Tutorial

Flight mechanics

N°1 : Twin-engine aircraft performances

We will study the performances of a twin-engine airliner in cruise flight.

Data :

Reference wing surface: $S = 120\ m^2$

Initial mass in cruise flight: $m_0 = 61.2\ t$ $\left(\frac{mg}{S} = 5000\ Pa\right)$

Polar curve: $C_X = C_{X_0} + k\,C_Z^2 + \Delta C_X(C_Z, M)$

$C_{X_0} = 1.753\ 10^{-2}$ $k = 3.881\ 10^{-2}$

$\Delta C_X(C_Z,M)$ is a compressibility term :

$\Delta C_X(C_Z,M) = 0$ if the fluid is considered to be incompressible

$\Delta C_X(C_Z,M) > 0$ if the fluid is considered to be compressible (increasing function of C_Z and Mach number)

Separation between the two domains is shown in graph 1.

$C_{Zmax} = C_{Zmax}(M)$ decreasing function of Mach number

$= 1.30$ if $M \leq 0.30$

Engine :

We will consider that:

Maximum thrust with 2 engines operating (independent of speed) :

$Fu = \sigma\, Fu_0$ $Fu_0 = 150\ kN$

$\sigma = \frac{\rho}{\rho_0}$ is relative density

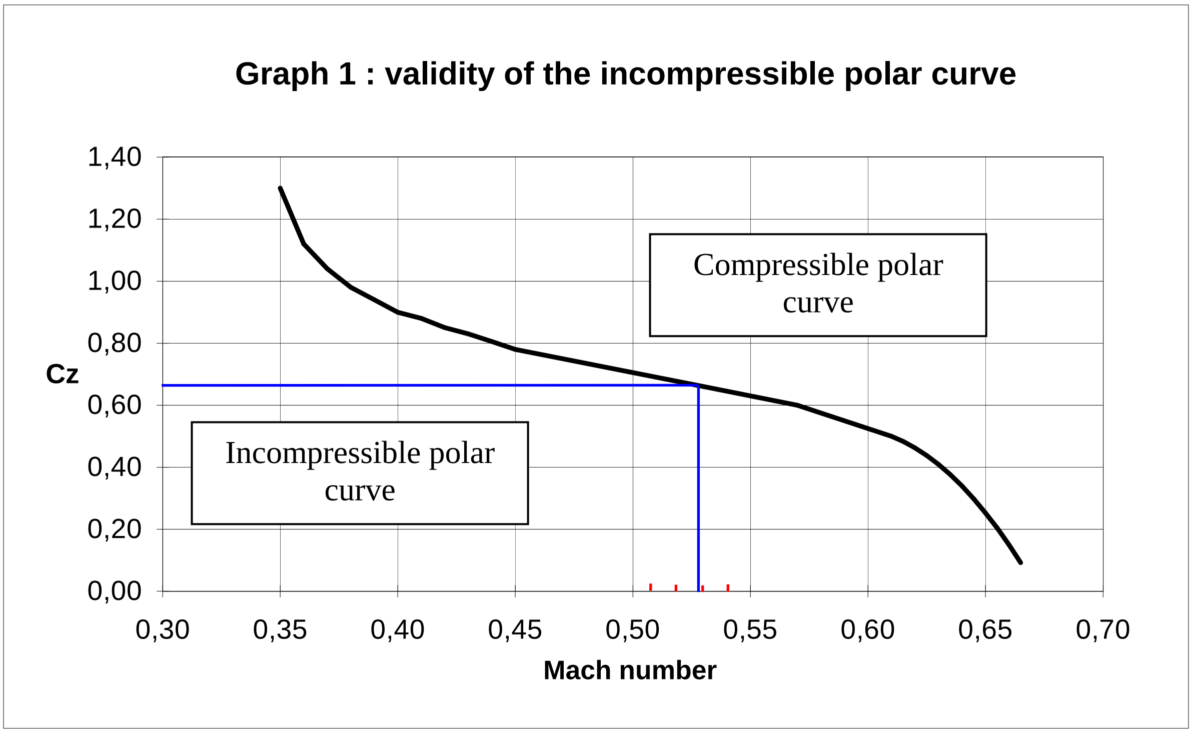
Limitations :

Calibrated airspeed $CAS \leq VMO = 350$ kts

Mach number $M \leq MMO = 0.82$

At low altitude, we will consider that $CAS \cong EAS$

Limit normal load factor $n_Z = 2.50$



1 Required thrust for steady level flight at sea level (standard conditions) T_R

1.1 Determine the limits of the thrust diagram Fn(TAS) at sea level (Pressure Altitude Z_P=0)

- stall speed with a load factor of 1g : V_{S1g} ,
- maximum speed,
- numerical application for mass m_0.

Show that in this domain, we can disregard the influence of compressibility on drag (incompressible domain).

1.2 Give the literal expression of the thrust required for level flight as a function of true airspeed TAS.

1.3 Using the expressions of T_R and TAS (as functions of C_X and C_Z), determine noticeable points of the curve:

- minimum of Fn,
- tangent line to this curve at the origin: minimum of Fn/TAS.

1.4 Numerical application: draw the curve Fn(TAS) (mass m_0, Z_P =0, standard conditions)

2 Influence of altitude and mass

2.1 Show that in the incompressible domain, the curve Fn(EAS) is independent of altitude and temperature.

2.2 Show that beyond a certain altitude, the thrust required for the flight increases with compressibility (at a constant EAS).

2.3 At mass m_0, if speed is V_0, lift coefficient is C_{Z0} and thrust required for the flight is Fn(V_0,m_0, Z_P). Mass is now m_1, give the expression of V_1, and thrust required for the flight Fn(V_1,m_1,Z_P) if the lift coefficient is C_{Z0}. We will consider that the air is incompressible.

3 Climb performances

3.1 In a constant TAS climb, what speed should we adopt to have a maximum climb gradient?

Numerical application : maximum climb gradient at sea level.

3.2 Show that during a climb at constant EAS, climb gradient is:

$$\gamma_a \text{ (constant EAS)} = \frac{1}{1+\frac{V}{g}\frac{dV}{dH}}\,\gamma_a \text{ (constant TAS)}$$

Show that during a climb with constant EAS, the climb gradient decreases with altitude.

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N°2 : Low altitude cruise flight of a transonic twinjet

We will study the performances of a twinjet at low altitude.

Data

Reference wing surface: $S = 122\ m^2$

Specific fuel consumption of engines: $SFC = 0.065$ kg/N/h
(constant in the flight envelope)

Aerodynamics

Polar curve (incompressible flow): $100\ C_X = 1.753 + 3.881\ C_Z^2$

The domain of validity of this polar is shown on graph 1.

Units

Speeds will be expressed in knots (kts), distances in nautical miles (Nm) and specific range in nautical mile per ton (Nm/t).

We will use $g = 9.81\ m/s^2$.

Extract of standard atmosphere tables:

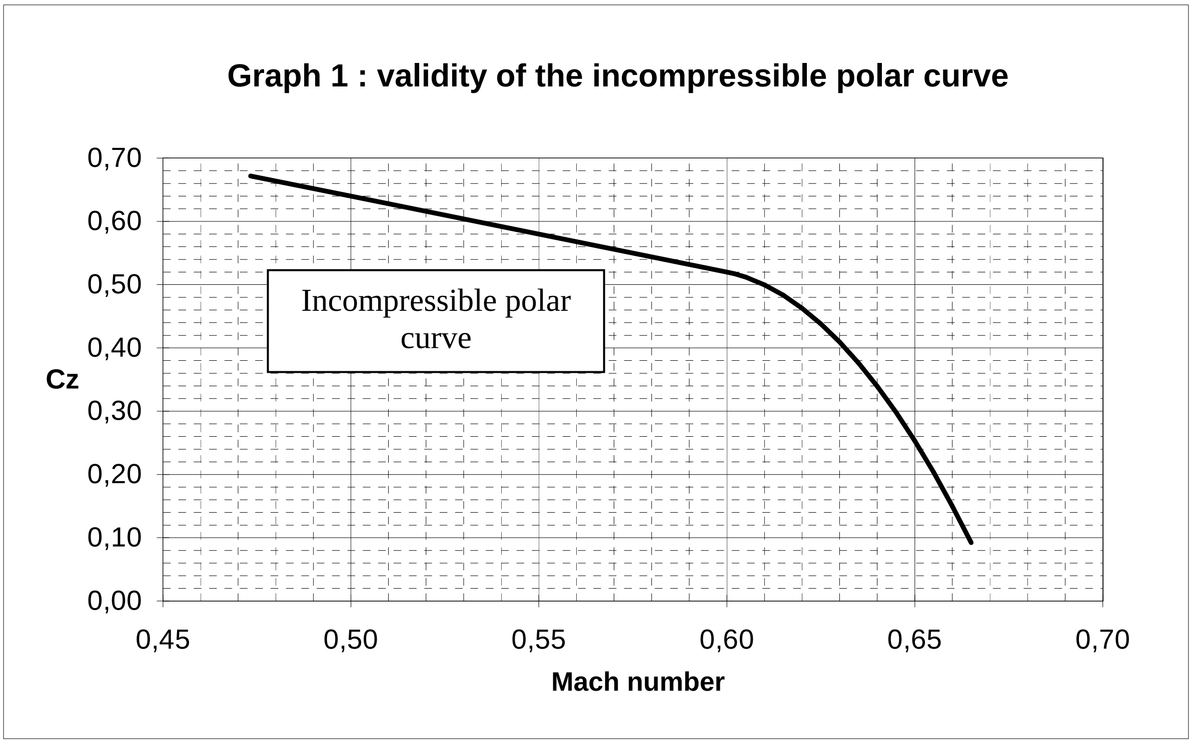
FL	0	100	200	210
$\sigma = \rho/\rho_0$	1.0000	0.7385	0.5328	0.5150
p (Pa)	101,325	69,681	46,563	44,645

Reminder: definitions and notations regarding endurance and range

Hourly consumption : Ch in kg/h

Specific range: $SR = \dfrac{G_S}{Ch}$ in Nm/kg

Initial mass: m_0
Final mass: $m_1 = m_0 - Q$ (Q : quantity of consumed fuel)
Endurance: τ
Range: D
Transport coefficient: $K = \dfrac{dm_0}{dm_1}$; distance D being fixed.



1 Preliminary: generalities about endurance and range

Hypothesis: level flight (constant altitude)

constant specific fuel consumption (SFC): $SFC = \dfrac{Ch}{F}$

incompressible polar (independent of Mach number) : $C_X = C_X(C_Z)$)

1.1 Determine the hourly consumption Ch as a function of mass m, altitude (relative density σ) and C_Z.

In what conditions (C_Z and altitude) is this consumption at its minimum value?

Deduce the expression of endurance τ.

1.2 Same question for specific range. Express the transport coefficient in function of ratio m_1/m_0.

1.3 In these conditions, why does the aircraft adopt a high cruise altitude?

2 Influence of wind

We consider that there is effective wind:

GS	=	V	+	We
Ground speed	=	true air speed	+	effective wind

We will note SR (V,We) the specific range as a function of true air speed and effective wind. Express SR (V,We) as a function of specific range when there is no wind (SR (V,0)) and effective wind.

By using diagrams representing hourly consumption Ch(V) and specific range SR (V,We), show the influence of effective wind. You will show how the best specific range speed V_{MR} ("maxi-range") is affected by effective wind.

Let us define "long range" cruise: at the long range speed V_{LR} with no wind, specific range is equal to 99% of the best specific range ($V_{LR} > V_{MR}$). What is the interest of long range cruise?

You will notice that the influence of the wind does not depend on the type of aircraft (jet engine or propeller) and is not due to any hypothesis on the polar curve and on specific fuel consumption.

3 Low altitude cruise, mass m=61t

3.1 Determine the equivalent airspeed EAS_{MR} that ensures the best specific range (if we consider an incompressible flow).

3.2 In these conditions, what is the hourly consumption Ch?
Deduce specific range SR as a function of relative density σ.

3.3 Up to what altitudes are the above formulae valid?

3.4 Deduce the specific range at flight level 100.

Tutorial
Flight mechanics
N°3 : Performances of a twinjet

We will study maneuvering performances of a twin-engine jet.

Data:

Reference wing surface: $S = 120\ m^2$

Mass: $m_0 = 53.8\ t \quad (\frac{mg}{S} = 4400\ Pa)$

Polar (valid for Mach numbers ≤ 0.45) : $C_X = C_{X_0} + k\, C_Z^2$

$C_{X_0} = 1.753\ 10^{-2}$ $\quad k = 3.881\ 10^{-2}$

Maximum C_Z as a function of Mach number:

Mach number	0.26	0.45
$C_{Z\ max}$	1.450	1.100

Engine:

We will consider that:

Maximum thrust with 2 engines operating (independent of speed) :

$Fu = \sigma\, Fu_0$ $\quad Fu_0 = 150\ kN$

$\sigma = \frac{\rho}{\rho_0}$ is relative density

Maximum normal load factor $n_Z = 2.50$ (structural limitation)

We are at flight level 50, equivalent air speed is 250 kt (we consider that EAS = CAS = 250 kt)

Extract of a table of standard atmosphere at FL50 (Zp = 5 000 ft)

p (Pa)	T (K)	T (°C)	a (kt)	ρ	$\sigma = \rho/\rho_0$
84307.1	278.2	5.1	650.01	1.0555	0.8617

1 Speed measurement at FL50

1.1 Determine the true air speed at FL50 (if EAS = 250kt) in ISA conditions. Deduce the Mach number.

1.2 Same question in ISA+10 conditions.

2 Thrust required for the flight – Acceleration performances

2.1 Determine the thrust required for the flight at mass m_0.

2.2 Deduce the acceleration in level flight at maximum thrust.

3 Turn with a bank angle of 30°

3.1 You perform a symmetrical level turn with a bank angle $\mu = 30°$.

Determine the normal load factor component n_z, deduce the rate of turn Ω (in degrees per second), and the turn radius.

3.2 What thrust should you apply to perform this turn while maintaining the same speed and altitude?

4 Avoidance maneuver with a g-load $n_z = 1.2$

Because of a resolution advisory (RA) of the Traffic Collision Avoidance System (TCAS), you pull up with a load factor $n_z = 1.2$, in order to have a rate of climb of 1500 ft/mn. Determine the time necessary to get this rate of climb.

5 Maximum load factor turn

5.1 Is it possible to start a symmetrical level turn under the maximum load factor n_z=2.5?

Deduce the corresponding bank angle, the rate of turn Ω (in degrees per second), and the turn radius.

5.2 Is it possible to continue this turn while maintaining altitude and speed?

5.3 Is it possible to reach the maximum load factor of 2.5 if the equivalent air speed equals 210 kt?

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Flight mechanics

N°4 : High altitude cruise of a transonic twinjet

We will deal with high altitude performances of the twinjet studied in the previous tutorial.

Data

Reference wing surface: $S=122\ m^2$

Engine specific fuel consumption : (constant in the flight envelope) SFC=0.065kg/N/h

Aerodynamics

1. Maximum lift-drag ratio as a function of Mach number:

Mach number	0.45	0.50	0.60	0.70	0.76	0.78	0.79	0.80	0.82
C_Z (max lift-drag ratio)	0.672	0.650	0.611	0.598	0.581	0.567	0.517	0.507	0.470
Maximum lift-drag ratio	19.17	18.98	18.90	18.13	17.54	17.12	16.59	16.07	13.81

2. Polar curve at Mach 0.78 : $C_{Zmax} = 0.844$

   Lift-drag ratio as a function of C_Z : graph 1.

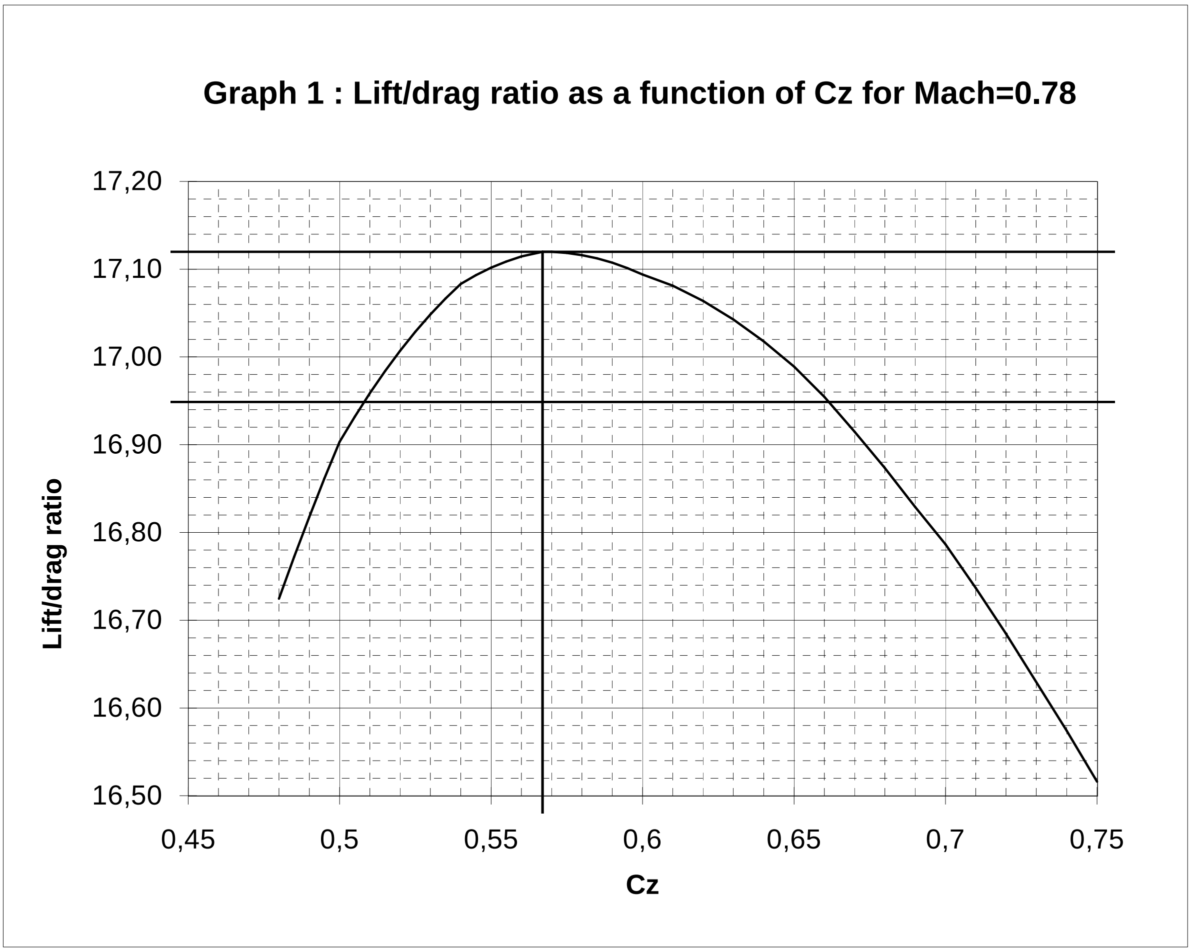
Units :

Speeds will be expressed in knots (kts), distances in nautical miles (Nm) and specific range in nautical mile per ton (Nm/t).

We will use $g = 9.81\ m/s^2$.

Extract of standard atmosphere tables:

FL	p (Pa)	T (K)	ρ (kg/m^3)	σ	a (kts)
0	101325.0	288.15	1.22500	1.00000	661.48
100	69681.4	268.34	0.90463	0.73848	638.33
350	23841.9	218.81	0.37959	0.30987	576.42
370	21662.4	216.65	0.34833	0.28435	573.57
390	19677.0	216.65	0.31640	0.25829	573.57
410	17873.5	216.65	0.28740	0.23461	573.57



1. Aerodynamic optimum Mach number and Cz

We want to determine the couple Mach number/Cz that optimizes specific range.

1.1 Express specific range SR as a function of Mach number M, speed of sound a, specific fuel consumption SFC, mass m and aerodynamic coefficients Cx and Cz.

1.2 For a given Mach number, for what altitude is specific range optimum?

1.3 Consider that speed of sound is constant. Determine the optimum Mach number/Cz couple.

2. Cruise flight at Mach 0.78

– initial mass $m_0 = 65$ t
– final mass $m_1 = 55$ t

2.1 Determine masses corresponding to the aerodynamic optimum at FL 350, FL 370, FL 390 and FL 410. Draw the curve of optimum flight level as a function of mass.

2.2 Ascending cruise

Consider that the speed of sound is constant. Express traveled distance D as a function of specific fuel consumption SFC, speed of sound a, aerodynamic coefficients Cx and Cz and initial and final masses.

Numerical application (justify the hypothesis: constant speed of sound). We disregarded climb gradient γ, justify this hypothesis.

2.3 Level cruise flight

The aircraft flies in RVSM airspace (vertical separation of 1000 ft), in the direction of odd flight levels (FL350/370/390…). Because of traffic, FL370 is not available.
Show that in this case, the optimum cruise flight is at FL390. Determine lift-drag ratio and the specific range with the following masses: 65 t, 63 t, 61 t, 59 t; 57 t, 55t .

Deduce distance traveled at this level D' (approximation using trapezoidal rule). Compare this result to the one of the previous paragraph.

3. Pressurization failure

In case of pressurization failure, the maximum cruise flight level is FL100 (unless terrain imposes a higher altitude).

If such a failure happens at mass m=61t, at FL390, compute the decrease of specific range due to the descent to FL100 (refer to Tutorial N°2).

Tutorial

Flight mechanics

N°5 : High altitude performances of a transonic twinjet

We will deal with high altitude performances of the twin-engine jet studied in the previous tutorials.

Data

Reference wing surface : $S = 122\ m^2$

Mass in cruise flight: $m = 67{,}100$ kg ($m/S = 550\ kg/m^2$)

Maximum thrust of an engine at high altitude: $Fu_1 = 22000 \dfrac{\rho}{0.3164}$ en N

(ρ is the density of the air in kg/m^3)

Aerodynamics

1. Maximum Cz as a function of Mach number:

Mach number	0.26	0.40	0.60	0.65	0.70	0.74
C_{Zmax}	1.410	1.100	0.925	0.905	0.889	0.884
$C_{Z\ max}*M^2$	0.0953	0.1760	0.3330	0.3824	0.4410	0.4841

Mach number	0.76	0.78	0.80	0.82	0.84	0.86
C_{Zmax}	0.871	0.844	0.785	0.678	0.499	0.319
$C_{Zmax}*M^2$	0.5031	0.5135	0.5024	0.4559	0.3521	0.2359

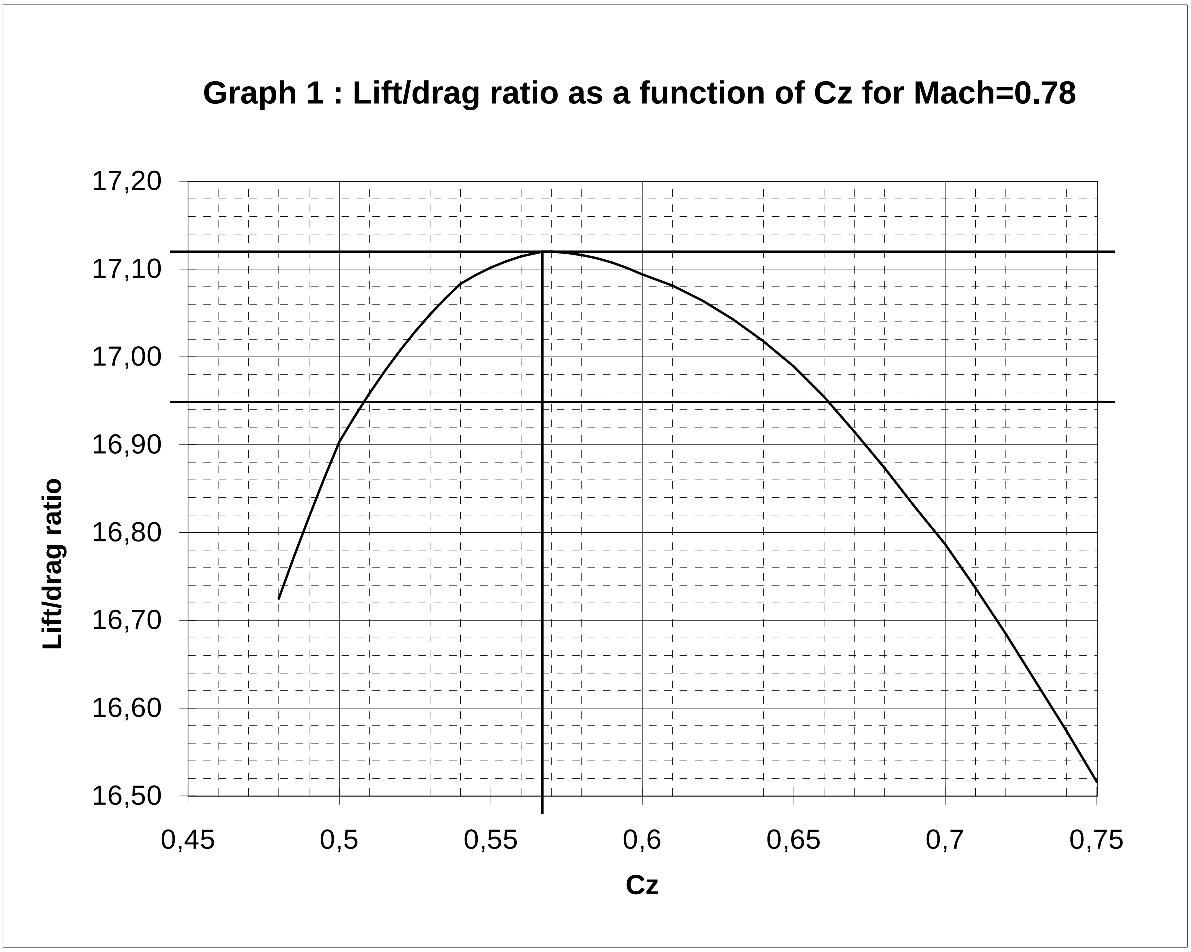
2. Polar curve at Mach 0.78 :

   $C_{Zmax} = 0.844$, lift-drag ratio as a function of C_Z : graph 1 (same as in tutorial N°4).

Extract of standard atmosphere tables:

FL	390	410	430	450	470
p (Pa)	19677	17874	16236	14748	13396
$\rho(kg/m^3)$	0.3164	0.2874	0.2611	0.2371	0.2154

At these altitudes, we consider that speed of sound a = 295.07 m/s.



1. Aerodynamic ceiling

Use the table that represents $C_{Z\ max}(M)$ and $C_{Zmax}*M^2(M)$ to determine:

- the Mach number that enables you to reach absolute ceiling
- aerodynamic ceiling and 1.3g buffet limited altitude (altitude where the instantaneous maneuvering limit is 1.3g).

2. Flight envelope at flight level 390

2.1 Show that in steady level flight, C_Z*M^2 depends on pressure altitude and mass. Do the numerical application for m = 67.1 t and FL 390. Deduce the instantaneous maneuver limit at Mach 0.78.

2.2 Draw the curve $C_{Z\ max}*M^2$ as a function of Mach number (Mach > 0.60) and deduce the limits of the flight envelope in level flight.

3. Maximum cruise thrust limited altitude

3.1 At flight levels 390 and 410, Mach 0.78, determine:

- C_Z and the corresponding lift-drag ratio (ref graph 1),
- required thrust Fn,
- maximum available thrust Fu (with 2 engines operating).

Deduce vertical speed Vz at constant Mach number (in m/s).

3.2 Using interpolations and extrapolations, deduce the service ceiling ($Vz = 1.5$ m/s $\cong$ 300 ft/mn) and the maximum thrust limited altitude at Mach 0.78.

4. Wind gradient at FL390

We consider that the aircraft is in level flight at FL390, Mach 0.78, with a mass m’ so that FL390 is the service ceiling.

We consider that wind gradient, that initially does not exist, increases. Determine maximum wind gradient that would authorize you to maintain Mach 0.78 in level flight.

Tutorial

Flight mechanics

N°6 : Power diagram of a twin-turboprop

We will study performances of a twin-turboprop, which has the following characteristics:

Reference wing surface : $S = 28\ m^2$

Maximum operating speed: VMO = 270 kts (calibrated air speed)

Mass: $m = 5600$ kg ($mg/S = 1962$ Pa)

Aerodynamics (landing configuration)

Maximum lift coefficient: $C_{Z\ max} = 1.55$

Polar curve: $100\ C_X = 2.70 + 6.00\ C_Z^2$

Propulsion: 2 turboprop units (TPU)

Each TPU has the following characteristics:

Maximum thermodynamic power (in kW) : $W_{max}(\sigma) = 910\ \sigma$; (σ is the air relative density)

Mechanical limits:

- Maximum propeller RPM: $N_{max} = 1998$ tr/mn (= 33.30 tr/s = 209.2 rd/s)
- Maximum shaft torque: $\Gamma_{max} = 3024$ N.m

Variable pitch / constant speed propeller:

- Diameter D = 2.50 m
- We consider that the propeller efficiency η_H is a function of the advance ratio γ according to the following table:

$$\gamma = \frac{V}{ND} \qquad \text{N in tr/s}$$

γ	0.50	0.625	0.75	1.00	1.25	1.50	1.70
η_H	0.65	0.74	0.79	0.84	0.86	0,.84	0.78

Speeds are expressed in knots (kts).

We initially are at sea level in standard conditions (ρ_0 =1.225 kg/m^3 , σ = 1)

1 Power diagram

1.1 Determine the stall speed V_{S1g} (with a load factor n_Z = 1).

1.2 What is the maximum motive power that a turbine can deliver in these conditions?

1.3 Draw the available power diagram as a function of true air speed: Wu(V).

1.4 Determine the necessary power for a level flight corresponding to the following conditions:

- Stall: V_{S1g},
- Minimum required power Wn $_{mini}$,
- Tangent at the origin,
- Maximum operating speed VMO.

1.5 Draw the required power diagram Wn(V).

2 Excess power

2.1 Draw the excess power ΔW = Wu-Wn as a function of speed V, at sea level.

2.2 Deduce:

- maximum speed in flight level,
- speed corresponding to maximum ΔW,
- speed corresponding to maximum ΔW/V.

3 Single engine ceiling

We study the flight in case of one turboprop unit failure. We disregard additional drag due to the failed engine and we suppose that propeller efficiency η_H is constant and equals 0.85.

We will use standard atmosphere tables to determine the relationship between altitude and relative density.

3.1 Determine maximum available power Wu and minimum required power (mass m_0, standard temperature conditions) as a function of relative density σ. Deduce gross single engine ceiling (maximum cruise thrust limited altitude). In these conditions, determine the equivalent air speed and the true air speed.

3.2 Same question for an ISA+10 atmosphere.