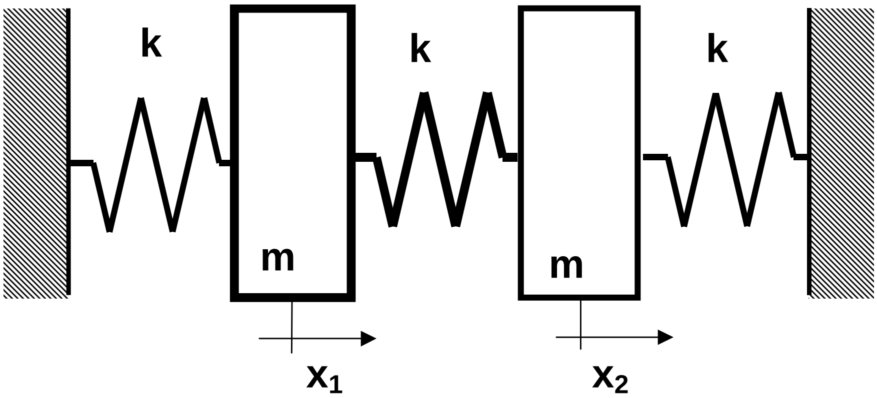
SYSTEME A 2 DEGRES DE LIBERTE



Calcul des fréquences et modes

1 - Equations du mouvement

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \left(\frac{\partial T}{\partial q_i}\right) + \left(\frac{\partial U}{\partial q_i}\right) = 0$$

avec ici

$$q_i \quad = \quad x_1 \quad \text{et} \quad x_2$$

1 – 1 Energie Cinétique

$$T = \sum T_i$$

$$T = \frac{1}{2}m\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2$$

1 – 2 *Energie de déformation*

$$U = \sum U_i$$

$$U_i = \frac{1}{2}k\,\Delta x^2$$

$$U = \frac{1}{2}k(x_1 - 0)^2 + \frac{1}{2}k(x_1 - x_2)^2 + \frac{1}{2}k(x_2 - 0)^2$$

Il y a deux équations :

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right)-\frac{\partial T}{\partial x_1}+\frac{\partial U}{\partial x_1}=0$$

et

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_2}\right)-\frac{\partial T}{\partial x_2}+\frac{\partial U}{\partial x_2}=0$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right)=?$$

$$\frac{\partial T}{\partial \dot{x}_1}=m\dot{x}_1$$

$$\frac{d}{dt}\left(-\right)=m\ddot{x}_1$$

$$\frac{\partial T}{\partial \dot{x}_2}=m\dot{x}_2$$

$$\frac{d}{dt}\left(-\right)=m\ddot{x}_2$$

$$\frac{\partial T}{\partial q_i}=?$$

$$\frac{\partial T}{\partial x_1}=0$$

$$\frac{\partial T}{\partial x_2}=0$$

$$\frac{\partial U}{\partial x_1} = ?$$

$$\frac{\partial U}{\partial x_1} = kx_1 + k(x_1 - x_2)$$

$$\frac{\partial U}{\partial x_2} = ?$$

$$\frac{\partial U}{\partial x_2} = -k(x_1 - x_2) + kx_2$$

$$m\ddot{x}_1 + kx_1 + k(x_1 - x_2) = 0$$

$$m\ddot{x}_2 - k(x_1 - x_2) + kx_2 = 0$$

En posant :

Matrice de masse

$$M = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$$

Matrice de raideur

$$K = \begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix}$$

Il vient

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \{0\}$$

<u>2 - Calcul des Pulsations Propres</u>

$$M\ddot{x} + Kx = 0$$

$x_i = X_i e^{rt}$

$$\begin{bmatrix} mr^2 + 2k & -k \\ -k & mr^2 + 2k \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \{0\}$$

$\forall t$ et $X_i \neq 0$

$$\left(mr^2 + 2k\right)^2 - k^2 = 0$$

$$\left(mr^2 + k\right)\left(mr^2 + 3k\right) = 0$$

$$r_1^2 = -\frac{k}{m} \quad \text{et} \quad r_2^2 = -\frac{3k}{m}$$

$$\omega_1 = \sqrt{\frac{k}{m}} \quad \text{et} \quad \omega_2 = \sqrt{\frac{3k}{m}}$$

Modes Propres

$$r_1^2 = -\frac{k}{m}$$

$$\begin{bmatrix} -m\frac{k}{m}+2k & -k \\ -k & -m\frac{k}{m}+2k \end{bmatrix} \begin{Bmatrix} X_{11} \\ X_{12} \end{Bmatrix} = \{0\}$$

$$\phi_1 = \begin{Bmatrix} X_{11} \\ X_{12} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

et

$$r_2^2 = -\frac{3k}{m}$$

$$\begin{bmatrix} -m\frac{3k}{m}+2k & -k \\ -k & -m\frac{3k}{m}+2k \end{bmatrix} \begin{Bmatrix} X_{21} \\ X_{22} \end{Bmatrix} = \{0\}$$

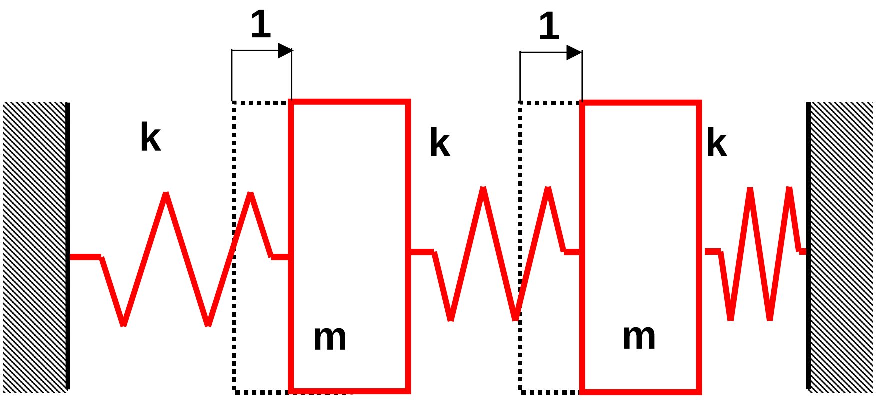
$$\phi_2 = \begin{Bmatrix} X_{21} \\ X_{22} \end{Bmatrix} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

<u>Matrice Modale des modes</u>

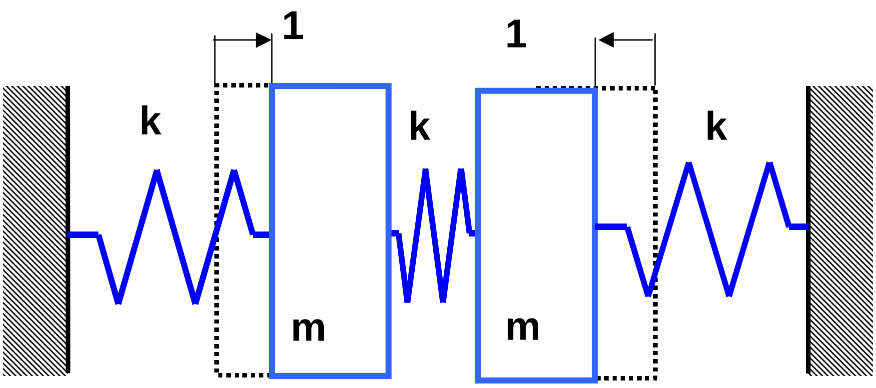
$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = [\phi_1 \quad \phi_2] \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$$

$$\{x\} = [\Phi]\{p\}$$

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$$



Premier Mode de Vibration



Second Mode de Vibration

$$M = [\phi_1 \quad \phi_2]^t \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} [\phi_1 \quad \phi_2] = \begin{bmatrix} 2m & 0 \\ 0 & 2m \end{bmatrix}$$

$$K = [\phi_1 \quad \phi_2]^t \begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix} [\phi_1 \quad \phi_2] = \begin{bmatrix} 2k & 0 \\ 0 & 6k \end{bmatrix}$$

$$\begin{bmatrix} 2m & 0 \\ 0 & 2m \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} + \begin{bmatrix} 2k & 0 \\ 0 & 6k \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \{0\}$$

M et K matrices diagonales (découplages)

$$\sqrt{\frac{K_{11}}{M_{11}}} = \sqrt{\frac{2k}{2m}} = \omega_1 \quad \text{et} \quad \sqrt{\frac{K_{22}}{M_{22}}} = \sqrt{\frac{6k}{2m}} = \omega_2$$