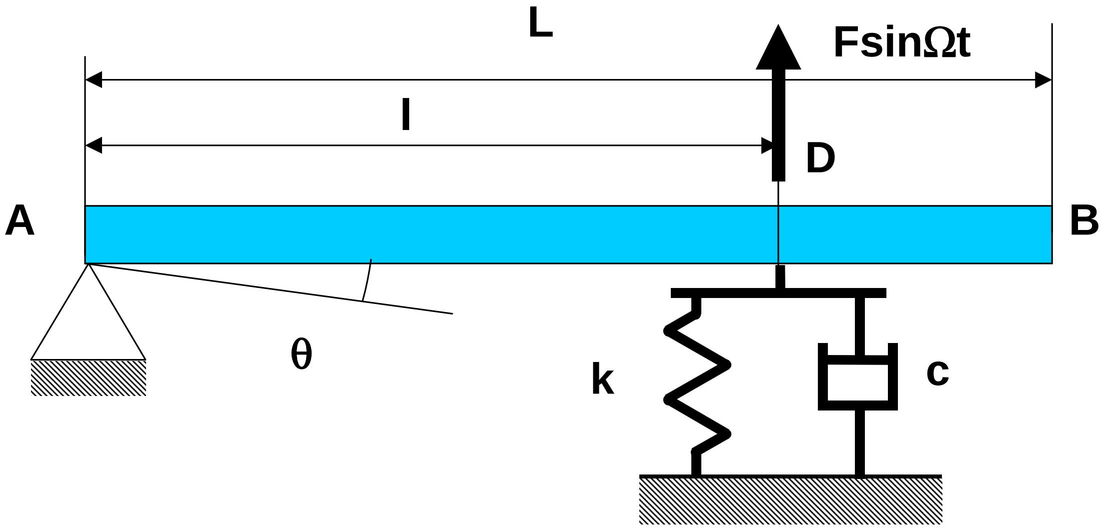
Réponse d'un Système à 1 degré de liberté



Energie cinétique :

$$T = \frac{1}{2} I_A \left(\dot{\theta}\right)^2$$

Energie de déformation :

$$U = \frac{1}{2} k \left(x_D\right)^2$$

Fonction de dissipation (Rayleigh) :

$$R = \frac{1}{2} c \left(\dot{x}_D\right)^2$$

Equation de Lagrange :

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} + \frac{\partial R}{\partial \dot{q}_i} = Q$$

<u>Equation en θ</u>

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = \frac{d}{dt}\left(I_A \dot{\theta}\right) = I_A \ddot{\theta}$$

$$x_D = l\theta$$

$$U = \frac{1}{2}k\left(x_D\right)^2 = \frac{1}{2}kl^2\theta^2$$

$$\frac{\partial U}{\partial \theta} = kl^2\theta$$

$$\dot{x}_D = l\dot{\theta}$$

$$R = \frac{1}{2}c\left(\dot{x}_D\right)^2 = \frac{1}{2}c\left(l\dot{\theta}\right)^2$$

$$\frac{\partial R}{\partial \dot{\theta}} = cl^2\dot{\theta}$$

$$\boxed{I_A\ddot{\theta} + cl^2\dot{\theta} + kl^2\theta = F\,l\sin\Omega t}$$

Application numérique :

$$
\begin{aligned}
I_A &= 0.065\,kg.m^2 \\
k &= 3.10^4\,N/m \\
c &= 1Ns/m \\
l &= 0.4m \\
L &= 0.5m
\end{aligned}
$$

$$I_A\ddot{\theta} = 0.065\ddot{\theta}$$

$$kl^2\theta = 0.48\,10^4\,\theta$$

$$cl^2\dot{\theta} = 0.16\dot{\theta}$$

$$\boxed{0.065\ddot{\theta} + 0.16\dot{\theta} + 0.48\,10^4\,\theta = 0.4F\sin\Omega t}$$

Par analogie avec

$$\boxed{\alpha = \frac{\text{amortissement réalisé}}{\text{amortissement critique}} = \frac{c}{2\sqrt{km}}}$$

Il vient

$$
\begin{aligned}
\alpha &= \frac{0.16}{\text{amortissement critique}} \\
&= \frac{0.16}{2\sqrt{0.065 \cdot 0.48\,10^4}} \\
&= 0.0045 \\
&= .45\%
\end{aligned}
$$