PS41 Propulsion – Chapter 1

Basic Concepts of Thermodynamics and Energy

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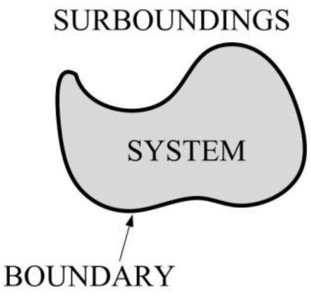
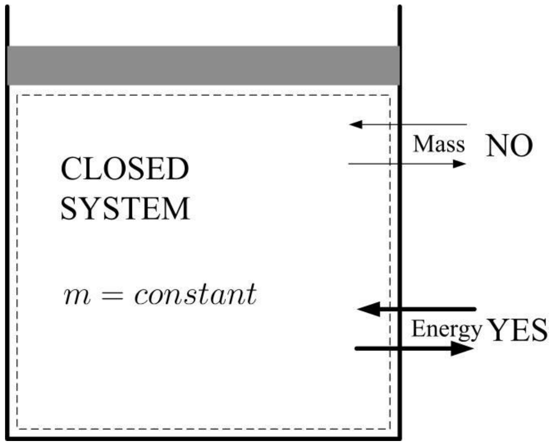
Objectives

- Review thermodynamics and one-dimensional gas dynamics

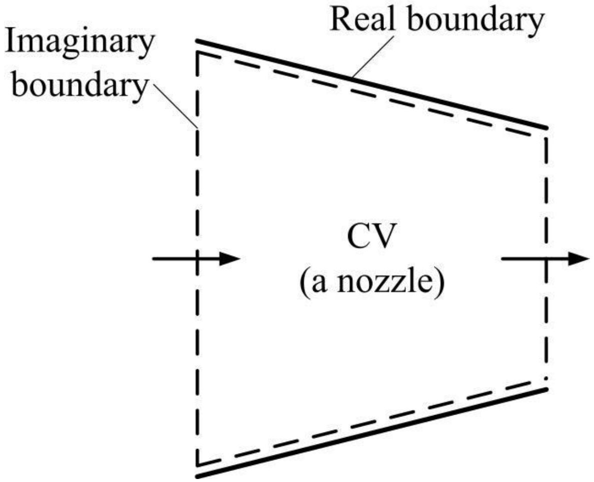
1 Systems and Control Volumes

- A system is defined as *a quantity of matter or a region in space chosen for study*.





Mass cannot cross the boundaries of a closed system, but energy can.



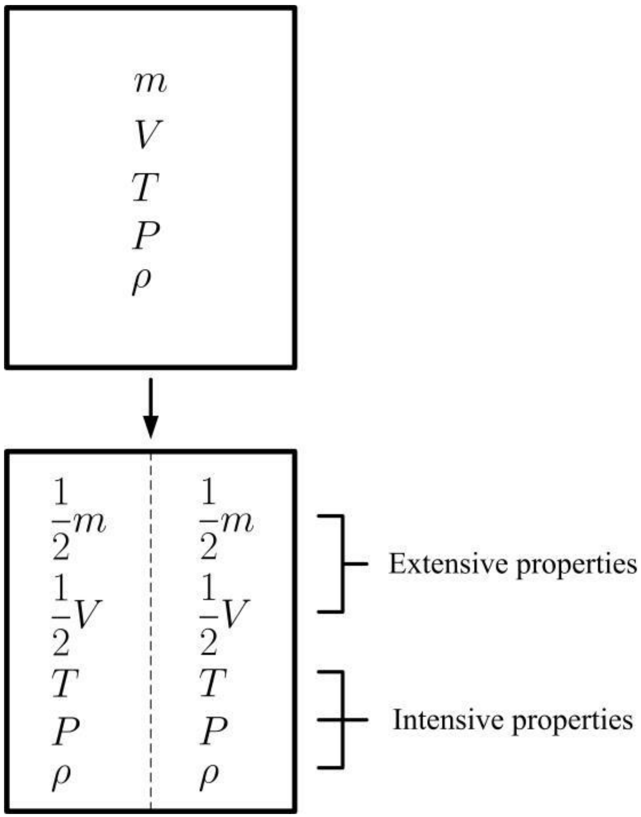
A control volume with real and imaginary boundaries.

2 State and Equilibrium

- State
- Equilibrium
  - Thermal Equilibrium
  - Mechanical Equilibrium
  - Phase Equilibrium
  - Chemical Equilibrium

3 Properties of System

- Intensive properties: pressure P, temperature T, density ρ
- Extensive properties: volume V, and mass m, energy E, Enthalpy H, Entropy S
- Specific properties



4 Energy

- Energy can exist in numerous forms such as thermal, mechanical, kinetic, potential, electric, magnetic, chemical, and nuclear, and their sum constitutes the total energy E of a system.
  - Internal energy: $U = f(T, v), U = f(T, P), U = f(P, v)$
  - Kinetic energy: $E_k = \frac{1}{2}mc^2$
  - Potential energy: $E_p = mgz$
- $E = U + E_k + E_p = U + \frac{1}{2}mc^2 + mgz$
- Specific total energy: $e = \frac{E}{m}$ (kJ/kg)

5 Work and Enthalpy

- Moving boundary work

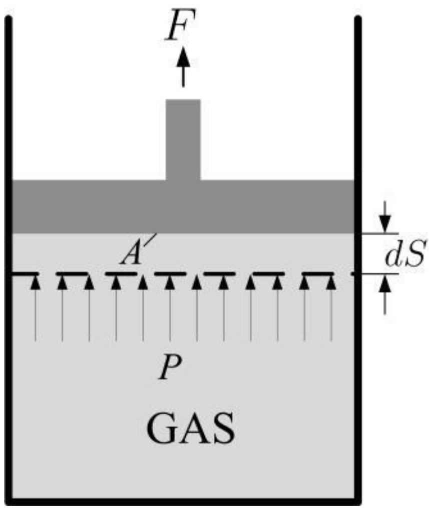
$$\delta W = FdS = PAdS = PdV$$

- Enthalpy

$$\mathrm{H} = U + PV \quad (\mathrm{J})$$

- Specific Enthalpy

$$h = u + Pv \quad (\mathrm{J/kg})$$



6.1 The Ideal-Gas Equation of State

- The ideal-gas equation of state：

$$Pv = RT$$

- R is called the gas constant.
  - The gas constant R is different for each gas.
  - $R = \frac{R_u}{M}$ (kJ/kg·K or kPa·m^3/kg·K)，R_u is the universal gas constant, M is the molar mass.
  - $R_u = \begin{cases} 8.31447\ \mathrm{kJ/kmol \cdot K} \\ 1.98588\ \mathrm{Cal/mol \cdot K} \end{cases}$
  - $m = MN\ \mathrm{(kg)}$

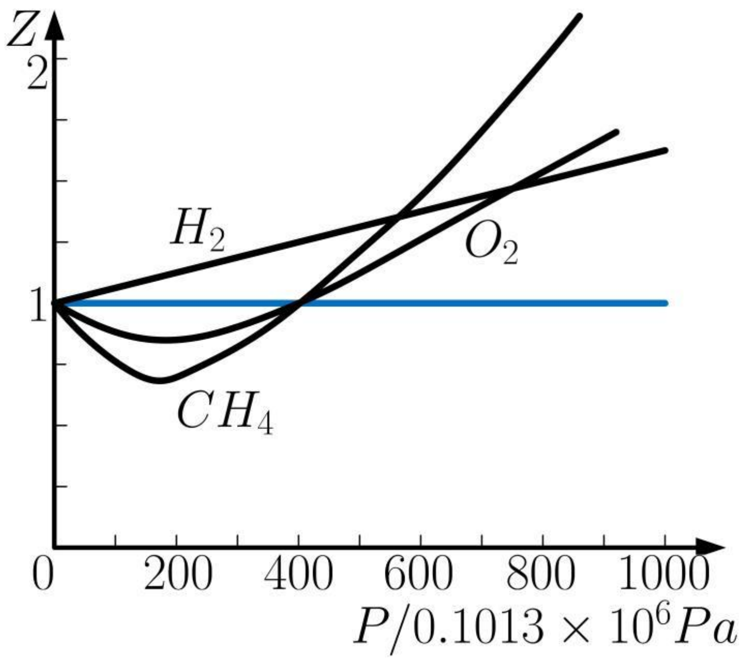
- $V = mv, PV = mRT$

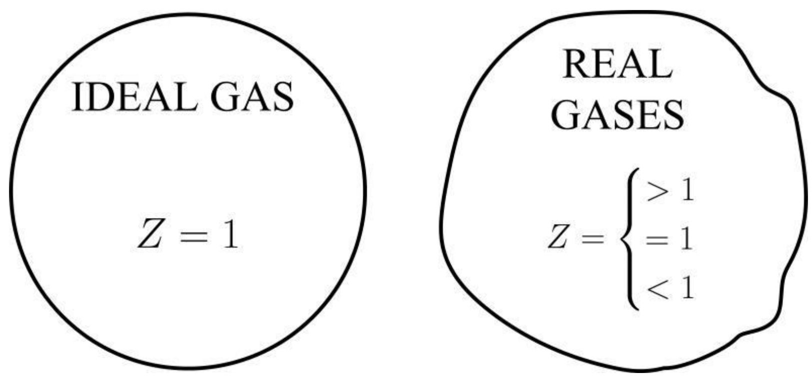
$$mR = (MN)R = NR_u, PV = NR_uT$$

6.2 Deviation from Ideal-Gas Behavior

- Compressibility factor Z:

$$Z = \frac{Pv}{RT}, \qquad Z = \frac{v_{\text{actual}}}{v_{\text{ideal}}}$$





6.3 Other Equations of State

- Van der Waals Equation of State

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT$$

- Redlich - Kwong Equation of State

$$P = \frac{RT}{v - b} - \frac{a}{T^{0.5}v(v + b)}$$

- Virial Equation of State

$$P = \frac{RT}{v} + \frac{a(T)}{v^2} + \frac{b(T)}{v^3} + \frac{c(T)}{v^4} + \frac{d(T)}{v^5} + \cdots$$

7 Specific Heats

- Specific Heat is defined as the energy required to raise the temperature of a unit mass of a substance by one degree.
  - Specific heat at constant volume $c_v = \left(\frac{\delta q}{dT}\right)_v = \left(\frac{du+pdv}{dT}\right)_v = \left(\frac{\partial u}{\partial T}\right)_v$
  - Specific heat at constant pressure $c_p = \left(\frac{\delta q}{dT}\right)_p = \left(\frac{dh-vdP}{dT}\right)_p = \left(\frac{\partial h}{\partial T}\right)_p$
- Specific heat relations of ideal gases

$$\frac{dh}{dT} = \frac{du}{dT} + R, \text{ then } c_p - c_v = R$$

$$\gamma = \frac{c_p}{c_v}, c_v = \frac{1}{\gamma - 1} R, c_p = \frac{\gamma}{\gamma - 1} R$$

8.1 Energy Analysis of Closed Systems

- Energy balance for any system undergoing any kind of process is expressed as

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{By heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}} \quad \text{(kJ)}$$

Or in the rate form, as

$$\dot{E}_{in} - \dot{E}_{out} = dE_{system}/dt \quad \text{(kW)}$$

- Energy balance for a closed system

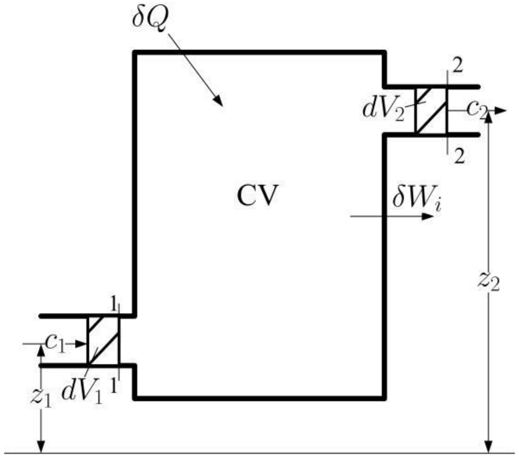
$$Q - W = \Delta U, \delta q = du + \delta w$$

- $\delta w = Pdv, \delta q = du + Pdv$

8.2 Mass and Energy Analysis of Control Volumes

- $E_{in} - E_{out} = \Delta E_{system}$
  $E_{in} = dE_1 + P_1 dV_1 + \delta Q$
  $E_{out} = dE_2 + P_2 dV_2 + \delta W_i$

  $\Delta E_{system} = dE_{CV}$



- $\delta Q = dE_{CV} + \sum_j \left(h + \frac{c^2}{2} + gz\right)_{out} \delta m_{out} - \sum_i \left(h + \frac{c^2}{2} + \right.$

9 Entropy

- Entropy

$$dS = \left(\frac{\delta Q}{T}\right)_{int\ rev}$$

Specific Entropy for ideal gas

$$ds = \frac{\delta q}{T}_{int\ rev} = \frac{c_p dT - vdP}{T}$$

$$\Delta s_{1-2} = \int_{T_1}^{T_2} c_p \frac{dT}{T} - R\ln\frac{P_2}{P_1} = \int_{T_1}^{T_2} c_v \frac{dT}{T} + R\ln\frac{V_2}{V_1}$$

10.1 One-dimensional gas dynamics

- Conservation of Mass Equation

$$q_m = \rho_1 v_1 A_1 = \rho_2 v_2 A_2$$

- Energy Equation

$$\pm q \pm l = (h_2 - h_1) + \frac{v_2^2 - v_1^2}{2} = h_2^* - h_1^*$$

$$\pm \mathrm{l}_u = \frac{v_2^2 - v_1^2}{2} + \int_1^2 \frac{dp}{\rho} + l_f$$

- Momentum Equation

$$\sum \vec{F} = q_m(\overrightarrow{v_2} - \overrightarrow{v_1})$$

10.2 Sound speed and Mach number

- $a = \sqrt{\frac{dp}{d\rho}} = \sqrt{\gamma RT}$
- $Ma = \frac{v}{a}$
- $\frac{dA}{A} = (Ma^2 - 1)\frac{dv}{v}, \frac{dp}{\rho} + d\left(\frac{v^2}{2}\right) = 0$
- $q = 0, l = 0$,then $h_2^* = h_1^*$（绝能过程）
  $dq > 0, dl = 0$, then $q = h_2^* - h_1^*$
  $dq = 0, dl > 0$, then $l = h_2^* - h_1^*$
- $\sigma = \frac{P_2^*}{P_1^*} < 1.0$（总压恢复系数）
- $\lambda = \frac{v}{a_{cr}}, a_{cr} = \sqrt{\frac{2\gamma}{\gamma+1} RT^*}$