Application of Darcy's law in a 2D porous embankment

By

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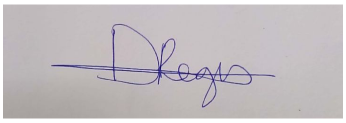
DECLARATION

This work was carried out at AIMS Rwanda in partial fulfilment of the requirements for a Master of Science Degree.

I hereby declare that except where due acknowledgement is made, this work has never been presented wholly or in part for the award of a degree at AIMS Rwanda or any other University.



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I especially don't forget my friends who gave me help and comfort.

DEDICATION

I dedicate this work to my family who have been my moral support.

Abstract

Given the population growth expected in the coming years and the rapid rate at which we are consuming water resources, the UN predicts that by 2025, 1.8 billion people will be living in countries or regions affected by total water scarcity and about 5 billion in regions where it will be difficult to meet all freshwater needs, the future remains uncertain. The objective of this work is to apply Darcy's law, which is the most fundamental law for flows in porous media, to a porous embankment in order to find out how much water will be able to pass through the embankment to be conserved and to avoid the risk of flooding. In the first part, we present a general overview to find out what work has already been done on a dam or in a porous medium. The second part consists of a general overview of porous media. In this part, Darcy's law defines as well as its parameters and the properties that characterize the porous medium. In the last part, we prove and calculate the flow of water in a horizontal or non-horizontal porous embankment. We explore the application to real-world situations.

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List of abbreviations

REV	Representative elementary volume
∇	Gradient operator ,$\nabla(.) = \frac{(.)}{\partial x}e_x + \frac{(.)}{\partial y}e_y + \frac{(.)}{\partial z}e_z$
ϕ	Total porosity
ϕ_{eff}	Effective porosity
θ	Water content
S	Saturation
K	Permeability
Q	Flow rate
V_T	Total volume
V_p	Void volume
V_s	Volume of the solid material
V_W	Volume of water
L	Length
μ	Dynamic viscosity
ν	Kinematic viscosity
ρ	Density
ψ	Potential
A	Constant cross-section
p	Pressure of water
U	Darcy velocity
v	Flow velocity
h	Piezometric height
g	Force of gravity

1. Introduction

1.1 Background

Flows in soils have been studied extensively for about fifty years to understand how a liquid flows in a porous medium. A porous medium is a solid that contains voids or pores. A typical example could be sandstone or the bones in our bodies. When the voids are interconnected, they permit the passage of gasses and fluids. Such motion through the medium is often called flow in porous media. A porous medium can be deformable or static, it can be made up of loose or cemented grains, rigid or elastic fibers, it can be uniform or diverse, and it can be ordered or disordered. Pores are the spaces that are not occupied by the solid constituents. Thus soil can be considered a porous medium because it is formed by solid aggregates (mineral particles, organic elements) between which there are voids spaces that can be filled with gases or liquids. The Figure 1.1 illustrates the intermediate size relative to the sizes of the flow domain and the pores

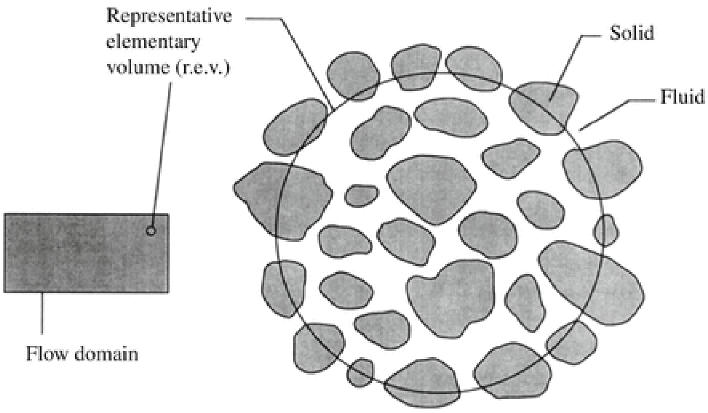

Figure 1.1: The representative elementary volume (r.e.v.)

There are numerous instances where porous media play a vital role or are required as a tool by technology.

- In Petroleum Engineering, porous medium (reservoir rock) stores crude oil and natural gas.
- In Chemical Engineering, a porous medium is applied as a filter or catalyst bed.
- In Soil Science, the porous medium (soil) contains and transports water and nutrients to plants.
- In Hydrology, the porous medium is a water-bearing and sealing layer.

These examples are omnipresent not only in nature (geological rocks), but also in many fields such as construction (concrete, pavements) or the pharmaceutical industry. In fields such as aeronautical and automotive engineering, composite materials are also widely used as porous media. Depending on the nature of the medium considered (composed of grains, fibers, etc.), the organization of the pore space can be very complex.

Darcy's law is the most fundamental law in groundwater science. It is one of the first mathematical relationships proposed for water flow through saturated porous media. It was proposed by Henry Darcy. Some of the important problems which are based on Darcy's law are:

1. Groundwater flow in saturated confined and unconfined aquifer.
2. Exploration of energy and resources.
3. Darcy's law is widely used for the basic quantification of any groundwater system.

Water is a very important and rare natural resource, without which no creature can exist. Although the earth's surface is 70% water, less than 3% of this is freshwater, of which 2.2% is contained in glaciers and groundwater. This leaves less than 1% of the earth's water to meet the needs of humans and species. It has therefore become everyone's responsibility to try to conserve this very precious resource. One of the ways of conservation is to build embankments or dams. However, if not properly managed, these embankments can eventually dry out due to groundwater seepage, resulting in embankment failure. Results from laboratory tests and in situ experiments show that embankments deform over time. In this thesis, we are interested in calculating the flow rate through a porous medium using Darcy's law.

1.2 Problem Statement

Among the oldest forms of civil engineering, structures are embankments. Embankments can be used as dikes for flood control along rivers, roads, railways, airport runways in the transport sector, and as check dams for reservoirs.The requirements on performance of embankments depend mainly on their purposes. In railway and road construction, settlement, particularly the differential settlement of embankments, is a major concern. In the field of hydraulic engineering and flood control, embankments are used to hold water back and for flood control respectively. Therefore their performance requirements depend mainly on their purpose. The stability of embankments must be ensured as embankment failure can have serious consequences. Depending on the types of construction materials used, embankments are classified into several categories. These include reinforced embankments, earthfill embankments, and rockfill embankments. To improve the performance of an embankment one can calculate the flow rate of water through it using Darcy's law.

1.3 Research Objectives

1.3.1 Main Objectives. The main objective of this research is to apply Darcy's law to a porous embankment.

1.3.2 Specific Objectives. Specifically, the study is guided by the following objectives.

- State Darcy's Law.

- Explain the different parameters of Darcy's law.
- Calculate the water flow rate through an embankment.

1.4 Overview

Chapter 2 reviews previous research papers related to this study. Chapter 3 presents a generalization on porous media. Chapter 4 is an application of Darcy's law to different rectangular shapes of embankment. Finally, the last chapter presents the main conclusions, results and perspectives arising from this work.

2. Literature Review

This chapter reviews various works carried out by other researchers related to the study of embankments and some studies carried out in porous media.

Guyon et al. (2015) in her book on Physical hydrodynamics, has shown that the flow in a porous embankment of Length L between two water tables at different levels y_0 and y_1 can be written as

$$Q = Kg\frac{y_0^2 - y_1^2}{2\nu L} \tag{2.0.1}$$

with, ν kinematic viscosity.

Lehbab (2019) in her article on earthen dams evaluated the flow rate knowing the flow network thanks to Darcy's law. She indicated that the total flow per unit width would be :

$$Q = \frac{K\rho g}{\mu}\frac{h}{bN_h}aN_c \tag{2.0.2}$$

With, N_h Number of drops in equipotentiality, N_c Number of flow channels , $\frac{h}{N_h}$pressure drop between two adjacent equipotential lines, K permeability , ρ density , μ, dynamic viscosity , a distance between two current lines and b distances between two equipotential lines.

As in general, flow networks are drawn with: $a = b$,then

$$Q = \frac{K\rho g}{\mu}h\frac{N_c}{N_h} \tag{2.0.3}$$

This calculation was made per unit Length.For a dam of Length L, the total leakage rate is :

$$Q = \frac{K\rho g}{\mu}h\frac{N_c}{N_h}L \tag{2.0.4}$$

Other methods taking into account the angle of the slope with the horizontal and also based on Darcy's law are applied for the calculation of the resurgence flow.

$$Q = \frac{K\rho g}{\mu}b\sin^2\theta, \quad \text{for} \quad \theta < \frac{\pi}{6} \tag{2.0.5}$$

$$Q = \frac{K\rho g}{\mu}y_0, \quad \text{for} \quad \frac{\pi}{6} \leq \theta \leq \frac{\pi}{2} \tag{2.0.6}$$

with,y_0 is the ordinate of the exit point of the free surface.

Narita (2000) identified several important issues associated with the design and construction of embankment dams, which engineers often encounter on dam projects, and discussions are held on these issues by presenting recent developments in design procedures and construction technologies. It showed that most catastrophic failures of embankment dams are caused by the overflow of water from the reservoir due to flooding or loss of the freeboard. Other main factors to cause embankment failures are hydraulic erosion, high pore-water pressure, earthquake forces, and so forth. More than 50 percent of embankment failures are due to hydraulic erosion.Hall et al. (2012) also reviewed all common types of embankment dams, construction materials, construction control, maintenance, failures, and causes of failure, analysis and design, and future trends.

Berrabah (2015) evaluated the effect of geosynthetic reinforcement on an settlement of the embankment over a localized weak zone. The influence of some parameters, namely the compressibility parameter of the localized weak zone, the axial stiffness of the geosynthetic reinforcement, the geometry of the localized weak zone and the friction angle of the embankment material are also analysed. These results show that a large deformation calculation is more appropriate than a small deformation calculation for this problem and the improvement of the differential settlement is due to the combination of the membrane resistance effect of the geosynthetic and the arch effect in the embankment material. It also showed that the reinforcement has no significant influence on the absolute settlement of the embankment. Furthermore, the use of geosynthetics in the construction of this embankment showed very relevant effects in increasing the bearing capacity of the supporting soil necessary for the placement of the first layers of the embankment and in improving the quality of compaction of the embankment layers.

Al Bitar (2007) in his work on the modeling of flows in hydro-systems comprising of geologically complex and heterogeneous soils and aquifers considered, for example, the case of a coastal aquifer subject to saline intrusion, with density coupling (freshwater/saltwater), a phenomenon to which other couplings can be added (variable saturation flows, surface / underground couplings) To carry out this work, he chose an approach with two characteristics: the model is spatially distributed to represent the heterogeneity of the environment and the model is strongly coupled to understand the flows in their physical complexity. To this end, it uses a highly integrated model with a single generic PDE equation,based on a generalized Darcy's law that allows the description of different flow regimes co-existing in the same domain. The model is based on a generalized Darcy's law that allows the description of different flow "regimes" co-existing in the same domain while maintaining robustness and efficiency.

The objective of Hussein (2002) was to develop efficient and reliable numerical tools for groundwater management and prediction of pollutant transport in porous media. The studies focus on two fundamental aspects of solving groundwater flow and pollutant transport equations. In the first part, he studied the behaviour of mixed finite element methods under the influence of spatial discretization of the domain, heterogeneity of the medium and abrupt boundary conditions. The second part of the thesis was devoted to the solution of the convection equation using a discontinuous finite element method. In order to stabilise this method, he developed new slope limiters for unstructured meshes. The last part of this work consisted of using the numerical tools developed to simulate the evolution of a radioactive waste repository. The studies focus on the migration of multi-species radionuclides along their path from the storage containers through artificial barriers and the geological environment to the biosphere.

El Dine (2017) is interested in Brinkman's modification, of Darcy's law,where are adds a velocity dissipation term. the modification which consists of modifying Darcy's law by adding a velocity dissipation term. This system leads to an elliptical equation in pressure and a non-standard parabolic equation in saturation, regularising in time. To do this, he first studied the mathematics of the Darcy-Brinkman system and regularity of solutions. To numerically simulate the solutions to this problem, he first proposed a convergence study for a finite volume scheme on an admissible mesh for homogeneous porous media. Then, a combined finite volume-non-conforming finite element method to take into account the anisotropy of the medium. The second part of his thesis aims to take into account the compressibility of fluids. He proposed to describe the Darcy-Brinkman model in the compressible single-phase case. He showed that this model is well-posed everwhere in one dimension. Then, under Bear (1972) assumption, it is also shown that the model is well-posed in dimension $d \geq 2$.

3. General background on flows in porous media

3.1 Basic Concepts

The study of flow in porous media is based on the determination of media properties such as granulometry, porosity, permeability and other media quantities. These appear in Darcy's law. In this part, we briefly describe some fundamental notions about porous media.

3.1.1 Representative elementary volume. The difficulty in determining the size of the volume to be connected with the material point gave rise to the concept of a representative elementary volume. It allows the microscopic properties of the porous medium to be treated with macroscopic quantities that provide average properties (Bear, 1972). A representative elementary volume (REV) is a volume for which the average characteristic properties (such as porosity, and permeability in the case of a porous medium) can be deduced. In reality, a porous medium is made up of solid and empty seeds for which it is only possible to attribute concepts such as porosity and permeability on a scale several orders of magnitude larger than the pore scale (Al Bitar, 2007).

The choice of the REV should therefore meet the following criteria (Marsily, 1981):

• The REV must contain a large number of pores so that an overall average property can be defined.

• The REV must be small enough so that the variations of properties from one domain to another can be approximated by continuous functions to introduce infinitesimal analysis, without introducing errors detectable by measuring instruments at the macroscopic scale.

According to the above criteria, a REV depends not only on the structure of the porous medium but also on the physical phenomena being studied. A REV must be large enough to represent the structure of the porous medium, but also small enough so that the variations in properties, sometimes non-linear (water content), Can be treated as continuous. Such a definition applied to hydro-geology is certainly subjective because heterogeneity exists at all scales of a natural porous medium, and several modeling assumptions exist for each problem.

3.1.2 Granulometry. The simplest property of a porous medium is provided by particle size analysis or granulometry. It is the measure of the size distribution in a collection of grains.

3.1.3 Porosity. The porosity of porous media is defined as the ratio of the volume of the pores to the total bulk volume of the media (usually expressed as fraction or percent). Let us select any point of the porous media and its environment with a sufficiently large volume V_T, where

$$V_T = V_p + V_s \tag{3.1.1}$$

where V_p is the void volume (pore volume) and V_s is the volume of the solid material.

Porosity is defined as the ratio of pore volume to total volume, which can be expressed as:

$$\phi = \frac{V_p}{V_T} \tag{3.1.2}$$

According to Heinemann (2005),One must distinguish between two kinds of porosities:

- Total porosity (isolated pores are considered also) and
- Effective porosity ϕ_{eff} (effective in the sense of fluid transport)

3.1.4 Water content . The Water content is the ratio of the volume of water V_W contained in the pores (or voids) of the soil, to the total volume V_T of the soil considered.It is expressed by the relation

$$\theta = \frac{V_W}{V_T} \tag{3.1.3}$$

The water content is a quantity strictly between 0 and 1 and is denoted by θ

3.1.5 Saturation. The Saturation is defined as the ratio of the volume of water present in the given soil mass to the volume of voids present in it. It is expressed by the relation

$$S = \frac{\theta}{\phi} \tag{3.1.4}$$

Saturation can vary from 0 (dry soil) 1 to (saturated soil). The water content is related to the saturation by the relation :

$$\theta = S\phi \tag{3.1.5}$$

3.1.6 Permeability. The Permeability or the constant of proportionality $K[L^2]$ is a property of the porous medium that measures the capacity and ability of the formation to transmit fluids.. This characteristic constant of a porous medium is linked to the shape of the grains and the porosity. It is therefore independent of the characteristics of the fluid and depends only on the structure and connectivity of the pores.

Medium	Permeability (**Darcy**)
Soil	0.3-15
Brick	0.005-0.2
Limestone	0.002-0.5
Sandstone	0.0005-5
Gravel	10-1000
Cigarette	1000
Sand	20-200
Silica Powder fibres	0.01-0.05

Table 3.1: A few typical values of the permeability for porous media.

Table 3.1 shows the values of permeability for different materials in Darcy (1 Darcy $\approx 1\mu m^2$).

3.2 Darcy's law

Darcy's law is an equation describing the fluid flow through a solid material with holes and pores filled with fluid. In 1840s engineer Henry Darcy, by working on the fountains system in Dijon, investigated the flow of water in a saturated homogeneous sand filter (Darcy, 1856). He deduced that the total flow Q through the column of length L can be calculated :

$$Q = \frac{K}{\mu} A \frac{\Delta p}{L} \tag{3.2.1}$$

where K is the permeability, A constant cross-section, Δp pressure gradient oriented along the length, L is the length of the sand column, μ Dynamic Viscosity of the fluid.

Dividing both sides of (3.2.1) by A, we obtain the filtration velocity $U = \frac{Q}{A}$, called the Darcy velocity. The law is then written as

$$U = \frac{K}{\mu} \frac{\Delta p}{L} \tag{3.2.2}$$

Defining the pressure gradient $\frac{\Delta p}{L} =: -\nabla p$, the law can be written as

$$U = -\frac{K}{\mu} \nabla p \tag{3.2.3}$$

Note that (3.2.3) is only valid in the case of horizontal flow.

U is the filtration velocity obtained as an average over the REV, and p is fluid pressure at the pore scale. The symbol K represents the medium permeability, and μ is the dynamic viscosity of the fluid.

3.2.1 Inhomogeneous Media. The pore space dimensions may vary from one location to another. If an average pore dimension can be identified at the scale of the REV, one can define a permeability function $K(x)$ and the corresponding form of Darcy's law:

$$U = -\frac{K(x)}{\mu} \nabla p \tag{3.2.4}$$

3.2.2 Anisotropic Media. Darcy's law (3.2.3) can be expressed in index notation as

$$U_i = -\frac{K}{\mu} \frac{\partial p}{\partial x_i} \tag{3.2.5}$$

Permeability (K) is a scalar function that is a constant. Equation (3.2.5) also admits tensor forms of permeability wherein permeability is a Cartesian second-order tensor, namely

$$U_i = -\frac{K_{ij}}{\mu} \frac{\partial p}{\partial x_j} \tag{3.2.6}$$

In a Cartesian coordinate system, Equation (3.2.6) extends as

$$\begin{cases} u = \frac{1}{\mu}\left(-k_{11}\frac{\partial p}{\partial x} - k_{12}\frac{\partial p}{\partial y} - k_{13}\frac{\partial p}{\partial z}\right) \\ v = \frac{1}{\mu}\left(-k_{21}\frac{\partial p}{\partial x} - k_{22}\frac{\partial p}{\partial y} - k_{23}\frac{\partial p}{\partial z}\right) \\ w = \frac{1}{\mu}\left(-k_{31}\frac{\partial p}{\partial x} - k_{32}\frac{\partial p}{\partial y} - k_{33}\frac{\partial p}{\partial z}\right) \end{cases} \tag{3.2.7}$$

K_{ij} is a permeability tensor and has nine components, with $i, j = 1, 2, 3$.

In the case of a three-dimensional, the hydraulic conductivity coefficient is defined by a symmetric tensor of the form

$$\begin{pmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{pmatrix} \tag{3.2.8}$$

The off-diagonal components of permeability in an orthotropic medium are zero, while the principal components are nonzero and distinct. Here :

$$K_{ij} = \begin{pmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & 0 \\ 0 & 0 & k_{33} \end{pmatrix} \tag{3.2.9}$$

In homogeneous media, the permeability tensor is symmetric, and the nine components reduce to six.

In isotropic homogeneous media, the nine components of the permeability tensor reduce to one, making permeability a scalar quantity. This means

$$K = K_{11} = K_{22} = K_{33} \tag{3.2.10}$$

For anisotropic media, one can identify principal directions with respect to which the off-diagonal components of permeability are zero. The Cartesian coordinate system can then be aligned with the principal directions of the porous medium being studied.

3.2.3 Effect of gravity. Gravity forces become important in large-scale reservoirs and affect the flow field gravity-related corrections will appear in the momentum equation, namely Darcy's law becomes

$$U = -\frac{K}{\mu}\left(\nabla p - \rho g\right) \tag{3.2.11}$$

If K and μ are constant, equation (3.2.11) becomes

$$U = -\nabla \psi \tag{3.2.12}$$

$$\text{where:} \quad \psi = \frac{K}{\mu}\left(p + \rho g z\right) \tag{3.2.13}$$

with ψ is the potential.

Therefore

$$U = -\frac{K}{\mu}\nabla\left(p + \rho g z\right) \tag{3.2.14}$$

Here z represents for the vertical direction opposed to the direction of gravity. For an incompressible fluid,the equation (3.2.14) becomes

$$U = -\frac{K\rho g}{\mu}\nabla\left(\frac{p}{\rho g} + z\right) \tag{3.2.15}$$

3.2.4 Piezometric height. Bernoulli's equation for incompressible media is given by

$$h = \frac{p}{\rho g} + z + \frac{v^2}{2g} \tag{3.2.16}$$

Where $\frac{p}{\rho g}$ is the pression head, $\frac{v^2}{2g}$ is the velovity head and z is the elevation head or potential.

Due to the low flow velocities in porous media (see Figure 3.1), the term $\frac{v^2}{2g}$ which represents the kinetic energy is neglected. Therefore the piezometric height or piezometric or potentiometric head h results from the action of the fluid pressure p and of the local height z

$$h = \frac{p}{\rho g} + z \tag{3.2.17}$$

The hydraulic head can be used to determine a hydraulic gradient between two or more points (see Figure 3.1).

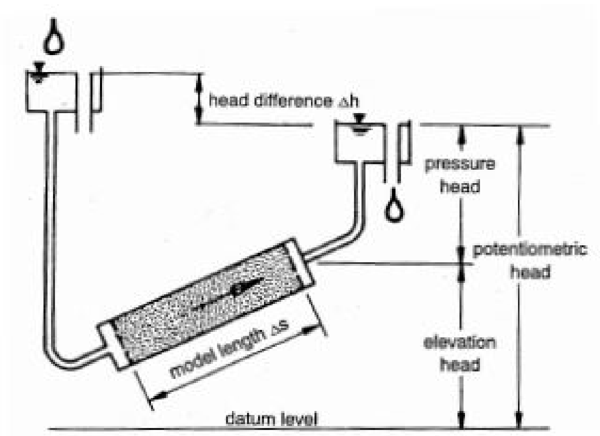

Figure 3.1: Darcy's experiment

Taking into account equation (3.2.17) then the equation (3.2.15) can also be written as follows

$$U = -\frac{K\rho g}{\mu}\nabla h \tag{3.2.18}$$

3.3 Two-phase flow

Two-phase flows are more complex than single-phase flows because of the abruptly varying dynamics of the interfaces (deformation, rupture, coalescence) and the evolution of these interfaces on a wide range of scales from a few microns to a few meters.

Principle of classification of two-phase flows.

Several authors, including Aloui (1994) recall that two-phase flows can be classified according to the phases present (liquid, solid, gas or plasma (ionised gas)).

- Mixing of two immiscible liquids (saline intrusion into the freshwater table)
- Solid-liquid mixing (sludge transport)
- Solid gas mixture (pneumatic transport)
- Mixing liquid gas (boiling in nuclear reactors)
- Solid plasma mixing (nanoparticle synthesis)

Two-phase flow is a flow in which two different aggregate states of a substance or of two different substances are simultaneously present. Two-phase flows can also be classified according to the spatial distribution of the interfaces (Figure 3.2).

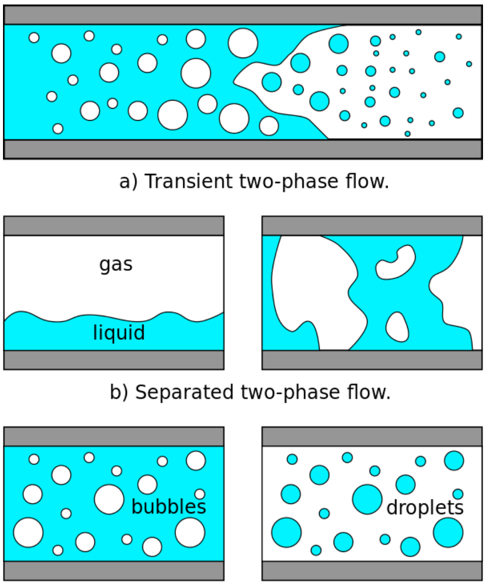

Figure 3.2: Different modes of two-phase flows

The following flows are distinguished:

- Transcient two-phase flow

- Separated two-phase flow
- Dispersed two-phase flow

An interface is a singular surface that separates two fluid media that are characterized by different phases.

In our study, we work essentially with two liquid-gas phases: a water phase (saturated zone) and an air phase (non-saturated zone) separated by a mobile interface.

3.4 Embankments

Based on the types of construction materials used, embankments are classified into several categories. These include rockfill embankments, earthfill embankments and reinforced embankments Hall et al. (2012).

3.4.1 Rockfill embankment. When we use rock up to 300 mm in embankment as a fill material, it is called rock fill embankment. The voids which left in embankment should be property filled with binding material preferably may be soil or rock dust.

Figure 3.3: Rockfill embankment,Noumea

3.4.2 Reinforced embankment. Introduced in the 1960s, the technique of reinforced embankments is now increasingly used as a retaining wall solution. In reinforced embankments, horizontal layers of reinforcement are installed within a fill material to create a reinforced soil mass that will resist the earth pressure forces developing behind it. Reinforcements can be made of steel (bars, nets, ladders etc ...) or geosynthetic (geogrids, woven geotextiles or polyester strips).

Figure 3.4: Reinforced Embankment, Taiwan Pavilion Expo Park, Hsinchu

3.4.3 Eartfill embankment. An earthen embankment is a raised confining structure made from compacted soil to confine runoff either for surface storage or for ground water recharge. These are also used for increasing infiltration; detention and retention of water to facilitate deep percolation and also to provide additional storage as in the case of semi dug-out ponds. When constructed across natural channel to induce channel storage, the embankment is also called an earthen dam.

Figure 3.5: Eartfill Embankment,The Aswan High Dam, Aswān, Egypt,

4. Applying Darcy's Law to the flow in a porous embankment

4.1 Horizontal flow porous embankment

The figures used in this section were made with GeoGebra available at $https://www.geogebra.org/$.

4.1.1 Flow between two water tables at different levels $(y = y_0, y_1)$. Let us analyze the flow through a porous embankment of permeability K and length L which separates two water volumes at different levels y_0 and y_1 (see Figure 4.1). One assumes a stationary regime in which the water level $y_s(x)$ within the medium is constant in time, with a flow rate Q per unit distance along z constant.

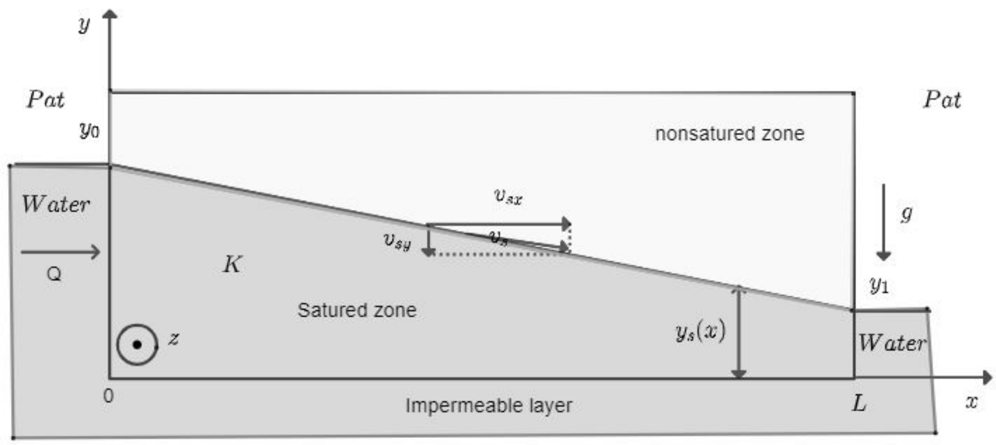

Figure 4.1: Flow between two water tables at different levels $(y = y_0, y_1)$ through a porous embankment bounded by the planes $x = 0, x = L$.

Guyon et al. (2015) have shown that the flow rate is

$$Q = Kg\frac{y_0^2 - y_1^2}{2L\nu} \tag{4.1.1}$$

We will prove using Guyon's Approch.

Proof of 4.1.1 The part of the porous medium which is not saturated by water is filled with air; if one neglects the effects of the surface tension, the water pressure just below the surface $y_s(x)$ is therefore equal to the atmospheric pressure patm. Let us now write the equality of the pressure between two points (flows at low Reynolds number) on the surface located at x and x $+$ δx. One obtains p(x, $y_s(x)$) $=$ p(x $+$ δx, y_s(x $+$ δx)) $\iff$ p(x $+$ δx, y_s(x $+$ δx)) - p(x, $y_s(x)$) $=$0.

By applying Taylor's formula, we get

$$p(x, y_s(x)) + \frac{\partial p}{\partial x}\delta x + \frac{\partial p}{\partial y}\delta y_s - p(x, y_s(x)) \approx 0 \tag{4.1.2}$$

whence, by expanding we get

$$\frac{\partial p}{\partial x}\delta x + \frac{\partial p}{\partial y}\delta y_s \approx 0 \tag{4.1.3}$$

If $\frac{\partial y_s}{\partial x}$ is sufficiently small, one can consider that the component v_{sy} of the surface velocity v_s to be much smaller than the component v_{sx} and that the pressure gradient $\frac{\partial p}{\partial y}$ in the y-direction reduces to the hydrostatic pressure gradient $-\rho g$ ($\frac{\partial p}{\partial y} = -\rho g$). One then obtains, from (4.1.3), the horizontal pressure-gradient:

$$\frac{\partial p}{\partial x} = -\frac{\partial p}{\partial y}\frac{dy_s}{dx} = \rho g \frac{dy_s}{dx} \tag{4.1.4}$$

By applying Darcy's law (3.2.3)(because the flow is horizontal), the horizontal component $v_{sx}(x)$ of the flow velocity is

$$v_{sx}(x) = -\frac{K}{\mu}\frac{\partial p}{\partial x} \tag{4.1.5}$$

The flow rate is

$$Q = v_{sx}(x) y_s(x) \tag{4.1.6}$$

From (4.1.4) and (4.1.5) , Q becomes :

$$Q = -\rho g \frac{K}{\mu} y_s(x) \frac{dy_s(x)}{dx} \tag{4.1.7}$$

We have $\nu = \frac{\mu}{\rho} \longrightarrow \frac{1}{\nu} = \frac{\rho}{\mu}$ and $y_s(x)dy_s(x) = \frac{1}{2}dy_s^2(x)$, (equation 4.1.7) becomes

$$Q = -\frac{K}{2\nu} g \frac{dy_s^2(x)}{dx} \tag{4.1.8}$$

by integrating (4.1.8) respectively between 0 and L, we get

$$Q = Kg\frac{y_o^2 - y_1^2}{2L\nu} \tag{4.1.9}$$

4.1.2 Flow between three water tables at different levels $(y = y_0, y_1, y_2)$**.** Let us analyze the flow through a porous embankment separated into 2 zones with permeabilities K_1 , K_2 and lengths L_1, $L_2 - L_1$ which separates three water volumes at different levels y_0, y_1 and y_2 (see Figure 4.2).

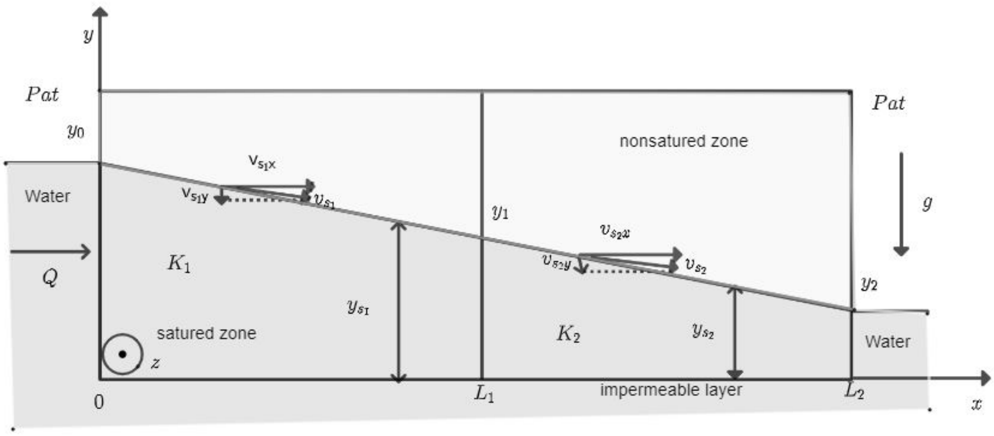

Figure 4.2: Flow between three water tables at different levels $(y = y_0, y_1, y_2)$ through a porous embankment bounded by the planes $x = 0, x = L_1, x = L_2$.

Here the flow rate is

$$Q = K_1 g \frac{y_0^2 - y_1^2}{2\nu L_1} + K_2 g \frac{y_1^2 - y_2^2}{2\nu(L_2 - L_1)} \tag{4.1.10}$$

Proof of 4.1.9 Consider a porous embankment of permeability K_1 and length L_1 which separates two water volumes at different levels y_0 and y_1.

According to (4.1.8), the flow rate Q_1 is

$$Q_1 = -\frac{K_1}{2\nu} g \frac{dy_{s_1}^2(x)}{dx} \tag{4.1.11}$$

and by integrating both sides of (4.1.11) between 0 and L_1, we get

$$Q_1 = K_1 g \frac{y_o^2 - y_1^2}{2\nu L_1} \tag{4.1.12}$$

Consider the flow between two water tables at different levels $(y = y_1, y = y_2)$ through a porous embankment bounded by the planes $x = L_1, x = L_2$ with a flow rate Q_2.

According to (equation 4.1.8), the flow rate Q_2 is

$$Q_2 = -\frac{K_2}{2\nu} g \frac{dy_{s_2}^2(x)}{dx} \tag{4.1.13}$$

and by integrating both sides of (4.1.13) respectively between L_1 and L_2, we get

$$Q_2 = K_2 g \frac{y_1^2 - y_2^2}{2\nu(L_2 - L_1)} \tag{4.1.14}$$

Hence, the flow rate Q is

$$Q = Q_1 + Q_2 \tag{4.1.15}$$

Therefore,

$$Q = K_1 g \frac{y_0^2 - y_1^2}{2\nu L_1} + K_2 g \frac{y_1^2 - y_2^2}{2\nu (L_2 - L_1)} \tag{4.1.16}$$

4.1.3 Flow between n water tables at different levels $(y = y_0, y_1 \cdot y_n)$**.** Let us analyze the flow through a porous embankment separated into n zones with permeabilities K_1 , K_2 ... K_n and lengths L_1, $L_2 - L_1$ ... $K_n - K_{n-1}$ which separates n water volumes at different levels $(y = y_0, y_1 \cdot y_n)$ (see Figure 4.3).

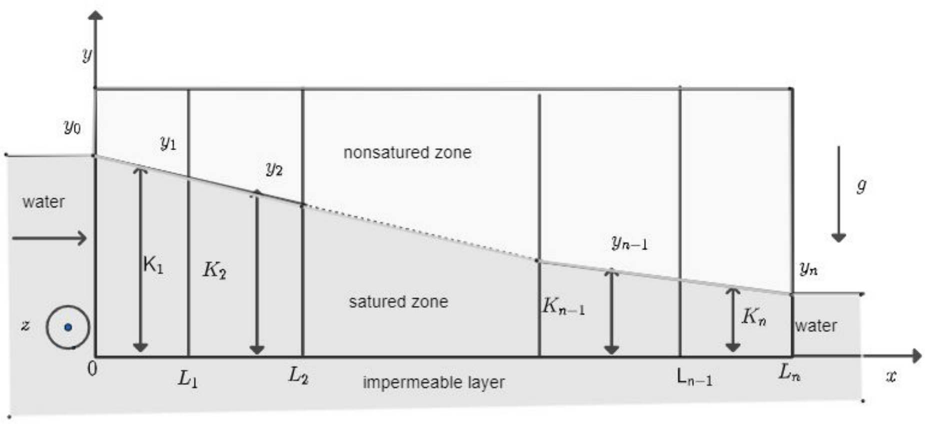

Figure 4.3: Flow between n water tables at different levels $(y = y_0, y_1, y_2, \ldots y_n)$ through a porous embankment bounded by the planes $x = 0, x = L_1, x = L_2 \ldots x = L_n$.

Here, the flow rate is

$$Q = \frac{g}{2\nu} \left(\sum_{i=1}^{n} K_i \frac{y_{i-1}^2 - y_i^2}{L_i - L_{i-1}} \right) \quad \text{with } L_0 = 0 \tag{4.1.17}$$

Proof of 4.1.16 Consider a porous embankment of permeability K_1 and length L_1 which separates two water volumes at different levels y_0 and y_1.

According to (4.1.8), the flow rate Q_1 is

$$Q_1 = -\frac{K_1}{2\nu} g \frac{dy_{s_1}^2(x)}{dx} \tag{4.1.18}$$

and by integrating both sides of (4.1.18) between 0 and L_1, we get

$$Q_1 = K_1 g \frac{y_o^2 - y_1^2}{2\nu L_1} \tag{4.1.19}$$

Consider the flow between two water tables at different levels $(y = y_1, y = y_2)$ through a porous embankment bounded by the planes $x = L_1, x = L_2$ with a flow rate Q_2.

According to (4.1.8), the flow rate Q_2 is

$$Q_2 = -\frac{K_2}{2\nu} g \frac{dy_{s_2}^2(x)}{dx} \tag{4.1.20}$$

and by integrating both sides of (4.1.20) between L_1 and L_2, we get

$$Q_2 = K_2 g \frac{y_1^2 - y_2^2}{2\nu(L_2 - L_1)} \tag{4.1.21}$$

Consider the flow between two water tables at different levels $(y = y_{n-1}, y = y_n)$ through a porous embankment bounded by the planes $x = L_{n-1}, x = L_n$ with a flow rate Q_n.

According to (4.1.8), the flow rate Q_n is

$$Q_n = -\frac{K_n}{2\nu} g \frac{dy_{s_n}^2(x)}{dx} \tag{4.1.22}$$

and by integrating both sides of (4.1.22) between L_{n-1} and L_n, we get

$$Q_n = K_n g \frac{y_{n-1}^2 - y_n^2}{2\nu(L_n - L_{n-1})} \tag{4.1.23}$$

Hence, the flow rate Q is

$$Q = Q_1 + Q_2 + \cdots + Q_n \tag{4.1.24}$$

Therefore,

$$Q = \frac{g}{2\nu} \left(\sum_{i=1}^{n} K_i \frac{y_{i-1}^2 - y_i^2}{L_i - L_{i-1}} \right) \quad \text{with } L_0 = 0 \tag{4.1.25}$$

4.2 Non-horizontal flow to a porous embankment

Now, we consider θ such that $\theta = 0, \theta = \frac{\pi}{2}$ and $0 < \theta < \frac{\pi}{2}$. Note that for $\theta = 0$, we find the horizontal case.

4.2.1 Flow between two water tables at different levels $(y = y_0, y_1)$**.** Let us analyze the flow through a porous embankment of permeability K and length L which separates two water volumes at different levels y_0 and y_1 (see Figure 4.4). One assumes a stationary regime in which the water level $y_s(x)$ within the medium is constant in time, with a flow rate Q per unit distance along z constant.

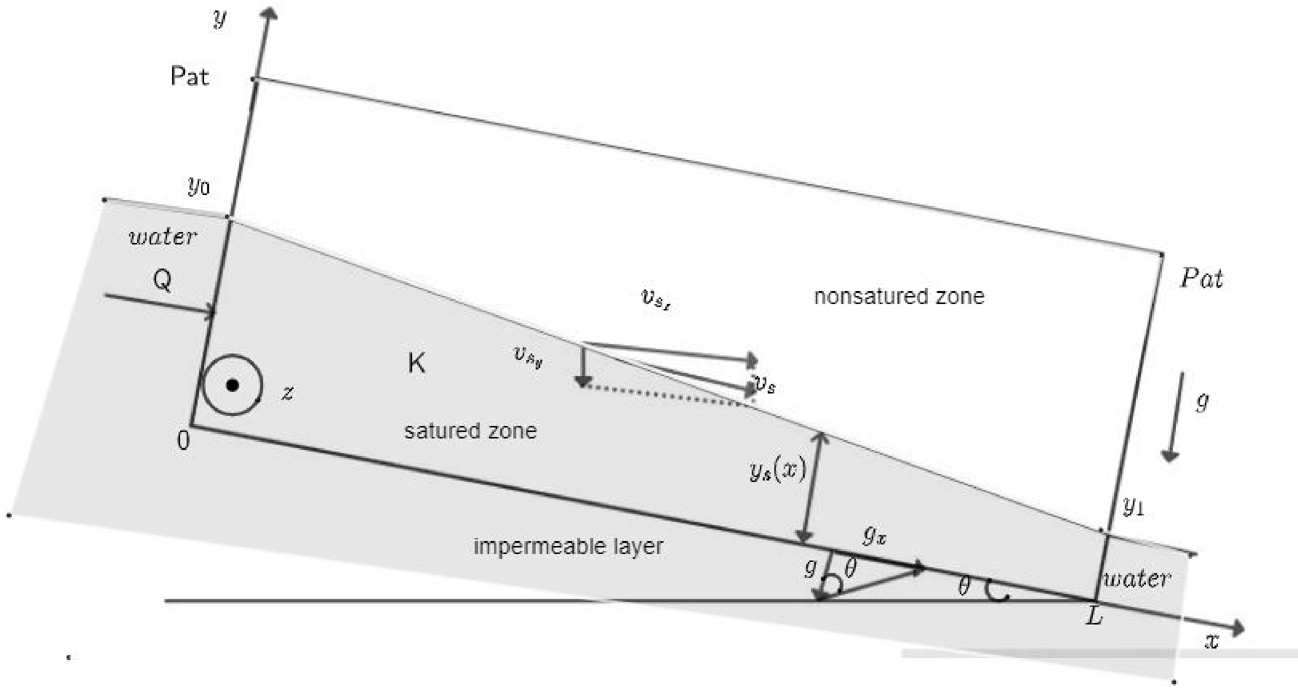

Figure 4.4: Flow between two water tables at different levels $(y = y_0, y_1)$ through a porous embankment bounded by the planes $x = 0, x = L$.

Here, the flow rate is

$$Q = Kg\frac{y_0^2 - y_1^2}{2\nu L} + \frac{Kg}{\nu}y_s(x)\sin\theta, \quad \text{for} \quad 0 < \theta < \frac{\pi}{2} \tag{4.2.1}$$

For $\theta = 0$, the flow rate is

$$Q = Kg\frac{y_0^2 - y_1^2}{2\nu L} \tag{4.2.2}$$

For $\theta = \frac{\pi}{2}$, the flow rate is

$$Q = Kg\frac{y_0^2 - y_1^2}{2\nu L} + \frac{Kg}{\nu}y_s(x) \tag{4.2.3}$$

Proof of 4.2.1 By applying Darcy's law (3.2.11)(because the flow is non-horizontal), the horizontal component $v_{sx}(x)$ of the flow velocity is

$$v_{sx}(x) = -\frac{K}{\mu}\left(\frac{\partial p}{\partial x} - \rho g_x\right) \tag{4.2.4}$$

The flow rate is

$$Q = v_{sx}(x)y_s(x) \tag{4.2.5}$$

From (4.1.4) and (4.2.4) , Q becomes

$$Q = -\frac{K}{\mu}\left(\rho g\frac{dy_s}{dx} - \rho g_x\right)y_s(x) \tag{4.2.6}$$

With $\sin\theta = \frac{g_x}{g} \implies g_x = g\sin\theta$

Hence, (4.2.6) becomes,

$$Q = -\frac{K}{\nu}gy_s(x)\frac{dy_s(x)}{dx} + \frac{K}{\nu}gy_s(x)\sin\theta \tag{4.2.7}$$

(4.2.7) can also be written as :

$$Q = -\frac{K}{2\nu}g\frac{dy_s^2}{dx} + \frac{K}{\nu}gy_s(x)\sin\theta \tag{4.2.8}$$

and by integrating both sides of (4.2.8) between 0 and L_1, we get

$$Q = Kg\frac{y_0 - y_1}{2\nu L} + \frac{K}{\nu}gy_s(x)\sin\theta \quad \text{for} \quad 0 < \theta < \frac{\pi}{2} \tag{4.2.9}$$

For $\theta = 0$, the flow rate (4.2.9) becomes

$$Q = Kg\frac{y_0^2 - y_1^2}{2\nu L} \tag{4.2.10}$$

For $\theta = \frac{\pi}{2}$, the flow rate(4.2.9) becomes

$$Q = Kg\frac{y_0^2 - y_1^2}{2\nu L} + \frac{Kg}{\nu}y_s(x) \tag{4.2.11}$$

4.2.2 Flow between three water tables at different levels $(y = y_0, y_1, y_2)$. Let us analyze the flow through a porous embankment separated into 2 zones with permeabilities K_1 , K_2 and lengths L_1, $L_2 - L_1$ which separates three water volumes at different levels y_0, y_1 and y_2 (see Figure 4.5).

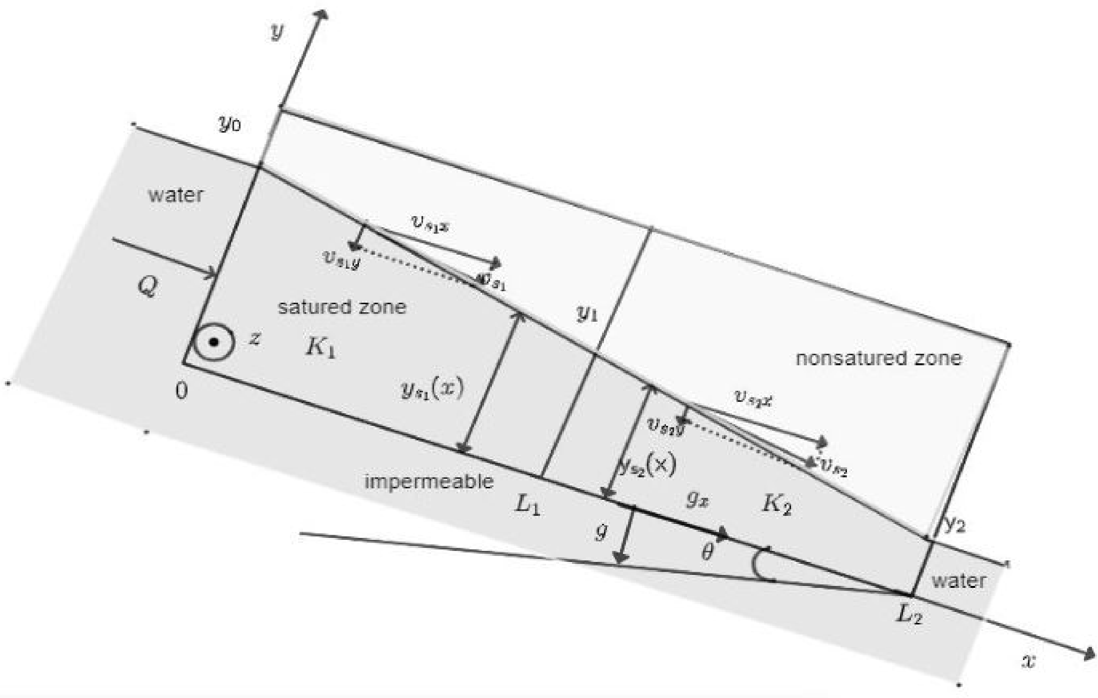

Figure 4.5: Flow between three water tables at different levels $(y = y_0, y_1, y_2)$ through a porous embankment bounded by the planes $x = 0, x = L_1, x = L_2$.

Here, the flow rate is

$$Q = K_1 g \frac{y_0^2 - y_1^2}{2\nu L_1} + K_2 g \frac{y_1^2 - y_2^2}{2\nu(L_2 - L_1)} + \frac{g \sin\theta}{\nu} \left(K_1 y_{s_1}(x) + K_2 y_{s_2}(x) \right) \quad 0 < \theta < \frac{\pi}{2} \qquad (4.2.12)$$

For $\theta = 0$, the flow rate is

$$Q = K_1 g \frac{y_0^2 - y_1^2}{2\nu L_1} + K_2 g \frac{y_1^2 - y_2^2}{2\nu(L_2 - L_1)} \qquad (4.2.13)$$

For $\theta = \frac{\pi}{2}$, the flow rate is

$$Q = K_1 g \frac{y_0^2 - y_1^2}{2\nu L_1} + K_2 g \frac{y_1^2 - y_2^2}{2\nu(L_2 - L_1)} + \frac{g}{\nu} \left(K_1 y_{s_1}(x) + K_2 y_{s_2}(x) \right) \qquad (4.2.14)$$

Proof of 4.2.12 Consider a porous embankment of permeability K_1 and length L_1 which separates two water volumes at different levels y_0 and y_1.

According to (4.2.8), the flow rate Q_1 is

$$Q_1 = -\frac{K_1}{2\nu} g \frac{dy_{s_1}^2}{dx} + \frac{K_1}{\nu} g y_{s_1}(x) \sin\theta \tag{4.2.15}$$

and by integrating both sides of (4.2.15) between 0 and L_1, we get

$$Q_1 = K_1 g \frac{y_0^2 - y_1^2}{2\nu L} + \frac{K_1}{\nu} g y_{s_1}(x) \sin\theta \tag{4.2.16}$$

Consider the flow between two water tables at different levels $(y = y_1, y = y_2)$ through a porous embankment bounded by the planes $x = L1, x = L2$ with a flow rate Q_2.

According to (4.2.8), the flow rate Q_2 is

$$Q_2 = -\frac{K_2}{2\nu} g \frac{dy_{s_2}^2}{dx} + \frac{K_2}{\nu} g y_{s_2}(x) \sin\theta \tag{4.2.17}$$

and by integrating both sides of (4.2.17) between L_1 and L_2, we get

$$Q_2 = K_2 g \frac{y_1^2 - y_2^2}{2\nu(L_2 - L_1)} + \frac{K_2}{\nu} g y_{s_2}(x) \sin\theta \tag{4.2.18}$$

Hence, the flow rate Q is

$$Q = Q_1 + Q_2 \tag{4.2.19}$$

Therefore,

$$Q = K_1 g \frac{y_0^2 - y_1^2}{2\nu L_1} + K_2 g \frac{y_1^2 - y_2^2}{2\nu(L_2 - L_1)} + \frac{g \sin\theta}{\nu} \left(K_1 y_{s_1}(x) + K_2 y_{s_2}(x)\right) \quad 0 < \theta < \frac{\pi}{2} \tag{4.2.20}$$

For $\theta = 0$, the flow rate is

$$Q = K_1 g \frac{y_0^2 - y_1^2}{2\nu L_1} + K_2 g \frac{y_1^2 - y_2^2}{2\nu(L_2 - L_1)} \tag{4.2.21}$$

For $\theta = \frac{\pi}{2}$, the flow rate is

$$Q = K_1 g \frac{y_0^2 - y_1^2}{2\nu L_1} + K_2 g \frac{y_1^2 - y_2^2}{2\nu(L_2 - L_1)} + \frac{g}{\nu} \left(K_1 y_{s_1}(x) + K_2 y_{s_2}(x)\right) \tag{4.2.22}$$

4.2.3 Flow between n water tables at different levels $(y = y_0, y_1 \cdot y_n)$**.** Let us analyze the flow through a porous embankment separated into n zones with permeabilities K_1 , K_2 and lengths L_1, $L_2 - L_1$ and $L_n - L_{n-1}$ which separates n water volumes at different levels $(y = y_0, y_1 \cdot y_n)$ (see Figure 4.6).

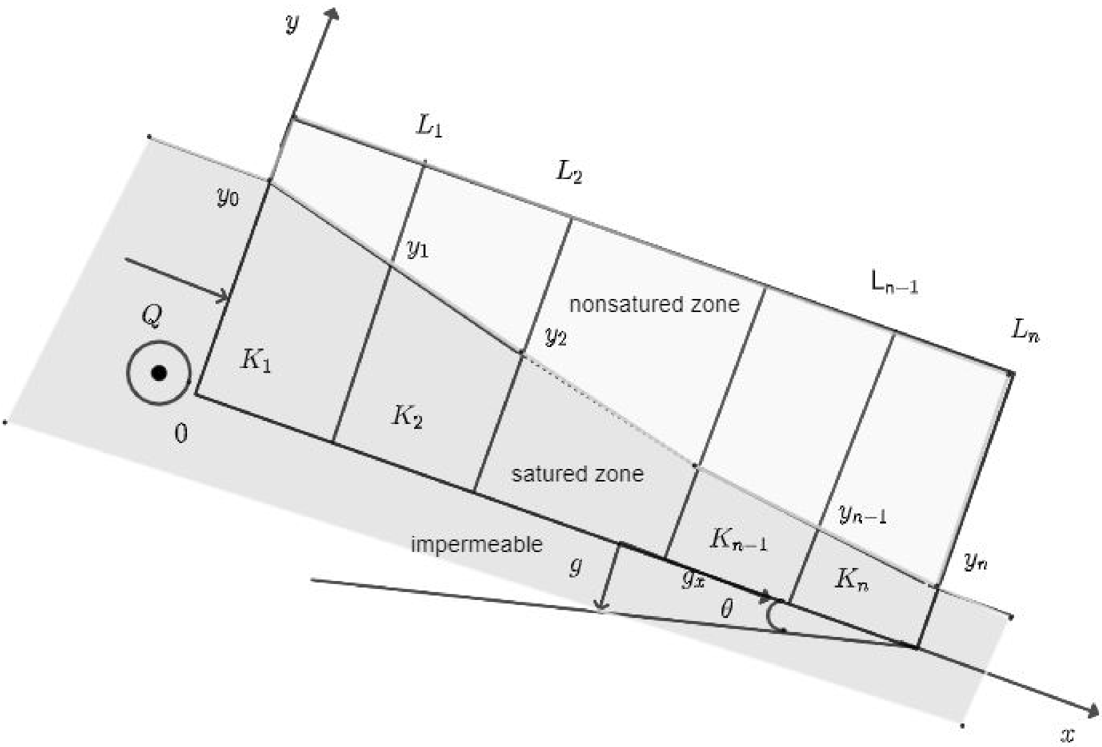

Figure 4.6: Flow between n water tables at different levels $(y = y_0, y_1, y_2, \ldots y_n)$ through a porous embankment bounded by the planes $x = 0, x = L_1, x = L_2 \ldots x = L_n$.

Here, the flow rate is

$$Q = \frac{g}{2\nu}\left(\sum_{i=1}^{n} K_i \frac{y_{i-1}^2 - y_i^2}{L_i - L_{i-1}}\right) + \frac{g\sin\theta}{\nu}\left(\sum_{i=1}^{n} K_i y_{s_i}(x)\right) \quad \text{with } L_0 = 0 \quad \text{and} \quad 0 < \theta < \frac{\pi}{2} \tag{4.2.23}$$

For $\theta = 0$, the flow rate is

$$Q = \frac{g}{2\nu}\left(\sum_{i=1}^{n} K_i \frac{y_{i-1}^2 - y_i^2}{L_i - L_{i-1}}\right) \quad \text{with} \quad L_0 = 0 \tag{4.2.24}$$

For $\theta = \frac{\pi}{2}$, the flow rate is

$$Q = \frac{g}{2\nu}\left(\sum_{i=1}^{n} K_i \frac{y_{i-1}^2 - y_i^2}{L_i - L_{i-1}}\right) + \frac{g}{\nu}\left(\sum_{i=1}^{n} K_i \quad y_{s_i}(x)\right) \quad \text{with } L_0 = 0 \tag{4.2.25}$$

Proof of 4.2.23 Consider a porous embankment of permeability K_1 and length L_1 which separates two water volumes at different levels y_0 and y_1.

According to (4.2.8), the flow rate Q_1 is

$$Q_1 = -\frac{K_1}{2\nu}g\frac{dy_{s_1}^2}{dx} + \frac{K_1}{\nu}gy_{s_1}(x)\sin\theta \tag{4.2.26}$$

and by integrating both sides of (4.2.26) between 0 and L_1, we get

$$Q_1 = K_1 g\frac{y_0^2 - y_1^2}{2\nu L} + \frac{K_1}{\nu}gy_{s_1}(x)\sin\theta \tag{4.2.27}$$

Consider the flow between two water tables at different levels $(y = y_1, y = y_2)$ through a porous embankment bounded by the planes $x = L1, x = L2$ with a flow rate Q_2.

According to (4.2.8), the flow rate Q_2 is

$$Q_2 = -\frac{K_2}{2\nu}g\frac{dy_{s_2}^2}{dx} + \frac{K_2}{\nu}gy_{s_2}(x)\sin\theta \tag{4.2.28}$$

and by integrating both sides of (4.2.28) between L_1 and L_2, we get

$$Q_2 = K_2 g\frac{y_1^2 - y_2^2}{2\nu(L_2 - L_1)} + \frac{K_2}{\nu}gy_{s_2}(x)\sin\theta \tag{4.2.29}$$

Consider the flow between two water tables at different levels $(y = y_{n-1}, y = y_n)$ through a porous embankment bounded by the planes $x = L_{n-1}, x = L_n$ with a flow rate Q_n.

According to (4.2.8), the flow rate Q_n is

$$Q_n = -\frac{K_n}{2\nu}g\frac{dy_{s_n}^n}{dx} + \frac{K_n}{\nu}gy_{s_n}(x)\sin\theta \tag{4.2.30}$$

and by integrating both sides of (4.2.30) between L_1 and L_2, we get

$$Q_n = K_n g\frac{y_{n-1}^2 - y_n^2}{2\nu(L_n - L_{n-1})} + \frac{K_n}{\nu}gy_{s_n}(x)\sin\theta \tag{4.2.31}$$

Hence, the flow rate Q is

$$Q = Q_1 + Q_2 + \cdots + Q_n \tag{4.2.32}$$

Therefore,

$$Q = \frac{g}{2\nu}\left(\sum_{i=1}^{n} K_i \frac{y_{i-1}^2 - y_i^2}{L_i - L_{i-1}}\right) + \frac{g\sin\theta}{\nu}\left(\sum_{i=1}^{n} K_i y_{s_i}(x)\right) \quad \text{with } L_0 = 0 \quad \text{and} \quad 0 < \theta < \frac{\pi}{2} \tag{4.2.33}$$

For $\theta = 0$, the flow rate is

$$Q = \frac{g}{2\nu}\left(\sum_{i=1}^{n} K_i \frac{y_{i-1}^2 - y_i^2}{L_i - L_{i-1}}\right) \quad \text{with} \quad L_0 = 0 \tag{4.2.34}$$

For $\theta = \frac{\pi}{2}$, the flow rate is

$$Q = \frac{g}{2\nu}\left(\sum_{i=1}^{n} K_i \frac{y_{i-1}^2 - y_i^2}{L_i - L_{i-1}}\right) + \frac{g}{\nu}\left(\sum_{i=1}^{n} K_i \quad y_{s_i}(x)\right) \quad \text{with } L_0 = 0 \tag{4.2.35}$$

The calculation method shown below is the general formula for flow rate in a porous embankment.

4.3 Application

4.3.1 Method for calculating water flow rate in a porous embankment. The application of Darcy's law in a real porous embankment requires knowledge of

- The permeabilities of the materials used.
- The medium, i.e. isotropic or anisotropic.
- The length(s) (depending on the different water levels considered) of the embankment.
- The different water levels considered.
- The kinematic viscosity of the water at a given temperature.
- The force of gravity.
- The angle, if it is a non-horizontal embankment.

Note that $\theta \in [0, \frac{\pi}{2}]$.

The length (L), the permeability (K), and the water level (y) are the quantities to be measured while the kinematic viscosity (ν) is a quantity that can be calculated.

4.3.2 Example: Water Entering an Embankment. Consider a porous earthfill embankment consisting of a homogeneous mass of isotropic permeability K_1 and K_2, resting on an impermeable horizontal bedrock (Figure 4.7). In the case of a flow network represented by the figure below, calculate the flow rate of water passing through the embankment.

Assume $y_0 = 26m$, $y_1 = 22m$, $y_2 = 18m$, $K_1 = 34 \times 10^{-12} m^2$, $K_2 = 60 \times 10^{-12} m^2$, $g = 9.81 m.s^{-2}$,$L_1 = 32m$, and $L_2 - L_1 = 28m$ and The kinematic viscosity of water at $20°$ is 10^{-6} $m^2.s^{-1}$.

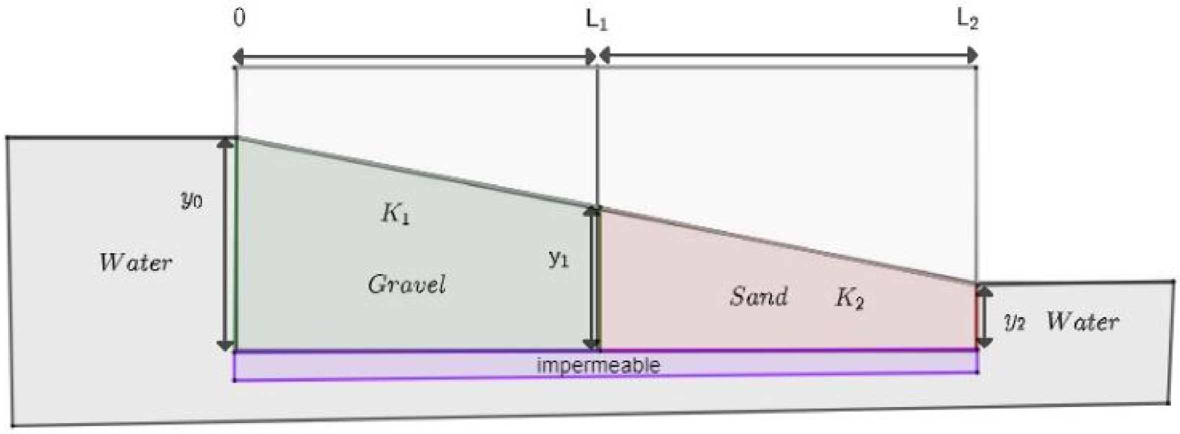

Figure 4.7: Horizontal Eartfill embankment resting on impermeable bedrock.

Here the flow rate is

$$Q = K_1 g \frac{y_0^2 - y_1^2}{2\nu L_1} + K_2 g \frac{y_1^2 - y_2^2}{2\nu (L_2 - L_1)} \tag{4.3.1}$$

Simplifying, we obtain

$$Q = 34 \times 10^{-12} \times 9.81 \frac{26^2 - 22^2}{2 \times 10^{-6} \times 32} + 60 \times 10^{-12} \times 9.81 \frac{22^2 - 18^2}{2 \times 10^{-6} \times 28}$$

Therefore

$$Q = 2.68 \times 10^{-3} m^2.s^{-1} \tag{4.3.2}$$

5. Conclusion

In this work, we investigated the application of Darcy's law to the study of a porous embankment. This study involves the notions of two-phase flow, porous medium, and interface because we consider that water (saturated zone) and air (non-saturated zone) are immiscible phases. We proceeded with a detailed and evolutionary study to solve our problem.

In the first chapter with a literature review and the second chapter defines and explains the different parameters such as porosity, granulometry, saturation, water content, hydraulic conductivity tensor, hydraulic head, piezometric height, and different types of embankment and Darcy's and Bernoulli's laws. In Chapter 4 we derived the water flow rate in a rectangular embankment for $\theta = 0$,$\theta = \frac{\pi}{2}$ and $0 < \theta < \frac{\pi}{2}$, which led to some interesting results.

This work aimed to apply Darcy's law to a porous embankment in the case of horizontal and non-horizontal flow. Darcy's Law is the primary equation that governs flow in porous embankment. Without it, we would not be able to know the flow rate of water through an embankment.

To build on this, one could be continued by implementing a fairly robust numerical scheme. Several methods can be used, such as finite element analysis, to discretize and simulate the results obtained in Chapter 4.

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