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1. Before we start

-Guidelines

A. The lectures will be given in Chinese, the Lecture Notes will be written in English though. The bilingual approach is anticipated to offer surprising advantages such as lowering the language barrier and getting familiar with English vocabularies. The use of original English teaching materials could help students to understanding relevant knowledge.

B. Attendance on class will take 10% of the overall score, experiment score will take 10%, while the final exam will take the rest 80%.

C. There is no homework at this course. But it does not mean you can pay less time on it after class.

-Who Cares?

This course, we call elastostatics, is come from the traditional course – elasticity but concisely. Elasticity is the most important course in the field of solid mechanics, and also tops the list for the key courses in many fields of engineering.

Elasticity goes a long way in its applications, spanning length scales from nanometers for a carbon nanotube to kilometers of a geological formation. Without the mastery of elasticity, you are incompetent in doing any serious works as a professional in engineering mechanics, even not ranked as a capable engineer.

Beside its contents, elasticity is a course that exemplifies the beauty of mathematical physics. Many concepts and methods in modern science and engineering are originated from elasticity. However, elasticity is also a quite difficult course to learn, please be prepared for it. You definitely need to spend 3 times as the lecture hours to grasp the materials. Without competitors, it will be the course that burdens you the most in this semester.

-Pre-request

The students are required to have first courses on Calculus, General Physics (including rational mechanics). In the following 3 years, you will learn more than 10 courses which are related to mechanics. Remember, this is the base of all those courses. You should be ready for hardworking in this.

2. Content of Elastostatics

-What is it?

Generally, mechanics is a subject which studies regulations of object's mechanical motion. [Mechanical motion includes not only external effect which is object's motion, but also internal effect we call deformation.]

According to J.R.BARBER's book [1], "The subject of Elasticity is concerned with the determination of the stresses and displacements in a body as a result of applied mechanical or thermal loads, for those cases in which the body reverts to its original state on the removal of the loads."

According to Xu's book [2], "Elasticity, or elastic theory, is to study stress, strain, and displacement of an elastomer caused by foreign loads or temperature change."

In this context, we should focus on three main parts: firstly, our purpose is looking for the stress, strain, and displacement; secondly, research object is an elastomer; and finally, all of these are resulted from foreign loads or temperature change. It should be retentioned because these decide boundaries and content of this subject.

- Its relations with physics and classic mechanics (rational mechanics).

Like in physics, we also looking for the regulations correlate with force, motion, etc. But, we study only particles or object which can be regarded as particle with little influence to researched problem. Similarly, we consider all objects as rigid or particle in classic mechanics.

Based on knowledge we learned in the past, we now allow object to change its dimension and shape while it is suffering a foreign load or temperature change. It is more practical, although it is still limited in elastic period of an object.

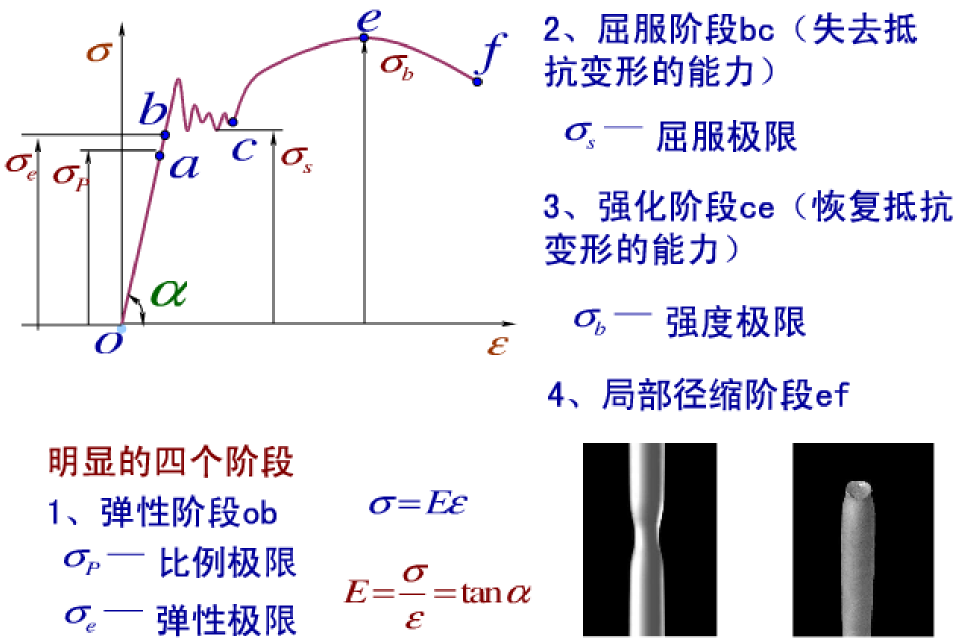

Figure1.1 Stress-strain curvatures of materials

In this course, we will study a part of elasticity, which we call “Elastostatics”. Actually, we will only study deformation of elastomer under foreign loads in context of statics.

As an entrance of a series of mechanics, we also will not talk about any content about energy (including variational method, and difference method), neither engineering methods like FEM, and thermo-elasticity.

- Our place in the syllabus of mechanics knowledge

In Chinese universities, there are three important mechanics: elasticity, material mechanics and mechanics of structures. They concern about the same question of different objects with different methods. You will learn all of these mechanics.

As you can see, there are many mechanical courses in your course system, but this course is an entrance.

- Knowledge involved in.

In the following several weeks, we will study below content.

[2nd]-- Stresses in elastic solid

[3rd]-- Linear solid strain

[4th]-- Elastic linear isotropic Hooke’s law

[5th]-Elastostatic methods

- Analytical methods in linear elastostatics

[6th, 7th]- Analytical methods in elastic plane problems

[8th]- Experimental elastic methods

- How we do?

Hypothetically, we split an object with a plane to achieve a section. On this section, load distribution on a point is studied. Enlarging this point, an infinitesimal-hexahedron that calls micro unit is used to indicate the point. Considering the loads on the cut part, balance equations can be written down as statics. According to Hooke's law, physics equations can be built up to correlate stress with strain. Also, accumulations of all points' strains can result in deformation of object. With boundary conditions and equations achieved, we can look for solutions for every point on the object. Many analytical methods are developed at this process. Also, these methods constructed the base to comprehension and utilizing elasticity. Sometimes, experimental methods and engineering computation methods are more important than analytical ones.

In this course, problems are classified in different coordinate systems, solution methods, steps of solving and complexity.

2, Basic concepts of elasticity

-External force: Force issued by other objects (not by object studied)

Or calls outside loads, is divided to two kinds: volume force and surface force.

a. Volume force (because of mass) - In an elastomer, volume force is distributed according to its volume, such as gravity, and inertia force. In a continuum, a limit vector is introduced to express distribution of volume force at a point that is "volume force density".

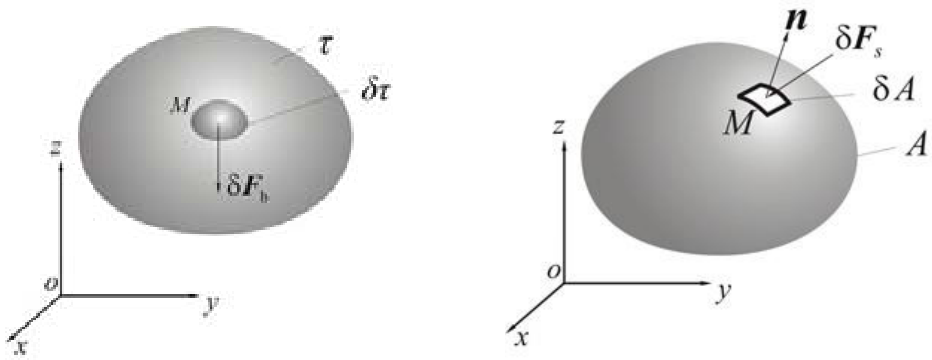

Figure1.2 Two kinds of loads

$$\vec{F} = \lim_{\Delta V \to 0} \frac{\Delta \vec{F_b}}{\Delta V} [\text{Force}] \cdot [\text{Length}]^{-3} \qquad \textbf{(1.1)}$$

b. Surface force (because of touch) – surface force is distributed on the surface of an object, such as pressure of wind and liquids. Similarly, "surface force density" can be used to descript the distribution of a continuous surface force function at a point.

$$\vec{f} = \lim_{\Delta S \to 0} \frac{\Delta \vec{F_s}}{\Delta S} [\text{Force}] \cdot [\text{Length}]^{-2} \qquad \textbf{(1.2)}$$

As vectors, they can be projected to coordinate axis.

-Internal force and Stress

Force issued by a part of studied object towards other parts. Section method is used to imaginarily separate an object. To descript distribution of internal force on a section, a limit equation is used to express the “internal force density” at a point, which is called **stress**.

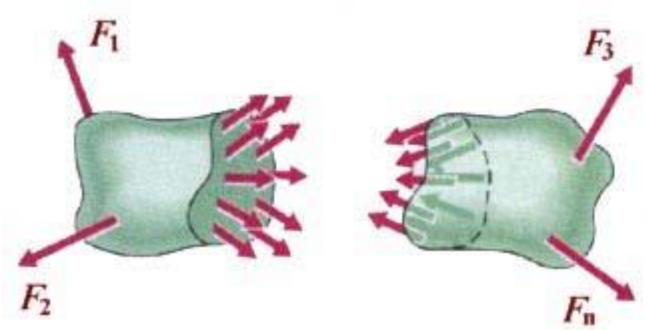

Figure1.3 Internal force and stress

$$\vec{s} = \lim_{\Delta A \to 0} \frac{\Delta \vec{Q}}{\Delta A} [\text{Force}] \cdot [\text{Length}]^{-2} \qquad \textbf{(1.3)}$$

Stress is also vector. However, it is always unhelpful to project it towards coordinate axis. Actually, values of stress at a point along normal direction and tangential directions of section surface are more meaningful. Hence, stress is projected towards external normal direction and two tangential directions.

Because of uncertain section direction, values of stress are different. To analyze and express stress state of a point, an infinitesimal unit is used.

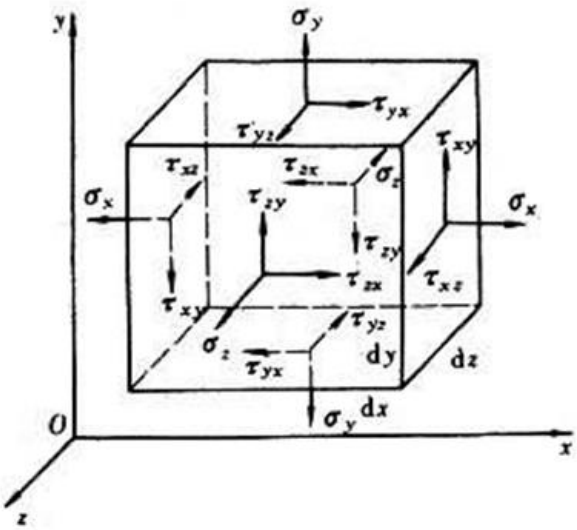

Figure1.4 An infinitesimal unit

-Deformation and Strain

Deformation refers to changes of dimension or shape of an object. To express deformation of an infinitesimal unit, normal strain and shear strain are defined to descript relative deviation of dimension and shape of a point.

-Displacement

Position change of a point in an object calls displacement. It is a result of force.

3. Basic assumptions of elasticity

To simplify the problems, elasticity is based on 5 assumptions which are acceptable in engineering domains:

-Continuity

Although an object is consisted with particles and spaces between them, elastomer in elasticity is full of continued materials without any empty. Based on this, we can use continuous functions and their derivatives (partial derivatives) to express and analyze our problems.

- Absolute elastic body

The word elasticity refers to characteristic that an object could restore its original state when factors (cause deformation) disappear. An absolute elastic body means object that can restore its original state completely without any residual deformation. This makes deformation only relates to current factors.

-Uniformity

If each part of object has same physics properties, any piece from object can be researched by the same method. Then, results from these researches can be extended to the entire object. -

Isotropism

Elastic constants of a point on an object are same at all directions.

Object matches four above properties calls ideal elastic body.

-Hypothesis of small deformation

Assuming deformations are far small than object geometric dimensions, balance equations can be built up according to rigid modal. This handling makes superposition principle possible to use.

4. Short history of elasticity (Mechanics) [Contents below are directly borrowed from book 4]

Era of the Exploration (1600－1700)

Elasticity developed in the outpouring of mathematical and physics studies following the era of Newton, although it has earlier roots. The need to understand and control the fracture seems to have been a first motivation. Leonardo da Vinci sketched in his notebooks a possible test of the tensile strength of a wire that might be crucial in hanging his paintings. He was aware of the possible length dependence of the wire strength due to statistical defects distribution. The classical mechanics is sometimes labeled Galileo-Newton mechanics. The reason is quite clear. Galileo, who dies in the year of Newton’s birth (1642), proposed the Principle of Inertia, while Newton extended it to Three Basic Laws of Mechanics. The classical works of Galileo “A Dialogue of Two New Sciences (1683)” is a milestone in the development of mechanics. Beside the Principle of Inertia now known in every house-hold, Galileo conducted a lengthy discussion on the deformation and strength of solids in that book. He investigated the breaking loads of rods under tension and concluded that the load was independent of length (contrast to the perception of da Vinci which was based on the statistical distribution of defects along the wire length) and proportional to the cross section area. It was quite unusual in an era when astronomy dominated the science. He took on a main target of nowadays strength of materials, the bending of a beam. A historic account of Galileo’s methodology is described in the book “History of Strength of Materials” by Timoshenko. The basic schematics are shown in his book. A bucket of water (or any form of weight) is hanged on one end of a beam of length L and a rectangular cross-section. The other end of the beam is clamped in a wall. Galileo performed the mechanics analysis for a cantilever beam. The latter is treated, the first time in the history, as a deformable body. The analysis provided correct geometric dependence of the strength on the parameters such as the beam length and the bending resistance of its cross-section. Unfortunately, the stress distribution across the height of the beam is wrong. The longitudinal stress was assumed to be zero by Galileo at the bottom of the beam, rather than at the neutral plane as was found later.

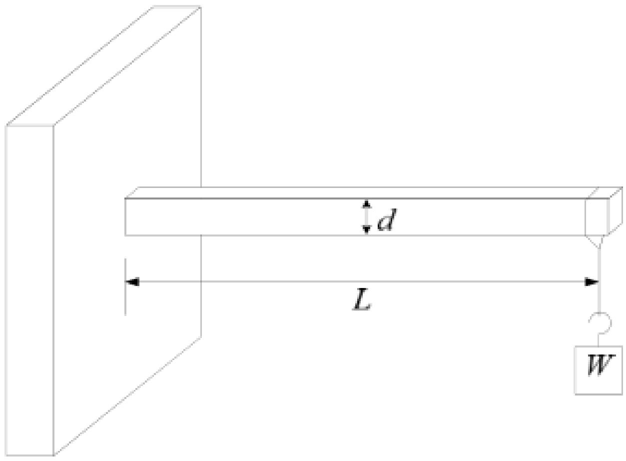

Figure 1.5 Galileo's perception of beam bending

The concept of an elastic relation was pioneered by the English scientist Robert Hooke. The law was discovered in 1660, but published only in 1678. In his paper entitled "Of Spring", the elastic relation, in its most primitive form, was phrased in an anagram of Latin "ceiiiosssttuu". Rearranging the letters, one finds the real meaning of so-called Hooke's Law "Ut tensio, sic vis", or "tension is proportional to the stretch" in English. Hooke's law established the notion of (linear) elasticity but not yet in a way that was expressible in terms of stress and strain.

Strokes of the Genius (1700-1880)

Early developments of elasticity recorded many works of scientific giants. The concepts of stress and strain were introduced by the Bernoulli brothers. It was the Swiss mathematician and mechanician Jakob Bernoulli who observed, in the final paper of his life, in 1705, that the proper way of describing a deformation was to give force per unit area, or stress, as a function of elongation per unit length, or strain, of a material fiber under tension. The Swiss mathematician and mechanician Leonard Euler, who was taught mathematics by Jakob's brother Johann Bernoulli, proposed a linear relation between stress σ and strain ε, in 1727, of the form σ=Eε. The coefficient E is now generally called Young's modulus after the British naturalist Thomas Young, who developed a related idea in 1807.

In 1744, Leonard Euler analyzed the buckling of a compressed bar. The deflection of the bar was shown to obey the following equation:

$$C' \frac{y''}{(1+y'^2)^{2/3}} = Px \tag{1.4}$$

where C' is the stiffness of a Bernoulli beam, P the compressive load, and x and y delineate the profile of the bar. As a classical example to illustrate the historic importance of elasticity, we mention that two important mathematical concepts were born as the by-products of this elasticity analysis. The first one is the “Variational Calculus” which Euler used to derive the equation. The second is the concept of “Bifurcation” that initiates the whole theme of nonlinear analysis. The solution of (1.4), known as the “elastica” in the literature, was found by Euler. As shown in Fig. 1.6, under different stages of “bar compression”, the bar would flip over to become a folded bar in stretching! Several possible folding patterns shown in Fig. 1.6 exist for different loading parameters.

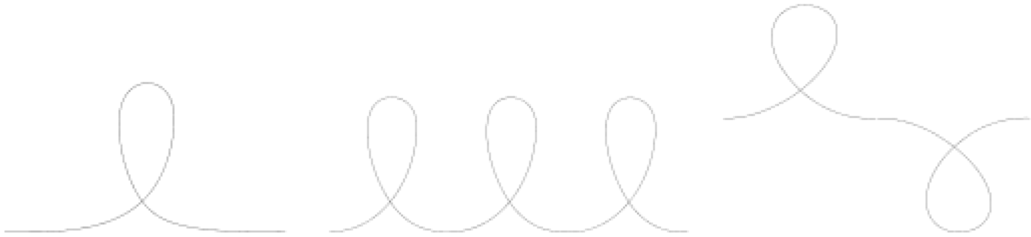

Figure 1.6 Euler’s elastica

Shortly after the time of Euler, most gifted scientists were gathered in France. One active research thrust in the Academy of France is the pursuit of elasticity. Several scientific giants got involved. The list includes Navier, Poisson, Columb, Cauchy and Saint Venant. In 1821, Claude-Louis-Marie-Henri Navier published an essay entitled “Equilibrium and motion of elastic solids”, in which the governing equation of elasticity is first formulated as

$$C(\nabla^2 \mu_i + 2\mu_{k,ki}) + f_i = 0 \qquad (1.5)$$

Where μ_i is the displacement vector, C a measure of elastic modulus and f_i the body force vector. This equation is called since as the displacement equation of elasticity, or simply the “Navier equation”.

Equation (1.5) is not exactly in the same form as we are using today. It was only valid for a special material, namely when two Lamè constants are equal. It was Simon Denis Poisson (1829) who brought the issue of lateral contraction, and named after him is the Poisson’s ratio ν that represents the negative ratio between extension and the induced lateral contraction. In its original form (1.5), the Navier equation is valid only for a Poisson’s ratio of one quarter. Poisson also contributed on finding the longitudinal and transverse waves that opened the gate to elastodynamics.

It was the great French mathematician Augustin-Louis Cauchy, originally educated as an engineer, who in 1822 formalized the concept of stress in the context of a generalized three-dimensional theory, showed its properties as consisting of a three by three symmetric array of numbers that transform as a tensor. Cauchy is responsible for relating traction vector and stress tensor in the form of Cauchy's stress principle, the concepts of principal stresses and principal strains, the generalized Hooke's law, and the equations of motion for a continuum in terms of components of stress with their boundary conditions. Cauchy also introduced the equations that express the six components of strain (three extensional and three shear) in terms of derivatives of displacements for the case where all those derivatives are much smaller than unity; though similar expressions had been given earlier by Euler in expressing rates of straining in terms of the derivatives of the velocity field in a fluid. Not only a rigorous mathematician, Cauchy also explored the atomistic aspect of elasticity and derived, by using the paired inter-particle potential, the so-called Cauchy's relation of the elasticity tensor. That relation inferred a complete symmetry of the elasticity tensor whose limitation will be discussed in the next chapter. For the special case of isotropic solids examined in detail by Cauchy, the theory of linear elastic response only requires the knowledge of one elastic constant.

Controversies concerning the maximum possible number of independent elastic moduli in the most general anisotropic solid were settled by the British mathematician George Green in 1837. Green pointed out that the existence of elastic strain energy required that of the 36 elastic constants relating the 6 symmetric stress components and the 6 strains, at most 21 could be independent. The Scottish physicist Lord Kelvin put this consideration on sounder ground in 1855 as part of his achievement of macroscopic thermodynamics, showing that a strain energy function must exist for reversible isothermal or adiabatic (isentropic) response. The middle and late 1800s were a period in which many basic elastic solutions were derived and applied to technology and to the explanation of natural phenomena. Adhemer Jean Claude Barre de Saint-Venant, a student of Navier, contributed significantly to this endeavor. He invented, among other things, the method of semi-inverse solution (1853) and solved the problems of beam bending and torsion of a non-circular prismatic rod in an accurate manner. We will devote the whole Chapter 4 on this subject. His solutions evaluated the accuracy of the bending and torsion solutions obtained earlier from a simplified methodology of Strength of Materials. Moreover, he proposed the famous Saint-Venant Principle, which gives numerous tasks for the mathematicians and mechanicians to engage.

The contributions from the German scientists at the end of the 19th century, then replaced France as the intellectual center of the world, are also worth of mentioning. The versatile

Prussian physicist Gustov Robert Kirchhoff, the founding master of electro-magnetism, left his marks on elasticity. In his book "Mechanik" published in 1876, Kirchhoff extended the application domain of elasticity to a new type of geometry, plates. Kirchhoff applied virtual work and variational calculus procedure in the framework of simplifying the kinematical assumptions that fibers initially perpendicular to the plate middle surface remain so after deformation of that surface. In one dimensional version, the Kirchhoff plate theory assembles the Euler-Bernoulli beam theory. It finds immense applications in civil and mechanical engineering as the structures in the form of plates and shells emerge. Another founding master of electro-magnetism theory, Helmholtz, also contributed to elasticity by including the concept of elastic free energy, named after him as the Helmholtz free energy, and the solution for stress waves in terms of Helmholtz transformation.

Forming of the Mansion (1880-1950)

All pieces of understandings on elasticity were assembled and systemized in that period of time to form a formidable mansion of elasticity knowledge. Beside his own contributions on the point source theory and Love wave, Love completed a comprehensive book "A Treatise on the Mathematical Theory of Elasticity (1892-1893)". The symbol is clear; elasticity took the center-stage among all branches of mathematical physics at the end the 19th century. The introduction of elasticity to various areas of engineering was greatly benefited by the enthusiastic effort by S. P. Timoshenko. The formerly Russian noblety used to work with Prandtl, the father of aerodynamics. Timoshenko was particularly keen to the engineering aspects of elasticity, and made enormous contributions to the engineering elasticity, such as beams on elastic foundation, Timoshenko beam theory, mechanics of plates and shells and elastic vibration. Timoshenko is not only a scientist and an engineer, but also a great educator. He wrote many books that dominated the engineering college education for decades. Along with Von Karman, Timoshenko was responsible for the prosperity of Applied Mechanics in the United States of America.

Two additional developments in that period of time are worth mentioning. The first is along the line of structures in large deflection and buckling, tackled by the great Theodere Von Karman, along with his students, noticeably H. S. Tsien and W. Z. Chien. One of the founding masters of quantum mechanics, Werner Heisenberg, also used the subject of buckling as his Ph. D. thesis. The second major development came from the formerly Soviet Union, in the school represented by Kolosov and Muskhelishivilli. They developed the complex potential method of elasticity. That subject was brilliantly summarized in two monographs of

Muskhelishivilli, namely “Some Basic Problems in Mathematical Theory of Elasticity” and “Singular Integral Equations”. The methods of analytical function, Cauchy integral, singular integral equation, conformal mapping, Riemann-Hilbert linear relationship problem were concisely put together to get a rationale for the plane and anti-plane problems of linear elasticity. The complex potential method was extended later on from the isotropic elasticity to anisotropic elasticity.

Expanding the Horizon of Elasticity (Since 1950)

Various branches of elasticity were prospered in the past half century. The field of elastic stability was advanced by the Dutch applied mechanician and engineer K. T. Koiter (1950), the concepts of static, kinematical and dynamic stabilities are proposed. The issue of defect sensitivity is extensively explored.

The field of fracture mechanics is pioneered by the British aeronautical engineer A. A. Griffith (1921). Griffith made his famous proposition that a spontaneous crack growth would occur when the energy released from the elastic field just balanced the work required to separate surfaces in the solid. The field became the center-stage in solid mechanics since the mid-20th century, largely due to the re-evaluation for the naval loss in World War II and the enthusiastic effort by George R. Irwin, an American engineer and physicist. Irwin (1957) proposed an alternative measure of the stress intensity factor for the severity near a crack tip. Largely under the impetus of Irwin, a major body of work on the mechanics of crack growth and fracture, including fracture by fatigue and stress corrosion cracking, started in the late 1940s and is continuing into the 21st century. The foundation of nonlinear fracture mechanics was laid down largely due to the works of American mechanician and geologist J. R. Rice (1968). The critical parameters in fracture mechanics, such as the energy release rate G, the stress intensity factor K, and the J-integral J are named after Griffith, Irwin and Rice, respectively.

Another important development which now forms the basic solution strategy in engineering is the invention of the Finite Element Method (FEM). This method was outlined by the mathematician Richard Courant (1943) and was developed independently, and put to practical use on computers, from 50s to 60s, by the aeronautical engineers M. J. Taylor, Ray W. Clough and others in United States, the civil engineers J. H. Argyris and O. C. Zienkiewicz in Britain, and the mathematician FENG Kang in China. The method originates in solving elasticity problems, and expands beyond imagination to form the basic building blocks of computational mechanics. The newest applications of FEM include materials microstructures, biological structures and medical procedures.

The theoretical aspect of elasticity has also been enriched in the recent years. The classical development of elasticity never fully confronted the problem of finite elastic straining, in which material fibers change their lengths by other than very small amount. The widespread use of natural polymeric materials, such as natural rubbers put the analysis of finite deformation elasticity in large demand. The works of British-born engineer and applied mathematician Ronald S. Rivlin (1960) for finite deformation elasticity provided solutions for tension, torsion, bending and inflection at extremely large deformation. He also participated in introducing the tensor representation theorem (Rivlin-Ericksen theorem) for isotropic elasticity. The so-called the Mooney-Rivlin theory gives a fair description for the subject of rubber elasticity.

Another important arena for the development of elasticity is targeted for anisotropic materials. In this area, the works of J. D. Eshel by, S. G. Lehnitskii and A. N. Stroh (1959－1962) made a revolutionary change of the field. The tragic life of Stroh and his brilliant contribution in a short professional period of ten years was prescribed in the eulogy in the comprehensive book of T. C. T. Ting (1996). The scope of this lecture note is unfortunately not able to cover this subject, and the readers may consult the book by Ting as a reference.

5. Applications of Elasticity

Construction

Elasticity finds applications everywhere. Its applications for basic infrastructure are the first on the list. The students may read an interesting book "Mechanics and Engineering" (2000) edited by Wu Yousheng to explore the wonders of elasticity. The examples include the integrity for Three Gorges Dam, critical rotating speed for an electric power generator, and the design to suppress the wind-induced wriggling of the TV antenna pole, known as the "East Pearl", in Shanghai. Of particular interest are the elasticity problems in the four basic construction projects for China to exploit her west. The first project concerns the gas-line construction from the west to the east. The creep of the sand domes may induce unpredictable high stress on the gas-line pipes, causing their dynamic rupture. Another issue concerns with the building of super-size gas tanks that impose severe structure reliability problem. The second project concerns the railway from Qinghai Province to Tibet, a good portion of it will pass through the highland with altitude higher than 5000m above sea level. The difficulty largely lies on the mechanics of soil under extremely low temperature. The elasticity of the problem is intertwined with the capillary fluid as one considers the maze of water-ice mixture.

The third project concerns the power transmission from the west to the east, involving elasticity problems such as the wriggling of high voltage electric cables and the critical rpm of the electric generators. The fourth project concerns highways construction, where the elasticity of the road layer, in combining with a reliable test method by stress waves, will safeguard the quality control of the road.

Earthquake

The civil engineers can quantify the event of earthquakes, as well as their effect on structures via elasticity. An earthquake manifests itself as stress waves of dilatational and shear features. Elasticity can predict the source and the amplitude of the quake, as well as its power of destruction. Investigation on faulting dynamics (an excellent review was given by J. R. Rice as the Opening Lecture of ICTAM 2000, with a Chinese translation by Gaofeng Guo in Advances in Mechanics, 2001.8.25). Most earthquakes are subsonic. The 1999 earthquake in Turkey, however, is intersonic. The study for such problem promotes the recent researches of intersonic fracture mechanics. The prediction of earthquakes relies on the detectable signals by the precursor of the earthquakes. Does the precursor exist and detectable, or not detectable in principle? This is the current research subject in elasticity. The phenomenon of failure waves was just discovered several years ago. The dispersion for the propagation of the failure wave will shed lights to the predictability of an earthquake. Another issue in which elasticity will provides a helping hand is to suppress the vibration induced by earthquakes. A good example for suppressed vibration under earthquake in ancient Japan can be found in the 33 Palace, Kyoto, where layered foundation was utilized to absorb and diffuse the shaken of earthquakes over hundreds of years. The modern example concerns with the approach of active control. A closely looped system could first sense the amplitude and frequency of the vibration, then counter-balance it by out-of-phase motion of the actuators. Largely attributed to this technique, the skyscrapers can now be built in Japan.

Astronautic Engineering

The same technique of vibration suppression is in ample demand in aeronautic and astronautic engineering. The vibration of a rocket and its payloads (such as satellites) during launching is a critical issue. The proof tests on a real size vibration table usually consume a large amount of the expenses of satellite making and occupy a substantial duration in its production period. Up on air, the fluttering of air-frame is a critical issue which endorsed the establishment of areoelasticity. Further up in the space, the gravity force diminishes, leaving only the elasticity effect for space-platforms. The vibration for the solar-energy cell panel dictates the success of a space mission. The functionality of a missile alert satellite depends on the accuracy to tone down its frame vibration when a gigantic radar antenna rotates to scan the sky. For the scientific exploration in the space, the FAST project utilizes the unique geological formation in China. A downscaled (one to ten) experiment is being undertaken for the FAST cable suspension system just on the east side of our department building. Our design has to meet a stringent requirement of controlling the position of the feedback source at the accuracy of 4 millimeters for a cable system with a more than 500m span. Vibration suppression of multiple levels is crucial for the design.

Integrated Circuit

We now enter a new application area of elasticity, the integrated circuits (IC) of microelectronic devices. By mismatched layers of tiny dimension, thermal stress is induced during the manufacture and operation of the microelectronic devices. Elasticity plays a central role in estimating the integrity of ICs. As a matter of fact, the current technology utilizes the stress of multi-layer epitaxy to create stresses in different layers so that the energy gap can be modulated. We refer the forthcoming book by L. B. Freund for this subject. The stress due to elasticity also control the self-assembly of quantum dots, the future generation of microelectronic devices.

Nanotechnology

Nanotechnology spearheads the industrial development in the 21st century. You will be surprised that the theory of elasticity, established in the context of bulk continuum, can be applied even in nanometer scale. Several interesting examples have been demonstrated in the literatures. A carbon nanotube of a few nanometer in diameter can be used, by its elastic vibration resonance, to weigh a gene. A concept of GHz mechanical oscillator is proposed by exciting the nested shells in a multiwalled carbon nanotube. The mismatch bi-layer thin film of a few nanometers of thickness can curl into a cylindrical tube with insulator on the outside and metal in the inside. The nano-lines of this type are proposed by Russian scientists as the new generation of microelectronic interconnection. The interested readers may find further examples from a forthcoming book "Interfacial and Nanoscale Failure, eds. by W. Gerberich and W. Yang (2003)".

Biology and Biomechanics

The area of biological materials provides endless opportunity for elasticity. The force generated by a mussel relies on the elasticity of various crossbridges acting in the retractable myomeres. The elasticity of a single molecule was measured by the "laser frozen technique" by Steven Zhu. Biological structures provide marvelous examples to inspire man-made structures. To resist wind induced bending, a bamboo has highly anisotropic and inhomogeneous structure: stronger in the longitudinal direction than the radial direction, and denser in the outside than the inside. The wing of a dragonfly, composed of a complicated structure of reinforced ribs, provides a wonder of the natural evolution that coincides with the result of topological optimization.

Sports

Various sport facilities rely on the development of elasticity. An exciting sport for young peoples is cliff jumping, in which your excitement critically depends on the length and the elasticity of a string which ties you and the diving platform. One homework problem will be assigned on this subject so that you can get the feeling for the controlling power of elasticity. In archery, the selection for the string and the bow is also important for shooting your arrows farther and faster. Tennis, as one of the most popular sports, advances a lot in the past 10 years. The serving speed increase from 100 mph in the John McEnroe era to 138 mph for current power severs. The increment is contributed slightly to the athletes strength, a certain degree to enhanced elasticity behavior of the rackets, and largely due to the advance for a study on the interaction between racket and the tennis ball, both treated in the FEM program ABAQUS as bodies of large elastic deformation. In the same category we should mention the pole vault where the graphite fiber poles play dominant role to grasp the elasticity during vaulting. The work-out machines are important for people's health and body muscle building. The state-of-arts work-out machine is switched from the weight lifting type (based on rigid body statics) to the flexible bar type (based on elasticity). The advantage is rather obvious: quiet rather than noisy; compact and sleek rather than bulky; versatile rather than single function. The sport of gymnastic relies on another aspect of elasticity. Strain gauges based on the principle of elastic-electric interaction can serve to correct the routines. Such researches are currently under way in our department for preparing the 2008 Olympics.

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Source 4：Elasticiy – Yang Wei, (English Version), 2006

Book 5：COURS D’ELASTICITE

Book 6：Mechanics of deformable solids - Elasticity

6. Figures here: http://www.chinabaike.com/article/316/416/2007/20070507109825.html