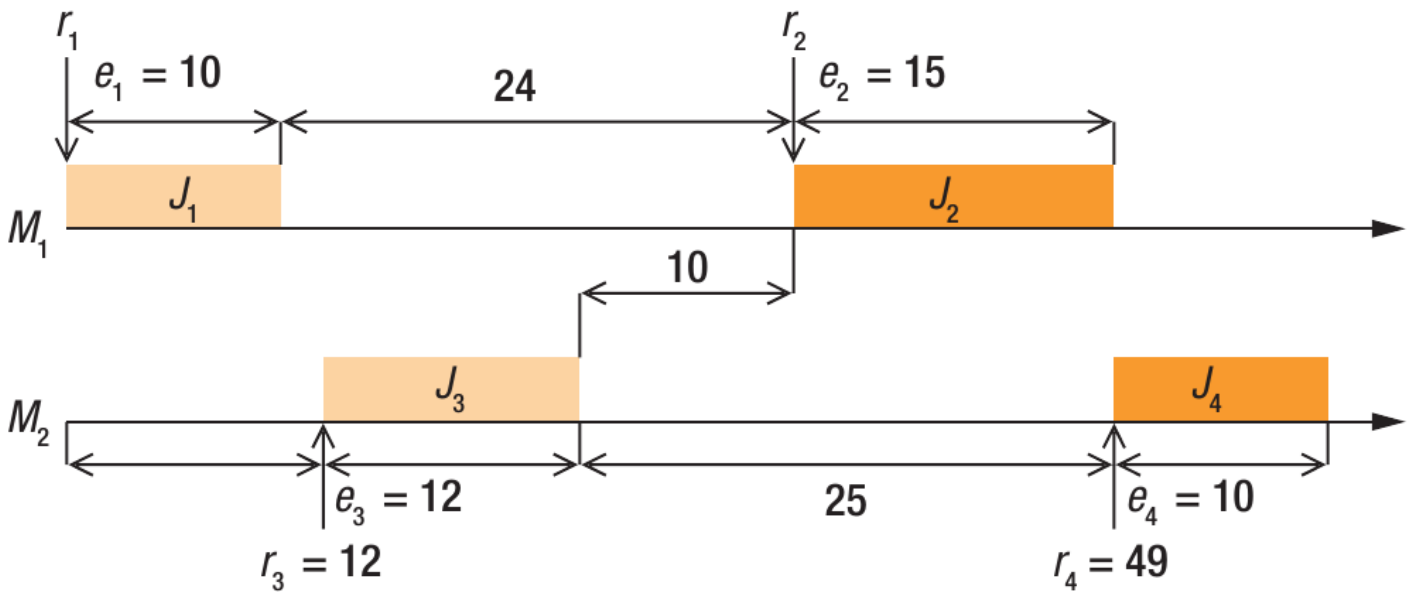
Online scheduling in a biological analysis system (2/5)

- An example of the scheduling problem
  - 4 jobs J_1, J_2, J_3, J_4 allocated on two machines M_1, M_2
  - e indicates an execution time, r indicates a release time
  - J_2 has a mutual exclusion with J_4
  - J_2 has a precedence constraints wrt the completion of J_1 and J_3
  - J_4 has a precedence constraints wrt the completion of J_3



Online scheduling in a biological analysis system (3/5)

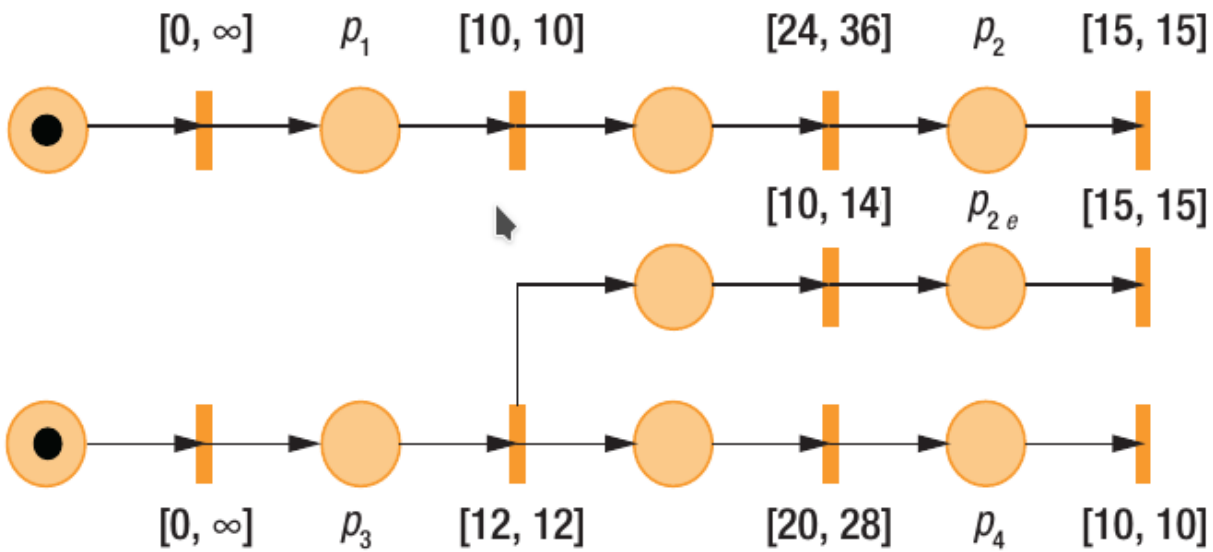
- A schedule that minimizes the overall completion time



- How to compute such a schedule?

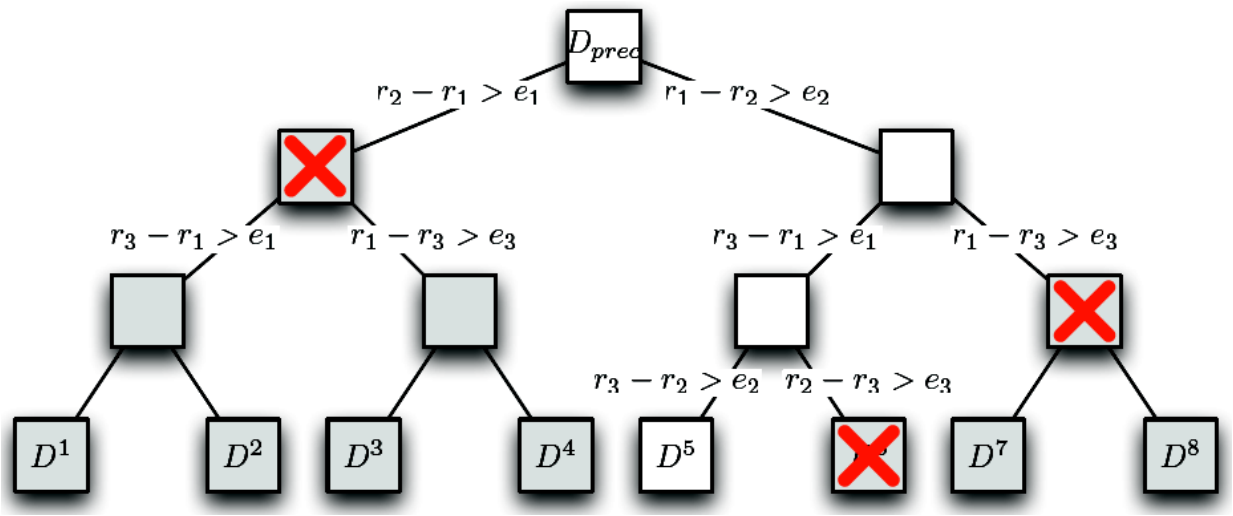
Online scheduling in a biological analysis system (4/5)

- TPN encoding sequencing and timing constraints of the scheduling problem
- An execution is correct if and only if it never visits a state class where:
  - place p_2 has a token while place p_{2e} is empty
  - both places p_2 and p_4 contain a token



Online scheduling in a biological analysis system (5/5)

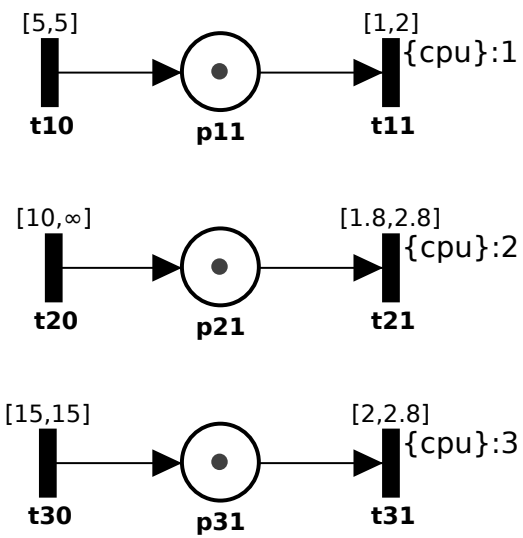
- State class graph enumeration to derive a schedule
- Optimization: heuristic-based enumeration of the state class graph
- Tests on challenging cases (e.g., 24 pipettor actions, 8 read head actions)
- Faster compared to traditional model checking approaches
- Software implementation running on a real system!



Preemptive Time Petri Nets

PTPN syntax

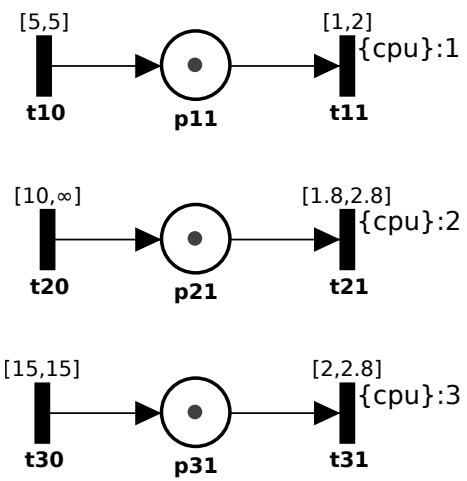
- Preemptive Time Petri Nets (PTPNs) extend TPNs with **resources**
- A PTPN is a tuple $\langle P, T, A^-, A^+, A^\cdot, EFT, LFT, Res, Req, Prio \rangle$
  - P, T, A^-, A^+, $A^\cdot$, EFT, LFT are the elements of a TPNs
  - Res is a set of resources
  - Req associates each transition t with a subset of resources $Req(t) \subseteq Res$
  - $Prio$ associates each transition t with a priority $Prio(t) \in \mathbb{N}$
    (the lower the priority number is, the higher the priority level is)
- Example
  - The model includes a resource cpu
  - `t11`, `t21`, and `t31` are all enabled by the initial marking `p11` `p21` `p31`...
  - ...but only `t11` can be assigned cpu



G. Bucci, A. Fedeli, L. Sassoli, E. Vicario, "Timed State Space Analysis of Real-Time Preemptive Systems," IEEE Transactions on Software Engineering, 2004.

State of a PTPN

- The **state** of a PTPN is a pair $\langle m, \tau \rangle$ where
  - m is a marking (representing the logical location of the system)
  - τ associates each enabled transition t with a **time to fire** $\tau(t) \in \mathbb{R}_{\geq 0}$
- Example
  - t10 can be associated with a time to fire $\tau(\texttt{t10}) = 5$
  - t20 can be associated with a time to fire $\tau(\texttt{t20}) \in [10, \infty]$
  - t30 can be associated with a time to fire $\tau(\texttt{t30}) = 15$
  - t11 can be associated with a time to fire $\tau(\texttt{t11}) \in [1, 2]$
  - t21 can be associated with a time to fire $\tau(\texttt{t21}) \in [1.8, 2.8]$
  - t31 can be associated with a time to fire $\tau(\texttt{t31}) \in [2, 2.8]$

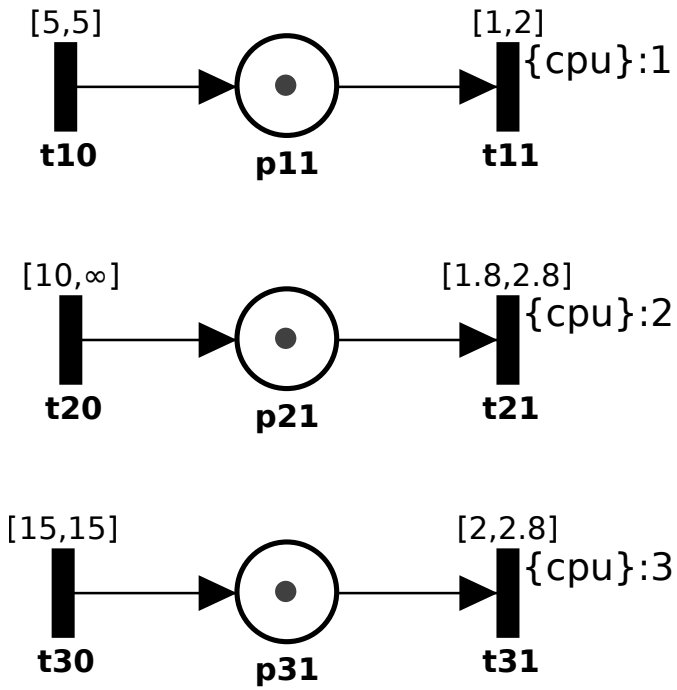


PTPN semantics (1/5)

- As in PNs with inhibitor arcs, a transition t is **enabled** by a marking m iff:
  - $m(p) > 0 \; \forall \, p \in P \mid (p, t) \in A^-$
    (i.e., m assigns at least one token to each input place of t)
  - $m(p) = 0 \; \forall \, p \in P \mid (p, t) \in A^\bullet$
    (i.e., m assigns no token to each inhibitor place of t)
- A transition t is **progressing** in a marking m iff:
  - it is enabled by m
  - $Req(t) \cap Req(t') \neq \varnothing \Rightarrow Prio(t) < Prio(t') \; \forall \, t' \in T_e(m)$ (i.e., every required resource is not required by another enabled transition with higher priority level)
- A transition t is **suspended** in a marking m iff:
  - it is enabled by m
  - it is not progressing in m
- A transition t is **firable** in a state $s = \langle m, \tau \rangle$ iff:
  - it is progressing in m
  - $\tau(t) \leq \tau(t') \; \forall \, t' \in T \mid t'$ is progressing in m (i.e., the time to fire of t is not larger than the time to fire of any other progressing transition)

PTPN semantics (2/5)

- In any possible initial state of the example PTPN:
  - t10, t20, t30, and t11 are progressing
  - t21 and 31 are suspended

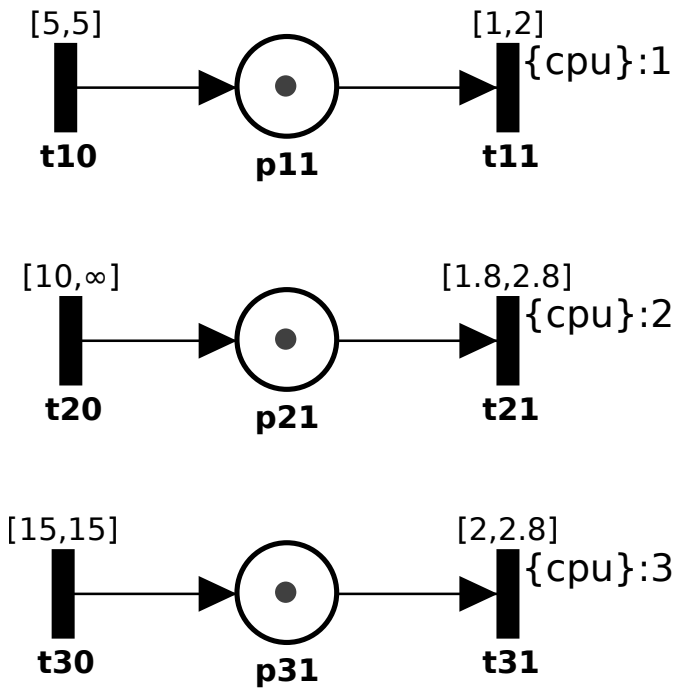


PTPN semantics (3/5)

- The firing of a transition t replaces $s_0 = \langle m_0, \tau_0 \rangle$ by $s_1 = \langle m_1, \tau_1 \rangle$
  - As in PNs, m_1 is derived from m_0 by:
    1. removing a token from each input place of t
       (i.e., $m_{\text{tmp}} = m_0(p) - 1 \; \forall \, p \,|\, (p, t) \in A^-$)
    2. adding a token to each output place of t
       (i.e., $m_1 = m_{\text{tmp}}(p) + 1 \; \forall \, p \,|\, (t, p) \in A^+$)
  - As in TPNs, a transition t' enabled by m_1 is termed:
    - **persistent** if it is not t and it is enabled both by m_0 and m_{tmp}
    - **newly enabled** if it is not enabled by m_0 or by m_{tmp},
      or if it is the fired transition t and it is enabled by m_1
  - As in TPNs, a transition is termed **disabled** if enabled by m_0 but not by m_1
  - τ_1 is derived from τ by:
    1. reducing the time to fire of each **persistent progressing** transition by $\tau_0(t)$
       (i.e., $\tau_1(t') = \tau_0(t') - \tau_0(t) \; \forall \, t' \in T \,|\, t'$ is persistent and progressing in m_1)
    2. leaving the time to fire of each **persistent suspended** transition unchanged
       (i.e., $\tau_1(t') = \tau_0(t') \; \forall \, t' \in T \,|\, t'$ is persistent and suspended in m_1)
    3. eliminating the time to fire of each transition **disabled** by the firing of t
    4. sampling the time to fire of each **newly enabled** transition t' in its firing
       interval (i.e., $\tau_1(t') \in [EFT(t'), LFT(t')] \; \forall \, t' \in T \,|\, t'$ is newly enabled in m_1)

PTPN semantics (4/5)

- Example PTPN
  - t11 is the only firable transition in any initial state with marking p11 p21 p31
  - The firing of t11 in state $\langle \mathtt{p11}\,\mathtt{p21}\,\mathtt{p31}, \tau_0 \rangle$ yields state $\langle \mathtt{p21}\,\mathtt{p31}, \tau_1 \rangle$ where $\tau_1(\mathtt{t10}) = \tau_0(\mathtt{t10}) - \tau_0(\mathtt{t11})$, $\tau_1(\mathtt{t20}) = \tau_0(\mathtt{t20}) - \tau_0(\mathtt{t11})$, $\tau_1(\mathtt{t20}) = \tau_0(\mathtt{t30}) - \tau_0(\mathtt{t11})$, $\tau_1(\mathtt{t21}) = \tau_0(\mathtt{t21})$, $\tau_1(\mathtt{t31}) = \tau_0(\mathtt{t31})$

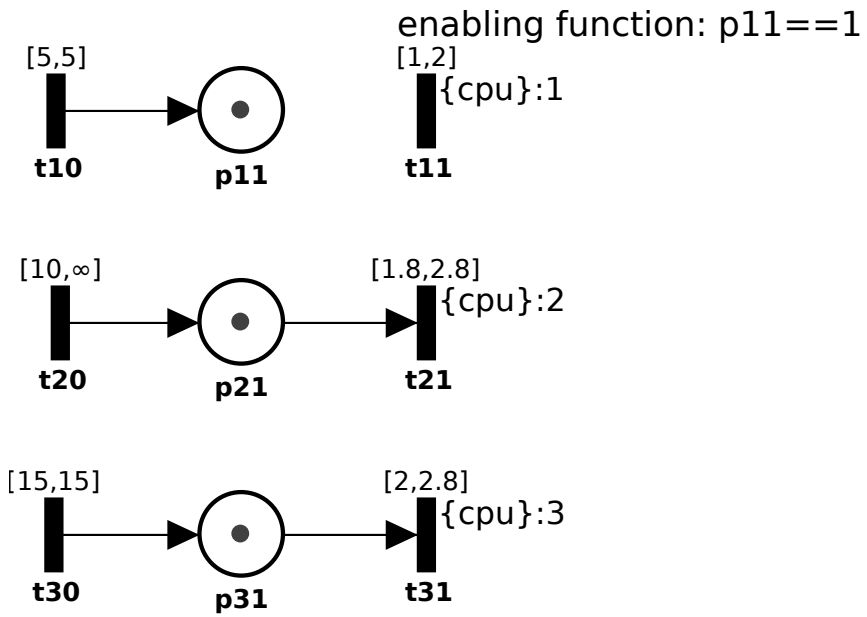


PTPN semantics (5/5)

- An **execution** of a PTPN is a path $\omega = s_0 \xrightarrow{\gamma_1} s_1 \xrightarrow{\gamma_2} s_2 \xrightarrow{\gamma_3} \dots$ such that:
  - $s_0 = \langle m_0, \tau_0 \rangle$ is the initial state
  - γ_i is the i-th fired transition
  - $s_i = \langle m_i, \tau_i \rangle$ is the state reached after the firing of γ_i
  - γ_i is selected among the transitions **firable** in state $s_{i-1} = \langle m_{i-1}, \tau_{i-1} \rangle$
  - m_i is derived from m_{i-1} by
    1. removing a token from each input place of γ_i
       (i.e., $m_{\text{tmp}} = m_{i-1}(p) - 1 \; \forall \, p \,|\, (p, \gamma_i) \in A^-$)
    2. adding a token to each output place of γ_i
       (i.e., $m_i = m_{\text{tmp}}(p) + 1 \; \forall \, p \,|\, (\gamma_i, p) \in A^+$)
  - τ_i is derived from τ_{i-1} by:
    1. reducing the time to fire of each persistent progressing transition by $\tau_{i-1}(\gamma_i)$
       (i.e., $\tau_i(t') = \tau_{i-1}(t') - \tau_{i-1}(\gamma_i) \; \forall \, t' \in T \,|\, t'$ is persistent progressing in s_i)
    2. leaving the time to fire of each persistent suspended transition unchanged
       (i.e., $\tau_i(t') = \tau_{i-1}(t') \; \forall \, t' \in T \,|\, t'$ is persistent suspended in s_i)
    3. eliminating the time to fire of each transition disabled by the firing of γ_i
    4. sampling the time to fire of each newly enabled transition in its firing interval
       (i.e., $\tau_i(t') \in [EFT(t), LFT(t)] \; \forall \, t' \in T \,|\, t$ is newly enabled in m_i)
  - ω is a finite or infinite path

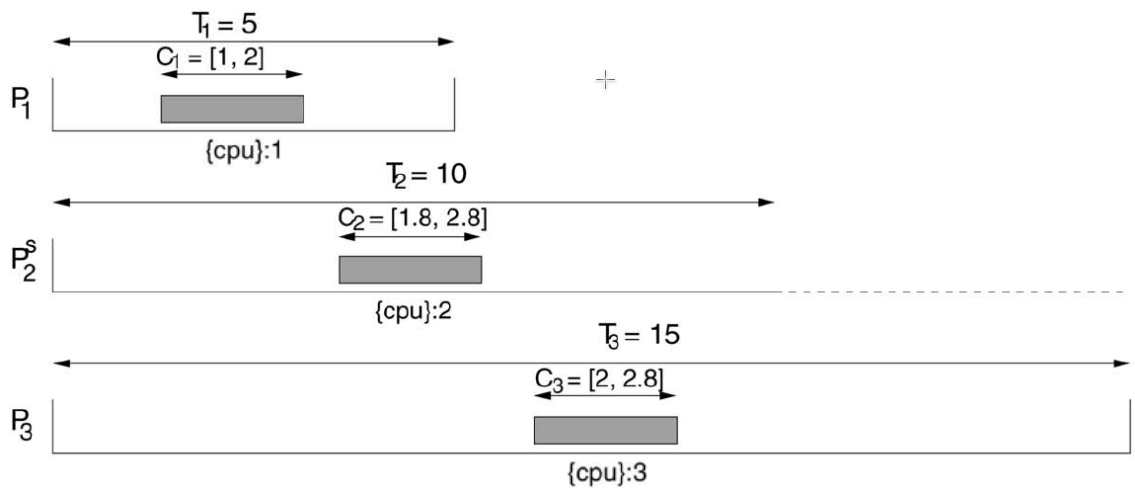
Extension of syntax and semantics of PTPNs

- PTPNs could be extended with enabling functions, update functions, priorities (intended to resolve nondeterminism, not associated with resources)
  - These features would improve the modeling convenience wrt PTPNs
  - These features would not increase the model expressivity wrt PTPNs



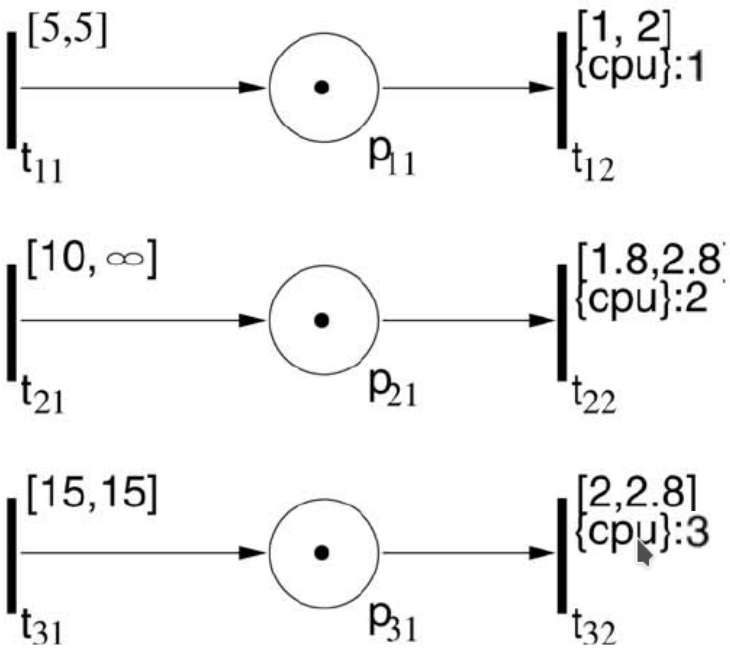
Modeling real-time task-sets with PTPNs: an example (1/4)

- 3 **independent** tasks P_1, P_2, P_3 contending for a **preemptable** resource cpu
  - P_1, P_2, P_3 run under **fixed-priority preemptive scheduling** P_1 is periodic: period $T_1 = 5$, relative deadline $D_1 = 5$, WCET $C_{11} \in [1, 2]$, requires resource cpu with priority 1
  - P_2 is sporadic: min inter-arrival time $T_2 = 15$, relative deadline $D_2 = 15$, WCET $C_{11} \in [1.8, 2.8]$, requires resource cpu with priority 2
  - P_3 is periodic: period $T_3 = 15$, relative deadline $D_3 = 15$, WCET $C_{11} \in [2, 2.8]$, requires resource cpu with priority 3
- Timeline representation of the task-set



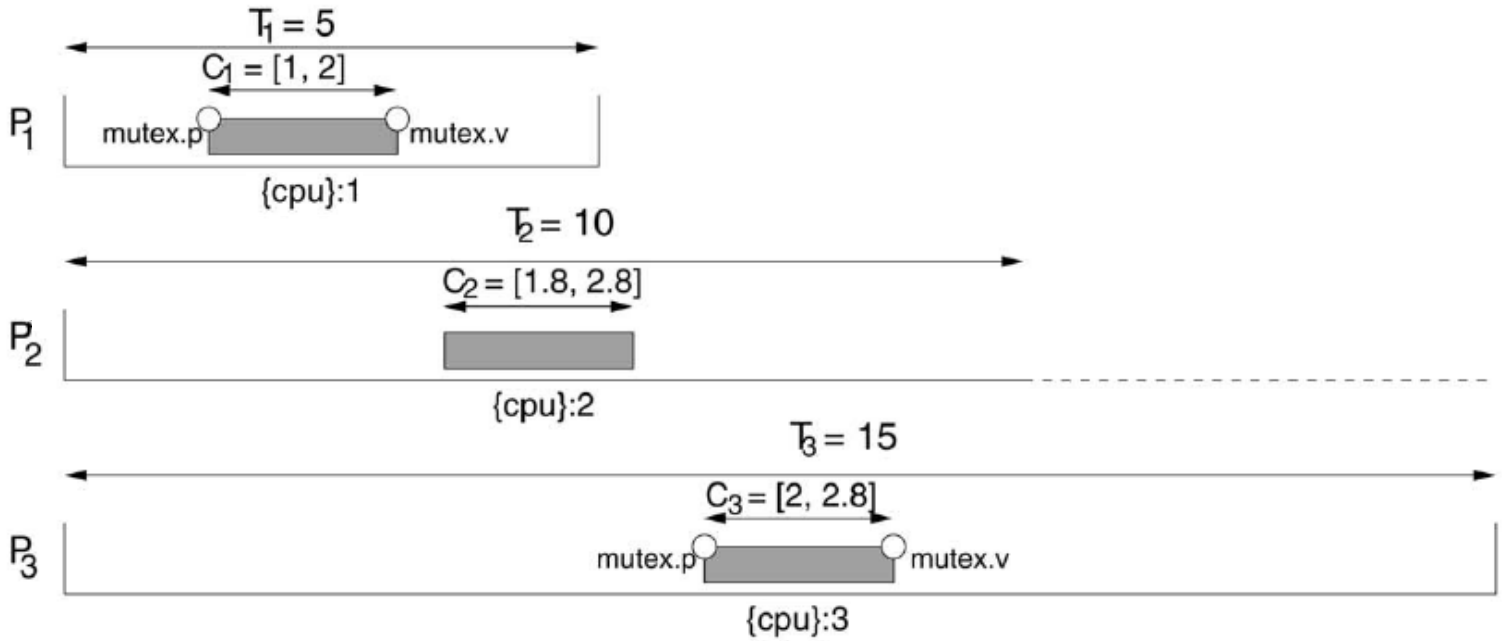
Modeling real-time task-sets with PTPNs: an example (2/4)

- Corresponding PTPN model
  - Are deadlines met? Which is the min/max completion time of each task?
  - If deadlines were not met, what could the software designer do?
  - How could preemptive Earliest Deadline First (EDF) scheduling be modeled?



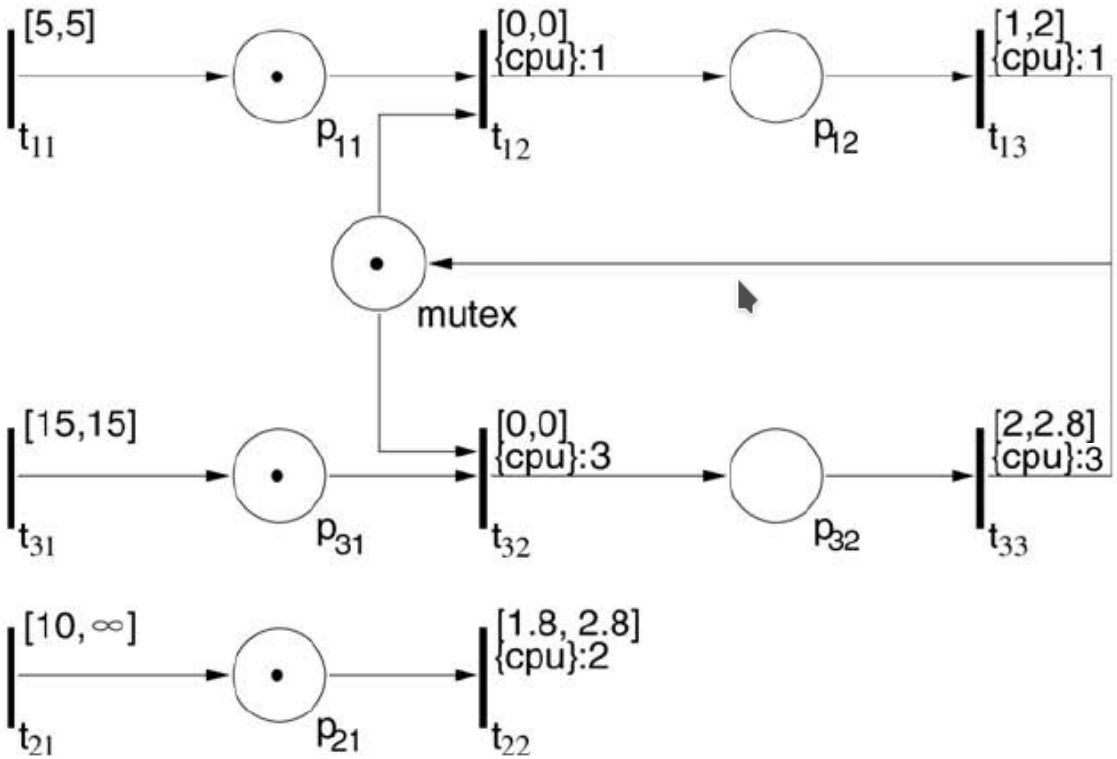
Modeling real-time task-sets with PTPNs: an example (3/4)

- A variant of the task-set where tasks P_1 and P_3 share an **exclusive resource** protected against concurrent accesses by a **binary semaphore** mutex
- Timeline representation of the task-set



Modeling real-time task-sets with PTPNs: an example (4/4)

- Corresponding PTPN model
  - Are deadlines met? Which is the min/max completion time of each task?
  - If deadlines were not met, what could the software designer do?
  - How could preemptive Earliest Deadline First (EDF) scheduling be modeled?



PTPN analysis: preliminaries

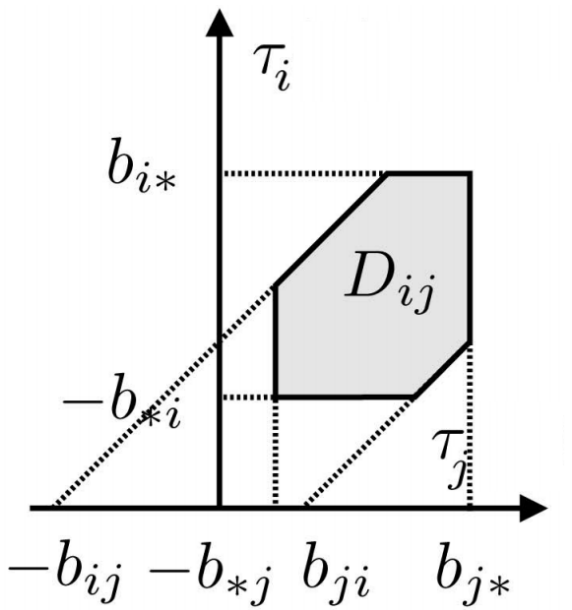
- As in TPNs, a **state class** is a pair $S = \langle m, D \rangle$ where:
  - m is a marking (representing the logical location of the system)
  - D is a continuous set of values for the times to fire of the enabled transitions
- As in TPNs, a state $s = \langle m_s, \vec{\tau_s} \rangle$ is included in a state class $S = \langle m, D \rangle$ if:
  - they have the same marking (i.e., $m_s = m$)
  - the times to fire vector $\vec{\tau_s}$ satisfies the constraints of D (i.e., $\vec{\tau_s} \in D$)
- As in TPNs, a state class S' is **reachable** from class S through transition t (i.e., $S \xrightarrow{t} S'$) if and only if it contains all and only the states that are reachable from some state collected in S through some feasible firing of t, i.e., S' is reachable from S through t if and only if:
  - $\forall\, s' \in S'\ \exists\, s \in S \mid s \xrightarrow{t} s'$
  - $s \in S \wedge s \xrightarrow{t} s' \Rightarrow s' \in S'$

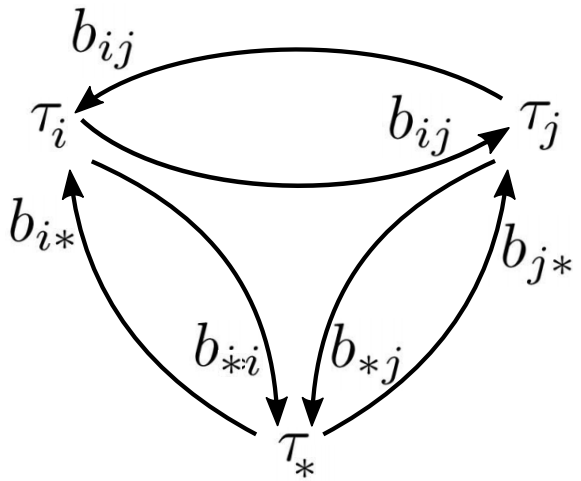
PTPN analysis: an intuitive introduction

TBD

PTPN analysis: initial state class

- As in TPNs, we assume that the firing domain D of the initial state class $S = \langle m, D \rangle$ is represented as a set of linear inequalities in DBM normal form, i.e., $D = \{\tau(t_i) - \tau(t_j) \leq b_{ij} \ \forall\, t_i, t_j \in T_e(m) \cup \{t_\star\}$ with $i \neq j$, where:
  - $b_{ij} \in \mathbb{R} \cup \{\infty\}$
  - $T_e(m)$ is the set of transitions enabled by marking m
  - $\tau(t_i)$ is the time to fire of transition $t_i \ \forall\, t_i \in T_e(m)$
  - $t_\star$ is the fictitious event of entrance into the class
  - $\tau(t_\star)$ is the **ground** time at which the class is entered





PTPN analysis: successor existence

- A state class $S = \langle m, D \rangle$ with D in DBM normal form has a successor through transition t_0 if and only if t_0 is **progressing** in m and D accepts solutions where $\tau(t_0)$ is not larger than the time to fire of any other **progressing** transition, i.e., the restricted firing domain D_{t_0} is non-empty:

$$D_{t_0} = \begin{cases} \tau(t_i) - \tau(t_j) \leq b_{ij} \\ \tau(t_0) - \tau(t_h) \leq 0 \\ \forall\, t_i, t_j \in T_{\mathrm{e}}(m) \cup \{t_\star\} \text{ with } t_i \neq t_j,\ \forall\, t_h \in T_{\mathrm{pr}}(m) \setminus \{t_0\} \end{cases}$$

where $T_{\mathrm{pr}}(m)$ is the set of transitions that are **progressing** in m

Lemma 9

A state class $S = \langle m, D \rangle$ with D in DBM normal form has a **successor** *through transition t_0 if and only if $t_0 \in T_{\mathrm{pr}}(m)$ and $b_{h0} \geq 0\ \forall\, t_h \in T_{\mathrm{pr}}(m)$.*

- Successor detection: linear complexity wrt the number of enabled transitions
- Given $\langle m, D \rangle \xrightarrow{t_0} \langle m', D' \rangle$, how to compute the min embedding DBM of D'?

PTPN analysis: successor computation - conditioning

- Condition D to the fact that t_0 fires first:

$$D_{t_0} = \begin{cases} D \\ \tau(t_0) - \tau(t_h) \leq 0 \\ \forall\, t_h \in T_{\mathrm{pr}}(m) \setminus \{t_0\} \end{cases} = \begin{cases} \tau(t_i) - \tau(t_j) \leq b_{ij} \\ \tau(t_0) - \tau(t_h) \leq 0 \\ \forall\, t_i, t_j \in T_{\mathrm{e}}(m) \cup \{t_\star\} \,|\, t_i \neq t_j \\ \forall\, t_h \in T_{\mathrm{pr}}(m) \setminus \{t_0\} \end{cases}$$

$$= \begin{cases} \tau(t_i) - \tau(t_j) \leq b_{ij} \\ \tau(t_0) - \tau(t_h) \leq \min\{0, b_{0h}\} \\ \forall\, t_i, t_j \in T_{\mathrm{e}}(m) \cup \{t_\star\} \,|\, t_i \neq t_j,\ \forall\, t_h \in T_{\mathrm{pr}}(m) \setminus \{t_0\} \end{cases}$$

- Represent D_{t_0} in normal form and let B_{ij} be its coefficients (quadratic complexity wrt the number of enabled transitions):

$$D_{t_0} = \begin{cases} \tau(t_i) - \tau(t_j) \leq B_{ij} \\ \tau(t_0) - \tau(t_i) \leq B_{0i} \\ \forall\, t_i, t_j \in T_{\mathrm{e}}(m) \cup \{t_\star\} \,|\, t_i \neq t_j,\ t_i \neq t_0 \end{cases}$$

PTPN analysis: successor computation - time adv. (1/4)

- Reduce by the time elapsed in S the time to fire of transitions that are **progressing** in m and **persistent** in S',
  i.e., $\tau'(t_i) = \tau(t_i) - \tau(t_0) \; \forall \, t_i \in T_{\mathrm{pr}}(m) \cap T_{\mathrm{p}}(S') \setminus \{t_0\}$
- Do not change the time to fire of transitions that are **suspended** in m and **persistent** in S',
  i.e., $\tau'(t_x) = \tau(t_x) \; \forall \, t_x \in T_{\mathrm{s}}(m) \cap T_{\mathrm{p}}(S') \setminus \{t_0\}$,
  where $T_{\mathrm{s}}(m)$ is the set of transitions that are **suspended** in m
- Before substituting variables, rewrite the inequalities in order to:
  - make explicit the constraints involving t_0 (fired transition) and $t_\star$ (ground)
  - distinguish progressing transitions from suspended transitions

PTPN analysis: successor computation - time adv. (2/4)

- Rewrite the inequalities in D_{t_0}:

$$
D_{t_0} = \begin{cases}
\tau(t_i) - \tau(t_j) \leq B_{ij} & \text{(progr,progr)} \\
\tau(t_i) - \tau(t_\star) \leq B_{i\star} & \text{(progr,}\star\text{)} \\
\tau(t_\star) - \tau(t_i) \leq B_{\star i} & \text{(}\star\text{,progr)} \\
\tau(t_i) - \tau(t_0) \leq B_{i0} & \text{(progr,0)} \\
\tau(t_0) - \tau(t_i) \leq B_{0i} & \text{(0,progr)} \\
\tau(t_i) - \tau(t_x) \leq B_{ix} & \text{(progr,susp)} \\
\tau(t_y) - \tau(t_j) \leq B_{yj} & \text{(susp,progr)} \\
\tau(t_x) - \tau(t_\star) \leq B_{x\star} & \text{(susp,}\star\text{)} \\
\tau(t_\star) - \tau(t_x) \leq B_{\star x} & \text{(}\star\text{,susp)} \\
\tau(t_y) - \tau(t_0) \leq B_{y0} & \text{(susp,0)} \\
\tau(t_0) - \tau(t_x) \leq B_{0x} & \text{(0,susp)} \\
\tau(t_x) - \tau(t_y) \leq B_{xy} & \text{(susp,susp)} \\
\tau(t_0) - \tau(t_\star) \leq B_{0\star} & \text{(0,}\star\text{)} \\
\tau(t_\star) - \tau(t_0) \leq B_{\star 0} & \text{(}\star\text{,0)} \\
\forall\, t_i, t_j \in T_{\mathrm{pr}}(m) \cap T_{\mathrm{p}}(S') \setminus \{t_0\} \mid t_i \neq t_j,\ t_i, t_j \neq t_0,\ t_i, t_j \neq t_\star \\
\forall\, t_x, t_y \in T_{\mathrm{s}}(m) \cap T_{\mathrm{p}}(S') \mid t_x \neq t_y,\ t_x, t_y \neq t_0,\ t_x, t_y \neq t_\star
\end{cases}
$$

PTPN analysis: successor computation - time adv. (3/4)

- Substitute variables in D_{t_0}:

$$D_{t_0} = \begin{cases} \tau(t_i) - \tau(t_j) \leq B_{ij} \\ \tau(t_i) - \tau(t_\star) \leq B_{i\star} \\ \tau(t_\star) - \tau(t_i) \leq B_{\star i} \\ \tau(t_i) - \tau(t_0) \leq B_{i0} \\ \tau(t_0) - \tau(t_i) \leq B_{0i} \\ \tau(t_i) - \tau(t_x) \leq B_{ix} \\ \tau(t_y) - \tau(t_j) \leq B_{yj} \\ \tau(t_x) - \tau(t_\star) \leq B_{x\star} \\ \tau(t_\star) - \tau(t_x) \leq B_{\star x} \\ \tau(t_y) - \tau(t_0) \leq B_{y0} \\ \tau(t_0) - \tau(t_x) \leq B_{0x} \\ \tau(t_x) - \tau(t_y) \leq B_{xy} \\ \tau(t_0) - \tau(t_\star) \leq B_{0\star} \\ \tau(t_\star) - \tau(t_0) \leq B_{\star 0} \\ \\ \forall\, t_i, t_j \in \ldots,\ \forall\, t_x, t_y \in \ldots \end{cases} \qquad D'_{t_0} = \begin{cases} \tau'(t_i) - \tau'(t_j) \leq B_{ij} \\ \tau(t_0) - \tau(t_\star) \leq B_{i\star} - (\tau'(t_i) - \tau'(t_\star)) \\ \tau(t_\star) - \tau(t_0) \leq B_{\star i} + \tau'(t_i) - \tau'(t_\star) \\ \tau'(t_i) - \tau'(t_\star) \leq B_{i0} \\ \tau'(t_\star) - \tau'(t_i) \leq B_{0i} \\ \tau(t_0) - \tau(t_\star) \leq B_{ix} - (\tau'(t_i) - \tau'(t_x)) \\ \tau_(t_\star) - \tau(t_0) \leq B_{xi} + \tau'(t_i) - \tau'(t_x) \\ \tau'(t_x) - \tau'(t_\star) \leq B_{x\star} \\ \tau'(t_\star) - \tau'(t_x) \leq B_{\star x} \\ \tau(t_\star) - \tau(t_0) \leq B_{y0} + \tau'(t_\star) - \tau'(t_y) \\ \tau(t_0) - \tau(t_\star) \leq B_{0x} - (\tau'(t_\star) - \tau'(t_0)) \\ \tau'(t_x) - \tau'(t_y) \leq B_{xy} \\ \tau(t_0) - \tau(t_\star) \leq B_{0\star} \\ \tau(t_\star) - \tau(t_0) \leq B_{\star 0} \\ \\ \forall\, t_i, t_j \in \ldots,\ \forall\, t_x, t_y \in \ldots \end{cases}$$

PTPN analysis: successor computation - time adv. (4/4)

- Reorder the equations in D'_{t_0}:

$$D'_{t_0} = \begin{cases} \tau'(t_i) - \tau'(t_j) \leq B_{ij} \\ \tau(t_0) - \tau(t_\star) \leq B_{i\star} - (\tau'(t_i) - \tau'(t_\star)) \\ \tau(t_\star) - \tau(t_0) \leq B_{\star i} + \tau'(t_i) - \tau'(t_\star) \\ \tau'(t_i) - \tau'(t_\star) \leq B_{i0} \\ \tau'(t_\star) - \tau'(t_i) \leq B_{0i} \\ \tau(t_0) - \tau(t_\star) \leq B_{ix} - (\tau'(t_i) - \tau'(t_x)) \\ \tau_(t_\star) - \tau(t_0) \leq B_{xi} + \tau'(t_i) - \tau'(t_x) \\ \tau'(t_x) - \tau'(t_\star) \leq B_{x\star} \\ \tau'(t_\star) - \tau'(t_x) \leq B_{\star x} \\ \tau(t_\star) - \tau(t_0) \leq B_{y0} + \tau'(t_\star) - \tau'(t_y) \\ \tau(t_0) - \tau(t_\star) \leq B_{0x} - (\tau'(t_\star) - \tau'(t_0)) \\ \tau'(t_x) - \tau'(t_y) \leq B_{xy} \\ \tau(t_0) - \tau(t_\star) \leq B_{0\star} \\ \tau(t_\star) - \tau(t_0) \leq B_{\star 0} \\ \\ \forall\, t_i, t_j \in \ldots,\ \forall\, t_x, t_y \in \ldots \end{cases} \quad D'_{t_0} = \begin{cases} \tau'(t_i) - \tau'(t_j) \leq B_{ij} \\ \tau'(t_x) - \tau'(t_y) \leq B_{xy} \\ \tau'(t_x) - \tau'(t_\star) \leq B_{x\star} \\ \tau'(t_\star) - \tau'(t_x) \leq B_{\star x} \\ \tau'(t_i) - \tau'(t_\star) \leq B_{i0} \\ \tau'(t_\star) - \tau'(t_i) \leq B_{0i} \\ \\ \tau(t_0) - \tau(t_\star) \leq B_{0\star} \\ \tau(t_0) - \tau(t_\star) \leq B_{i\star} - (\tau'(t_i) - \tau'(t_\star)) \\ \tau(t_0) - \tau(t_\star) \leq B_{ix} - (\tau'(t_i) - \tau'(t_x)) \\ \tau(t_0) - \tau(t_\star) \leq B_{0x} - (\tau'(t_\star) - \tau'(t_0)) \\ \\ \tau(t_\star) - \tau(t_0) \leq B_{\star 0} \\ \tau(t_\star) - \tau(t_0) \leq B_{\star i} + \tau'(t_i) - \tau'(t_\star) \\ \tau(t_\star) - \tau(t_0) \leq B_{xi} + \tau'(t_i) - \tau'(t_x) \\ \tau(t_\star) - \tau(t_0) \leq B_{y0} + \tau'(t_\star) - \tau'(t_y) \\ \\ \forall\, t_i, t_j \in \ldots,\ \forall\, t_x, t_y \in \ldots \end{cases}$$

PTPN analysis: successor computation - projection

- Eliminate the time to fire of the fired transition t_0 through a projection:

$$
D''_{t_0} = \begin{cases}
\tau'(t_i) - \tau'(t_j) \leq C_{ij} = B_{ij} \\
\tau'(t_x) - \tau'(t_y) \leq C_{xy} = B_{xy} \\
\tau'(t_x) - \tau'(t_\star) \leq C_{x\star} = B_{x\star} \\
\tau'(t_\star) - \tau'(t_x) \leq C_{\star x} = B_{\star x} \\
\tau'(t_i) - \tau'(t_\star) \leq C_{i\star} = B_{i0} \\
\tau'(t_\star) - \tau'(t_i) \leq C_{\star i} = B_{0i} \\
\tau'(t_i) - \tau'(t_x) \leq C_{ix} = B_{ix} + B_{\star 0} \\
\tau'(t_x) - \tau'(t_i) \leq C_{xi} = B_{xi} + B_{0\star} \\
\\
(\tau'(t_i) - \tau'(t_x)) + (\tau'(t_y) - \tau'(t_j)) \leq C_{ix} + C_{yj} - c_{ixyj} & \text{with } c_{ixyj} = B_{\star 0} + B_{0\star} \\
(\tau'(t_\star) - \tau'(t_x)) + (\tau'(t_y) - \tau'(t_j)) \leq C_{\star x} + C_{yj} - c_{\star xyj} & \text{with } c_{\star xyj} = B_{0\star} + B_{\star x} - B_{0x} \\
(\tau'(t_i) - \tau'(t_\star)) + (\tau'(t_y) - \tau'(t_j)) \leq C_{i\star} + C_{yj} - c_{i\star yj} & \text{with } c_{i\star yj} = B_{i0} + B_{0\star} - B_{i\star} \\
(\tau'(t_i) - \tau'(t_x)) + (\tau'(t_\star) - \tau'(t_j)) \leq C_{ix} + C_{\star j} - c_{ix\star j} & \text{with } c_{ix\star j} = B_{\star 0} + B_{0j} - B_{\star j} \\
(\tau'(t_i) - \tau'(t_x)) + (\tau'(t_y) - \tau'(t_\star)) \leq C_{ix} + C_{y\star} - c_{ixy\star} & \text{with } c_{ixy\star} = B_{y\star} + +B_{\star 0} - B_{y0} \\
(\tau'(t_\star) - \tau'(t_x)) + (\tau'(t_\star) - \tau'(t_j)) \leq C_{\star x} + C_{\star j} - c_{\star x\star j} & \text{with } c_{\star x\star j} = B_{0j} + B_{\star x} - B_{\star j} - \\
(\tau'(t_i) - \tau'(t_\star)) + (\tau'(t_y) - \tau'(t_\star)) \leq C_{i\star} + C_{y\star} - c_{i\star y\star} & \text{with } c_{i\star y\star} = B_{i0} + B_{y\star} - B_{\star j} - \\
\\
\forall\, t_i, t_j \in T_{\mathrm{pr}}(m) \cap T_{\mathrm{p}}(S') \setminus \{t_0\} \mid t_i \neq t_j,\ t_i, t_j \neq t_0,\ t_i, t_j \neq t_\star \\
\forall\, t_x, t_y \in T_{\mathrm{s}}(m) \cap T_{\mathrm{p}}(S') \mid t_x \neq t_y,\ t_x, t_y \neq t_0,\ t_x, t_y \neq t_\star
\end{cases}
$$

- If coefficients B are in normal form $\Rightarrow$ coefficients C are in normal form

PTPN analysis: successor computation - newly enabling

- Add the time to fire of each newly-enabled transition:

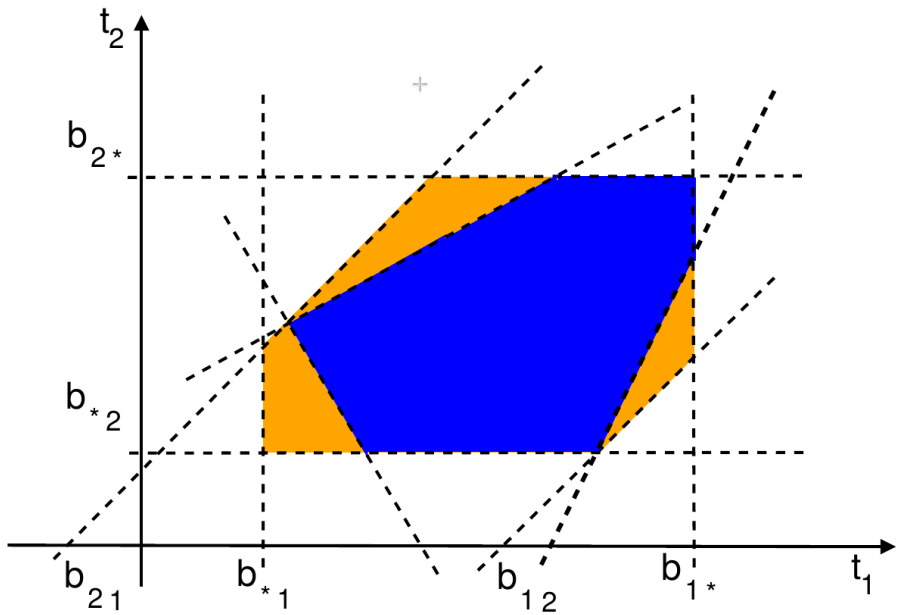
$$D' = \begin{cases} D''_{t_0} \\ \tau'(t_j) - \tau'(t_\star) \leq LFT(t_j) \\ \tau'(t_\star) - \tau'(t_j) \leq -EFT(t_j) \\ \forall\, t_j \in T_{\mathrm{n}}(S') \end{cases}$$

- The firing domain D' is **not in DBM form**!$\Rightarrow$
  $\Rightarrow$ The DBM form is **not closed** wrt the steps of successor computation!
- Repeated derivation of successor classes yields more complex firing domains, with exponential number of inequalities in the number of enabled transitions!

PTPN analysis: successor approximation

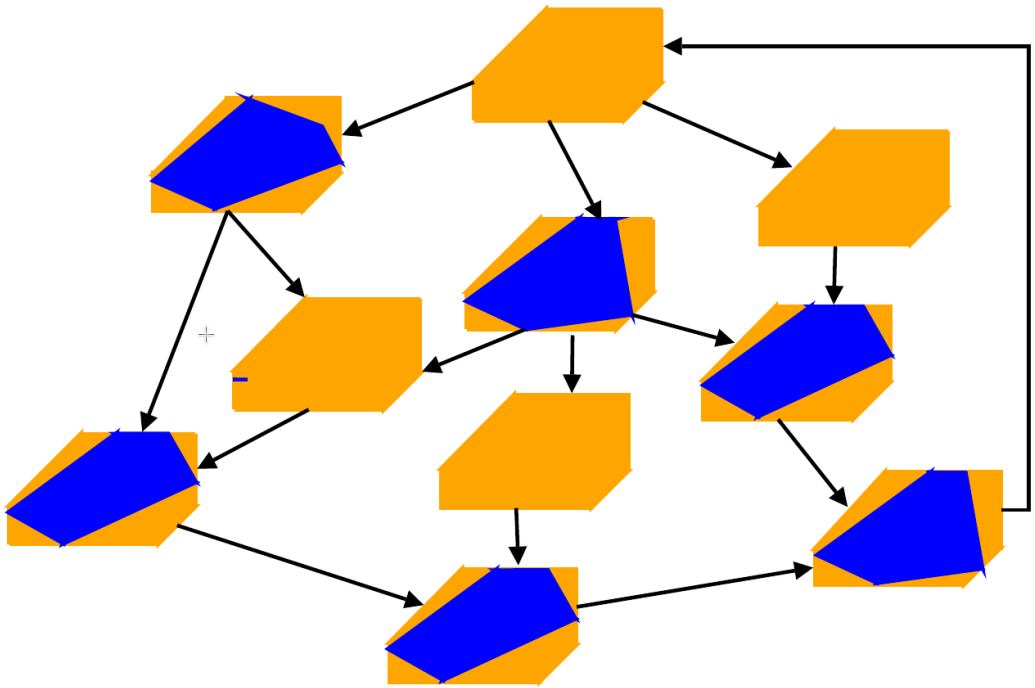
- D' is approximated by its **tightest embedding DBM** $\bar{D}'$:

$$\bar{D}' = \begin{cases} \tau'(t_i) - \tau'(t_j) \leq C_{ij} \\ \tau'(t_x) - \tau'(t_y) \leq C_{xy} \\ \tau'(t_x) - \tau'(t_\star) \leq C_{x\star} \\ \tau'(t_\star) - \tau'(t_x) \leq C_{\star x} \\ \tau'(t_i) - \tau'(t_\star) \leq C_{i\star} \\ \tau'(t_\star) - \tau'(t_i) \leq C_{\star i} \\ \tau'(t_i) - \tau'(t_x) \leq C_{ix} \\ \tau'(t_x) - \tau'(t_i) \leq C_{xi} \\ \\ \tau'(t_n) - \tau'(t_\star) \leq LFT(t_j) \\ \tau'(t_\star) - \tau'(t_n) \leq -EFT(t_j) \\ \\ \forall\, t_i, t_j \in T_{\mathrm{pr}}(m) \cap T_{\mathrm{p}}(S') \setminus \{t_0\} \mid t_i \neq t_j,\ t_i, t_j \neq t_0,\ t_i, t_j \neq t_\star \\ \forall\, t_x, t_y \in T_{\mathrm{s}}(m) \cap T_{\mathrm{p}}(S') \mid t_x \neq t_y,\ t_x, t_y \neq t_0,\ t_x, t_y \neq t_\star \\ \forall\, t_n \in T_{\mathrm{n}}(S') \end{cases}$$



PTPN analysis: state space enumeration

- Enumeration of the approximated reachability relation between state classes yields the **approximated state class graph**



Domain of timings of a symbolic run (1/2)

- Consider a symbolic run $\rho = S_0 \overset{t^0_{f(0)}}{\rightarrow} S_1 \overset{t^1_{f(1)}}{\rightarrow} S_2 \ldots S_{N-1} \overset{t^{N-1}_{f(N-1)}}{\rightarrow} S_N$
  - $S_0 = \langle m_0, D_0 \rangle$ is the initial state class
  - $S_n = \langle m_n, D_n \rangle$ is the n-th state class visited by $\rho \; \forall \, n \in \{1, \ldots, N\}$
  - $f(n)$ is the index of the transition fired in state class $S_n \; \forall \, n \in \{0, 1, \ldots, N-1\}$
  - t_i^k is a transition newly enabled in state class S_k
    (it is denoted as t_i^k in any class S_h such that it is persistent from S_k to S_h)
  - $l(t_i^k)$ is the index of the last class in which t_i^k is persistent
  - $c_n(t_i)$ is 1 or 0 depending on whether t_i is progressng or suspended in S_n, resp.
  - σ_n is the sojourn time in $S_n \; \forall \, n \in \{1, \ldots, N\}$
- The domain of timings of ρ satisfies the following constraints:

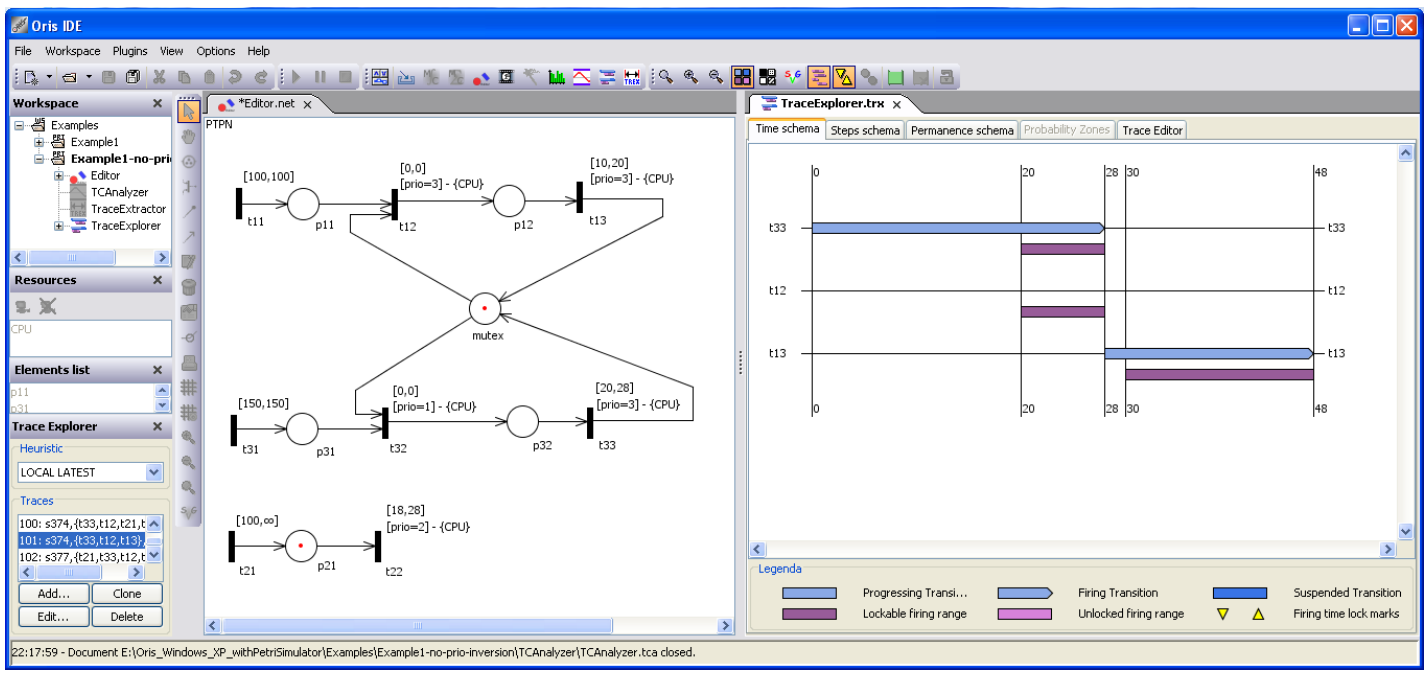
$$
D_\rho = \begin{cases}
-b^k_{\star i} \leq \sum_{n=k}^{l(t_i^k)} c_n(t_i)\sigma_n \leq b^k_{i\star} & \forall \; t_i^k \text{ fired along } \rho \\
\sum_{n=k}^{l(t_w^k)} c_n(t_w)\sigma_n \leq b^k_{w\star} & \forall \; t_w^k \text{ enabled but not fired along } \rho \\
\sum_{n=0}^{l(t_j^0)} c_n(t_j)\sigma_n - \sum_{n=0}^{l(t_z^0)} c_n(t_z)\sigma_n \leq b^0_{ij} & \forall \; t_j^0, t_z^0 \text{ enabled in } S_0 \text{ and fired along } \rho \\
-b^k_{i\star} \leq \sum_{n=0}^{l(t_w^0)} c_n(t_w)\sigma_n - \sum_{n=0}^{l(t_i^0)} c_n(t_i)\sigma_n \leq b^0_{wi} & \forall \; t_i^0 \text{ enabled in } S_0 \text{ and fired along } \rho, \; \forall \; t_w^k \text{ enabled but not fired along } \rho
\end{cases}
$$

Domain of timings of a symbolic run (2/2)

- If D_ρ does not admit solution $\Rightarrow$ ρ is a **false behavior**
- If D_ρ admits solution $\Rightarrow$ any min/max bound on the time elapsed between any two (not necessarily consecutive) transition firings can be derived by solving a linear programming problem (e.g., through the simplex method) so as to **clean up** false behaviors introduced by the approximation
  - Compute the minimum time spent in any feasible execution of ρ:
$$\min_{D_\rho} \left\{ \sum_{n=0}^{N-1} \sigma_n \right\}$$
  - Compute the minimum time spent in any feasible execution of ρ:
$$\max_{D_\rho} \left\{ \sum_{n=0}^{N-1} \sigma_n \right\}$$

ORIS 1: a tool for modeling and analysis of TPNs and PTPNs

- Developed by the **Software Technologies lab** of the University of Florence
  - Home page: `https://stlab.dinfo.unifi.it/oris1.0`
- Main features
  - **Modeling**: graphical editing of TPNs and PTPNs
  - **State space analysis**: state space enumeration of TPNs and PTPNs
  - **Trace analysis**: evaluation of the tight timing profile of any symbolic run selected in the state space of a TPN and in the state space of a PTPN



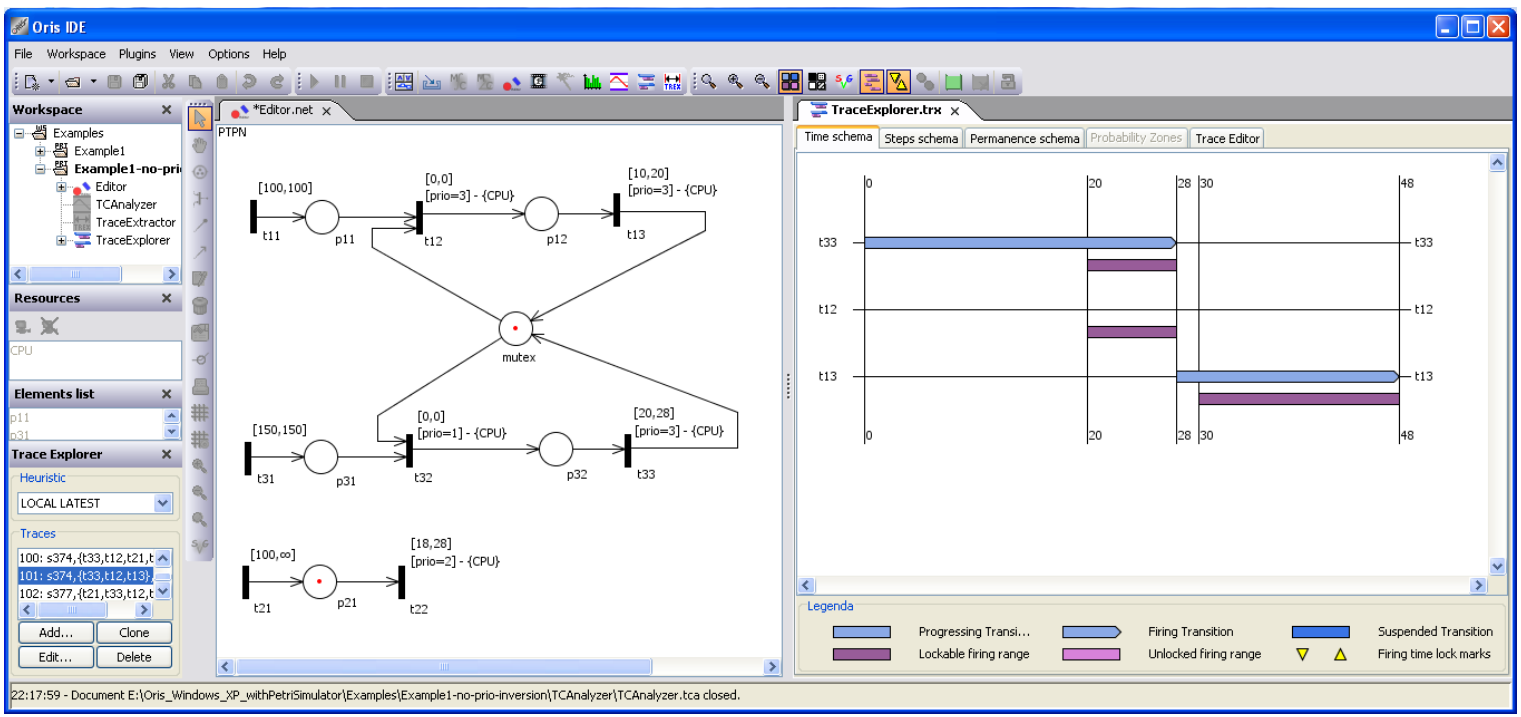
ORIS 1: how to install and use the tool

- How to install ORIS 1
  - Install Virtual Box (a free and open-source hosted hypervisor for virtualizing the x86 computing architecture): `https://www.virtualbox.org`
  - Download the ORIS 1 virtual machine: `https://stlab.dinfo.unifi.it/carnevali/misc/WinXP.ova`
  - Launch Virtual Box and open the ORIS 1 virtual machine
- How to use ORIS 1
  - Manual: `https://stlab.dinfo.unifi.it/carnevali/misc/Oris1.pdf`
  - Note: for PTPNs, the higher the priority number the higher the priority level
- A new release of the tool: ORIS 2
  - Home page: `http://www.oris-tool.org`
  - Support for the analysis of TPNs and Stochastic Time Petri Nets (STPNs)
  - Ongoing porting of PTPN features from ORIS 1 to ORIS 2

Using Preemptive Time Petri Nets in a V-Model SW life cycle

Modeling and analysis of real-time task-sets with PTPN theory

- Representation of task-sets running under **preemptive** scheduling policies
  - e.g., task-sets running under fixed-priority preemptive scheduling
- **Timed reachability**: can the task-set reach a specific logical condition?
  - e.g., verify sequencing correctness such as absence of priority inversion
- **Timeliness analysis**: which is the min/max time between specific events?
  - e.g., compute the best/worst case completion time of each task



Is this only a matter of verification?

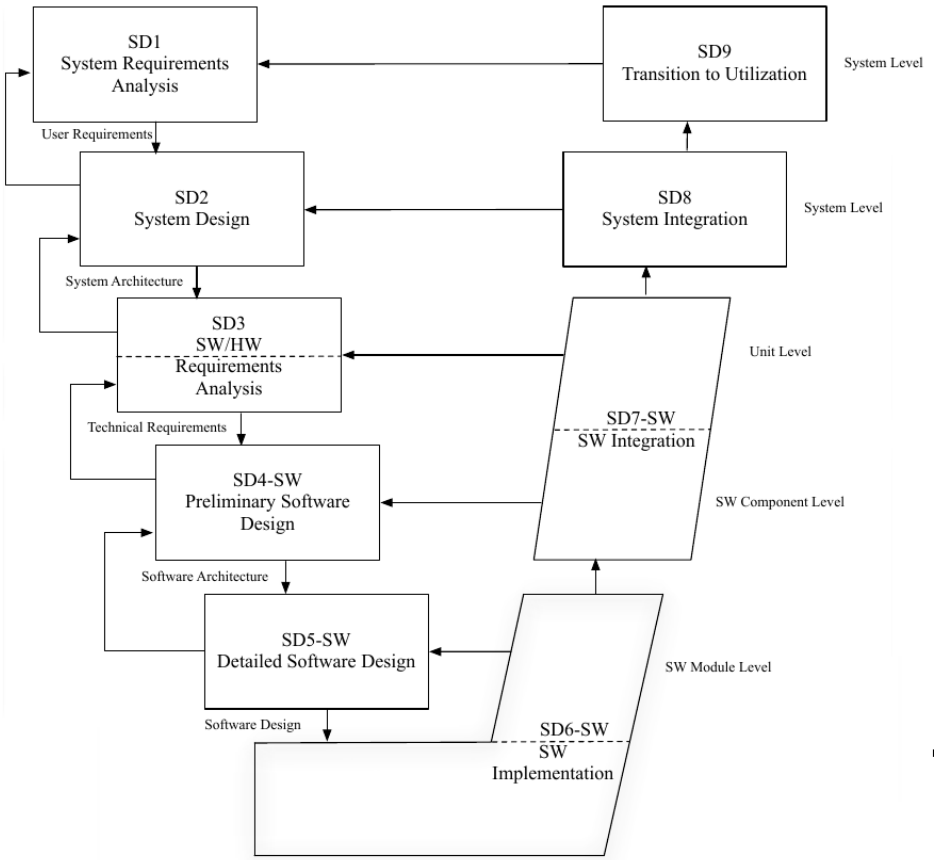
- PTPNs as formal core of a Model Driven Development (MDD) approach...
  - automated derivation of pTPN from timelines
  - automated code generation
  - execution time measurement of entry-point functions
  - test-case selection and execution
  - oracle verdict
- ...organized in a comprehensive software (SW) development life cycle:
  - referred to the practice of industrial development (V-Model)
  - referred to applicable regulatory standards (RTCA-178B)
  - formalized in terms of artifacts of the UML profile for MARTE (Modeling and Analysis of Real-Time and Embedded Systems)

L. Carnevali, L. Ridi, E. Vicario, "Putting preemptive Time Petri Nets to work in a V-Model SW life cycle", IEEE Transactions on Software Engineering, 2011.

I. Bicchierai, G. Bucci, L. Carnevali, E. Vicario, "Combining UML-MARTE and preemptive Time Petri Nets: An Industrial Case Study", IEEE Transactions on Industrial Informatics, 2013.

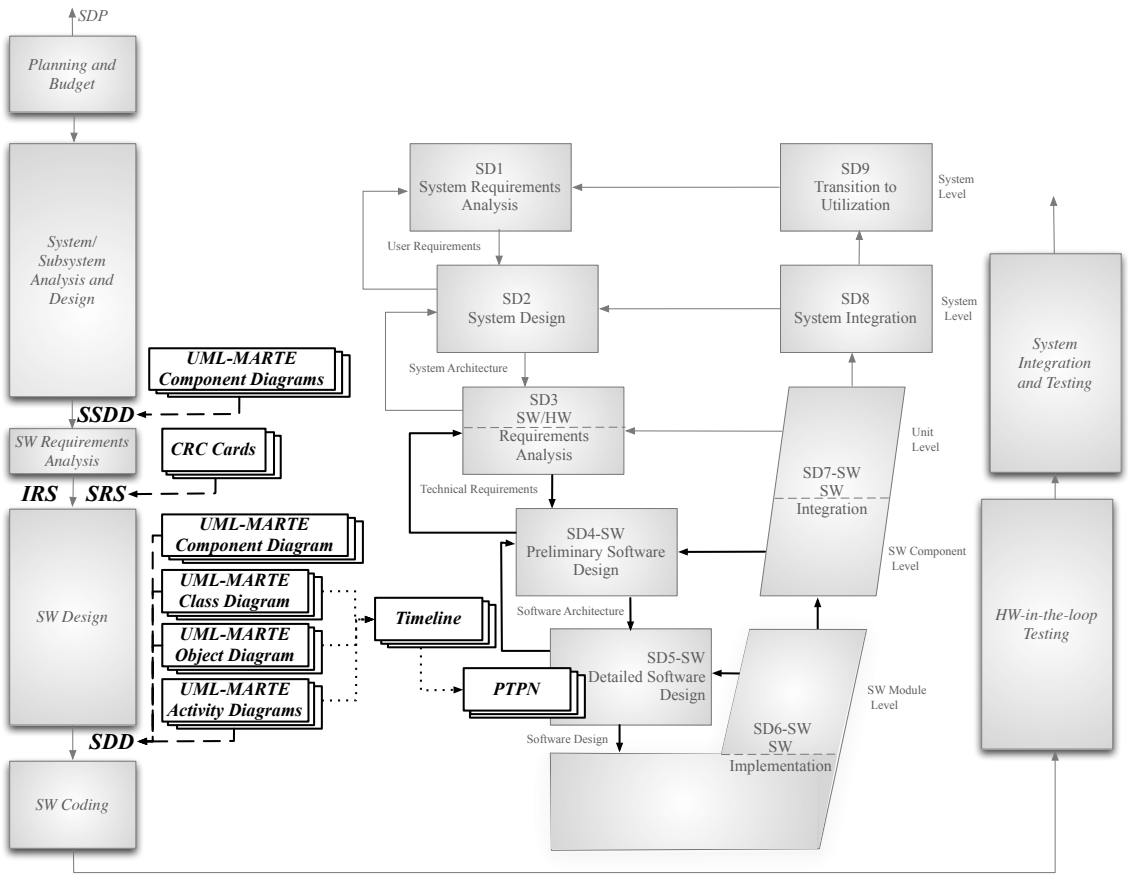
V-Model

- Reference SW development life cycle for safety critical systems
- Integration of *design* and *verification* (left/right)
- Decomposition from *System* to *Modules* (up/down)



V-Model tailored according to MIL-STD-498

- Integration of formal methods within the development life cycle requires SW engineering methods not to disrupt industrial practices
  - V-model development process supported by the pTPN theory
  - MIL-STD-498 documentation process supported by UML-MARTE

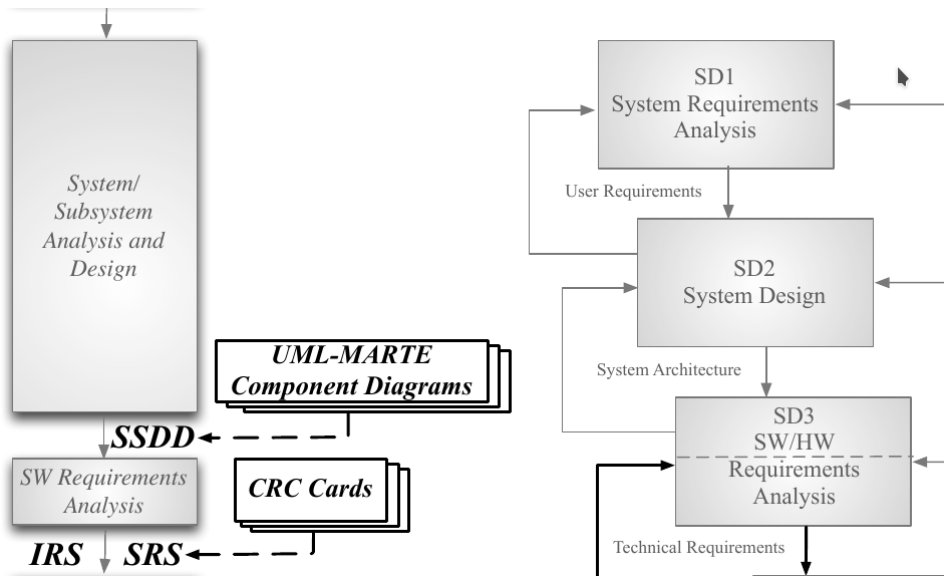


A running case study

- An **electro-optical system** of a **military vehicle**
  - Developed by Selex-Galileo of the FinMeccanica Group (now Leonardo)
  - Guarantees battlefield advantage using visual, infra-red and thermal imaging, long-range target acquisition and illumination, precise aiming
- The system consists of 3 units
  - An **Optical Unit (OU)** made of sensors, cameras, and servo-motors
  - An **Electronic Unit (EU)** responsible for sensor control and image processing
  - A **System Monitoring Unit (SMU)** managing the entire system
- EU plays the role of a bridge in the communication between SMU and OU
  - Forwards the commands periodically sent by SMU to OU
  - Sending the corresponding OU replies to SMU
- EU processes images acquired by OU and sends obtained results to SMU
- Experimentation of the formal methodology in the development of the EU in a one-year-long research collaboration with Selex-Galileo (now Leonardo) supported by the SW-Initiative of the FinMeccanica Group

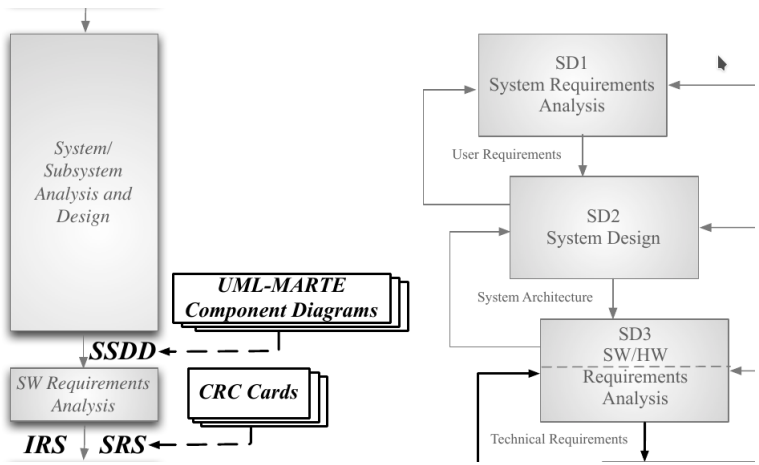
V-Model: SD1, SD2, SD3

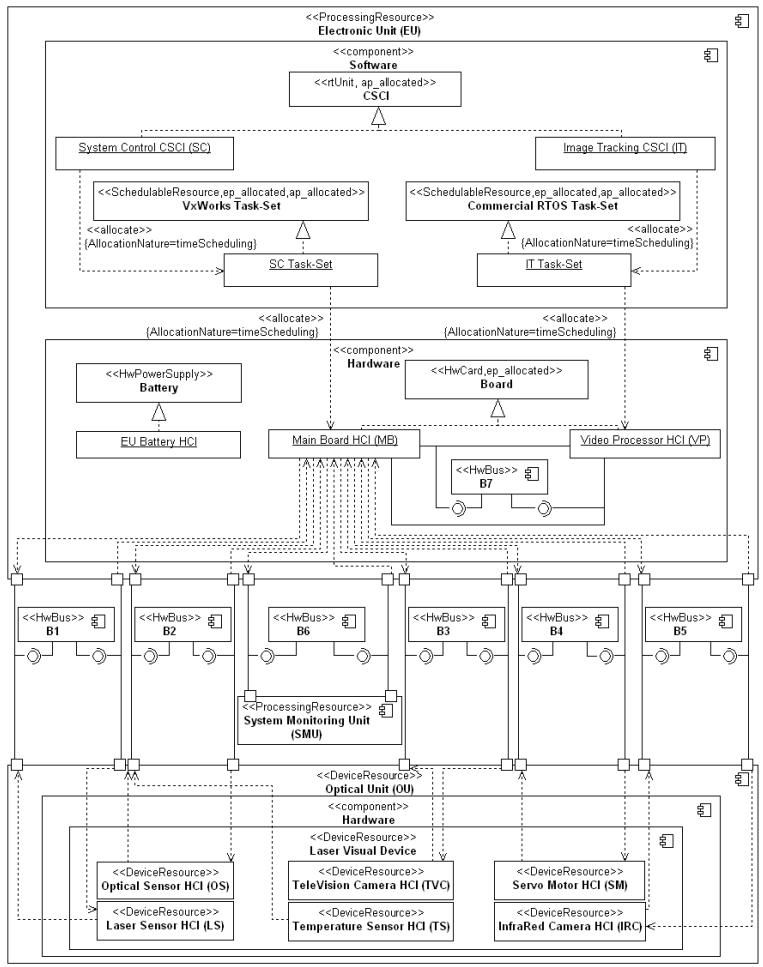
- SD1 (System Requirements Analysis) - system scope
  - Determines high level user requirements of the system
- SD2 (System Design) - System scope
  - Identifies system units and allocates user requirements to them
- SD3 (SW/HW Requirements Analysis) - from system to unit scope
  - Decomposes each unit into HW Configuration Items (HCIs), Computer SW Configuration Items (CSCIs), and Firmware Configuration Items (FCIs)



MIL-STD-498: System/Subsystem Analysis and Design

- Integrates SD1, SD2, and the first part of SD3 (system scope)
- Produces the System/Subsystem Design Description (SSDD)
  - Here in the form of a UML-MARTE component diagram of system units





MIL-STD-498: SW Requirements Analysis

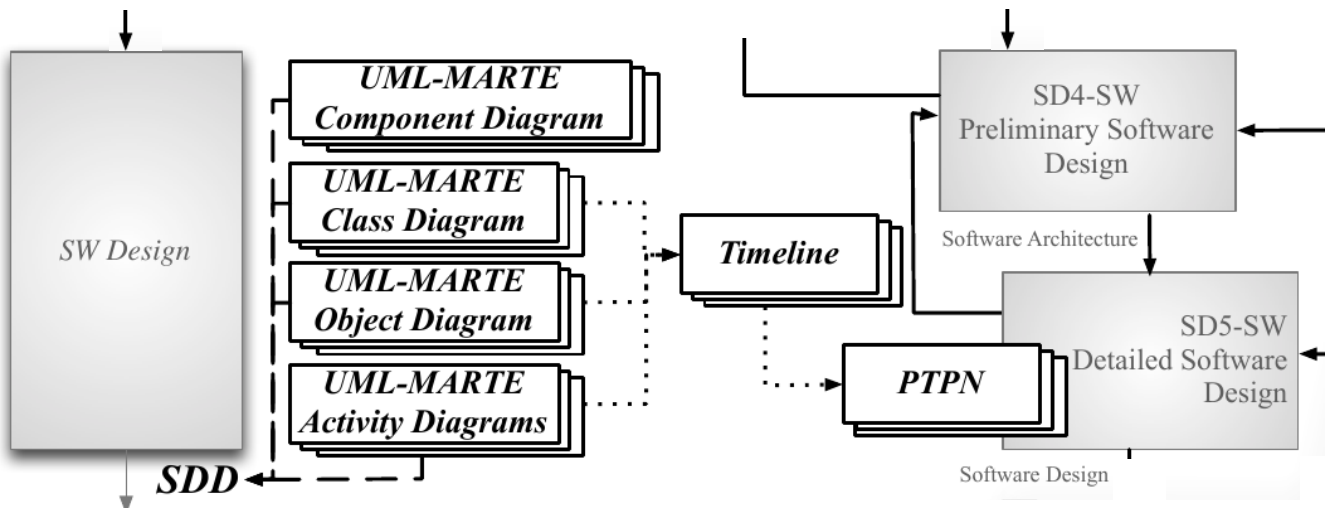
- Corresponds to the second part of SD3 (CSCI scope)
- Produces the Software Requirements Specification (SRS) of each CSCI and the Interface Requirements Specification (IRS) of each unit interface
  - Here SRS in a form similar to Class Responsibility Collaboration (CRC) cards
    - e.g., CRC card specifying capabilities of the System Control CSCI (left)
    - e.g., CRC card specifying sub-capabilities of SMU-OU_COMMANDS (right)

Capability	Description	Collaboration
Init	HW and SW initialization	-
LS-IRC_Power	Switching on/off LS and IRC	LS, IRC
TVC_Config	Management of the TVC configuration	TVC
SMU-OU_Cmd	Management of the messages exchanged by SMU and OU	-
SM_Comm	Communication with SM	SM
SMU_Comm	Communication with SMU	SMU
IT_Comm	Communication with IT	IT
IRC_Comm	Communication with IRC	IRC
TVC_Comm	Communication with TVC	TVC
LS_Comm	Communication with LS	LS

Sub-Capability	Description	Collaboration
SMU_Cmd	Management of the SMU commands and the OU replies	-
TVC_Cmd	Management of the TVC configuration parameters	-
HCIs_Trasm	Activation of the data transmission to IRC, TVC, and LS	-
LS-IRC_State	Management of the switched on/off state of IRC and LS	-
HCIs_Data	Processing of the HCIs data	-
SM_LocationData	Processing of the SM location data elaborated by IT	-
OperationModes	Management of Operation Modes	-

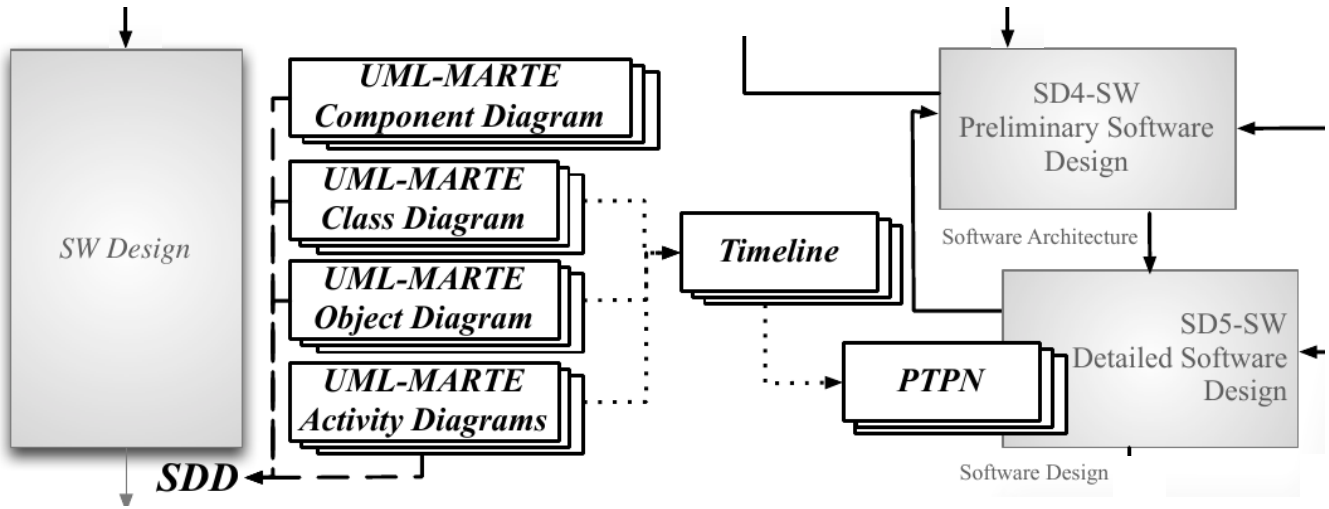
V-Model: SD4-SW, SD5-SW

- SD4-SW (Preliminary SW Design) - SW component scope
  - Defines the **SW architecture** of each CSCI as set of communicating tasks with assigned functional modules and prescribed release times and deadlines
- SD5-SW (Detailed SW Design) - SW module scope
  - Defines the **SW design** of each CSCI allocating resources and time requirements to each software module
  - Includes SD5.2-SW (Analysis of Resources and Time Requirements)



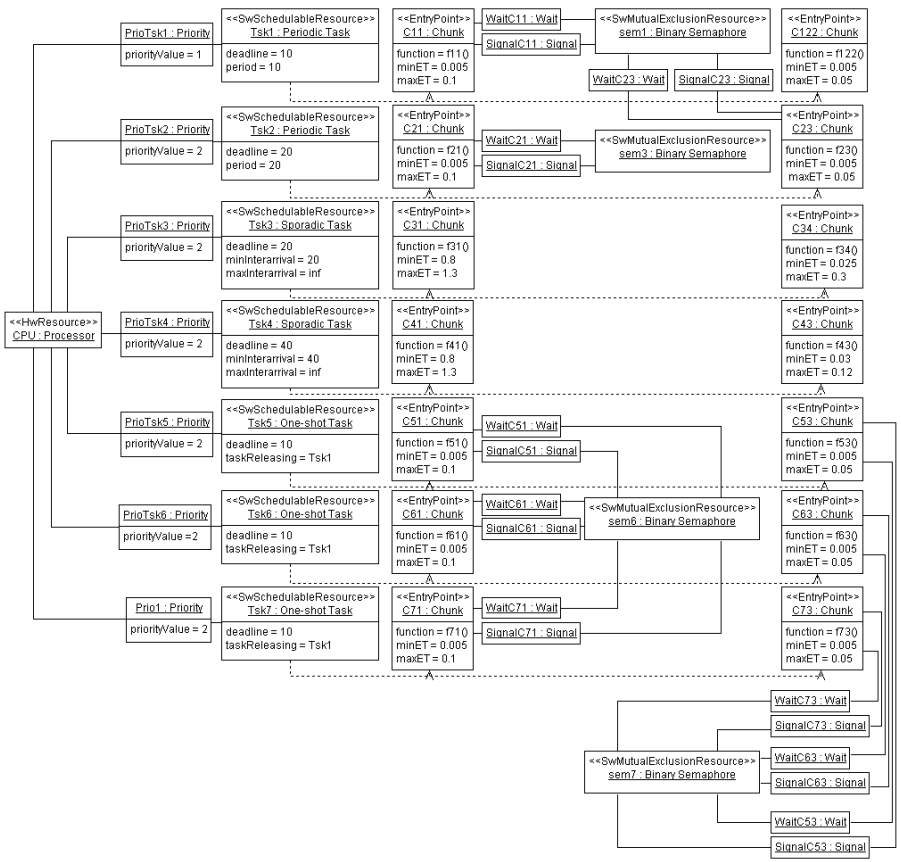
MIL-STD-498: SW Design

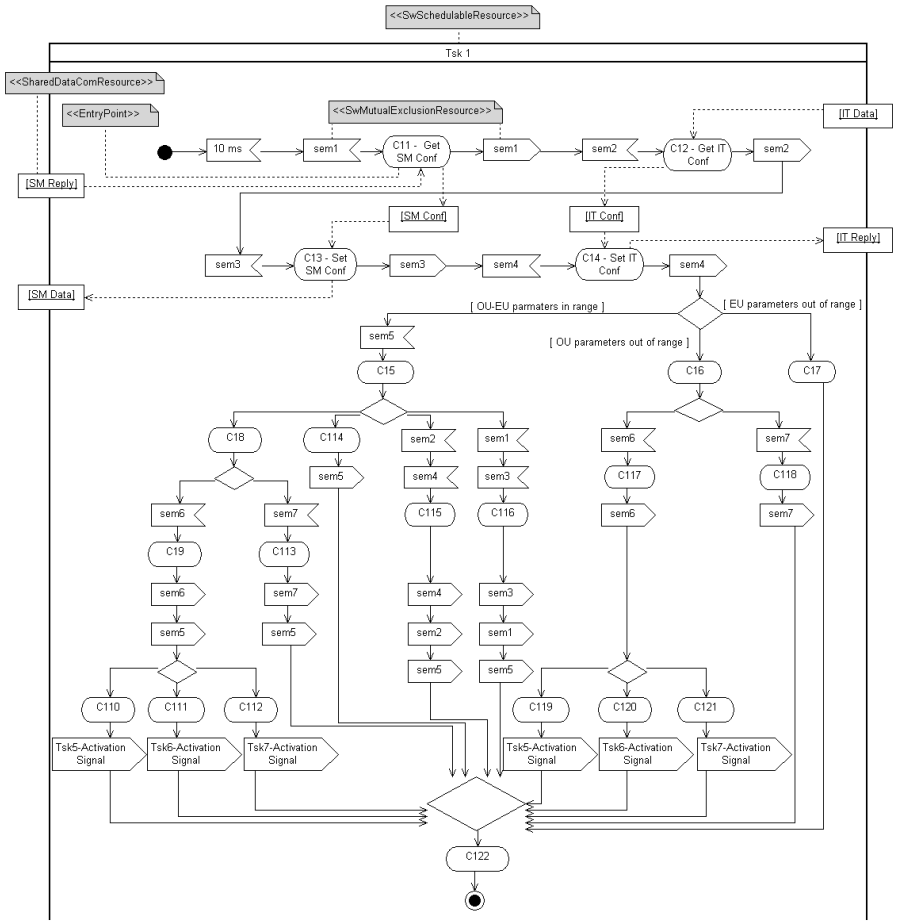
- Integrates SD4-SW and SD5-SW
- Produces the Software Design Descripton (SDD)



SW Design: semi-formal specification through UML-MARTE

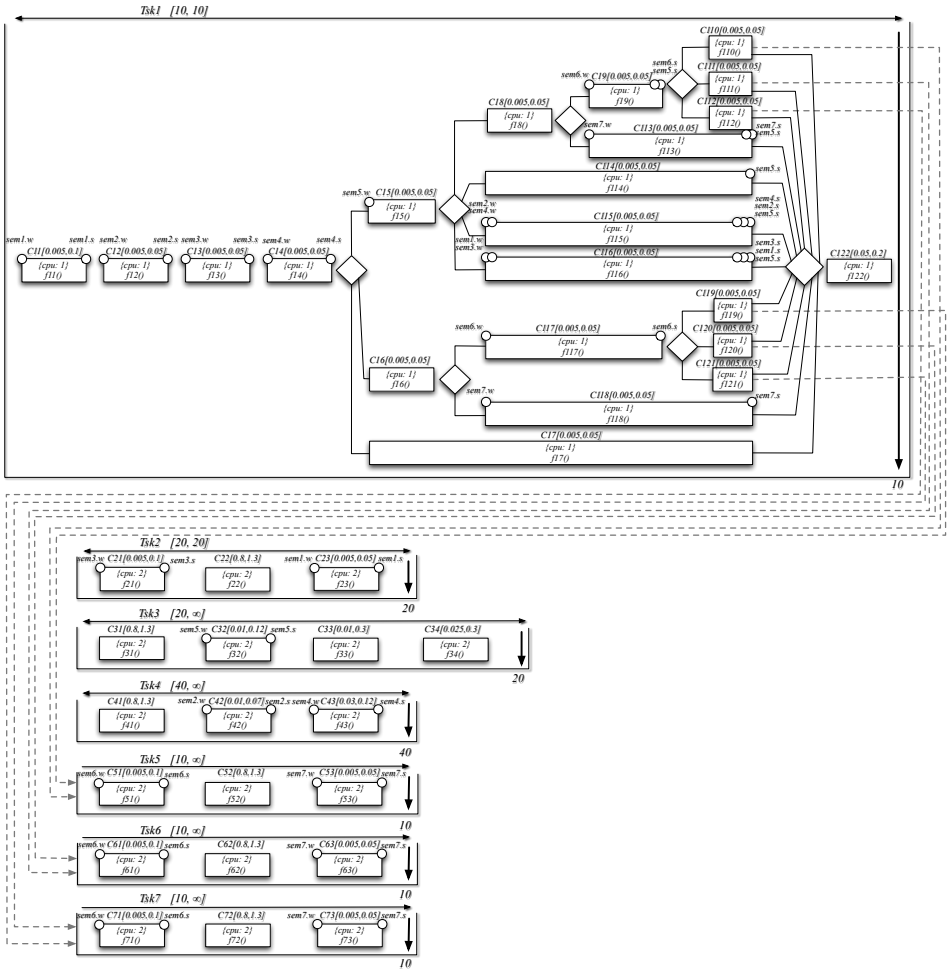
- Specification of non-functional requirements of each CSCI
  - Here in the form of a UML-MARTE object diagram for the whole task-set
- Specification of functional requirements of each CSCI
  - Here in the form of a UML-MARTE activity diagram for each task





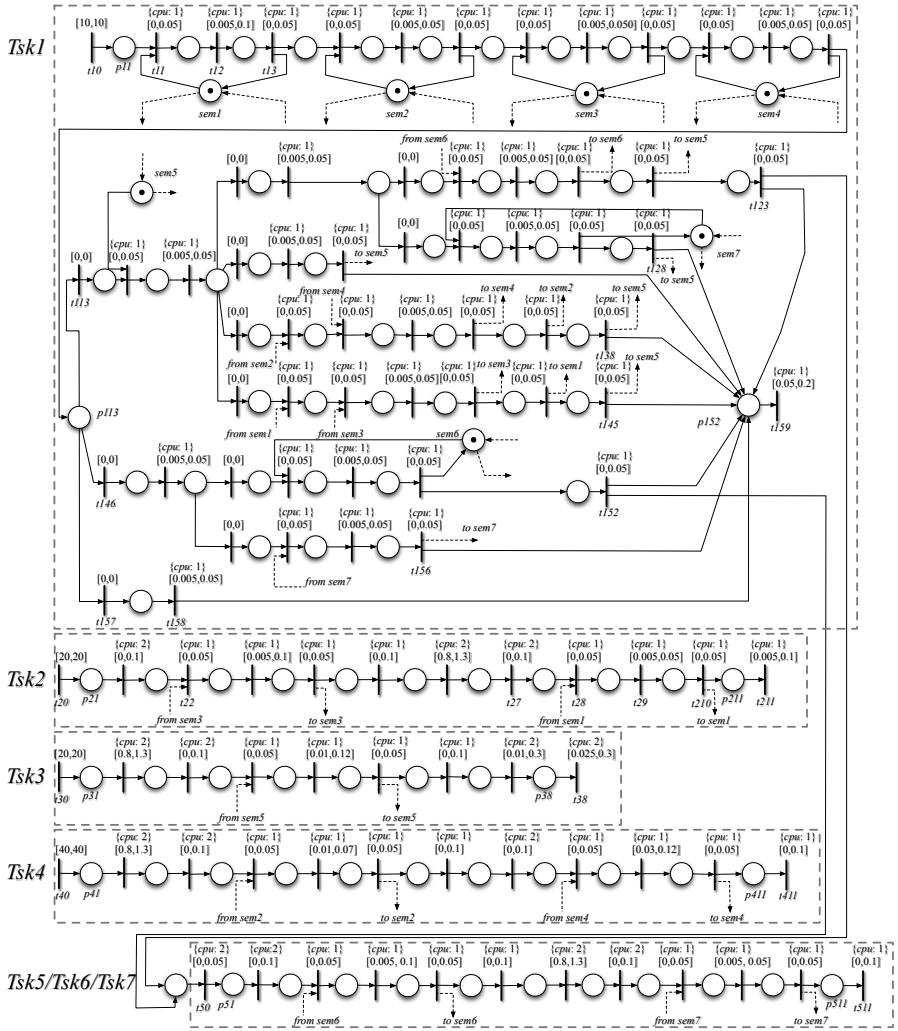
SW Design: semi-formal specification through timelines

- Can be (manually/automatically) derived from UML-MARTE diagrams
- Provide a synthetic and intuitive representation of a task-set
- Do not support automated verification of the model



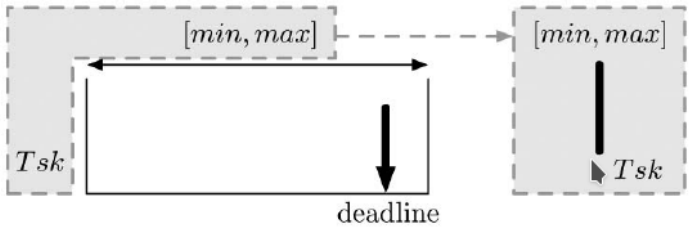
SW Design: formal specification through PTPNs

- Can be (manually/automatically) derived from timelines
- Support formal verification and subsequent development steps

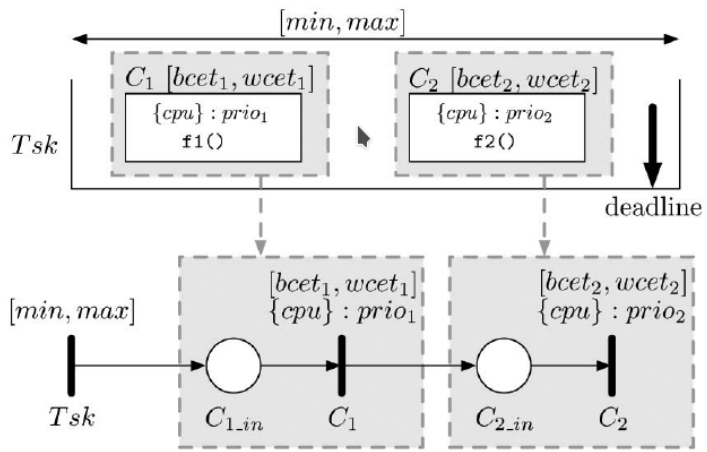


SW Design: derivation of PTPNs from timelines (1/3)

- Representation of task releases

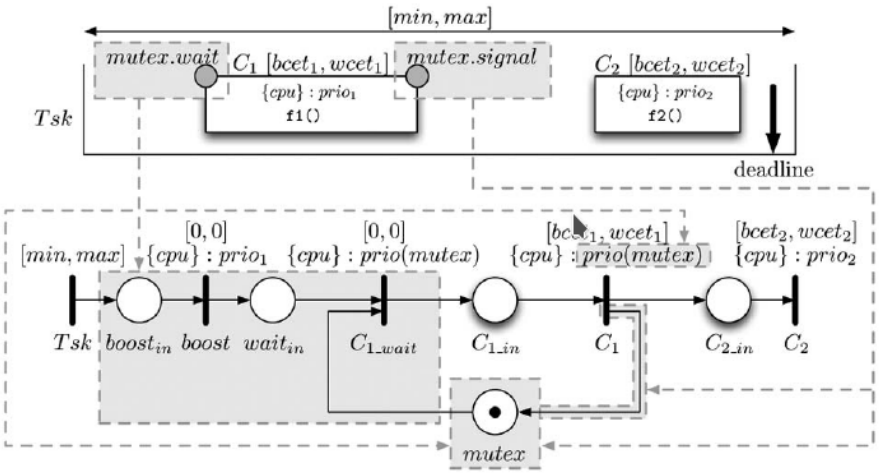


- Representation of tasks with multiple sequential chunks



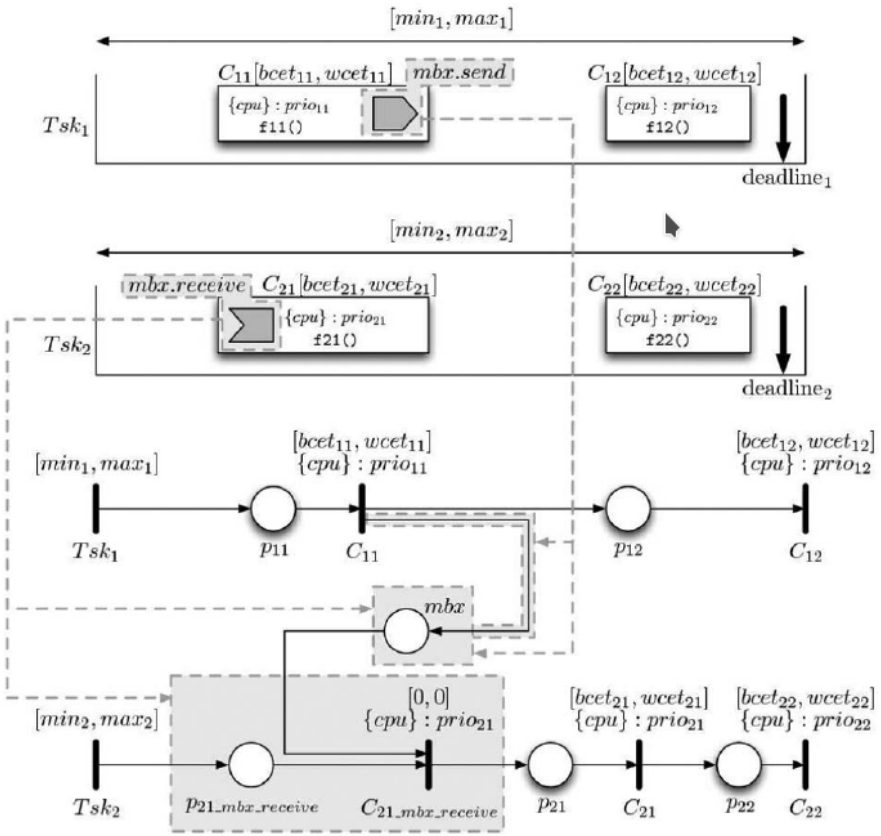
SW Design: derivation of PTPNs from timelines (2/3)

- Representation of tasks synchronized on a semaphore



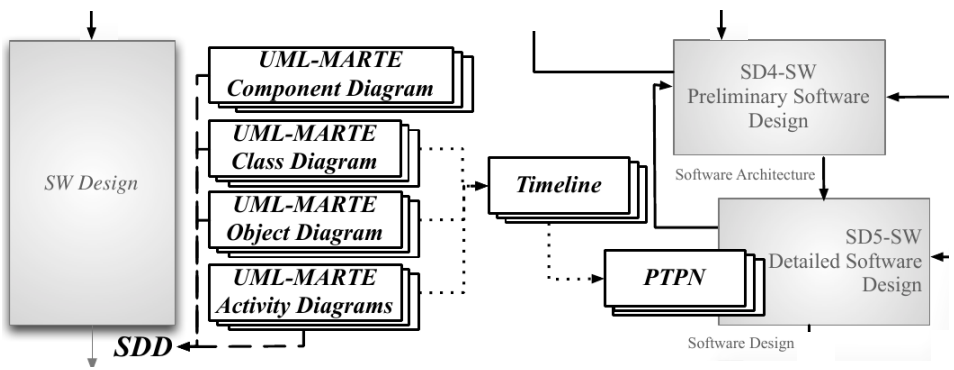
SW Design: derivation of PTPNs from timelines (3/3)

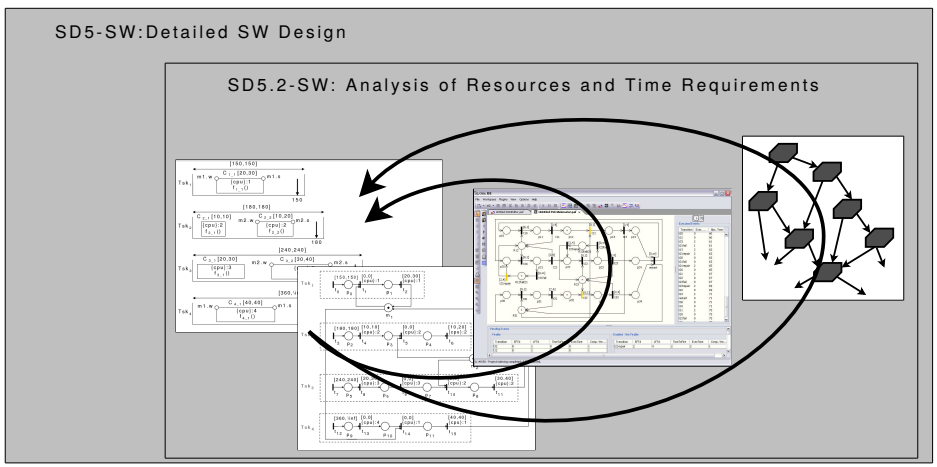
- Representation of tasks synchronized on a mailbox



SD5.2-SW: Analysis of Resources and Time Requirements

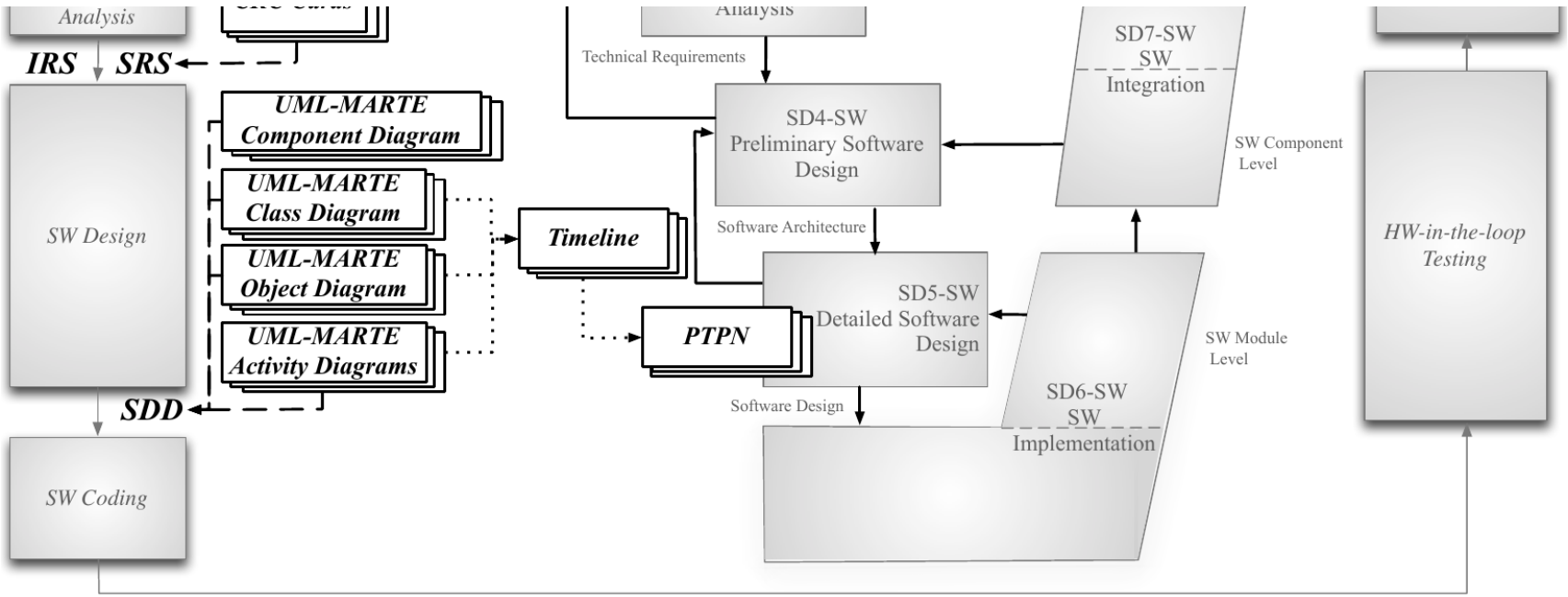
- Simulation of the PTPN model
  - Online interactive token game
  - Offline derivation of statistics
- State-space analysis of the PTPN model
  - Exhaustive state space coverage (unless the state space explodes)
  - Verification of sequencing and timeliness properties





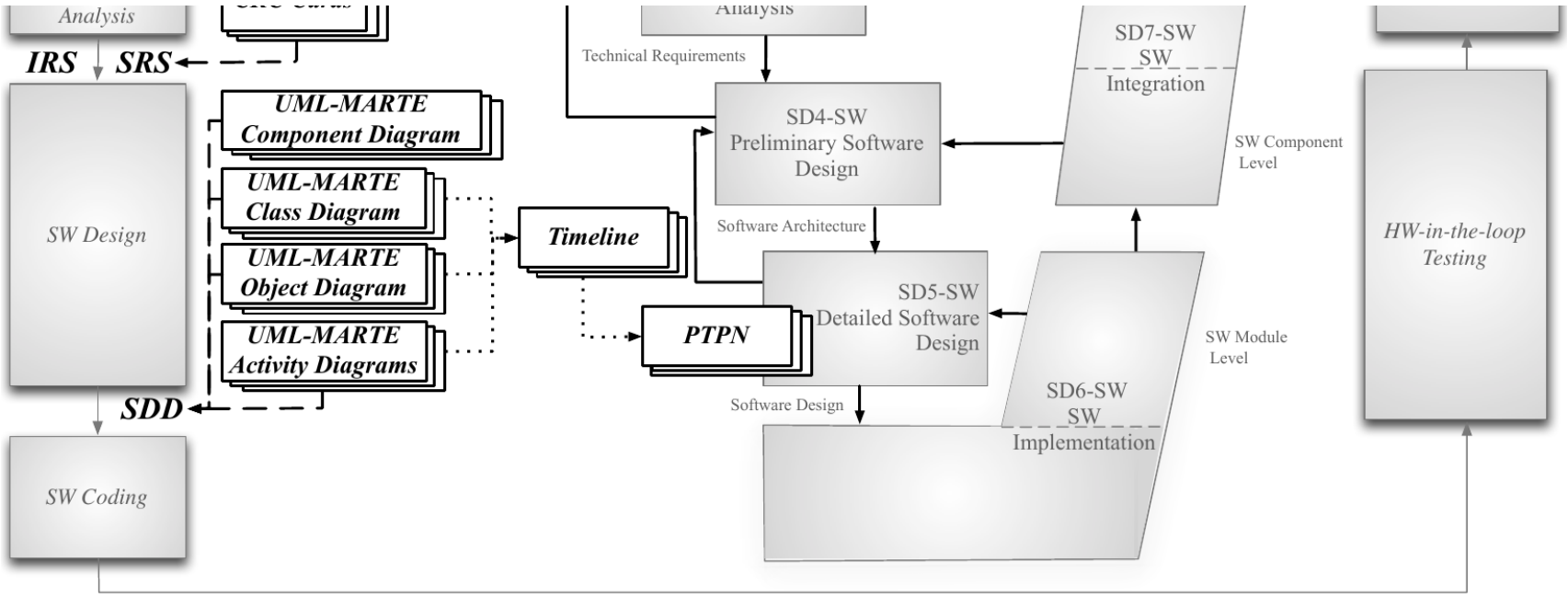
V-Model: SD6-SW

- SD6-SW: SW Implementation - SW module scope
  - Implementation of the task set architecture (executable architecture)
  - Unit testing (execution time profiling of low-level modules)
  - Implementation of entry-point functions (not in the scope of the approach)



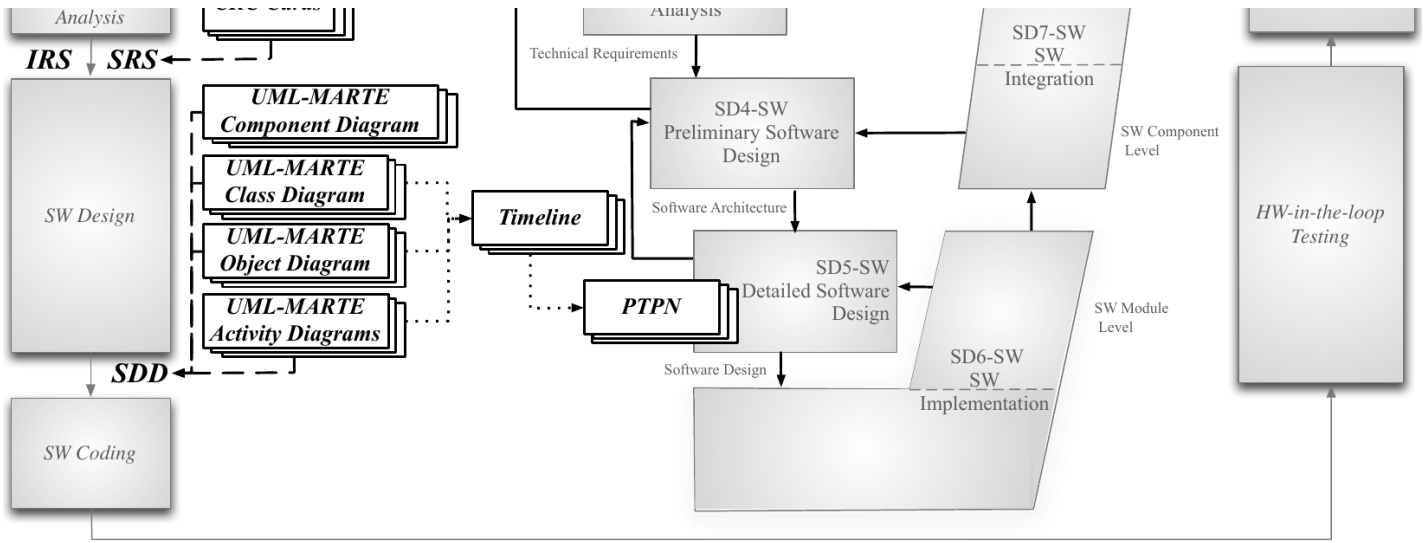
MIL-STD-498: SW Coding

- Corresponds to the first part of SD6-SW
- Produces the task-set architecture of each CSCI (**dynamic architecture**)
  - A skeleton of **control code** that is responsible of implementing **non-functional behavior** of tasks as well as invoking **functional code** of entry-point functions



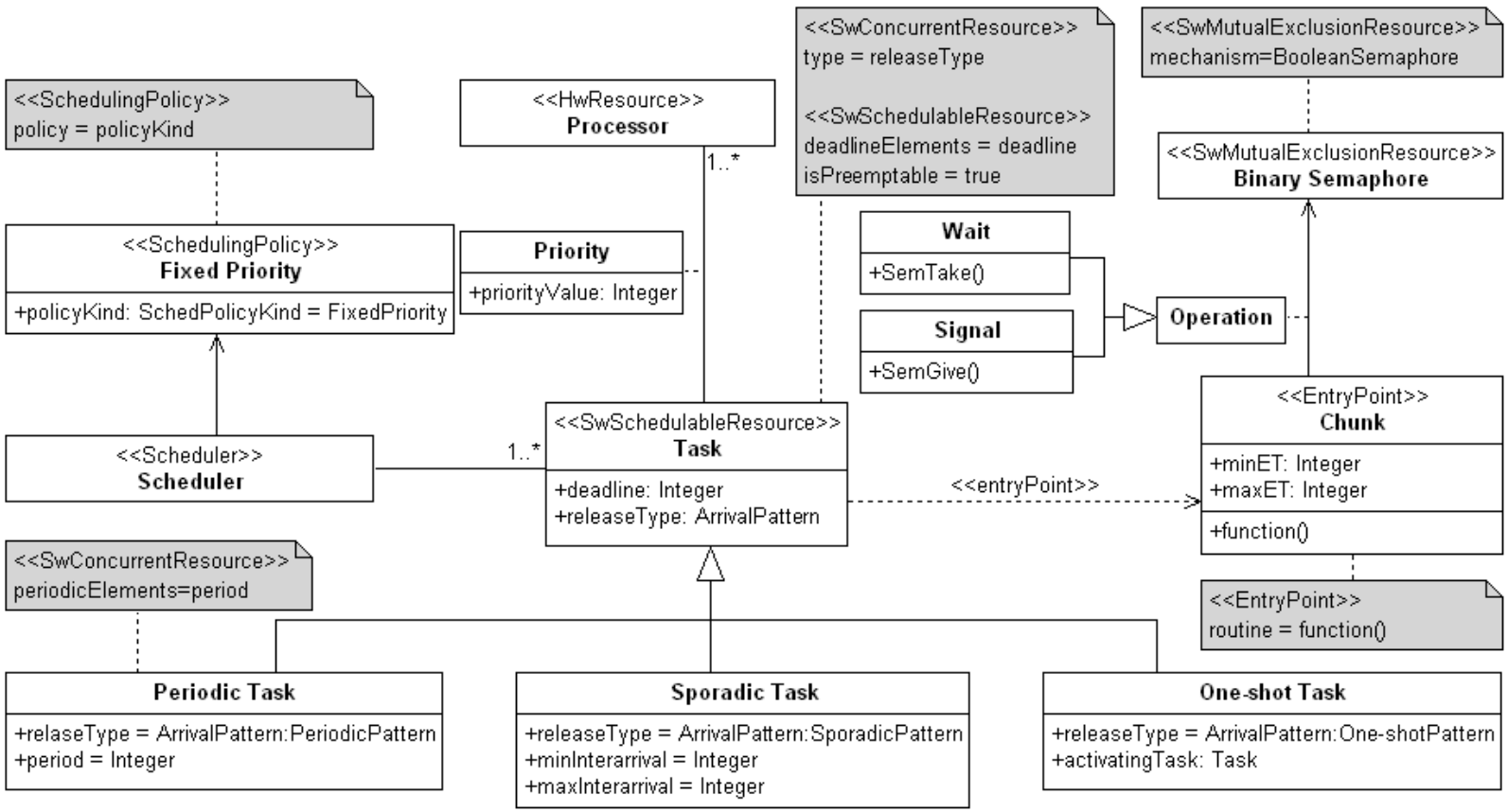
SW Coding: implementation of the task-set architecture (1/5)

- Responsibilities
  - Release task jobs according to their policies
  - Implement semaphore operations and priority handling
  - Sequence the invocation of entry point functions
- Structure
  - Reflects the timeline specification
  - Relies on conventional primitives of a real-time operating system
  - Experiments on c-code of single-processor applications (VxWorks, Linux RTAI)
  - Is this disciplined coding or Model Driven Development (MDD)?



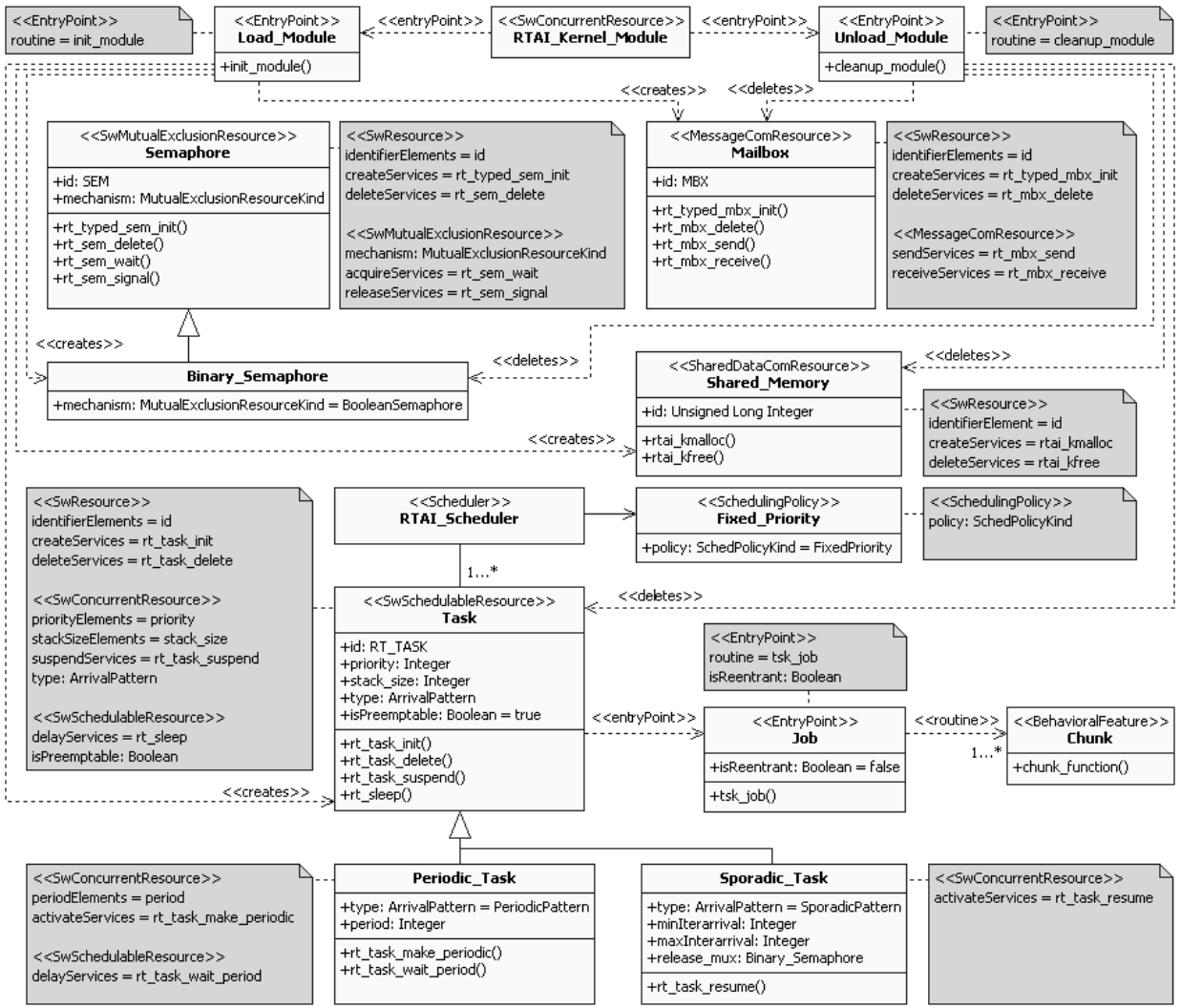
SW Coding: implementation of the task-set architecture (2/5)

- UML-MARTE class diagram of the implementation structure of the dynamic architecture of a CSCI running on **VxWorks 6.5**



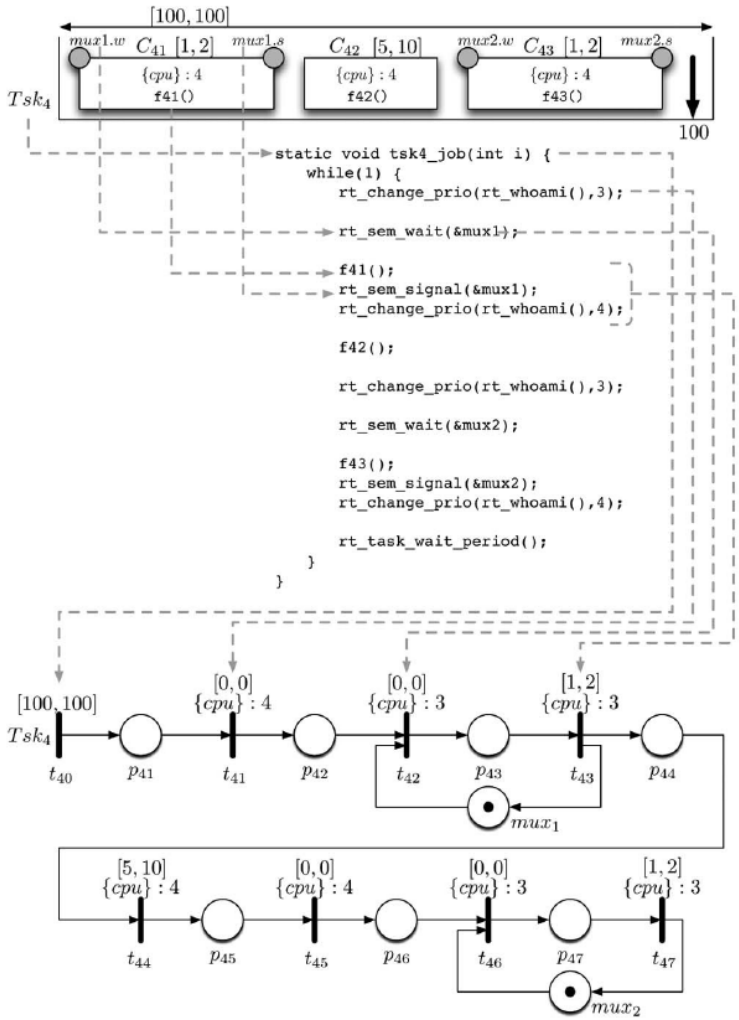
SW Coding: implementation of the dynamic architecture (3/5)

- UML-MARTE class diagram of the implementation structure of the dynamic architecture of a CSCI running on **Linux-RTAI 3.3**



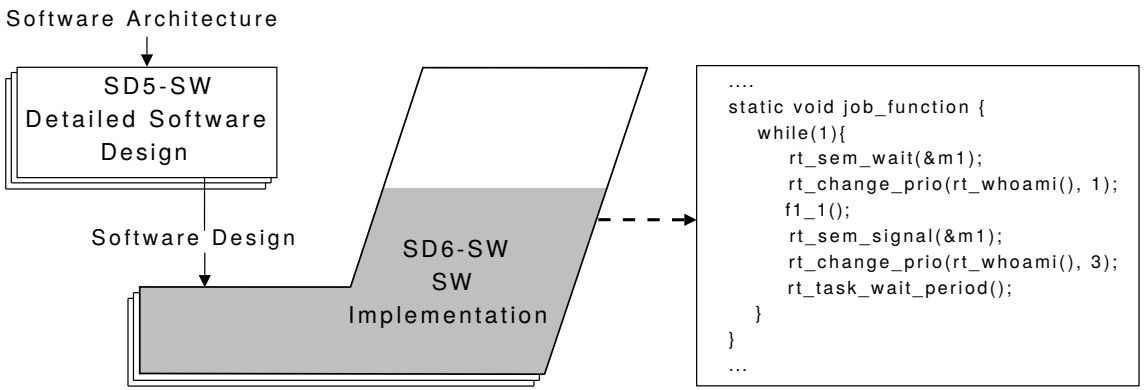
SW Coding: implementation of the dynamic architecture (4/5)

- Fragment of the dynamic architecture of a CSCI running on **Linux-RTAI 3.3**
  - The architecture of the code reflects the PTPN model
  - The developer maintains full control over the code



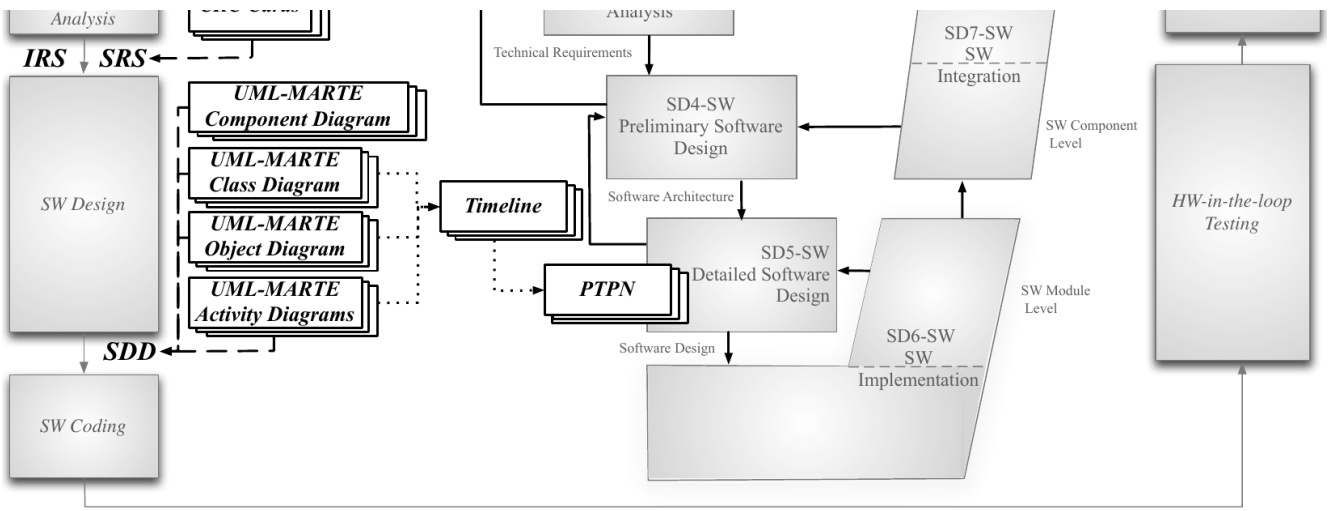
SW Coding: implementation of the dynamic architecture (5/5)

- Emulates the Execution Time (ET) of entry-point functions through a busy-sleep function (e.g., deterministic loop using cpu for a controlled ET)
- Comprises a kind of **executable architecture** (Unified Process)
- Provides a baseline for incremental integration of entry-point functions



MIL-STD-498: first part of HW-in-the-loop testing

- Corresponds to the second part of SD6-SW
- Performs **unit testing**, i.e., testing of each SW module (also referred to as low-level unit) in the dynamic architecture of each CSCI
  - Includes other SW modules emulated through busy-sleep functions
  - Embeds each SW module within the range of behaviors that are feasible for the specification ⇒ permits circumventing the known problem of **determinization of the implementation** with respect to the specification

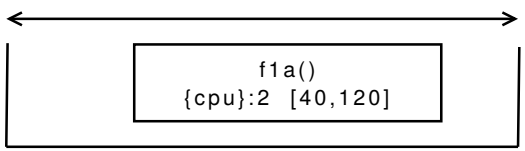


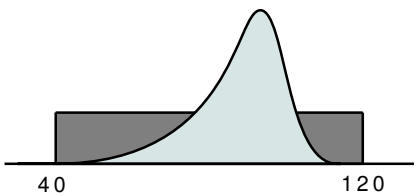
HW-in-the-loop testing: execution time profiling (1/6)

- The executable architecture can be automatically instrumented to produce **time-stamped logs** of the events corresponding to transitions in the PTPN
  - Allows reconstruction of the execution time of each entry-point function:

$$ET(t_i^n) = \sum_{k=n}^{K_{i,n}-1} c_{i,k} \cdot (\tau_{k+1} - \tau_k)$$

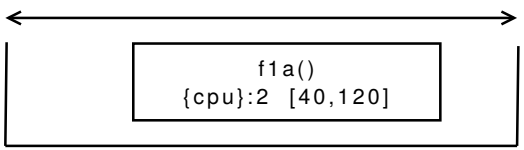
    - t_i^n is the **instance** of transition t_i newly-enabled in the n-th visited state
    - $c_{i,k}$ is 1 or 0 if t_i is progressing or suspended in the k-th visited state, resp.
    - $K_{i,n}$ is the index (in the trace) of the state reached through the firing of t_i^n
    - τ_k is the entrance time in the k-th visited state, i.e. the k-th timestamp
  - Identifies the **implementation behavior** with respect to the specification

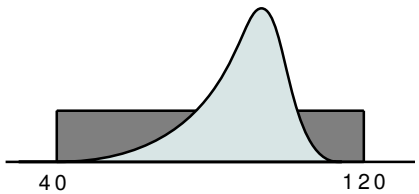




HW-in-the-loop testing: execution time profiling (2/6)

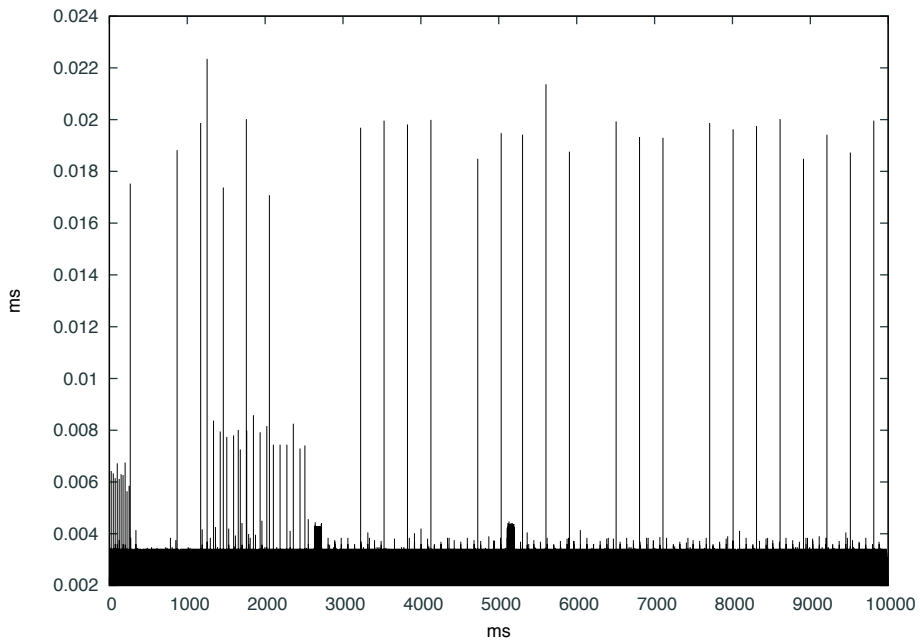
- This is a **measurement-based** approach to execution time analysis
- Runs in interrupted mode ⇒ accounts for context switches, preemption events, HW interrupts, SW interrupts, cache and pipeline effects
- What is the impact of the logging operation on measured execution times?

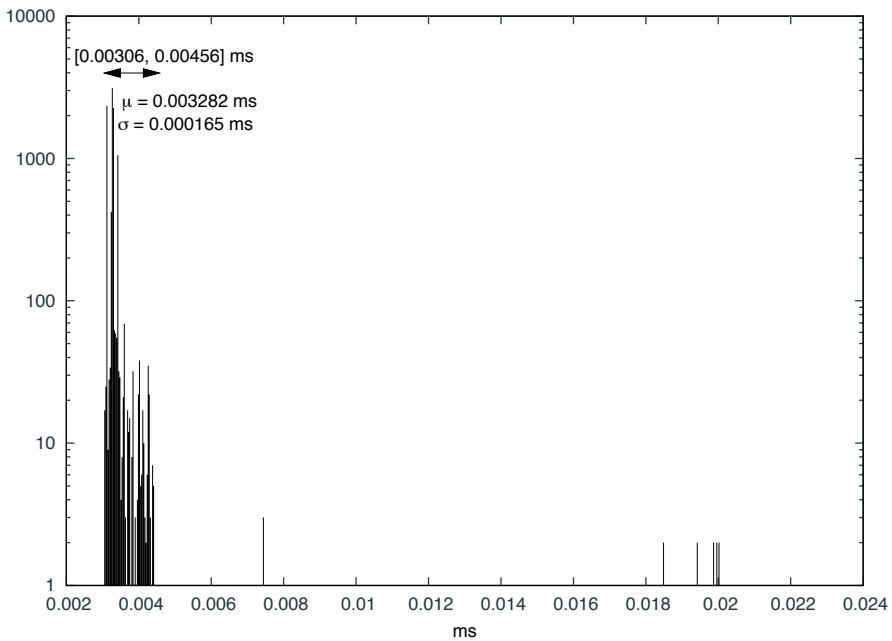




HW-in-the-loop testing: execution time profiling (3/6)

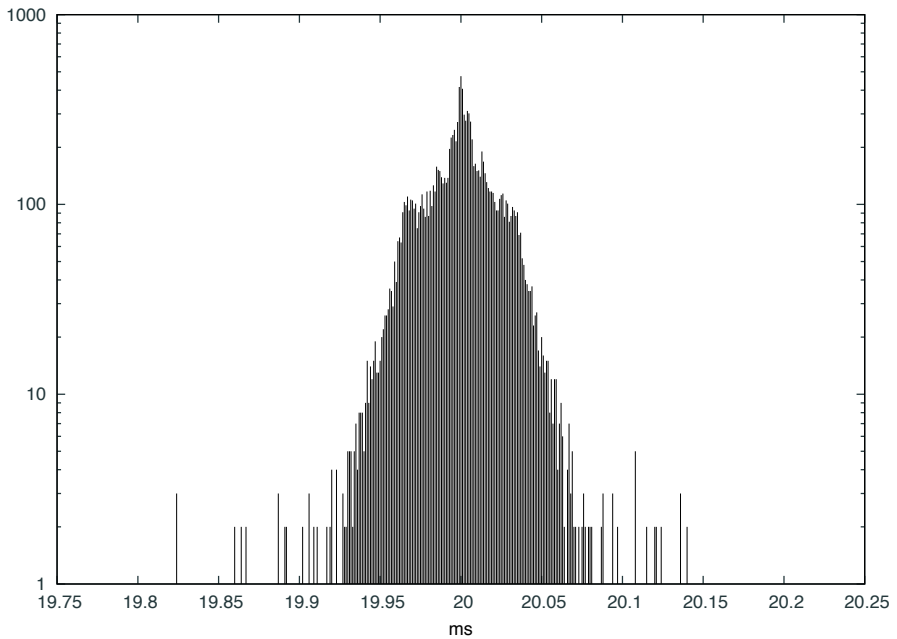
- Perform a large number of repetitions of the logging operation (10,000)
  - Sequence of observed execution times of the logging operation (left)
  - Histogram of observed execution times of the logging operation (right)
- Simple but coarse-grained approach (logging requires tens of microseconds)
  - Min-max intervals associated with temporal parameters were enlarged during iterative refinements of the dynamic architecture of each CSCI





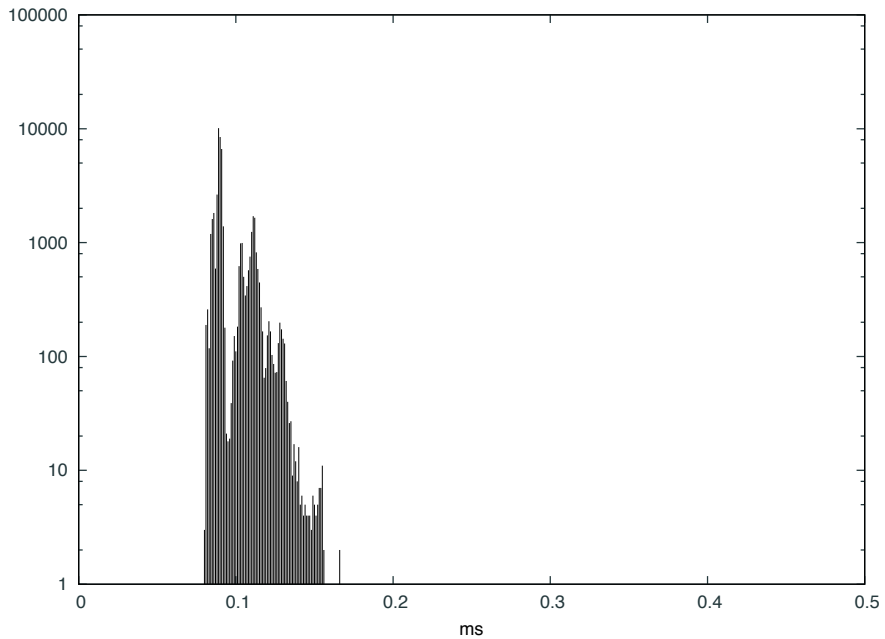
HW-in-the-loop testing: execution time profiling (4/6)

- Histogram of observed inter-release times of the periodic task *Tsk2*
  - The nominal period is equal to 20 ms
  - The observed period falls within $[19.920, 20.069]$ ms in 98.9% of the cases
- The observed variability is not negligible: what can we do?
  - Fix the implementation: not viable given that jitters depend on the RTOS!
  - Refine the model: not convenient due to state-space increase!
  - Raise a warning to subsequent testing stages!



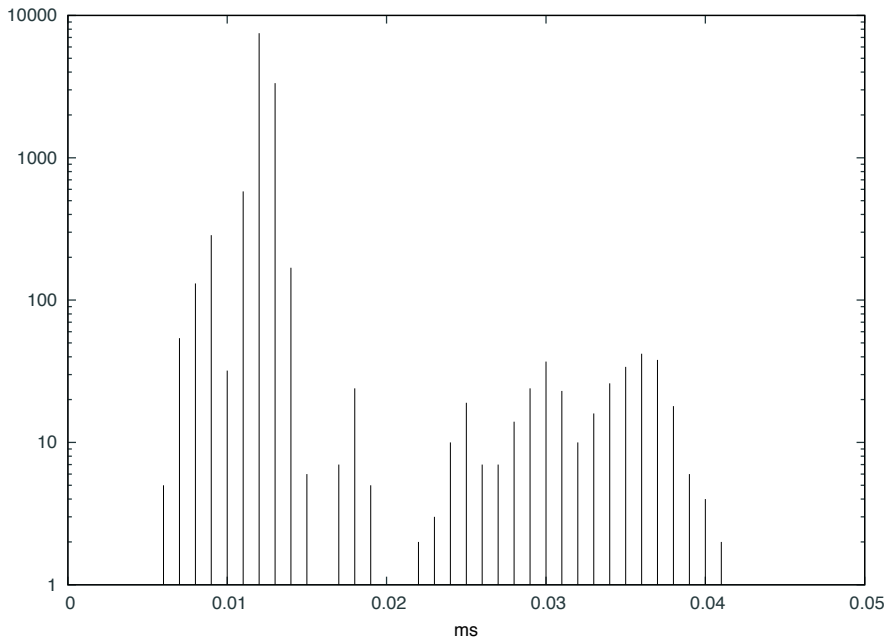
HW-in-the-loop testing: execution time profiling (5/6)

- Histogram of observed execution times of entry-point `f122` of $Tsk1$
  - Nominal execution time within the interval $[0.05, 0.2]$ ms
  - Reflects the absence of data-dependent alternatives in the implementation of the entry-point function (otherwise the histogram would have larger support)



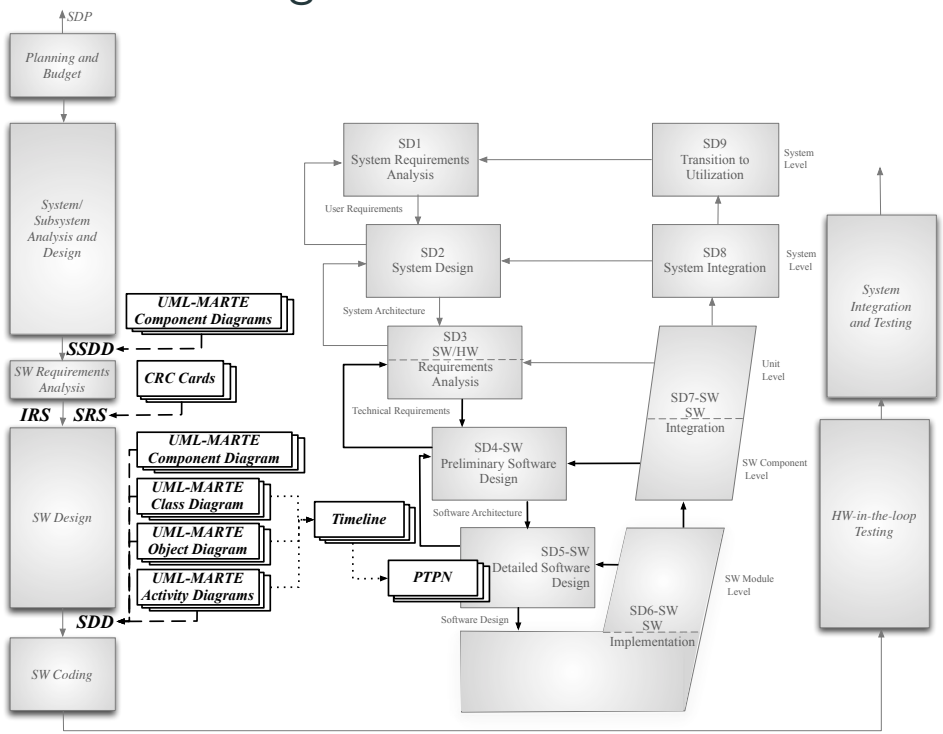
HW-in-the-loop testing: execution time profiling (6/6)

- Histogram of observed execution times of the wait operation performed by the periodic task *Tsk2* on the binary semaphore *sem3*
  - The time spent for semaphore operations on VxWorks 6.5 is not negligible with respect to the order of the execution time of entry-point functions



V-Model: first part of SD7-SW

- First part of SD7-SW: SW Integration - SW component scope
- Corresponds to the second part of HW-in-the-loop Testing in MIL-STD-498
- Activities supported by the formal core of PTPNs:
  - Test case selection and execution
  - Oracle verdict and coverage evaluation

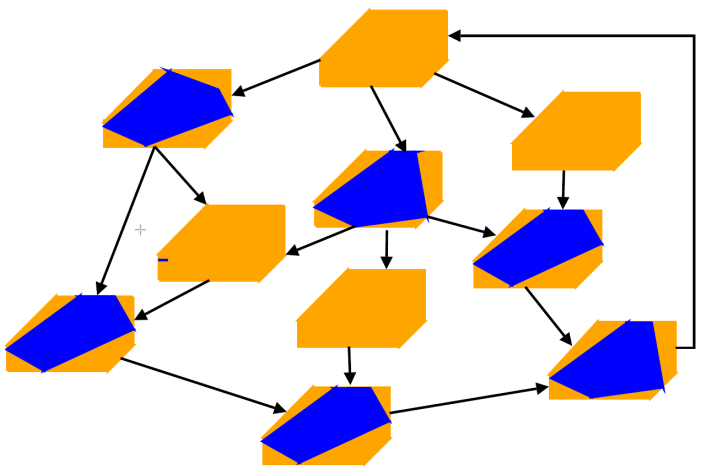


Failure and defect model

- Failures (deviations of a component from its expected delivery, service, result)
  - **Unsequenced execution**: a run breaking sequencing requirements (e.g., chunk sequence, semaphore and priority handling operations, IPC mechanisms, . . .)
  - **Timeframe violation**: a temporal parameter taking values out of its nominal interval (e.g., untimely job release, execution time $\notin$ [BCET, WCET])
  - **Deadline miss**: a job breaking its end-to-end timing requirement
- Defects (flaws in a component that can cause failures of the system)
  - **Task programming defect**: flaw in concurrency control and task interactions (e.g., semaphore operations not properly combined with priority handling, wrong priority assignment, unsequenced invocation of entry-points)
  - **Cycle stealing**: detraction of computational resources due to additional tasks (intentionally or not intentionally omitted in the specification)
- Remarks
  - Deadlines missed by tasks have contractual relevance
  - Failures in chunk sequencing/timing have relevance for design and unit testing
- Functional behavior is not addressed

SD7-SW: which abstraction for test case selection?

- Certification standards prescribe structural coverage criteria measured on the **control-flow graph** of the code
  - e.g., all-statemens, all-decisions, modified condition decision coverage, ...
  - Effectively exert control structures and data-flow dependencies
  - Limited coverage of the variety of behaviors that result from concurrent and interrupted execution of the dynamic architecture of each CSCI
- The **State Class Graph (SCG)** of the PTPN model of a CSCI is an effective abstraction of the variety of behaviors of the CSCI dynamic architecture
  - Qualified code generator $\Rightarrow$ SCG provides a **structural** coverage measure
  - Not qualified code generator $\Rightarrow$ SCG provides a **functional** abstraction



SD7-SW: criteria for test case selection on the SCG (1/3)

- All-markings
  - Requires coverage of each reachable marking
  - Guarantees coverage of all possible **states of concurrency** (i.e., all possible combinations of chunks that are concurrently running/ready/blocked)
  - For each reachable marking m, select any class S_m with marking m and include in the test suite any path from a **Controllable Starting Point (CSP)** to S_m
  - Complexity proportional to the number of reachable markings
- All-marking-edges (includes all-markings)
  - Requires coverage of each edge between any two reachable markings
  - Guarantees coverage of all possible **transitions between concurrency states** (e.g., preemptions following asynchronous releases, chunk completions, semaphore and mailbox operations, priority boost/deboost operations)
  - Complexity proportional to the number of reachable markings multiplied by the maximum number of events in a marking (the latter is $\leq$ # tasks)
  - For each edge between any two markings m_1 and m_2, select any class S_1 with marking m_1 with an event leading to class S_2 with marking m_2, and include in the test suite any path starting from a CSP and covering the edge

SD7-SW: criteria for test case selection on the SCG (2/3)

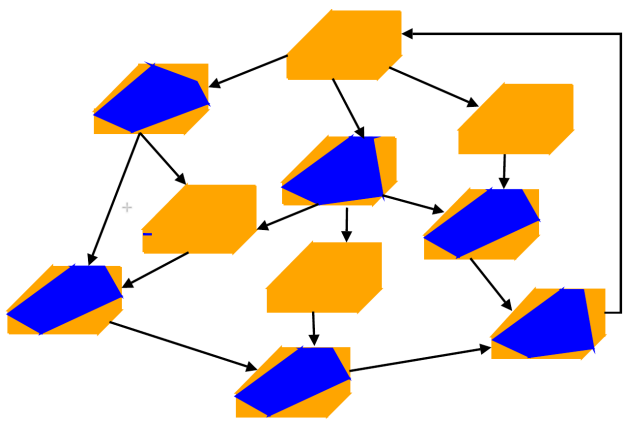
- All-classes (includes all-markings)
  - Requires coverage of all reachable state classes
  - Distinguishes states of concurrency associated with different timings yielded by the execution of different sequences of transition firings
  - Increases complexity with respect top all-markings
- All-class-edges (includes all-marking-edges)
  - Requires coverage of all edges between reachable state classes
  - Distinguishes states of concurrency associated with different timings yielded by the execution of different sequences of transition firings
  - Increases complexity with respect top all-marking-edges

SD7-SW: criteria for test case selection on the SCG (3/3)

- All-symbolic-runs (includes all-class-edges)
  - Requires coverage of all symbolic runs that start with a job release and terminate either with its completion or with a deadline miss
  - Distinguishes concurrency states and transitions visited in $\neq$ job executions
  - Increases complexity with respect top all-class-edges
- All-symbolic-executions
  - Requires coverage of any event sequence that starts with a job release and terminates either with its completion or with a deadline miss
  - Does not distinguish paths with same firing sequence and $\neq$ starting class
  - Reduces the complexity of all-symbolic runs

SD7-SW: test case execution

- How can we determine **timed inputs** that force the **Implementation Under Test (IUT)** to execute each selected test case?
  - Periodic and asynchronous releases times can be controlled
  - Computation times are often impractical to control
  - Problem of **determinization** of the implementation wrt the specification
    - e.g., the specification neglects dependencies in the execution times of chunks, nominal ranges of variation of timers are enlarged to make the model robust, ...
- **Guided testing** (as opposed to **randomized testing**)
  - Exploit SCG to identify state classes that can be selected as starting points
  - Exploit PTPN trace analysis to derive timing restrictions for starting classes, that can lead to the execution of specific sequences of transition firings



SD7-SW: oracle verdict and coverage evaluation

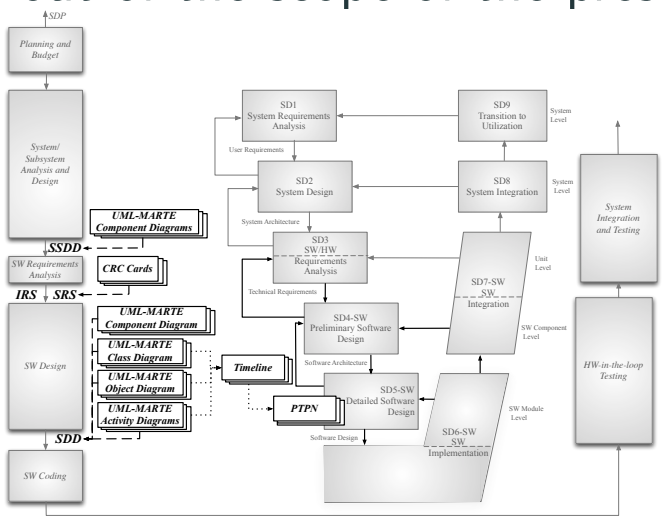
- End-to-end oracle
  - Verifies that end-to-end task deadlines are met
  - Requires time-stamped logging of job releases and completions
- Sequencing oracle
  - Verifies that the qualitative ordering of events conforms with the specification (i.e., that there exists at least one timing makes the logged sequence feasible)
  - Requires logging of all events corresponding to a transition in the PTPN model
- Timing oracle
  - Verifies that the logged sequence is a feasible execution for the specification (i.e., verifies the **timed trace inclusion relation**)
  - Requires time-stamped logging of all events corresponding to a transition
  - Detects all failures detected by the sequencing oracle

SD7-SW: results on the running case study

- Testing in a simulated environment of the code of the System Control (SC) CSCI, code integrated with **functional** entry-points of its chunks
- Detection of an unsequenced execution
  - Caused by a task programming defect: two chunks synchronizing on a semaphore not explicitly represented in the dynamic architecture
  - Fixed by representing the semaphore in the task-set specification and then repeating the formal verification
- Detection of time-frame violations
  - Caused by a cycle stealing due to a VxWorks task named `tNetTask` providing packet-processing network services
  - Fixed by assigning SC tasks larger priority than `tNetTask`

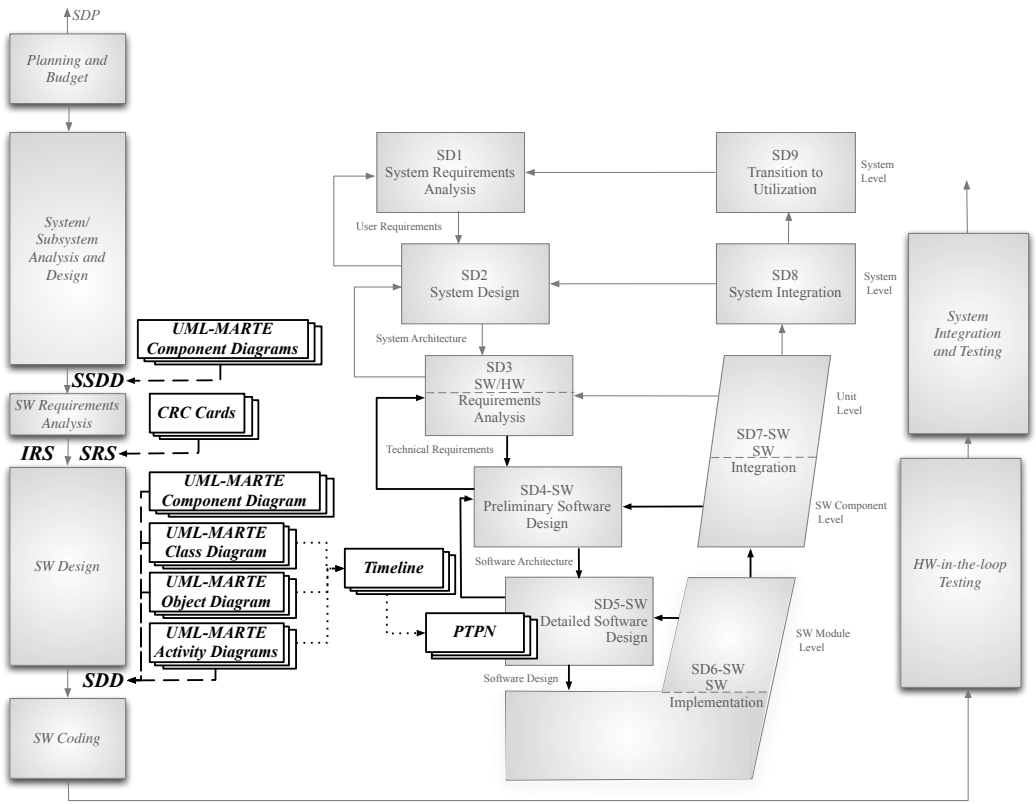
V-Model: second part of SD7-SW, SD8, SD9

- Second part of SD7-SW: SW Integration - unit scope
  - Tests the integration of CSCIs, HCIc, and FCIs within each unit
- SD8: System Integration - system scope
  - Tests the integration of all units within the system
- SD9: Transition to Utilization - system scope
  - Puts the completed system into operation at the intended application site
- The second part of SD7-SW and SD8 correspond to System Integration and Testing in MIL-STD-498
- All these activities are out of the scope of the presented methodology



Is testing really needed in Model Driven Development (MDD)?

- Architectural verification abstracts from a number of details
  - Execution time of operations (execution time may be negligible but not zero), additional tasks, deployment platform, context switches, ...
- Architectural verification could be not exhaustive (state space explosion)
- Testing is in any case needed for certification purposes (for good reasons)



Timed Automata

Introduction to TA (1/2)

- Timed Automata support the representation of **concurrent timed** systems
  - Expressivity comparable with that of Time Petri Nets (TPNs)
- A Timed Automaton (TA) is a finite automaton enriched with:
  - a set of **clocks**
  - **timing constraints** on transitions (also termed guards or enabling conditions)
  - **clock resets** on transitions
- A minimal TA: two locations `H` and `K`, two clocks `x` and `y`, a transition from `H` to `K` (enabled if $x > 0$, labeled with action `a`, resetting clock `y` to 0)

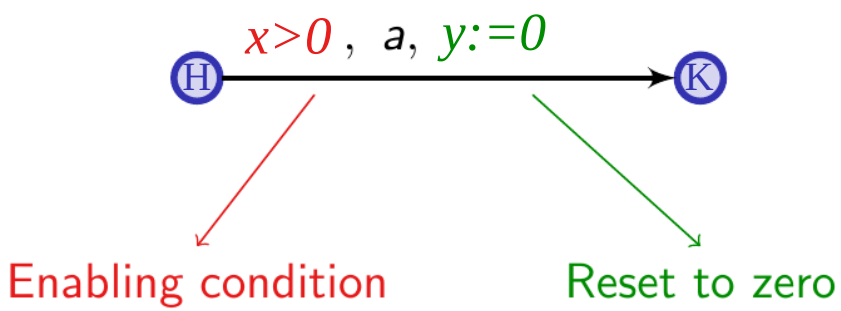
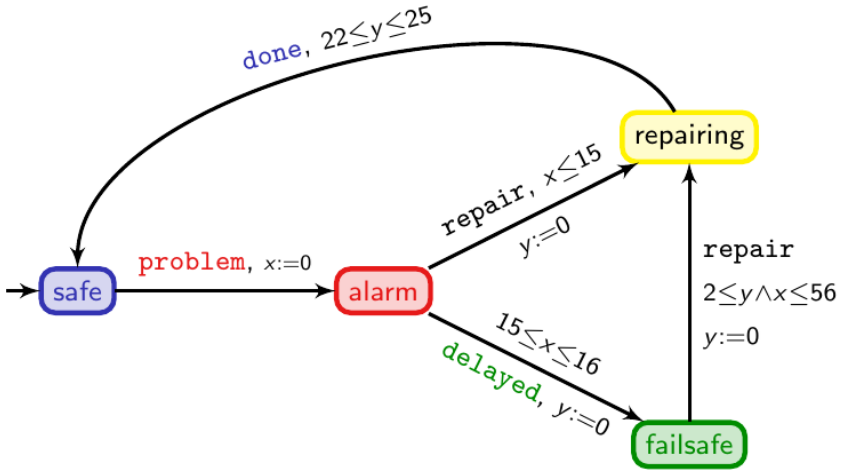


Image adapted from Patricia Bouyer-Decitre, CNRS & ENS Paris-Saclay

R. Alur and D. L. Dill, "A theory of timed automata," Theoretical Computer Science, 1994.

Introduction to TA (2/2)

- The TA of a repair process



- A possible execution of the TA

	safe	23 →	safe	problem →	alarm	15.6 →	alarm	delayed →	failsafe	
x	0		23		0		15.6		15.6	⋯
y	0		23		23		38.6		0	

	failsafe	2.3 →	failsafe	repair →	repairing	22.1 →	repairing	done →	safe
⋯	15.6		17.9		17.9		40		40
	0		2.3		0		22.1		22.1

Image by Patricia Bouyer-Decitre, CNRS & ENS Paris-Saclay

TA syntax

- A TA is a tuple $\langle L, l_0, \Sigma, X, E \rangle$ where:
  - L is a finite set of **locations**
  - $L_0 \subseteq L$ is the set of **initial location**
  - Σ is a finite **action alphabet**
  - X is a set of **clocks**
  - $E \subseteq L \times \Sigma \times G(X) \times 2^X \times L$ is a set of **edges** where $G(X)$ is a set of **guards** with syntax $g := x \sim c \mid g \wedge g$ where $x \in X$, $\sim \in \{<, \leq, =, >, \geq\}$, $c \in \mathbb{N}$ (i.e., a guard is a conjunction of atomic constraints of the form $x \sim c$)
- An example with three locations, two actions, two clocks, and two transitions

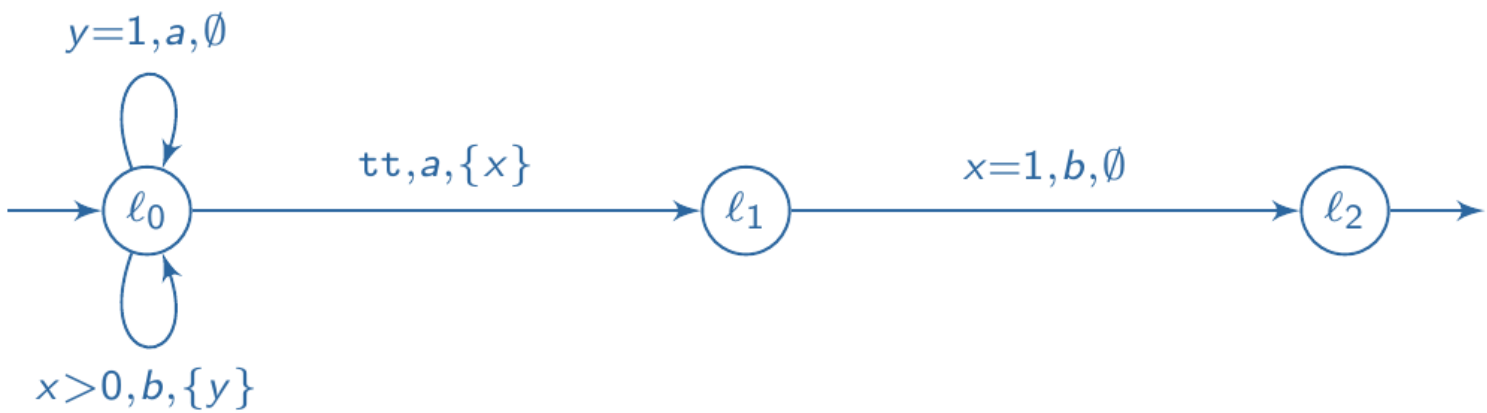


Image by Natalie Bertrand, Inria Rennes Bretagne

TA semantics (1/3)

- A **clock valuation** $v \in \mathbb{R}^X_{\geq 0}$ assigns a value to each clock
  - If a valuation $v \in \mathbb{R}^X_{\geq 0}$ satisfies a guard $g \in G(X)$, then we write $v \models g$
  - $v + \tau$ denotes the valuation $(v + \tau)(x) = v(x) + \tau \; \forall x \in X, \forall \tau \in \mathbb{R}_{\geq 0}$
- The **state** of a TA is a pair (l, v) where:
  - l is a location
  - v is a valuation
- There exist two types of **transition** between states
  - **Delay transition**: $(l, v) \xrightarrow{\tau} (l, v + \tau)$ for any state (l, v) and any delay $\tau \in \mathbb{R}_{\geq 0}$
  - **Discrete transition**: $(l, v) \xrightarrow{a} (l', v')$ if $\exists (l, a, g, R, l') \in E$ such that $v \models g$ and $v'(x) = 0$ if $x \in R$ and $v'(x) = v(x)$ if $x \notin R$
- A **run** of a TA is a sequence of alternating delay and discrete transitions: $(l_0, v_0) \xrightarrow{\tau_1} (l_0, v_0 + \tau_1) \xrightarrow{a_1} (l_1, v_1) \xrightarrow{\tau_2} (l_1, v_1 + \tau_2) \xrightarrow{a_2} \ldots \xrightarrow{a_k} (l_k, v_k)$
  - Equivalently written as $(l_0, v_0) \xrightarrow{\tau_1, a_1} (l_1, v_1) \xrightarrow{\tau_2, a_2} \ldots \xrightarrow{\tau_k, a_k} (l_k, v_k)$

TA semantics (2/3)

- A **time sequence** is a finite non-decreasing sequence of non-negative real values i.e., $s = (t_1)(t_2)\cdots(t_k)$ with $t_i \in \mathbb{R}_{\geq 0}\ \forall\, i \in \{1, \ldots, k\}$
- A **timed word** is a finite sequence of pairs consisting of an action and a time value belonging to a time sequence, i.e., $w = (a_1, t_1)(a_2, t_2)\cdots(a_k, t_k)$ with $a_i \in \Sigma$ and with $(t_1)(t_2)\cdots(t_k)$ being a time sequence
- A timed word $w = (a_1, t_1)(a_2, t_2)\cdots(a_k, t_k)$ is **accepted** by a TA if $\exists\ \rho = (l_0, v_0) \xrightarrow{\tau_1, a_1} (l_1, v_1) \xrightarrow{\tau_2, a_2} \ldots \xrightarrow{\tau_k, a_k} (l_k, v_k)$ with $l_0 \in L_0$ and $t_i = \sum_{1 \leq j \leq i} \tau_j$
- The **accepted timed language** of a TA is the set of its accepted timed words

TA semantics (3/3)

- A run of the example TA is $(l_0,(0,0)) \xrightarrow{0.1,b} (l_0,(0.1,0)) \xrightarrow{0.2,b} (l_0,(0.3,0)) \xrightarrow{1,a} (l_0,(1.3,1)) \xrightarrow{0.2,b} (l_0,(1.5,0)) \xrightarrow{0,a} (l_1,(0,0)) \xrightarrow{1,b} (l_2,(1,1))$
- An accepted timed word is $w = (b,0.1)(b,0.3)(a,1.3)(b,1.5)(b,2.5)$
- Always true guards and empty reset sets are omitted in the representation

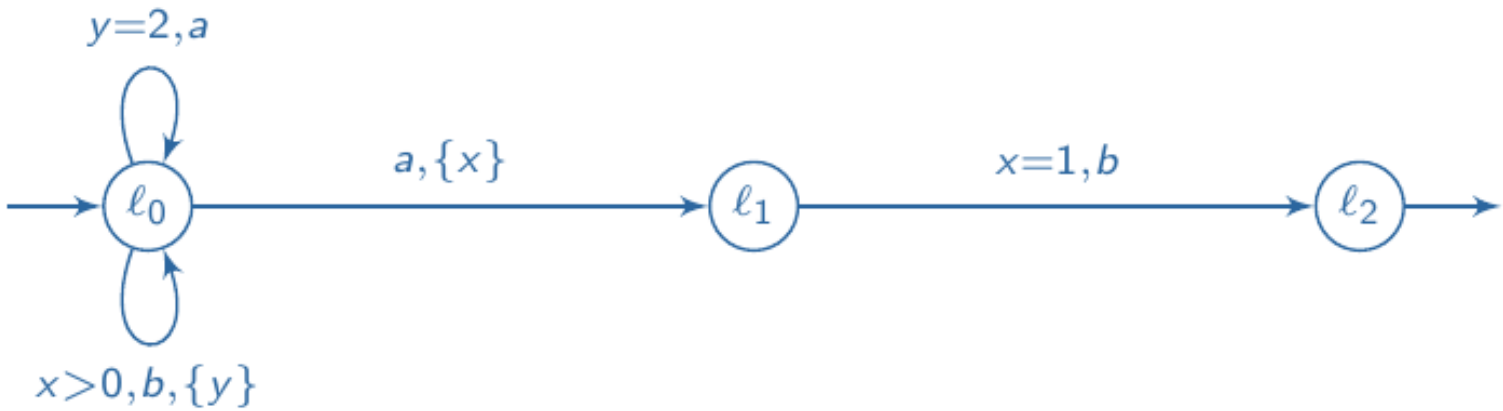
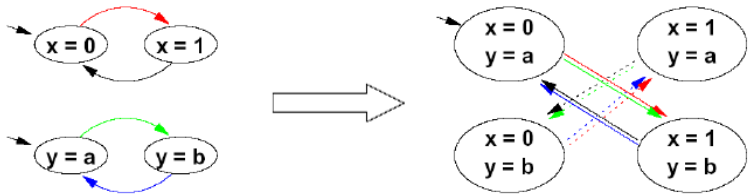


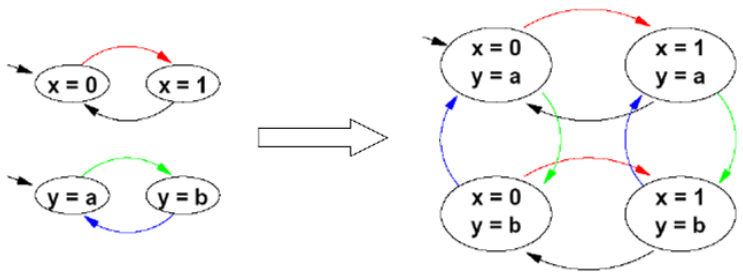
Image by Natalie Bertrand, Inria Rennes Bretagne

Many variants of TA in the literature (1/2)

- Networks of TA (NTA)
  - Product (parallel composition) of two or more TA with different sets of clocks
  - Different semantics depending on the type of composition
    - Synchronous composition



    - Asynchronous composition



    - Composition with explicit synchronization

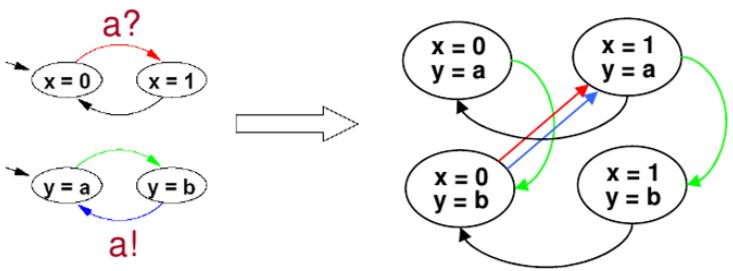


Image by Prof. Marco Di Natale, Scuola Superiore Sant'Anna, Pisa

Many extensions of TA in the literature (2/2)

- TA with diagonal constraints
  - Guards have syntax $g := x \sim c \,|\, x - y \sim c \,|\, g \wedge g$ where $x, y \in X$, $\sim \in \{<, \leq, =, >, \geq\}$, and $c \in \mathbb{N}$ (i.e., a guard is a conjunction of atomic constraints of the form $x \sim c$ and $x - y \sim c$)
- TA with epsilon transitions
  - Epsilon transitions are silent or unobservable transitions
- ...

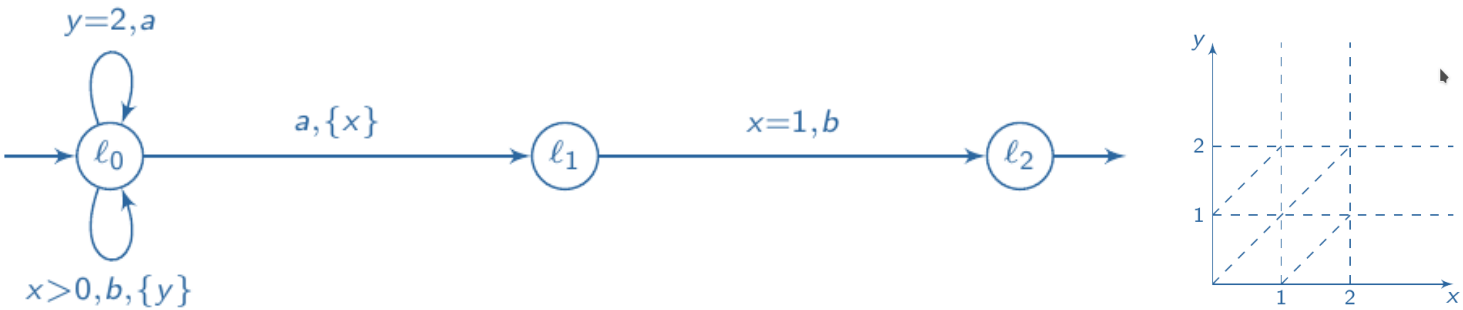
Region partitioning (1/2)

- $\forall\, g \in G$, let $[\![g]\!]$ be the set of valuations $\{v \in \mathbb{R}^X_{\geq 0} \mid v \models g\}$
- $\forall\, Y \subseteq X$, let $[Y \leftarrow 0]v$ be the valuation such that
  $[Y \leftarrow 0]v(x) = 0$ if $x \in Y$ and $[Y \leftarrow 0]v(x) = v(x)$ if $x \in X \setminus Y$

- A finite partition $\mathcal{R}$ of $\mathbb{R}^X_{\geq 0}$ is a **set of regions** (for the set of guards G) if:
  1. $\forall\, g \in G$ and $\forall\, R \in \mathcal{R}$, $[\![g]\!] \subseteq R$ or $[\![g]\!] \cap R = \varnothing$
  2. $\forall\, R, R' \in \mathcal{R}$, if $\exists\, v \in R$ and $t \in \mathbb{R}_{\leq 0}$ with $v + t \in R' \Rightarrow$
     $\Rightarrow \exists\, t' \in \mathbb{R}_{\geq 0}$ with $v' + t' \in R'\ \forall\, v' \in R$
  3. $\forall\, R, R' \in \mathcal{R}$, $\forall\, Y \subseteq X$, if $R_{[Y \leftarrow 0]} \cap R' \neq \varnothing \Rightarrow R_{[Y \leftarrow 0]} \subseteq R$
     (where $R_{[Y \leftarrow 0]}$ is the region obtained from R by resetting clocks in $Y \in X$)
- $\mathcal{R}$ defines an **equivalence relation** over valuations
  - An equivalence class is termed a **region**
- If two valuations are equivalent, then their future behaviors are equivalent
  - condition 1: two equivalent valuations satisfy the same clock constraints
  - condition 2: elapsing of time does not distinguish two equivalent valuations
  - condition 3: resetting clocks does not distinguish two equivalent valuations

Region partitioning (2/2)

- Two valuations v and v' with $v, v' \in \mathbb{R}^X_{\geq 0}$ are equivalent if $\forall\, x, y \in X$:
  - $v(x) > M \Leftrightarrow v'(x) > M$ where M is the **maximal constant** for all clocks
  - $v(x) \leq M \Rightarrow \lfloor v(x) \rfloor = \lfloor v'(x) \rfloor \wedge (\{v(x)\} = 0 \Leftrightarrow \{v'(x)\} = 0)$
    where $\{v(x)\}$ is the fractional part of $\{v(x)\}$
  - $v(x) \leq M \wedge v(y) \leq M \Rightarrow (\{v(x)\} \leq \{v(y)\} \Leftrightarrow \{v'(x)\} \leq \{v'(y)\})$
- The partition is compatible with conditions 1, 2, and 3
- Diagonal-free region partitioning for 2 clocks and maximal constant 2

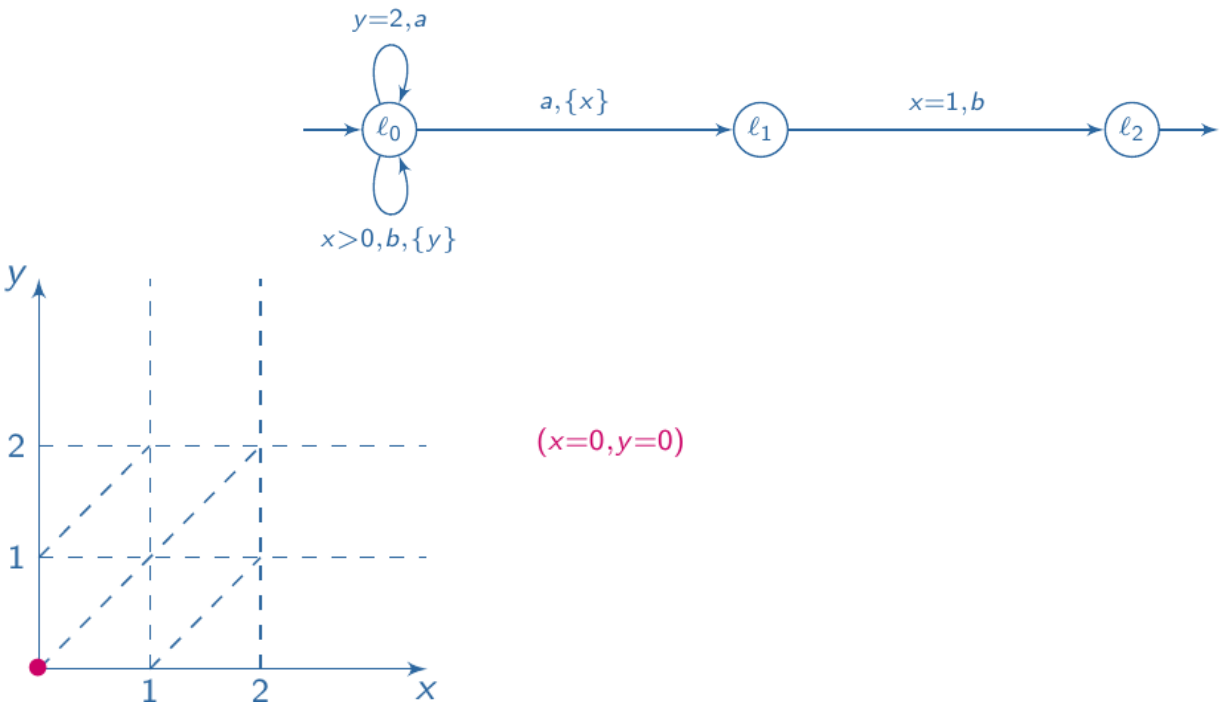


- Shape of **bounded** regions with 2 clocks and no diagonal constraints:

Images by Natalie Bertrand, Inria Rennes Bretagne

Region graph

- Is a finite automaton whose states are the regions and whose transitions are:
  - $R \xrightarrow{\tau} R'$ if $\exists\, v \in R,\ v' \in R',\ \tau \in \mathbb{R}_{\geq 0}$ such that $v' = v + \tau$
    (in this case, R' is termed time successor of R)
  - $R \xrightarrow{Y} R'$ if $R_{[Y \leftarrow 0]} \subseteq R'$
- Represents the possible timing evolutions of the system
- Diagonal-free region partitioning for 2 clocks and maximal constant 2



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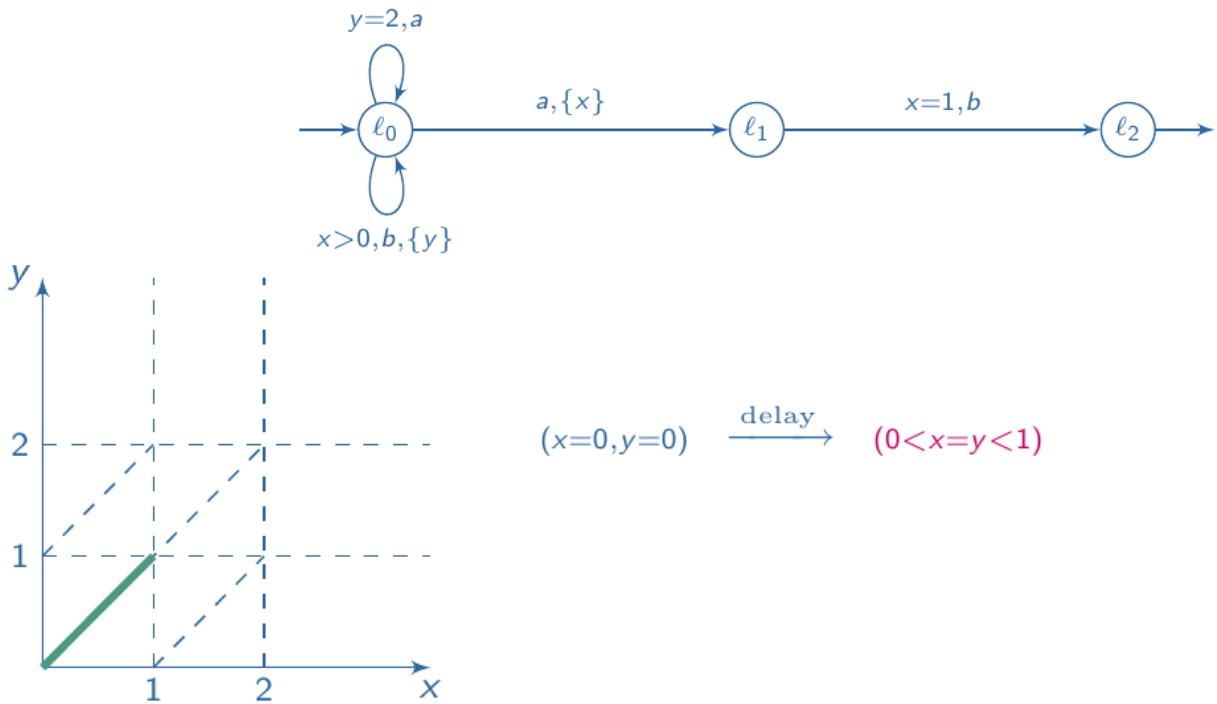

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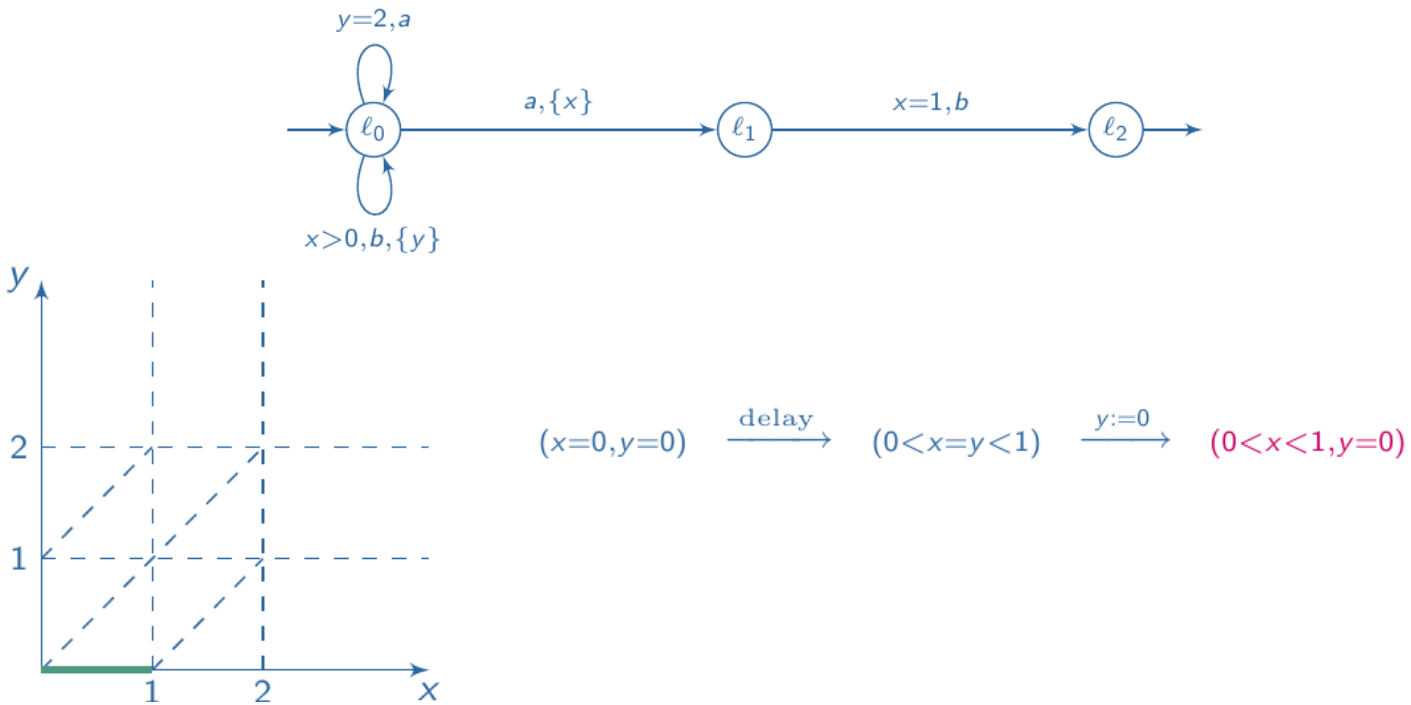

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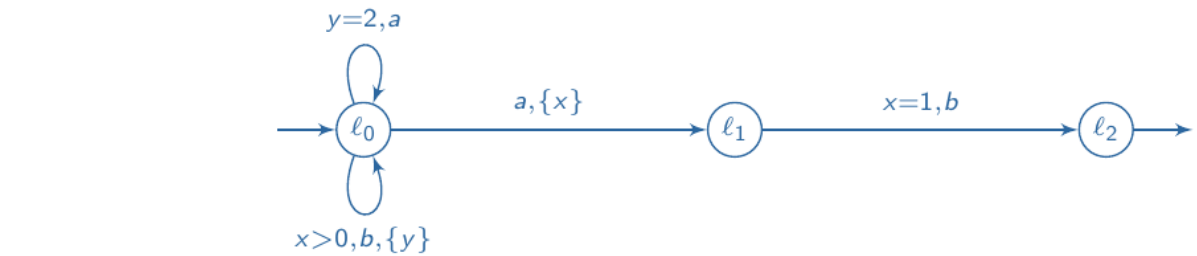

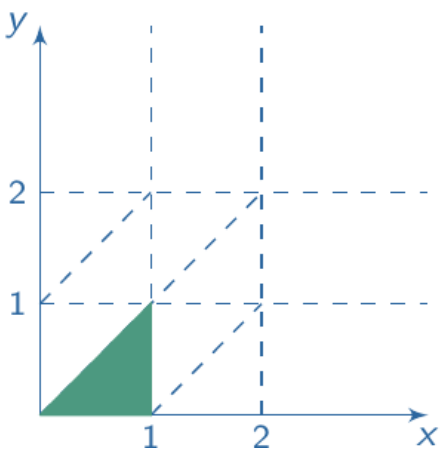

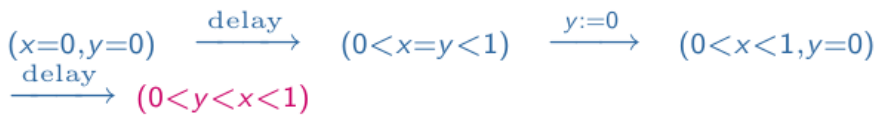

Images by Natalie Bertrand, Inria Rennes Bretagne

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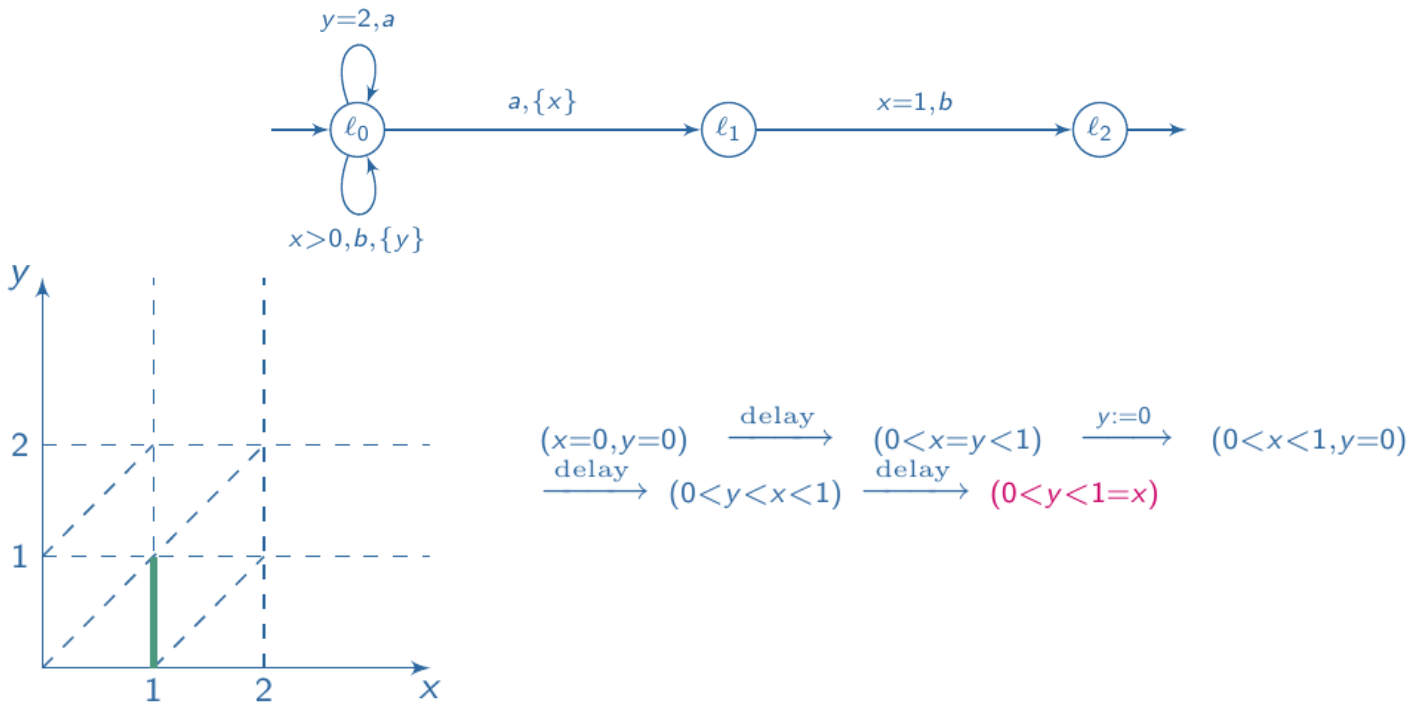

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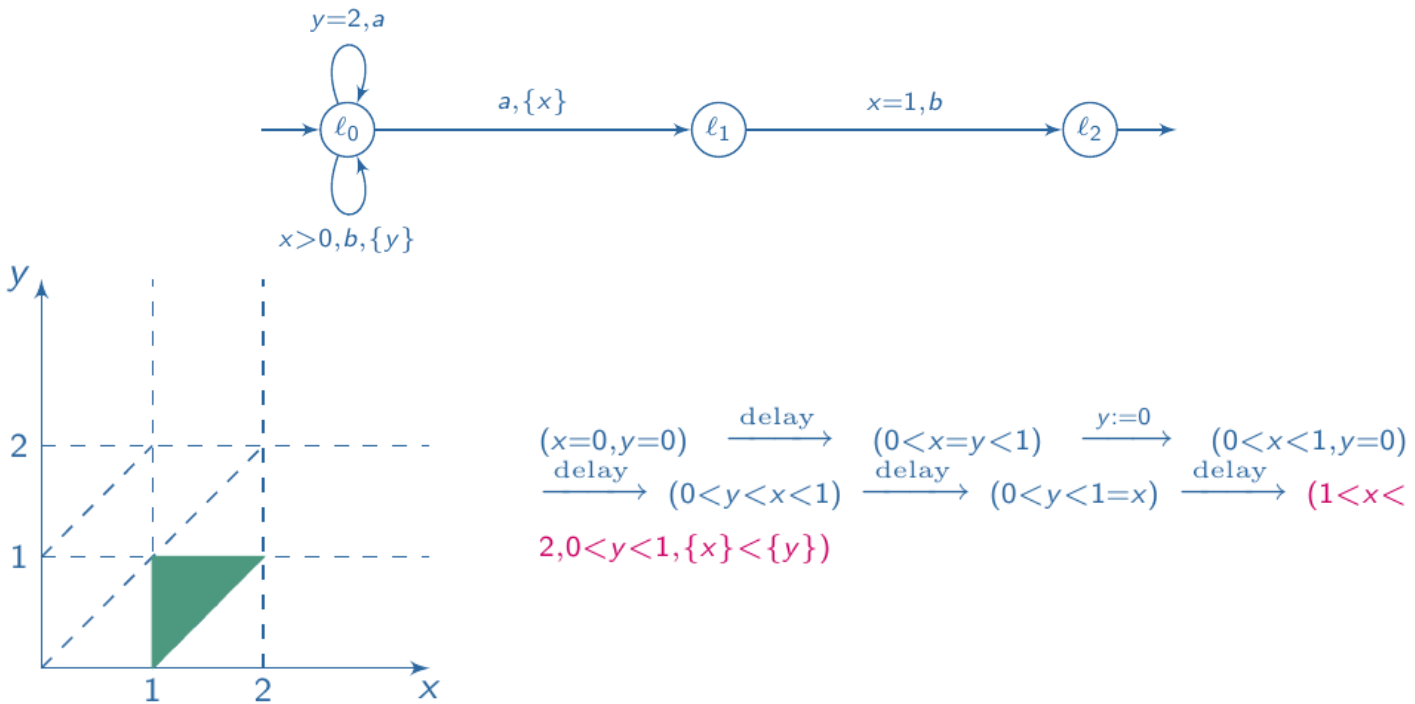

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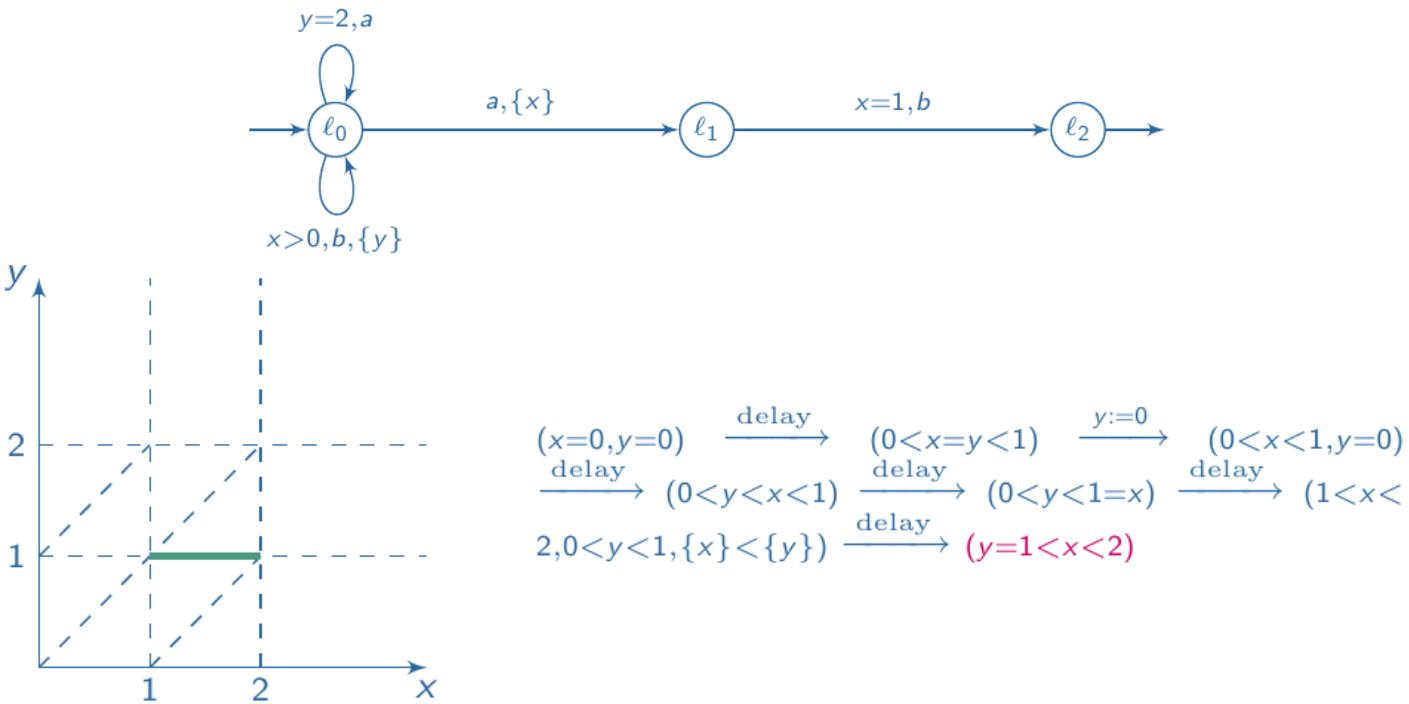

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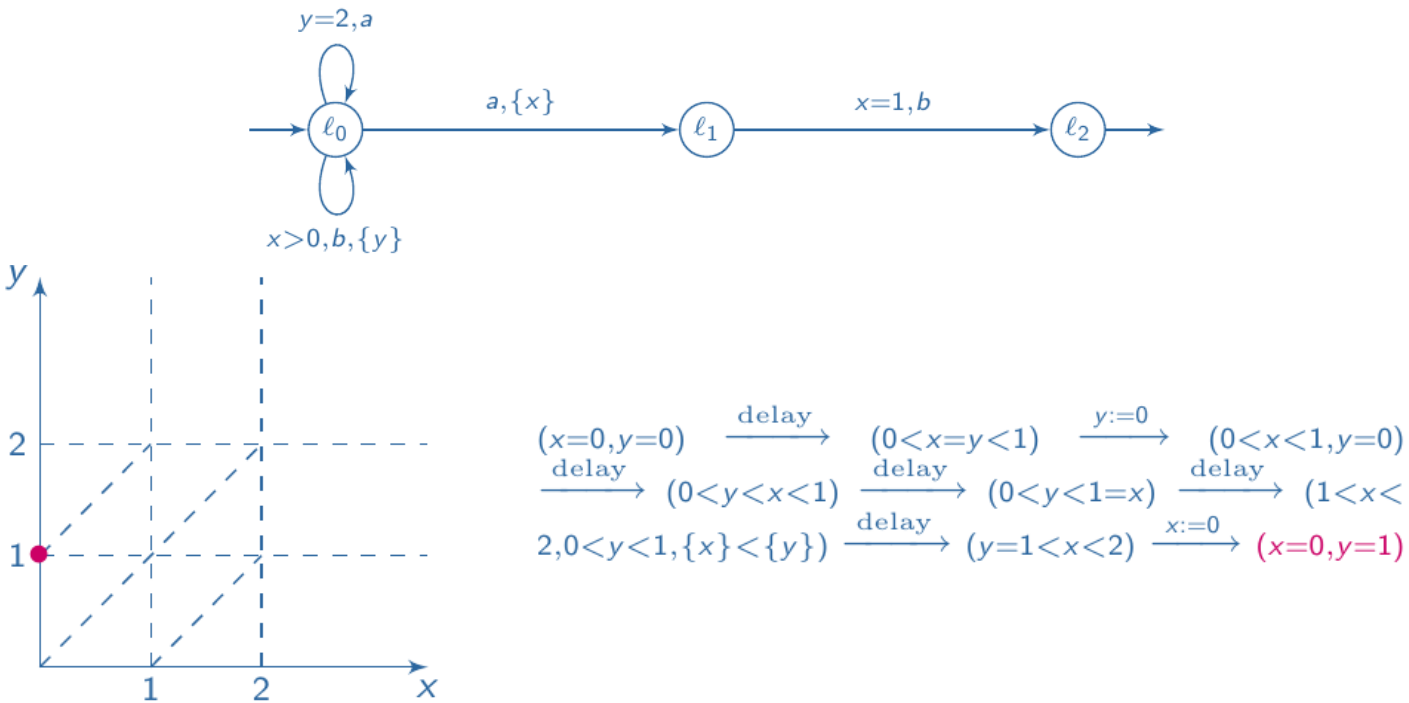
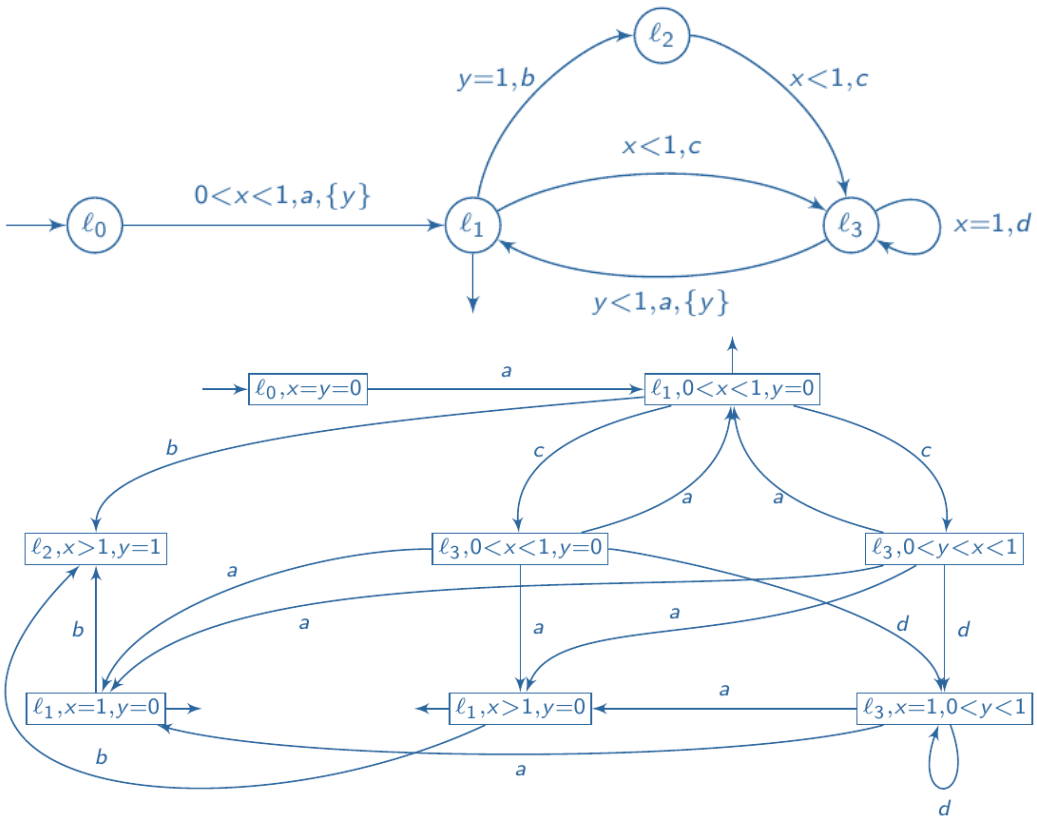


Images by Natalie Bertrand, Inria Rennes Bretagne

Region automaton

- Is a finite automaton whose set of states is $L \times \mathcal{R}$ and whose transitions are:
  - $(l, R) \xrightarrow{a} (l', R')$ if $\exists\, l \xrightarrow{g,a,Y} l'$ is a transition in the TA with $R \subseteq [\![g]\!]$ and $R \xrightarrow{Y} R'$ is a transition in the region graph
  - $(l, R) \xrightarrow{\tau} (l', R')$ if $R \xrightarrow{\tau} R'$ is a transition of the region graph
- An example



Images by Natalie Bertrand, Inria Rennes Bretagne

Zone graph

- The number of regions is **exponential** in the number of clocks and the value of the maximal constants in the guards!
- The **zone graph** provides a more efficient encoding
  - Zones are DBMs representing the union of regions
- An example: a TA and its zone graph

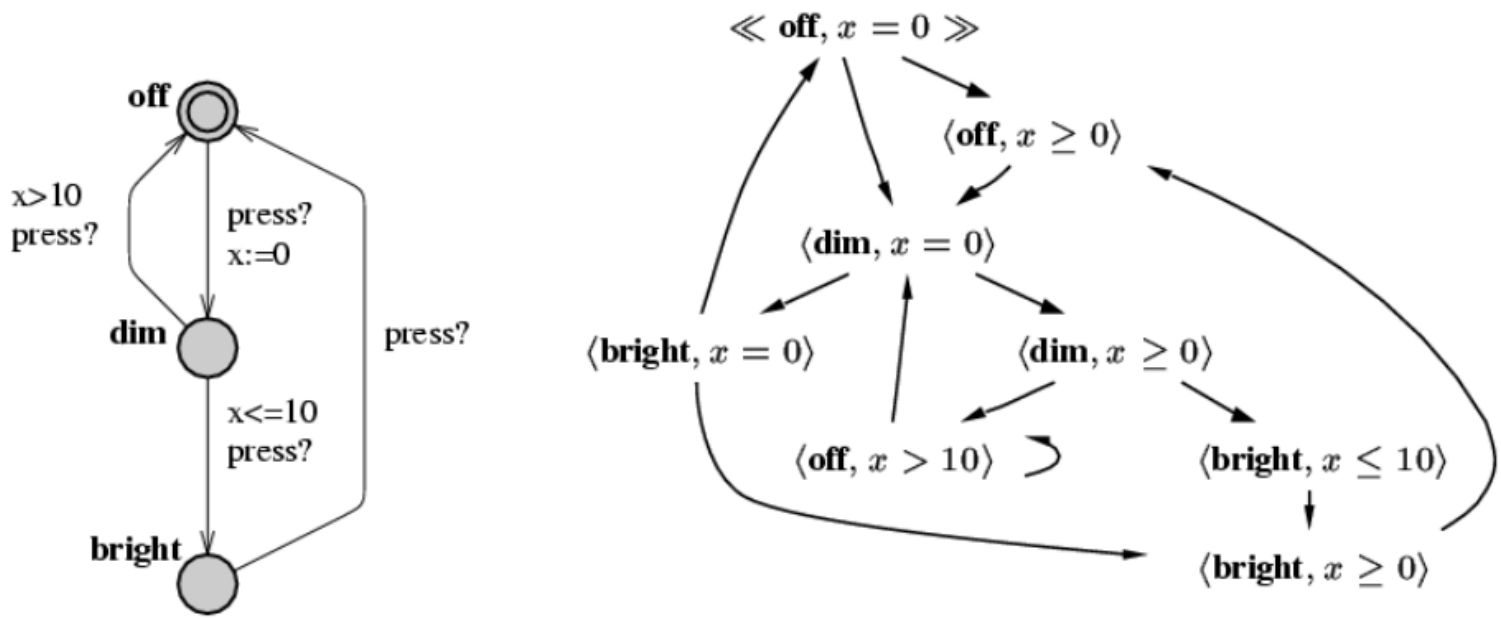
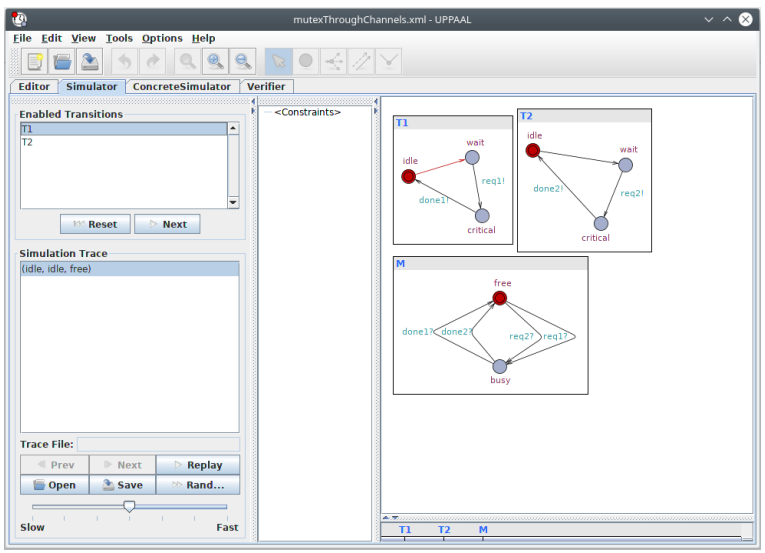


Image by Prof. Marco Di Natale, Scuola Superiore Sant'Anna, Pisa

Uppaal: a tool for modeling and analysis of NTA

- Jointly developed by the Department of Information Technology at Uppsala University and the Department of Computer Science at Aalborg University
  - Home page: `http://www.uppaal.org`
- Main features
  - **Modeling**: graphical editing of NTA (with templates, constants, ...)
  - **Simulation**: manual or random execution of runs of NTA
  - **Analysis**: verification of reachability, safety, and liveness properties



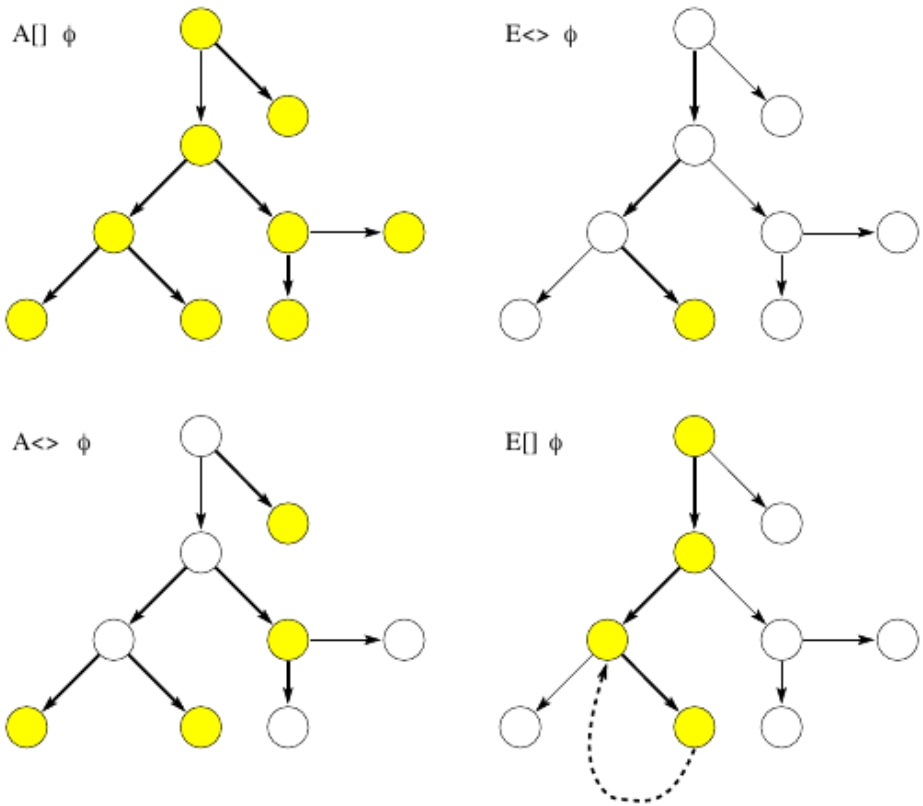
G. Behrmann, A. David, and K. G. Larsen, "A Tutorial on Uppaal 4.0," November, 2006.

Uppaal: verification of temporal logic formulas (1/2)

- The first temporal connective can be:
  - A: on all paths from the current state
  - E: on at least one path from the current state
- The second temporal connective can be:
  - X: the next state
  - G: all future states (denoted as [] in Uppaal)
  - F: some future state (denoted as <> in Uppaal)
  - U: until
- Examples (assuming the system is in some state s)
  - ϕ is true iff it is satisfied by s
  - AXϕ is true iff ϕ is true for every immediate successor state of s
  - AGϕ is true iff ϕ is true for every successor state of s
  - AFϕ is true iff on all paths originating from s there is a state where ϕ holds
  - AϕUθ is true iff all paths from s satisfy ϕ until they reach a state where θ holds
  - EXϕ is true iff ϕ is true for at least one immediate successor state of s
  - ...

Uppaal: verification of temporal logic formulas (2/2)

- Examples (assuming the system is in some state s)
  - `A[]`ϕ is true iff ϕ is true in all states of all paths from s
  - `E<>`ϕ is true iff ϕ is true in some state of at least one path from s
  - `A<>`ϕ is true iff ϕ is true in some state of all paths from s
  - `E[]`ϕ is true iff ϕ is true in all states of at least one path from s



Images by Prof. Marco Di Natale, Scuola Superiore Sant'Anna, Pisa

Credits

Credits

- Part of materials on Petri nets and time Petri nets presented in these slides is taken from the notes of the course "Methods for Verification and Testing" ("Metodi di verifica e testing") given by Prof. Enrico Vicario: `https://stlab.dinfo.unifi.it/vicario/Teaching/Verification_and_Testing_Metho/verification_and_testing_metho.html`
- Part of materials on timed automata presented in these slides is taken from:
  - the slides "An introduction to timed systems" by Patricia Bouyer: `http://www.lsv.fr/~bouyer`
  - the slides "Timed Automata" by Natalie Bertrand: `http://people.rennes.inria.fr/Nathalie.Bertrand`
  - the slides of the course "Embedded Systems - Model-Based Design" given by Prof. Marco Di Natale: `http://retis.sssup.it/~marco/teaching/embeddedsystems`