PLANAR LAMINAR PREMIXED FLAME

I)

- Réaction $-R+P\rightarrow 0$ transformation instantanée : $\delta_{ch}\approx 0$
- Diffusion des espèces négligée : pas de mélange entre R et P : $\varepsilon_k = Y_k$

1-a) $$\Delta=\frac{\lambda}{\dot{m}c_p}=\frac{3.10^{-2}}{0.385X1300}=6.10^{-5}m=0.06mm$$

1-b) Convection négligée devant la diffusion :

$$\frac{d}{dx}\left(\dot{m}\sum Y_k h_k-\lambda\frac{dT}{dx}\right)\approx\frac{d}{dx}\left(-\lambda\frac{dT}{dx}\right)=0\Rightarrow\frac{dT}{dx}=cte\Rightarrow\boxed{T=ax+b}$$

$x=0 \qquad T=T_b\Rightarrow b=T_b$

$x=-\Delta \qquad T=T_f\Rightarrow T_f=-a\Delta+T_b\Rightarrow \qquad a=\dfrac{T_b-T_f}{\Delta}$

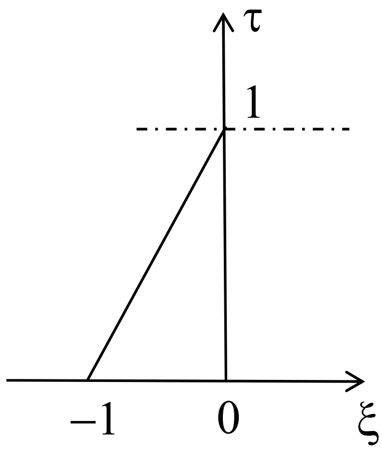
$$\boxed{T=\frac{T_b-T_f}{\Delta}x+T_b}$$

$$\tau=\frac{T-T_f}{T_b-T_f}=\frac{\left(\frac{T_b-T_f}{\Delta}x+T_b\right)-T_f}{T_b-T_f}=\frac{x}{\Delta}+1$$

en posant $\xi=\dfrac{x}{\Delta}$: $\xi=0$, $\tau=1$ $\qquad \xi=-1$, $\tau=0$

$\boxed{\tau=\xi+1}$ pente =1 en $\xi=0$

1-c)



Flamme stationnaire $u_f = V_F$ $\qquad \dot{m} = \rho_f u_f = \rho_f V_F$

$$\rho_f = \frac{pM}{RT} = \frac{10^5}{8.314}\frac{32.10^{-3}}{300} = 1.28 kg/m^3 \qquad V_F = 0.3 m/s$$

1-d)

$$\omega = \frac{\dot{m}}{\Delta} = \frac{0.385}{6.10^5} = 6.4\ 10^3 kg/m^3/s$$

grand car $1m^3$ de flamme correspond à $1.67\ 10^4 m^2$

1-e)

$$T_b = T_f + \frac{q}{c_p} = T_f + \frac{\left|-\Delta_r h^*\right|}{c_p} = 300 + \frac{2630}{1300} = 2323K$$

1-f)

$$\omega \propto p^{\Sigma \alpha_k} = p^2 \qquad \omega = \omega_1 \left(\frac{p}{p_1}\right)^2 = 10^{-2}\omega_1$$

$$V_F = \frac{1}{\rho_f}\sqrt{\frac{\lambda\omega}{c_p}} \propto \frac{1}{p}\sqrt{p^2} = 1 \Rightarrow V_F \text{ inchangée}$$

$$\Delta \propto \sqrt{\frac{1}{\omega}} \propto \frac{1}{p} \Rightarrow \boxed{\Delta(0.1bar) = 10\Delta(1bar)}$$

$$\dot{m} = \omega\Delta \propto p^2\frac{1}{p} = p \Rightarrow \boxed{\dot{m}(0.1bar) = 0.1\dot{m}(1bar)}$$

2)

$$\frac{d}{dx}\left(\dot{m}\sum Y_k h_k - \lambda\frac{dT}{dx}\right) = 0 \qquad \text{Gaz parfaits : } \sum Y_k h_k = h = c_p\left(T - T^*\right)$$

$$\frac{d}{dx}\left(\dot{m}c_p T - \lambda\frac{dT}{dx}\right) = 0 \Rightarrow \boxed{\dot{m}c_p T - \lambda\frac{dT}{dx} = cte}$$

$$x \to -\infty \qquad \frac{dT}{dx} = 0 \qquad T = T_f \Rightarrow \boxed{\dot{m}c_p T_f = cte}$$

$$\boxed{\dot{m}c_p\left(T - T_f\right) - \lambda\frac{d\left(T - T_f\right)}{dx} = 0} \text{ sachant que } \Delta = \frac{\lambda}{\dot{m}c_p} \Rightarrow : \boxed{\frac{d\left(T - T_f\right)}{T - T_f} = d\left(\frac{x}{\Delta}\right)}$$

$$\boxed{T - T_f = Ae^{\frac{x}{\Delta}}} \qquad x = 0 \qquad T = T_b \Rightarrow A = T_b - T_f \quad \boxed{\tau = e^{\frac{x}{\Delta}} = e^{\xi}}$$

Même pente $= 1$ en $\xi = 0$ que la solution précédente (voir **<u>1-b)</u>**)

II)

$-R+P+Q^* \rightarrow 0$ order 1

1) $\omega_p = \nu_p M_p \mathrm{V_r} = \nu_p M_p B T^b e^{-E_a/RT}\left(\dfrac{p}{RT}\right)^{\sum_R \alpha_k} \Pi_R X_k^{\alpha_k}$

$$\nu_p = 1 \qquad M_p = M_r = M = \frac{\rho RT}{p} \qquad Y_R = \frac{x_R M_R}{x_R M_R + x_P M_P} = \frac{x_R}{x_R + x_P} = X_R$$

$$\Pi_R X_k^{\alpha_k} = X_R = Y_R = 1 - Y_P \text{ and } \boxed{\omega_P = \rho B T^b e^{-E_a/RT}(1-Y_P)}$$

2) Reduced temperature equation (See the course)

$$\xi = \frac{x}{\Delta} \qquad \Delta = \frac{\lambda}{\dot{m} c_p}$$

$$\frac{\partial \tau}{\partial \xi} - \frac{\partial^2 \tau}{\partial \xi^2} = \frac{\lambda}{\dot{m}^2 c_p}\omega_p$$

identical to the equation of the mass fraction of products:

$$\frac{\partial Y_P}{\partial \xi} - \frac{\partial^2 Y_P}{\partial \xi^2} = \frac{\lambda}{\dot{m}^2 c_p}\omega_p$$

For an adiabatic system: same boundary conditions $\Rightarrow \boxed{Y_P \equiv \tau}$

$$\boxed{\frac{\partial \tau}{\partial \xi} - \frac{\partial^2 \tau}{\partial \xi^2} = \Lambda_0 (1-\tau)}$$

With :

$$\Lambda_0 = \frac{\lambda}{\dot{m}^2 c_p} \rho B T^b e^{-T_a/T}$$

where $T_a = E_a / R$ is the activation temperature.

3)

a) <u>Thermal zone:</u> $T << T_a$ $\qquad e^{-T_a/T} \rightarrow e^{-\infty} \rightarrow 0 \Rightarrow \Lambda_0 \rightarrow 0$ and $\omega \rightarrow 0$

$\tau << \tau_i$

$$\boxed{\frac{\partial \tau}{\partial \xi} - \frac{\partial^2 \tau}{\partial \xi^2} = 0}$$

If we put $u = \dfrac{\partial \tau}{\partial \xi} \Rightarrow u - \dfrac{\partial u}{\partial \xi} = 0 \Rightarrow \dfrac{\partial u}{u} = \partial \xi \Rightarrow u = Ae^{\xi}$

$$Ae^{\xi} = \frac{\partial \tau}{\partial \xi} \Rightarrow \partial \tau = Ae^{\xi}\partial \xi \Rightarrow \tau = Ae^{\xi} + B$$

$\xi \rightarrow -\infty \qquad T = T_f \Rightarrow \tau = 0 \Rightarrow B = 0$

$\xi \rightarrow 0 \qquad T = T_i \Rightarrow \tau = \tau_i \Rightarrow A = \tau_i$

$\boxed{\tau = \tau_i e^{\xi}}$ pour $\xi < 0$

b) <u>Chemical zone</u>: $T \approx T_b$ and $\tau_i < \tau < 1$

$$\Lambda_0 = \frac{\lambda}{\dot{m}^2 c_p}\rho B T^b e^{-T_a/T} \qquad \frac{\lambda}{\dot{m}^2 c_p}\rho B T^b \approx cte \Rightarrow \Lambda_0 = e^{-T_a/T}.cte$$

$$\Lambda_b = e^{-T_a/T_b}.cte$$

$$\frac{\Lambda_0}{\Lambda_b} = e^{-T_a\left(\frac{1}{T}-\frac{1}{T_b}\right)} = e^{-\frac{T_a}{T_b}\left(\frac{T_b}{T}-1\right)} \approx e^{-\beta\left(\frac{1}{\tau}-1\right)} \approx e^{-\beta(1-\tau)} \approx 1 \quad \text{then: } \boxed{\Lambda_0 = \Lambda_b}$$

$$\boxed{\frac{\partial \tau}{\partial \xi} - \frac{\partial^2 \tau}{\partial \xi^2} = \Lambda_b (1-\tau)}$$

$$\theta = 1-\tau \qquad \boxed{\begin{aligned}&\frac{\partial^2 \theta}{\partial \xi^2} - \frac{\partial \theta}{\partial \xi} - \Lambda_b \theta = 0\\ &\theta = Ae^{r_1\xi} + Be^{r_2\xi}\end{aligned}} \qquad \begin{aligned}&r_{1,2} = \frac{1 \pm \sqrt{1+4\Lambda_b}}{2}\\ &\theta = (1-\tau) = Ae^{r_1\xi} + Be^{r_2\xi}\end{aligned} \qquad \begin{aligned}&r_1 > 0\\ &r_2 < 0\end{aligned}$$

$$\begin{aligned}&\xi \to 0 \qquad && \tau = \tau_i \Rightarrow 1-\tau_i = A+B\\ &\xi \to +\infty && \tau = 1 \Rightarrow A = 0\end{aligned}$$

$$\boxed{(1-\tau) = (1-\tau_i)e^{\frac{1-\sqrt{1+4\Lambda_b}}{2}\xi}}$$

c) <u>Case of</u> $\Lambda_b >> 1$

$$\boxed{(1-\tau) = (1-\tau_i)e^{-\sqrt{\Lambda_b}\xi}}$$

Which corresponds to the case where the convection is negligible, the equation is then reduced to: $\boxed{\frac{\partial^2 \theta}{\partial \xi^2} - \Lambda_b \theta = 0} \Rightarrow \boxed{\theta = (1-\tau) = (1-\tau_i)e^{-\sqrt{\Lambda_b}\xi}}$

d) $\xi = 0 \qquad \tau_{th} = \tau_{ch} = \tau_i$

- Thermal zone: $\tau = \tau_i e^{\xi} \Rightarrow \left(\frac{\partial \tau}{\partial \xi}\right)_{\xi=0} = \tau_i$
- Chemical zone: $(1-\tau) = (1-\tau_i)e^{-\sqrt{\Lambda_b}\xi} \Rightarrow \left(\frac{\partial \tau}{\partial \xi}\right)_{\xi=0} = (1-\tau_i)\sqrt{\Lambda_b}$

Equality of both slopes $\Rightarrow \tau_i = (1-\tau_i)\sqrt{\Lambda_b} \Rightarrow \boxed{\tau_i = \frac{\sqrt{\Lambda_b}}{1+\sqrt{\Lambda_b}}}$

$$1-\tau_i = 1 - \frac{\sqrt{\Lambda_b}}{1+\sqrt{\Lambda_b}} = \frac{1}{1+\sqrt{\Lambda_b}} \approx \frac{1}{\sqrt{\Lambda_b}} \Rightarrow \boxed{\tau_i \approx 1 - \frac{1}{\sqrt{\Lambda_b}}} \qquad \boxed{\Rightarrow \tau_i \approx 1}$$

4) <u>Chemical zone</u>: $\Delta T = T_b - T_i = (T_b - T_f)(1-\tau_i) = \frac{T_b - T_f}{\sqrt{\Lambda_b}}$

(a) $\Delta T = T_b - T_i = \frac{T_b - T_f}{\sqrt{\Lambda_b}}$ small value

(b) $\Lambda_b = 4938$ $\sqrt{\Lambda_b} \approx 70$ $\boxed{\Delta T \approx 29K}$

III)

1.

<u>In the thermal zone</u>, $\Lambda_0 \to 0$ and $\omega \to 0$

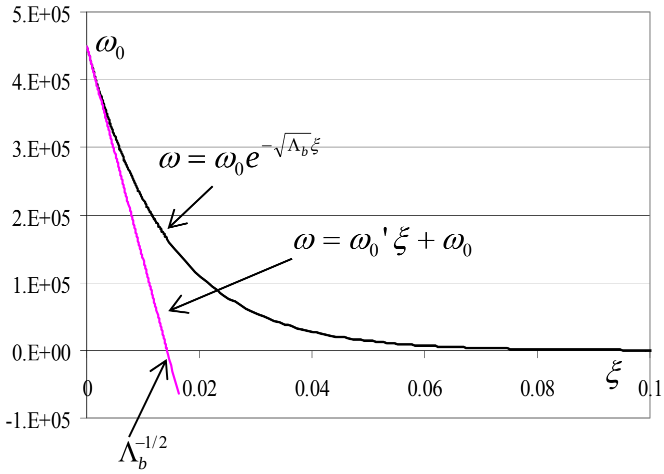
<u>In the chemical zone,</u>

The reduced temperature is given by: $\tau = 1-(1-\tau_i)e^{-\sqrt{\Lambda_b}\xi}$ and the chemical production is given by: $\omega = (1-\tau)\rho B T^b e^{-E_a/T}$

$$\Lambda_0 = \frac{\lambda}{\dot{m}^2 c_p}\rho B T^b e^{-E_a/T} \Rightarrow \omega = \frac{\dot{m}^2 c_p}{\lambda}\Lambda_0(1-\tau)$$

$\Lambda_0 = \Lambda_b$ and $1-\tau = \frac{1}{\sqrt{\Lambda_b}}e^{-\sqrt{\Lambda_b}\xi}$ Then $\boxed{\omega = \frac{\dot{m}^2 c_p}{\lambda}\sqrt{\Lambda_b}e^{-\sqrt{\Lambda_b}\xi} = \omega_0 e^{-\sqrt{\Lambda_b}\xi}}$

2. $\omega' = \frac{d\omega}{d\xi} = -\sqrt{\Lambda_b}\omega$ the tangent of curve ω at the point $\xi = 0$ has the equation: $\omega = \omega_0'\xi + \omega_0$



For $\omega = 0 \Rightarrow \xi_{ch} = \frac{\delta_{ch}}{\Delta} = -\frac{\omega_0}{\omega_0'} = -\frac{\omega_0}{-\sqrt{\Lambda_b}\omega_0} = \frac{1}{\sqrt{\Lambda_b}} = 0.01428571 \approx 0$

The chemical thickness $\boxed{\delta_{ch} = \frac{\Delta}{\sqrt{\Lambda_b}}}$

Thermal zone <u>$(\xi < 0)$</u>:

$\tau = \tau_i e^{\xi}$ with $\tau_i = 1 - \frac{1}{\sqrt{\Lambda_b}} = \tau_0 = \tau(0)$

$\tau' = \tau_i e^{\xi} = \tau$ and $\tau'_0 = \tau_i$

The tangent of the curve τ at $\xi = 0$ has the equation $\tau = \tau_i(1+\xi)$, then for $\tau = 0 \Rightarrow \xi_{th} = \frac{\delta_{th}}{-\Delta} = -1$ then $\boxed{\delta_{th} = \Delta}$

The final result is that $\boxed{\dfrac{\delta_{th}}{\delta_{ch}} = \sqrt{\Lambda_b} \approx 70}$