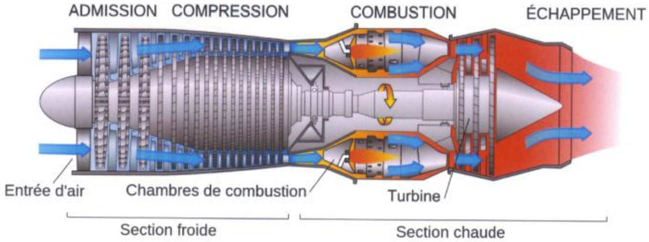
Chapitre V

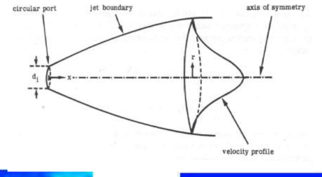
Flammes de Diffusion de Type Jet

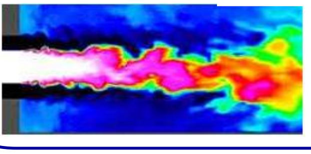
1

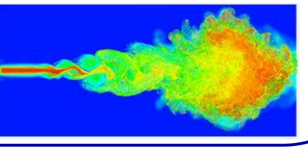
Turboréacteurs



Jet issu d'une buse circulaire en régime turbulent

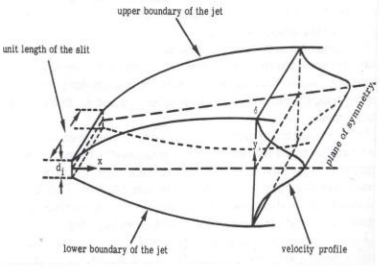




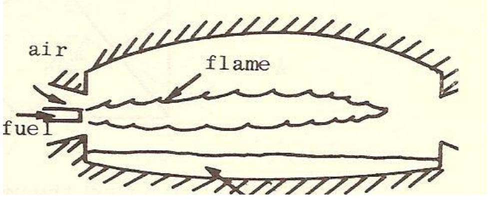


2

Jet issu d'une buse rectangulaire

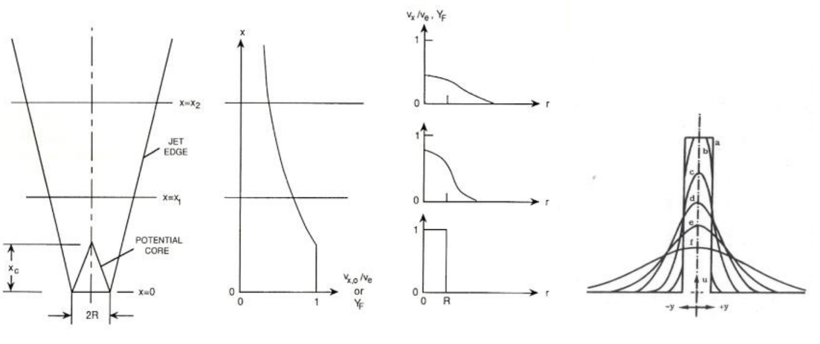


Four industriel avec un seul orifice pour injection du combustible liquide ou de gaz



3

Profils des variables : $\dfrac{u}{u_m}$, $\dfrac{T - T_\infty}{T_m - T_\infty}$ et $\dfrac{Y_F}{Y_{Fm}}$



Cœur potentiel, x<xc

Expansion, x>xc

4

Jet issu d'une buse rectangulaire de type couche limite non réactif - 2D

4 Hypothèses
$$\begin{cases} 1) & \rho = \text{constante} \\ 2) & \Delta Y_i \quad \text{(loi de Fick)} \\ 3) & Sc = \nu / D_{AB} \equiv 1 \quad \text{(viscosité cinématique /diffusivité des espèces)} \\ 4) & \dfrac{\partial^2}{\partial y^2} >> \dfrac{\partial^2}{\partial x^2} \quad \text{(diffusion radiale importante)} \end{cases}$$

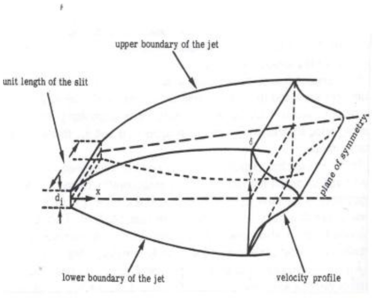
Equation de conservation de la masse

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Conservation de la quantité de mouvement

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (\nu + \nu_t)\frac{\partial^2 u}{\partial y^2}$$

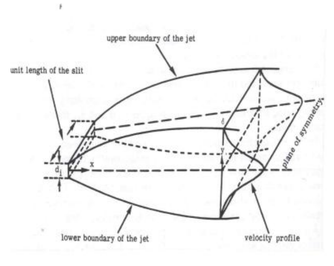
$$P = P_\infty$$



Viscosité turbulente : $\nu_t = C'\delta^2 \left|\frac{\partial u}{\partial y}\right|$

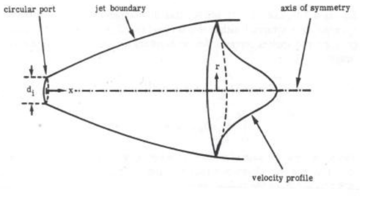
Jet rectangulaire :

C'=0.0185



Jet circulaire :

C'=0.0128



Conservation de l'énergie

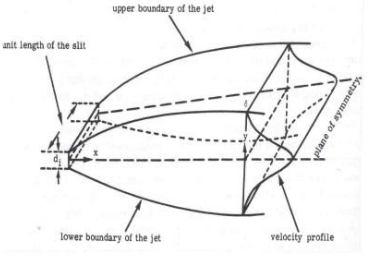
$$u\frac{\partial}{\partial x}(T-T_\infty)+v\frac{\partial}{\partial y}(T-T_\infty)=(\alpha+\alpha_t)\frac{\partial^2}{\partial y^2}(T-T_\infty)$$

Conservation des espèces chimiques

$$u\frac{\partial Y_F}{\partial x}+v\frac{\partial Y_F}{\partial y}=D_{AB}\frac{\partial^2 Y_F}{\partial y^2}$$

$$Y_O=1-Y_F$$

Conditions limites

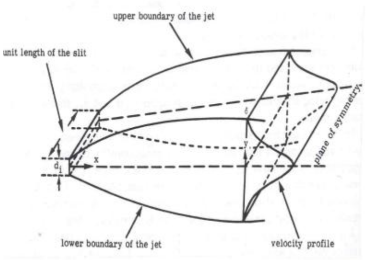


Sur l'axe du jet (y=0)

$$0\le x\le\infty,\qquad v=0,\qquad \left.\begin{array}{l}\partial u/\partial y\\ \partial(T-T_\infty)/\partial y\\ \partial Y_F/\partial y\end{array}\right\}=0$$

Loin de l'axe $(y\ge\delta)$ (l'air est au repos)

$$0\le x\le\infty,\qquad \left.\begin{array}{l}u\\ T-T_\infty\\ Y_F\end{array}\right\}=0,\quad \left.\begin{array}{l}\partial u/\partial y\\ \partial(T-T_\infty)/\partial y\\ \partial Y_F/\partial y\end{array}\right\}=0$$



Loin de la tuyère de sortie $(x \to \infty)$

$$0 \le y \le \infty, \quad \left.\begin{matrix} u \\ (T - T_\infty) \\ Y_F \end{matrix}\right\} = 0, \quad \left.\begin{matrix} \partial u / \partial y \\ \partial (T - T_\infty) / \partial y \\ \partial Y_F / \partial y \end{matrix}\right\} = 0$$

À la sortie de la tuyère (x=0)

$$0 \le y \le d_i / 2, \quad u = u_i, \quad T = T_i, \quad Y_F = 1$$

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Solutions analytiques intégrales

$$u \frac{\partial u}{\partial x} = -u \frac{\partial v}{\partial y}$$

$$\frac{\partial (vu)}{\partial y} = u \frac{\partial v}{\partial y} + v \frac{\partial u}{\partial y}$$

$$v \frac{\partial u}{\partial y} = u \frac{\partial u}{\partial x} + \frac{\partial (uv)}{\partial y}$$

Après quelques substitutions et mises en formes

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = (\nu + \nu_t) \frac{\partial^2 u}{\partial y^2} \quad \Rightarrow \quad \frac{\partial u^2}{\partial x} + \frac{\partial (uv)}{\partial y} = (\nu + \nu_t) \frac{\partial^2 u}{\partial y^2}$$

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Intégration de y=0 à $y=\delta$ et y=0 à $y=\delta/2$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \Rightarrow \quad v_\delta=\frac{d}{dx}\int_0^\delta u\,dy \qquad \text{(opposée à y)}$$

$$v_{\delta/2}=\frac{d}{dx}\int_0^{\delta/2} u\,dy$$

$$\frac{\partial u^2}{\partial x}+\frac{\partial (uv)}{\partial y}=(\nu+\nu_t)\frac{\partial^2 u}{\partial y^2} \quad \Rightarrow \quad \frac{d}{dx}\int_0^\delta u^2\,dy+\left[uv\right]_\delta=\left[(\nu+\nu_t)\frac{\partial u}{\partial y}\right]_\delta$$

$$\frac{d}{dx}\int_0^{\delta/2} u^2\,dy+\left[uv\right]_{\delta/2}=\left[(\nu+\nu_t)\frac{\partial u}{\partial y}\right]_{\delta/2}$$

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Conditions aux limites : $y=\delta$ $u=0$ et $\frac{\partial u}{\partial y}=0$

Quantité de mouvement du jet en fonction de x

$$\frac{d}{dx}\int_0^\delta u^2\,dy=0 \Leftrightarrow \frac{d}{dx}\int_0^\delta \rho u^2\,dy=0$$

Quantité de mouvement initiale, propriétés à la sortie de la tuyère:

$$\int_0^\delta u^2\,dy=\frac{u_i^2 d_i}{2}$$

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Conservation de l'énergie

$$\frac{\partial u(T-T_\infty)}{\partial x}+\frac{\partial v(T-T_\infty)}{\partial y}=u\frac{\partial (T-T_\infty)}{\partial x}+v\frac{\partial (T-T_\infty)}{\partial y}+(T-T_\infty)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

$$\frac{\partial u(T-T_\infty)}{\partial x}+\frac{\partial v(T-T_\infty)}{\partial y}=(\alpha+\alpha_t)\frac{\partial^2 (T-T_\infty)}{\partial y^2}$$

Intégration de y=0 à $y=\delta$

$$\frac{d}{dx}\int_0^\delta u(T-T_\infty)dy+\left[v(T-T_\infty)\right]_\delta=\left[(\alpha+\alpha_t)\frac{\partial (T-T_\infty)}{\partial y}\right]_\delta$$

Conditions limites : $y=\delta$ $\quad (T-T_\infty)=0$ et $\frac{\partial (T-T_\infty)}{\partial y}=0$

$$\frac{d}{dx}\int_0^\delta u(T-T_\infty)dy=0 \quad\Leftrightarrow\quad \frac{d}{dx}\int_0^\delta \rho C_p u(T-T_\infty)dy=0$$

Energie initiale à la tuyère: $\int_0^\delta u(T-T_\infty)dy=u_i(T_i-T_\infty)\frac{d_i}{2}$

Conservation de la concentration du combustible

$$\frac{\partial uY_F}{\partial x}+\frac{\partial vY_F}{\partial y}=D_{AB}\frac{\partial^2 Y_F}{\partial y^2}$$

Intégration de y=0 à $y=\delta$

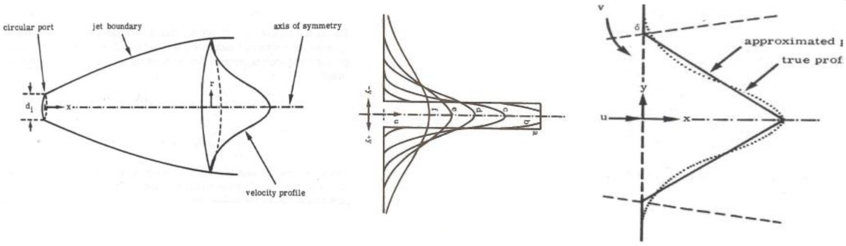
$$\frac{d}{dx}\int_0^\delta uY_F dy+\left[vY_F\right]_\delta=\left[D_{AB}\frac{\partial Y_F}{\partial y}\right]_\delta$$

Conditions limites : $y=\delta$ $\quad Y_F=0$ et $\frac{\partial Y_F}{\partial y}=0$

$$\frac{d}{dx}\int_0^\delta uY_F dy=0 \quad\Leftrightarrow\quad \frac{d}{dx}\int_0^\delta \rho uY_F dy=0$$

Combustible pur à la tuyère : $\int_0^\delta uY_F dy=u_iY_{Fi}\frac{d_i}{2}=u_i\frac{d_i}{2}$

Forme intrinsèque du profil de vitesse axiale issu d'un jet



Principe de similitude

$$\frac{u(x,y)}{u_m(x)} = 1 - \frac{y}{\delta} \quad \rightarrow \quad u(x,y) = \left(1 - \frac{y}{\delta}\right) u_m(x) \qquad 0 \le |y| \le \delta$$

$$\frac{d\,u_m(x)^n}{dx} \quad \Rightarrow \quad \int \frac{d\,u_m(x)^n}{dx} \Rightarrow \quad \text{solution analytique}$$

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Forme intrinsèque du profil de vitesse axiale

$$\frac{u}{u_m} = 1 - \frac{y}{\delta} \quad \rightarrow \quad u = \left(1 - \frac{y}{\delta}\right) u_m \qquad 0 \le |y| \le \delta$$

Conservation de la masse

$$v_\delta = \frac{d}{dx}\int_0^\delta u\,dy \qquad \Rightarrow \qquad v_\delta = \frac{d}{dx}\left(u_m \frac{\delta}{2}\right)$$

$$v_{\delta/2} = \frac{d}{dx}\int_0^{\delta/2} u\,dy \qquad \Rightarrow \qquad v_{\delta/2} = \frac{d}{dx}\left(3u_m \frac{\delta}{8}\right)$$

Quantité de mouvement

$$\frac{d}{dx}\int_0^\delta u^2\,dy = 0 \Rightarrow \qquad \frac{d}{dx}(u_m^2 \delta) = 0$$

$$\frac{d}{dx}\int_0^{\delta/2} u^2\,dy + [uv]_{\delta/2} = \left[(\nu + \nu_t)\left|\frac{\partial u}{\partial y}\right|\right]_{\delta/2} \quad \Rightarrow \quad \frac{7}{24}\frac{d}{dx}(u_m^2 \delta) + u_m \frac{v_{\delta/2}}{2} = (\nu + \nu_t)\frac{u_m}{\delta}$$

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Quelques substitutions et mises en formes

1) $v_\delta = \frac{d}{dx}(u_m \frac{\delta}{2})$

2) $u_m^2 \delta = \frac{u_i^2 d_i}{2} \quad \Rightarrow \quad \delta = \frac{u_i^2 d_i}{2 u_m^2}$ $\qquad (x = 0,\ u_m = u_i,\ \delta = d_i / 2)$

3) $\frac{3}{16}\frac{d}{dx}(u_m \delta) = \frac{(\nu + \nu_t)}{\delta}$

Expression différentielle en régime laminaire, $\nu_t = 0$, éliminer δ

$$\frac{d}{dx}(1/u_m) = \frac{64\nu}{3u_i^4 d_i^2} u_m^2$$

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Intégration :

$$\frac{1}{3u_m^3} = \frac{64\nu}{3u_i^4 d_i^2} x + C$$

Conditions aux limites : x=0, u_m=u_i et $\delta = d_i / 2$

$$\frac{u_m}{u_i} = \left[1 + \frac{64\nu}{u_i d_i}\frac{x}{d_i}\right]^{-1/3} \quad \Leftrightarrow \quad \frac{u_m}{u_i} = \left[1 + \frac{64}{Re_i}\frac{x}{d_i}\right]^{-1/3} \qquad \text{avec} \qquad Re_i \equiv \frac{u_i d_i}{\nu}$$

$$u_m^2 \delta = \frac{u_i^2 d_i}{2} \qquad \Rightarrow \quad \delta$$

$$v_\delta = \frac{d}{dx}(u_m \frac{\delta}{2}) \qquad \Rightarrow \quad v_\delta$$

$$\frac{u}{u_m} = 1 - \frac{y}{\delta} \qquad \Rightarrow \quad \frac{u}{u_i}$$

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Epaisseur de la couche limite

$$\frac{2\delta}{d_i} = \left[1 + \frac{64}{Re_i}\frac{x}{d_i}\right]^{2/3}$$

Vitesse d'expansion ou d'entraînement

$$\frac{v_\delta}{u_i} = \frac{16}{3Re_i}\left[1 + \frac{64}{Re_i}\frac{x}{d_i}\right]^{-2/3}$$

Vitesse axiale

$$\frac{u}{u_i} = \left[1 + \frac{64}{Re_i}\frac{x}{d_i}\right]^{-1/3}\left[1 - 2\left(\frac{y}{d_i}\right)\left(1 + \frac{64}{Re_i}\frac{x}{d_i}\right)^{-2/3}\right]$$

Débit massique (kg/s)

$$\dot{W} = 2\int_0^\delta \rho u dy \Rightarrow \frac{\dot{W}}{\rho u_i d_i} = 1 + 0.25\left[1 + \frac{64}{Re_i}\frac{x}{d_i}\right]^{1/3}$$

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Loin de la sortie d'un jet, soit $\frac{64}{Re_i}\frac{x}{d_i} >> 1$

Epaisseur de la couche limite : $\frac{2\delta}{d_i} = \left(\frac{64x}{Re_i d_i}\right)^{2/3}$

Vitesse d'expansion ou d'entraînement : $\frac{v_\delta}{u_i} = \left(\frac{16}{3Re_i}\right)\left(\frac{Re_i d_i}{64x}\right)^{2/3}$

Vitesse axiale : $\frac{u}{u_i} = \left(\frac{Re_i d_i}{64x}\right)^{1/3}\left[1 - 2\left(\frac{y}{d_i}\right)\left(\frac{Re_i d_i}{64x}\right)^{2/3}\right]$

Débit massique (kg/s) : $\frac{\dot{W}}{\rho u_i d_i} = 1 + 0.25\left[\frac{64}{Re_i}\frac{x}{d_i}\right]^{1/3}$

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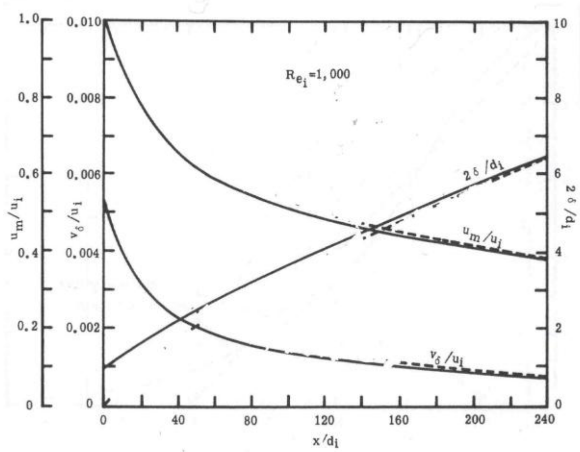
$\alpha = \nu = D \quad$ et $\quad \dfrac{64}{Re_i}\dfrac{x}{d_i} >> 1$

$$\frac{T - T_\infty}{T_i - T_\infty} = \frac{Y_F}{Y_{Fi}} = \frac{u}{u_i} = \left(\frac{Re_i d_i}{64x}\right)^{1/3}\left[1 - 2\left(\frac{y}{d_i}\right)\left(\frac{Re_i d_i}{64x}\right)^{2/3}\right]$$



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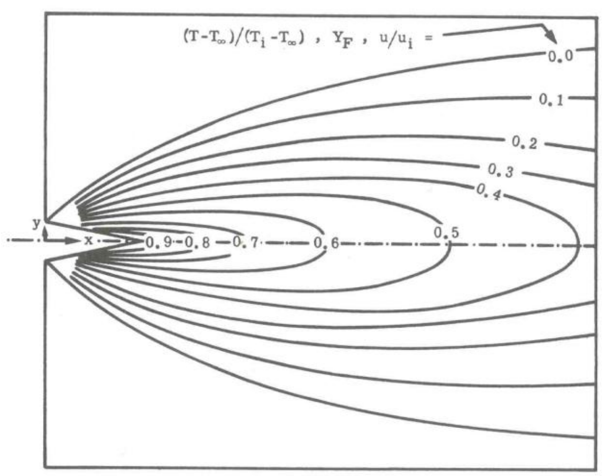
$$\frac{T - T_\infty}{T_i - T_\infty} = \frac{Y_F}{Y_{Fi}} = \frac{u}{u_i} = \left(\frac{Re_i d_i}{64x}\right)^{1/3}\left[1 - 2\left(\frac{y}{d_i}\right)\left(\frac{Re_i d_i}{64x}\right)^{2/3}\right]$$



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Ecoulement en régime turbulent

$Re_i > 2000 \quad \nu_t >> \nu \quad$ et $\quad \nu_t = C' u_m \delta$

$$\frac{3}{16}\frac{d}{dx}(u_m\delta) = \frac{(\nu+\nu_t)}{\delta} \quad \Rightarrow \quad \frac{3}{16}\frac{d}{dx}(u_m\delta) = C' u_m$$

$$u_m^2 \delta = \frac{u_i^2 d_i}{2}$$

$$-\frac{1}{u_m^3} d u_m^2 = \frac{32}{3}\frac{C'}{u_i^2 d_i} dx \quad \Rightarrow \quad u_m^{-2} = \frac{64}{3}\frac{C'}{u_i^2 d_i} x + C$$

Conditions aux limites : $\quad x=0,\ u_m=u_i$ et $\quad \delta = d_i / 2$

Vitesse maximale sur l'axe :

$$\frac{u_m}{u_i} = \left[1 + \frac{64}{3} C' \frac{x}{d_i}\right]^{-1/2}$$

$$v_\delta = \frac{d}{dx}(u_m \frac{\delta}{2}) \quad \Rightarrow \quad v_\delta$$

$$u_m^2 \delta = \frac{u_i^2 d_i}{2} \quad \Rightarrow \quad \delta$$

$$\frac{u}{u_m} = 1 - \frac{y}{\delta} \quad \Rightarrow \quad \frac{u}{u_i}$$

Vitesse maximale sur l'axe :

$$\frac{u_m}{u_i}=\left[1+\frac{64}{3}C'\frac{x}{d_i}\right]^{-1/2}$$

Epaisseur de la couche limite :

$$\frac{2\delta}{d_i}=\left[1+\frac{64}{3}C'\frac{x}{d_i}\right]$$

Vitesse d'expansion ou d'entraînement :

$$\frac{v_\delta}{u_i}=\frac{8C'}{3}\left[1+\frac{64}{3}C'\frac{x}{d_i}\right]^{-1/2}$$

Vitesse axiale

$$\frac{u}{u_i}=\left[1+\frac{64}{3}C'\frac{x}{d_i}\right]^{-1/2}\left[1-2\frac{y}{d_i}\left(1+\frac{64}{3}C'\frac{x}{d_i}\right)^{-1}\right]$$

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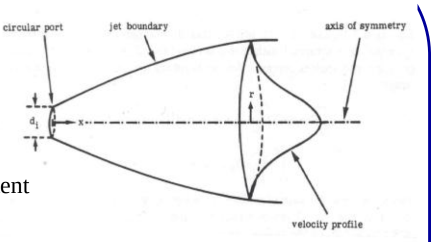
Jet issu d'une buse circulaire (2D) non réactif



Conservation de la masse

$$\frac{\partial ru}{\partial x}+\frac{\partial rv}{\partial r}=0$$

Conservation de la quantité de mouvement

$$ru\frac{\partial u}{\partial x}+rv\frac{\partial u}{\partial r}=(\nu+\nu_t)\frac{\partial}{\partial r}r\frac{\partial u}{\partial r}$$

Conservation de l'énergie

$$ru\frac{\partial (T-T_\infty)}{\partial x}+rv\frac{\partial (T-T_\infty)}{\partial r}=(\alpha+\alpha_t)\frac{\partial}{\partial r}r\frac{\partial (T-T_\infty)}{\partial r}$$

Conservation des espèces chimiques

$$ru\frac{\partial Y_F}{\partial x}+rv\frac{\partial Y_F}{\partial r}=D_{AB}\frac{\partial}{\partial r}\left(r\frac{\partial Y_F}{\partial r}\right)$$

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Intégration de r=0 à $r = \delta$

$$v_\delta = \frac{1}{\delta}\frac{d}{dx}\int_0^\delta r u\,dr$$

$$\frac{d}{dx}\int_0^\delta r\,u^2\,dr = 0$$

$$\int_0^\delta 2\pi r \rho u C_p (T - T_\infty) dr = \pi d_i^2 \rho u_i C_p (T - T_\infty)/4$$

$$\int_0^\delta 2\pi r \rho u Y_F dr = \pi d_i^2 \rho u_i Y_{Fi}/4$$

Intégration de r=0 à $r = \delta/2$

$$v_{\delta/2} = \frac{2}{\delta}\frac{d}{dx}\int_0^{\delta/2} r u\,dr$$

$$\frac{d}{dx}\int_0^{\delta/2} r u^2\,dr + [ruv]_{\delta/2} = \left[(\nu + \nu_t) r \frac{\partial u}{\partial r}\right]_{\delta/2}$$

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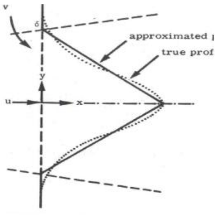
Forme intrinsèque du profil de vitesse axiale



$$\frac{u}{u_m} = 1 - \frac{y}{\delta} \quad \rightarrow \quad u = \left(1 - \frac{y}{\delta}\right) u_m \qquad 0 \le |y| \le \delta$$

Vitesse axiale

$$\frac{u}{u_i} = \left[1 + \frac{48}{Re_i}\frac{x}{d_i}\right]^{-1} \left[1 - 2\left(\frac{r}{d_i}\right)\left(1 + \frac{48}{Re_i}\frac{x}{d_i}\right)^{-1}\right]$$

Vitesse d'expansion

$$\frac{v_\delta}{u_i} = \left(\frac{4}{Re_i}\right)\left(1 + \frac{48}{Re_i}\frac{x}{d_i}\right)^{-1}$$

Epaisseur de la couche limite

$$\frac{2\delta}{d_i} = \left(1 + \frac{48}{Re_i}\frac{x}{d_i}\right) \qquad Re_i = \frac{u_i d_i}{\nu}$$

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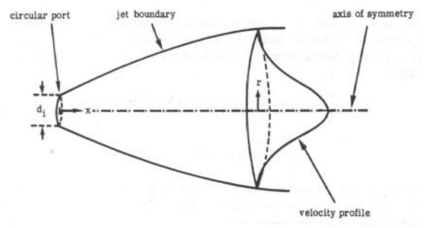
Jet circulaire en régime turbulent $(\nu_t >> \nu \quad \text{et} \quad \nu_t = C' u_m \delta)$

Vitesse axiale

$$\frac{u}{u_i} = \left[1+24C'\frac{x}{d_i}\right]^{-1}\left[1-\frac{2r}{d_i}\left(1+24C'\frac{x}{d_i}\right)^{-1}\right]$$

Vitesse d’expansion

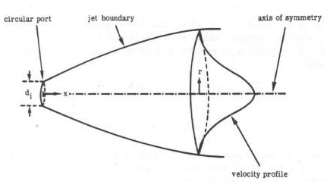


$$\frac{v_\delta}{u_i} = 2C'\left[1+24C'\frac{x}{d_i}\right]^{-1}$$

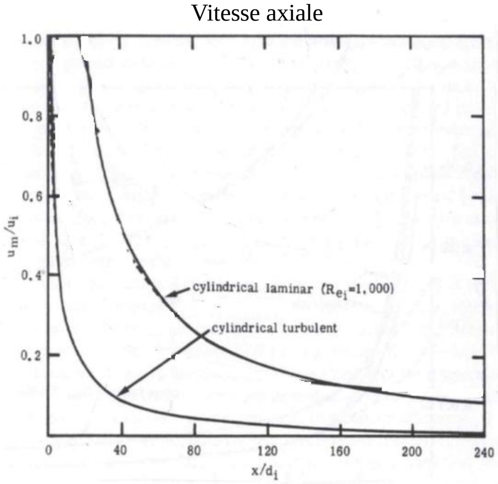
Epaisseur de la couche limite

$$\frac{2\delta}{d_i} = \left[1+24C'\frac{x}{d_i}\right]$$

Comparaison entre un jet circulaire laminaire et celui turbulent

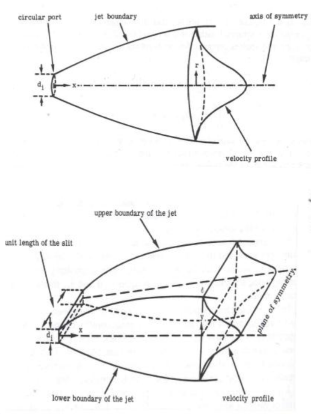


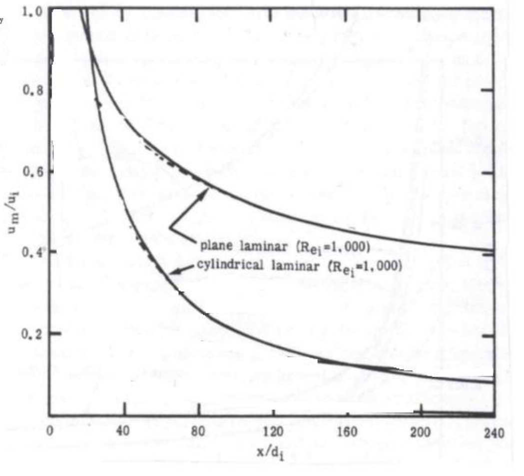
Vitesse axiale



Comparaison entre un jet circulaire et celui rectangulaire en régime laminaire

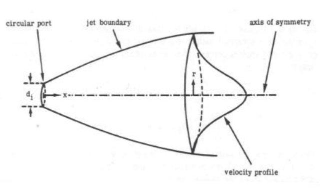
Vitesse axiale

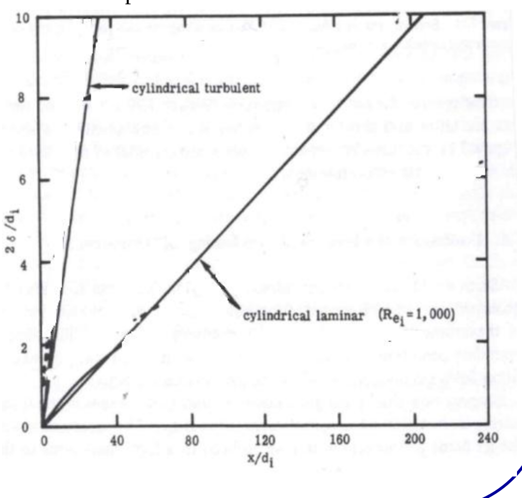




Comparaison entre un jet circulaire laminaire et celui turbulent

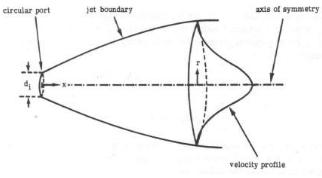
Epaisseur de la couche limite

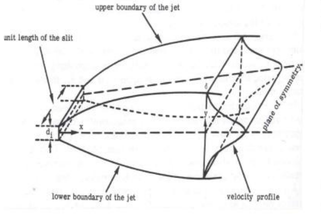


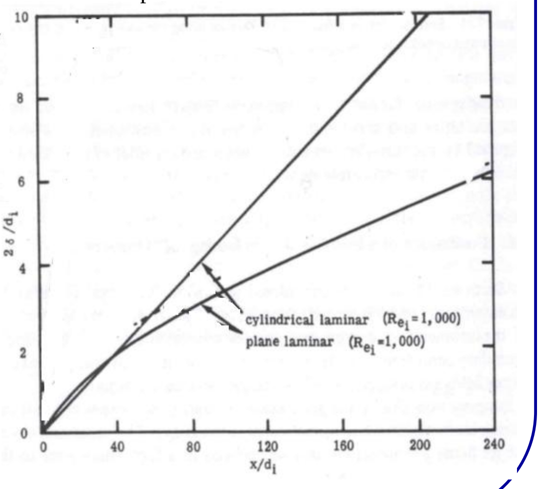


Comparaison entre un jet circulaire et celui rectangulaire en régime laminaire

Epaisseur de la couche limite

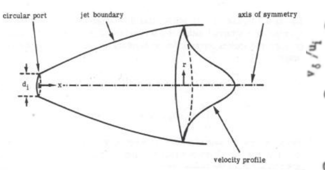


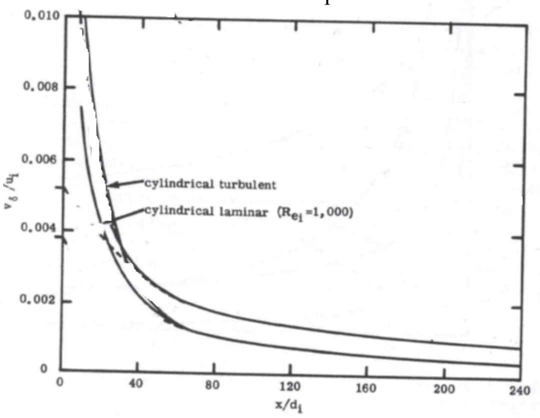




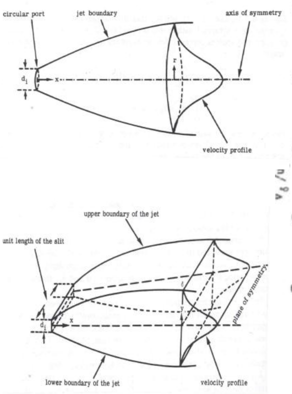
Comparaison entre un jet circulaire laminaire et celui turbulent

Vitesse d'expansion

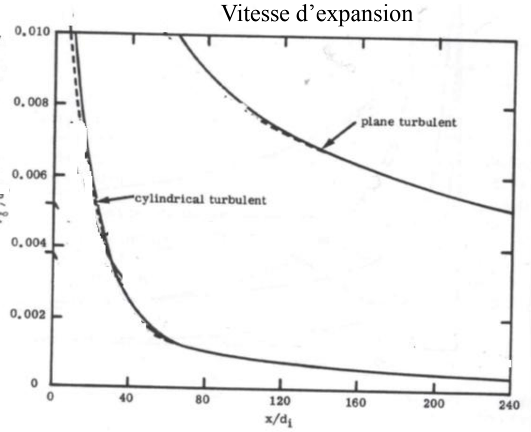




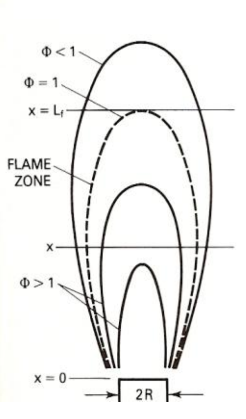
Comparaison entre un jet circulaire et celui rectangulaire en régime turbulent

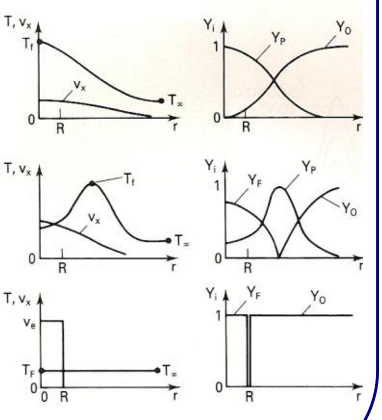


Vitesse d'expansion

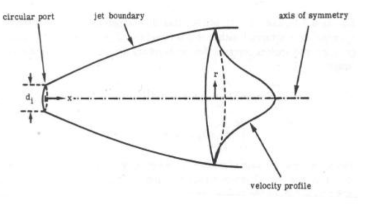


Description physique d'un jet réactif issu d'une buse circulaire (2D)





Equations de conservation de base



8 hypothèses

1) Jet combustible, écoulement axisymétrique
2) Trois espèces : F, O et P
3) Réaction stoechiométrique, infiniment rapide
4) Transfert par diffusion binaire (loi de Fick)
5) $Le = \alpha / D_{AB} \equiv 1$
6) On néglige le rayonnement
7) Diffusion radiale importante, diffusion axiale négligé
8) Axe de la flamme ouverte

$Le = \alpha / D_{AB} \equiv 1$

$\alpha = \nu = D_O = D_F = D_P = D$

Conservation de l'énergie

$$ru\frac{\partial(T - T_\infty)}{\partial x} + rv\frac{\partial(T - T_\infty)}{\partial r} = (\nu + \nu_t)\frac{\partial}{\partial r} r\frac{\partial(T - T_\infty)}{\partial r} + \frac{\dot{q}''' r}{\rho C_p}$$

Conservation des espèces chimiques

$$ru\frac{\partial Y_j}{\partial x} + rv\frac{\partial Y_j}{\partial r} = (\nu + \nu_t)\frac{\partial}{\partial r} r\frac{\partial Y_j}{\partial r} + \frac{W_j''' r}{\rho}$$

$j = F, O, P$

Conservation des espèces chimiques (F, O, P)

$$ru\frac{\partial Y_F}{\partial x}+rv\frac{\partial Y_F}{\partial r}=(\nu+\nu_t)\frac{\partial}{\partial r}r\frac{\partial Y_F}{\partial r}+\frac{W_F''' r}{\rho}$$

$$ru\frac{\partial (Y_O-Y_{O\infty})}{\partial x}+rv\frac{\partial (Y_O-Y_{O\infty})}{\partial r}=(\nu+\nu_t)\frac{\partial}{\partial r}r\frac{\partial (Y_O-Y_{O\infty})}{\partial r}+\frac{W_O''' r}{\rho}$$

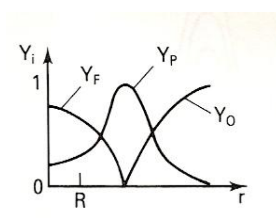
$$ru\frac{\partial \left[Y_P-(1-Y_{O\infty})\right]}{\partial x}+rv\frac{\partial \left[Y_P-(1-Y_{O\infty})\right]}{\partial r}=(\nu+\nu_t)\frac{\partial}{\partial r}r\frac{\partial \left[Y_P-(1-Y_{O\infty})\right]}{\partial r}+\frac{W_P''' r}{\rho}$$

$$Y_{N2}=(1-Y_{O\infty})$$

Approximation de la flamme mince, infiniment rapide

$$\nu_{F/O}\_kg\ F+1\_kg\ O(O_2)\rightarrow(1+\nu_{F/O})\_kg\ P+\nu_{F/O}\Delta h_c$$

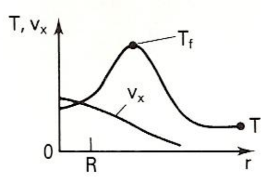
$$-\frac{W_F'''}{\nu_{F/O}}=-\frac{W_O'''}{1}=+\frac{W_P'''}{1+\nu_{F/O}}=+\frac{\dot{q}'''}{\nu_{F/O}\Delta h_c}$$



$$\frac{W_F'''}{\nu_{F/O}}+\frac{W_P'''}{1+\nu_{F/O}}=0 \qquad \frac{W_F'''}{\nu_{F/O}}+\frac{\dot{q}'''}{\nu_{F/O}\Delta h_c}=0$$

$$W_O'''+\frac{W_P'''}{1+\nu_{F/O}}=0 \qquad W_O'''+\frac{\dot{q}'''}{\nu_{F/O}\Delta h_c}=0$$

$$\frac{W_F'''}{\nu_{F/O}}-W_O'''=0$$



$$\frac{\dddot{W}_F}{\nu_{F/O}} + \frac{\dot{q}'''}{\nu_{F/O}\Delta h_c} = 0 \quad \Rightarrow \quad \dddot{W}_F + \frac{\dot{q}'''}{\Delta h_c} = 0$$

$$ru\frac{\partial Y_F}{\partial x} + rv\frac{\partial Y_F}{\partial r} = (\nu + \nu_t)\frac{\partial}{\partial r} r \frac{\partial Y_F}{\partial r} + \frac{r}{\rho}\dddot{W}_F$$

$$ru\frac{\partial C_p(T - T_\infty)/\Delta h_c}{\partial x} + rv\frac{\partial C_p(T - T_\infty)/\Delta h_c}{\partial r} = (\nu + \nu_t)\frac{\partial}{\partial r} r \frac{\partial C_p(T - T_\infty)/\Delta h_c}{\partial r} + \frac{r}{\rho}\frac{\dot{q}'''}{\Delta h_c}$$

$$ru\frac{\partial \phi_{FT}}{\partial x} + rv\frac{\partial \phi_{FT}}{\partial r} = (\nu + \nu_t)\frac{\partial}{\partial r} r \frac{\partial \phi_{FT}}{\partial r}$$

$$\phi_{FT} \equiv \left(Y_F + \frac{C_p(T - T_\infty)}{\Delta h_c} \right)$$

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$$\phi_{FP} \equiv \left(Y_F + \frac{\nu_{F/O}}{1 + \nu_{F/O}} Y_P \right) - \frac{\nu_{F/O}}{1 + \nu_{F/O}}(1 - Y_{O\infty})$$

$$\phi_{OP} \equiv \left(Y_O + \frac{1}{1 + \nu_{F/O}} Y_P \right) - \left(\frac{1 + \nu_{F/O} Y_{O\infty}}{1 + \nu_{F/O}} \right)$$

$$\phi_{FO} \equiv \left(Y_O - \frac{Y_F}{\nu_{F/O}} \right) - Y_{O\infty}$$

$$\phi_{FT} \equiv \left(Y_F + \frac{C_p(T - T_\infty)}{\Delta h_c} \right)$$

$$\phi_{OT} \equiv \left(Y_O + \frac{C_p(T - T_\infty)}{\nu_{F/O}\Delta h_c} \right) - Y_{O\infty}$$

Equation de conservation générique

$$ru\frac{\partial \phi}{\partial x} + rv\frac{\partial \phi}{\partial r} = (\nu + \nu_t)\frac{\partial}{\partial r} r \frac{\partial \phi}{\partial r}$$

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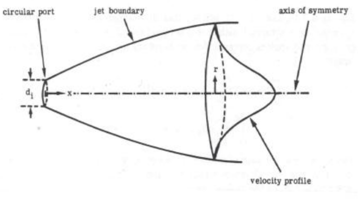
Conditions aux limites

$$x = 0, \quad 0 \le r \le d_i/2, \quad \phi = \phi_i$$

$$x = \infty, \quad 0 \le r \le \infty, \quad \phi = 0, \quad \frac{\partial \phi}{\partial r} = 0$$

$$r = 0, \quad 0 \le x \le \infty, \quad v = 0, \quad \frac{\partial \phi}{\partial r} = 0$$

$$r \ge \delta, \quad 0 \le x \le \infty, \quad \phi = 0, \quad \frac{\partial \phi}{\partial r} = 0$$



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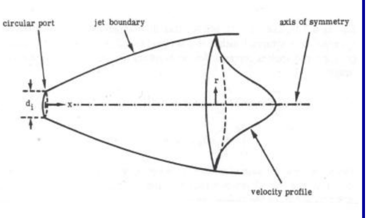
Profils des variables : $\frac{u}{u_i} \Rightarrow \frac{\phi_{FO}}{\phi_{FO,i}}, \frac{\phi_{FP}}{\phi_{FP,i}}, \frac{\phi_{OP}}{\phi_{OP,i}}, \frac{\phi_{FT}}{\phi_{FT,i}}, \frac{\phi_{OT}}{\phi_{OT,i}}$

Approche théorique : $\frac{u}{u_i} = \frac{\phi}{\phi_i} = F(x, r)$



Jet circulaire en régime laminaire

$$F(x,r) = \left[1 + \frac{48}{Re_i}\frac{x}{d_i}\right]^{-1}\left[1 - 2\frac{r}{d_i}\left(1 + \frac{48}{Re_i}\frac{x}{d_i}\right)^{-1}\right]$$

Jet circulaire en régime turbulent

$$F(x,r) = \left[1 + 24C'\frac{x}{d_i}\right]^{-1}\left[1 - 2\frac{r}{d_i}\left(1 + 24C'\frac{x}{d_i}\right)^{-1}\right]$$

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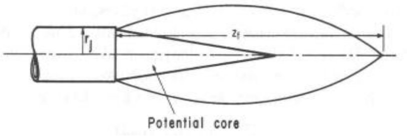
LONGUEUR DE LA FLAMME (pilotée par la convection forcée)

$$\phi_{FO} \equiv \left(Y_O - \frac{Y_F}{\nu_{F/O}} \right) - Y_{O\infty}$$



$$\phi_{FO,\,stoe} = -Y_{O\infty} \quad \text{et} \quad \phi_{FO,\,i} = -\left(\frac{1}{\nu_{F/O}} + Y_{O\infty} \right)$$

$$\frac{\phi_{FO,\,stoe}}{\phi_{FO,\,i}} = \frac{\nu_{F/O} Y_{O\infty}}{\nu_{F/O} Y_{O\infty} + 1} = F(x_c, r_c)$$

$$r_c \to 0 \quad \Rightarrow \quad \overline{x}_c$$

En régime laminaire

$$\frac{\nu_{F/O} Y_{O\infty}}{\nu_{F/O} Y_{O\infty} + 1} = \left[1 + \frac{48}{Re_i} \frac{\overline{x}_C}{d_i} \right]^{-1}$$

$$\frac{\overline{x}_C}{d_i} = \frac{u_i d_i}{48 \nu Y_{O\infty} \nu_{F/O}} \equiv \frac{Re_i}{48 Y_{O\infty} \nu_{F/O}}$$

En régime turbulent

$$\frac{\nu_{F/O} Y_{O\infty}}{\nu_{F/O} Y_{O\infty} + 1} = \left[1 + 24C' \frac{\overline{x}_C}{d_i} \right]^{-1}$$

$$\frac{\overline{x}_C}{d_i} = \frac{1}{24C' \nu_{F/O} Y_{O\infty}}$$

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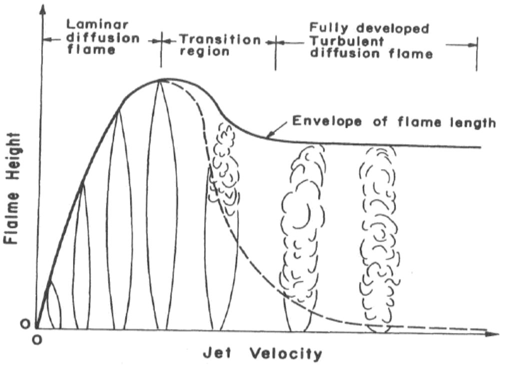
Influence de la turbulence sur la longueur de la flamme

En régime laminaire

$$\frac{\overline{x}_C}{d_i} = \frac{Re_i}{48 Y_{O\infty} \nu_{F/O}}$$

En régime turbulent

$$\frac{\overline{x}_C}{d_i} = \frac{1}{24C' \nu_{F/O} Y_{O\infty}}$$



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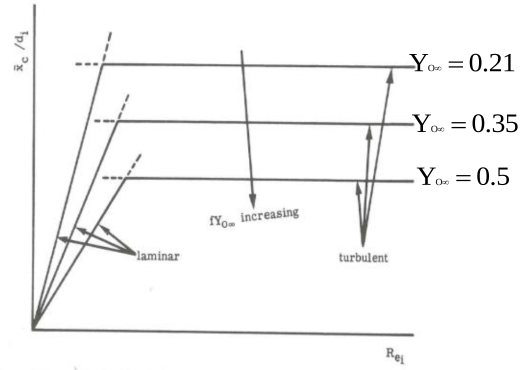
Influence de la concentration en O_2 sur la longueur de flamme

En régime laminaire

$$\frac{\bar{x}_C}{d_i} = \frac{Re_i}{48 Y_{O\infty} \nu_{F/O}}$$

En régime turbulent

$$\frac{\bar{x}_C}{d_i} = \frac{1}{24C' \nu_{F/O} Y_{O\infty}}$$



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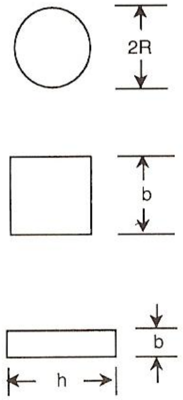
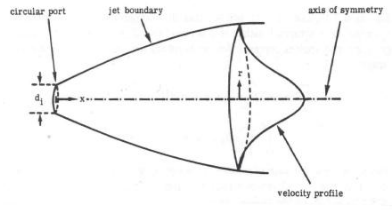
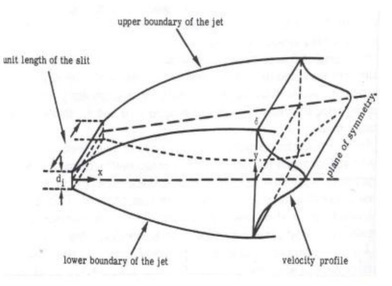
Longueur de flamme pour brûleurs circulaires et en fente de surfaces de sortie identiques







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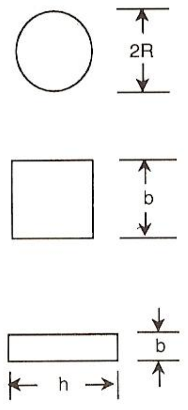
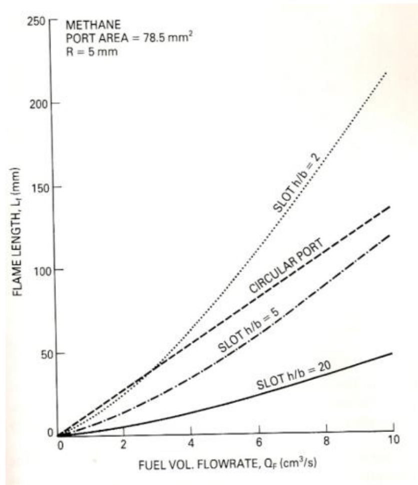
Influences du débit massique et de la géométrie sur la longueur de flamme





Influence de la stoéchiométrie sur la longueur de flamme

En régime laminaire

$$\frac{\bar{x}_C}{d_i} = \frac{Re_i}{48 Y_{O\infty} \nu_{F/O}}$$

En régime turbulent

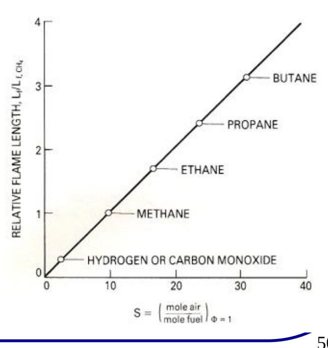
$$\frac{\bar{x}_C}{d_i} = \frac{1}{24 C' \nu_{F/O} Y_{O\infty}}$$

L_f croit avec la diminution du rapport H/C (dépendance linéaire)

CH_4 C_2H_6 C_3H_8 C_4H_8

CO et H_2 donnent les flammes les plus courtes

Longueur de flamme en fonction
de la stoéchiométrie
pour différents combustibles
et par rapport CH_4



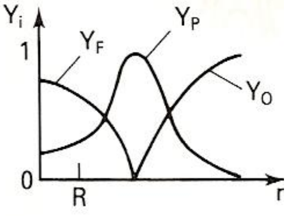
Distribution de combustible à l'intérieur de la flamme

$$\phi_{FO} \equiv \left(Y_O - \frac{Y_F}{\nu_{F/O}} \right) - Y_{O\infty}$$

A l'intérieur de la flamme

$$(0 \le |r| \le r_C): \quad Y_O = 0 \quad \Rightarrow \phi_{FO} \equiv -\left(\frac{Y_F}{\nu_{F/O}} + Y_{O\infty} \right)$$

Profils des espèces chimiques



Condition limites : $x = 0, Y_O = 0$ et $Y_F = 1$

$$\phi_{FO,\,i} = -\left(Y_{O\infty} + \frac{1}{\nu_{F/O}} \right)$$

$$\frac{\phi_{FO}}{\phi_{FO,i}} = \frac{Y_F + \nu_{F/O} Y_{O\infty}}{1 + \nu_{F/O} Y_{O\infty}} = F(x,r) \quad \Rightarrow \quad Y_F = (1 + \nu_{F/O} Y_{O\infty}) F(x,r) - \nu_{F/O} Y_{O\infty}$$

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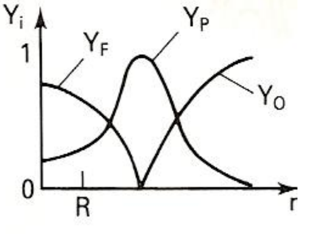
Distribution d'oxygène à l'extérieur de la flamme

$$\phi_{FO} \equiv \left(Y_O - \frac{Y_F}{\nu_{F/O}} \right) - Y_{O\infty}$$

Profils des espèces chimiques

A l'extérieur de la flamme

$$(r_C \le |r| \le \delta): \quad Y_F = 0 \quad \Rightarrow \phi_{FO} \equiv Y_O - Y_{O\infty}$$



Condition limites : $x = 0, Y_O = 0$ et $Y_F = 1$

$$\phi_{FO,\,i} = -\left(Y_{O\infty} + \frac{1}{\nu_{F/O}} \right)$$

$$\frac{\phi_{FO}}{\phi_{FO,i}} = \frac{Y_O - Y_{O\infty}}{-(1 + \nu_{F/O} Y_{O\infty}) / \nu_{F/O}} = F(x,r) \quad \Rightarrow \quad Y_O = Y_{O\infty} - \left(\frac{1 + \nu_{F/O} Y_{O\infty}}{\nu_{F/O}} \right) F(x,r)$$

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Distribution de produit à l'extérieur de la flamme

$$\phi_{FP} \equiv \left(Y_F + \frac{\nu_{F/O}}{1+\nu_{F/O}} Y_P \right) - \frac{\nu_{F/O}}{1+\nu_{F/O}} (1 - Y_{O\infty})$$

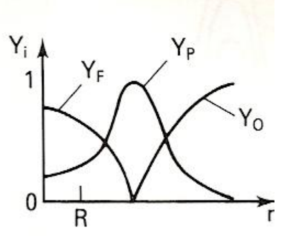
A l'extérieur de la flamme $(r_C \le |r| \le \delta)$ $Y_F=0$

$$\phi_{FP} \equiv \frac{\nu_{F/O}}{1+\nu_{F/O}} Y_P - \frac{\nu_{F/O}}{1+\nu_{F/O}} (1 - Y_{O\infty})$$

Conditions limites à $x = 0, Y_P = 0$ et $Y_F = 1$ $\phi_{FP,i} = \frac{1+\nu_{F/O} Y_{O\infty}}{1+\nu_{F/O}}$

$$\frac{\phi_{FP}}{\phi_{FP,i}} = \frac{\frac{\nu_{F/O} Y_P}{1+\nu_{F/O}} - \frac{\nu_{F/O}(1-Y_{O\infty})}{1+\nu_{F/O}}}{\frac{1+\nu_{F/O} Y_{O\infty}}{1+\nu_{F/O}}} = F(x,r)$$

$$Y_P = \left(\frac{1+\nu_{F/O} Y_{O\infty}}{\nu_{F/O}} \right) F(x,r) + (1 - Y_{O\infty})$$



Distribution de produit à l'intérieur de la flamme

$$\phi_{OP} \equiv \left(Y_O + \frac{1}{1+\nu_{F/O}} Y_P \right) - \left(\frac{1+\nu_{F/O} Y_{O\infty}}{1+\nu_{F/O}} \right)$$

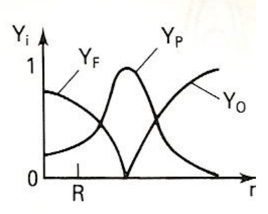
A l'intérieur de la flamme $0 \le |r| \le r_C$ $Y_O=0$

$$\phi_{OP} = \frac{Y_P}{1+\nu_{F/O}} - \left(\frac{1+\nu_{F/O} Y_{O\infty}}{1+\nu_{F/O}} \right)$$

Conditions limites à $x = 0, Y_O = 0$ et $Y_P = 0$ $\phi_{OP,i} = -\frac{1+\nu_{F/O} Y_{O\infty}}{1+\nu_{F/O}}$

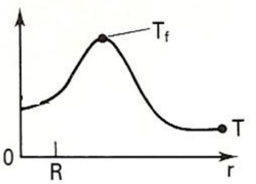
$$\frac{\phi_{OP}}{\phi_{OP,i}} = \frac{\frac{Y_P}{1+\nu_{F/O}} - \frac{(1+\nu_{F/O} Y_{O\infty})}{1+\nu_{F/O}}}{-\frac{(1+\nu_{F/O} Y_{O\infty})}{1+\nu_{F/O}}} = F(x,r)$$

$$Y_P = (1+\nu_{F/O} Y_{O\infty})(1 - F(x,r))$$



Profils de la température à l'intérieur de la flamme

$$\phi_{OT} \equiv \left(Y_O + \frac{C_p(T - T_\infty)}{\nu_{F/O}\Delta h_c} \right) - Y_{O\infty}$$



A l'intérieur de la flamme $0 \leq |r| \leq r_c$ $Y_O=0$

$$\phi_{OT} = \frac{C_p(T - T_\infty)}{\nu_{F/O}\Delta h_c} - Y_{O\infty}$$

$$x = 0, Y_O = 0, \quad \phi_{OT,\,i} = \frac{C_p(T_i - T_\infty)}{(\nu_{F/O}\Delta h_c)} - Y_{O\infty}$$

$$\frac{\phi_{OT}}{\phi_{OT,i}} = \frac{\dfrac{C_p(T - T_\infty)}{\nu_{F/O}\Delta h_c} - Y_{O\infty}}{\dfrac{C_p(T_i - T_\infty)}{\nu_{F/O}\Delta h_c} - Y_{O\infty}} = F(x,r)$$

$$(T - T_\infty) = \left[(T_i - T_\infty) - \frac{Y_{O\infty}\nu_{F/O}\Delta h_c}{C_p} \right] F(x,r) + \frac{Y_{O\infty}\nu_{F/O}\Delta h_c}{C_p}$$

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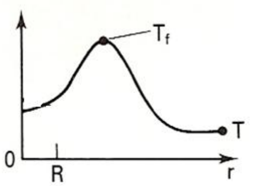
Profils de la température à l'extérieur de la flamme

$$\phi_{FT} \equiv \left(Y_F + \frac{C_p(T - T_\infty)}{\Delta h_c} \right)$$



A l'extérieur de la flamme $r_c \leq |r| \leq \delta$ $Y_F=0$

$$\phi_{FT} = \frac{C_p(T - T_\infty)}{\Delta h_c}$$

$$x = 0, Y_F = 1, \quad \phi_{FT,\,i} = \frac{C_p(T_i - T_\infty)}{\Delta h_c} + 1$$

$$\frac{\phi_{FT}}{\phi_{FT,i}} = \frac{\dfrac{C_p(T - T_\infty)}{\Delta h_c}}{\dfrac{C_p(T_i - T_\infty)}{\Delta h_c} + 1} = F(x,r)$$

$$(T - T_\infty) = \left[(T_i - T_\infty) + \frac{\Delta h_c}{C_p} \right] F(x,r)$$

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Température adiabatique de flamme

$$Y_P = \left(\frac{1+\nu_{F/O}Y_{O\infty}}{\nu_{F/O}}\right)F(x,r) + (1 - Y_{O\infty})$$

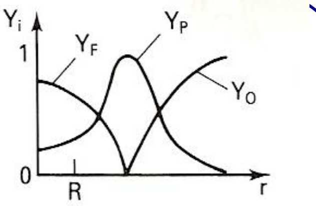
$$x = x_c,\ r = r_c \quad \Rightarrow \quad Y_F = Y_O = 0 \text{ et } Y_P = 1$$

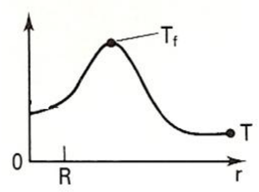
$$F(x_c, r_c) = \frac{\nu_{F/O}Y_{O\infty}}{1+\nu_{F/O}Y_{O\infty}}$$

$$(T - T_\infty) = \left[(T_i - T_\infty) + \frac{\Delta h_c}{C_p}\right]F(x,r)$$

$$F(x,r) = F(x_c, r_c) \quad \Rightarrow \quad T_f$$

$$(T_f - T_\infty) = \left[(T_i - T_\infty) + \frac{\Delta h_c}{C_p}\right]\frac{\nu_{F/O}Y_{O\infty}}{1+\nu_{F/O}Y_{O\infty}} = \left[(T_i - T_\infty) + \frac{\Delta h_c}{C_p}\right]f_{stoe}$$





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Jet liquide et vaporisation des gouttes dans une chambre de combustion

Diamètre dynamique du jet

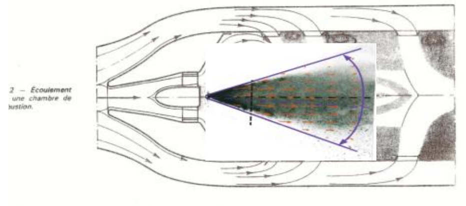
$$d_i' = d_i\left(\frac{\rho_F}{\rho_a}\right)^n$$

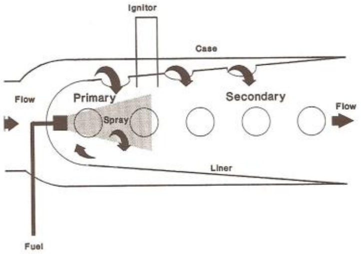
n=0.5 pour un jet circulaire

n=1 pour un jet rectangulaire

ρ_F (masse volumique de liquide)

ρ_a (masse volumique d'air)





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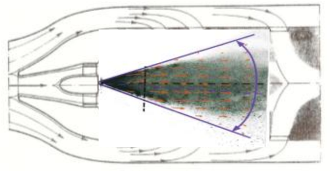
Longueur de flamme de type jet liquide issu d'une buse circulaire en régime turbulent



$$\frac{\nu_{F/O} Y_{O\infty}}{\nu_{F/O} Y_{O\infty}+1} = \left[1+24C'\frac{\bar{x}_C}{d_i'}\right]^{-1}$$

Loin de la sortie du jet : $\frac{24C' x_C}{d_i'} >> 1$

$$\bar{x}_C \approx \frac{d_i'}{24C'}\left(\frac{\nu_{F/O}Y_{O\infty}+1}{\nu_{F/O}Y_{O\infty}}\right) = \frac{d_i'}{24C'}\left(1+\nu_{A/F}\right) = \frac{d_i'}{24C'}\cdot\frac{1}{f_{stoe}}$$

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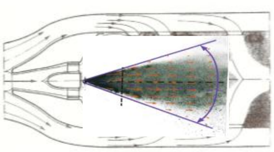
Longueur d'évaporation des gouttelettes dans le jet

$$\frac{u}{u_i} = \left[1+24C'\frac{x}{d_i'}\right]^{-1}\left[1-\frac{2r}{d_i'}\left(1+24C'\frac{x}{d_i'}\right)^{-1}\right]$$

Loin de la sortie du jet sur l'axe: $\frac{24C' x}{d_i'} >> 1 \quad \text{et} \quad r = 0$

Vitesse d'injection : $\frac{u_m}{u_i} \approx \frac{d_i'}{24C' x}$



$$u_m = \left(\frac{\partial x}{\partial t}\right)_{r=0} \quad \Rightarrow \quad \frac{x}{d_i'} = \left(\frac{u_i t}{12C' d_i'}\right)^{1/2}$$

$$t_{tot} = t_{évap} + t_{pré} \approx \frac{D_0^2}{K} + \left[\frac{\rho_l C_{pl}}{12k_g}\ln\left(\frac{T_g - T_0}{T_g - T_s}\right)\right]D_0^2$$

$$t \rightarrow t_{tot} \quad \Rightarrow \quad \frac{\bar{x}_b}{d_i'} = \left(\frac{u_i t_{tot}}{12C' d_i'}\right)^{1/2}$$

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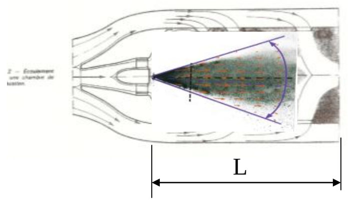
Longueur de la chambre de combustion



Longueur de flamme : $\dfrac{\bar{x}_C}{d_i} \approx \dfrac{1}{24C'}\left(\dfrac{\nu_{F/O}Y_{O\infty}+1}{\nu_{F/O}Y_{O\infty}}\right) = \dfrac{1}{24C' f_{stoe}}$

Longueur d'évaporation des gouttes : $\dfrac{\bar{x}_b}{d_i} = \left(\dfrac{u_i t_{tot}}{12C' d_i'}\right)^{1/2}$

$$L = \max(\bar{x}_C,\ \bar{x}_b)$$

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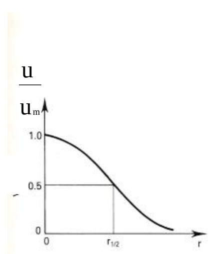
Forme de jet circulaire en régime laminaire

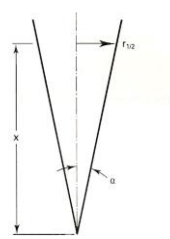
Loin de la sortie du jet :

$$\frac{u}{u_i} = \frac{Re_i}{48}\frac{d_i}{x}\left[1-2\left(\frac{r}{d_i}\right)\frac{Re_i}{48}\frac{d_i}{x}\right]$$

$$\frac{u_m}{u_i} = \frac{Re_i}{48}\frac{d_i}{x} \qquad \text{(sur l'axe, r=0)}$$

$$\frac{u}{u_m} = \frac{1}{2} \quad \Rightarrow \quad \frac{r_{1/2}}{x}$$





$$\begin{cases} \dfrac{r_{1/2}}{x} = 12\,Re_i^{-1} & \text{Rapport entre demi rayon et x} \\ \alpha = \tan^{-1}\left(\dfrac{r_{1/2}}{x}\right) & \text{Demi angle d'ouverture} \end{cases}$$

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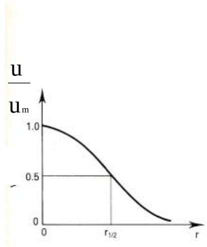
Forme de jet circulaire en régime turbulent

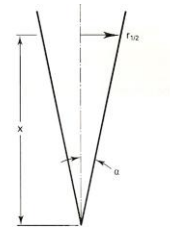
Loin de la sortie du jet :

$$\frac{u}{u_i} = \frac{1}{24C'}\frac{d_i}{x}\left(1 - \frac{1}{12C'}\frac{r}{x}\right)$$

$$\frac{u_m}{u_i} = \frac{1}{24C'}\frac{d_i}{x} \quad \text{(sur l'axe, r=0)}$$

$$\frac{u}{u_m} = \frac{1}{2} \quad \Rightarrow \quad \frac{r_{1/2}}{x}$$





$$\begin{cases} \dfrac{r_{1/2}}{x} \approx 0.077 & \text{Rapport entre demi rayon et x} \\ \alpha = \tan^{-1}\left(\dfrac{r_{1/2}}{x}\right) \approx 4.5° & \text{Demi angle d'ouverture} \end{cases}$$

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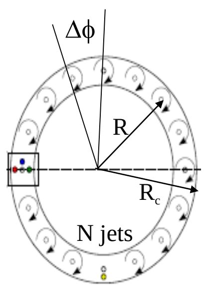
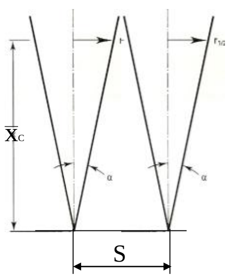
Interaction entre les jets





1) $\bar{x}_C \approx \dfrac{d_i^*}{24C'}\left(\dfrac{\nu_{F/O} Y_{O\infty} + 1}{\nu_{F/O} Y_{O\infty}}\right)$ $\Rightarrow$ Distance entre les jets : $S = 2\bar{x}_C \tan(\alpha)$

2) $\Delta\phi(\text{rad}) = \dfrac{S}{R}$ $\Rightarrow$ Nombre des jets : $N = \dfrac{2\pi}{\Delta\phi}$

3) $\dot{q}'' = \dfrac{\dot{Q}}{0.25\pi d_i^2 N} = \dfrac{\dot{m}_F \Delta h_c}{0.25\pi d_i^2 N}$ $\dot{q}'' \le \dot{q}''_{max}$ (puissance maximale de jet)

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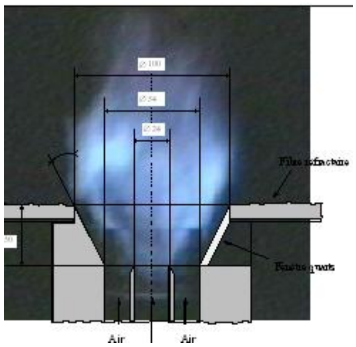
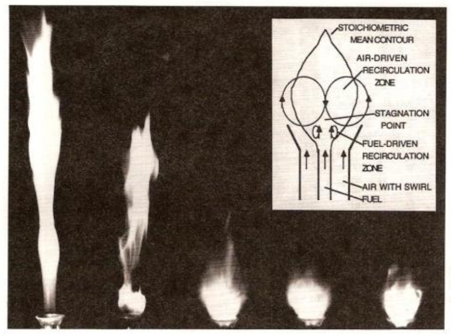
STABILISATION DE LA FLAMME EN RÉGIME TURBULENT
(Réduction de la longueur de flamme à l'aide

des écoulements tourbillonnaires hélicoïdaux - Swirl)

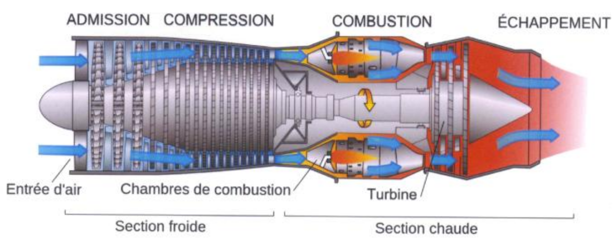


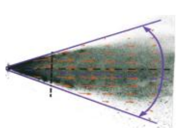


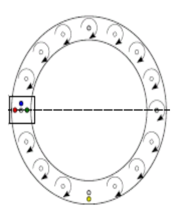
$$\overline{X}_{C,\ \text{swirl}} \approx \frac{\overline{X}_C}{5}$$

Applications

Moteur aéronautique : chambre de combustion annulaire avec multiples jets en présence des écoulements tourbillonnaires hélicoïdaux (oxydant) - Swirl

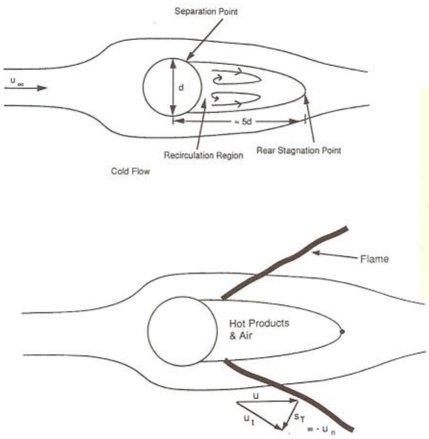


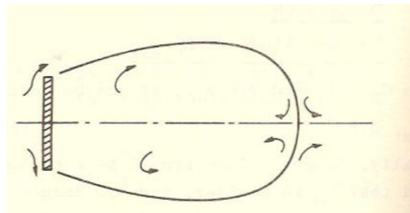




STABILISATION DE LA FLAMME EN RÉGIME TURBULENT

(Réduction de la longueur de flamme à l'aide de l'effet de sillage)





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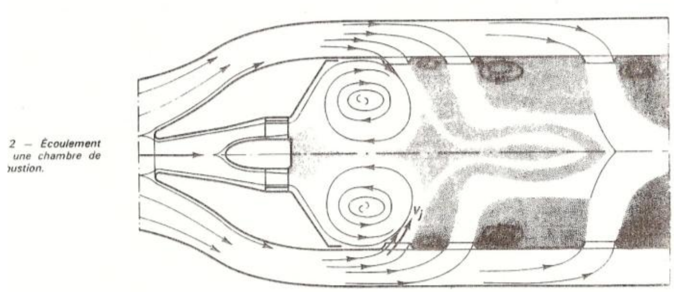
STABILISATION DE LA FLAMME EN RÉGIME TURBULENT

(Réduction de la longueur de flamme à l'aide de l'élargissement brusque)



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