Chapitre V

Etude des régimes compressibles en Fluide Parfait

→ Part 1 : Introduction
Description of compressible and inviscid flows
Steady unidirectional compressible flows

→ Part 2 : From pressure waves to shock waves
Normal and oblique shock waves
Examples

Avertissement : Ce chapitre est tiré du cours de Master donné dans le cadre du « Summer Program » associant les écoles aéronautiques du GEA. Seule la première partie sera traitée.

Introduction (1)

→ Compressibility phenomena are associated with large velocities or large accelerations in a gas flow.

→ The development of this field of fluid mechanics is linked with the evolution of aeronautics in the middle of the 20th century.

→ In fact, it concerns a lot of applications :

$\Rightarrow$ Pneumatic transport

$\Rightarrow$ flows in Intake or exhaust ports of automotive engines

$\Rightarrow$ **…...**

Description of compressible and inviscid flows (1)

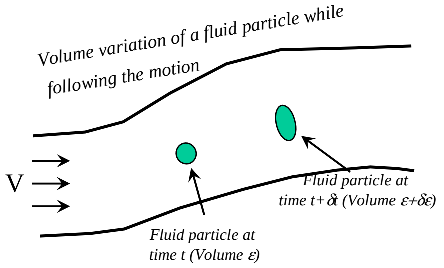
→ Goals :

⇒ Physical description

⇒ Mathematical model

⇒ Definition and properties of stagnation quantities

Description of compressible and inviscid flows (2)



→ What are the consequences of fluid motion on the variation of the volume of an elementary fluid particle ?

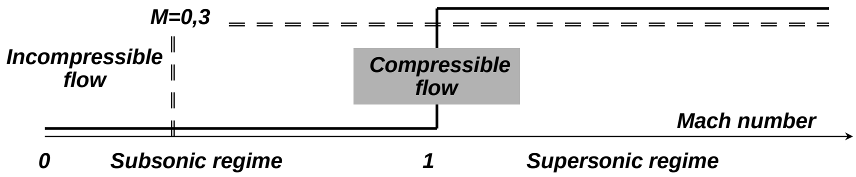
→ Important parameter : Mach Number $M = V/a$

→ First estimation ($m = \rho\,\varepsilon$ *constant mass of the fluid particle)*

- $\left|\frac{\delta\varepsilon}{\varepsilon}\right| = \left|\frac{\delta\rho}{\rho}\right| = \frac{1}{a^2}\left|\frac{\delta p}{\rho}\right|$
- If $\rho \approx$ constant : $\delta p \approx \rho \frac{V_\infty^2}{2}$

$$\left|\frac{\delta\varepsilon}{\varepsilon}\right| \approx \frac{M_\infty^2}{2}$$

→ Regimes of compressible flows



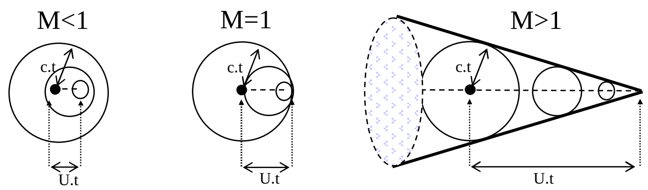
Mach number : M=V/c (c: speed of sound)

(For air in standard conditions of temperature and pressure, M=1 corresponds to 330 m/s)

→ In some flows, for example on airfoils, both subsonic and supersonic regions can co-exist. We say that the flow regime is transsonic

→ On both sides of Mach ONE !

→ SOURCE IN MOTION



⇒ Subsonic regime : Information arrives before the source

⇒ Supersonic regime : Information arrives after

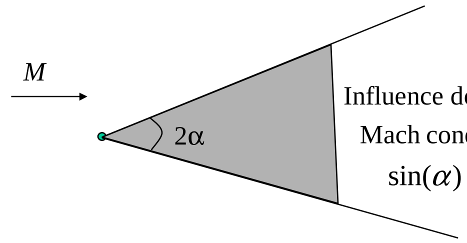
- Mach cone of angle α / $\sin(\alpha)=1/M$

⇒ The properties of the equations of motion are changing :
* Subsonic : System elliptic in space
* Supersonic : System hyperbolic in space

Description of compressible and inviscid flows (5)

→ On both sides of Mach ONE

→ FIXED SOURCE in a Supersonic flow



Influence domain :
Mach cone of angle 2α
$\sin(\alpha) = 1/M$

$\Rightarrow$ In a supersonic flow, the fluid particle is not “informed” that there is an obstacle !!
- This explains why we observe very sharp transitions
- Explains the apparition of shock waves.

Mathematical model for compressible and inviscid flows. (1)

→ Large Reynolds numbers

→ Non buoyant fluid , No volumetric heat transfer

→ We assume that the fluid is a perfect gaz.

* Equation of state : $p/\rho = rT$ where $r = R/M$

* Joule Laws : $de = c_v dT$; $dh = c_p dT$

c_v and c_p supposed constant ; $\gamma = c_p / c_v$

Meyer relation : $r = c_p - c_v$

* Entropy : $s = c_v \log\left(p/\rho^{\gamma}\right)$

* Velocity of sound : $a^2 = \left.\dfrac{\partial p}{\partial \rho}\right|_s = \gamma\, rT = \gamma \dfrac{p}{\rho}$

r is the material specific gaz constant.
c_p and c_v are specific heats
at constant pressure and volume

→ Tables and numerical integration must be used for more complex thermodynamics.

Mathematical model for compressible and inviscid flows. (2)

- Mass, Momentum and energy balance are written .

- If there are no irreversibilities and no volumetric heat flux,
For a general unsteady flow :

$$\Rightarrow \frac{Ds}{Dt} = 0$$

In a compressible and inviscid flow, the entropy is constant along trajectories.

- BEWARE : This is only true « pieces by pieces » if there are shock waves.

Mathematical model for compressible and inviscid flows. (3)

- Mass, Momentum and energy balance are written .

- If there are no irreversibilities and no volumetric heat flux
Case of a permanent flow (which will be considered afterwards))

$$\Rightarrow \vec{V}.g\vec{r}ad(s) = 0$$

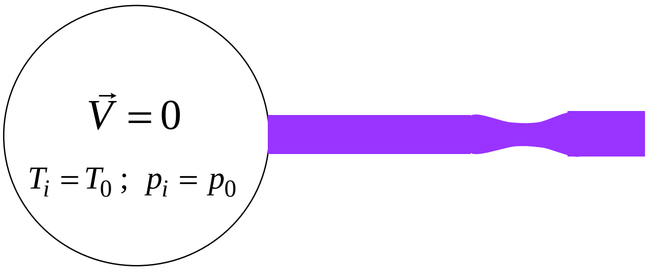
$$\vec{V}.g\vec{r}ad(h_i) = 0 \quad \text{where} \quad h_i = h + \frac{V^2}{2}$$

In a permanent compressible and inviscid flow the entropy and the stagnation enthalpy are constant along streamlines.

- BEWARE : This is again only true « pieces by pieces » if there are shock waves.

Permanent regime: practical definitions of the stagnation quantities (1).

→ Practical point of view : In a tank : $h_i = c_p T_i = c_p T_0$



→ Following theoretical results, along streamlines :

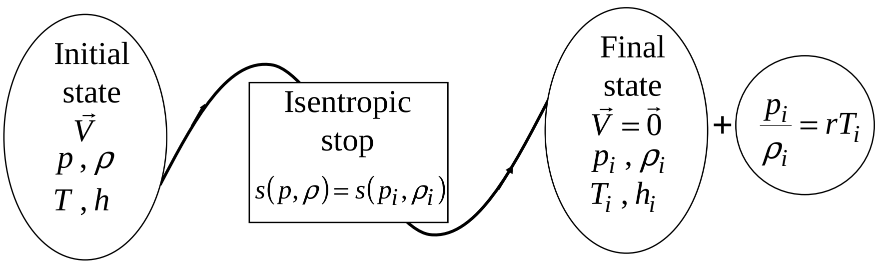
$* \; h_i = h + \dfrac{V^2}{2}$ or $T_i = T + \dfrac{V^2}{2c_p}$ are constant quantities

→ If then the properties of the gaz are uniform in the tank :

$\Rightarrow$ h_i et T_i are constant EVERYWHERE.

→ Note that : $h_i = \dfrac{a^2}{(\gamma-1)} + \dfrac{V^2}{2}$ and $\dfrac{T_i}{T} = \left(1 + \dfrac{(\gamma-1)}{2} M^2\right)$

Permanent regime: practical definitions of the stagnation quantities (2).



→ Following theoretical results, along streamlines :

$* \; h_i(p_i, \rho_i) = c_p T_i$ AND $s(p_i, \rho_i)$ are independant constant quantities.

$\Rightarrow$ T_i ; p_i et ρ_i are constant quantities along streamlines

→ If then the properties of the gaz are uniform in the tank :

$\Rightarrow$ T_i ; p_i et ρ_i are constant EVERYWHERE.
(Not valid if there are shock waves !!)

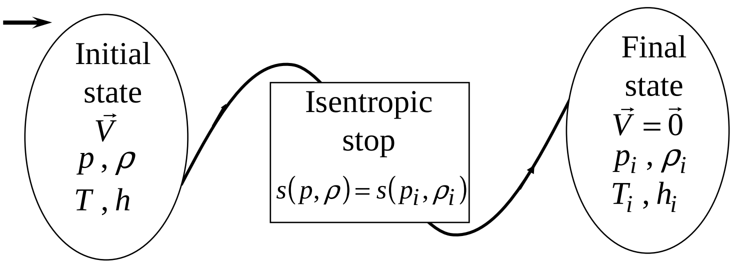
→ At a given Mach number M, the ratio between a quantity and the corresponding stagnation quantity are given by :

SAINT-VENANT RELATIONS

$$\frac{T}{T_i} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1} \qquad \frac{p}{p_i} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\gamma/(\gamma-1)}$$

$$\frac{\rho}{\rho_i} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1/(\gamma-1)} \qquad \frac{a}{a_i} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-1/2}$$

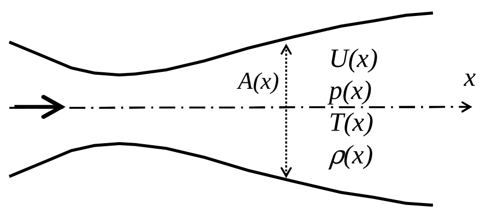
→



→ In a permanent compressible and inviscid flow
If the boundary conditions are UNIFORM, and if there are NO shock waves :

Initial Conditions + Local Mach number $\Rightarrow$ Local Thermodynamic State of the Gas

Permanent compressible and inviscid
Monodimensional flow
-
Laval Nozzle Flow



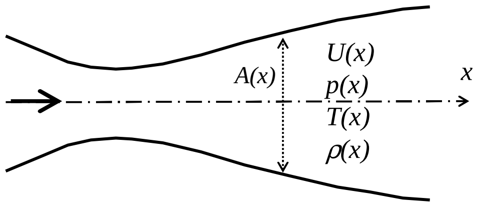
* Quasi-monodimensional flow along the x coordinate

* Slow variations of $A(x)$ with $dA/A << 1$

* Weak curvatures $A/R^2 << 1$

* Uniform boundary conditions

Permanent compressible and inviscid monodimensional flow: Laval nozzle flow (2)



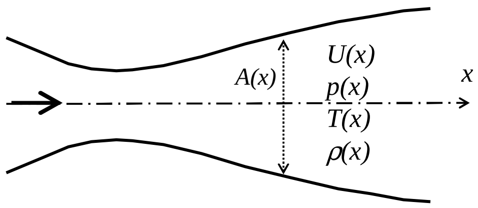
* Continuity : $\rho.U.A = cste$

* Momentum : $\rho.U.\frac{dU}{dx} = -\frac{dp}{dx}$

* Enthalpy : $h_i = cste$

* Entropy : $s = cste \Rightarrow \frac{p}{\rho^\gamma} = cste$

Permanent compressible and inviscid monodimensional flow: Laval nozzle flow (3)



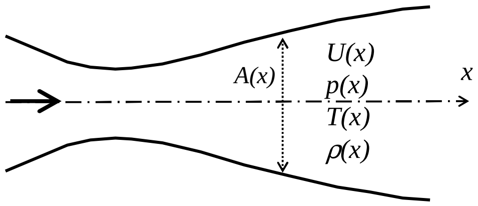
* Continuity : $\frac{d\rho}{\rho} + \frac{dU}{U} + \frac{dA}{A} = 0$

* Momentum : $\rho.U.\frac{dU}{dx} + \frac{dp}{dx} = 0$

* Enthalpy : $h_i = cste$

* Entropy : $\frac{dp}{p} = \gamma\frac{d\rho}{\rho} \Rightarrow dp = a^2 d\rho$

Permanent compressible and inviscid monodimensional flow: Laval nozzle flow (4)



Hugoniot Theorem

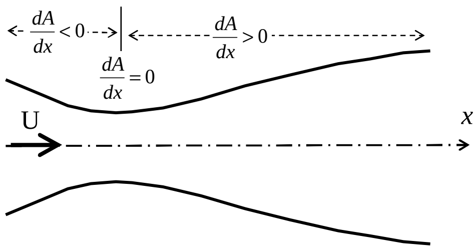
* Reducing variables :

$$\left(M^2-1\right)\frac{dU}{U}=\frac{dA}{A}$$

* In a SUBSONIC FLOW, When section increases, The velocity decreases (and vice versa)

* In a SUPERSONIC FLOW, When section increases, The velocity increases (and vice versa)

Laval nozzle flow (5)



At the throat ??

$$\left(M^2-1\right)\frac{dU}{U}=\frac{dA}{A}$$

* If M=1 , then dA=0

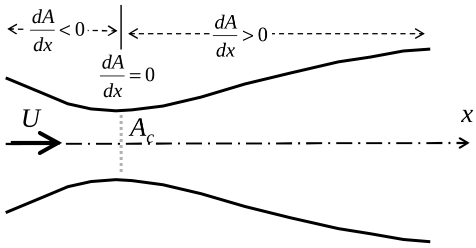
If an isentropic monodimensional flow is sonic, then we are at a minimum of the cross-section.

* Conversely, if dA=0 :

$\Rightarrow$ Either dU=0

$\Rightarrow$ Either M=1. In this case, we are at a MINIMUM of the section

Laval nozzle flow(6)



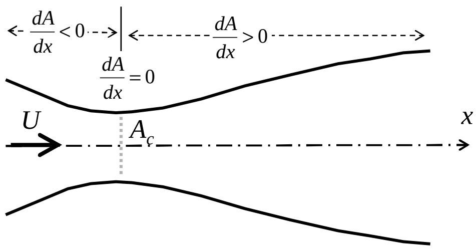
$M = M_c = 1$
at $A = A_c$

* With a sonic flow at the throat, we obtain for the mass flux :

$$\Rightarrow \qquad Q_m = \sqrt{\frac{\gamma}{r}} \left(\frac{\gamma+1}{2} \right)^{-\frac{\gamma+1}{2(\gamma-1)}} \frac{p_i}{\sqrt{T_i}} A_c = \left(4.04.10^{-2} \right) \frac{A_c \, p_i}{\sqrt{T_i}} \qquad (\gamma = 1.4)$$

* This relation has a lot of practical applications if one wants to regulate a mass flux just by controlling the initial stagnation pressure p_i

Laval nozzle flow (7)



$$\left(M^2 - 1 \right) \frac{dU}{U} = \frac{dA}{A}$$

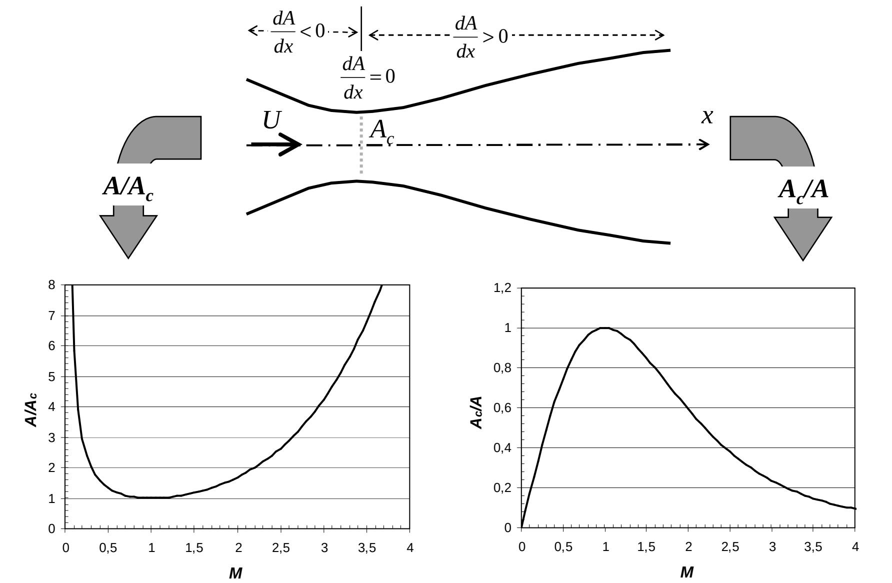
* By definition, the throat conditions are defined by :

$M = M_c = 1$ at $A = A_c$ *(This section can be virtual !!)*

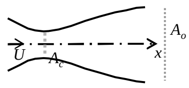
* An important theoretical link between A/A_c and the local Mach number is :

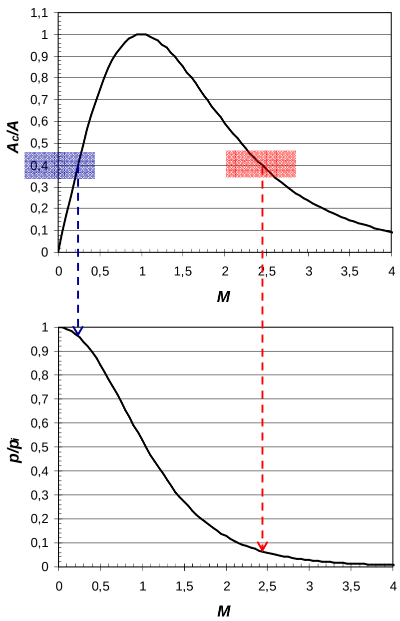
$$\Rightarrow \qquad \frac{A}{A_c} = \frac{1}{M} \left[\frac{2}{\gamma+1} \left(1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

Laval nozzle flow (7)



Laval nozzle flow (8)





* Applications :

- Atmospheric conditions at entrance
- Varying outlet pressure
- Outlet section : $A_o / A_c = 2.5$

* Subsonic flow and *M= 1* at the minimum section. M_o , p_o ???

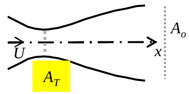
$$\Rightarrow M_o = 0.24 \text{ and } p_o / p_i = 0.96$$

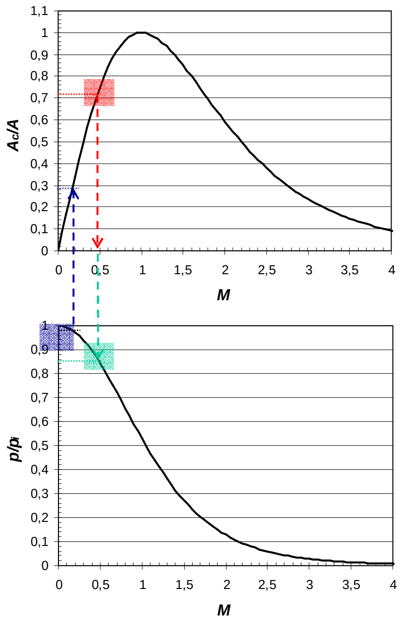
* Supersonic flow and M=1 at the minimum section. M_o , p_o ???

$$\Rightarrow M_o = 2.44 \text{ and } p_o / p_i = 0.064$$

AND FOR OTHER VALUES OF p_o / p_i ???

Laval nozzle flow (9)





* Conditions :

- Atmospheric conditions at entrance
- Varying outlet pressure
- Outlet section : $A_o / A_T = 2.5$

* $p_o / p_i = 0.98$

value of M_o, M_s ???

$\Rightarrow M_o = 0.17$

$\Rightarrow A_c / A_o = 0.29$ $* A_o / A_T = 2.5$

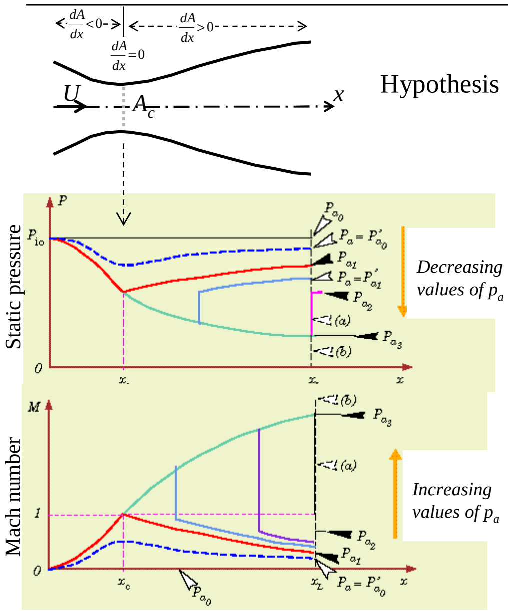
$\Rightarrow A_c / A_T = 0.72$

$\Rightarrow M_T = 0.48$

$\Rightarrow p_T / p_i = 0.854$

The section "C" is here virtual.

Laval nozzle flow (10)



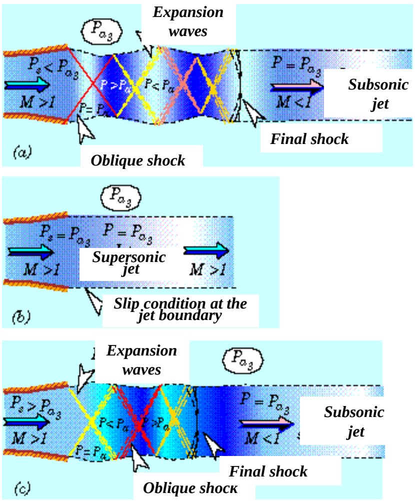
(From Chassaing 1997)

Hypothesis : Fixed upstream stagnation pressure
p_o : Decreasing static pressure at outlet

* For : $p_{a1} < p_o$
- Isentropic subsonic flow

* For : $p_s = p_{a1}$
- Isentropic flow
M=1 at the throat

* For : $p_{a2} < p_s < p_{a1}$
- NON isentropic flow
- Formation of straight shock waves in the diverging section



* For : $p_{a3} < p_s < p_{a2}$

- Compression at the oulet by oblique shock waves

* For : $p_s = p_{a3}$

- Isentropic Supersonic flow

* For : $p_s < p_{a3}$

- Expansion wave at the outlet reflecting on the boundary of the jet *(p is a constant on this surface)*

(From Chassaing 1997)