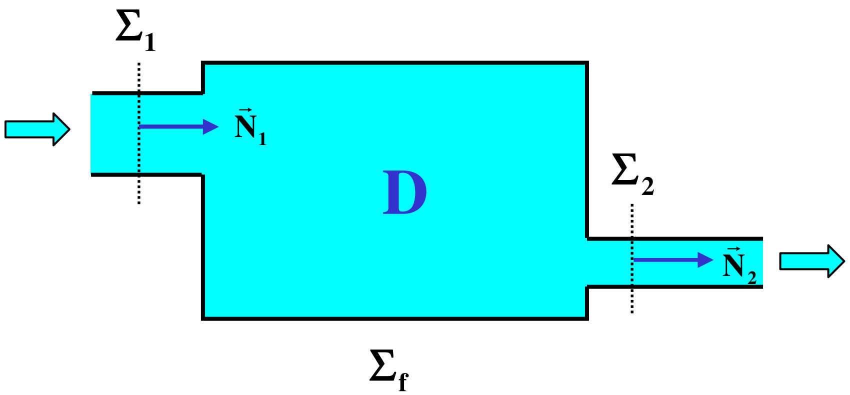
CHAPITRE 3
Bilans intégraux

Bilan de masse

$$\frac{\partial}{\partial t}\int_D \rho\, dv + \int_\Sigma \rho \vec{V}\cdot\vec{n}\, d\sigma = 0$$

Ecoulement permanent d 'un fluide incompressible



$$S_1\overline{V}_1 = S_2\overline{V}_2$$

avec $$\overline{V} = \frac{1}{S}\int_\Sigma \vec{V}\cdot\vec{n}\, d\sigma$$

Bilan de quantité de mouvement

$$\frac{\partial}{\partial t}\int_D \rho\vec{V}\,dv+\int_\Sigma \rho\vec{V}(\vec{V}\cdot\vec{n})\,d\sigma=-\int_\Sigma p\cdot\vec{n}\,d\sigma+\int_\Sigma \vec{\tau}\,d\sigma+\int_D \rho\vec{f}\,dv$$

Ecoulement permanent d 'un fluide incompressible

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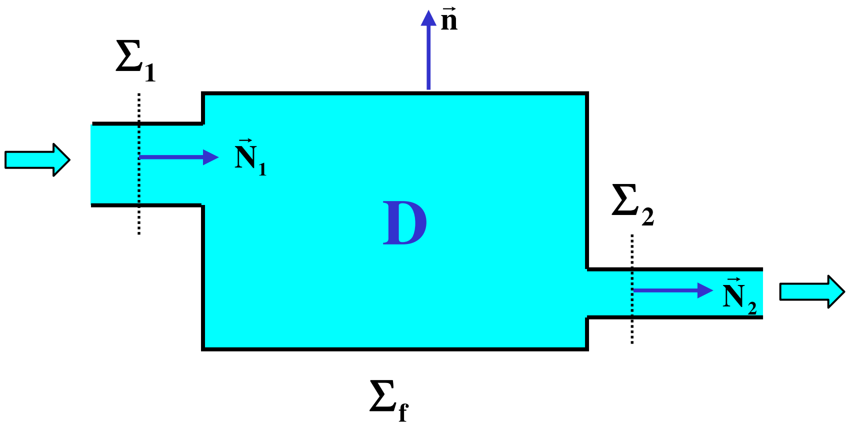


$$\vec{F}\approx-mg\vec{z}+\left[S_i\left(\rho\beta_i\overline{V}_i^2+\overline{p}_i\right)\vec{N}_i\right]_{i=2}^{i=1}$$

ou

$$\vec{F}\approx-\int_{\Sigma_f}\rho g z\,\vec{n}\,d\sigma+\left[S_i\left(\rho\beta_i\overline{V}_i^2+p_i^*\right)\vec{N}_i\right]_{i=2}^{i=1}$$

$$\beta S\overline{V}^2\vec{n}=\int_\Sigma \vec{V}(\vec{V}\cdot\vec{n})\,d\sigma$$

avec:

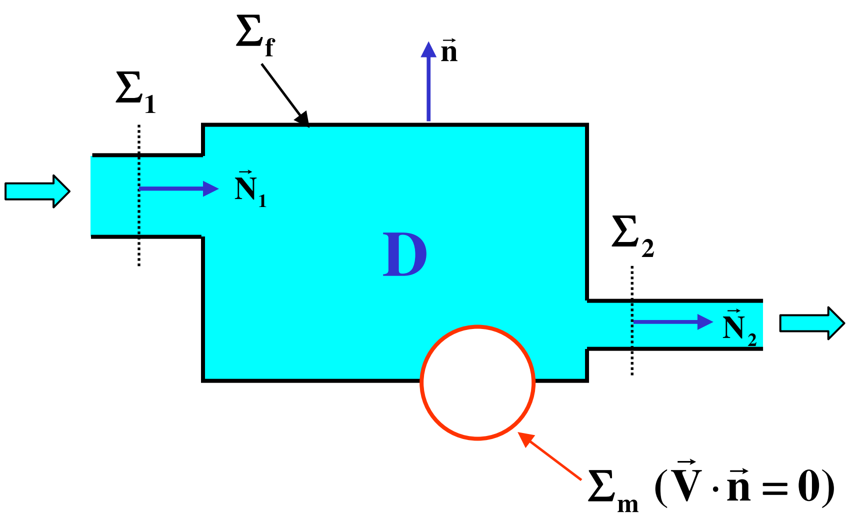
$$p^*=p+\rho g z$$

$$\overline{p}=\frac{1}{S}\int_\Sigma p\,d\sigma$$

Bilan d 'énergie cinétique

$$\frac{\partial}{\partial t}\int_D \tfrac{1}{2}\rho V^2\,dv + \int_\Sigma \tfrac{1}{2}\rho V^2(\vec{V}\cdot\vec{n})\,d\sigma = -\int_\Sigma p\vec{V}\cdot\vec{n}\,d\sigma + \int_\Sigma \vec{\tau}\cdot\vec{V}\,d\sigma$$
$$+\int_D \rho\vec{f}\cdot\vec{V}\,dv + \int_D p\,\mathrm{div}\vec{V}\,dv - \int_D \varphi_1\,dv$$

Ecoulement permanent d 'un fluide incompressible



$$q_m\left[\frac{p_i^*}{\rho}+\tfrac{1}{2}\alpha_i\overline{V}_i^2\right]_{i=2}^{i=1} = \Phi_1 - P_m$$

$$\alpha = \frac{1}{S\overline{V}^3}\int_\Sigma V^2(\vec{V}\cdot\vec{n})\,d\sigma$$

avec: $\Phi_1 = \int_D \varphi_1\,dv$

$$P_m = \int_{\Sigma_m} \vec{\tau}\cdot\vec{V}\,d\sigma$$

Pertes de charge

Charge moyenne dans une section

$$\boxed{\overline{k}_i} = \frac{p_i^*}{\rho} + \tfrac{1}{2}\alpha_i \overline{V}_i^2$$

(Dimension: carré d 'une vitesse)

Charge hydraulique moyenne dans une section

$$\boxed{\overline{H}_i} = \frac{\overline{k}_i}{g} = \frac{p_i^*}{\rho g} + \frac{\alpha_i \overline{V}_i^2}{2g}$$

(Dimension: longueur)

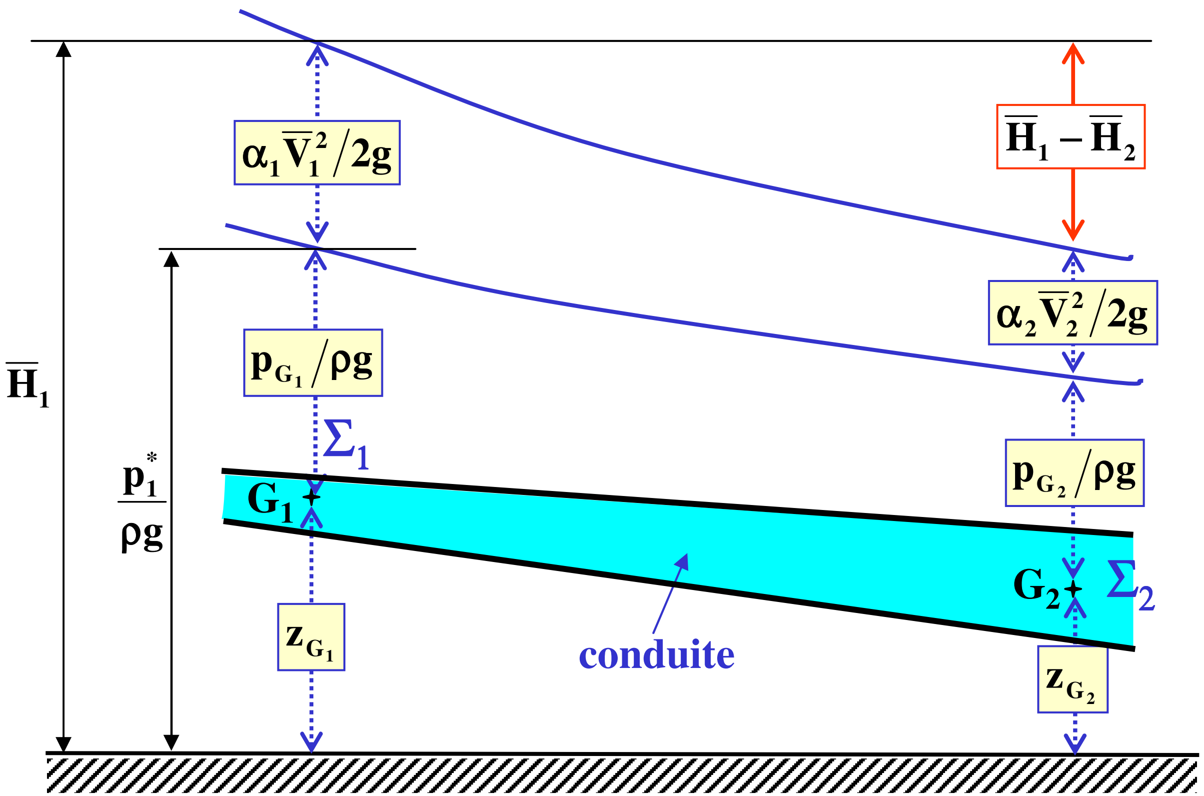
Coefficient de perte de charge dans un domaine (entre 2 sections)

$$\frac{\Phi_1}{q_m} = \frac{1}{2}\boxed{\zeta} V_r^2$$

Vr : vitesse de référence

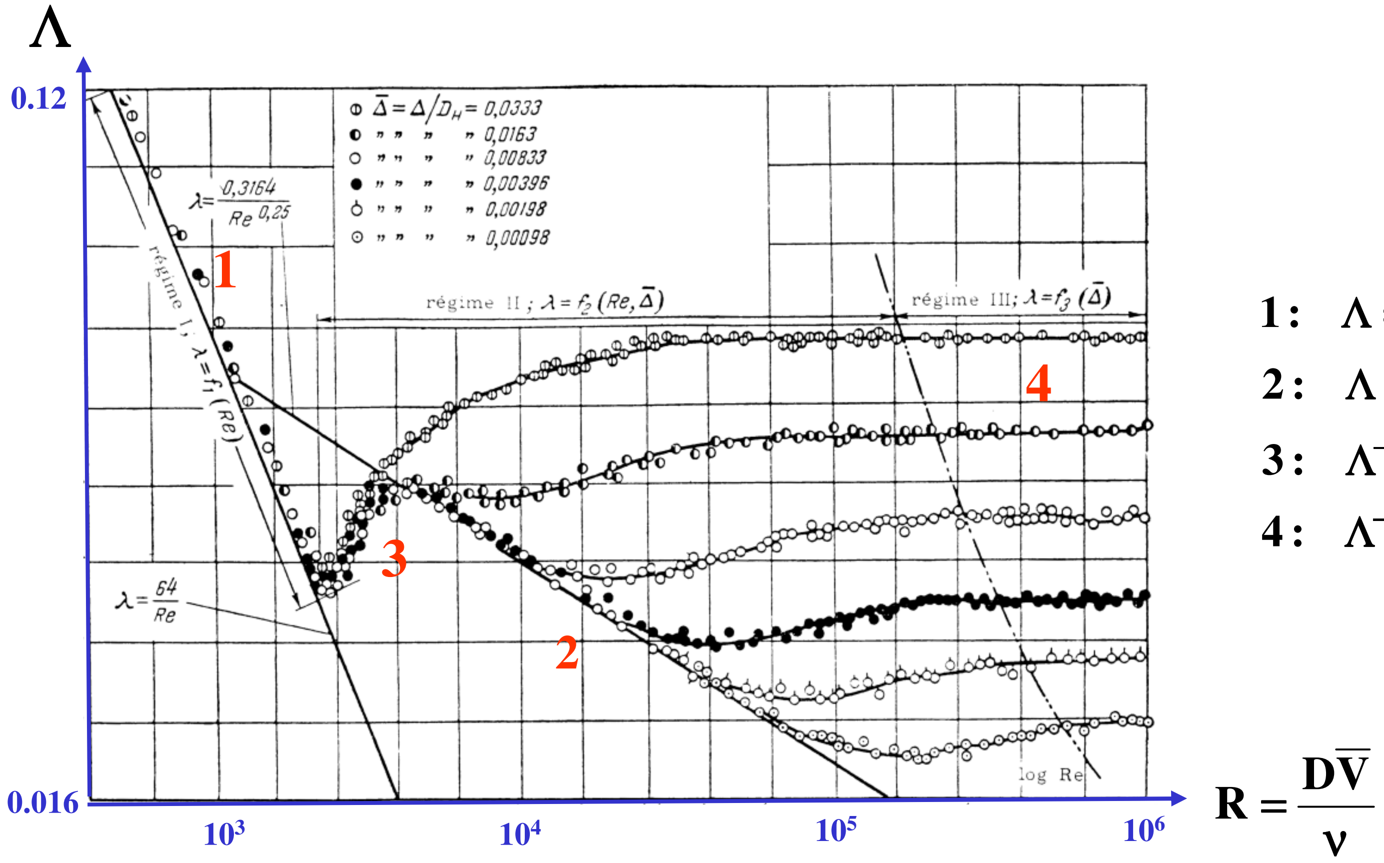
$$\begin{aligned} q_m \, \Delta\overline{k} &= \Phi_1 - P_m \\ &= \tfrac{1}{2}\zeta V_r^2 \, q_m - P_m \end{aligned}$$

Représentation géométrique des pertes de charges



$$\overline{H} = z_G + \frac{p_G}{\rho g} + \frac{\alpha \overline{V}^2}{2g}$$

Conduite cylindrique



$$1: \quad \Lambda = 64/R$$

$$2: \quad \Lambda = 0.316/R^{0.25}$$

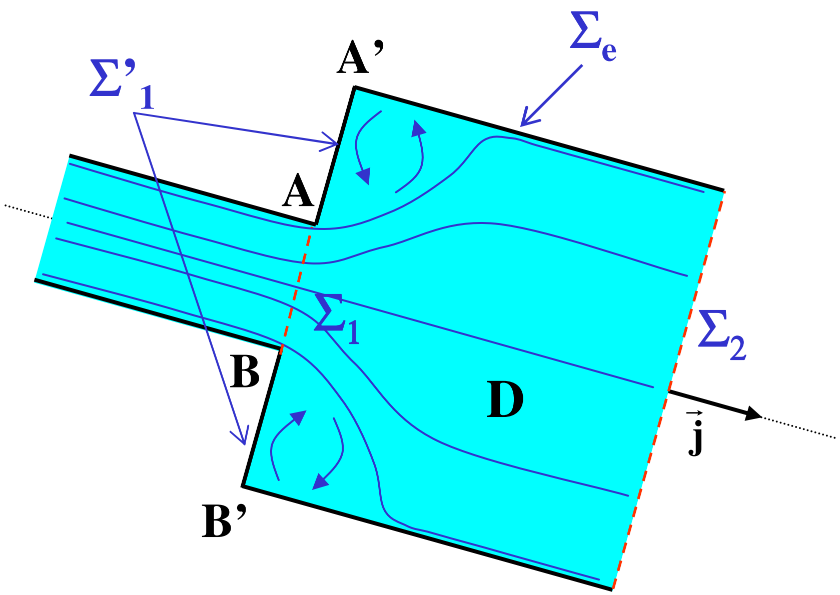
$$3: \quad \Lambda^{-0.5} = 2\log R\sqrt{\Lambda} - 0.8$$

$$4: \quad \Lambda^{-0.5} = 2\log(D/h) + 1.14$$

Figure d 'après l 'ouvrage « Mémento des pertes de charge» de I.E. IDEL 'CIK, Eyrolles, 1986.

Elargissement brusque

<u>Hypothèses:</u>
Ecoulement permanent en moyenne
Fluide incompressible
Conduites cylindriques coaxiales



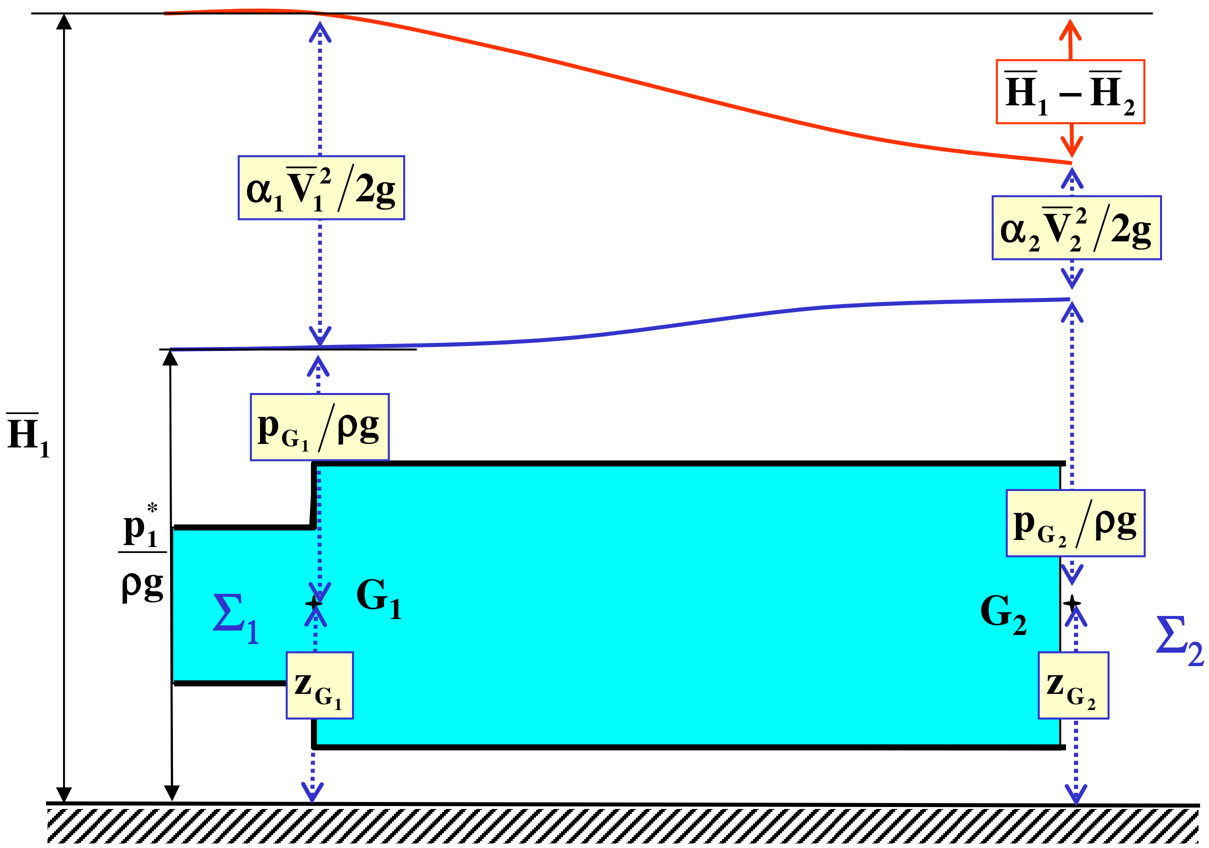
$$\Delta\bar{k} = \frac{\bar{V}_1^2}{2}(1-\omega)^2$$

$$\Rightarrow \quad \zeta = (1-\omega)^2$$

$$\text{avec :} \quad \omega = \frac{S_1}{S_2}$$

Diagramme de pertes de charge

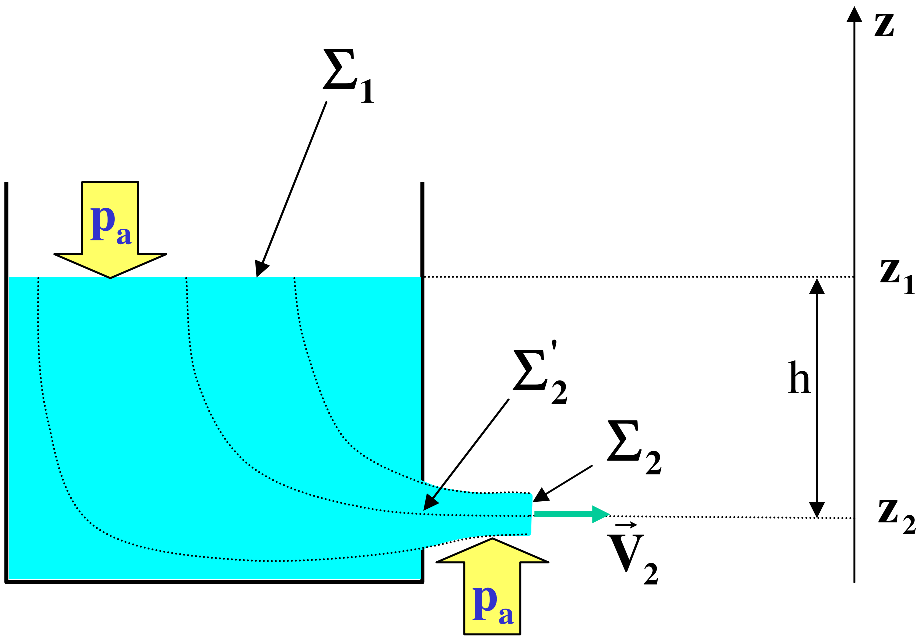
Elargissement brusque



$$\overline{H} = z_G + \frac{p_G}{\rho g} + \frac{\alpha \overline{V}^2}{2g}$$

Vidage d 'un réservoir

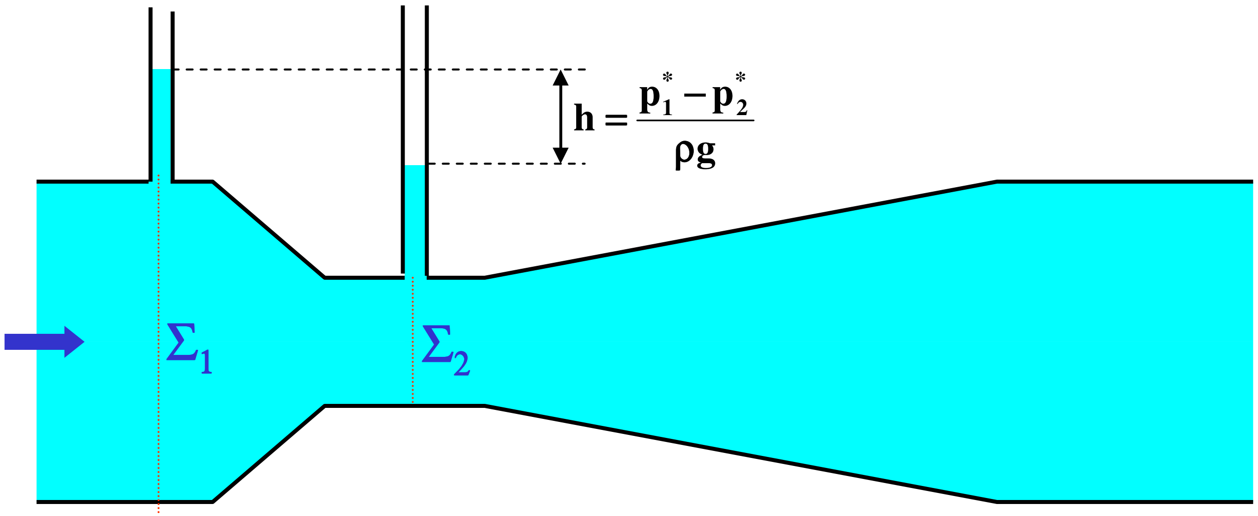
<u>Hypothèses:</u>
Ecoulement permanent
d 'un fluide incompressible



$$\Rightarrow \quad \boxed{V_2 = \sqrt{\frac{2gh}{\alpha_2}}}$$

Tube de Venturi

Hypothèses:
Ecoulement permanent
d ’un fluide incompressible



$$q_m = \rho S_2 \left[\frac{2gh}{\alpha_2 - \alpha_1 \omega^2} \right]^{\frac{1}{2}}$$

avec : $\omega = \dfrac{S_2}{S_1}$