Introduction to Operations Management
运营管理概论

5- Inventory Management

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Inventory Management

The objective of inventory management is to strike a balance between inventory investment and customer service

- One of the most expensive assets of many companies representing up to as much as 50% of total invested capital
- Operations managers must **balance** inventory investment and customer service

Functions of Inventory

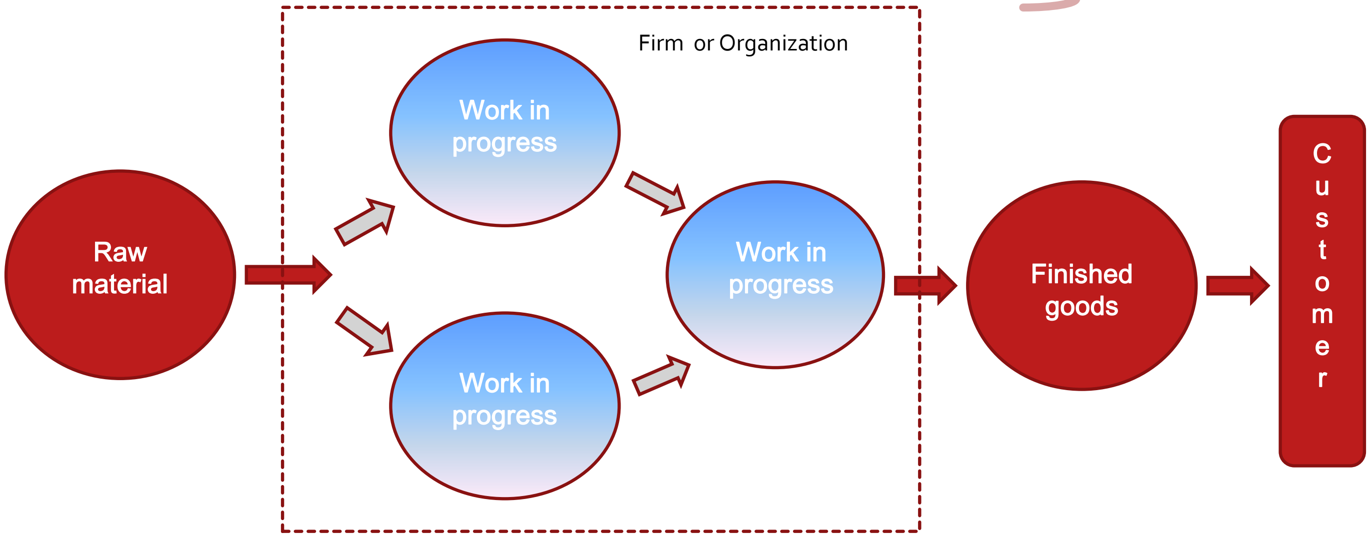
1. To **decouple** or separate various parts of the production process
2. To **decouple** the firm from fluctuations in demand and provide a stock of goods that will provide a selection for customers
3. To take advantage of quantity **discounts**
4. To **hedge** against inflation

Types of Inventory

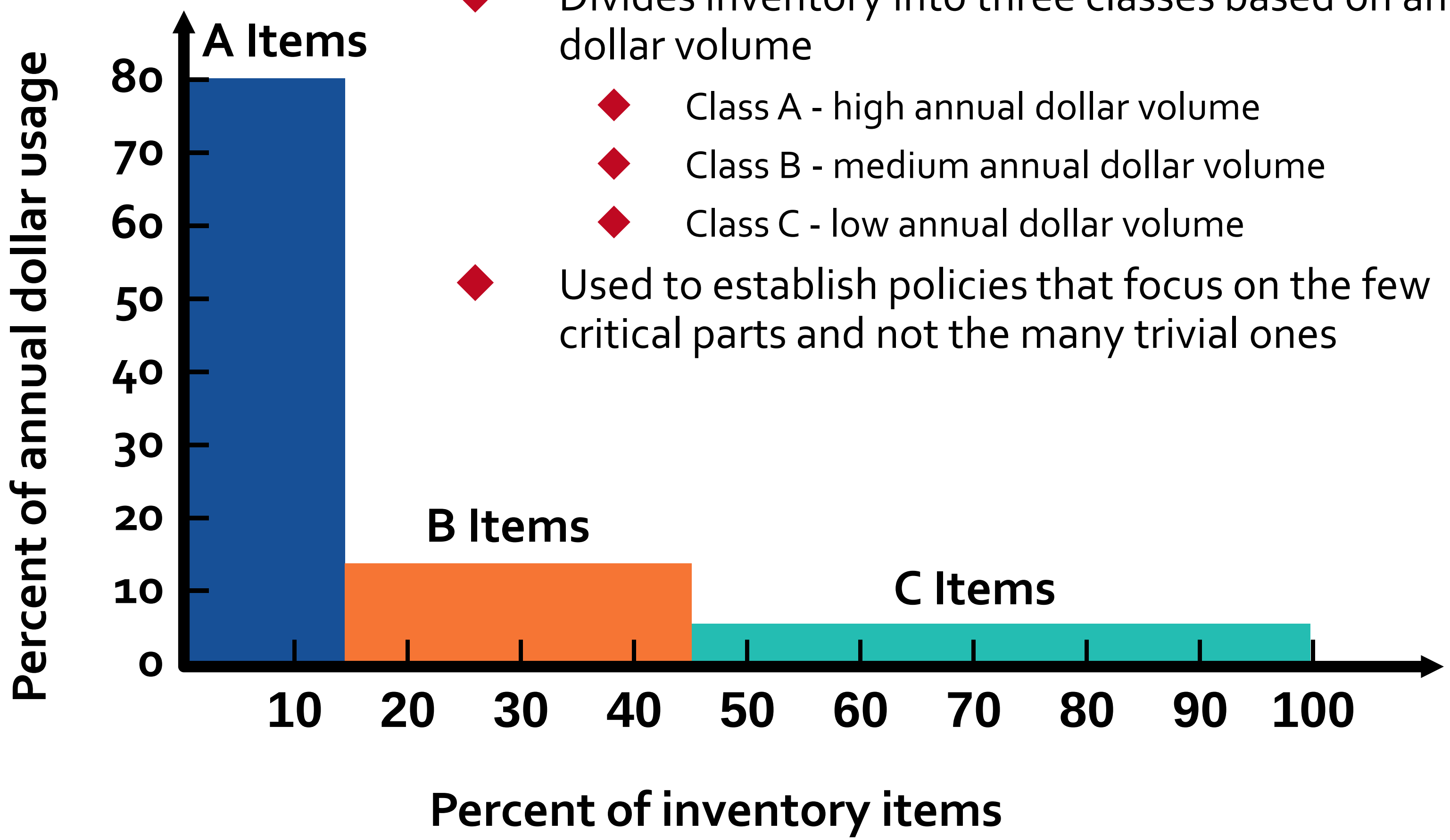
- **Raw material**
  - Purchased but not processed
- **Work-in-process**
  - Undergone some change but not completed
  - A function of cycle time for a product
- **Maintenance/repair/operating (MRO)**
  - Necessary to keep machinery and processes productive
- **Finished goods**
  - Completed product awaiting shipment

How to classify?

How to keep accurate records?



ABC Analysis



- Divides inventory into three classes based on annual dollar volume
  - Class A - high annual dollar volume
  - Class B - medium annual dollar volume
  - Class C - low annual dollar volume
- Used to establish policies that focus on the few critical parts and not the many trivial ones

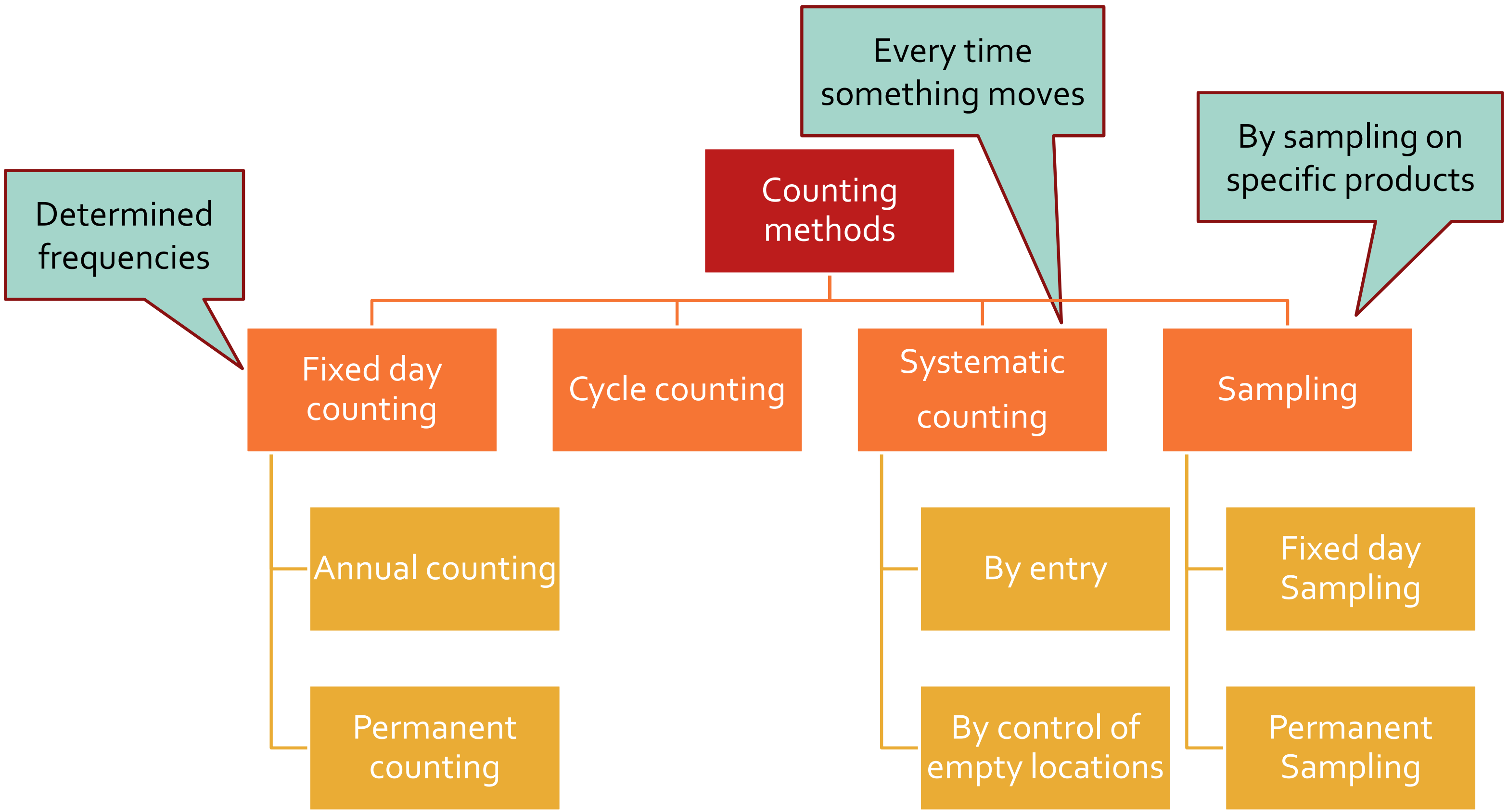
ABC Analysis

- Other criteria than annual dollar volume may be used
  - Anticipated engineering changes
  - Delivery problems
  - Quality problems
  - High unit cost
- Policies employed may include
  - More emphasis on **supplier development** for A items
  - Tighter physical **inventory control** for A items
  - More care in **forecasting** A items

Record Accuracy

- Accurate records are a critical ingredient in production and inventory systems
- Allows organization to focus on what is needed
- Necessary to make precise decisions about ordering, scheduling, and shipping
- Incoming and outgoing record keeping must be accurate
- Stockrooms should be secure

Don’t start complex calculations before you are sure your CURRRENT inventory is right !



Cycle Counting

- Items are counted and records updated on a periodic basis
- Often used with ABC analysis to determine cycle
- Has several advantages

1. Eliminates shutdowns and interruptions
2. Eliminates annual inventory adjustment
3. Trained personnel audit inventory accuracy
4. Allows causes of errors to be identified and corrected
5. Maintains accurate inventory records

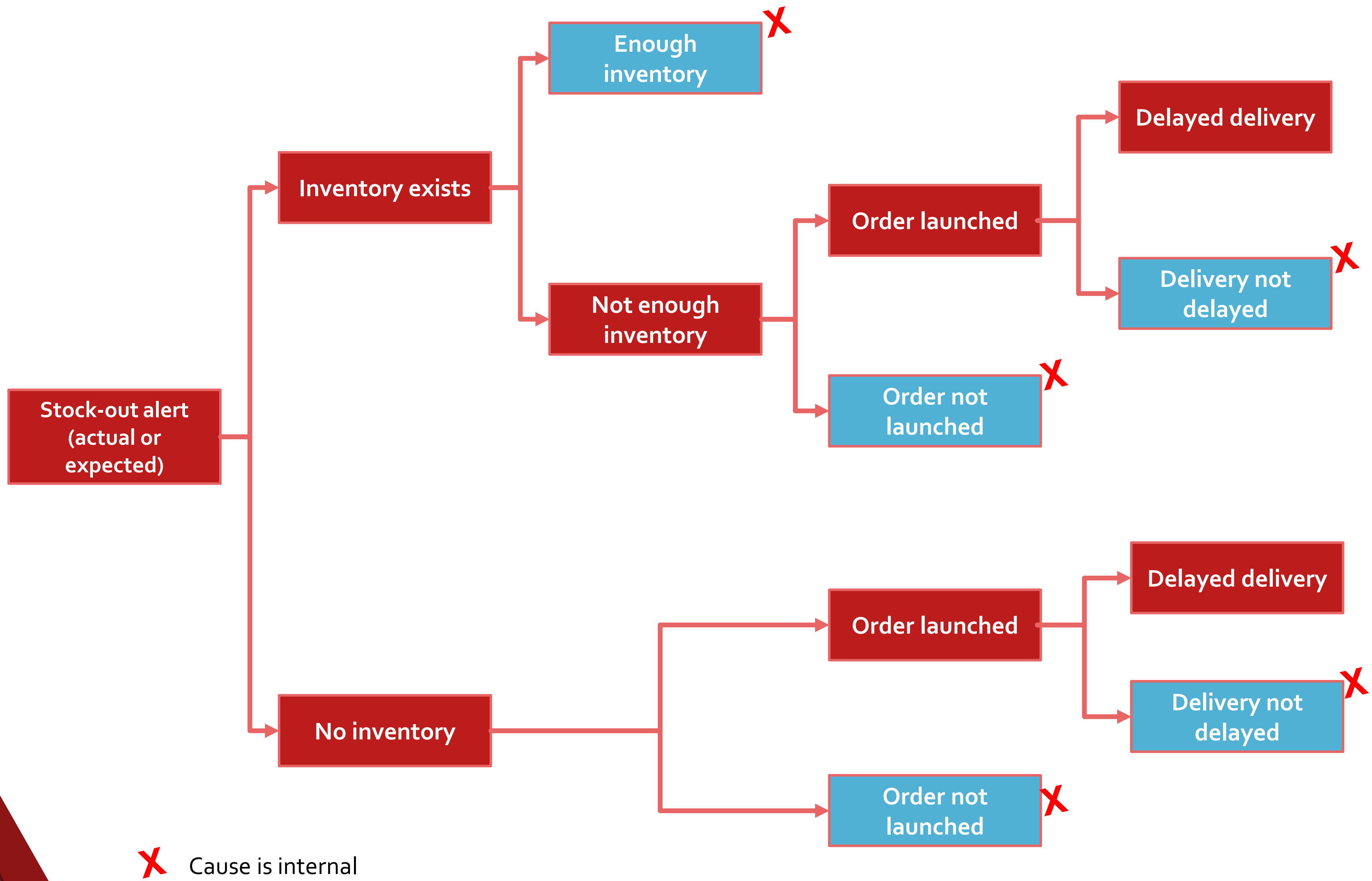
Cycle Counting Example

5,000 items in inventory, 500 A items, 1,750 B items, 2,750 C items

Policy is to count A items every month (20 working days), B items every quarter (60 days), and C items every six months (120 days)

Item Class	Quantity	Cycle Counting Policy	Number of Items Counted per Day
A	500	Each month	500/20 = 25/day
B	1,750	Each quarter	1,750/60 = 29/day
C	2,750	Every 6 months	2,750/120 = 23/day
			77/day

When stockout occurs or is expected, you must find WHY



Independent Versus Dependent Demand

- Independent demand - the demand for item is independent of the demand for any other item in inventory
- Dependent demand - the demand for item is dependent upon the demand for some other item in the inventory

Holding, Ordering, and Setup Costs

- Holding costs - the costs of holding or "carrying" inventory over time
- Ordering costs - the costs of placing an order and receiving goods
- Setup costs - cost to prepare a machine or process for manufacturing an order

Inventory Models for Independent Demand

Need to determine when and how much to order

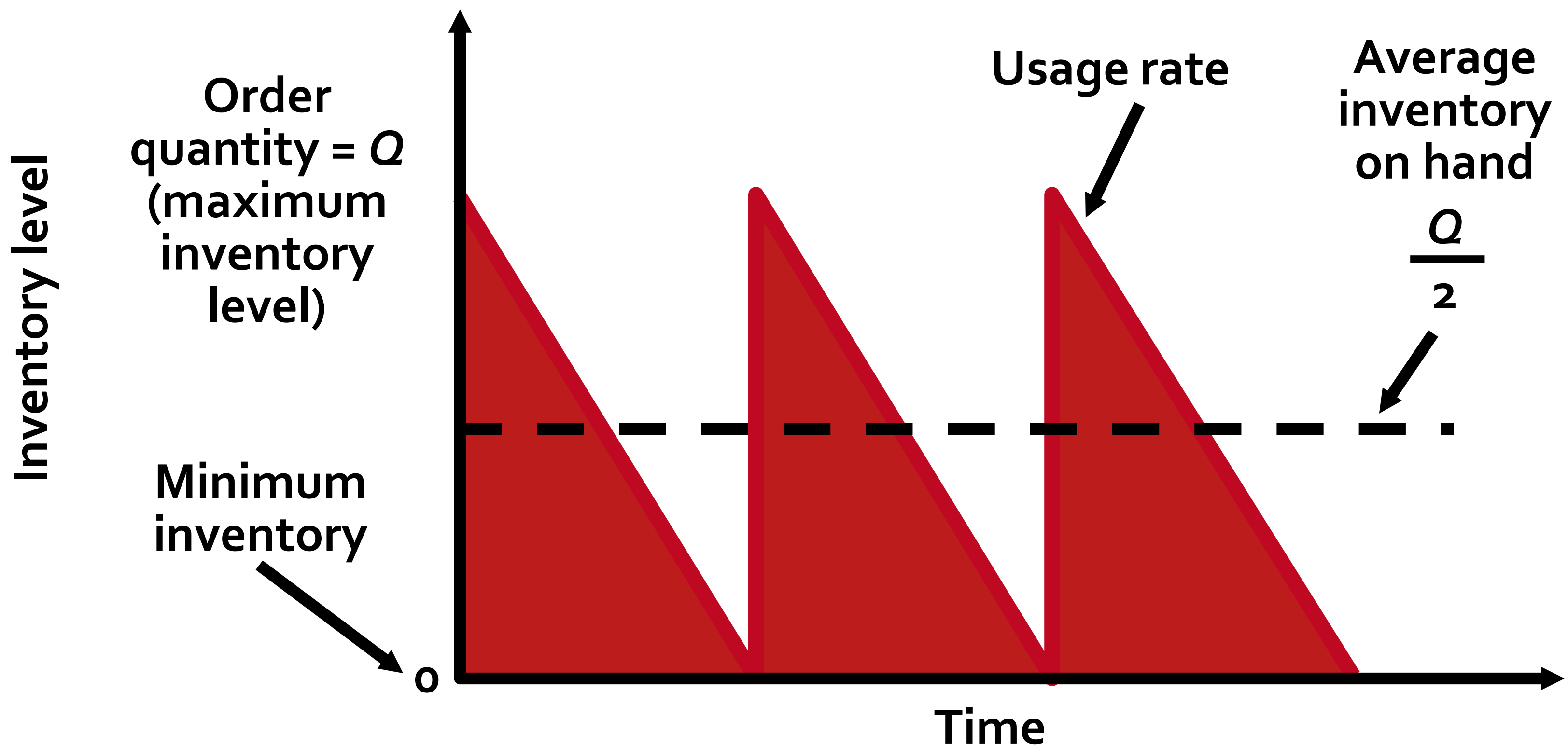
1. **Basic economic order quantity**
2. **Production order quantity**
3. **Quantity discount model**

Basic EOQ Model (Wilson)

Important assumptions

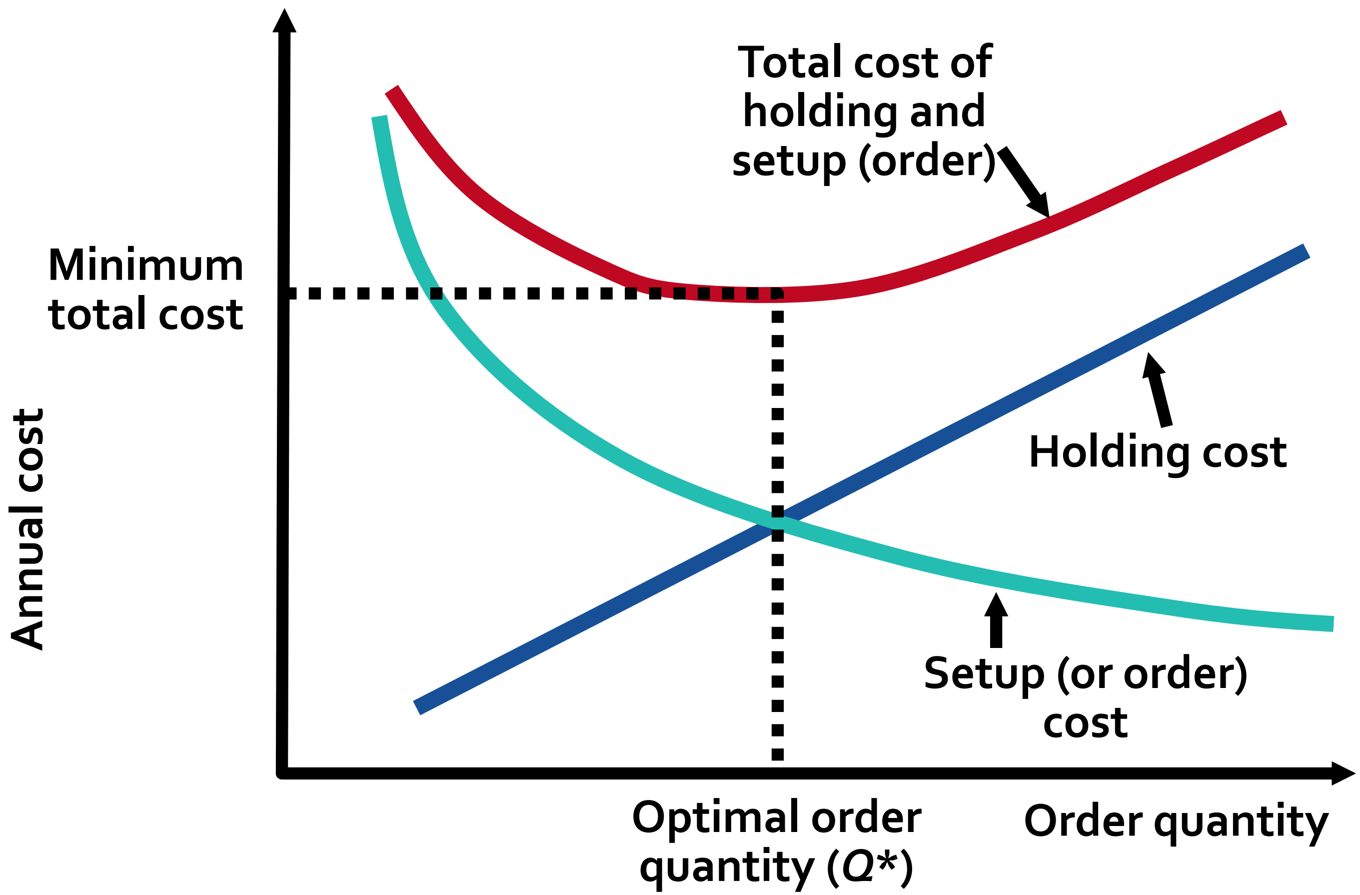
1. Demand is known, constant, and independent
2. Lead time is known and constant
3. Receipt of inventory is instantaneous and complete
4. Quantity discounts are not possible
5. Only variable costs are setup and holding
6. Stockouts can be completely avoided

Inventory Usage Over Time



Minimizing Costs

Objective is to minimize total costs



The EOQ Model

Q = Number of pieces per order
Q^* = Optimal number of pieces per order (EOQ)
D = Annual demand in units for the inventory item
S = Setup or ordering cost for each order
H = Holding or carrying cost per unit per year

Annual setup cost = (Number of orders placed per year) x (Setup or order cost per order)

$$= \left(\frac{\text{Annual demand}}{\text{Number of units in each order}}\right)\left(\begin{array}{c}\text{Setup or order}\\ \text{cost per order}\end{array}\right)$$

$$= \left(\frac{D}{Q}\right)(S)$$

The EOQ Model

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*Q** = Optimal number of pieces per order (EOQ)
D = Annual demand in units for the inventory item
S = Setup or ordering cost for each order
H = Holding or carrying cost per unit per year

Annual holding cost = (Average inventory level) x (Holding cost per unit per year)

$$= \left(\frac{\text{Order quantity}}{2}\right)(\text{Holding cost per unit per year})$$

$$= \left(\frac{Q}{2}\right)(H)$$

The EOQ Model

Q = Number of pieces per order
Q^* = Optimal number of pieces per order (EOQ)
D = Annual demand in units for the inventory item
S = Setup or ordering cost for each order
H = Holding or carrying cost per unit per year

Optimal order quantity is found when annual setup cost equals annual holding cost

$$\frac{D}{Q}S = \frac{Q}{2}H$$

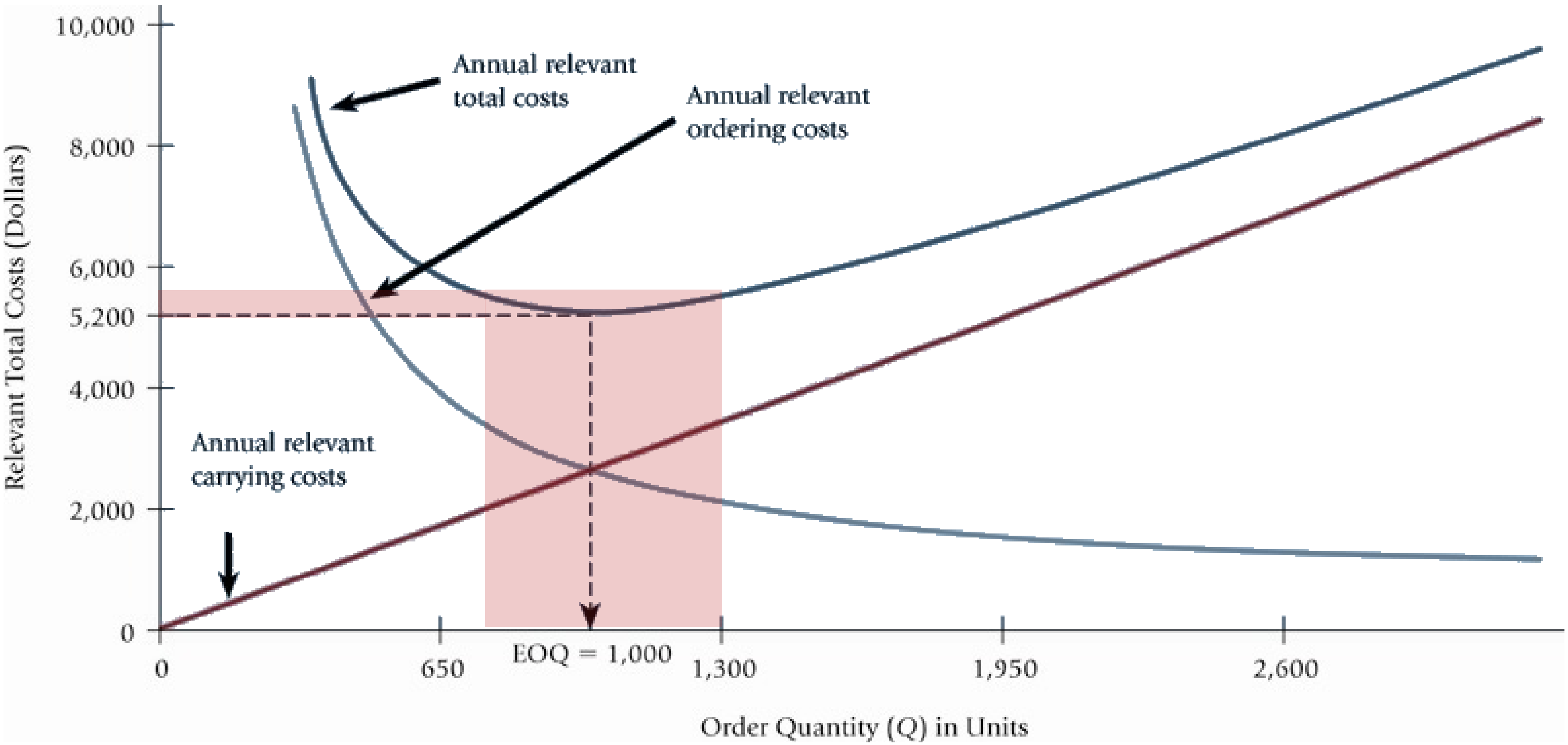
Solving for Q^*

$$2DS = Q^2H$$

$$Q^2 = 2DS/H$$

$$Q^* = \sqrt{2DS/H}$$

The EOQ model is quite robust



It means the EOQ can be rounded to a meaningful value (industrial lot size, container size, …)

Another EOQ Example, when demand is changing over time (D from 1000 to 1500)

Actual EOQ for new demand is 244.9 units

D = ~~1,000 units~~ 1,500 units

S = \$10 per order

H = \$.50 per unit per year

Q^* = 244.9 units

N = 5 orders per year

T = 50 days

$$TC = S\frac{D}{Q} + H\frac{Q}{2}$$

$$TC = (\$10) + \frac{1,500}{244.9} \quad (\$.50)\frac{244.9}{2}$$

$$TC = \$61.24 + \$61.24 = \$122.48$$

Only 2% less than the total cost of \$125 when the order quantity was 200

Reorder Points

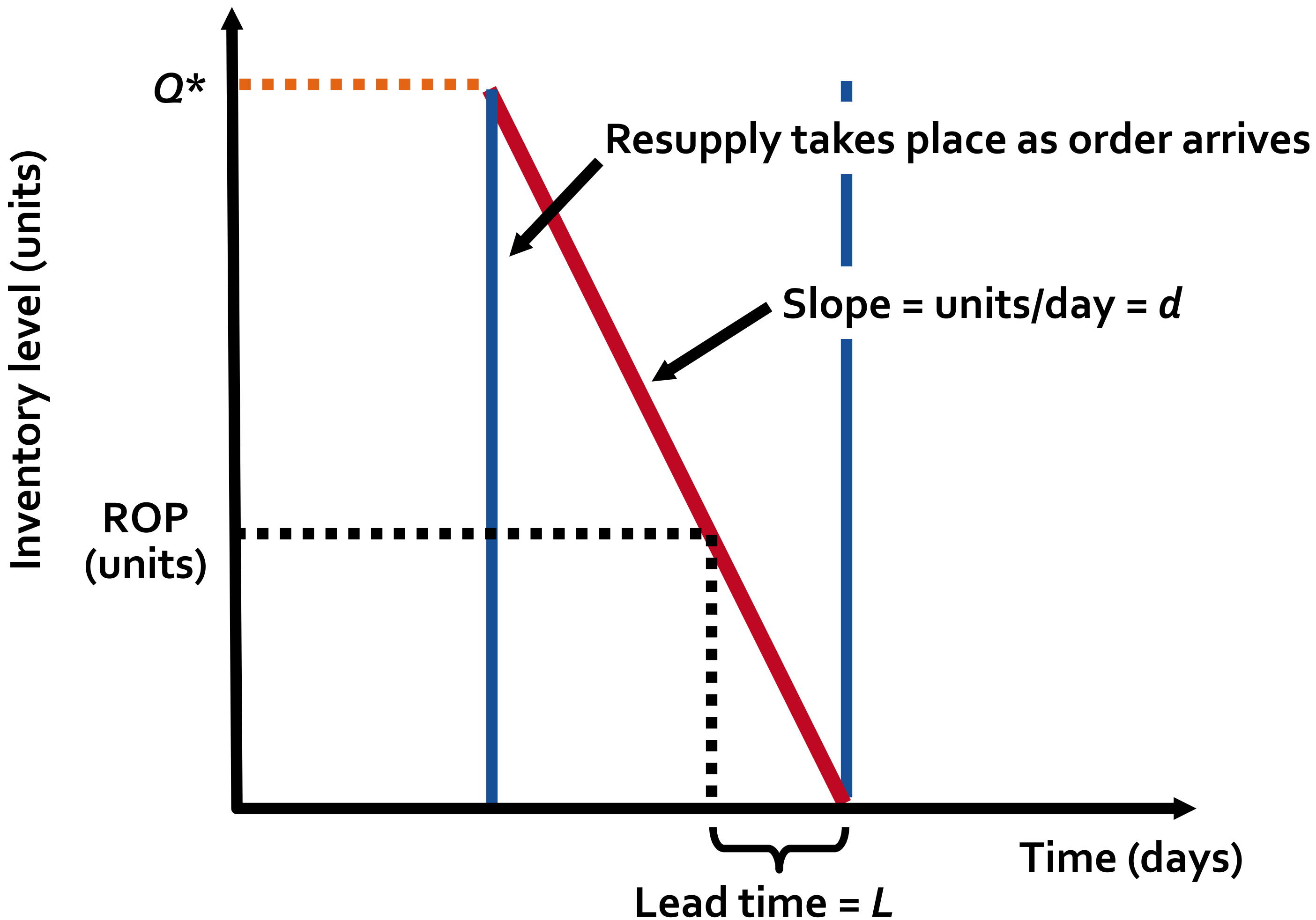
- **EOQ answers the "how much" question**
- **The reorder point (ROP) tells "when" to order**

$$\text{ROP} = \begin{pmatrix}\text{Demand}\\ \text{per day}\end{pmatrix}\begin{pmatrix}\text{Lead time for a new}\\ \text{order in days}\end{pmatrix}$$

$$= d \times L$$

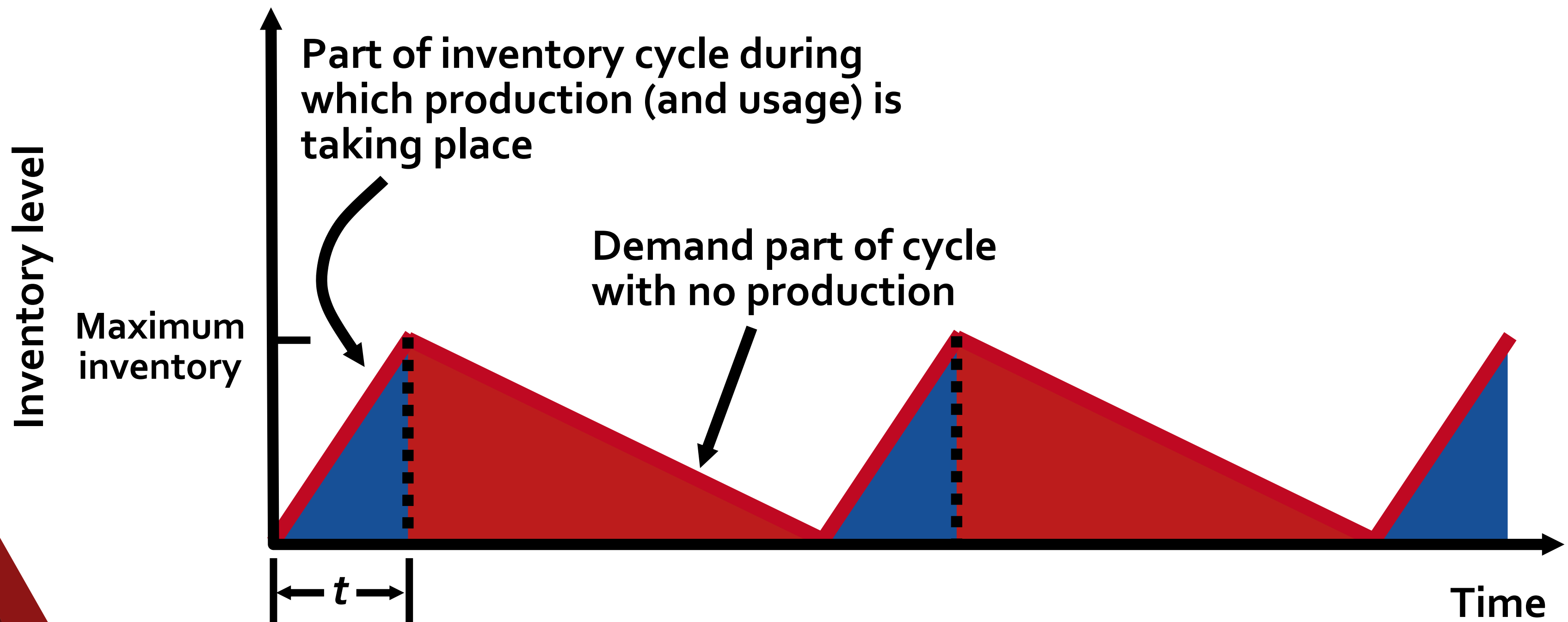
$$d = \frac{D}{\text{Number of working days in a year}}$$

Reorder Point Curve



Production Order Quantity Model

- Used when inventory builds up over a period of time after an order is placed
- Used when units are produced and sold simultaneously



Production Order Quantity Model

Q = Number of pieces per order
H = Holding cost per unit per year
t = Length of the production run in days
p = Daily production rate
d = Daily demand/usage rate

$$\left(\begin{array}{c}\text{Annual inventory}\\ \text{holding cost}\end{array}\right) = (\text{Average inventory level}) \times \left(\begin{array}{c}\text{Holding cost}\\ \text{per unit per year}\end{array}\right)$$

$$\left(\begin{array}{c}\text{Annual inventory}\\ \text{level}\end{array}\right) = (\text{Maximum inventory level})/2$$

$$\left(\begin{array}{c}\text{Maximum}\\ \text{inventory level}\end{array}\right) = \left(\begin{array}{c}\text{Total produced during the}\\ \text{production run}\end{array}\right) - \left(\begin{array}{c}\text{Total used during the}\\ \text{production run}\end{array}\right)$$

$$= pt - dt$$

Production Order Quantity Model

Q = Number of pieces per order
H = Holding cost per unit per year
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$$\left(\begin{array}{c}\text{Maximum}\\\text{inventory level}\end{array}\right) = \left(\begin{array}{c}\text{Total produced during the}\\\text{production run}\end{array}\right) - \left(\begin{array}{c}\text{Total used during the}\\\text{production run}\end{array}\right)$$

$$= pt - dt$$

However, Q = total produced = pt ; thus $t = Q/p$

$$\left(\begin{array}{c}\text{Maximum}\\\text{inventory level}\end{array}\right) = \left(\frac{Q}{p}\right) - d\left(\frac{Q}{p}\right) = Q\left(1 - \frac{d}{p}\right)$$

$$\text{Holding cost} = \frac{\text{Maximum inventory level}}{2} H = \frac{Q}{2}\left[1 - \left(\frac{d}{p}\right)\right]$$

Production Order Quantity Model

Q = Number of pieces per order
H = Holding cost per unit per year
D = Annual demand
p = Daily production rate
d = Daily demand/usage rate

$$\textbf{Setup cost} = (D/Q)S$$

$$\textbf{Holding cost} = \frac{1}{2}HQ[1 - (d/p)]$$

$$(D/Q)S = \frac{1}{2}HQ[1 - (d/p)]$$

$$Q^2 = \frac{2DS}{H[1 - (d/p)]}$$

$$Q^*_p = \sqrt{\frac{2DS}{H[1 - (d/p)]}}$$

Quantity Discount Models

- **Reduced prices are often available when larger quantities are purchased**
- **Trade-off is between reduced product cost and increased holding cost**

Total cost = Setup cost + Holding cost + Product cost

$$TC = S + \frac{D}{Q} H + PD \frac{Q}{2}$$

Quantity Discount Models

A typical quantity discount schedule

Discount Number	Discount Quantity	Discount (%)	Discount Price (*P*)
1	0 to 999	no discount	$5.00
2	1,000 to 1,999	4	$4.80
3	2,000 and over	5	$4.75

Quantity Discount Models

Steps in analyzing a quantity discount

1. For each discount, calculate *Q**
2. If *Q** for a discount doesn't qualify, choose the smallest possible order size to get the discount
3. Compute the total cost for each *Q** or adjusted value from Step 2
4. Select the *Q** that gives the lowest total cost

Quantity Discount Models

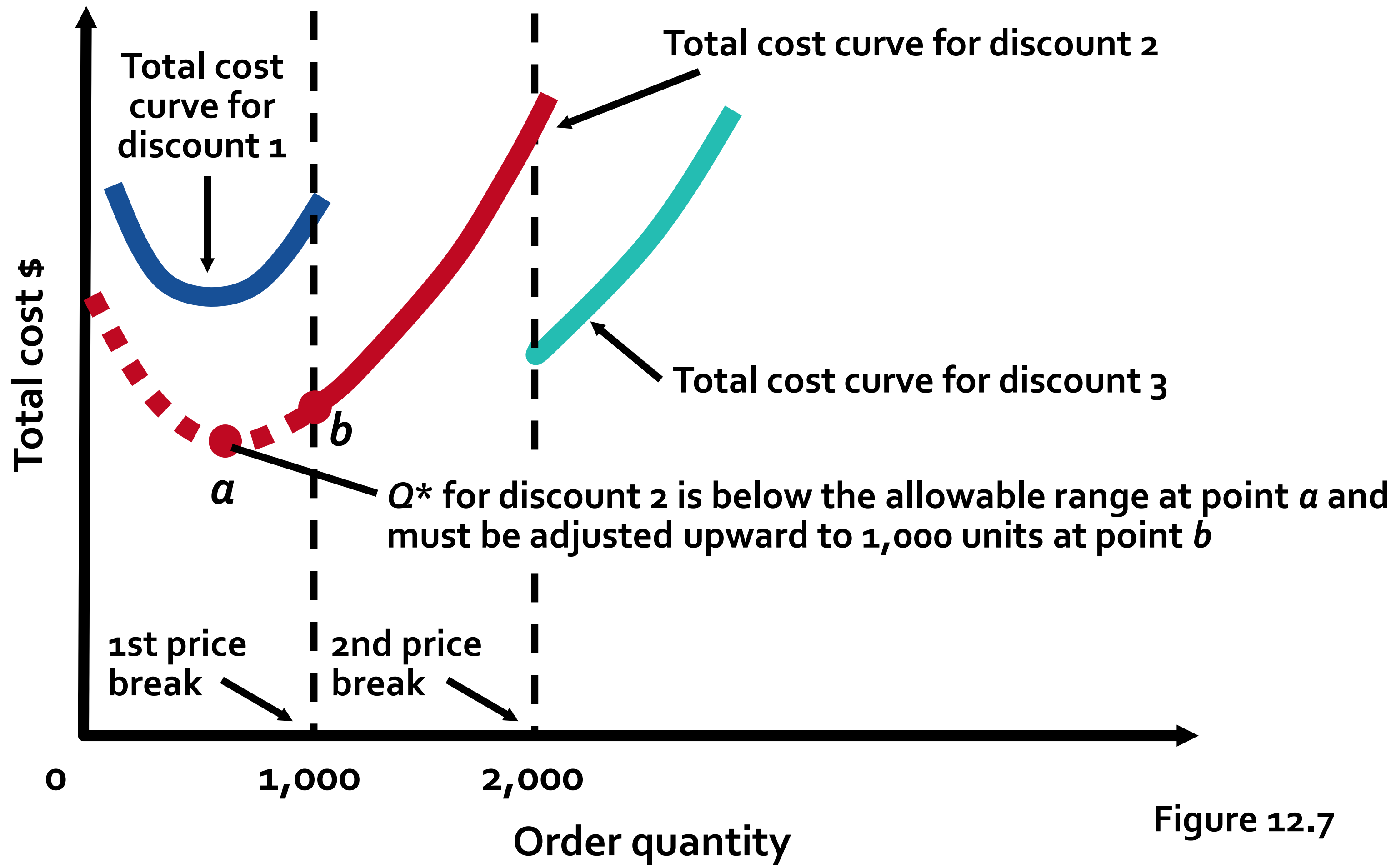


Figure 12.7

Quantity Discount Example

Calculate Q^* for every discount

$$Q^* = \sqrt{\frac{2DS}{HP}}$$

$$Q_1^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(5.00)}} = 700 \text{ cars/order}$$

$$Q_2^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(4.80)}} = 714 \text{ cars/order}$$

$$Q_3^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(4.75)}} = 718 \text{ cars/order}$$

Quantity Discount Example

Calculate *Q** for every discount

$$Q^* = \sqrt{\frac{2DS}{IP}}$$

$$Q_1^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(5.00)}} = 700 \text{ cars/order}$$

$$Q_2^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(4.80)}} = \cancel{714} \text{ cars/order}$$

1,000 — adjusted

$$Q_3^* = \sqrt{\frac{2(5{,}000)(49)}{(.2)(4.75)}} = \cancel{718} \text{ cars/order}$$

2,000 — adjusted

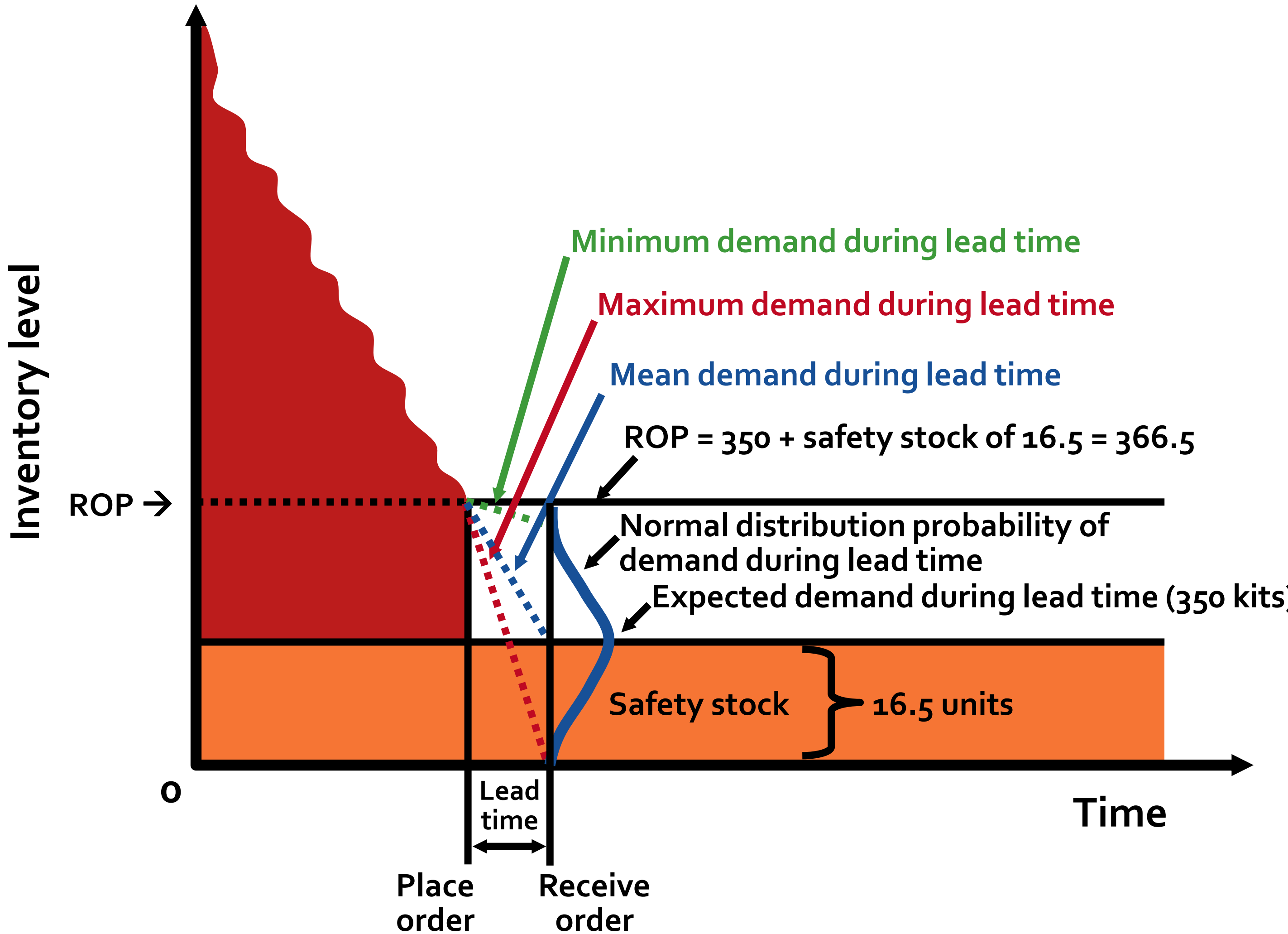
Quantity Discount Example

Discount Number	Unit Price	Order Quantity	Annual Product Cost	Annual Ordering Cost	Annual Holding Cost	Total
1	$5.00	700	$25,000	$350	$350	$25,700
2	$4.80	1,000	$24,000	$245	$480	$24,725
3	$4.75	2,000	$23.750	$122.50	$950	$24,822.50

Choose the price and quantity that gives the lowest total cost

Buy 1,000 units at $4.80 per unit

Probabilistic Demand



Probabilistic Demand

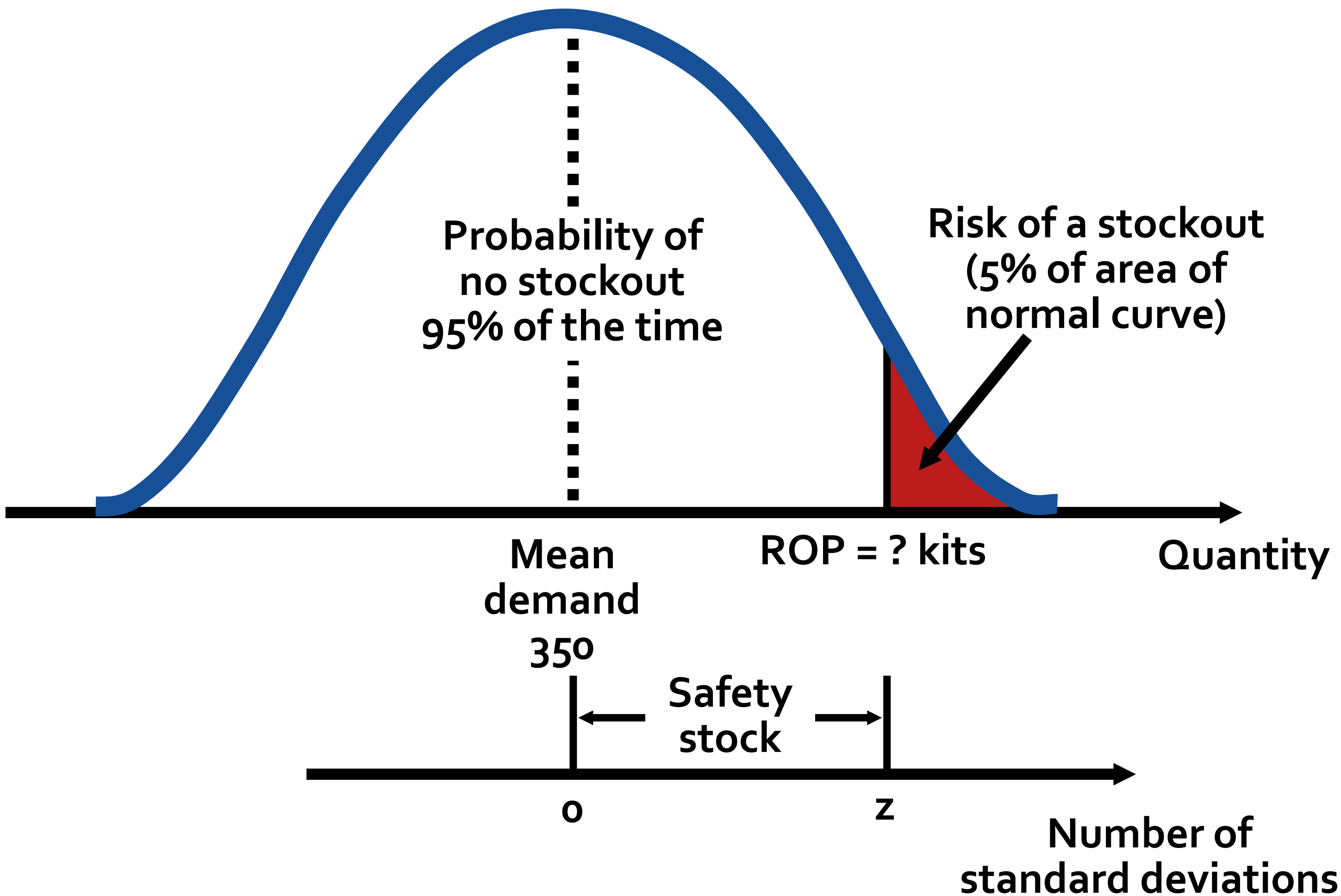
Use prescribed service levels to set safety stock when the cost of stockouts cannot be determined

$$\text{ROP} = \text{demand during lead time} + Z\sigma_{dLT}$$

where Z = number of standard deviations

σ_{dLT} = standard deviation of demand during lead time

Probabilistic Demand



Probabilistic Example

Average demand = μ = 350 kits
Standard deviation of demand during lead time = σ_{dLT} = 10 kits
5% stockout policy (service level = 95%)

Sce Level	Std Deviation
90%	1.28
95%	1.65
99%	2.33
99.9%	3.09

Using the table, for an area under the curve of 95%, the Z = 1.65

Safety stock = $Z\sigma_{dLT}$ = 1.65(10) = 16.5 kits

Reorder point = expected demand during lead time + safety stock
= 350 kits + 16.5 kits of safety stock
= 366.5 or 367 kits

Other Probabilistic Models

When data on demand during lead time is not available, there are other models available

1. **When demand is variable and lead time is constant**
2. **When lead time is variable and demand is constant**
3. **When both demand and lead time are variable**

Other Probabilistic Models

Demand is variable and lead time is constant

$$ROP = (\text{average daily demand} \times \text{lead time in days}) + Z\sigma_{dLT}$$

where σ_d = standard deviation of demand per day

$$\sigma_{dLT} = \sigma_d \sqrt{\text{lead time}}$$

Other Probabilistic Models

Lead time is variable and demand is constant

ROP = (daily demand x average lead time in days)

= Z x (daily demand) x σ_{LT}

where σ_{LT} = standard deviation of lead time in days

Other Probabilistic Models

Both demand and lead time are variable

$$\text{ROP} = (\text{average daily demand} \times \text{average lead time}) + Z\sigma_{dLT}$$

where

σ_d = standard deviation of demand per day

σ_{LT} = standard deviation of lead time in days

$$\sigma_{dLT} = \sqrt{(\text{average lead time} \times \sigma_d^2) + (\text{average daily demand})^2 \times \sigma_{LT}^2}$$

Single Period Model

- Only one order is placed for a product
- Units have little or no value at the end of the sales period

C_s **= Cost of shortage = Sales price/unit – Cost/unit**

C_o **= Cost of overage = Cost/unit – Salvage value**

$$\text{Service level} = \frac{C_s}{C_s + C_o}$$

Single Period Example

Average demand = μ = 120 papers/day

Standard deviation = σ = 15 papers

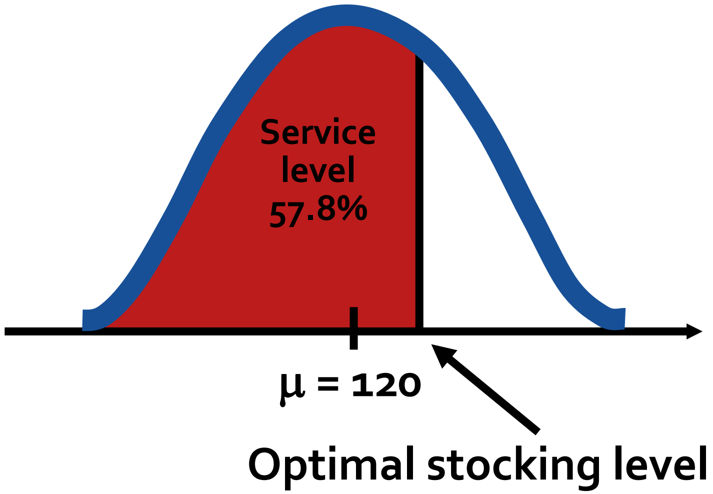
C_s = cost of shortage = \$1.25 - \$.70 = \$.55

C_o = cost of overage = \$.70 - \$.30 = \$.40

Service level = $$\frac{C_s}{C_s + C_o}$$

$$= \frac{.55}{.55 + .40}$$

$$= \frac{.55}{.95} = .578$$



Single Period Example

From the table, for the area .578, $Z \cong .20$

The optimal stocking level

$= 120$ copies $+ (.20)(\sigma)$

$= 120 + (.20)(15) = 120 + 3 = 123$ papers

The stockout risk = 1 – service level

$= 1 - .578 = .422 = 42.2\%$

Fixed-Period (P) Systems

- Orders placed at the end of a fixed period
- Inventory counted only at end of period
- Order brings inventory up to target level
  - Only relevant costs are ordering and holding
  - Lead times are known and constant
  - Items are independent from one another

Fixed-Period (P) Systems

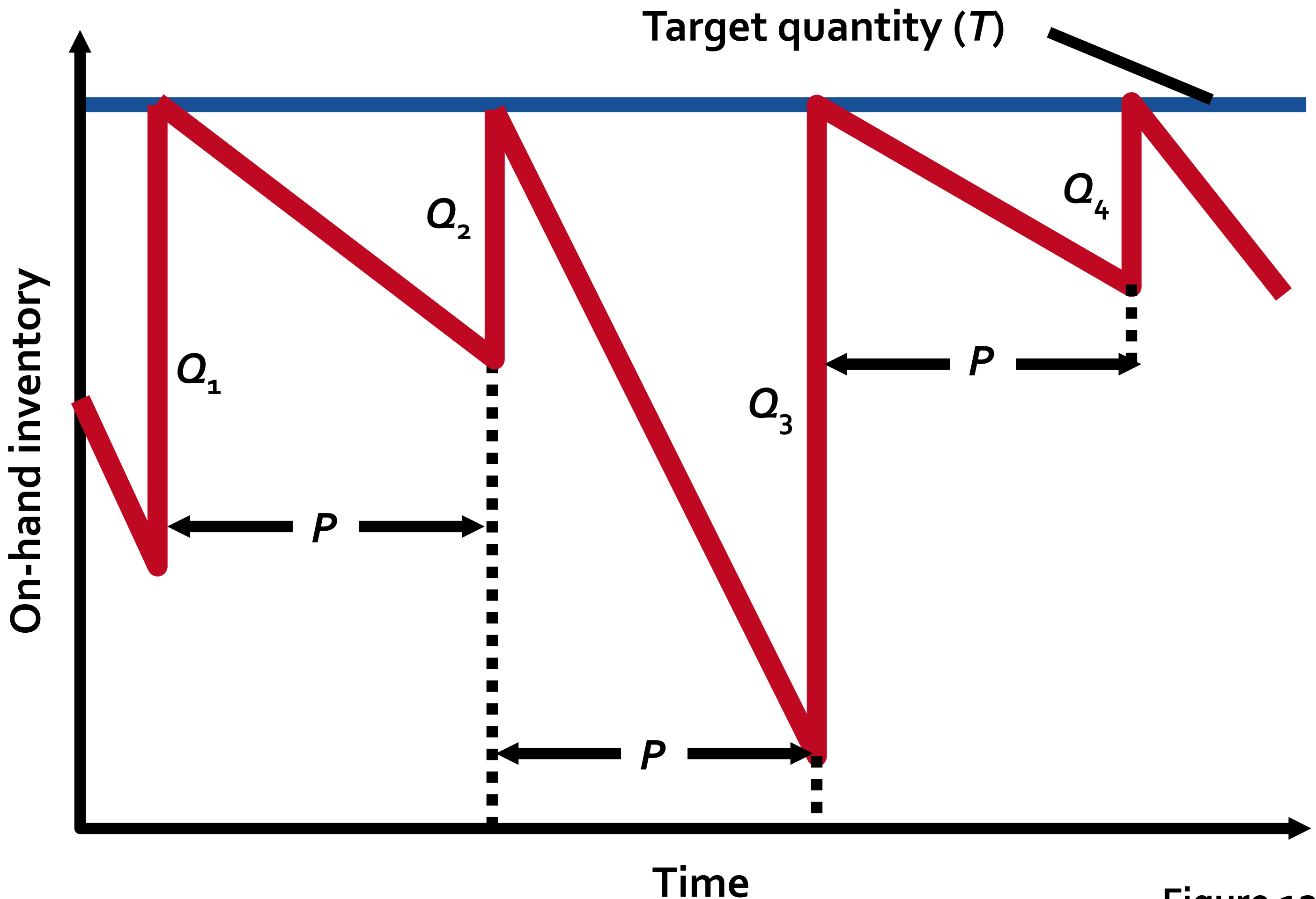


Figure 12.9

Fixed-Period Systems

- Inventory is only counted at each review period
- May be scheduled at convenient times
- Appropriate in routine situations
- May result in stockouts between periods
- May require increased safety stock

Summary of Management of Inventory Costs

1. **Purchasing costs**—the cost of goods acquired from suppliers, including freight
2. **Ordering costs**—the costs of preparing and issuing purchase orders, receiving and inspecting the items included in the orders, and matching invoices received, purchase orders, and delivery records to make payments
3. **Carrying costs**—the costs that arise while holding inventory of goods for sale. This includes the opportunity cost of the investment tied up in inventory, and costs associated with storage.
4. **Stockout costs**—the costs that result when a company runs out of a particular item for which there is customer demand (stockout) and the company must act quickly to meet the demand or suffer the costs of not meeting it.
5. **Quality costs**—the costs that result when features and characteristics of a product or service are not in conformance with customer specifications. These costs include:
    1. Prevention
    2. Appraisal
    3. Internal failure
    4. External failure
6. **Shrinkage costs**—costs that result from theft, embezzlement, and clerical errors

Safety Stock Computation Illustration

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Safety Stock Level in Units (1)	Demand Levels Resulting in Stockouts (2)	Stockout in Units[a] (3) = (2) - 500 - (1)	Probability of Stockout (4)	Relevant Stockout Costs[b] (5) = (3) x $4	Number of Orders per Year[c] (6)	Expected Stockout Costs[d] (7) = (4) x (5) x (6)	Relevant Carrying Costs[e] (8) = (1) x $5.20	Relevant Total Costs (9) = (7) + (8)
0	600	100	0.20	$ 400	13	$1,040		
	700	200	0.09	800	13	936		
	800	300	0.06	1,200	13	936		
						$2,912	$ 0	$2,912
100	700	100	0.09	400	13	$ 468		
	800	200	0.06	800	13	624		
						$1,092	$ 520	$1,612
200	800	100	0.06	400	13	$ 312	$1,040	$1,352
300	-	-	-	-	-	$ 0[f]	$1,560	$1,560

[a] Demand level resulting in stockouts – Inventory available during lead time (excluding safety stock), 500 units – Safety stock.

[b] Stockout in units x Relevant stockout costs of $4.00 per unit.

[c] Annual demand, 13,000 ÷ 1,000 EOQ = 13 orders per year.

[d] Probability of stockout x Relevant stockout costs x Number of orders per year.

[e] Safety stock x Annual relevant carrying costs of $5.20 per unit (assumes that safety stock is on hand at all times and that there is no overstocking caused by decreases in expected usage).

[f] At a safety stock level of 300 units, no stockout will occur and, hence, expected stockout costs = $0.