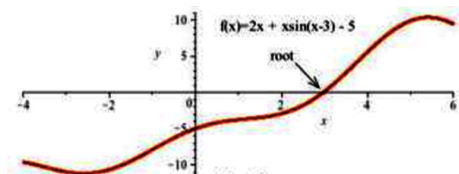


Computational Methods

Tutorial II

December 31, 2024

1. State the Intermediate Value Theorem
2. Consider the bisection method
 - a) State the two conditions required to apply the bisection root finding algorithm.
 - b) Write down the pseudo-code of the bisection root finding algorithm.
 - c) Use the bisection method to find the root of $f(x) = 2x^3 - 2x - 5$
3. State the Newton's root finding formula.
4. Use the Newton's root finding method to approximate $f(x) = x^7 - 1000$ correct to eight decimal places.
5. The figure below shows the curve of a function $f(x) = 2x + x \sin(x+3) - 5$. Use the Newton's root finding formula to find x_3 . **Hint:** Take $x_0 = 3$.



6. Suppose x_0, x_1, \dots, x_n are $n+1$ equispaced points.
 - b) State the, forward difference operator, backward difference operator, and central difference operator.
 - a) Show that $\Delta^2 y_3 = y_5 - 2y_4 + y_3$.
7.
 - a) State the Newton's forward difference interpolation formula.
 - b) Using the result in (a), show that the Newton's interpolation polynomial for the points $\{(0, 7), (10, 18), (20, 32), (30, 51), (40, 87)\}$ is given by

$$P_n(x) = 0.0000416x^4 - 0.0022x^3 + 0.05x^2 + 1.26x + 7$$

8. a) Explain the principle of Numerical differentiation.
- b) State the formulas for $\frac{df}{dx}$ and $\frac{d^2f}{dx^2}$ based on the Newton's forward interpolation.
- c) Find the value of $\frac{df}{dx}$ and $\frac{d^2f}{dx^2}$ at $x = 1.5$ using the data given in the table.

x	1.5	2.0	2.5	3.0	3.5	4.0
y	3.375	7.000	13.625	24.000	38.875	59.000

9. Suppose the function $y_i = f(x_i)$ is known at $(n+1)$ points $x_0, x_1, x_2, \dots, x_n$, $i = 0, 1, 2, \dots, n$ are known. Let $x_i = x_0 + ih$, $i = 0, 1, 2, \dots, n$ and $p = \frac{x - x_n}{h}$

- a) State the Newton's backward difference formula.
- b) Using (a), write down the formulas for approximating the first, second and third derivative at any point $x = x_n + ph$.
- c) A particle is moving along a straight line. The displacement x at some time increases t are given below:

t	0	1	2	3	4
x	5	8	12	17	26

Using the information in the table, find the velocity and acceleration of the particle $t = 4$. **Hint:** first compute backward difference table.

10. Consider the interval $[a, b]$. Say we wish to do seven function evaluations, $f_k = f(x_k)$ for $k = 0, 1, \dots, 4, 5, 6$. This implies six sub-intervals. $[x_0, x_1], [x_1, x_2], [x_2, x_3], [x_3, x_4], [x_4, x_5]$ and $[x_5, x_6]$.
- a) State the composite Trapezoidal rule and the composite Simpson's rule for the six sub-intervals.
- b) Consider the integration of $f(x) = 1 + e^{-x} \sin(4x)$ over $[0, 2]$. Use exactly seven function evaluations and compare the result from the composite trapezoidal rule, composite Simpson's rule.
11. Find the least-squares line $y = f(x) = mx + b$ for the data and calculate $E_2(f)$; $(-4, -3), (-1, -1), (0, 0), (2, 1), (3, 2)$
12. Define the following as used in linear algebra
- a) A matrix.
- b) A system of linear equations.
- c) A linear combination.

13. Let A be a matrix. What do the following notations stand for
- a) a_{ij} .
 - b) $|A|$
14. Let $A = [a_{ij}]_{m \times n}$ for $1 \leq i \leq 5$ and $1 \leq j \leq 5$.
- a) Write A in expanded form.
 - b) Define the minor of a_{ij}
 - c) Using (a) write an expression for $|A|$ for $i = 3$. **Hint:** apply cofactor.
15. Solve the following linear system using Gaussian elimination with partial pivoting.

$$x_1 + 20x_2 - x_3 + 0.001x_4 = 0$$

$$2x_1 - 5x_2 + 30x_3 - 0.1x_4 = 1$$

$$5x_1 + x_2 - 100x_3 - 10x_4 = 0$$

$$2x_1 - 100x_2 - x_3 + x_4 = 0$$