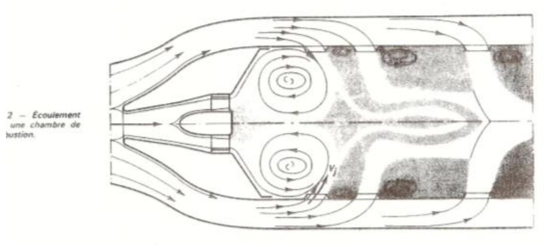
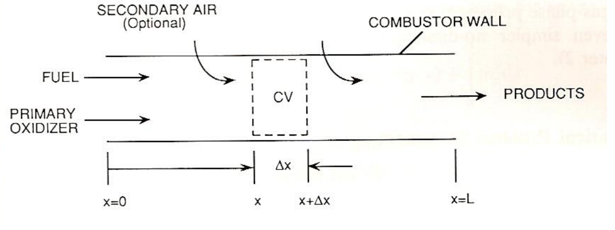
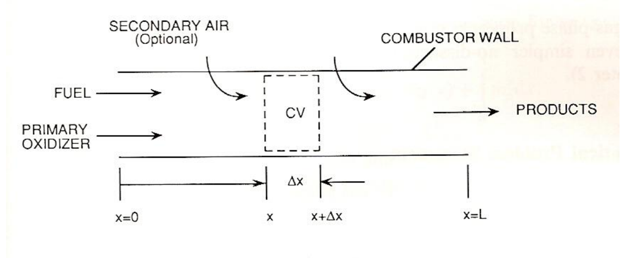
Chapitre IV

Approche Dimensionnelle sur Combustion Contrôlée par l'Évaporation des Gouttes

I) Approche monodimensionnelle stationnaire







PHASE GAZEUSE

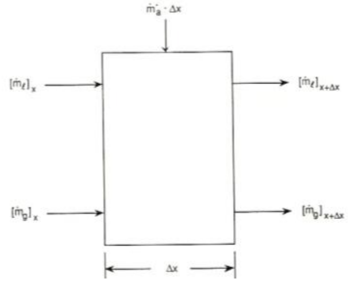
$$\begin{cases} T_g(x) & \text{température du gaz} \\ \dot{m}_g(x) & \text{débit massique} \\ \phi_g(x) & \text{richesse} \\ v_g(x) & \text{vitesse du gaz} \end{cases}$$

PHASE LIQUIDE

$$\begin{cases} D(x) & \text{diamètre de la gouttelette} \\ \dot{m}_l(x) & \text{vitesse d'évaporation du combustible} \\ v_d(x) & \text{vitesse de la gouttelette} \end{cases}$$

3

1) Conservation de la masse totale



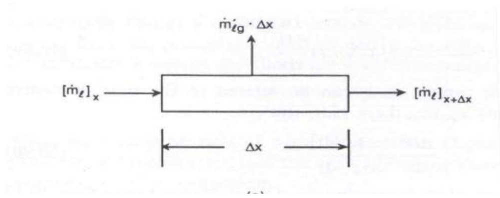
$$[\dot{m}_l]_x + [\dot{m}_g]_x + \dot{m}_a' \Delta x = [\dot{m}_l]_{x+\Delta x} + [\dot{m}_g]_{x+\Delta x} \quad \Rightarrow \quad \frac{[\dot{m}_l]_{x+\Delta x} - [\dot{m}_l]_x}{\Delta x} + \frac{[\dot{m}_g]_{x+\Delta x} - [\dot{m}_g]_x}{\Delta x} = \dot{m}_a'$$

$$\frac{d\dot{m}_l}{dx} + \frac{d\dot{m}_g}{dx} = \dot{m}_a' \qquad \int_0^x \frac{d\dot{m}_l}{dx} + \int_0^x \frac{d\dot{m}_g}{dx} = \int_0^x \dot{m}_a' \, dx$$

$$\dot{m}_g(x) = \dot{m}_g(0) + \dot{m}_l(0) - \dot{m}_l(x) + \int_0^x m_a' dx$$

4

2) Conservation de la masse du liquide



$$[\dot m_l]_{x+\Delta x} - [\dot m_l]_x = -\dot m'_{lg}\,\Delta x$$

$$\frac{d\dot m_l}{dx} = -\dot m'_{lg}$$

$$\dot m_l = \dot N m_d = \dot N \rho_l \pi D^3/6$$

$$\frac{d\dot m_l}{dx} = \frac{\pi}{4}\dot N \rho_l D \frac{dD^2}{dx}$$

(Loi en D^2)

$$\frac{dD^2}{dx} = \frac{1}{V_d}\frac{dD^2}{dt} \qquad \frac{dD^2}{dt} = -K \qquad K = \frac{8k_g}{\rho_l C_{pg}}\ln(B_C+1)$$

$$\frac{dD^2}{dx} = -\frac{K}{V_d} \quad \Rightarrow \quad D(x)$$

$$\dot m_l(0) = \dot N \rho_l \pi D_0^3/6 \quad \Rightarrow \quad \dot N(\text{nombre de gouttes}/s) = 6\dot m_l(0)/(\rho_l \pi D_0^3)$$

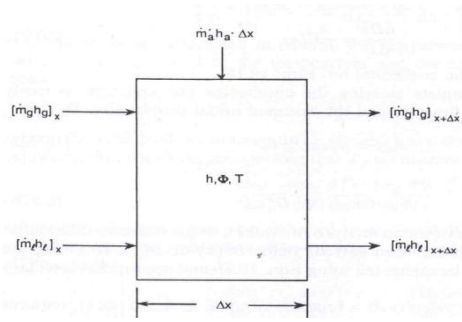
$$\dot m_l(x) \quad \Rightarrow \quad \dot m_g(x)$$

Vitesse du gaz

$$V_g = \frac{\dot m_g}{\rho_g A}$$

$$\rho_g = \frac{P}{(R_u/MW_g)T_g} \qquad \Rightarrow \qquad V_g = \frac{\dot m_g R_u T_g}{(MW_g P A)}$$

3) Conservation de l’énergie en phase gazeuse



$$\frac{d(\dot{m}_g h_g)}{dx} + \frac{d(\dot{m}_l h_l)}{dx} = \dot{m}'_a h_a \quad \Rightarrow \quad \frac{dh_g}{dx} = \frac{\left[\dot{m}'_a h_a - h_g \dfrac{d\dot{m}_g}{dx} - h_l \dfrac{d\dot{m}_l}{dx}\right]}{\dot{m}_g}$$

7

$$h_g = f(T, P, \phi)$$

$$\frac{dh_g}{dx} = \frac{\partial h_g}{\partial T}\frac{dT}{dx} + \frac{\partial h_g}{\partial \phi}\frac{d\phi}{dx} \qquad \text{Code UVFLAME} \begin{cases} \dfrac{\partial h_g}{\partial T} \\ \dfrac{\partial h_g}{\partial \phi} \\ P = cst \end{cases}$$

$$\frac{dT}{dx} = \frac{\left[\left(\dot{m}'_a h_a - h_g \dfrac{d\dot{m}_g}{dx} - h_l \dfrac{d\dot{m}_l}{dx}\right) / \ \dot{m}_g - \dfrac{\partial h_g}{\partial \phi}\dfrac{d\phi}{dx}\right]}{\dfrac{\partial h_g}{\partial T}}$$

substitution

$$\frac{d\dot{m}_g}{dx} = \dot{m}'_a - \frac{d\dot{m}_l}{dx}$$

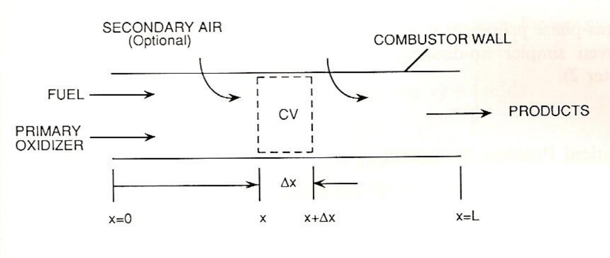
$$\frac{dT}{dx} = \frac{\left[\left((h_a - h_g)\dot{m}'_a + (h_g - h_l)\dfrac{d\dot{m}_l}{dx}\right) / \ \dot{m}_g - \dfrac{\partial h_g}{\partial \phi}\dfrac{d\phi}{dx}\right]}{\dfrac{\partial h_g}{\partial T}}$$

connu

$$\frac{d\dot{m}_l}{dx} = -\dot{m}'_{lg}$$

8

4) Composition de la phase gazeuse $\Rightarrow \dfrac{d\phi(x)}{dx}$



$$\dot{m}_g(x) = \dot{m}_g(0) + \dot{m}_l(0) - \dot{m}_l(x) + \int_0^x \dot{m}_a' dx$$

$$\dot{m}_g(0) = \dot{m}_F(0) + \dot{m}_a(0)$$

$$\left(\frac{F}{O}\right)_x = \frac{\dot{m}_{F,x}}{\dot{m}_{a,x}} = \frac{\dot{m}_g(x) - (\dot{m}_{a,0} + \int_0^x \dot{m}_a' dx)}{\dot{m}_{a,0} + \int_0^x \dot{m}_a' dx}$$

9

$$\left(\frac{F}{O}\right)_x = \dot{m}_g \left[\dot{m}_{a,0} + \int_0^x \dot{m}_a' dx\right]^{-1} - 1$$

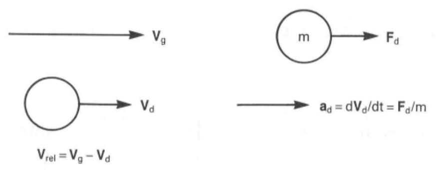
$$\frac{d\left(\frac{F}{O}\right)_x}{dx} = \frac{d\dot{m}_g}{dx}\left[\dot{m}_{a,0} + \int_0^x \dot{m}_a' dx\right]^{-1} - \dot{m}_g \dot{m}_a' \left[\dot{m}_{a,0}' + \int_0^x \dot{m}_a' dx\right]^{-2}$$

$$\phi = \frac{\left(\frac{F}{O}\right)_x}{\left(\frac{F}{O}\right)_{\phi=1}}$$

$$\frac{d\phi(x)}{dx} = \frac{1}{\left(\frac{F}{O}\right)_{\phi=1}} \frac{d\left(\frac{F}{O}\right)_x}{dx}$$

10

5) Conservation de la quantité de mouvement de la gouttelette



Loi de Newton à la gouttelette : $F_d = m_d \dfrac{dV_d}{dt} = m_d V_d \dfrac{dV_d}{dx}$

$$F_d = \frac{1}{2} C_D \rho_g V_{rel}^2 \left(\frac{1}{4}\pi D^2\right) \qquad m_d = \rho_l \frac{1}{6}\pi D^3$$

$$C_D = f(Re_{D,\,rel}) \approx \frac{24}{Re_{D,\,rel}} + \frac{6}{1+\sqrt{Re_{D,\,rel}}} + 0.4 \qquad (0 \le Re_{D,\,rel} \le 2\times 10^5)$$

$$\frac{dV_d}{dx} = \frac{3 C_D \rho_g V_{rel}^2}{4 \rho_l D V_d} \quad \Rightarrow \quad \frac{dV_d}{dx} = \frac{3 C_D \rho_g}{4 \rho_l D V_d} (V_g - V_d)\left|V_g - V_d\right|$$

11

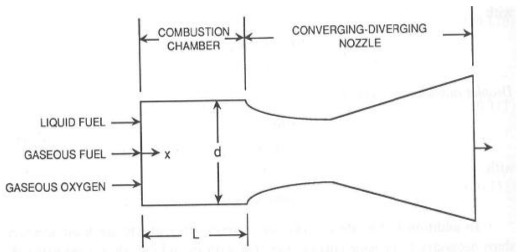
Application

Section droite de la chambre de combustion : 0.157 m^2
Longueur de la chambre de combustion : 0.725 m
Surface d'injection du combustible : 0.0157 m^2
Combustible : n-heptane (C_7H_{16})
Richesse globale : $\phi(F/A) = 2.3$
Richesse du pré-mélange initial : $\phi(0) = 0.45$

Température initiale: T(0)=801 K

Pression dans la chambre de combustion : P=3.5 MPa

Vitesse de goutte initiale : $V_d(0)$=10 m/s



12

Phase liquide (diamètre de la gouttelette)

$$\frac{dD^2}{dx} = -\frac{K}{V_d} \qquad D(0) = D_0$$

Phase gazeuse

a) Conservation de la masse

$$\dot{m}_g(x) = \dot{m}_g(0) + \dot{m}_l(0) - \dot{m}_l(x) + \int_0^x \dot{m}_a' dx \qquad \dot{m}_g(0) = \dot{m}_{F,0} + \dot{m}_{a,0}$$

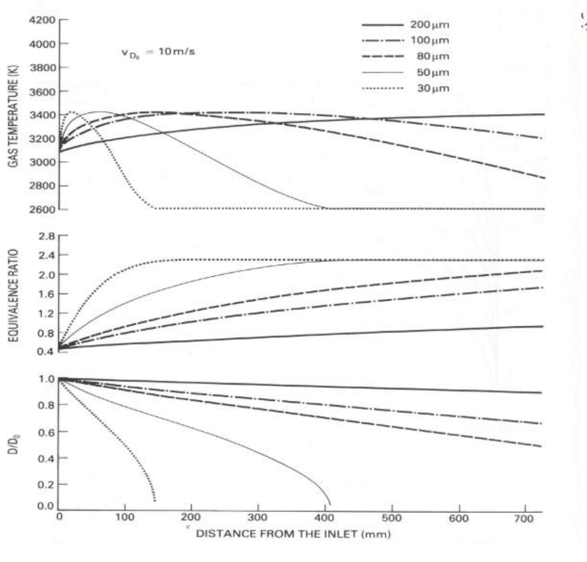
b) Conservation de l'énergie

$$\frac{dT}{dx} = \left[\left((h_a - h_g)\, \dot{m}_a' + (h_g - h_l) \frac{d\dot{m}_l}{dx} \right) / \ \dot{m}_g - \frac{\partial h_g}{\partial \phi} \frac{d\phi}{dx} \right] / \frac{\partial h_g}{\partial T} \qquad T_g(0) = T_{g,0}$$

c) Quantité de mouvement de la gouttelette

$$\frac{dV_d}{dx} = \frac{3 C_D \rho_g}{4 \rho_l D V_d} (V_g - V_d) \left| V_g - V_d \right| \qquad V_d(0) = V_{d,0}$$

13



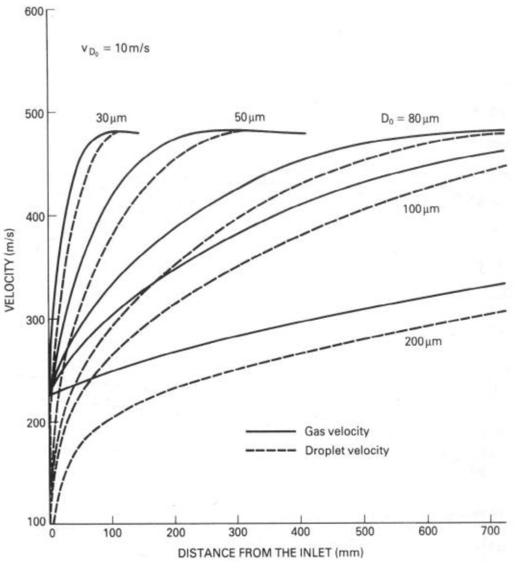
1) $D_0 = 30\,\mu m$

T_{max} trop proche de l'injecteur

2) $D_0 = 50 - 60\ \mu m$

Optimal

14



1) $D_0 \le 50\,\mu m$

$(V_g - V_d)$ petite

2) $D_0 \ge 100\,\mu m$

$(V_g - V_d)$ grande

Effets convectifs importants

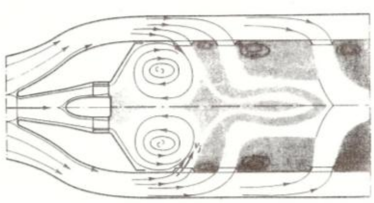
3) $V_g > V_d$

Accélération des gouttes

15

II) **Approche tridimensionnelle instationnaire**

- Connaître les évolutions selon les conditions initiales et aux limites (rapport de mélange, prémélange, diffusion, géométrie, débits, …)
- Comprendre les processus d'oxydation des hydrocarbures (taux de production des espèces, création d'espèces intermédiaires)
- Optimiser la combustion (Q élevée – pertes réduites, pollution minimisée)



Codes numériques (Fortrant 77, 90 ou C^{++}) :

Fluent, Saturne, FDS, CFX, FireFoam, OpenFoam…

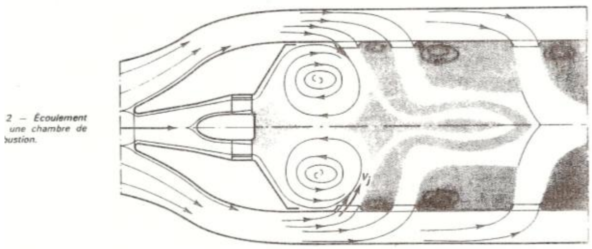
16

1) Résolution de la partie homogène et réactive du système

2) Résolution de la partie dispersée du système

3) Résolution du transfert radiatif

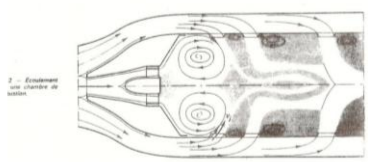
Phase gazeuse $\begin{cases} T_g(\vec{x},t) \\ \dot{m}_g(\vec{x},t) \\ \phi_g(\vec{x},t) \\ v_g(\vec{x},t) \end{cases}$ Phase liquide $\begin{cases} D(\vec{x},t) \\ \dot{m}_l(\vec{x},t) \\ v_d(\vec{x},t) \end{cases}$



17

Phase gazeuse

Ecoulement réactif turbulent



Conservation de la masse

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho = -\rho \nabla \cdot \mathbf{u}$$

Conservation de la quantité de mouvement

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u}\right) = -\nabla p + \rho \mathbf{g} + \nabla \cdot \tau + F$$

Viscosité turbulente

$$\mu_t = \rho C \Delta \sqrt{k}$$

Conservation des espèces chimiques

$$\frac{\partial \rho Y_i}{\partial t} + \nabla \cdot \rho Y_i \mathbf{u} = \nabla \cdot \rho D_i \nabla Y_i + \dot{\bar{W}}_i$$

Equation d'état

$$p = R_u \rho T \sum_{i=0}^{N} \frac{Y_i}{W_i}$$

Conservation de l'énergie

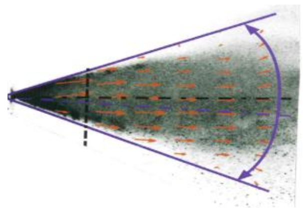
$$\frac{\partial \rho h}{\partial t} + \nabla \cdot \rho h \mathbf{u} = \frac{Dp}{Dt} + \dot{\bar{q}}_c + \nabla \cdot \sum_{i=0}^{N} h_i \rho D_i \nabla Y_i + \nabla \cdot \lambda \nabla T - \nabla \cdot \mathbf{q}_r$$

18

Phase dispersée

Trajectoire des gouttelettes

$$\frac{d}{dt}(m_d \mathbf{u}_d) = m_d \mathbf{g} - \frac{1}{2}\rho C_d \pi r_d^2 (\mathbf{u}_d - \mathbf{u})|\mathbf{u}_d - \mathbf{u}|$$



$$\frac{d\vec{x}_d}{dt} = \mathbf{u}_d \qquad \text{(position de goutte)}$$

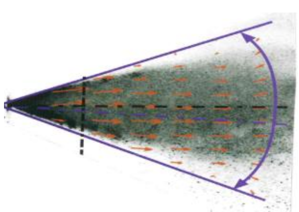
$$C_d = \begin{cases} 24/Re & Re < 1 \\ 24(1+0.15\,Re^{0.687})/Re & 1 < Re < 1000 \\ 0.44 & Re > 1000 \end{cases} \qquad Re = \frac{\rho|\mathbf{u}_d - \mathbf{u}|2r_d}{\mu}$$

Interaction fluide-particule

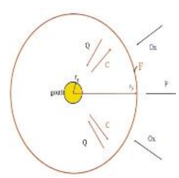
Force de traînée : $F = \dfrac{1}{2}\dfrac{\sum_i \rho C_d \pi r_{d,i}^2 (\mathbf{u}_{d,i} - \mathbf{u})|\mathbf{u}_{d,i} - \mathbf{u}|}{\delta V}$

19

Echange de masse entre gouttelette et gaz



$$\frac{dm_d}{dt} = 4\pi r_d^2 h_m \rho (Y_g - Y_d)$$



Nombre de Sherwood : $Sh = \dfrac{2h_m r_d}{D_m} = 2 + 0.6\,Re^{1/2} Sc^{1/3}$

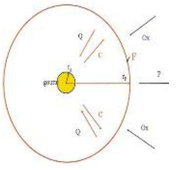
Fraction massique du fuel : $Y_d = \dfrac{X_d}{X_d(1 - MW_A/MW_F) + MW_A/MW_F}$

Equation de Clausius-Clapeyron : $X_d = \exp\left[\dfrac{L_v MW_F}{R_u}\left(\dfrac{1}{T_E} - \dfrac{1}{T_d}\right)\right]$

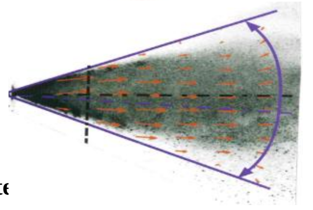
20

Echange d'énergie entre gouttelette et gaz

$$m_d C_{pl}\frac{dT_d}{dt} = 4\pi r_d^2 h_d (T_g - T_d) + \frac{dm_d}{dt} L_v$$



$$Nu = \frac{2h_d r_d}{k_g} = 2 + 0.6\, Re^{1/2}\, Pr^{1/3}$$



Echange de masse et d'énergie paroi-gouttelette

$$\frac{dT_d}{dt} = \frac{2\pi r_d^2 \left[h_s (T_s - T_d) + \dot{q}''_{rd}\right]}{C_{pl}\, m_d}$$ $T_d < T_E$ (Préchauffage des gouttes)

$$\frac{dm_d}{dt} = -\frac{2\pi r_d^2 \left[h_s (T_s - T_d) + \dot{q}''_{rd}\right]}{L_v}$$ $T_d = T_E$ (Evaporation des gouttes)

21

Distribution de taille des gouttelettes

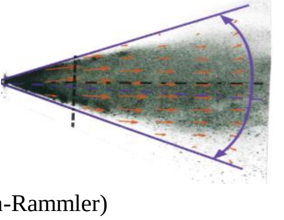
Rapport entre diamètre moyen de goutte (d_m) et celui de jet (D_j)

$$d_m / D_j \propto We^{-1/3} \qquad We = \frac{\rho_l U_d^2 D_j}{\sigma_w} \qquad \Rightarrow \quad d_m \propto \left(\frac{D_j}{U_d}\right)^{2/3}$$

Fonction de densité de probabilité (PDF) $\Rightarrow$ diamètre de goutte , d

$$f(d) = \frac{F'(d)}{d^3} \Bigg/ \int_0^\infty \frac{F'(d')}{d'^3} dd'$$

D_j



Fonction cumulative volumique

(combinaison des distributions Log-normal et Rosin-Rammler)

$$F(d) = \begin{cases} \dfrac{1}{\sqrt{2\pi}} \displaystyle\int_0^d \frac{1}{\sigma d'} \exp\left(-\frac{\left[\ln(d'/d_m)\right]^2}{2\sigma^2}\right) dd' & (d \le d_m) \\ 1 - \exp\left[-0.693 (d/d_m)^\gamma\right] & (d > d_m) \end{cases}$$

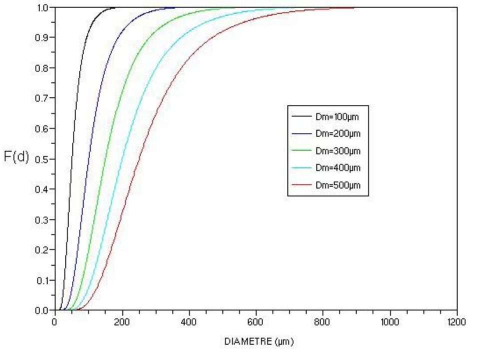
22

Conservation de masse assurée par un Facteur pondération C :

$$\dot{m}\delta t = C\sum_{i=1}^{N} g'(U_i)\frac{4}{3}\pi\rho_i\left(\frac{d_i}{2}\right)^3 \qquad g'(U) = \frac{1}{n}U^{1/n-1}$$

Variable aléatoire, $U(d)$ $[0,1]$ $\qquad U(d) = \int_0^d f(d')dd'$

Sensibilité de la cumulative combinée au diamètre moyen : $d_m \propto \left(\frac{D_j}{U_d}\right)^{2/3}$



$$\left.\begin{matrix} U_d \uparrow \\ D_j \downarrow \end{matrix}\right\} \quad d_m \downarrow$$

$$\left.\begin{matrix} U_d \downarrow \\ D_j \uparrow \end{matrix}\right\} \quad d_m \uparrow$$

23