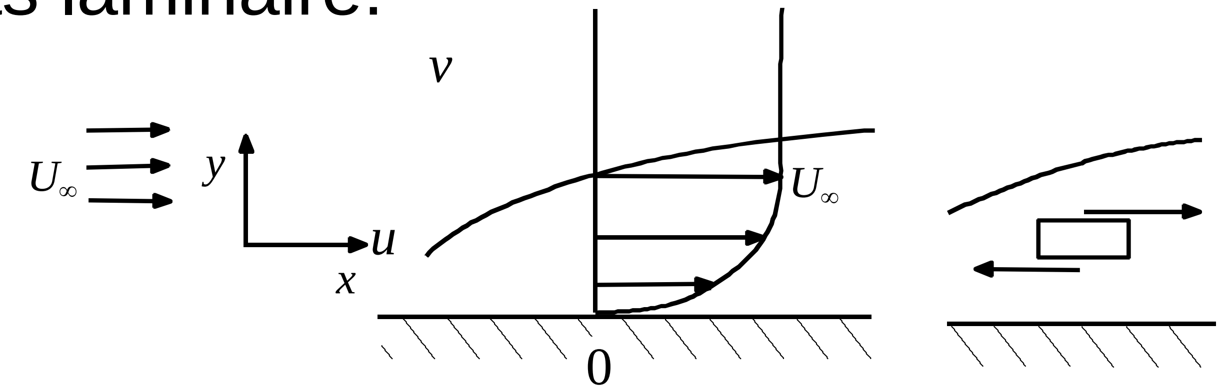
LEÇON 2: ANALOGIE DYNAMIQUE/CONVECTION

Relation vitesse/température du fluide convectant

CONTRAINTE DE CISAILLEMENT

- Cas laminaire:



  - $$\tau = \mu \frac{\partial u}{\partial y}$$
  - en paroi: $$\tau_0 = \mu \left.\frac{\partial u}{\partial y}\right)_{y=0}$$

CONTRAINTE DE CISAILLEMENT

- Cas turbulent:
  - Un autre mode de transport s'ajoute au transport moléculaire:
  - $\mu + \mu_t = \rho(\nu + \varepsilon_t^m)$

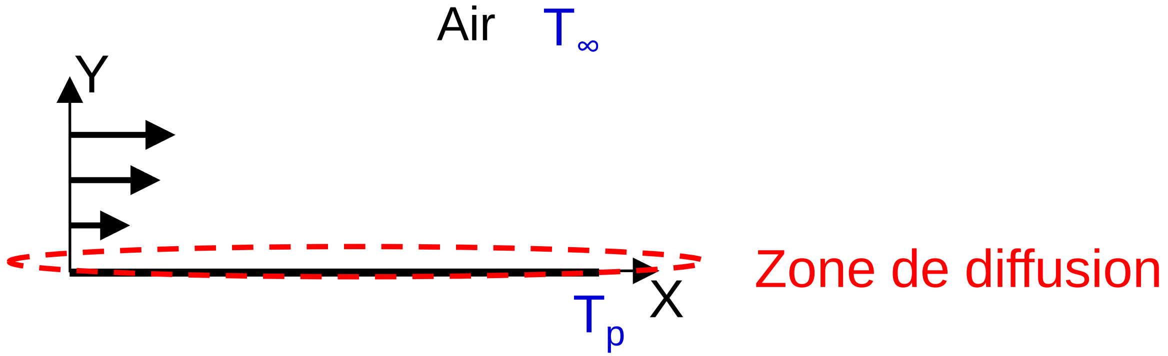
viscosité cinématique turbulente

$$\Rightarrow \quad \tau = \rho(\nu + \varepsilon_t^m)\frac{\partial u}{\partial y}$$

  - Mais en proche paroi: sous couche visqueuse:
  - $\tau \approx \rho\nu\dfrac{\partial u}{\partial y}$

FLUX PARIETAL ECHANGE

- Cas laminaire:



- En très proche paroi: zone de diffusion

$\Rightarrow$ Conduction

$$\Rightarrow \varphi_0 = -\lambda \left.\frac{\partial T}{\partial y}\right)_{y=0}$$

- $$\tau_o = \mu \left.\frac{\partial u}{\partial y}\right)_{y=0}$$

FLUX PARIETAL ECHANGE

- Cas laminaire:
  - $a = \dfrac{\lambda}{\rho\, c} = diffusivit\acute{e}\ thermique$
  - $\varphi = -\rho\, c\, a \dfrac{\partial T}{\partial y}$
- Cas turbulent:
  - $\varphi = -\rho\, c \left(a + \varepsilon_t^t\right) \dfrac{\partial T}{\partial y}$
  - $\Pr = \dfrac{\upsilon}{a} = \dfrac{\mu / \rho}{\lambda / \rho\, c} = nombre\ de\ \Pr andtl$
  - $\Pr_t = \dfrac{\varepsilon_t^m}{\varepsilon_t^t} = nombre\ de\ \Pr andtl\ turbulent$

FLUX PARIETAL ECHANGE

- Cas turbulent:
  - en paroi:
    - $\varphi_0 \approx -\lambda \left.\frac{\partial T}{\partial y}\right)_{y=0}$
    - $\tau_o \approx \mu \left.\frac{\partial u}{\partial y}\right)_{y=0}$

GRANDEUR ADIMENSIONNEES

- Coefficient de frottement:
  - $C_f = \dfrac{\tau_o}{\dfrac{1}{2}\rho U_\infty^2}$

- Le nombre de Nusselt: Flux adimensionné:
  - $Nu = \dfrac{h\,L}{\lambda}$
  - Or $\mathrm{h} = \dfrac{\varphi_0}{(\mathrm{T_p} - \mathrm{T_f})} = \dfrac{-\lambda \left.\dfrac{\partial T}{\partial y}\right)_{y=0}}{(\mathrm{T_p} - \mathrm{T_f})}$
  - $\Rightarrow Nu = \dfrac{-\lambda \left.\dfrac{\partial T}{\partial y}\right)_{y=0} L}{\lambda(\mathrm{T_p} - \mathrm{T_f})} = \dfrac{-\left.\dfrac{\partial T}{\partial y}\right)_{y=0} L}{(\mathrm{T_p} - \mathrm{T_f})}$

GRANDEUR ADIMENSIONNEES

- Le nombre de Nusselt: Flux adimensionné:
  - $$Nu = \frac{-\left.\frac{\partial T}{\partial y}\right)_{y=0} L}{(\mathrm{T_p} - \mathrm{T_f})}$$
  - On pose $\theta = \frac{T}{(\mathrm{T_p} - \mathrm{T_f})}$ et $y^+ = \frac{y}{L}$
  - $$Nu = -\left.\frac{\partial \theta}{\partial y^+}\right)_{y=0}$$

- Remarque: $Nu = \frac{hL}{\lambda} = \frac{h(\mathrm{T_p} - \mathrm{T_f})}{\frac{\lambda(\mathrm{T_p} - \mathrm{T_f})}{L}}$

  Convection

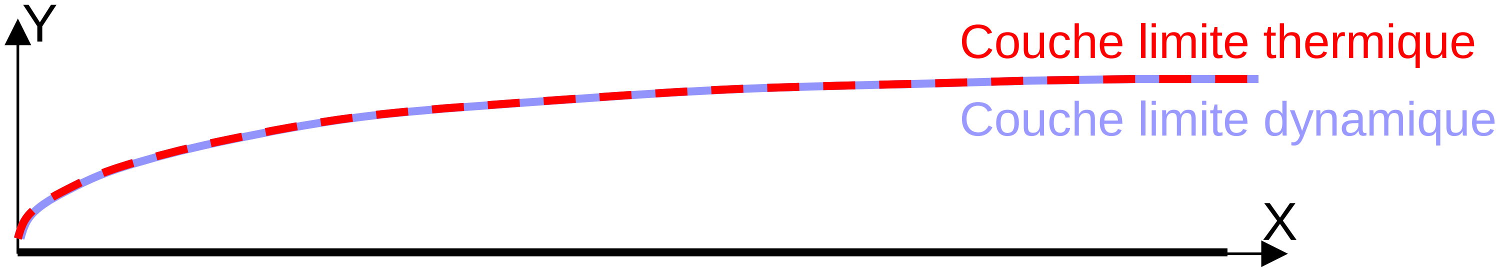
  Conduction

ANALOGIE DYNAMIQUE/THERMIQUE

Mécanique	Thermique
Contrainte de cisaillement pariétale, τ_o	Densité de flux pariétal, φ_o
Coefficient de frottement, C_f	Nombre de Nusselt

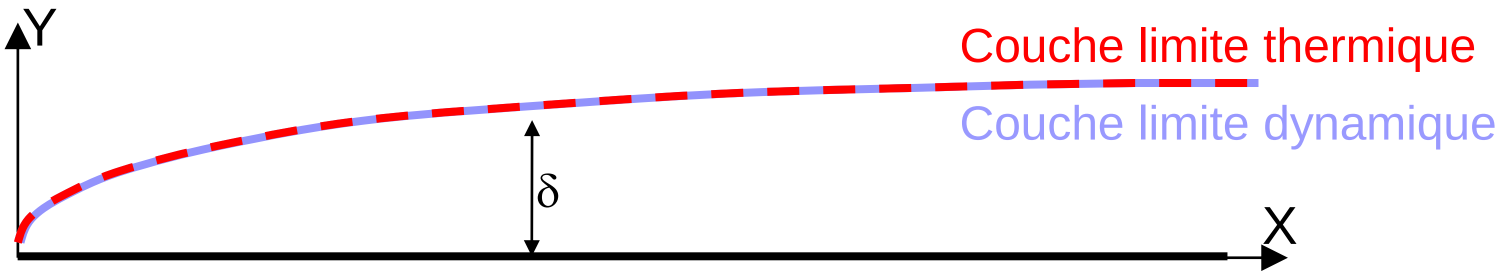
ANALOGIE DE REYNOLDS

- Hypothèses:
  - $\Pr = 1 \Rightarrow \frac{\upsilon}{a} = 1 \Rightarrow \upsilon = a$
    - Air : Pr≈0.7
  - $Pr_t=1$
  - Profil transversal de φ et τ semblables:
    - φ/τ indépendant de Y



ANALOGIE DE REYNOLDS

- Correspondances:
  - $$\partial u = \frac{\tau \partial y}{\rho(\nu + \varepsilon_t^m)} \Rightarrow U_\infty - 0 = \int_0^\delta \frac{\tau \partial y}{\rho(\nu + \varepsilon_t^m)}$$
  - $$\partial T = -\frac{\varphi\,\partial y}{\rho\, c\left(a + \varepsilon_t^t\right)} \Rightarrow T_f - T_p = \int_0^\delta \frac{-\varphi\,\partial y}{\rho\, c\left(a + \varepsilon_t^t\right)}$$



ANALOGIE DE REYNOLDS

-
  - $\upsilon = a$
  - $\varepsilon_t^m = \varepsilon_t^t = \varepsilon$
  - $\dfrac{\tau}{\varphi} = \dfrac{\tau_o}{\varphi_p}$

$$\Rightarrow \quad \frac{U_\infty}{\left|T_f - T_p\right|} = \frac{\dfrac{\tau_0}{\varphi_0}\displaystyle\int \frac{\varphi}{\rho(\nu+\varepsilon)}dy}{\displaystyle\int \frac{\varphi}{\rho c(a+\varepsilon)}dy} = \frac{\tau_0 c}{\varphi_0}$$

$$\Rightarrow \quad \frac{\Delta T}{U_\infty} = \frac{\varphi_0}{c\tau_0}$$

ANALOGIE DE REYNOLDS

- Autre expression :
  - Nombre de Stanton:
    - $St = \frac{Nu}{\mathrm{Re}\,\mathrm{Pr}} = \frac{h\,L}{\lambda} \times \frac{\upsilon}{u_\infty L} \times \frac{\lambda}{\upsilon\,\rho\,c}$
    - analogie: $\frac{h(T_f - T_p)}{\rho c U_\infty (T_f - T_p)} = \frac{\varphi_0}{\rho c U_\infty (T_f - T_p)}$

$$\Rightarrow \varphi_0 = \frac{c \Delta T \tau_0}{U_\infty}$$

$$\Rightarrow St = \frac{c \Delta T \tau_0}{U_\infty} \times \frac{1}{\rho c U_\infty \Delta T} = \frac{\tau_0}{\rho U_\infty^2} = \frac{C_f}{2}$$

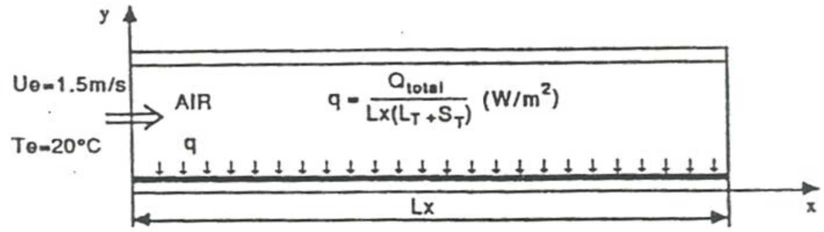
ANALOGIE DE REYNOLDS

- Validité
  - $Pr = 1$
  - $Pr_t = 1$
  - profils φ et τ similaires

- Extension:
  - Analogie de COLBURN:
  - $St\Pr^{2/3} = C_f / 2$
  - Valable pour $Pr \neq 1$

EXEMPLES: CARTES DE COMPOSANTS ÉLECTRONIQUES

- Plusieurs modélisations:
- De la moins précise à la plus précise



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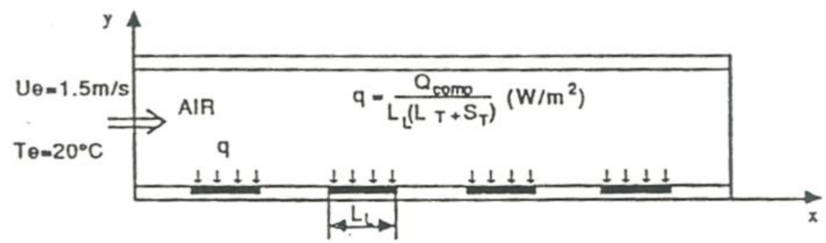

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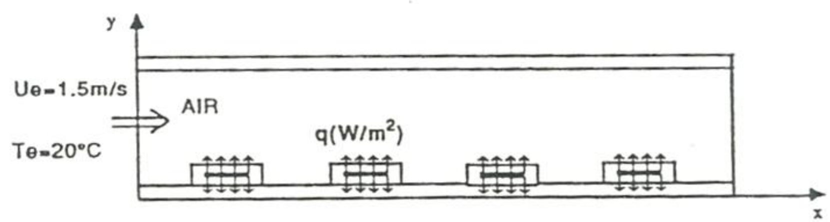

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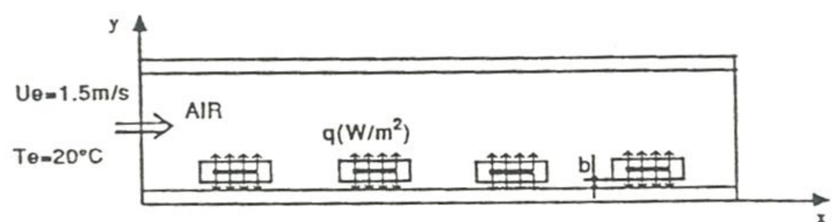

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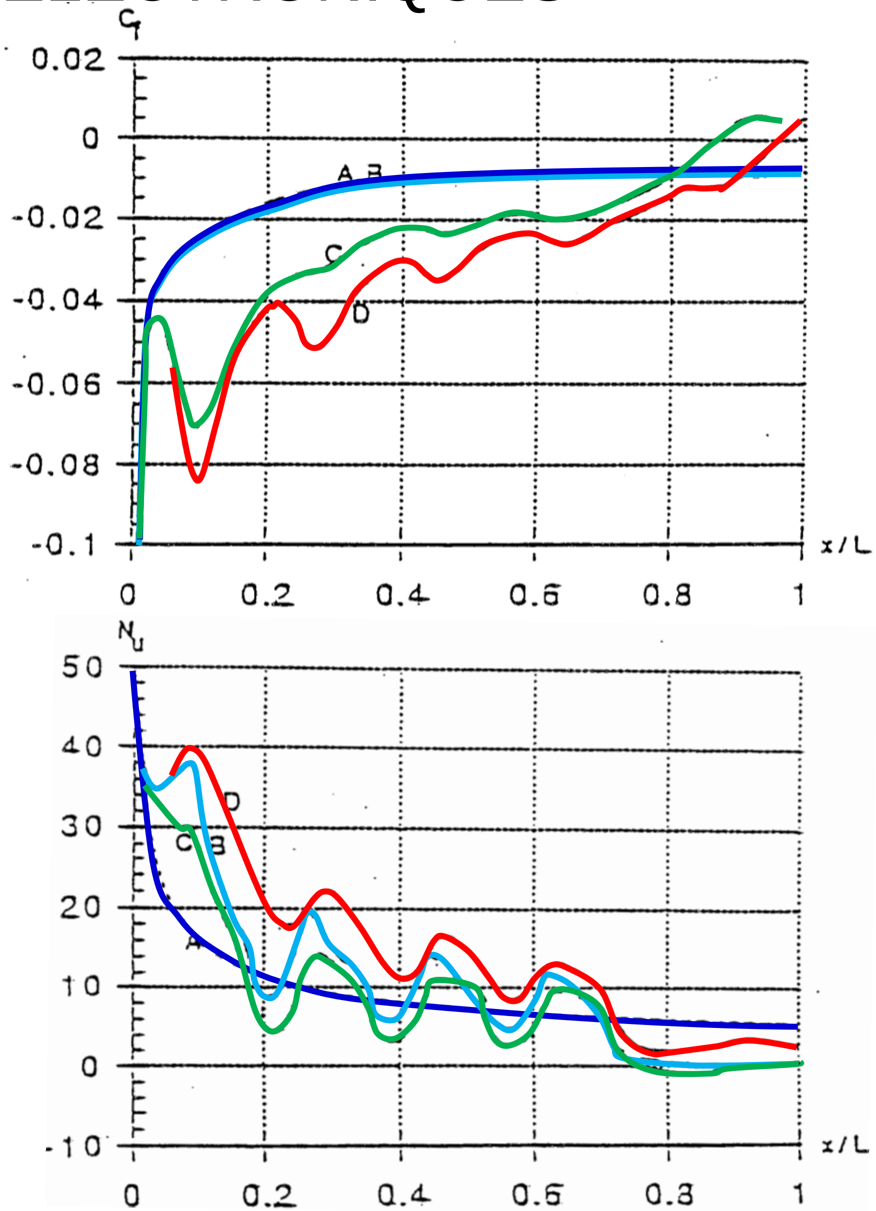
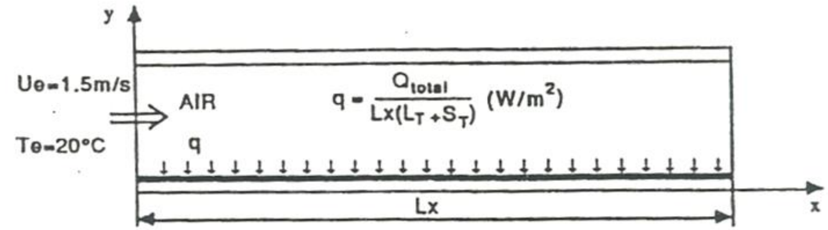
EXEMPLES: CARTES DE COMPOSANTS ÉLECTRONIQUES





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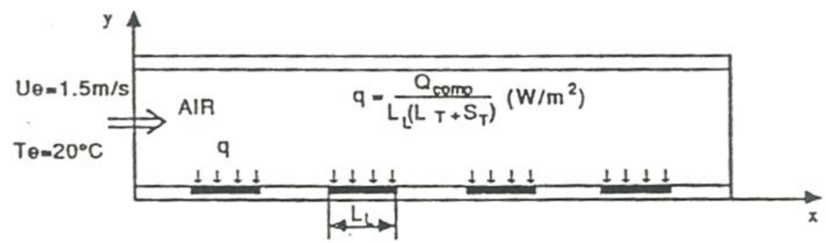

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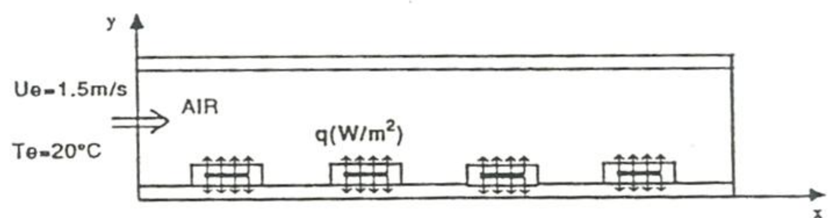

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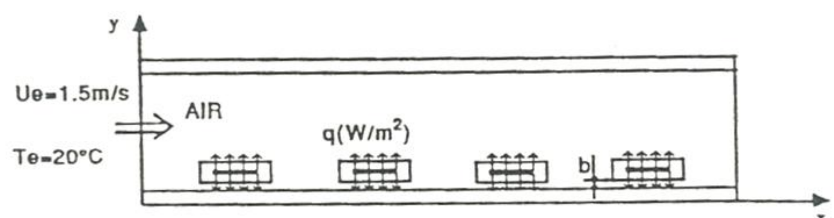

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