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Automatic control fundamentals
Flight control laws design

Content

- •Classical approach
  - Definition / Laplace transform
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  - Bode plot
  - Root locus
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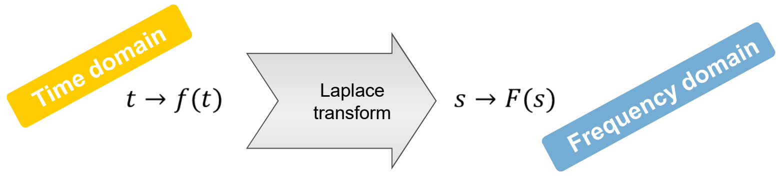
  - Limit of the classical approach
- •Modal approach
  - Brief definition
  - Application to the design of a Nz/C* law

Frequency domain approach: Definition / Laplace transform

• **Laplace Transform** = integral transform of a function:



• **Formal definition:**

$$F(s) = \int_0^{\infty} e^{-st} f(t)dt$$

• **Notation:**

F function is also noted $\mathcal{L}\{f\}$ or $\mathcal{L}\{f(t)\}$

Frequency domain approach: Definition / Laplace transform

• **Some properties** of the Laplace Transform:

- Linearity:

$$\mathcal{L}(a.f(t) + b.g(t)) = a.\mathcal{L}(f(t)) + b.\mathcal{L}(g(t))$$

- Derivation and integration: $\mathcal{L}(f'(t)) = \mathrm{s.F(s)} - f(0)$

$$\mathcal{L}\left(\int_0^t f'(\tau)d\tau\right) = \frac{1}{s}\mathrm{F(s)}$$

• **Usual functions Laplace transform:**

Time Domain Function		Laplace Domain Function
Name	Definition*	
Unit Impulse	$\delta(t)$	1
Unit Step	$\gamma(t)$ †	$\frac{1}{s}$
Unit Ramp	t	$\frac{1}{s^2}$
Parabola	t^2	$\frac{2}{s^3}$
Exponential	e^{-at}	$\frac{1}{s+a}$
Asymptotic Exponential	$\frac{1}{a}(1-e^{-at})$	$\frac{1}{s(s+a)}$
Dual Exponential	$\frac{1}{b-a}\left(e^{-at}-e^{-bt}\right)$	$\frac{1}{(s+a)(s+b)}$
Asymptotic Dual Exponential	$\frac{1}{ab}\left[1+\frac{1}{a-b}\left(be^{-at}-ae^{-bt}\right)\right]$	$\frac{1}{s(s+a)(s+b)}$
Time multiplied Exponential	te^{-at}	$\frac{1}{(s+a)^2}$

Sine	$\sin(\omega_0 t)$	$\frac{\omega_0}{s^2+\omega_0^2}$
Cosine	$\cos(\omega_0 t)$	$\frac{s}{s^2+\omega_0^2}$
Decaying Sine	$e^{-at}\sin(\omega_d t)$	$\frac{\omega_d}{(s+a)^2+\omega_d^2}$
Decaying Cosine	$e^{-at}\cos(\omega_d t)$	$\frac{s+a}{(s+a)^2+\omega_d^2}$
Generic Oscillatory Decay	$e^{-at}\left[B\cos(\omega_d t)+\frac{C-aB}{\omega_d}\sin(\omega_d t)\right]$	$\frac{Bs+C}{(s+a)^2+\omega_d^2}$
Prototype Second Order Lowpass, underdamped	$\frac{\omega_0}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin\left(\omega_0\sqrt{1-\zeta^2}t\right)$	$\frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$
Prototype Second Order Lowpass, underdamped - Step Response	$1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin\left(\omega_0\sqrt{1-\zeta^2}t+\phi\right)$ $\phi=\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$	$\frac{\omega_0^2}{s(s^2+2\zeta\omega_0 s+\omega_0^2)}$

Frequency domain approach: Definition / Laplace transform

- **Initial and final value theorems:**

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s).$$

Frequency domain approach: Definition / Laplace transform

- Application to IVP (initial value problem solving):
  - From IPV to transfer function:

$$\begin{cases} au'' + bu' + cu = g(t), \\ u(0) = \alpha, \\ u'(0) = \beta \qquad (\text{if } a \neq 0) \end{cases}$$

$$au'' + bu' + cu = g(t)$$

$$\downarrow \mathcal{L}$$

$$a\,\boxed{\mathcal{L}\{u''\}} + b\,\boxed{\mathcal{L}\{u'\}} + c\,\boxed{\mathcal{L}\{u\}} = \boxed{\mathcal{L}\{g(t)\}}$$

$$\mathcal{L}\{u''\} = s^2\mathcal{L}\{u\} - su(0) - u'(0) = s^2U(s) - \alpha s - \beta$$

$$\mathcal{L}\{u'\} = s\mathcal{L}\{u\} - u(0) = sU(s) - \alpha$$

$$\mathcal{L}\{u\} = U(s)$$

$$\mathcal{L}\{g(t)\} = G(s)$$

source: https://math.la.asu.edu/~bdw/MAT274/Su05/Laplace_transform_review.pdf

Frequency domain approach: Definition / Laplace transform

Note how the ICs are used right away in the determination of $U(s)$. Collecting terms yields

$$(as^2 + bs + c)U(s) - a\alpha s - (a\beta + b\alpha) = G(s),$$

i.e.,

$$U(s) = \underbrace{\frac{G(s)}{as^2+bs+c}}_{\substack{\downarrow \\ U(s) \text{ with trivial ICs} \\ \alpha=0,\ \beta=0 \\ \text{(start at rest)}}} + \underbrace{\frac{a\alpha s + a\beta + b\alpha}{as^2+bs+c}}_{\substack{\downarrow \\ U(s) \text{ with no forcing} \\ g(t)=0,\ G(s)=0 \\ \text{(HODE)}}}. \tag{16.12}$$

The quantity $\dfrac{1}{as^2+bs+c}$ is called a *transfer function*.

source: https://math.la.asu.edu/~bdw/MAT274/Su05/Laplace_transform_review.pdf

Frequency domain approach: Definition / Laplace transform

Find $U(s)$ if $u(t)$ is the solution of the IVP

$$\begin{cases} u' = 2u, \\ u(0) = 1. \end{cases}$$

$$u' \quad - \quad 2u \quad = \quad 0$$

$$\downarrow \mathcal{L}$$

$$\boxed{\mathcal{L}\{u'\}} - 2\boxed{\mathcal{L}\{u\}} = \boxed{\mathcal{L}\{0\}}$$

$$\boxed{\begin{aligned} &= s\mathcal{L}\{u\} - u(0) \\ &= sU(s) - 1 \end{aligned}} \qquad \boxed{= U(s)} \qquad \boxed{= 0}$$

Thus

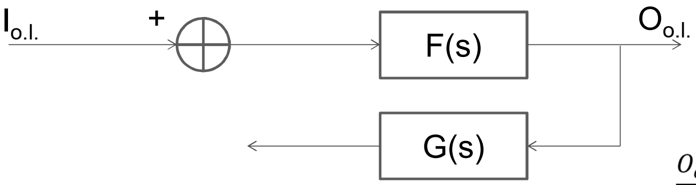
$$sU(s) - 1 - 2U(s) = 0 \quad \Rightarrow \quad U(s) = \frac{1}{s-2}.$$

This expression is of course the Laplace transform of $u(t) = e^{2t}$

source: https://math.la.asu.edu/~bdw/MAT274/Su05/Laplace_transform_review.pdf

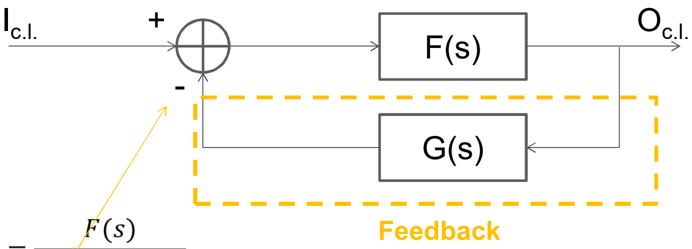
Open Loop/Closed Loop transfer function

•Open loop:



$$\frac{O_{o.l.}}{I_{o.l.}} = F(s)$$

•Closed loop :



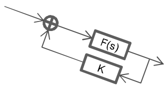
$$\frac{O_{c.l.}}{I_{c.l.}} = \frac{F(s)}{1 + F(s)G(s)}$$

Root locus

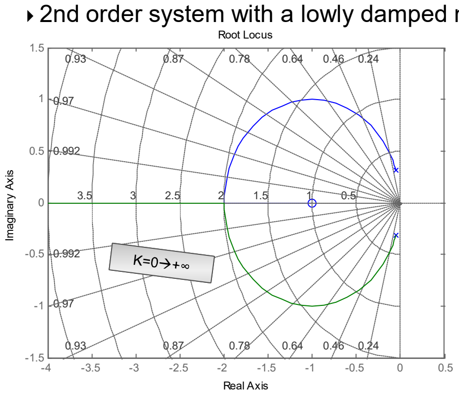
•Definition:

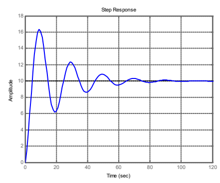
‣ The root locus is the plot in the complex plane of the location of a closed loop transfer function poles depending on one parameter (usually, the feedback gain)

•Example:



‣ 2nd order system with a lowly damped mode :





$$F(s) = \frac{1 + s}{s^2 + 0.1s + 0.1}$$

Open loop transfer function

Bode plot

•Definition:

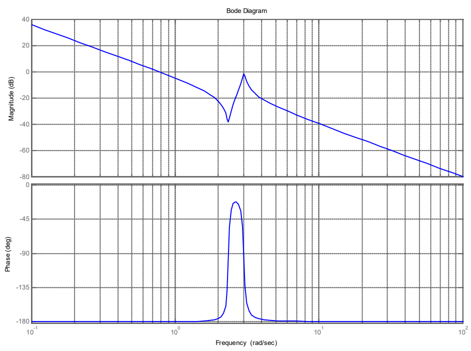
‣ The bode plot of F(jω) (complex function of the frequency) is a 2-part semi-log graph composed of:

– The magnitude (usually in dB) : $G_{dB} = 20\log|F|$

– The phase (in degree) : $Phase = arg\big(F(j\omega)\big)$

•Example:

$$F(j\omega) = \frac{s^2 + 0.1s + 5.5}{s^4 + 0.16s^3 + 9s^2}$$



Bode Stability Criterion / Stability margins

•2 options to assess the stability of a transfer in closed loop:

‣ Compute the closed loop transfer function, determine its pole

‣ Apply the Bode criteria to the open loop system (study in open loop → conclusion about the closed loop)

‣ Bode diagram allows to visualize stability margin (phase margin and gain margin)

•Bode stability criterion:

‣ Definitions:

– Phase crossover frequency = frequency where phase shift is equal to -180º.

– Gain crossover frequency = frequency where the amplitude ratio is 1, or when log modulus is equal to 0.

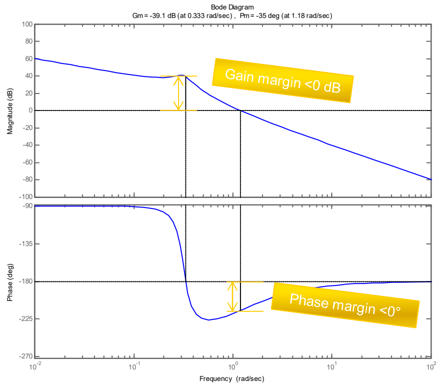
‣ Criterion statement:

– If at the crossover frequency the gain log modulus is lower then 0dB, the closed loop system is stable

Bode Stability Criterion / Stability margins

•Gain and phase margin in bode diagram:

- Definitions:
  - Gain margin = difference between 0 and log gain in dB when the phase reaches -180°
  - Phase margin = difference between -180° and the phase where the log gain in dB reaches 0



Application to the design of a yaw damper

•Objective of the yaw damper:

- Damp → Acquire or improve dynamic stability of one eigenmotion
- Need to improve « Dutch Roll » damping, in particular at low speed and high altitude

•We proceed in 4 steps:

1. Definition of the model
2. Open loop analysis
3. Choice of a corrector structure (which output measure to choose, which input to feed)
4. Tuning of the gain(s) of this corrector

Application to the design of a yaw damper

1 - Definition of the model

Aircraft linearized lateral model:

A and B numerical values not shown here, representative of a big airliner linearized at high altitude and high MACH

Sideslip

Yaw rate

$$X = \begin{pmatrix} \beta \\ r \\ p \\ \phi \end{pmatrix} \qquad \dot{X} = A.X + B.u \qquad u = \begin{pmatrix} dr \\ dp \end{pmatrix}$$

Roll rate

Roll angle

Rudder deflection

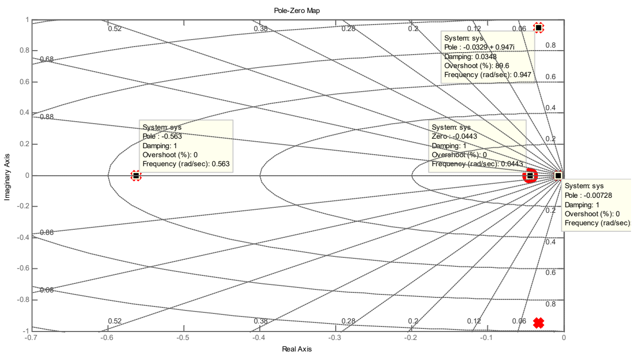
Aileron deflection

Application to the design of a yaw damper

1 – Open Loop Analysis

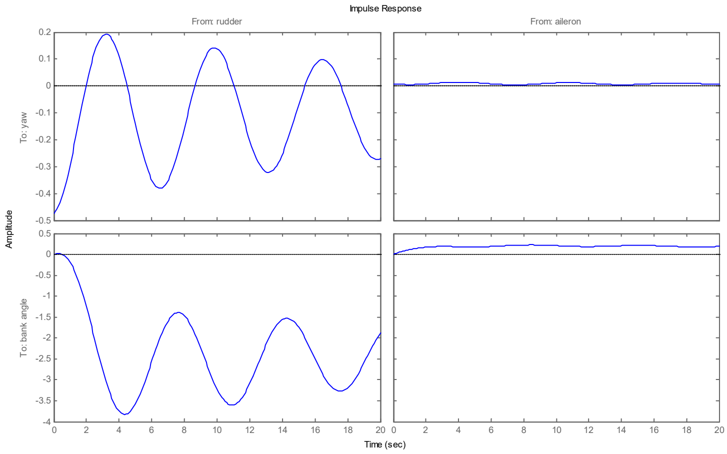
Get to know with the model – Identification of the dutch roll:

Pole-zero open loop map:



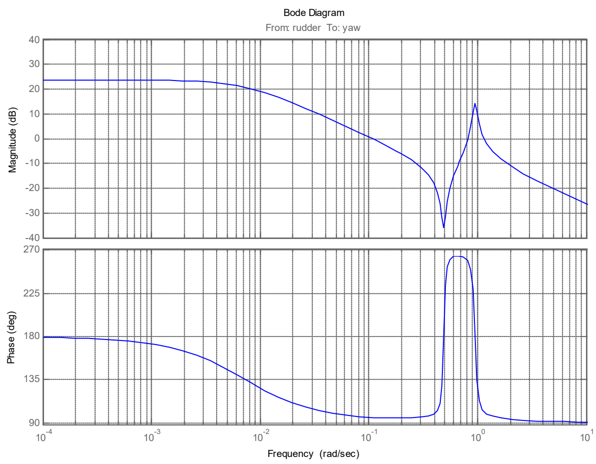
Application to the design of a yaw damper

•Impulse response:



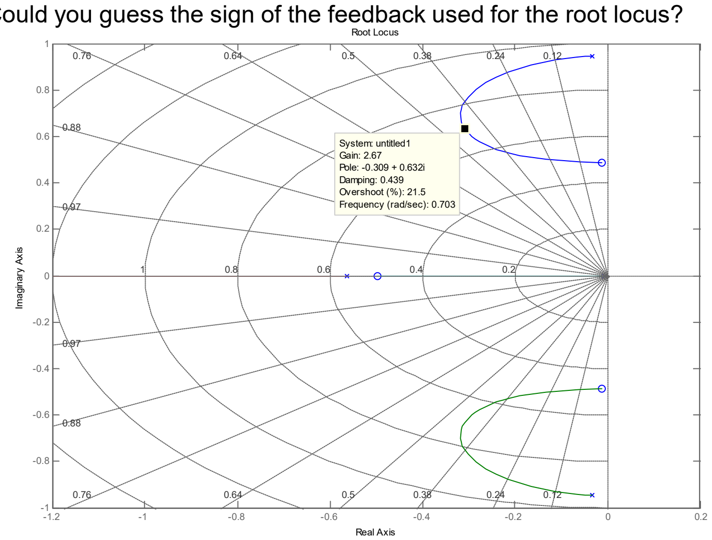
Application to the design of a yaw damper

•Bode diagram:



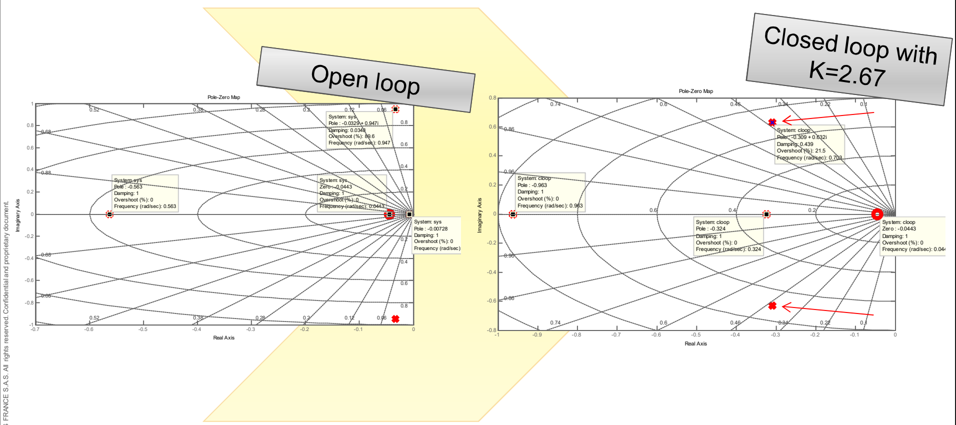
Application to the design of a yaw damper

- **Root locus:**
  - Could you guess the sign of the feedback used for the root locus?



Application to the design of a yaw damper

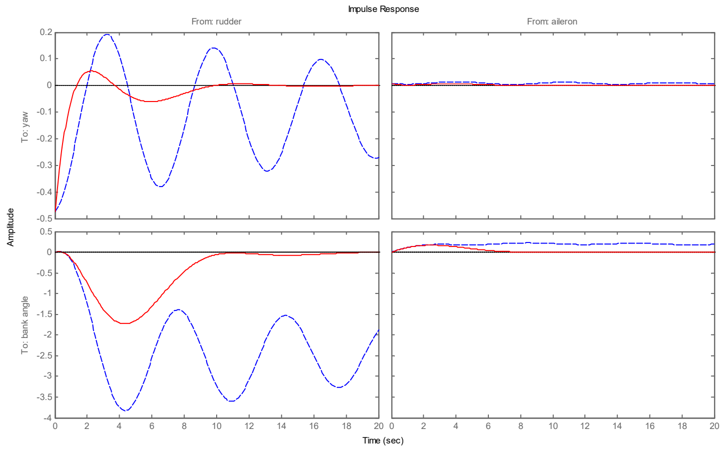
Eigenvalue	Damping	Freq. (rad/s)
-3.24e-001	1.00e+000	3.24e-001
-3.09e-001 + 6.32e-001i	4.39e-001	7.03e-001
-3.09e-001 - 6.32e-001i	4.39e-001	7.03e-001
-9.63e-001	1.00e+000	9.63e-001



Application to the design of a yaw damper

•Impulse response:

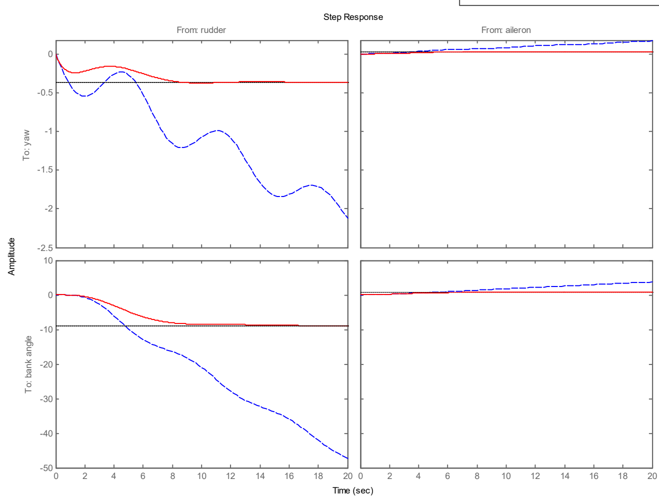
Open loop
Closed loop (K=2.67)



Application to the design of a yaw damper

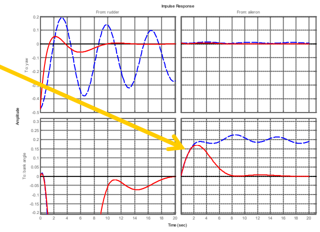
•Step response:

Open loop
Closed loop (K=2.67)



Application to the design of a yaw damper

- **Conclusion / limits of the standard approach**
  - Looking closer at the impulse and step responses:
    - The spiral mode has been overstabilized : The Aircraft roll shall not return to zero on an impulse sent to the aileron input : the natural aircraft will not do that, and pilots
    - From a pilot point of view, our yaw damper prevents the A/C from turning and keeping, a steady yaw rate



    - Consistent with the root locus : the frequency of the spiral mode was doubled

➔ Need to complement the law, introducing a WASH (high pass filter) for instance in the yaw rate feedback line.

Application to the design of a yaw damper

- **Conclusion / limits of the standard approach**
  - With the standard approach:
    - We could modify dutch roll mode damping quite easily
    - But we also affected the spiral mode whereas we did not want too
    - Nevertheless, adjustments are possible using an high pass filter
  - The <u>modal approach</u> applied to the same example allows to compute a corrector based on the MIMO lateral model imposing constraints on <u>the 2 modes</u> at the same time.

Modal Approach: Brief definition

•**What is modal synthesis:**

States vector Input vector

Let us consider the following linear system : $\begin{cases} \dot{X} = A.X + B.U = & (1) \\ Y = C.X + D.U & (2) \end{cases}$

Output vector

We fit this system with a simple state feedback, using a matrix a proportional gains note K, we define a precommand matrix H and the e the setpoint (reference) :

$$U = K.X + H.E$$

Then equation (1) becomes : $\dot{X} = (A + B.K)X + B.H.E$

Modal synthesis consists in setting the eigen values and eigen vectors of this new matrix A+B.K chosing K.

Doing so, we set the closed loop modes of the linear system.

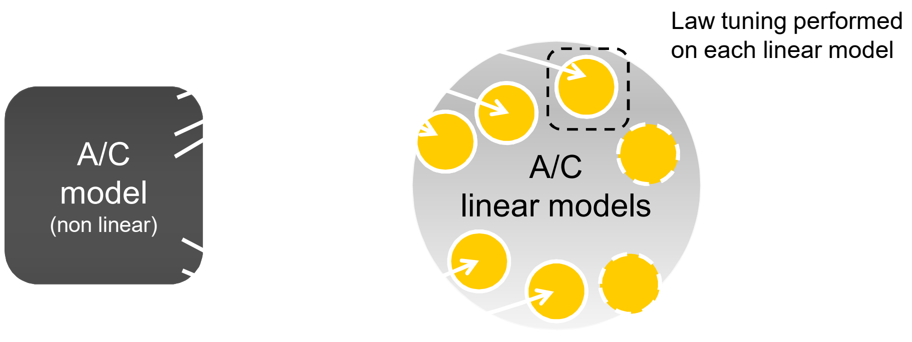
Application to the design of a Nz/C* law

• **How to build a NZ/C* LAW** based on an A/C model :

- A/C model definition
  Need to work on a simplified model to perform a modal synthesis
- Open loop modes analysis
  Check natural A/C modes, check the effect of the simplification on the modes to control
- Closed loop behaviour objectives
  Impose constraints on closed loop modes
- Controller structure definition
  Define the feedbacks, the precommand, the presence of integrator
- Controller gains computation (modal approach)
  Based on closed loop modes objectives
- Validation
  Validate modes placement on the full A/C model

Application to the design of a Nz/C* law

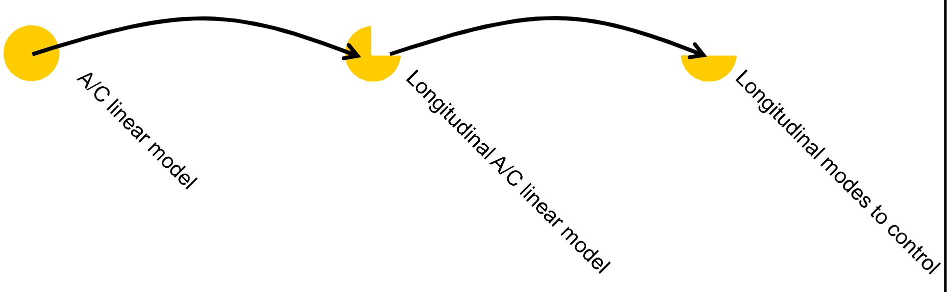
- A/C model definition : **1 - linearization**



- Linearization provides as many models as flight points characterized by altitude, MACH, weight, CG high-lift conf…

Application to the design of a Nz/C* law

- A/C model definition : **2 – model simplification**



3 longitudinal modes : Short period, Phugoïd, Engine
➔ **For the example, <u>focus short period</u> we need to stabilize**
➔ **Keep only Angle of Attack (alpha) and Pitch Rate (q)**

Application to the design of a Nz/C* law

- A/C model definition : **3 – model complement**

- Data acquisition from probes / computer is not immediate
- Law orders are not directly transformed into surface deflection.

➔**Without this, law synthesis would not be valid**

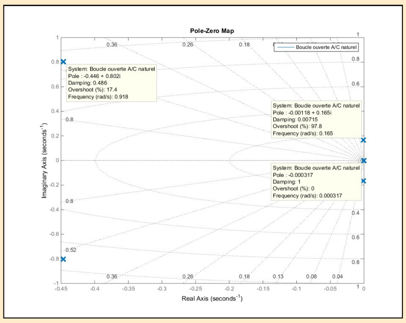
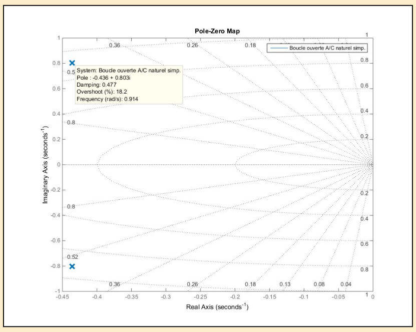
Longitudinal modes to control
+ actuator, probes and filter

Add delays and filter to model

…Then it is time for controller synthesis

Application to the design of a Nz/C* law

- Open loop modes analysis : **Poles / Zeros map**

FULL LONGI A/C MODEL	SIMPLIFIED MODEL
	

Keep only Angle of Attack (alpha) and Pitch Rate (q)
Only short period is present on simplified model

Application to the design of a Nz/C* law

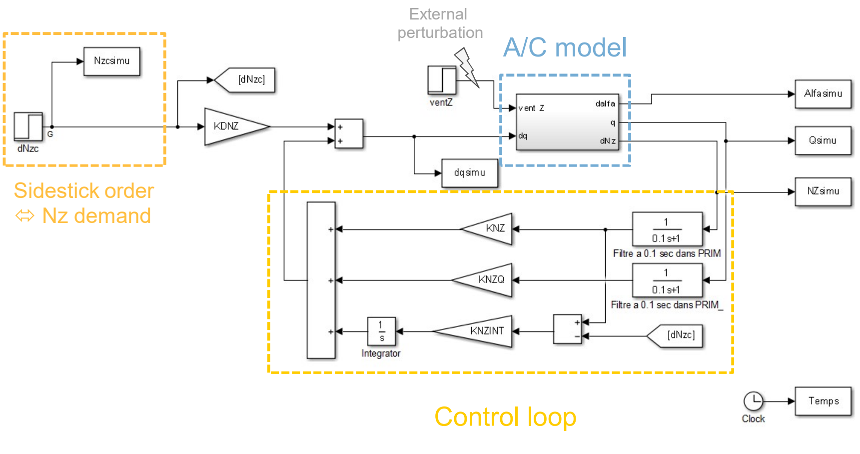
- **Closed loop behaviour objectives :**
  - By hypothesis, only one mode to control (short period)
    - ➔ increase stability
    - ➔ accelerate the response
    - ➔ ensure the precision of Nz response (no static error)
    - ➔ the robustness against external pertubation

Need for an integrator in the control loop
(new mode objective = as fast as short period or a bit slower)

Short period mode objective :
- Damping = 0.8
- Pulsation = 1 rad/sec

Application to the design of a Nz/C* law

- Controller struct. def. : **NZ and q feedback + NZ integrator**



Application to the design of a Nz/C* law

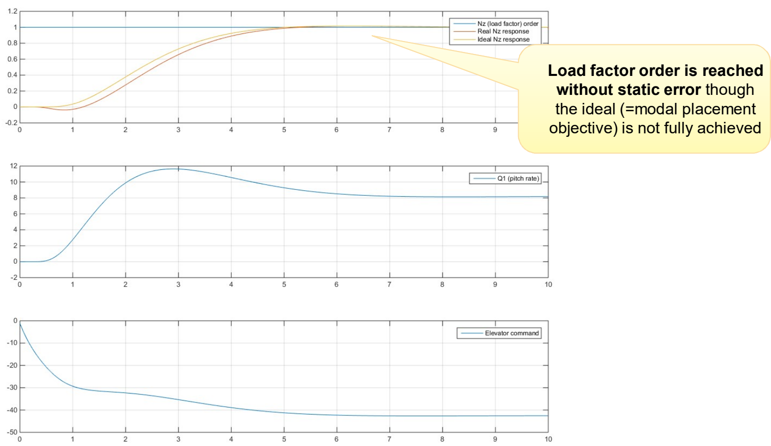
- Gains computation : **modal synthesis**

- **Modal synthesis provides a generic method** (not detailed here) to compute KNZ, KNZQ, KNZINT (the 3 feedback gains) having determined closed loop poles objectives
- **Other methods** (LQ, LQG, Hinf synthesis) could be used with the same feedback structure to compute the gains with other type of objectives than mode placement
- KDNZ (pre-command) is tuned independently, a dynamic pre-command could also be used (and usually is) to shape the response to manual order

  ➔With the integrator, no need to tune KDNZ to ensure precision

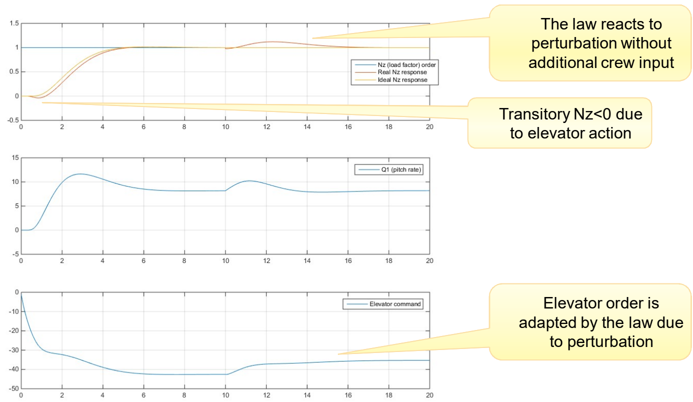
Application to the design of a Nz/C* law

- Law validation : **Closed loop step response**



Application to the design of a Nz/C* law

- Gains computation : **Robustness to perturbation**



With the integrator, **precision is ensured** and **robust to perturbation**, but the response time and the stability are degraded

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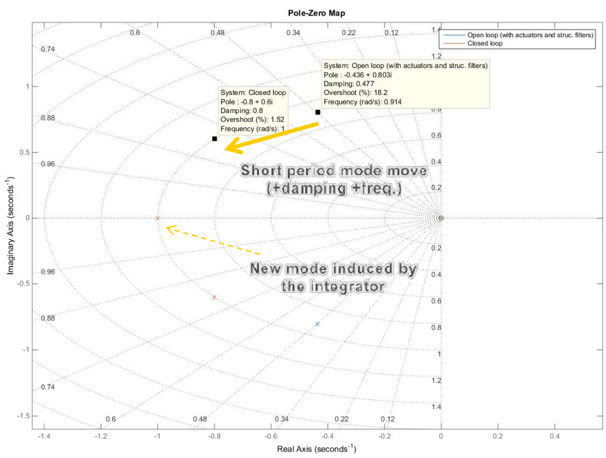
Application to the design of a Nz/C* law

- Law validation / closed loop : **Poles / Zeros map**



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Application to the design of a Nz/C* law

• **How to cover the whole domain ?**
➔Perform the same analysis on other flight point with a new linearized model (will give as many gain sets as flight points)

• **How to validate the law ?**
➔Need to extend validation beyond the simplified linear model:
- ➔ Check the mode placement is still consistent on the full longi linear A/C model (with all longi modes, not just the short period)
- ➔ Check linearity assumption validity (check the effect of non linearities with the full A/C model)
- ➔ Check the consistency of the domain slicing (smooth transition between 2 gain sets)
- ➔ Complement the law where non-linear phenomenon appears (pitch-up effect…)

More on the web

- **Automatic Flight Control Summary:**
http://aerostudents.com/files/automaticFlightControl/automaticFlightControlFullVersion.pdf

- **MathWorks : Design Yaw Damper for Jet Transport:**
https://mathworks.com/help/control/ug/design-a-yaw-damper-for-a-747-jet-transport.html

- **Lecture Series on Industrial Automation and Control by Prof. S. Mukhopadhyay, Dept.of Electrical Engineering, IIT Kharagpur**
https://www.youtube.com/playlist?list=PLE8F9BF5CB1201D23