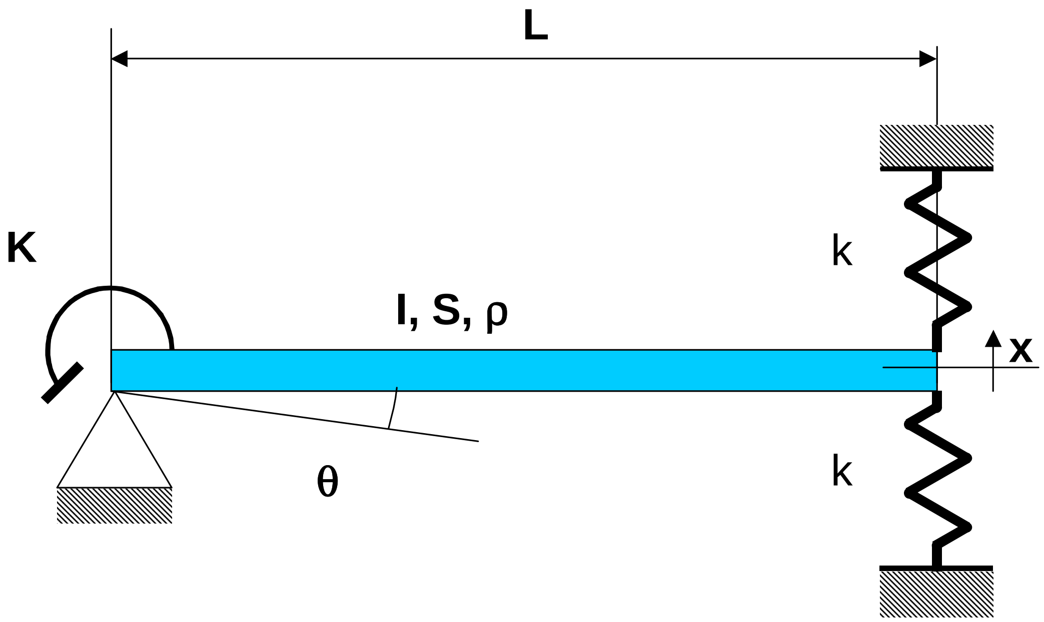
Système à 1 degré de liberté



Faibles déplacements

Energie Cinétique:

$$T = \frac{1}{2} I \left(\frac{d\theta}{dt} \right)^2$$

Energie de Déformation :

$$U = \sum U_i$$

$$\text{ressort k} \Rightarrow \frac{1}{2} k_{eq} x^2 \qquad \textbf{et} \qquad \text{ressort K} \Rightarrow \frac{1}{2} K\theta^2$$

Relations :

$$x = L\theta$$

$$T = \frac{1}{2} I \left(\frac{d\theta}{dt} \right)^2 \qquad U = kL^2\theta^2 + \frac{1}{2} K\theta^2$$

Equations de Lagrange

$$\boxed{\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right)-\frac{\partial T}{\partial \theta}+\frac{\partial U}{\partial \theta}=0}$$

$$I\frac{d^2\theta}{dt^2}+\left(2kL^2+K\right)\theta=0$$

$$\omega^2=\frac{2kL^2+K}{I}$$

<u>Rappels :</u>

$$I=\int_V x^2 dm=\rho S\int_0^L x^2 dx=\frac{ML^2}{3} \qquad dm=\rho S dx$$

$$K=[Nm] \quad \textbf{et} \quad k=[N/m]$$

$$\omega^2=\frac{3\left(2kL^2+K\right)}{ML^2}$$

Vérification :

$$\omega^2=\frac{\left[\frac{N}{m}m^2\right]+[Nm]}{\left[kgm^2\right]}=\left[\frac{kgm}{s^2}\frac{m}{kgm^2}\right]=\left[\frac{1}{s^2}\right]$$