Chapter 1 : GENERAL POINTS

1°) HANDLING QUALITIES DEFINITION (HQ)

Flight mechanics theory can be subdivided in 2 parts :

* the **performances** study : movements of the aircraft centre of gravity in the space *(it determines the speeds on trajectory, the slopes, the take-off and landing distances, the ceilings, ...)*

* the **HQ** study : movements of the aircraft **around** its centre of gravity *(generated by the deflections of the flight control surfaces or by gusts...)*

- We may also define the HQ as being :
  the **characteristics of stability and control** of the aircraft which have a significant bearing on flight safety and on pilot feelings about the easiness to fly and handle the aircraft in stabilised flight and in manoeuvre.

- The HQ study may be subdivided itself in 2 parts :

  – **STABILITY** : possibility to maintain an equilibrium state (trim)
  (characteristics of the aircraft movements when the pilot does not act on the controls)

  – **CONTROLLABILITY** : possibility to modify this trimmed position *(action of the pilot on the controls)*

- About the controllability, we can distinguish :
  - **Static manoeuvrability** : static effect of the controls (*flight control deflections or control forces necessary to maintain steady flight, at different load factors, for instance)*

  - **Dynamic manoeuvrability** : dynamic effects of the controls

Handling Qualities are based on
the **trade-off**
STABILITY / Dynamic MANOEUVRABILITY

The more an airplane is stable, the less it is manoeuvrable.

But with Electrical Flight Control Systems ("fly-by-wire"), this trade-off is easier to perform.

We will see that HQ intervene in the different steps of the life of the airplane :

- *preliminary studies and plan*
- *sizing of*
  - *horizontal and vertical tailplanes,*
  - *flight control surfaces*
  - *associated actuators (servo commands...)*
- *contribution to the design of flight control laws*
- *study of failure cases affecting aircraft handling*
- *definition of crew procedures*
- *certification*
- *piloting*
- *follow-up of incidents/ accidents in service*
- *modifications after Entry Into Service (new engines, new Flight Control Laws standard,…)*

2°) Some quick REMINDERS about the FLIGHT of an AIRPLANE

An aircraft does not escape to the classical law of the mechanics :

The **equations of motion** come from **"Newton's second law"** for the 6 degrees of freedom :

(Fundamental principal of the dynamics)

$$\begin{cases} m\vec{\Gamma} = \sum \vec{F}ext & (1) \\ d\vec{C}/dt = \sum \overrightarrow{M}ext & (2) \end{cases}$$

where	*Γ*	*is the airplane acceleration*
	C	*is the angular momentum of the airplane*
	m	*is the airplane mass*
	Fext	*external forces applied to the airplane*
	Mext	*moments resulting from external forces*

Piloting an aircraft is equivalent to act more or less directly on $\overrightarrow{\textbf{Fext}}$ and $\overrightarrow{\textbf{Mext}}$ in order to make the airplane follow a **determined trajectory.**

Fext and **Mext** are resulting from

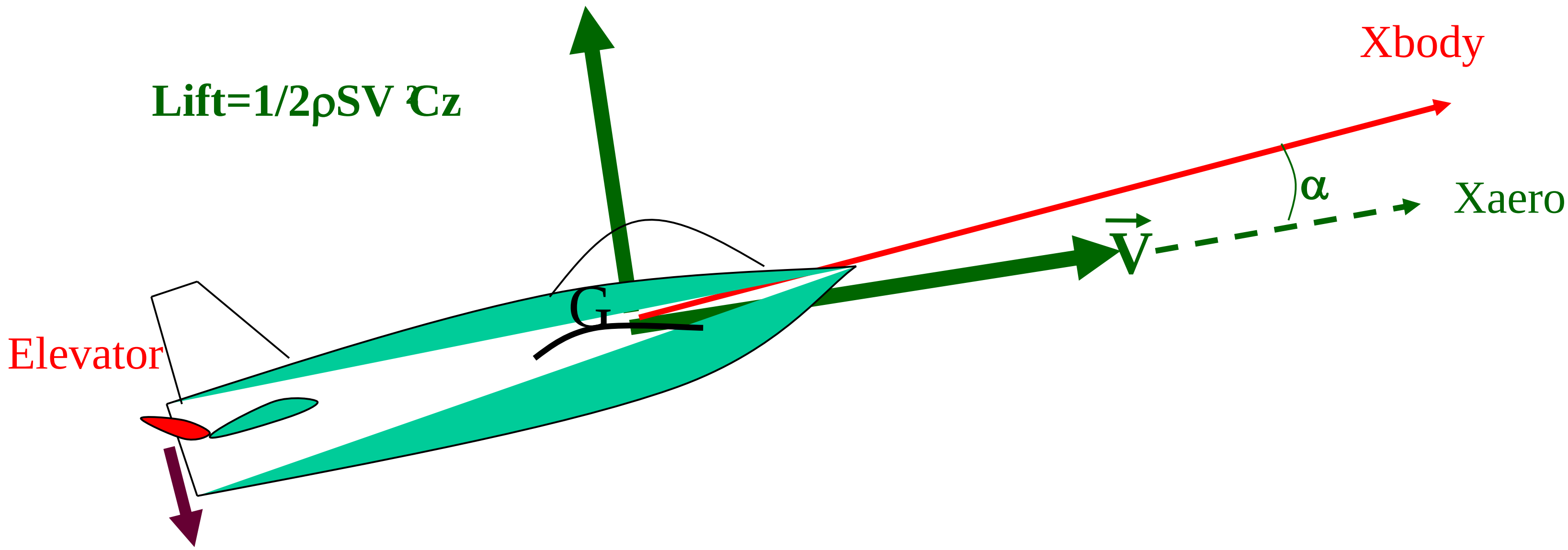
- the **gravitational** forces *(weight)*
- the **aerodynamic** forces *(lift, drag...)*
- the **propulsive** forces *(thrust)*

On almost all airplanes, there is an **indirect piloting in moment** : i.e. the pilot, thanks to the flight control surfaces, will create **moments** (aerodynamic $\overrightarrow{Mext}$) which will then modify **airplane attitude with regard to the velocity vector** : angle of attack (or incidence), sideslip angle and bank angle.

This attitude modification will induce **$\overrightarrow{Fext}$ variations** *(because aerodynamic forces mainly depends on plane attitude/velocity vector).*

This modifies the airplane **acceleration** and thus airplane trajectory.

Example of indirect piloting in moment :



*This **local downwards lift**, created by the elevator, induces a pitch up **moment**.*

*This moment **modifies** the airplane **attitude** and here increases the **angle of attack** (AOA) α.*

*This AOA modification increases the plane **lift** and therefore bends the **trajectory** (upwards in this example)*

3°) Flight controls :

a) Main **pilot controls and flight control surfaces**

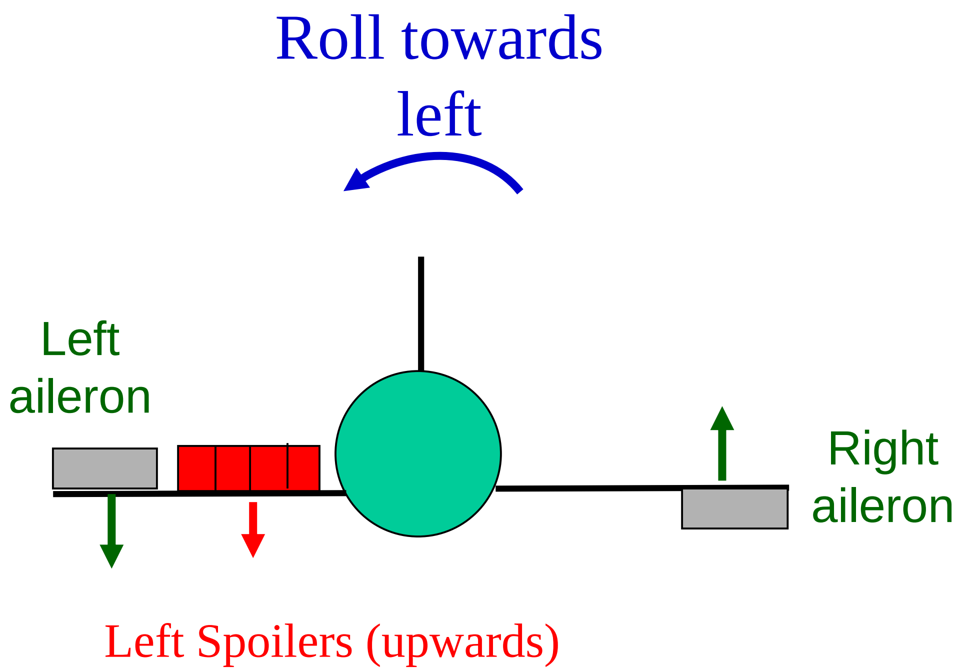
- **Pitch control** (*stick or wheel moved in pitch : longitudinally)*
  - Creates a pitch moment
  - by the intermediary of the elevators (and sometimes with elevons)

Standard control surfaces

Planes without stabilizer / « canard »)

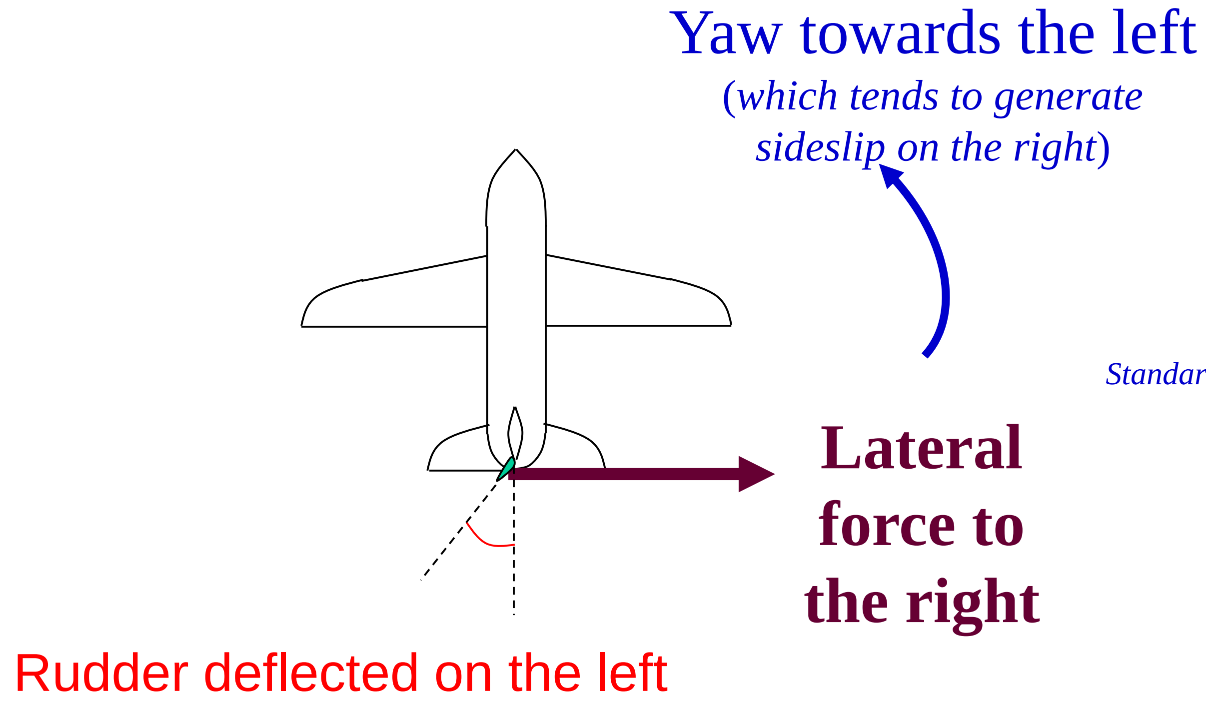
It allows to modify pitch attitude and angle of attack

- **Roll control** (*stick or wheel moved laterally)*
  - Creates a roll moment
  - by the intermediary of the ailerons (and sometimes with spoilers or also elevons)



Standard control surfaces

- **Yaw control** : rudder pedal
  - Creates a yaw moment
  - By the intermediary of the rudder



Standard flight control surfaces

These three controls allow the pilot to be **in control of the instantaneous attitude** of the airplane with regard to $\vec{\mathbf{V}}$.

Thus

- the **pitch** control (through elevator) allows to modify the **angle of attack α**

Wind axes

- the **yaw** control (through **rudder**) allows to modify the **sideslip β**

Body axes

- the **roll** control (ailerons…) makes turn the plane around its velocity vector : it allows to modify the **bank angle** and therefore makes the **LIFT vector** tilt.

Reminder : *the initial action of the flight control surfaces is to modify the plane attitude with regard to* $\vec{V}$*. The consequence is the modification of the aerodynamic forces and then the modification of the acceleration vector.*

TRIM controls

Standard flight control surfaces

Possibility to cancel the pilot control forces in case of almost constant long term forces.

Pitch trim : longitudinal trimming of the plane (*depends on aircraft speed, thrust, flaps settings, centre of gravity*…)

Roll trim : trimming in roll of the plane *(to counteract the potential fuel asymmetries in the wing tanks, the effect of the propeller : blown wing (slipstream effect) in case of engine failure*…)

Yaw trim : balance in yaw of the airplane (*engine thrust asymmetries*…)

Notations and sign conventions

	Notation	Sign Convention
Pitch	**δm** (elevator deflection) Dδm (wheel or stick deflection in pitch) Eδm (wheel or stick pilot force in pitch) δm_T (pitch trim deflection)	>0 Aircraft **nose down**
Roll	**δl** (aileron deflection) Dδl (wheel or stick deflection in roll) Eδl (wheel or stick pilot force in roll) δl_T (roll trim deflection)	>0 for a roll towards **left**
Yaw	**δn** (rudder deflection) Dδn (rudder pedal deflection) Eδn (rudder pedal force) δn_T (rudder trim deflection)	>0 on **left**

By convention, the flight control surfaces deflection is >0 downwards .

For ailerons, we use $\delta l = (\delta l_R - \delta l_L)/2$.

A **control surface law** is characterised by :

- a **gearing** :

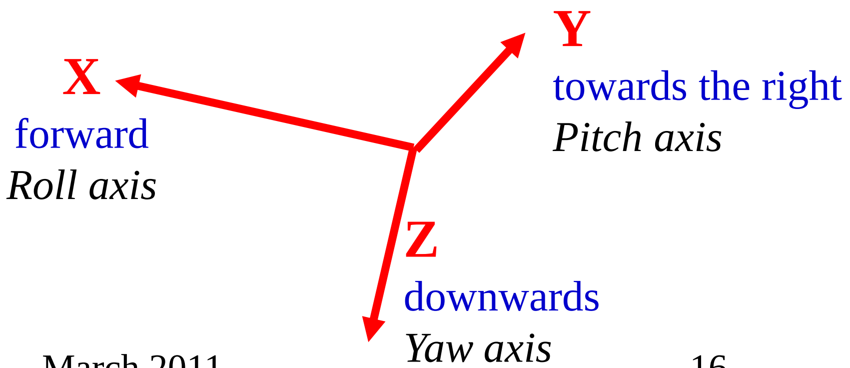
control surface deflection= f(pilot control deflection)

- a **force law** :

pilot control force= f(pilot control deflection).

Used letters :

	Roll axis	Pitch axis	Yaw axis
Axes	Gx	Gy	Gz
Angular speed rates	p	q	r
Control surfaces or pilot controls deflection, Moments	l, L	m, M	n,N

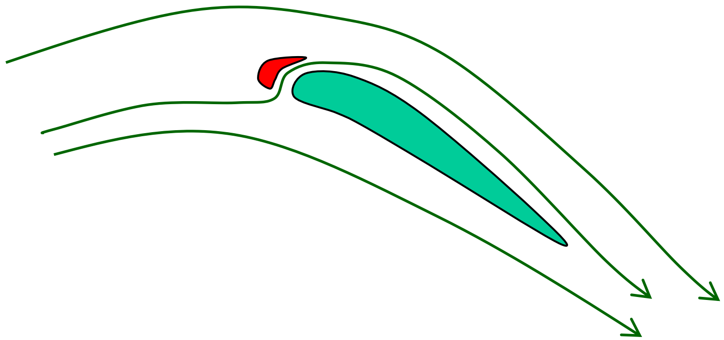


b) Secondary flight control surfaces

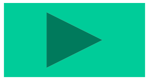
- **Airbrakes** : **drag** increase. (Descent slope increase, deceleration , emergency descent).

- **High Lift devices** : surfaces used in low speed area, allowing to fly with more **lift** :

  - **slats** (on leading edge) : delay the wing stall (*thanks to slot effect*)

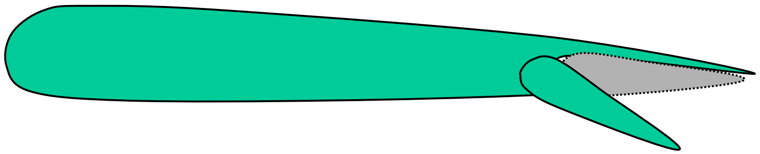


The slot is a boundary-layer control device: the air thus channeled energizes the boundary layer about the wing and retards the separation

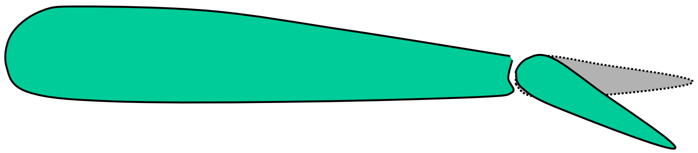


Standard flight control surfaces

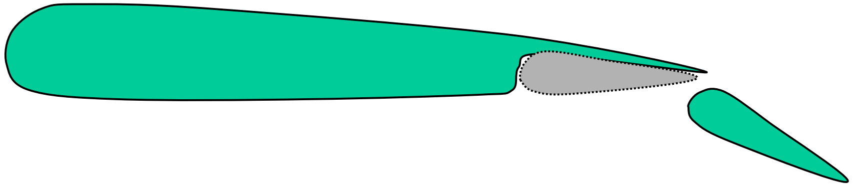
- flaps (trailing edge) : increase the lift by effect of camber



Split flap



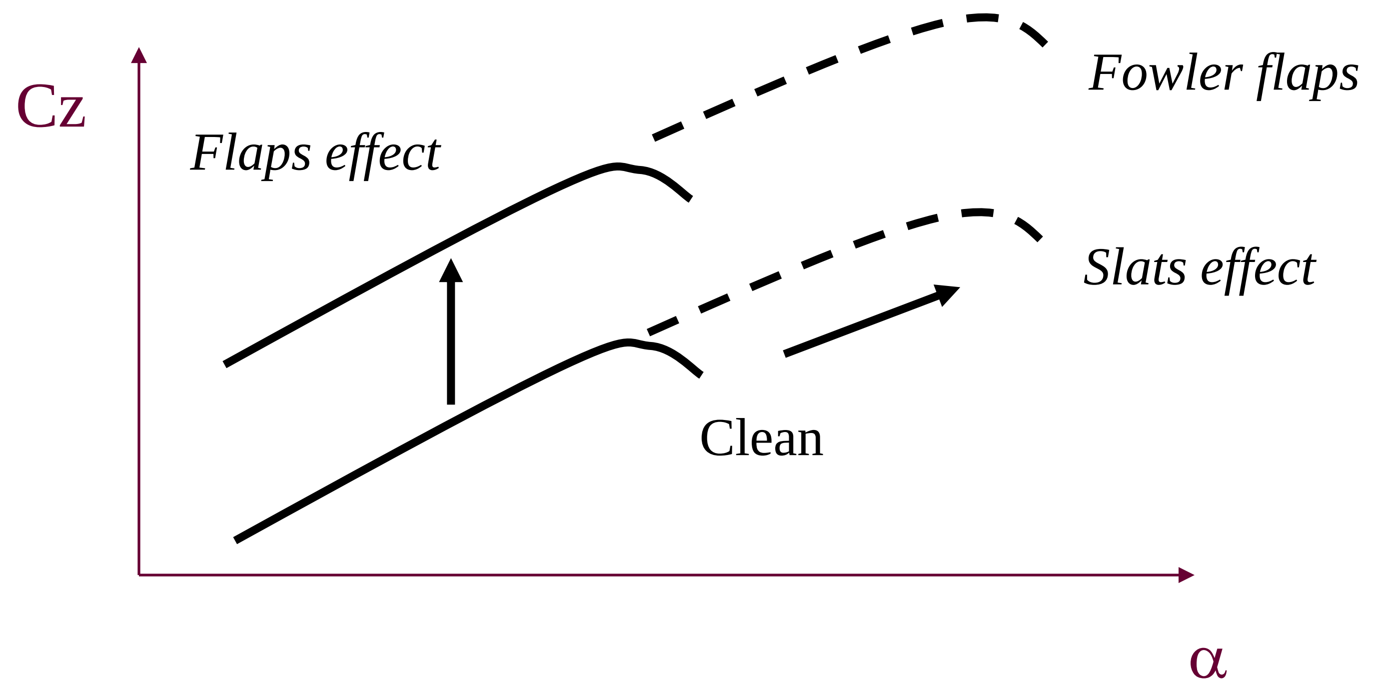
Plain flap



Fowler flap**:**

area effect (flap backwards) + camber effect (flap rotation)) + slot effect

Slats and flaps effects on lift Cz(α) or CL(α):



The high-lift configurations at take-off and landing combine the use of both slats and flaps on heavy aircraft.

Lift dumpers (ground spoilers…): to « **stick** » **the aircraft on ground** in order to improve the wheel **braking** efficiency during landing (or aborted take-off) . Also increase the drag.

Other controls are available for the pilot : even if they are not flight controls, they can modify the external forces applied to the airplane :

Landing gear : sometimes used to increase the drag.

Engines control : the engine thrust depends on :

- the altitude *(decreasing function)*
- the temperature *(decreasing function)*
- the airspeed (or Mach) *(decreasing function)*

4°) Flight controls types

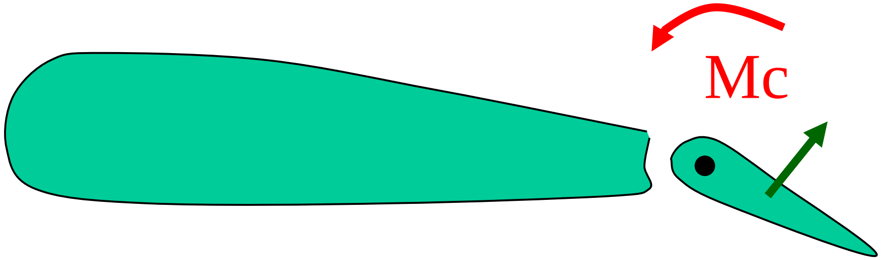
a) Mechanical flight controls :

The link between the pilot control and the flight control surface is mechanical (cables, push rods, pulleys, bellcranks...)

2 types can be distinguished:

a1) Direct controls :

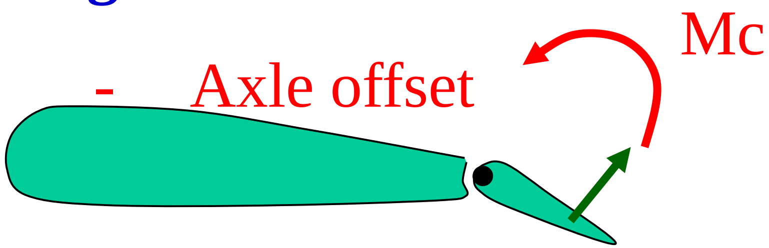
The link is **only** carry out **through cables** and the pilot has to counteract the **hinge moments** (moments resulting from aerodynamic forces applied on control surface /hinge of the surface). No actuators (servo-commands...)



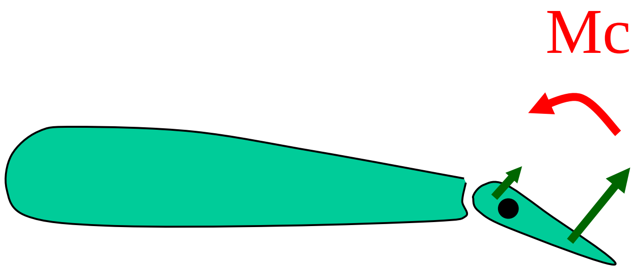
At the time of the aircraft design, **hinge moments** have to be **minimized :**

Either by moving the centre of pressure (*The centre of pressure is the point on a body where the sum total of the aerodynamic pressure field acts, causing a force and no moment about that point*) **closer to the surface hinge** :

- Axle offset

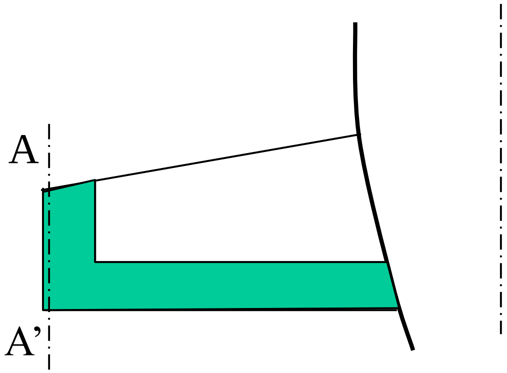


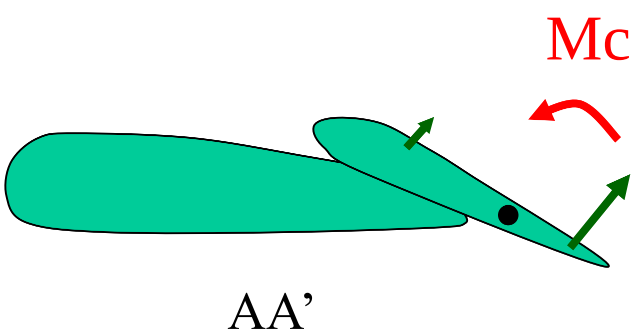
Unchanged hinge



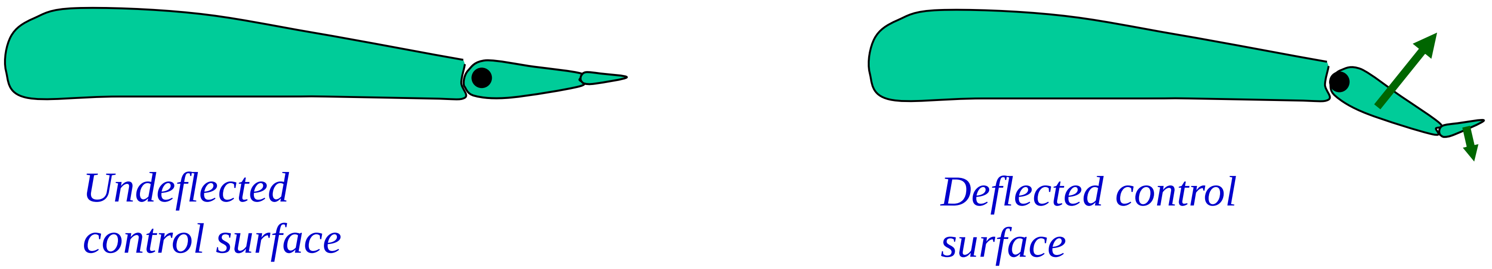
Hinge moved backwards :
Mc decreases

- Horn balance





or by generating an antagonistic moment : an auxiliary surface is added at the trailing edge of the control surface : Tab.



Mind the over-compensation :

Might occur if the center of pressure moves forward the surface hinge line, resulting in a softening, or even a stick force inversion at the large surface deflections. It can occur at high angle of attack (or sideslip) : potential airflow separation at the rear part of the control surface.

The control surface will go by itself on its stop if it is not maintained (by the pilot or a servo-actuator) .

a2) Servo-actuator :

Irreversible hydraulic device, which controls the surface at a fixed location depending on the position of the control. It implies the use of a "**springs box**" : **artificial feels systems** (*because it is the anchor-point of the servo which takes the hinge moment).*

It is used when the aerodynamic forces applied to the controls surfaces are too high (high speed or heavy airplanes ...)

Servo-command

b) « Electrical flight controls » : FLY-BY-WIRE :

The **link** between the pilot control and the surface (or more precisely the servo-valve) is **electrical**..

The words Fly-by-Wire imply only an electrically-signaled control system

It allows the implementation of devices which **helps the pilot : stabilizers/ dampers**, flight envelope **protections** (for speed, angle of attack, load factor, attitudes...)

The fly-by-wire, through the use of various **sensors** which measure the aircraft **response**, allows to elaborate flight control laws with a new aspect. For some aircraft, it is a **piloting by objectives** : load factor demand for pitch control and roll rate for roll control.

It enables to reinforce the **flight safety**, to optimize weight , to use a sidestick...

This requires a **redundancy** of computers, actuators, electrical devices, sensors…

5°) Sizing of flight controls and stabilizers (tailplane)

The Handling Qualities are involved in the design and sizing of flight controls :

a) For the **flight control surfaces** and the **tailplanes** (size/shape, maximum deflection).

Some **criteria** which can be **sizing** :

- **The elevator** sized by ***an acceptable manœuvrability*** or the ***flare***…

- **Pitch trim or horizontal tailplane** sized by the ***longitudinal balance (=trim)*** *(for all the combinations of center of gravity, speed, engine thrust, flap setting...)*

The horizontal tailplane can be sized by the ***push over*** manœuvre during which the tailplane has not to **stall**.

The **ailerons and spoilers** are sized by adequate ***roll rates***.

The **spoilers** and **speedbrakes** can be sized by the ***emergency descent***.

The **rudder** is sized by the ***engine failure*** (or sometimes the **crosswind** at take-off and landing).

The **fin** (= vertical stabilizer) can be sized by the ***dutch roll*** (rare) or the ***directional stability*** (for instance during manœuvres with one ***engine failure)***. A too large fin is detrimental to the crosswind capability (too large **weathercock** effect).

b) For the control surfaces **actuators**.

Their performances (static = stall load, dynamic , stroke) are linked to Handling Qualities criteria.

c) For the **stick/yoke and rudder pedal forces**.

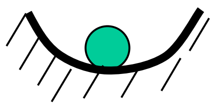
The regulation requires maximum forces not to exceed.

(*Example : stick forces <33 daN in pitch for short term application, permanent stick force <4.5 daN for long term application. CS 25.143 from EASA =Certification Specifications for Large Aeroplanes -European Aviation Safety Agency*)

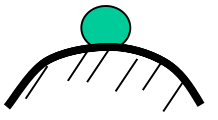
6°) Some notions of stability

a) Static stability :

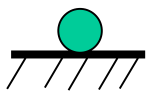
Tendency to come back or not to an equilibrium position.



Stable



Unstable

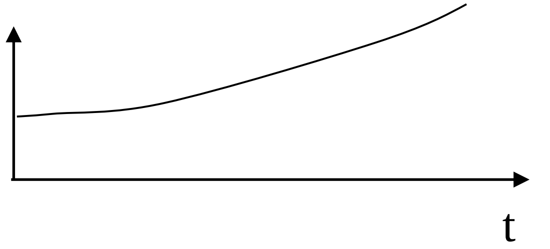


Neutral

b) Dynamic stability :

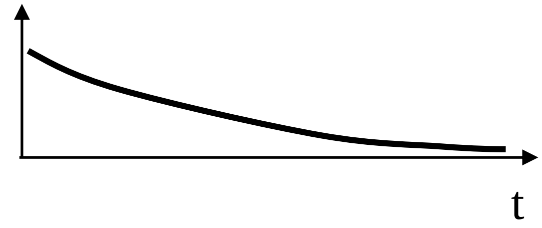
Manner used by the object to come back to its equilibrium position.

For **the static unstable**, the movement is ***aperiodic divergent***

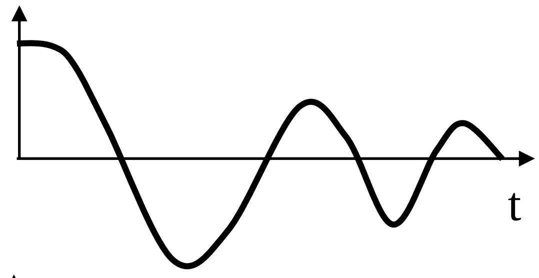


For **the stable static**, the movement may be :

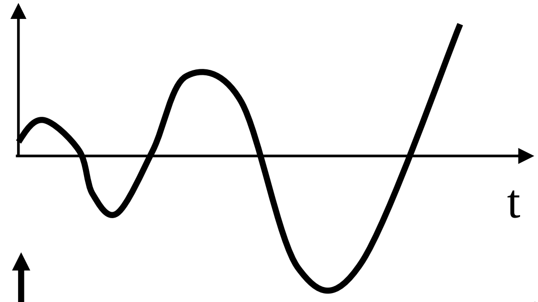
- ***aperiodic convergent***



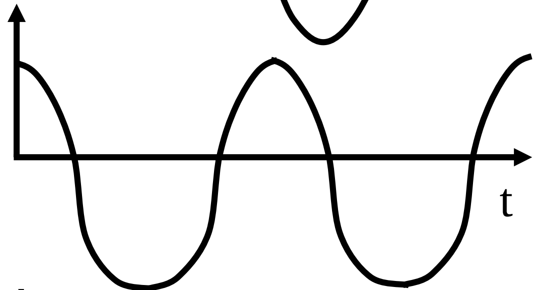
- ***oscillatory convergent***



- ***oscillatory divergent***



- ***Harmonic oscillation***



7°) Equations used for Handling Qualities study :

Hypotheses : *Rigid aircraft*
Symmetry plane Gxz

Newton's second law:

$$\begin{cases} \sum \vec{F}ext = d(m\vec{V})/dt & (1) \\ \sum \overrightarrow{M}ext = d(I_G \vec{\Omega})/dt & (2) \end{cases}$$

where $I_G = \begin{bmatrix} I_X & 0 & -I_{XZ} \\ 0 & I_Y & 0 \\ -I_{XZ} & 0 & I_Z \end{bmatrix}$ and $\vec{\Omega} = \begin{pmatrix} p \\ q \\ r \end{pmatrix}$

Roll rate
Pitch rate
Yaw rate

		Aero.	Propulsion	Weight
(1)	*Drag equation*	R_x	F_x	$m\,g_x$
	Lateral force equation	R_y	F_y	$m\,g_y$
	Lift equation	R_z	F_z	$m\,g_z$
(2)	*Roll moment equation (Gx)*	L	L_F	0
	Pitch moment equation (Gy)	M	M_F	0
	Yaw moment equation (Gz)	N	N_F	0

where

$$\begin{cases} Rx = -\,1/2\ \rho S V^2\ Cx \\ Ry = +\ 1/2\ \rho S V^2\ Cy \\ Rz = -\,1/2\ \rho S V^2\ Cz \end{cases}$$

and

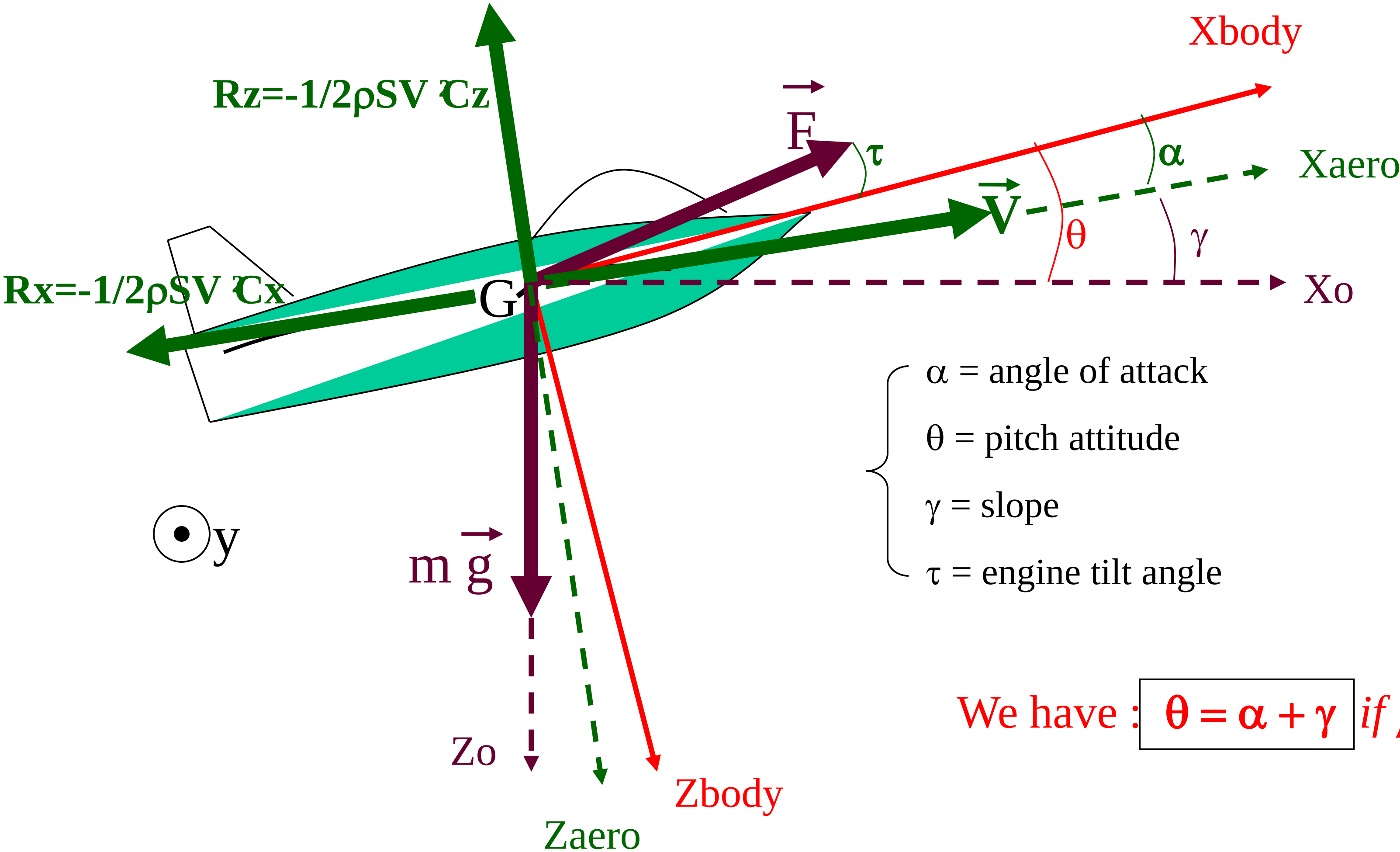
$$\begin{cases} L = 1/2\ \rho S \mathbf{l} V^2\ Cl \\ M = 1/2\ \rho S \mathbf{l} V^2\ Cm \\ N = 1/2\ \rho S \mathbf{l} V^2\ Cn \end{cases}$$

Note 1 : *S is the reference area, l is the reference length = mean aerodynamic chord (MAC) .*

Note 2 : *The **forces** equations (1) will be projected onto the **aerodynamic axes** (except the lateral equation) and the moments equations (2) onto the **aircraft axes***

7° a) Study of the longitudinal motion :

Assumes : $p=r=0$ $\beta=\phi=0$



- α = angle of attack
- θ = pitch attitude
- γ = slope
- τ = engine tilt angle

We have : $\boxed{\theta=\alpha+\gamma}$ *if $\beta=\phi=0$*

Projection of 2 the force equations onto the aerodynamic axes:

$$\begin{cases} m\dot{V} = - mg \sin\gamma - 1/2\, \rho S V^2 C_x + F \cos(\alpha + \tau) & \text{Drag equation} \\ - mV\dot{\gamma} = mg \cos\gamma - 1/2\, \rho S V^2 C_z - F \sin(\alpha + \tau) & \text{Lift equation} \end{cases}$$

Projection of the moment equation onto the pitch axis (aircraft axis) Gy

$$I_y\dot{q} = 1/2\, \rho S\, l\, V^2 (C_m + C_{mF}) \quad \text{Pitch moment equation}$$

Definition of the load factor $\vec{n}$:

Notion of the apparent weight . Sum of « mass forces ».

$$\vec{n}mg = m\vec{g} + \sum \vec{F_I} \qquad \text{i.e. :} \qquad \vec{n} = -1/mg\left(\sum \vec{F_E} - m\vec{g}\right)$$

Or also :

$$\vec{n} = -1/g\left(d\vec{V}/dt - \vec{g}\right)$$

It gives for the longitudinal load factors projected onto aerodynamic axes :

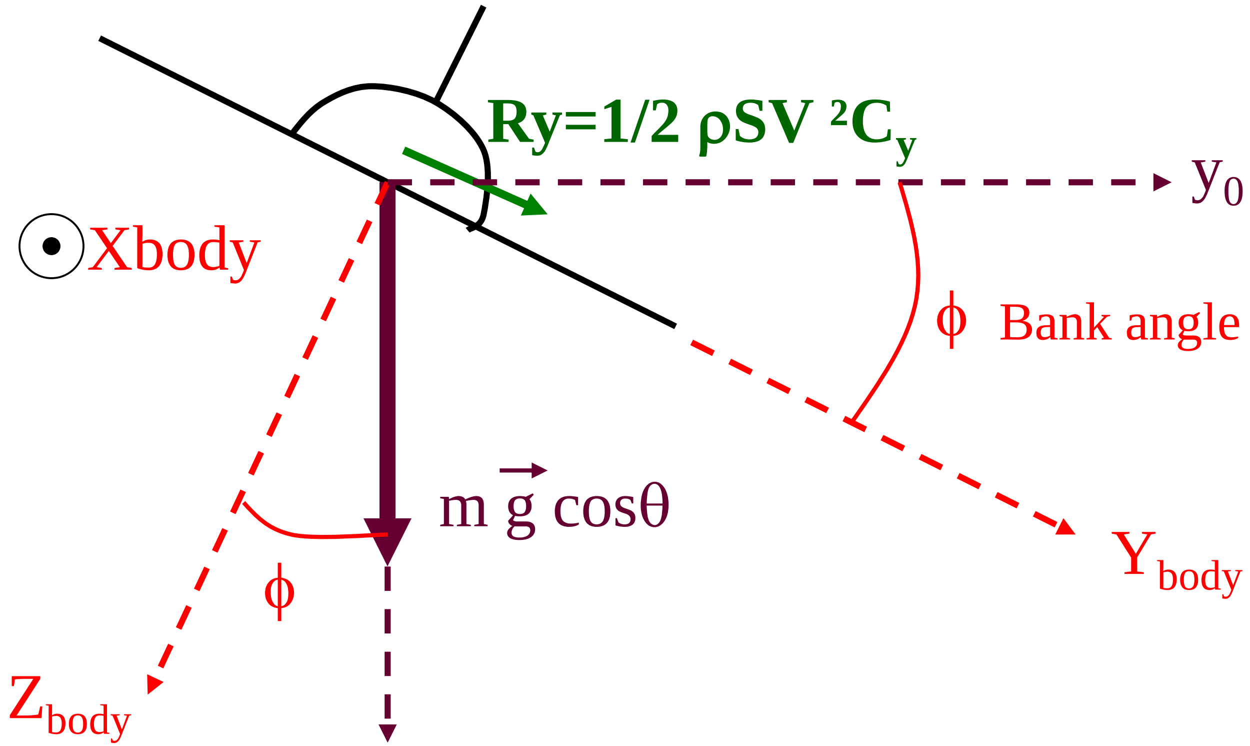
$$\begin{cases} n_{xa} = -1/g\ dV/dt - \sin\gamma \\ n_{za} = V/g\ d\gamma/dt + \cos\gamma \end{cases}$$

It is then possible to simplify the lift equation :

$$n_{za}\,mg = 1/2\ \rho S V^2 C_{za} + F\sin(\alpha+\tau)$$

i.e. $\boxed{\mathbf{n\ mg = 1/2\ \rho S V^2 C_z}}$ *(neglecting the thrust component)*

7° b) Study of the lateral motion :



Where the lateral aerodynamic force Ry can be positive, negative or null depending on the flight conditions…

Lateral force equation on <u>body axis</u> (aircraft axis):

$$m\,(d\vec{V}/dt\,.\,\vec{j}) = mg\,\sin\phi\,\cos\theta + 1/2\,\rho SV\,Cy$$

Note : it is often assumed $\cos\theta \simeq 1$

Roll and yaw moments equations :

$$\begin{cases} Ix\,dp/dt - Ixz\,dr/dt = 1/2\,\rho SlV\,Cl + L_F & \text{Roll axis X} \\ Iz\,dr/dt - Ixz\,dp/dt = 1/2\,\rho SlV\,Cn + N_F & \text{Yaw axis Z} \end{cases}$$

Note :

- reminder : p is the roll rate and r is the yaw rate.

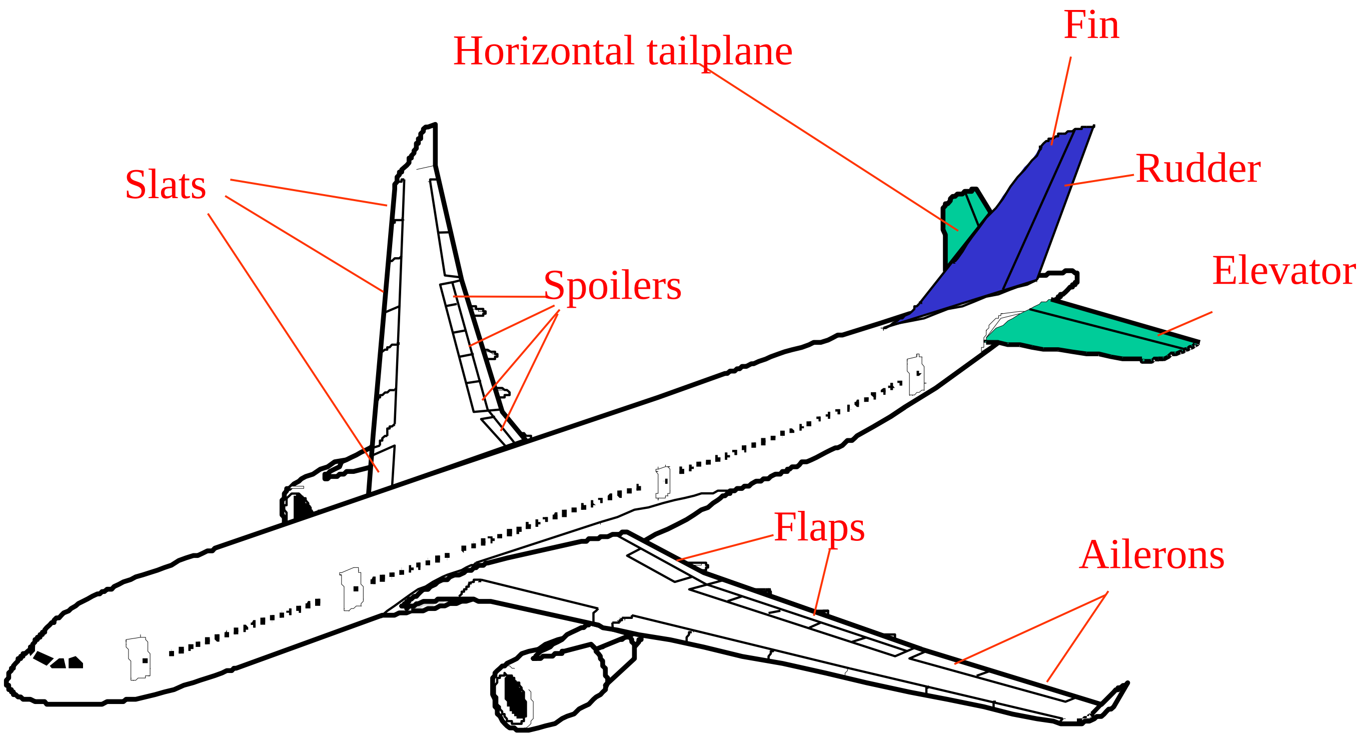
-The cross inertia Ixz is sometimes neglected

- the roll and yaw moments due to propulsion L_F. N_F have not to be disregarded in case of engine failure.

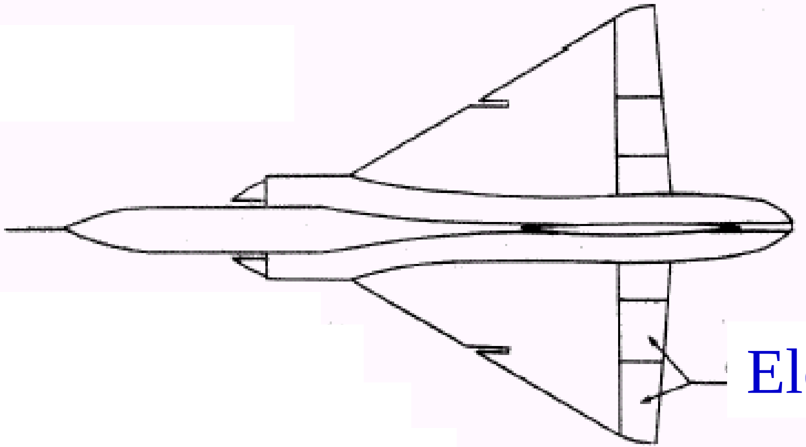
-Cl and Cn depend on sideslip, ailerons and rudder, roll and yaw rates…

-The lateral force equation may also be written : $n_y\,mg = -1/2\,\rho SV^2 Cy$

Example of flight controls on Airbus (A330):



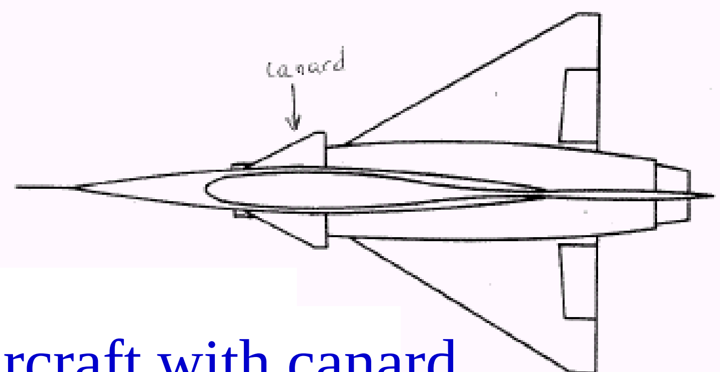
Example of aircraft fitted with elevons and canards



Elevons

Aircraft without horizontal tailplane

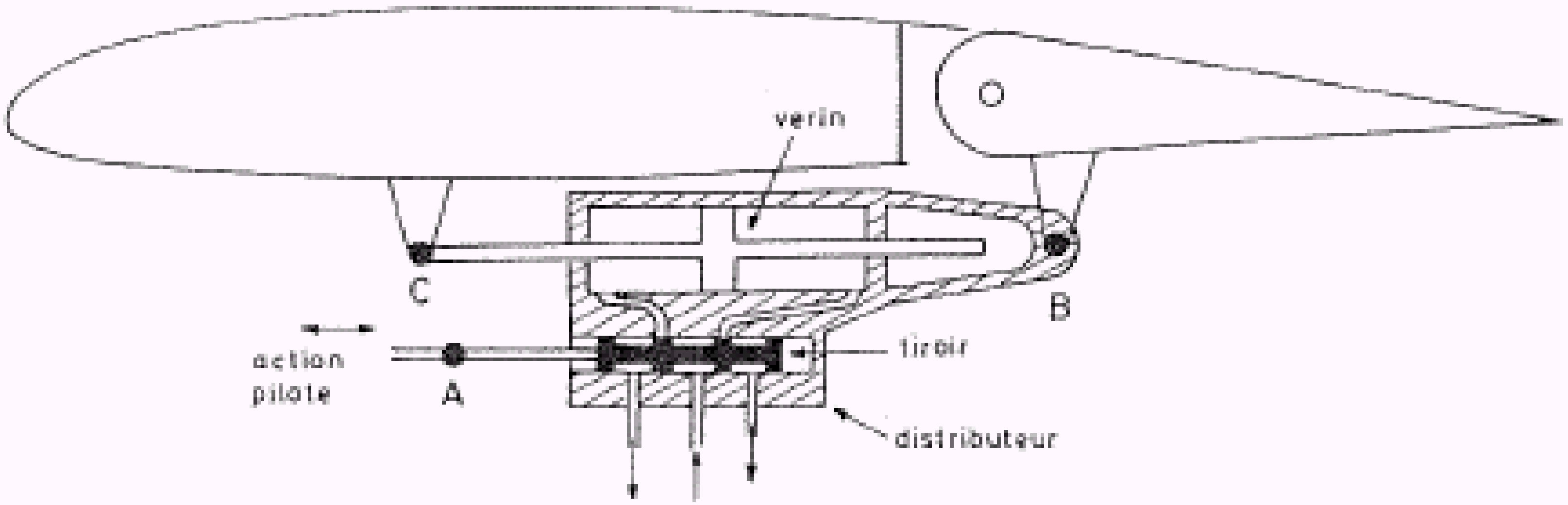
(Mirage III)



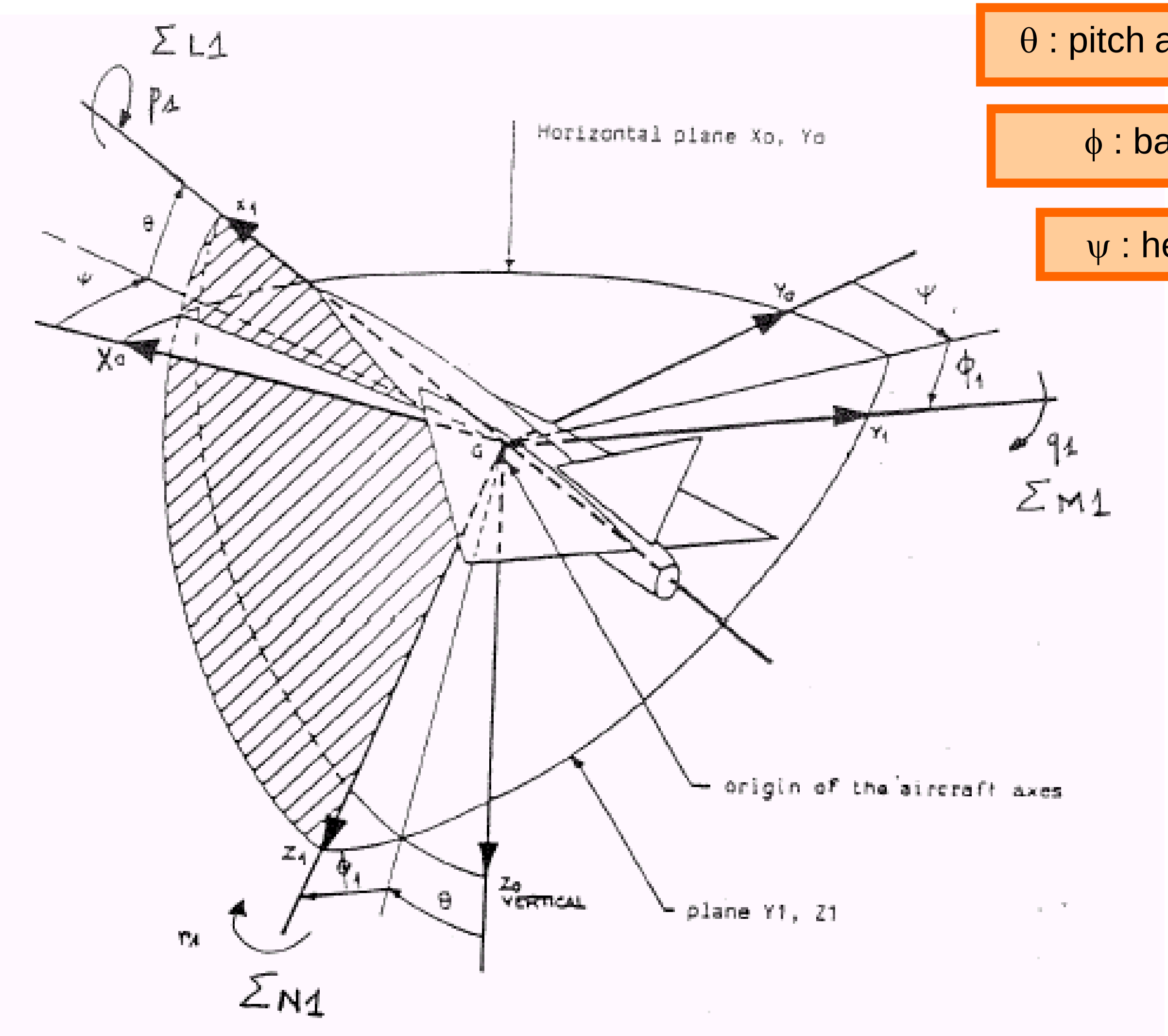
Aircraft with canard

(Griffon Nord 1500-02)

Mechanical flight control with **servo-commands**



Aircraft axes or body axis $(Gx_1y_1z_1)$ or $(Gx_{body}, y_{body}, z_{body})$ / **Earth axes** $(Gx_0y_0z_0)$

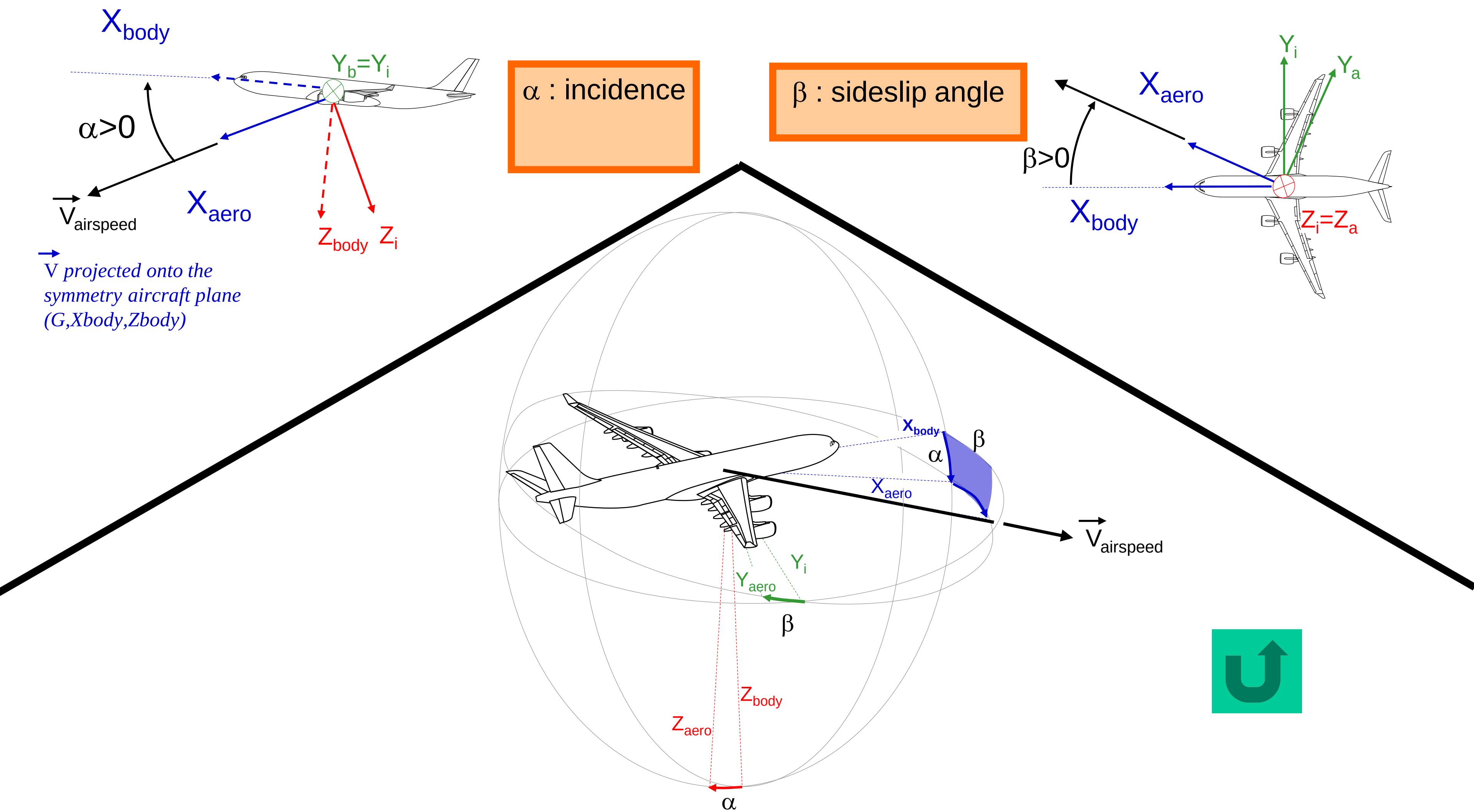


θ : pitch attitude

ϕ : bank angle

ψ : heading

Aerodynamic axes / body (aircraft) axes



Chapter 2 : Static longitudinal stability

- <u>1°) Lift coefficient Cz (or CL) :</u>

Reminder :

- The lift $Rz=-1/2\ \rho SV^2 Cz$
- The simplified lift equation : **n mg = 1/2 $\rho SV^2 Cz$**

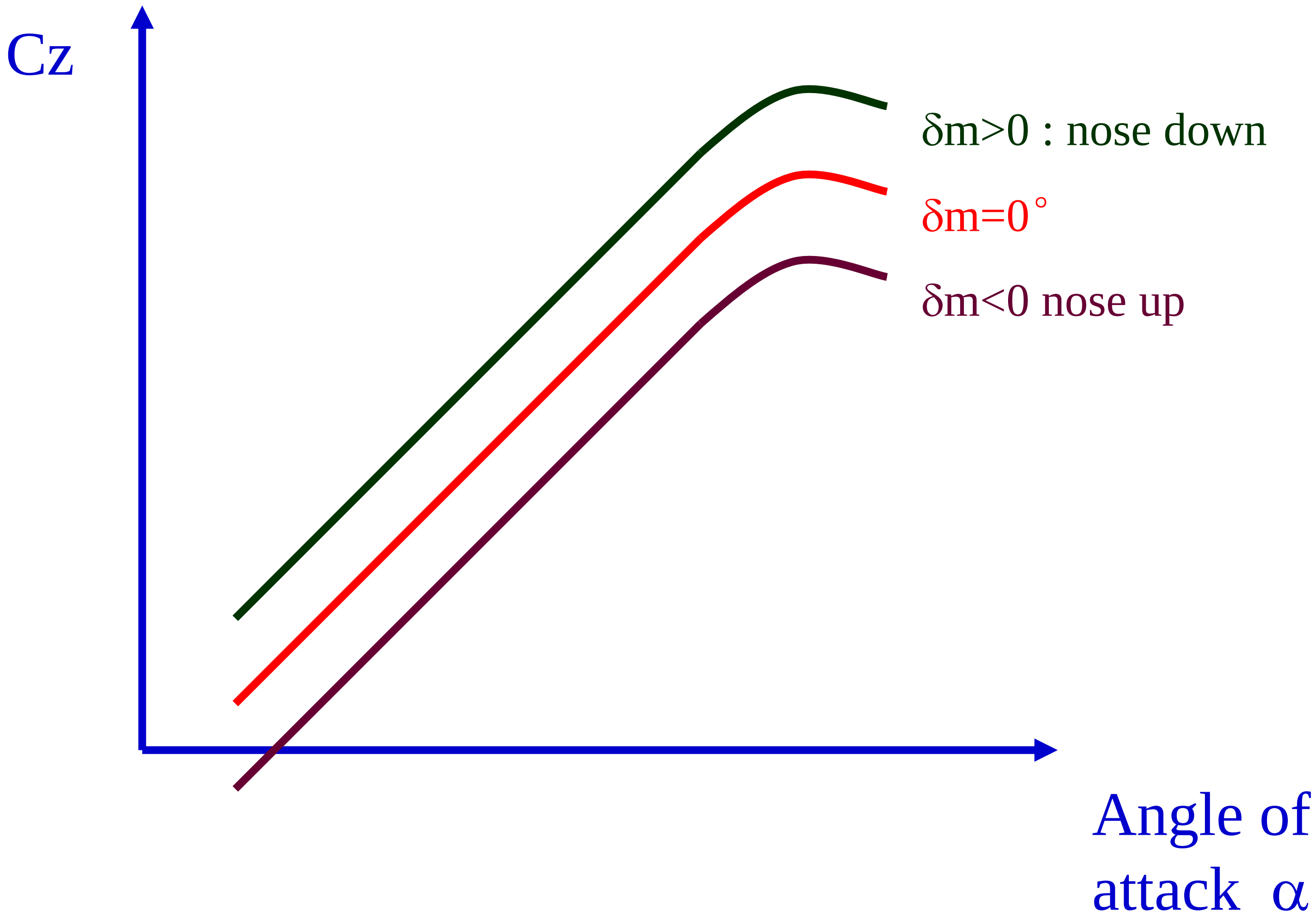
We may write : $Cz = Cz\alpha\ (\alpha - \alpha_0) + Cz\delta m \cdot \delta m + Cz\delta m_T \cdot \delta m_T$

where α_0 : incidence for null lift and $\delta m = \delta m_T = 0$

$Cz\delta m$: gradient (>0) due to elevator deflection δm

$Cz\delta m_T$: gradient (>0) due to pitch trim deflection δm_T

Example for Cz (where δm_T= cst, for a given slats/flaps configuration) :



•2°) Drag coefficient Cx (or CD):

Reminder :

•The drag $Rx=-1/2\ \rho S V^2 Cx$

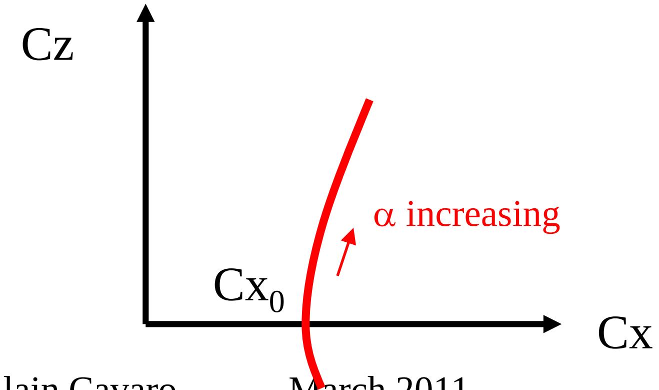
•The drag equation : $m\ dV/dt = -mg \sin\gamma - 1/2\ \rho S V^2 C_x + F \cos(\alpha + \tau)$

Cx may be written under the simplified expression $\mathbf{Cx = Cx_0 + k\ Cz^2}$

(as far as α is not too high).

- the term $\mathbf{k\ Cz^2}$ is called the (lift-) induced drag.

- Cx= f(Cz) is called the POLAR



•3°) Pitch moment coefficient Cm:

Reminder :

•The pitch moment equation is :

$$I_y\, dq/dt = M + M_F = 1/2\, \rho S l\, V^2 (C_m + C_{mF})$$

Cm may be expressed as follow :

$$\boxed{\mathbf{Cm = Cm_0 + Cm\alpha\,(\alpha - \alpha_0) + Cm\delta m\, .\, \delta m + Cm\delta m_T\, .\, \delta m_T + Cmq.ql/V}}$$

<u>3a) Cm_0 : pitch moment coefficient at null lift and zero elevator deflection :</u>

Cm_0 nose down (<0) : it depends on the airplane shape (asymmetry of the used airfoils, the fuselage) ; it depends on Mach number (in transonic flight).

3b) $C_{m\alpha}$: pitch moment coefficient due to angle of attack :

It is a torque which, for stability reasons, has to be negative : **back-pulling torque** mainly generated by the horizontal tailplane (or the delta wing).

It makes intervene the notion of **aerodynamic centre**.

DEFINITION of the AERODYNAMIC CENTRE or NEUTRAL POINT :

point F such as Cm around this point is constant, whatever α is.

In other words : the **aerodynamic centre** is the **point** where the **lift forces** generated by the incidence **variations** are applied.

The **aerodynamic centre** is a fixed point of the airplane (however it moves backwards in supersonic flight).

Take care to not confuse with the centre of pressure , variable, where the resultant of the aerodynamic forces are applied.

(*The centre of pressure is the point on a body where the sum total of the aerodynamic pressure field acts, causing a force and no moment about that point*)

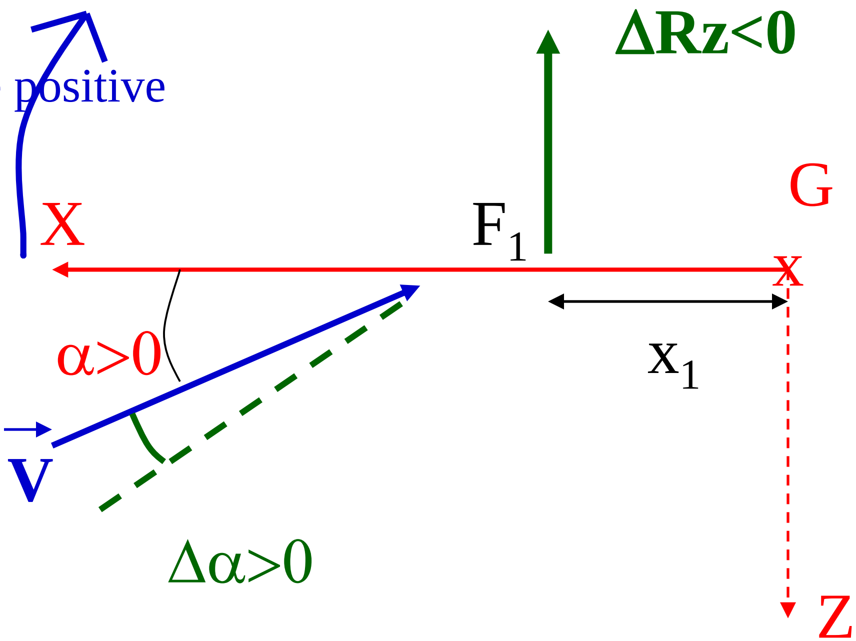
It is here the overall aerodynamic centre of the airplane, barycentre of the aerodynamic centre without tailplane and aerodynamic centre of the tailplane.

We can differentiate several aerodynamic centres, depending whether or not the elevators are at constant deflection.

We call F_1 the **aerodynamic centre controls fixed** : $\delta m = cst$.

We write $\mathbf{x_1 = \overline{G\,F_1}}$

Direction for the positive pitch moments



On this drawing, $x_1 > 0$, what is seldom the case in reality

Notion of AERODYNAMIC CENTRE

$M_{G\alpha} = - x_1 \Delta Rz = - x_1 (-1/2 \rho S V^2 Cz\alpha \Delta\alpha)$

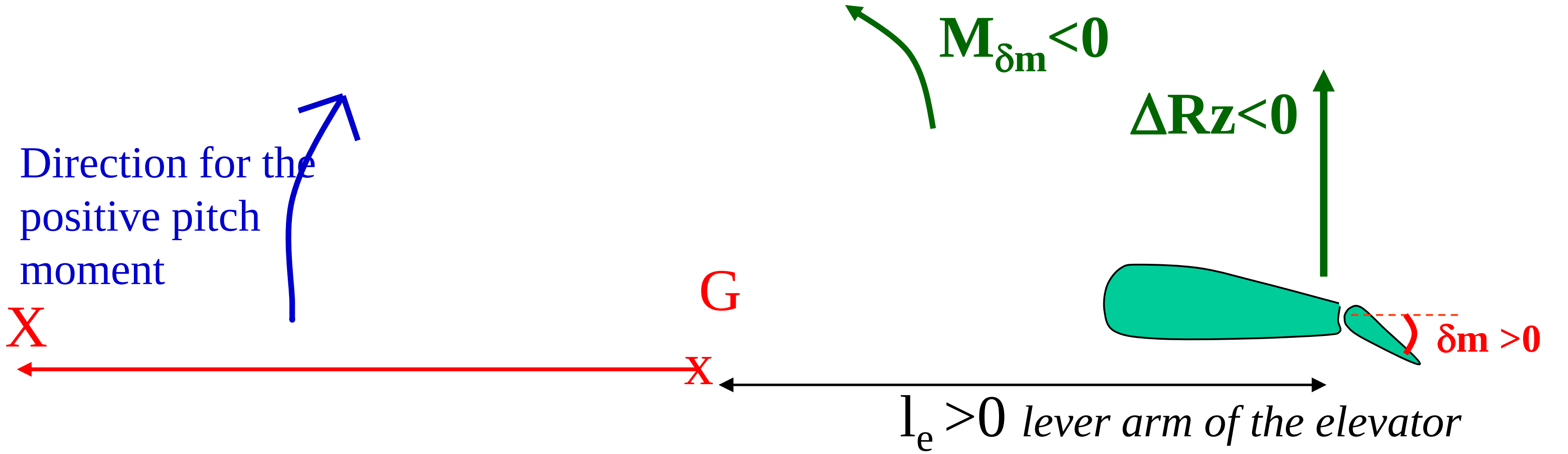
Now $M_{G\alpha} = 1/2 \rho S \, l \, V^2 Cm\alpha \Delta\alpha$

Whence : $\mathbf{Cm\alpha = x_1 / l \; Cz\alpha}$

*The sign of $Cm\alpha$ is the sign of x_1 because $Cz\alpha > 0$. We will see that **$Cm\alpha$ is negative** for stability reasons.*

3c) Cmδm: pitch moment coefficient due to elevator deflection δm:

In fact, it is the **efficiency of the elevator**. It is the raison d'être (purpose) of this control surface.



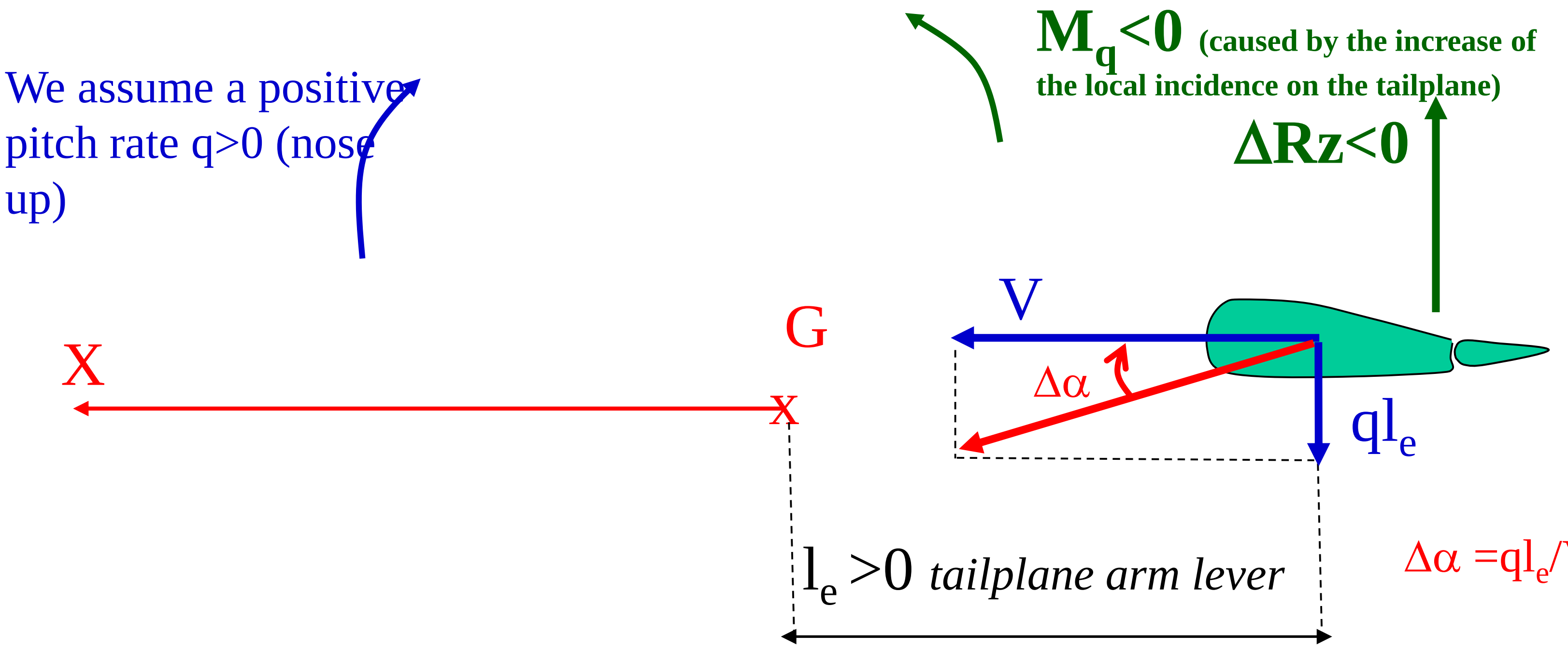
$$M_{G\delta m} = l_e \, \Delta Rz = l_e \, (-1/2 \, \rho S V^2 Cz\delta m \, . \, \delta m)$$

Now $M_{G\delta m} = 1/2 \, \rho S \, l \, V^2 Cm\delta m \, . \, \delta m$

Whence : **$Cm\delta m = -le/l \, Cz\delta m < 0$**

We may also define a CmδmT for the trim , in particular in the case of the trimmable horizontal stabilizer .

3d) Cmq: pitch moment coefficient due to pitch rate q:



$M_q = 1/2\ \rho S\ l\ V^2 C_{mq}\ (ql/V)$ (ql/V is dimensionless)

Cmq <0 : Pitch damping.

Note : *in addition to the tailplane, the fuselage and the wing participate to the pitch damping.*

3e) CmF: torque due to the propulsive forces (engines):

The thrust axis does not pass inevitably the centre of gravity.

The torque generated varies with α (*often destabilizing*).

The blow of the propellers or the jet modifies the C_{m0} or the $C_{m\delta m}$.

The study of the stability is done at constant engine command. For simplification, we will include the engines effects in the aerodynamic coefficients (which is not completely right because CmF varies with airspeed).

3f) $Cm\dot{\alpha}$: torque due to the variation of incidence:

Often neglected.

Due to the delay for the setting up of the downwash ε at the horizontal tailplane ($\alpha_{HTP}=\alpha-\varepsilon+\delta mT+ql/V$)

Conclusion:

We may write :

$$\mathbf{Cm = Cm_0 + Cm\alpha\,(\alpha - \alpha_0) + Cm\delta m\,.\,\delta m + Cmq.ql/V}$$

or also a relationship Cm function of Cz and the position of the aerodynamic centre :

$$\mathbf{Cm = Cm_0 + Cz\, x_1/l - (x_1 + l_e)/l\;\; Cz\delta m\,.\,\delta m + Cmq.ql/V}$$

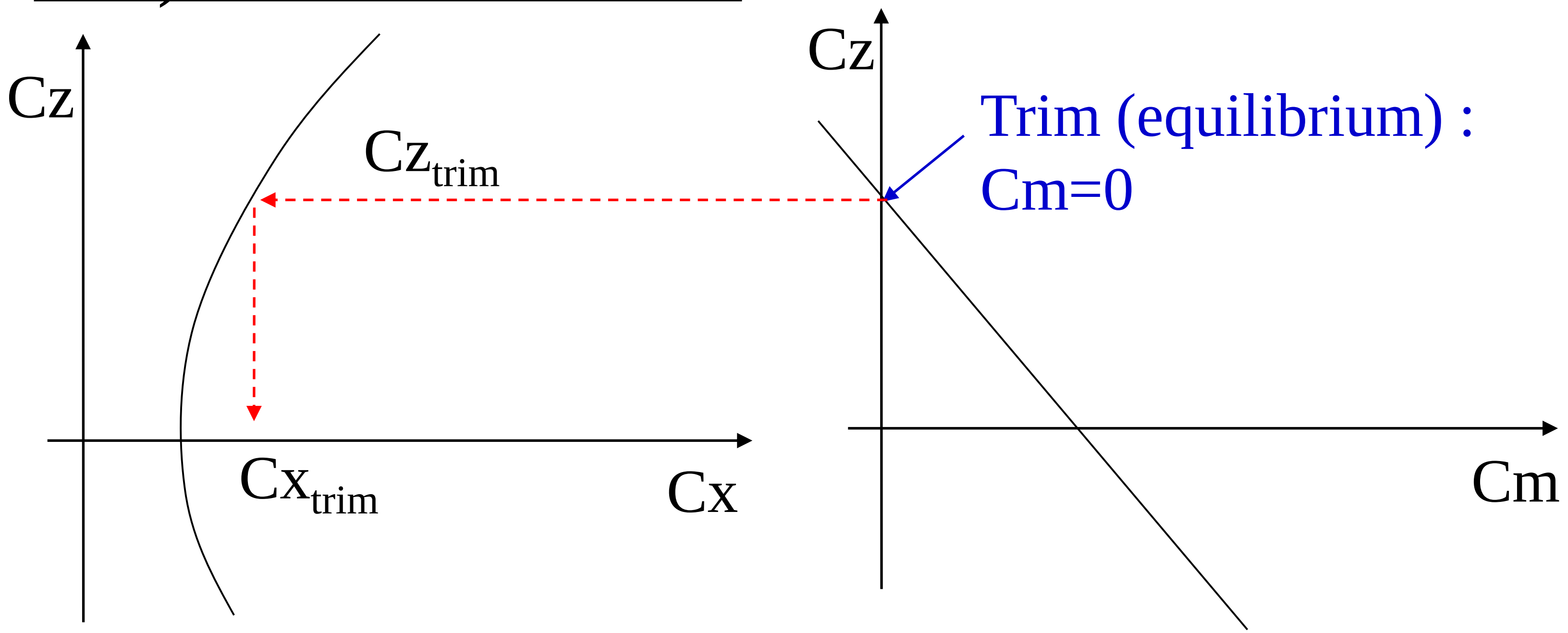
thanks to the relationships we previously referred :

$$Cz = Cz\alpha\,(\alpha - \alpha_0) + Cz\delta m\,.\,\delta m \quad (avec\,.\delta m_T = 0)$$

$$Cm\alpha = x_1/l\; Cz\alpha$$

$$and\;\; Cm\delta m = -le/l\; Cz\delta m$$

•4°) Trimmed Polar :



We have a polar Cx=f(Cz) for a given elevator deflection δm.

For a given centre of gravity (thus x_1 determined) and for a given aerodynamic configuration (flaps/slats), there is a single trimmed (balanced) point : Cm=0. This point determines Cx_{trim} and Cz_{trim}.

The set of the points (Cx_{trim}; Cz_{trim}) got when δm varies constitutes the **trimmed polar.**

It is used for the aircraft **performance** computations.

•5°) Criterion for longitudinal static stability :

It is a criterion for **stability** with regard to **angle of attack** : an incidence increase has to produce a pitch moment that tends to decrease this incidence increase :

a positive $\Delta\alpha$ (>0) has to produce a pitch down moment ($\Delta Cm<0$)

Thus, this stability may be written as :

$$\mathbf{dCm/d\alpha <0}$$

or $\mathbf{dCm/dCz<0}$ because $dCz/d\alpha>0$

•6°) Longitudinal static stability in level flight with controls fixed

6a) Stability criterion

Straight flight in level flight : *so q=0*
Controls fixed : $\delta m = constant$

In this case : **$dCm/d\alpha = Cm\alpha$** and also **$dCm/dCz = x_1/l$**

The stability criterion may be written :

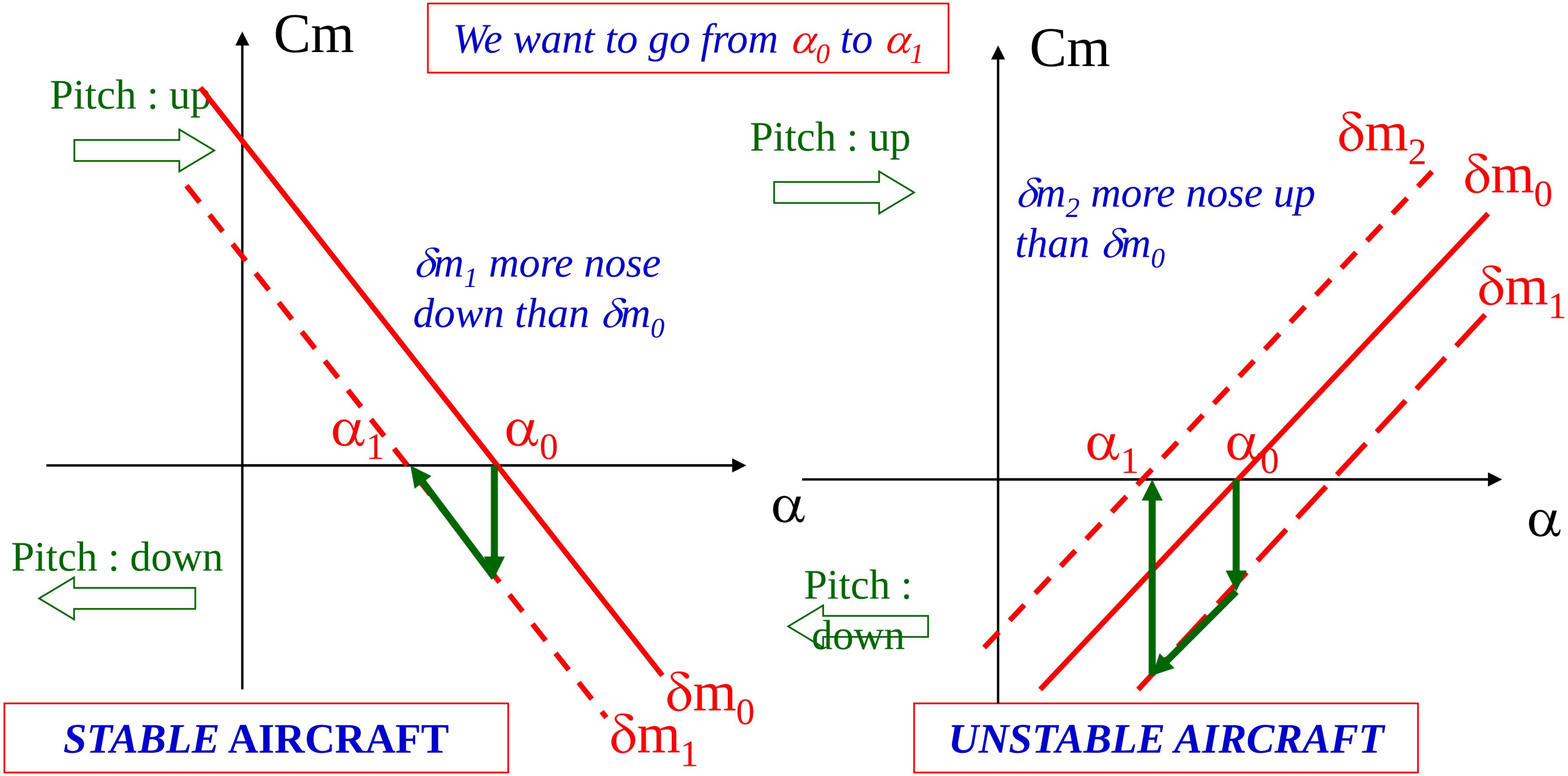
$$\boxed{Cm\alpha < 0 \quad \text{or} \quad x_1/l < 0}$$

Reminder : $Cm\alpha = Cz\alpha \; x_1/l$ *and* $\overline{x_1} = GF_1$

An airplane is statically STABLE, controls fixed, if the controls fixed AERODYNAMIC CENTRE F_1 is behind the centre of gravity G.

The airplane will be all the more stable so since the distance GF_1 is important *(or the static margin* x_1/l *is important).*

The **aft CG limit** is therefore often **determined** by a **stability** condition.

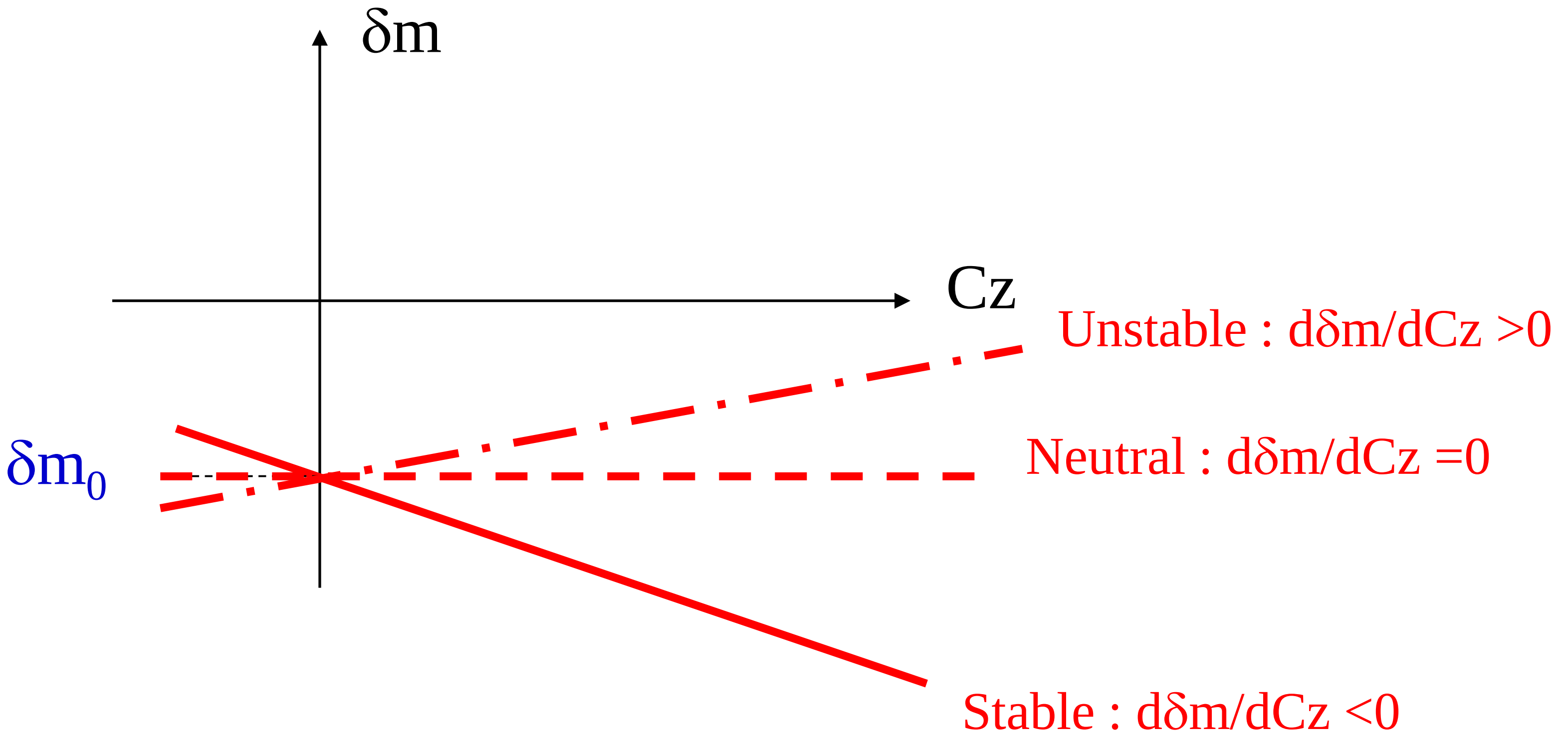


ATTENTION : whether the aircraft is **stable or unstable, push on the stick (or wheel) always cause a pitch down moment** (*except if the surface is on its stop or if there is a high speed protection …)*

6b) Stability assessment

On a stable airplane, we have seen that an increase of lift ($\Delta Cz>0$) will generate a pitch down moment, that the pilot has to counteract by a nose-up elevator deflection ($\delta m<0$) if he wants to balance the aircraft ($Cm=0$).

Thus, the airplane is **stable if $d\delta m/dCz<0$**



From the expression Cm=f(Cz,δm) (see § 3), we may deduce at the **equilibrium** (Cm=0 et q=0) the elevator deflection δm:

$$\boldsymbol{\delta m = \delta m_0 + (d\,\delta m\,/dCz)\,.\,Cz}$$

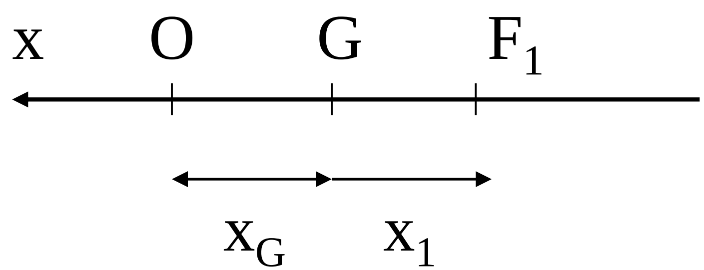
where $\boldsymbol{d\,\delta m\,/dCz = (x_1/\,(x_1+le)\,)\,/\,Cz\delta m}$

We notice that this slope is null when $x_1=0$, i.e. when $G=F_1$.

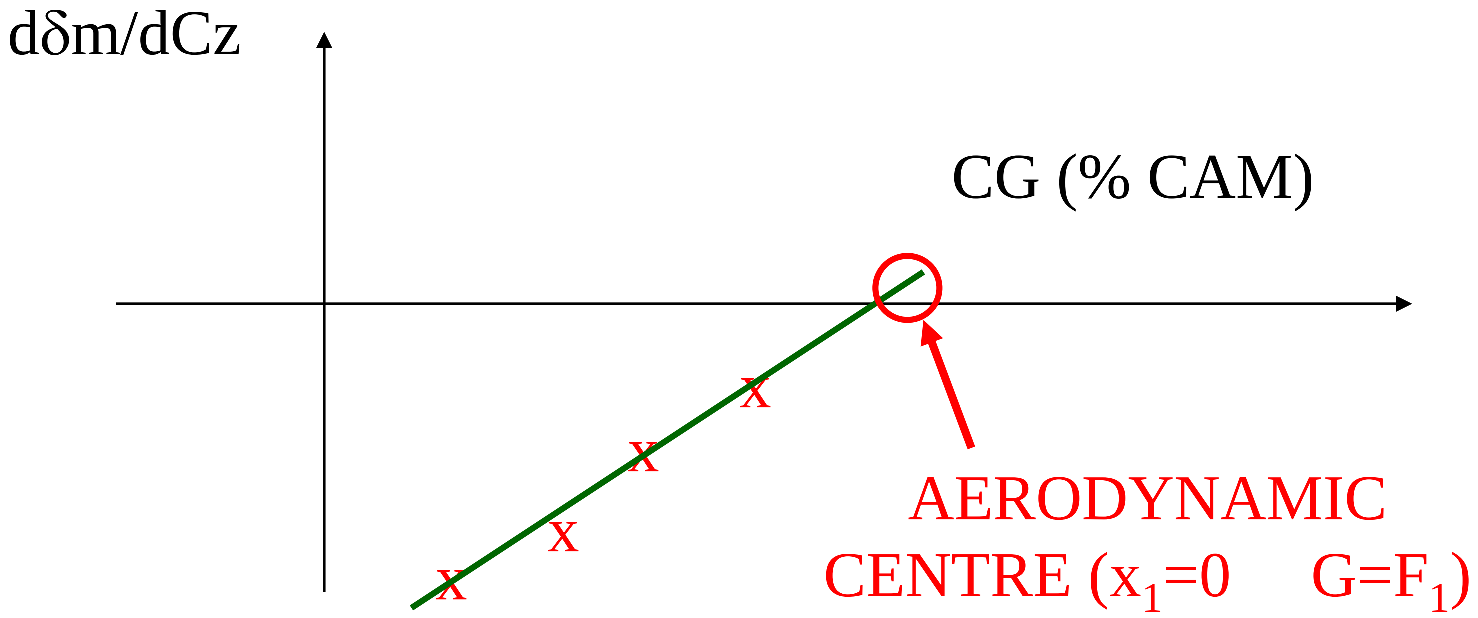
6c) Determination of the aerodynamic centre by flight tests

$d\delta m / dCz$ depends on the centre of gravity through $x_1 = GF_1$.

Definition of the centre of gravity :



Centring = $x_G / l = |OG| / l$ (*where O , fixed point of the aircraft, on the leading edge of the reference airfoil and l= MAC= Mean Aerodynamic Chord*)



dδm/dCz
CG (% CAM)
x
x
x
x
AERODYNAMIC
CENTRE (x_1=0 G=F_1)

6d) Flight tests for longitudinal static stability

Tests performed at max aft CG (the most unfavorable for the stability).

The airplane is trimmed (*balanced*) at a particular flight point (Zp=altitude, Vc=airspeed). Then the speed is changed without modifying the trim, the longitudinal balance being provided by the elevator.

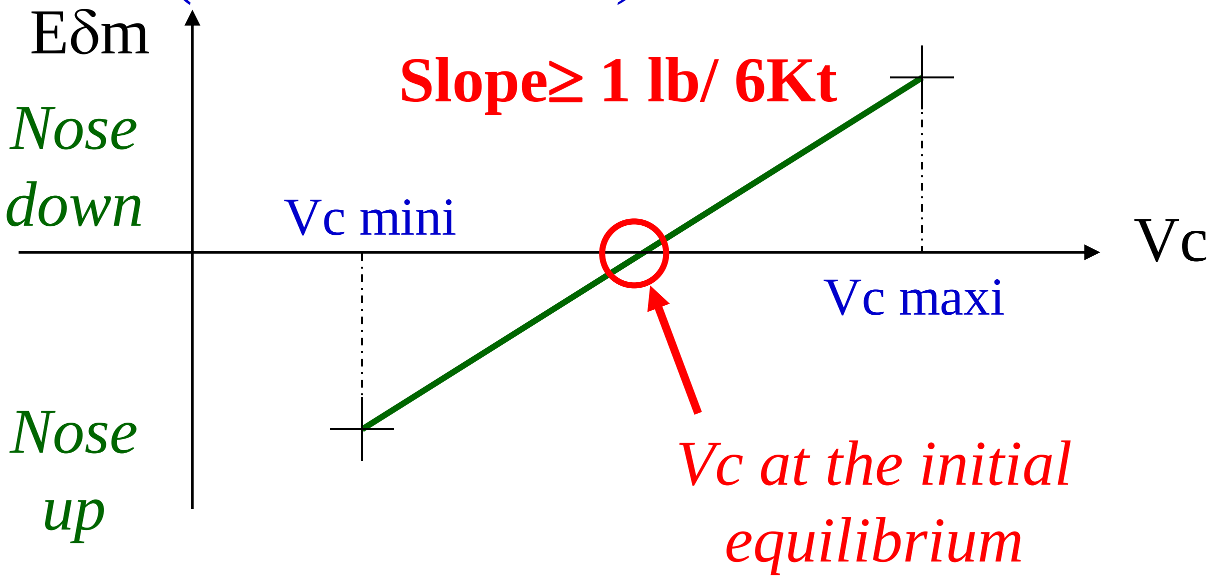
Qualitative analysis : the stick is progressively released. For a stable aircraft, Vc shall come back towards its equilibrium speed.

Quantitative analysis : it is done through the plots $\delta m = f(Vc)$ and $E\delta m = f(Vc)$.
Reminder : $E\delta m$ is the pilot pitch control force (on stick or wheel)

The slope $\delta m(Vc)$ shall be positive for a stable aircraft (*because the slopes $\delta m(Vc)$ and $\delta m(Cz)$ have an opposite sign, in accordance with the lift equation*).

The regulation (FAA or EASA) requires that the slope **$E\delta m(Vc)$ > 1 lb /6 Kt**

(1 lb= 0.454 daN)



6e) Aids for piloting

• In the case of an unstable airplane, we may use a " **Vc trim"** (or "Mach trim") : *automatic pitch trimming restoring an artificial stability (case of the A300-A310).*

• For fly-by-wire aircraft, there may be an autotrim function which ensures a neutral stability. Some protections at the limits of the flight envelope restores a positive stability *(even if the airplane is naturally unstable).*

• For aircraft fitted with servo-actuators that do not meet the criterion "1 pound for 6 knots", we can increase the **stiffness** of the artificial feels system.

•7°) Longitudinal static stability controls free

It has to be considered for aircraft without servo-actuators (the controls surfaces are free when the pilot controls are released).

The elevator will float into the "wind", in a way to nullify the hinge moment (created by the aerodynamic forces applied to the elevator).

Elevator hinge moment coefficient :

$$Cc = b_0 + b_1\,\alpha + b_2\,\delta m + b_3\,\delta m_T$$

Elevator free, we get : $Cc=0$

Controls fixed , we had : $\mathbf{dCm/dCz = x_1/l}$, where $x_1=GF_1$.

Controls free, we have (when using he relationship $Cm=f(Cz)$ provided in § 3) :

$$\mathbf{dCm/dCz = x_1/l - (x_1+l_e) / l \;\; Cz\delta m \; (d\delta m/dCz) = x_2/l} \, ,$$

$$\text{where } d\delta m/dCz = -\,(b_1/b_2)\,.\,(1/Cz\alpha)$$

We define the **controls free aerodynamic centre** $\mathbf{F_2}$ where $x_2=GF_2$.

F_2 is closer to CG than F_1. Aft CG limit

•8°) Difficulty to pilot a statically unstable airplane

In the case of a statically unstable airplane , the difficulty arises from the fact that the pilot has to constantly counteract the tendencies of angle of attack variations.

However, the piloting is not impossible, but unpleasant in the flight phases requiring a speed stability (approach,…). Nevertheless, we have to put nuances in the judgment : it depends also on the dynamic stability which partly depends on pitch inertia).

On the other hand, a too strong stability could be even embarrassing :

the pilot control forces variations or control surfaces variations may then become too large when we want to change the aircraft speed.

•9°) Longitudinal static stability in manœuvre with controls fixed

In **manœuvre**, damping due to pitch rate (Cmq) has to be taken into account in the expression :

$$\mathbf{Cm = Cm_0 + Cz\ x_1/l - (x_1+l_e)/l\ Cz\delta m\ .\ \delta m + Cmq.ql/V}$$

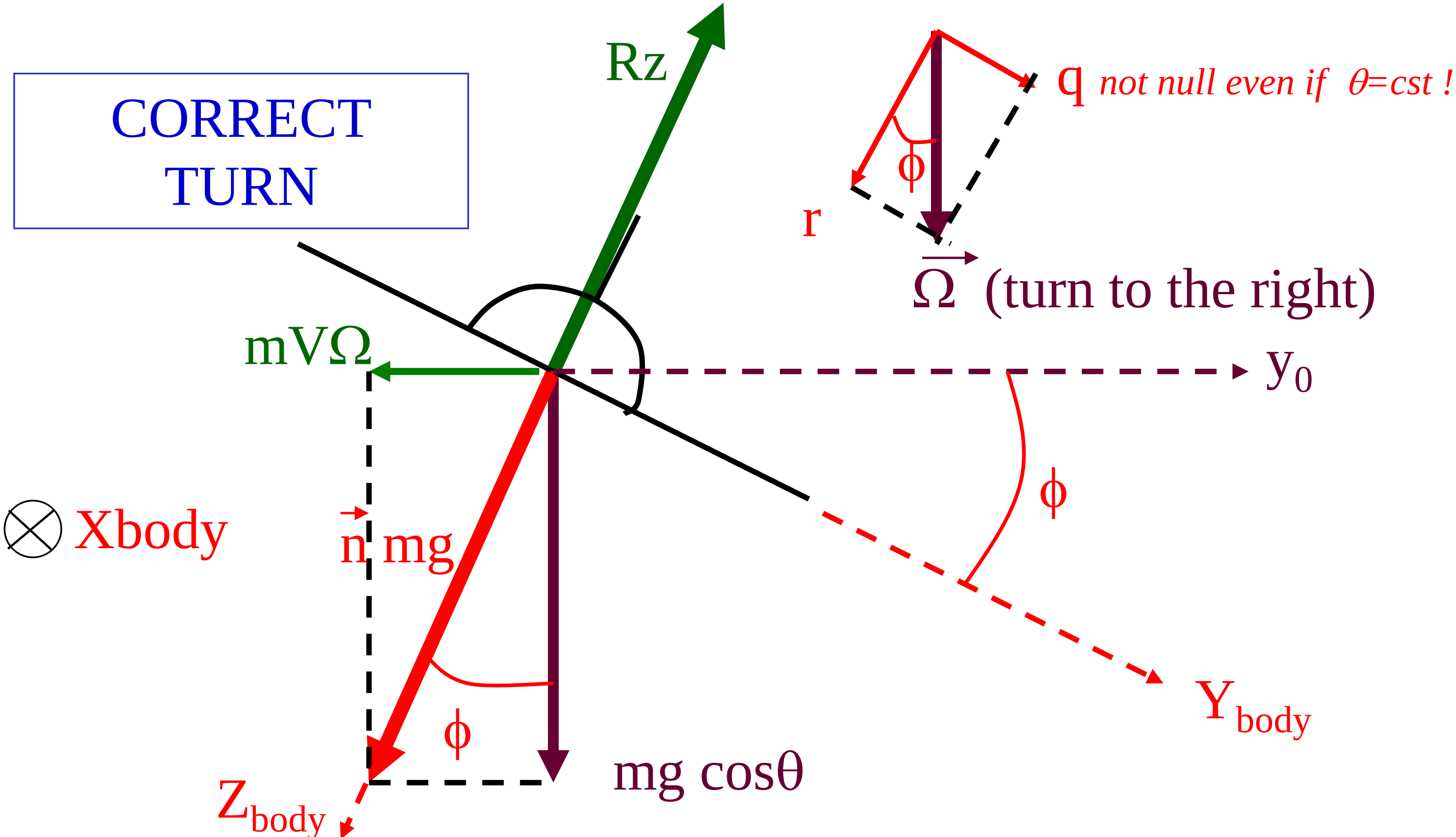
9a) Relationship between load factor n and pitch rate q :

Reminder : $\vec{n}mg = m\vec{g} + \sum \vec{F_I}$ *where F_I are fictitious forces*

A) IN TURN (in level flight or at constant slope) :

We assume a **correct turn** : $\vec{\mathbf{n}}$ **is in the aircraft symmetry plane** Gxz. This corresponds to a turn where Ry is null, i.e. a **sideslip nearly null.**

CORRECT TURN



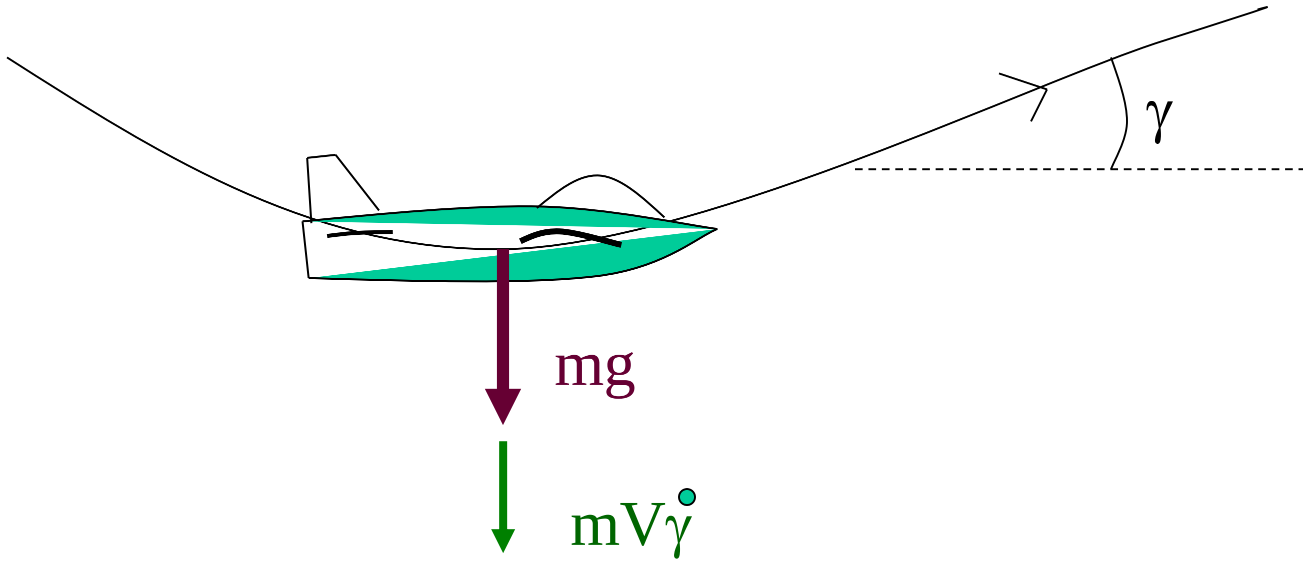
We get in this case : $\cos\phi = \cos\theta / n_z$ i.e. $\boxed{n_z = \cos\theta / \cos\phi}$

We notice that : $q = \Omega \sin\phi$ and $\sin\phi = mV\Omega / (n_z mg)$ i.e. $q = V\Omega^2/(ng)$

From $(nmg)^2 = (mV\Omega)^2 + (mg)^2$, we deduce $\Omega^2 = (n^2-1)g^2/V^2$

Whence : **$q = g/V\,(n - 1/n)$**

B) IN PULL UP MANOEUVRE (bottom point of the manoeuvre):



$nmg = mg + mV\dot{\gamma}$

But $\dot{\gamma} = \dot{\theta} - \dot{\alpha} = q$ because α little varies (and $\phi=0$)

So **$q = g/V\ (n-1)$**

For the 2 types of manœuvre, we may write q as follows :

$$\boxed{q = g/V\ k(n)}$$

9b) Stability criterion :

Reminder : stable if $\boxed{dCm/dCz<0}$

Cm may be written (*using the relationship found in a*) :

$$\mathbf{Cm = Cm_0 + Cz\, x_1/l - (x_1+l_e)/l\, Cz\delta m\, .\, \delta m + Cmq.\, gl/V^2 k(n)}$$

The stick is fixed, whence δm=cst.

In addition, $dk(n)/dCz = \rho SV^2/(2mg)\, dk(n)/dn$

(because $nmg=1/2\rho SV^2 Cz$)

Thus : $dCm/dCz = \underset{<0}{\mathbf{x_1/l}} + \rho Sl/(2m)\, \underset{<0}{Cmq}\, \underset{>0}{dk(n)/dn} = \underset{<0}{\mathbf{x_{m1}/l}}$

$\mathbf{x_{m1}}$ defines the controls fixed manœuvre point H_1 : $\mathbf{x_{m1}} = \overline{GH_1}$

The manœuvre point is **behind the aerodynamic centre** ($0 > x_{F1} > x_{m1}$).

Its position depends on flight conditions m, Zp (ρ) and manoeuvre type (*turn, pull up manoeuvre,   through dk(n)/dn*)

9c) Assessment of the stability in manœuvre. « elevator per g » :

The plot δm(n) (= *plot of the elevator per g*) characterises the static stability **in manœuvre**.

In fact, we have seen that δm(Cz) characterises the stability and in manœuvre, the increase in Cz characterises the load factor variation (*the lift balances the apparent weight : lift equation*).

At the equilibrium : Cm=0. In addition, Cz= 2n mg/ (ρSV ²). Then when deriving with respect to n the pitch equation (in Cm), we get :

Elevator per g =

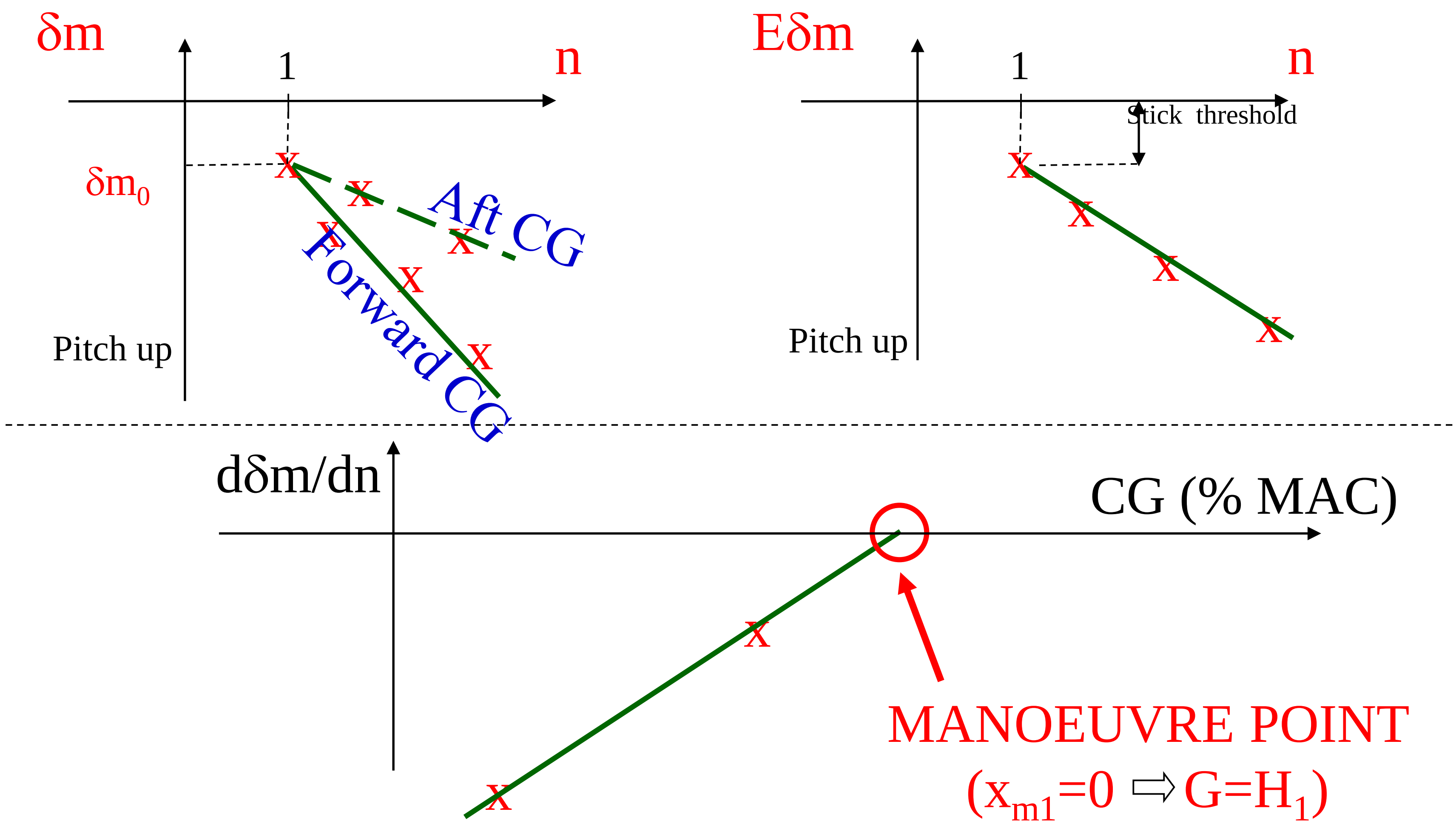
$$\boxed{\frac{d\delta m}{dn} = \frac{x_{m1}}{x_1 + l_e} \frac{1}{Cz_{\delta m}} \frac{2mg}{\rho S V^2}}$$

The "elevator per g" are therefore function of m, Zp, Vc, CG (through xm_1) and manœuvre type.

They increase at forward CG, at high speed (in absolute value).

9d) Determination of manœuvre point by flight tests :

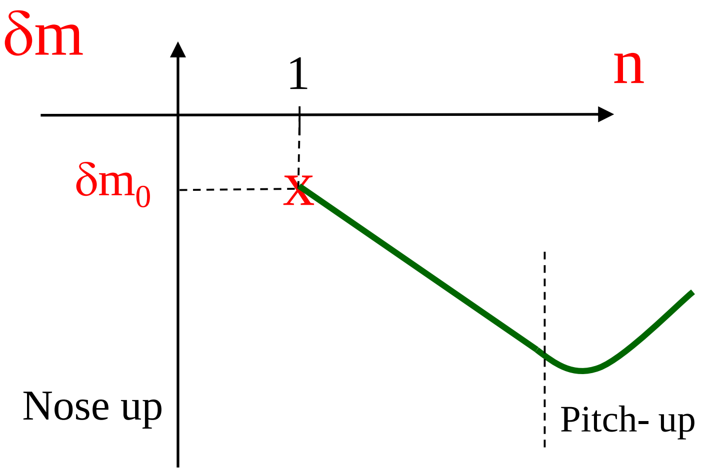
Stabilised turns are performed with increasing bank angles (so n increases) in order to be able to plot the elevator and stick forces per g . This is done at different CGs.



9e) **Pitch-up** phenomenon :

Phenomenon, which may be dangerous, observed at high incidence (*it is the case when the load factor n is high*), above all for the wing with sweep.

Caused by a flow separation close to the wing tip which creates a pitch up, reducing the elevator and forces per g.



To fight against pitch-up :

* **Aerodynamic means** : twist of the wing, cambered slats, fence (delays the stall and/or the flow separation).

•**Artificial stability** : **α trim**. Automatic trimming in the nose down direction from a given incidence, function of Mach.

•**Fly-by-wire** : dedicated function can be tuned against pitch-up

•10°) Longitudinal manoeuvrability

10a) General points :

Study based on the forces and elevators per g. It is therefore performed at the same time as the one of the stability in manœuvre.

OBJECTIVE : get forces per g practically constant in the whole flight envelope (about 15 daN/g ,except for A/C fitted with sidestick), and this, despite the large scattering of the elevators per g (dδm/dn).

$$\underbrace{\frac{dE\delta m}{\delta n}}_{\text{Forces per g}} = \underbrace{\frac{dE\delta m}{dD\delta m}}_{\text{Stiffness of the control}} \underbrace{\frac{dD\delta m}{d\delta m}}_{\text{Gearing of the pitch control}} \underbrace{\frac{d\delta m}{\delta n}}_{\text{Elevators per g}}$$

We can make **variable** the **gearing** of the control (*non-linearities in mechanical links*) and/or **the spring stiffness** of the control (*connecting rod with variable length wrt to aircraft speed …*)

10b) On an airplane fitted with servo-actuators :

We have here an artificial feels system. Therefore, we will rather use the stiffness of the control (inside the "springs box").

Thus, the stiffness will depend on Vc, Mach and trim (*to take into account the CG position : more nose-up trim when the CG moves forwards*)

10c) On an airplane fitted with direct controls :

The adjustment and tuning of the forces per g is then more fussy.

We play upon the compensation of the control surfaces to modulate the hinge moments and so the stick forces (*because there is no actuators*).

1°) General points :

We will study the dynamic response of the aircraft if it is moved aside from its equilibrium position.

Equations of the longitudinal motion:

$$\begin{cases} m\dot{V} = -mg\sin\gamma - 1/2\,\rho S V^2 C_x + F & \text{Drag equation} \quad (1) \\ -mV\dot{\gamma} = mg\cos\gamma - 1/2\,\rho S V^2 C_z & \text{Lift equation} \quad (2) \\ I_y\dot{q} = 1/2\,\rho S\, l\, V^2 C_m & \text{Pitch moment equation} \quad (3) \end{cases}$$

We neglect F in the equation (2) and we suppose F=cst.

We get **4 unknown variables :** **V, α, γ, q.**

We need a 4 fourth equation :

$\dot{\alpha} = q - \dot{\gamma}$ (4) that comes from $\theta = \alpha + \gamma$ *if $\beta=\phi=0$*

With the assumption of **small perturbations**, we may linearize these equations : we get a system of **differential linear equations**.

The solutions are :

$$\begin{cases} \alpha = \sum_{i=1}^{4} \alpha_i e^{\lambda_i t} \\ q = \sum_{i=1}^{4} q_i e^{\lambda_i t} \end{cases}$$

where λ_i : roots of the characteristic equation.

The matrix form is often used by defining the state vector X and the input vector U:

$$\mathbf{X} = \begin{bmatrix} \mathbf{V} \\ \boldsymbol{\alpha} \\ \boldsymbol{\gamma} \\ \mathbf{q} \end{bmatrix} \qquad \mathbf{U} = \begin{bmatrix} \boldsymbol{\delta}\mathbf{m} \\ \mathbf{F} \end{bmatrix}$$

The system comes down to : $\boxed{\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU}}$

STATE SPACE FORM

The resolution of the system of differential equations allows to distinguish 2 oscillatory motions :

- a **fast mode : the short period pitching oscillation**
- a **slow mode : the phugoïd**.

2°) The short period pitching oscillation or rapid incidence adjustment :

2a) Definition :

Rapid oscillatory motion, very **damped**, excited by an **incidence perturbation** around the equilibrium position.

2b) Mechanism :

If $\Delta\alpha > 0$ $\Rightarrow$ *pitch down moment (through $Cm\alpha$: stable A/C)*

$\Rightarrow$ *generates a nose-down pitch rate q*

$\Rightarrow$ *damped by the moment Cmq , (nose-up here).*

So we get an **oscillatory motion in α and q.**

2c) Approximate computation :

Assumes : *V cst, slope γ is negligibly small, Stick fixed, motions with small amplitude,*

$$nmg = mg(1+V/g\ (q - d\alpha/dt)) = 1/2\ \rho S V^2 (Cz\alpha(\alpha-\alpha_0) + Cz\delta m\ \delta m) \qquad (2)$$

$$I_y \dot{q} = 1/2\ \rho S\ l\ V^2 (Cm_0 + Cm\alpha(\alpha-\alpha_0) + Cm\delta m\ \delta m + Cmq\ ql/V) \qquad (3)$$

Deriving (2) and eliminating q in the above relationships, we get a 2nd order equation in α :

$$2I_y/(\rho S l V^2)\ \ddot{\alpha} - (Cmq\ l/V - I_y/(mlV)\ Cz\alpha)\ \dot{\alpha} - (Cm\alpha + Cmq\ Cz\alpha\ \rho S l/2m)\ \alpha = \mathbf{cst}$$

The term in $\dot{\alpha}$ is positive. Therefore it is an equation of an oscillatory convergent motion of which we may compute the damping and the period (about 1 to 2 s.)

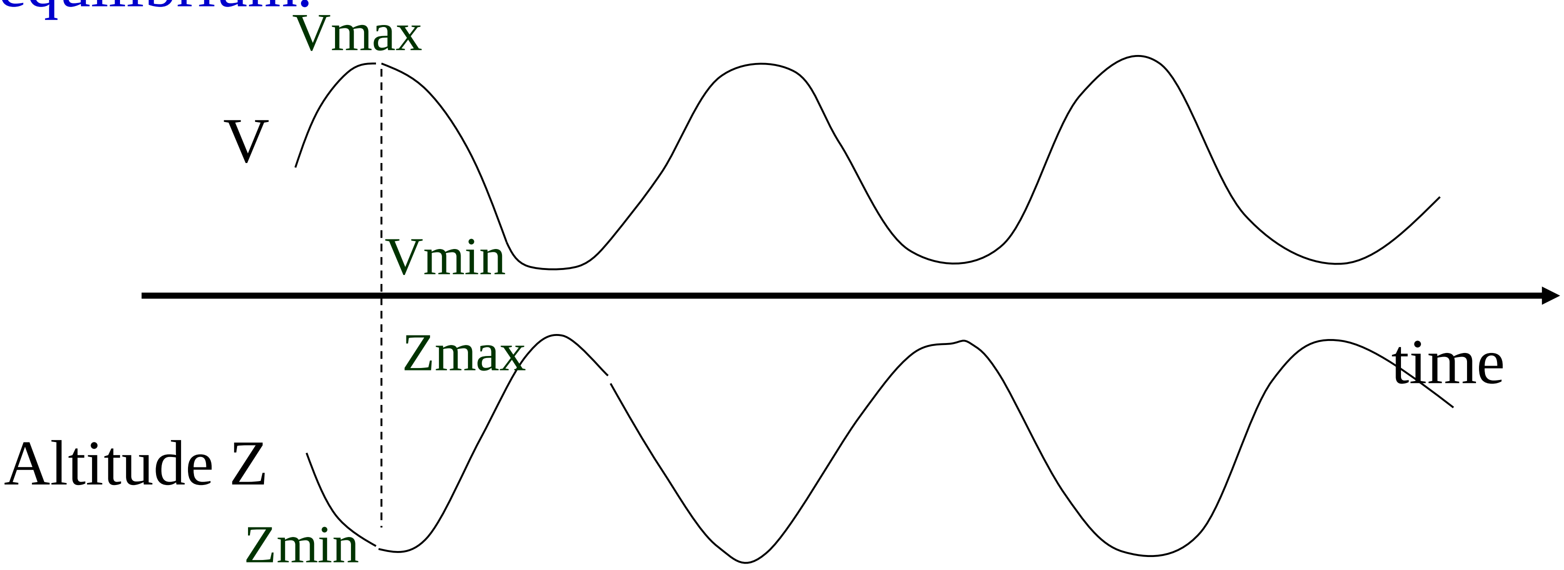
2d) Flight tests method :

Perturbation in incidence, generated for instance by a square input solicitation with the elevator.

<u>3°) Phugoïd :</u>

3a) Definition :

Slow oscillatory motion**,** very **weakly damped (or even undamped),** stimulated by a **speed perturbation** around the trim equilibrium.



3b) Mechanism :

It is basically an **exchange** between **potential energy and kinetic energy** : the **speed variations** will generate lift variations which explain the trajectory oscillations.

3c) Approximate computation :

We assume α constant, drag and thrust constant (which is not fully right : V varies…).

The drag equation becomes in this case : $m dV/dt = -mg \sin \gamma$ (1)

because $-1/2 \rho S V^2 Cx + F = cst = -1/2 \rho S V^2 Cx_0 + F_0 = 0$

Now $dZ/dt = V \sin \gamma$ allows to write (1) : $mV\, dV/dt + mg\, dZ/dt = 0$

We find **$1/2 mV^2 + mg Z = cst$** : i.e. **Total energy =constant**

So it is an **exchange** between **potential energy and kinetic energy .**

Lift equation :- $mV\dot{\gamma} = mg\cos\gamma - 1/2\,\rho S V^2 C_z$ (2)

Assuming the thrust component on the Z axis negligible

But if γ small, $V\dot{\gamma} = \ddot{Z}$ and (2) may be written : $-m\ddot{Z} = -1/2\,\rho S V^2 C_z + mg$

Moreover, we may write that at initial time (V_0, Cz_0) : $mg = 1/2\,\rho S V_0^2 Cz_0$

And then $Cz = Cz_0$ because α=cst.

Thus $\ddot{Z} - g(V^2/V_0^2 - 1) = 0$

i.e. $\mathbf{\ddot{Z} + 2g^2/V_0^2(Z - Z_0) = 0}$

It is the equation of a **maintained sinusoidal motion** with a period

$$T = \frac{\Pi\sqrt{2}V_0}{g}$$

Actual case: *total energy non constant, α non constant*

4°) Aircraft Pilot Coupling (APC) :

The natural motion of the aircraft may be modified by the pilot intervention who, by his reaction delay, generates a certain **phase lag.**

The oscillatory motion may become divergent : **Pilot Induced Oscillations** or **Aircraft Pilot coupling**.

The risks of APC can be decreased with a **pitch damper** : *system creating an automatic elevator deflection with a phased-advance, measuring the pitch rate q (feedback for stability augmentation).*

1°) General points :

Reminder : *the lateral motion uses 3 equations projected onto aircraft axes.*

1a) Lateral force equation.

$$m\,(d\vec{V}/dt \, . \, \vec{j}) = mg \sin\phi \cos\theta + Ry$$

where $Ry = 1/2\ \rho S V^2 Cy$

other expression : $n_y\, mg = -1/2\ \rho S V^2 Cy$

$$\mathbf{Cy = Cy\beta \, . \, \beta + Cy\delta n \, . \, \delta n}$$

1b) Rolling moment equation

Ix dp/dt – Ixz dr/dt = 1/2 $\rho S l V^2$ Cl (if L_F =0)

where **Cl= Clβ . β + Clδl . δl + Clδn . δn + Clp.pl/V + Clr.rl/V**

1c) Yawing moment equation

Iz dr/dt – Ixz dp/dt = 1/2 $\rho S l V^2$ Cn (if N_F =0)

where **Cn= Cnβ . β + Cnδl . δl + Cnδn . δn + Cnp.pl/V + Cnr.rl/V**

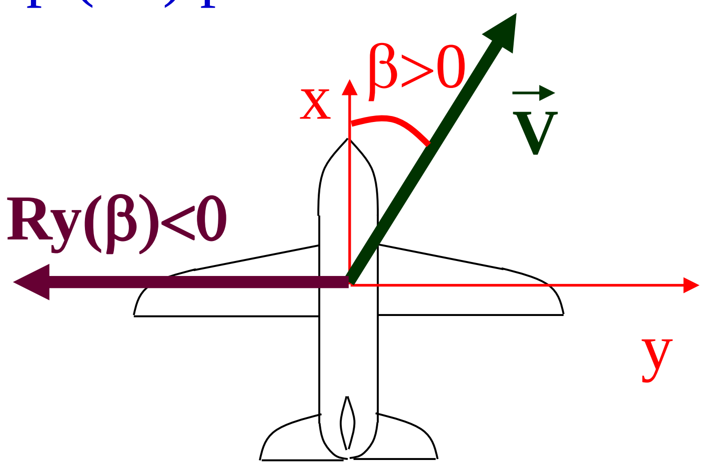
2°) Lateral aerodynamic force (or sideforce):

$$Cy = Cy\beta \cdot \beta + Cy\delta n \cdot \delta n$$

2a) Sideslip Influence $Cy\beta$:

Proportional to all vertical surfaces of the airplane (fin, fuselage, wing dihedral).

A right sideslip (>0) provides a lateral force towards the left (<0).

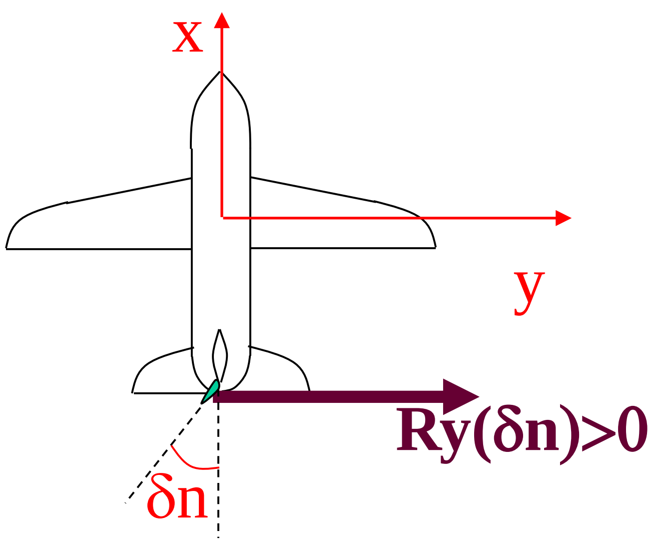


Whence **$Cy\beta<0$**

2b) **Rudder influence Cyδn:**

Influence less important than the sideslip one.

The rudder on left (δn>0) provides a lateral force towards the right (>0)



Whence **Cyδn>0**

3°) Roll aerodynamic moment $L = 1/2\, \rho S l V^2 Cl$:

$Cl = Cl\beta \,.\, \beta + Cl\delta l \,.\, \delta l + Cl\delta n \,.\, \delta n + Clp.pl/V + Clr.rl/V$

3a) **Dihedral effect $Cl\beta$:**

Rolling moment due to **sideslip**.
It is a combination of 4 influences :

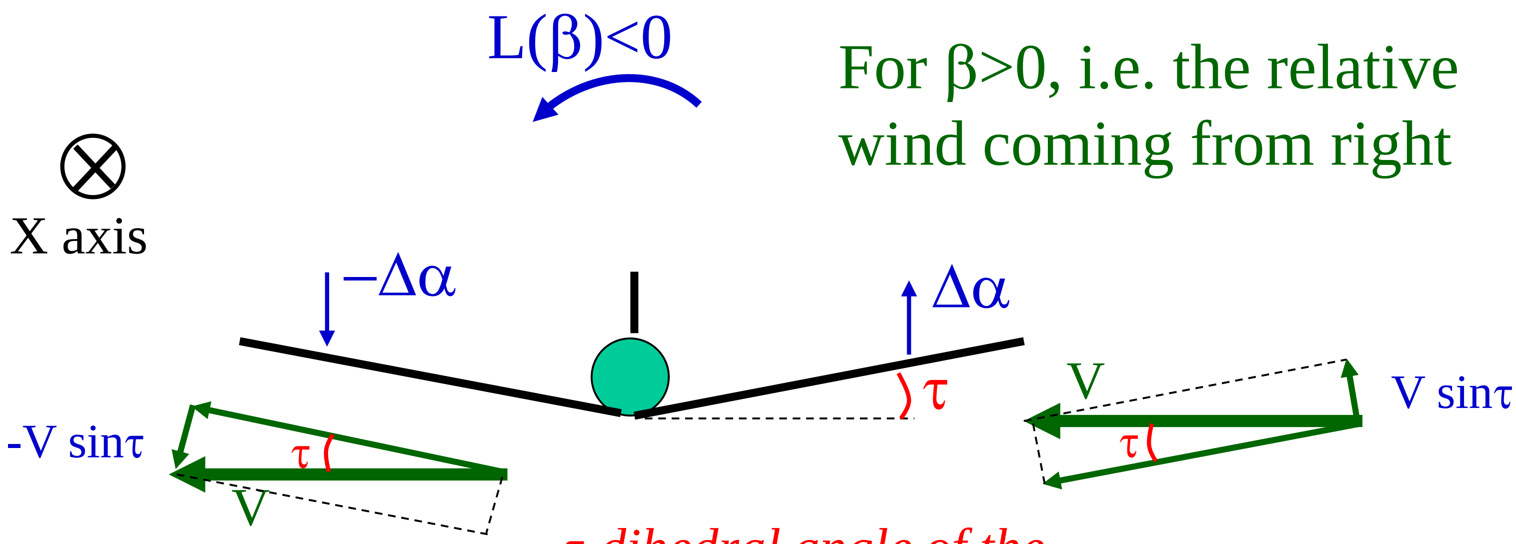
a1) **Influence of wing dihedral :**

For a positive dihedral angle (upwards), a right sideslip (>0) generates a vertical component of the airspeed perpendicular to the wing, upwards for the right wing.

It corresponds to an incidence increase, thus a lift increase on this wing. This provides a roll moment towards the left.

Thus Clβ (dihedral>0) **<0**

For β>0, i.e. the relative wind coming from right



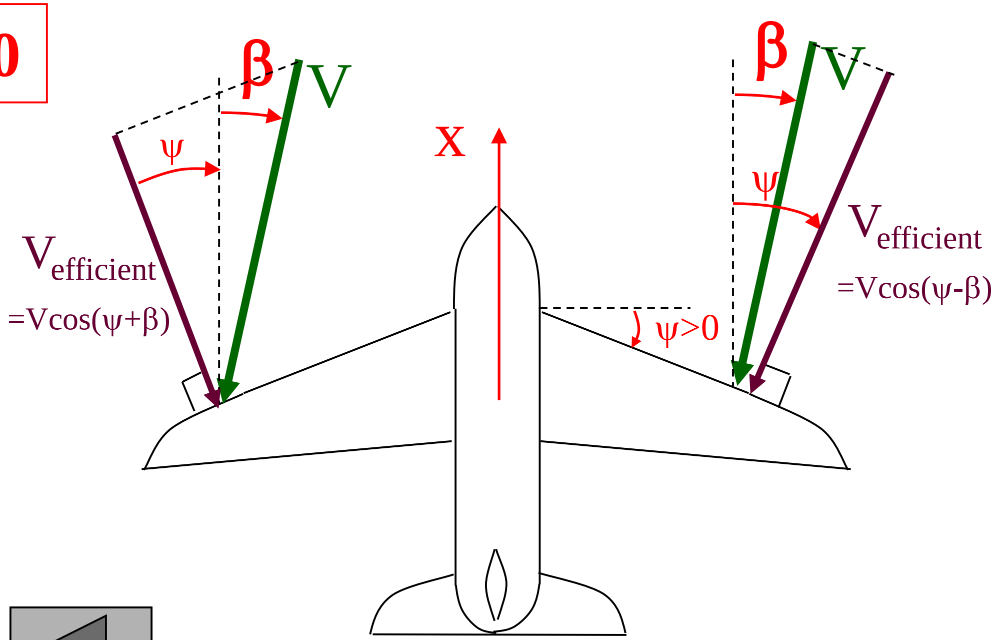
τ dihedral angle of the wing

a2) **Influence of the sweep of the wing :**

The "efficient speed" is the component of the airspeed perpendicular to the wing leading edge.

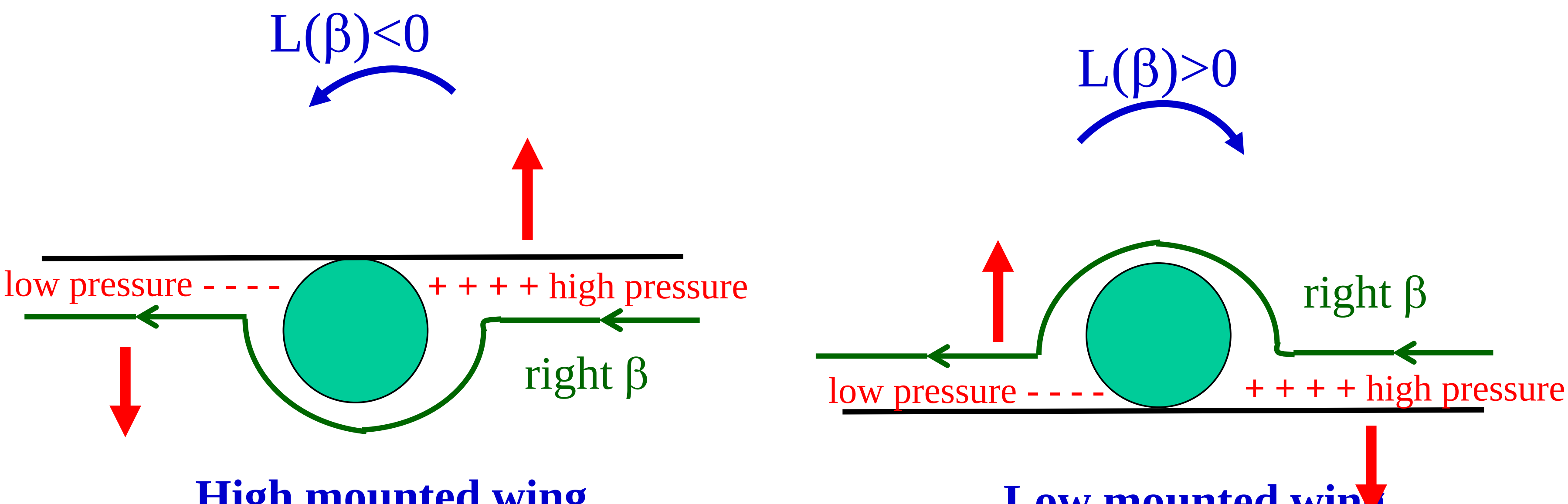
For a right sideslip (>0), the efficient speed of the right wing is greater than the left wing one. This wing side lifts more : it generates a left roll moment (<0).

Thus $Cl\beta_{(sweep>0=sweepback)}$ <0



a3) **Influence of the position of the wing:**

A right sideslip (>0) generates a high pressure region on the lower surface of the right wing if it is a high mounted wing : this side lifts more. This generates a left roll moment (<0).



High mounted wing **Low mounted wing**

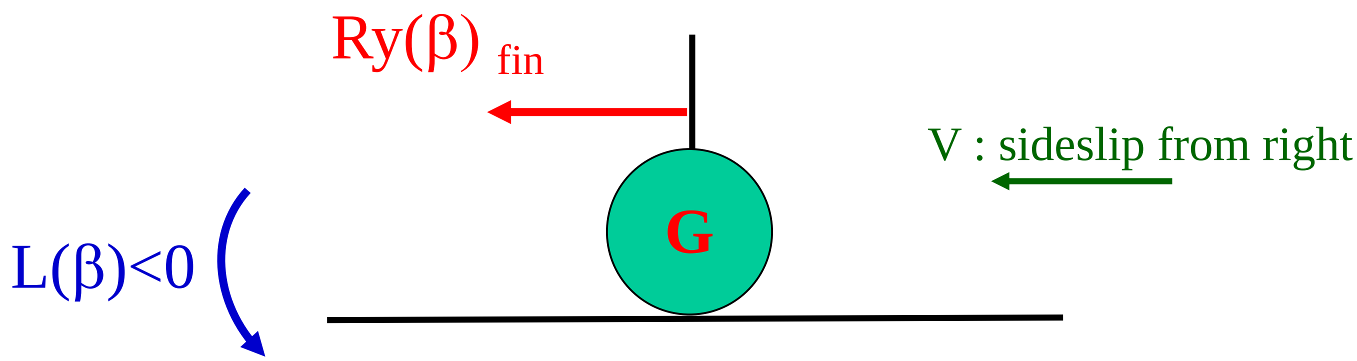
Thus Clβ (high wing) **<0**

Clβ (low wing) **>0**

a4) **Influence of the fin :**

Due to the fact that the centre of pressure of the fin is above the roll axis Gx.

A right sideslip (>0) generates a lateral force on the fin towards the left : this creates a left roll moment (<0).



Therefore Clβ $_{(fin)}$ **<0**

a5) Conclusion for the dihedral Clβ:

The dihedral effect Clβ is the sum of these 4 influences.

For stability reasons, we want a Clβ <0 (see next chapter) but not too much.

It implies a repercussion on the design of the airplane.

It is sometimes necessary to get a negative dihedral (or anhedral on the high mounted wings (and/or with sweepback).

We have to notice that the horizontal stabilizer also participates to the dihedral effect (dihedral, sweep, high or low position)

3b) **Influence of the roll control surfaces $Cl\delta l$:**

The roll control is obtained thanks to differential deflection of :

- the **ailerons** (for a conventional airplane)

- the outer **elevons** (for an aircraft with a delta wing)

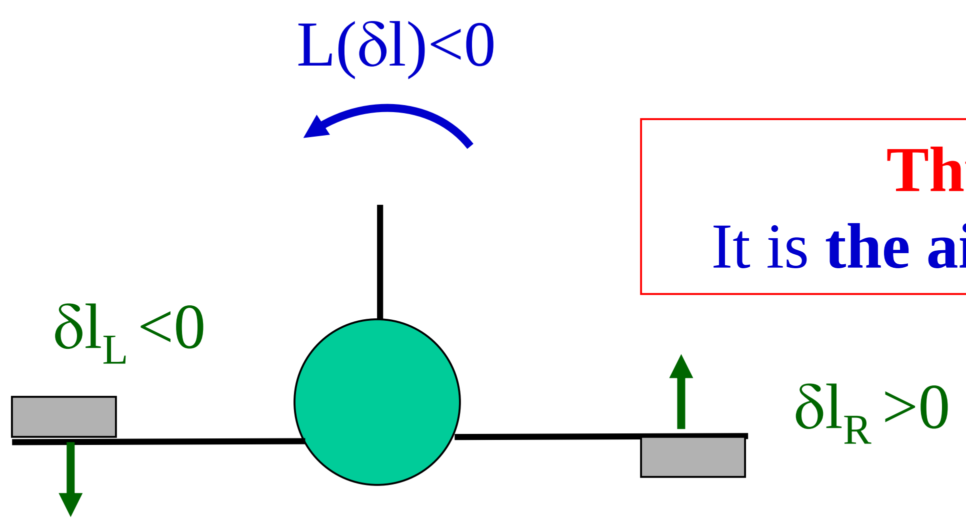
- the **elevators** (for supersonic aircraft, to increase ailerons efficiency)

- the **canards** (on the Rafale for instance)

Spoilers are often used **:** they are upwards deflected on **one wing side** (the side where it is intended to turn).

By definition: $\delta l = (\delta l_R - \delta l_L)/2$

If $\boldsymbol{\delta l > 0}$*, the right aileron is lowered (*$\delta l_R > 0$*) and the left aileron is upwards deflected (*$\delta l_L < 0$*).*



Thus Clδl <0.

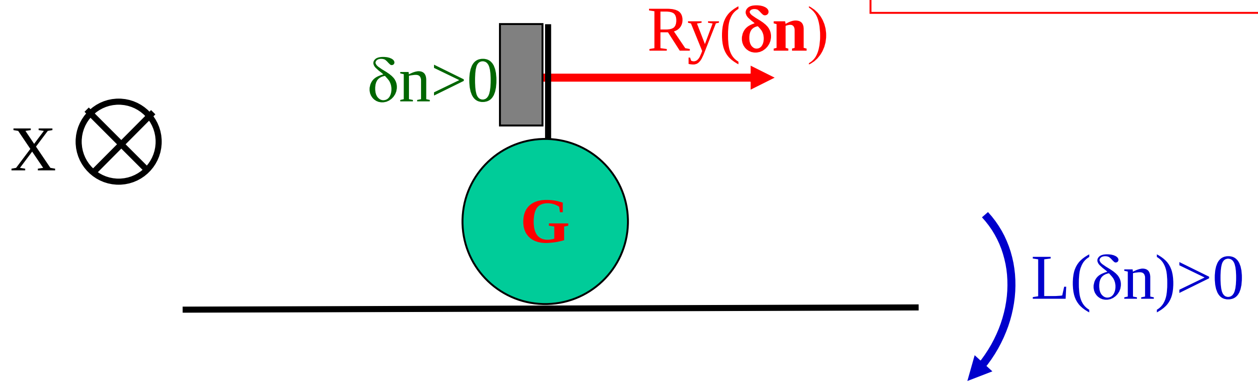
It is **the ailerons efficiency.**

3c) **Adverse roll (generated by the rudder) Clδn :**

Small effect.

*A rudder deflection towards the **left** (δn>0) causes a small **right** roll moment (>0)*

Thus Clδn >0.



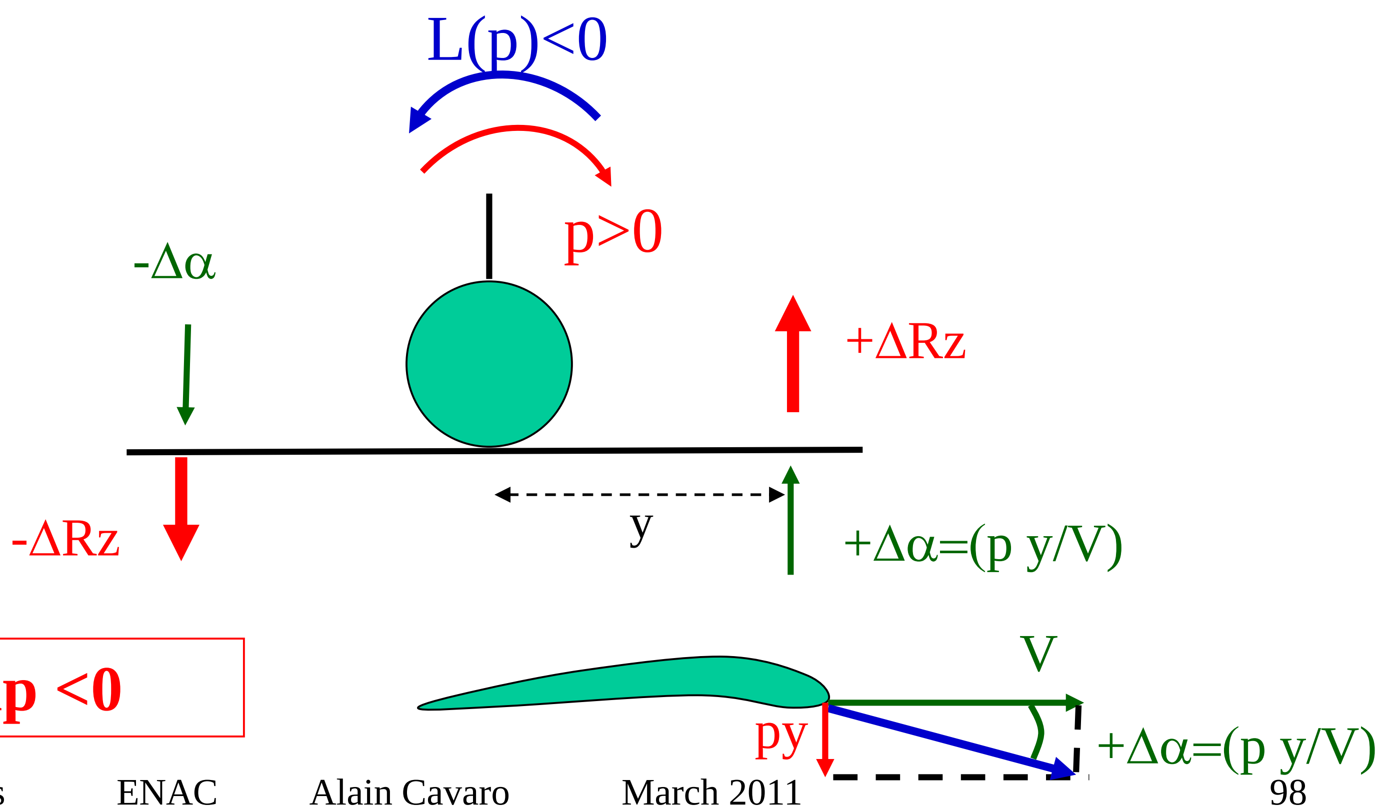
Important note :

*This adverse roll is the initial roll effect. But we must not forget that the primary effect of the rudder is to cause **sideslip** (on the right here) and therefore it will tend to create a roll moment on the « good » side (on the left here) thanks to the dihedral effect (Clβ).*

3d) **Rolling moment due to roll rate (roll damping) Clp :**

A roll rate towards the right (p>0) causes an angle of attack increase on right wing (and a decrease on left wing) : this generates a left roll moment (<0) that tends therefore to damp the roll rate.

So it is a **damping term**.



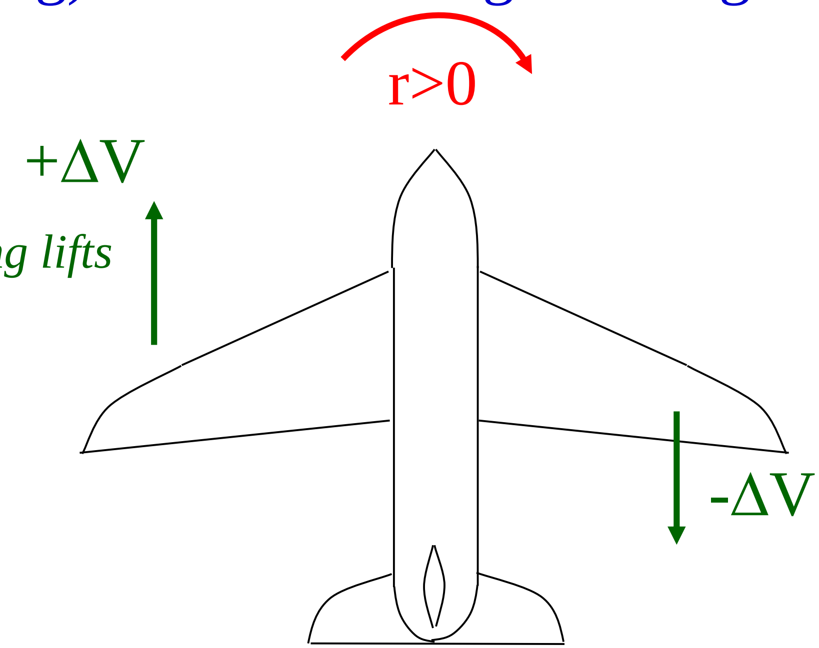
Thus Clp <0

3e) **Rolling moment due to yaw rate Clr :**

A yaw rate towards the right (r>0) provides a speed increase on the left wing ("moving forward") and a decrease on the right wing.

So we get a lift increase on the left wing (and a decrease on the right wing) that causes a right rolling moment towards (>0).

So the left wing lifts more



and the right wing lifts less

causing a right roll here

The fin contributes too.

Thus Clr >0

4°) Yaw aerodynamic moment N= $1/2\ \rho S l V^2 C_n$:

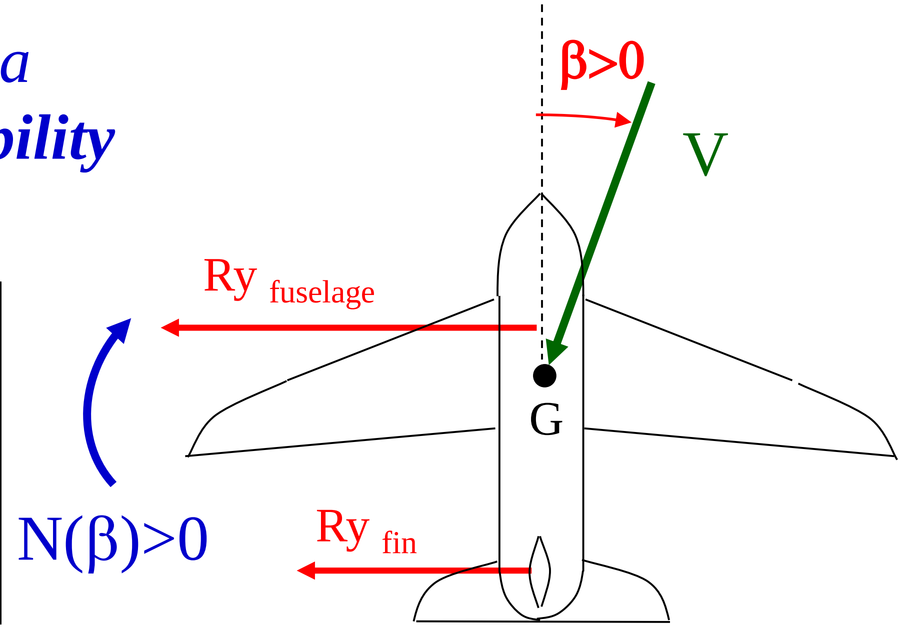
$C_n = C_{n\beta} \cdot \beta + C_{n\delta l} \cdot \delta l + C_{n\delta n} \cdot \delta n + C_{np} \cdot pl/V + C_{nr} \cdot rl/V$

4a) **Directional** stability **$C_{n\beta}$:**

It is the influence of the **sideslip** on the **fuselage (destabilizing**) and on the **fin** (**stabilizing effect** : it is the essential purpose of the fin). It is also called **the weathercock stability**.

*A right sideslip (>0) must cause a right yawing moment>0) for **stability** reason (to decrease it)*: $C_{n\beta}>0$

We have besides a stabilizing effect of the wing sweepback (if right sideslip, efficient speed on right wing is greater, so drag higher on right wing, and so yawing towards right).



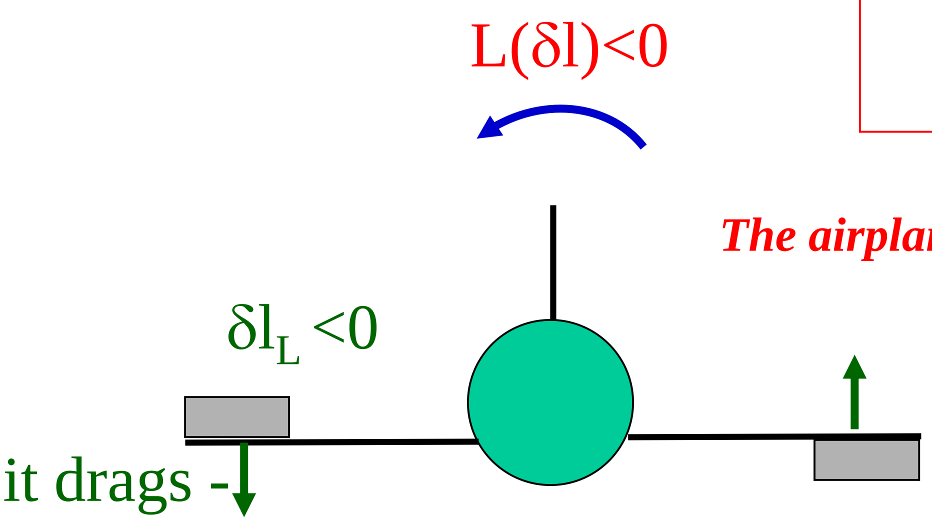
4b) **Adverse yaw** **Cnδl :**

To turn to the left (δl>0), the right wing, at the aileron level, gets a higher lift than the left one.

The induced drag on the right wing is then slightly greater than on the left one.

Whence a right yawing moment (>0) (« adverse » to the left turn)

Thus Cnδl >0.
Very small



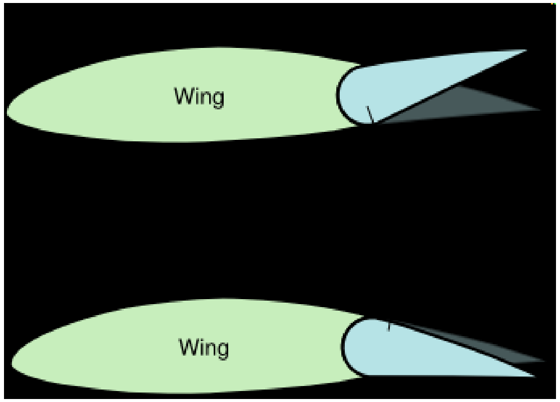
The airplane nose slightly turns towards the right

Minimizing the effect of Adverse yaw :

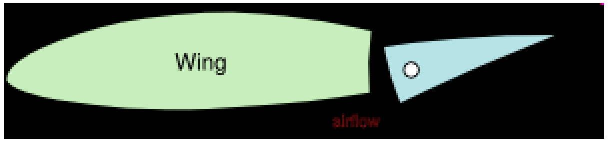
*Adverse yaw is countered by using the aircraft **rudder** to perform a **coordinated turn** (no sideslip), however an aircraft designer can reduce the amount of correction required by careful design of the aileron.*

Three methods are common:

- **differential ailerons** (smaller deflection for downwards aileron (less induced drag) than the upwards one)



- **frise ailerons** (when up aileron is applied, some of the aileron leading edge will protrude downward into the airflow, causing increased drag on this (down-going) wing. This will counter the drag produced by the other aileron, thus reducing adverse yaw.

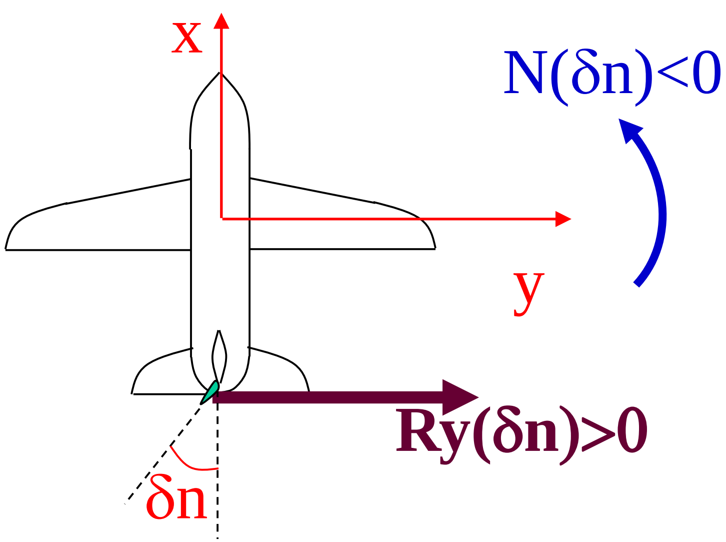


- **roll spoilers** (they are deflected only on the down-going wing to decrease the lift: they increase the drag too on this wing).

4c) **Rudder efficiency Cnδn :**

A rudder deflection towards the left (δn>0) causes a left yawing moment (<0).

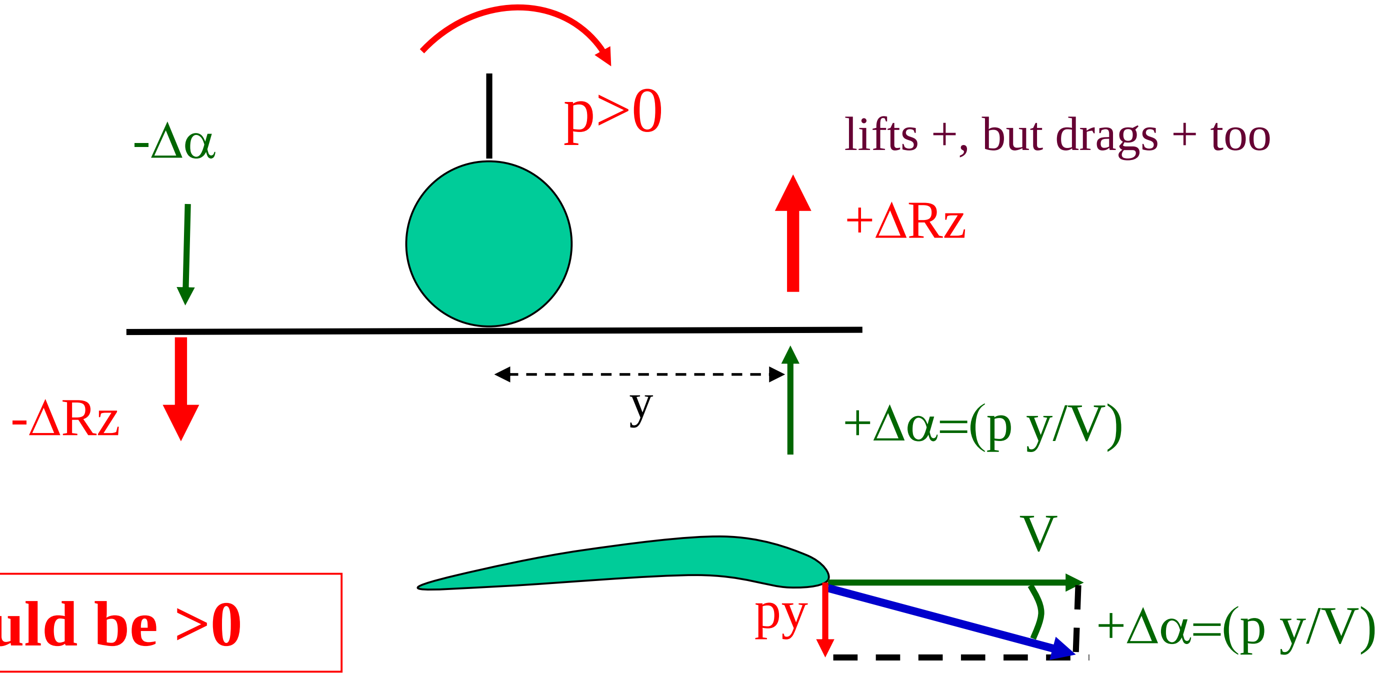
*This coefficient is the **essential purpose** of the rudder.*



Therefore Cnδn <0.

4d) Yawing moment induced by the roll rate p Cnp :

For a right roll rate (p>0), we get an incidence increase on the right wing (and a decrease on the left one). The induced drag is higher on the right wing. It generates a right yawing moment (>0).



Thus Cnp should be >0

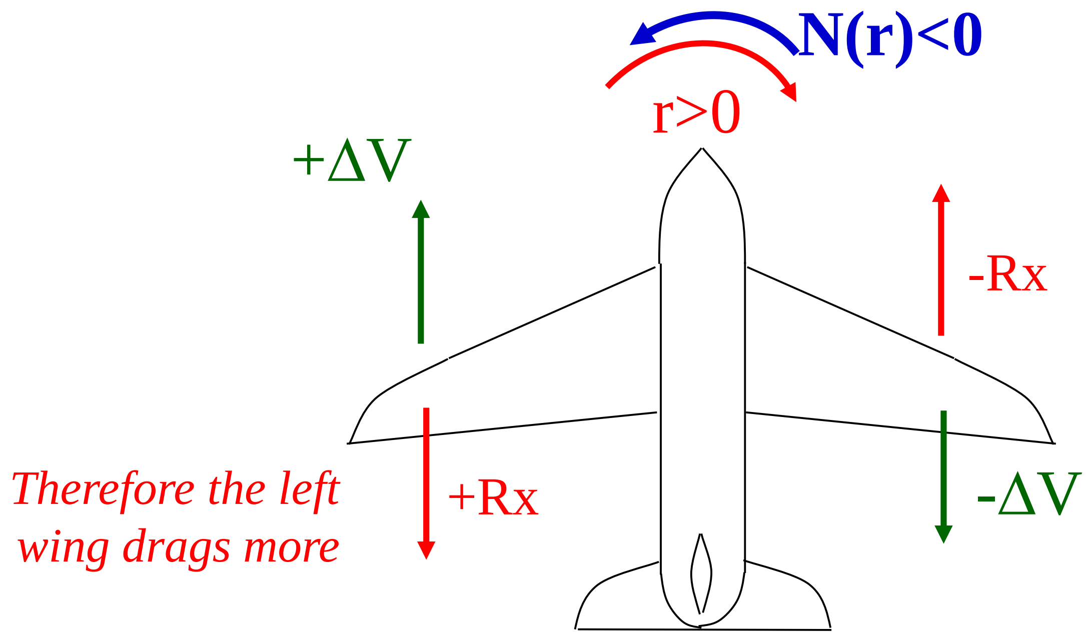
The aircraft nose slightly turns to the right

In practice, this coefficient is small with a variable sign. Another mechanism can be preponderant : As the right wing descends, its **lift vector**, which is perpendicular to the relative motion, **tilts forward** and therefore has a forward component.

4e) Yaw damping Cnr :

A yaw rate towards the right (r>0) causes an increase of airspeed on the left wing (moving forward) and a decrease on the right wing.

Therefore we get a drag increase on the left wing (moving forward) and a decrease on the right wing : this produces a left yawing moment (<0). This is a **damping term**.



Thus Cnr <0

The fin participates too (local sideslip counteracting the motion)

5°) SUMMARY TABLE for lateral aerodynamic coefficients

	Sideslip **β** > 0 on R	Ailerons **δl** > 0 on L	Rudder **δn** > 0 on L	Roll rate **p** >0 on R	Yaw rate **r** >0 on R
Sideforce Cy>0 on R	Cyβ < 0		Cyδn> 0		
ROLLING moment Cl>0 on R	Clβ < 0 Dihedral effect	Clδl<0 Ailerons efficiency	Clδn>0 Adverse roll	Clp<0 Roll damping	Clr>0 Induced roll
YAWING moment Cn>0 on R	Cnβ >0 Directional stability	Cnδl >0 Adverse yaw	Cnδn <0 Rudder efficiency	Cnp small Induced yaw	Cnr <0 Yaw damping

6°) Study of the stabilised turn in level flight.

6a) General points :

We look here the stabilised turn and not the turn initiation (roll rate).

Stabilised turn : V, α, β, ϕ, p,q,r are constant and p(=roll rate) =0.

Note 1: compared to straight flight, to maintain the **airspeed constant**, we need to increase the **thrust**.

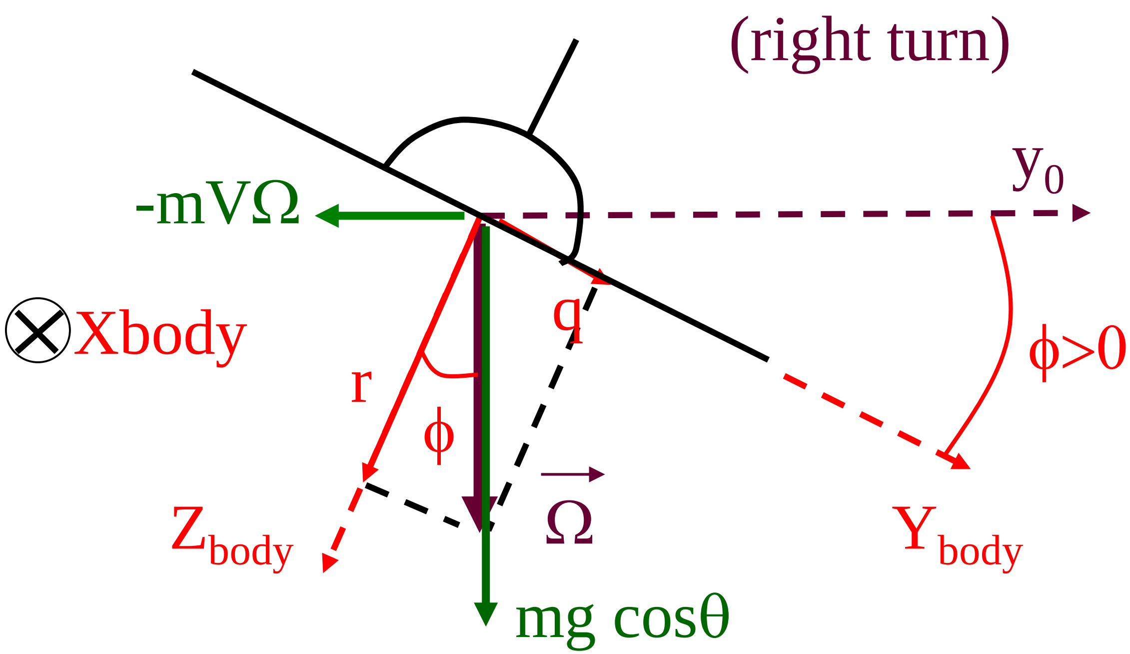
Indeed, we need to **increase Cz to counteract apparent weight nmg** (*in other words, the lift has to increase in order that its vertical component balances the weight*.)

And this lift increase results in an **incidence increase and therefore a drag increase** (whence a thrust increase to maintain V).

Note 2: the stick has to be **pulled** (despite the unchanged speed : to counteract the stabilising effect of $\Delta\alpha$ and q (see § about « elevator per g »

6b) Simplified equations for the stabilised turn :

$$\vec{\Omega} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 0 \\ \Omega \sin\phi \\ \Omega \cos\phi \end{bmatrix} = \overrightarrow{cste}$$



The 3 equations of the lateral motion can be simplified, in particular when using :

the assumptions of § 6.1 combined with L_F and N_F=0, which implies that **Cl=0 and Cn=0** (because dp/dt and dr/dt=0)

neglecting Cyδn and the cross coefficients **Clδn, Cnδl**

$$\left\{\begin{array}{ll} ny\ mg = mg \sin\phi \cos\theta - mV\,\Omega \cos\phi = -1/2\,\rho S V^2 Cy\beta .\,\beta & \text{(1) Force Y} \\ Cl = Cl\beta.\beta + Cl\delta l.\,\delta l + Clr.\,rl/V = 0 & \text{(2) roll axis X} \\ Cn = Cn\beta.\beta + Cn\delta n.\,\delta n + Cnr.\,rl/V = 0 & \text{(3) yaw axis Z} \end{array}\right.$$

with $r = \Omega \cos\phi$

In turn, the yaw rate r produces :

a **rolling moment Clr** tending to increase the bank angle ϕ

a **yawing moment Cnr** tending to bring back the aircraft nose towards the outside of the turn (yaw damping)

*To maintain a stabilised turn, these moments have to be counteracted : it will be used the **ailerons**, the **rudder** and/or the **sideslip** in this aim.*

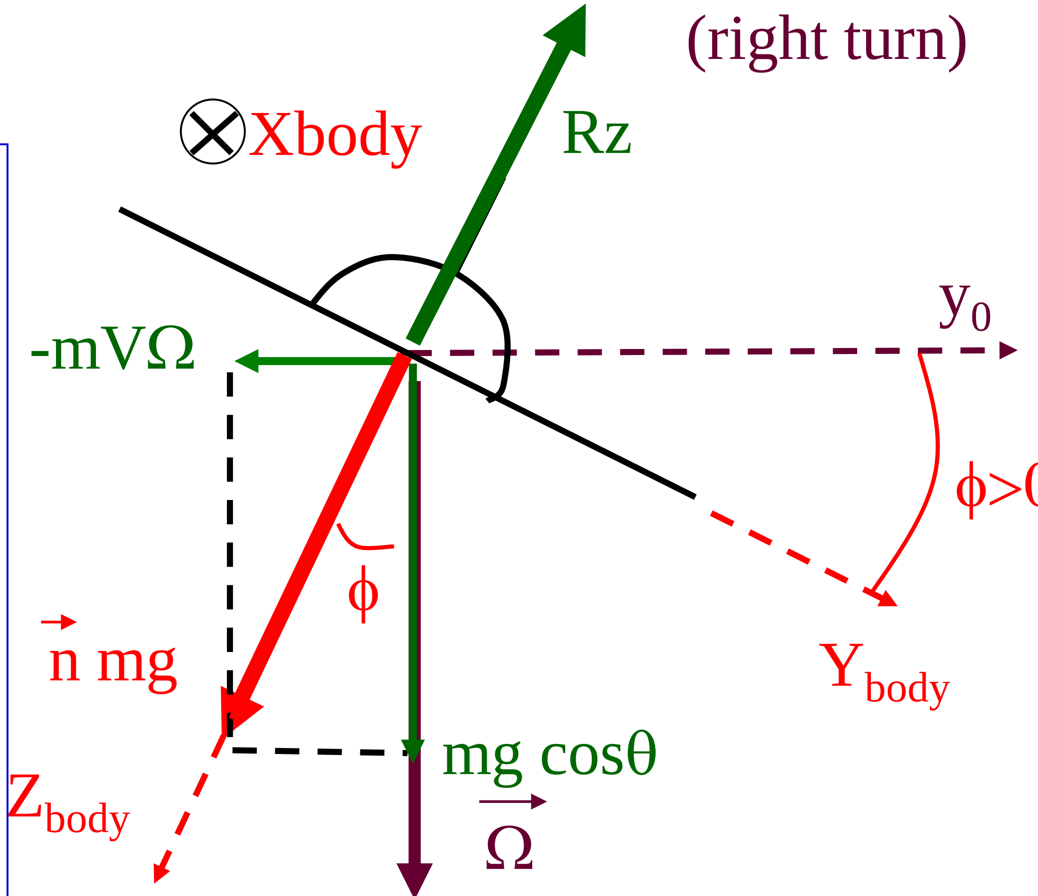
6c) Coordinated turn:

(right turn)

Coordinated or "correct" turn means that the apparent weight is in the aircraft symmetry plane Gxz.

n_y=0 results in a null sideforce Ry.

It **is therefore a turn with 0 sideslip** (if we neglect Ry(δn)



Reminder : in this case nz= cosθ/cosϕ

We can resolve the equations of the previous § with **β=0**.

We then find the deflections δl and δn necessary to balance the turn.

In the case of the stabilised coordinated turn, the **controls are crossed** :

the stick is deflected towards the outside of the turn and the rudder pedals towards the inner.

In turn, the yaw rate r produces :

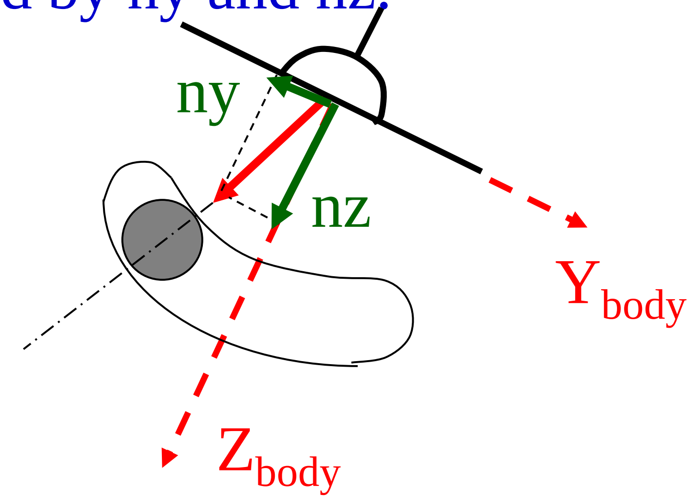
• a **roll moment Clr** tending to **increase the bank angle ϕ** : it has to be counteracted by deflecting the **stick towards the outside** of the turn *(for flight without sideslip), which may seem paradoxical.*

• a **yaw moment Cnr** tending to bringing back **the nose of the aircraft** towards the outside of the turn (yaw damping) : it has to be counteracted by pushing the **inner (to the turn**) **rudder pedal** to maintain the yaw rate constant *(for flight without sideslip)*

For a coordinated turn, **the slip indicator (the ball) will be centered.**

Indeed, the ball will be placed according to the sum of « mass forces » represented by ny and nz.

Turn and Bank Indicator, also called a Turn and Slip



And in coordinated turn, **ny=0**

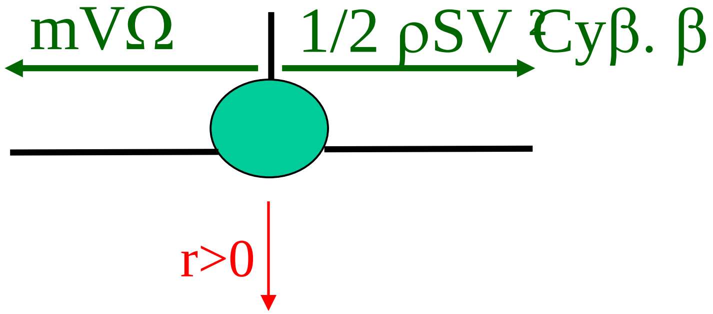
The ball is a sideslip indicator only if we can neglect lateral forces not generated by sideslip, in particular $Ry(\delta n)$

6d) Wings level turn

$\phi=0$. The lateral force equation indicates that, in this case, Ω is function of β : $mV\Omega = 1/2\ \rho S V^2 C_{y\beta}.\ \beta$

We may also translate this by the fact that the normal (horizontal) acceleration is no longer mainly due to the horizontal projection of the lift (which is the case of the classic turn), but only due to the lateral force generated by the sideslip :

We cannot turn very quickly with this method.



The sideslip has to come from the outside of the turn (skid) : the **controls (roll/yaw) will be strongly crossed** because the roll and yaw induced by r and β have the same direction.

It is a turn nearly never used.

6e) Turn with stick (wheel) only

Therefore, we assume that δn=0.

The balance of the yaw equation can be realised only when flying with **sideslip coming from the inner side of the turn** that allows to counteract, by the weathercock effect Cn(β), the yaw damping Cn(r).

This sideslip (from inner) causes, due to the dihedral effect Clβ, a rolling moment which is in the other direction than the rolling moment induced by the yaw rate r.

We can also find the ailerons deflection δl necessary to stabilise this turn, using pour the equations (2) et (3) with δn=0.

$$\delta l = \frac{1}{Cl\delta l.Cn\beta}\frac{\Omega l}{V}\cos\phi\left[Cl\beta.Cnr - Clr.Cn\beta\right]$$

The aircraft will be « **spirally stable** » if, to **maintain the stabilized turn**, without using rudder pedals, the **stick is deflected towards the turn** direction. If we release the stick, the aircraft will tend to come back wings level.

In the opposite case, it will be called « **spirally unstable** ».

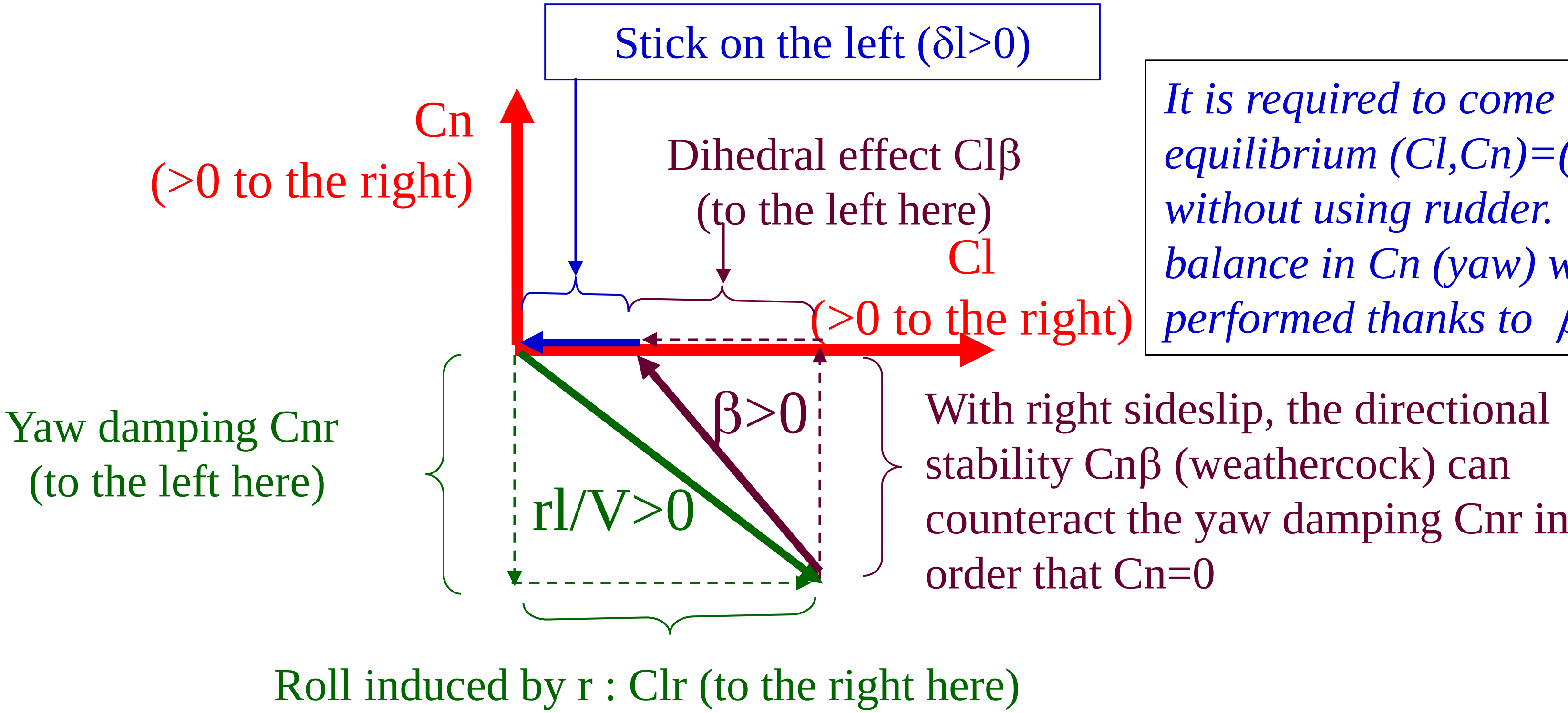
So, the spiral stability depends on the sign of δl and so of the sign of [Clβ.Cnr - Cnβ.Clr].

The spiral stability depends on the sign of the determinant : $\begin{vmatrix} Cl\beta & Clr \\ Cn\beta & Cnr \end{vmatrix}$

2 cases have to be foreseen, depending on the relative vectors (Clβ, Cnβ) and (Clr, Cnr).

We assume a **turn towards the right** (yaw rate r>0).

First case : $\begin{vmatrix} Cl\beta & Clr \\ Cn\beta & Cnr \end{vmatrix} < 0$

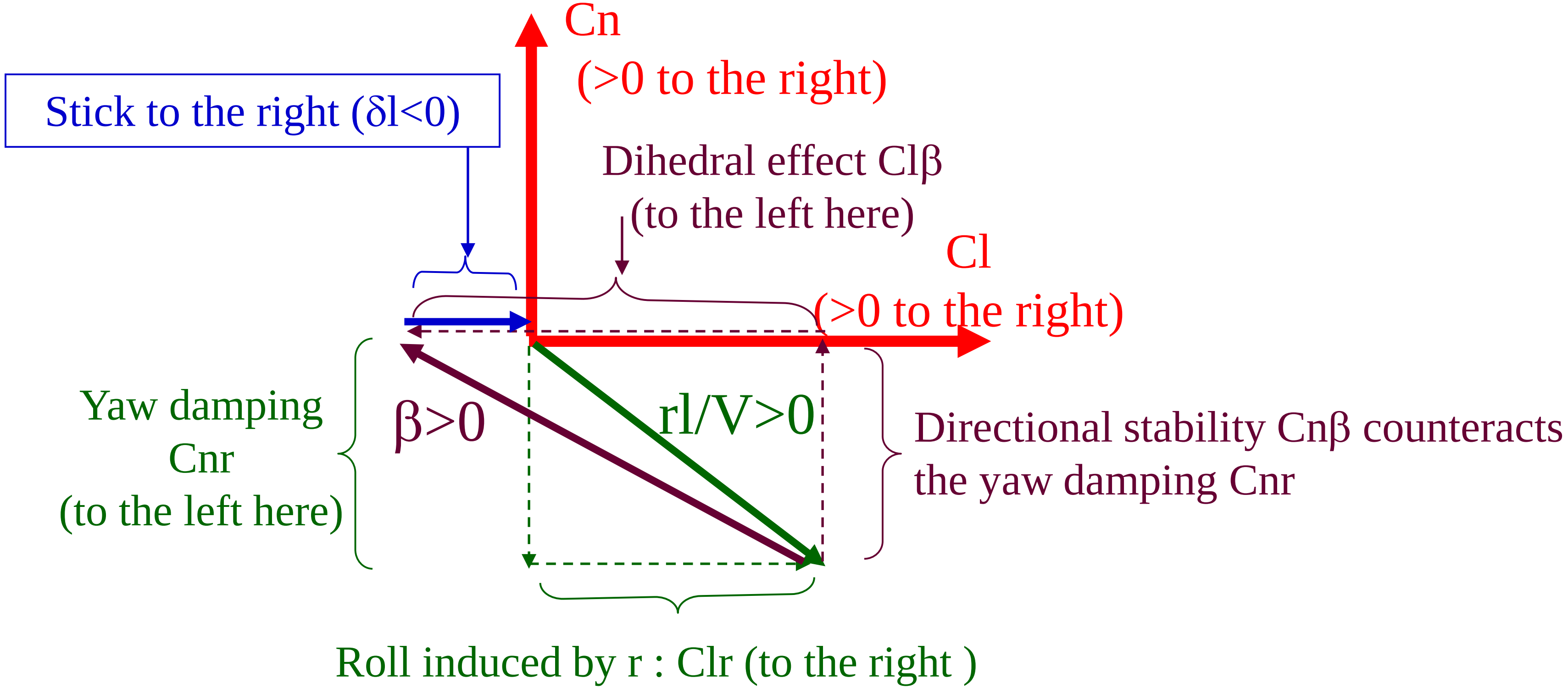


It is required to come back to equilibrium (Cl,Cn)=(0,0) without using rudder. The balance in Cn (yaw) will be performed thanks to β.

In this case, it is necessary to deflect the ***stick towards the outside*** *of the turn : the airplane is* ***spirally unstable*** *(if the stick is released, the aircraft will be entered in a more pronounced turn).*

Second case :

$$\begin{vmatrix} Cl\beta & Clr \\ Cn\beta & Cnr \end{vmatrix} > 0$$



In this case, it is necessary to deflect the ***stick towards the inside*** *of the turn : the airplane is* ***spirally stable*** *(if the stick is released, the aircraft will tend to leave the turn).*

Note : *Increasing the dihedral effect $Cl\beta$ (in | |) enhances this spiral stability.*

There is also the case of the **neutral spiral stability** when the 2 vectors ($Cl\beta$, $Cn\beta$) and (Clr, Cnr) are collinear.

It is the case of some aircraft fitted with electrical flight controls systems where flight control laws allow to get a neutral spiral stability when $|\phi|$ is not too high.

6f) Turn with rudder pedal only

As for the previous paragraph, the same kind of study for spiral stability may be undertaken, but here the **ailerons are at 0** ($\delta l=0$).

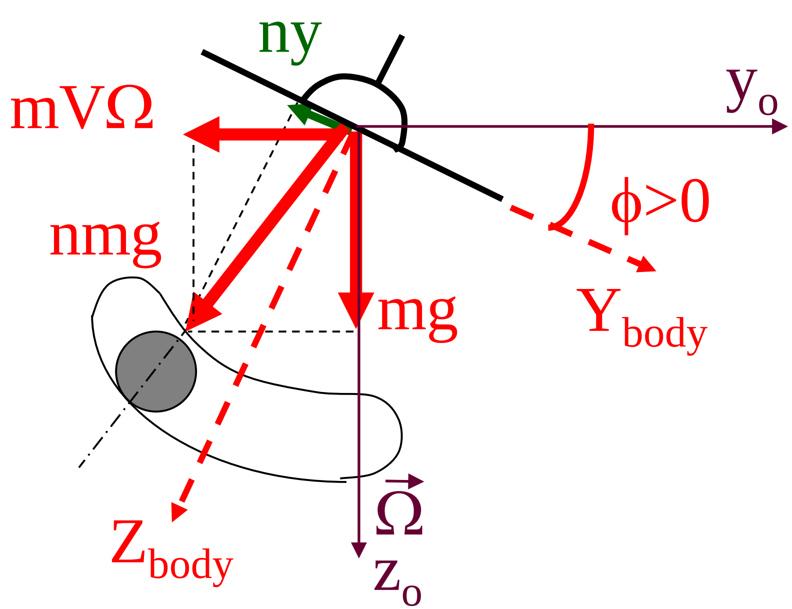
It is a turn seldom used because the roll initiation is not very efficient with rudder pedal only (*it generates sideslip, which then causes roll through dihedral effect*).

6g) Asymmetric turn

The sum of mass forces are not in the symmetry plane Gxz : $\beta \neq 0$

SKIDDING TURN

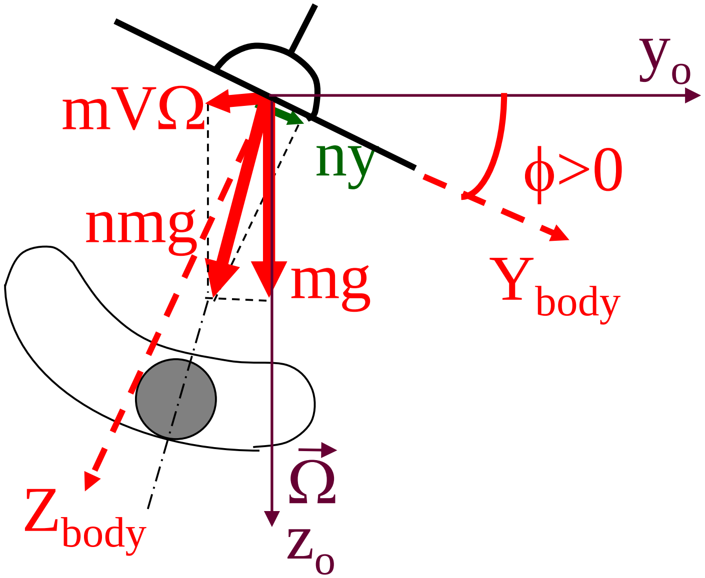
The sideslip is towards the outside of the turn ($\beta<0$ ici)



To center the ball, the sideslip has to be nullified either by increasing the bank (ϕ) (more on the right here) or by changing rudder pedal position (towards left here)

« The stick attracts the ball, the foot pushes it away »

SLIPPING TURN



The sideslip is towards the inside of the turn ($\beta>0$ here)

To center the ball, the sideslip has to be nullified either by decreasing the bank (ϕ) (on the left here) or by changing rudder pedal position (towards the right here)

Again, « *The stick attracts the ball, the foot pushes it away* »

7°) Study of the engine failure.

7a) General points :

The study of this failure is done in the case of a multi-motor (!).

The resulting **propulsion asymmetry** causes a **yawing moment** which needs to be balanced.

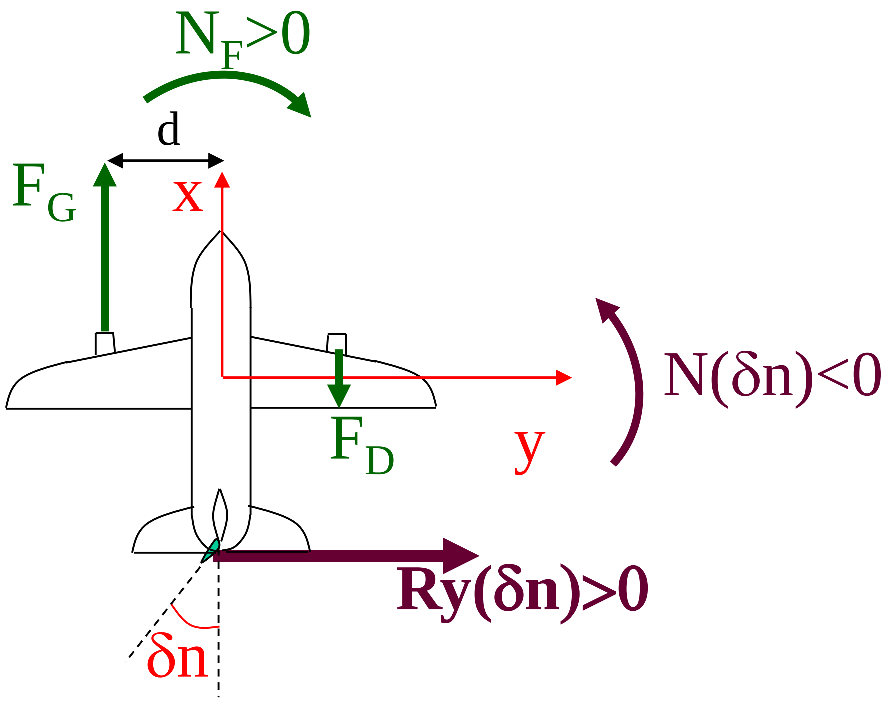
We will study the case of a **right engine failure** on a twin-engines.

$$N_F = (F_G - F_D) . d$$

where d is the distance between the thrust axis and the CG.

F_D <0 on our drawing.

Note : the HQ criticality of the engine failure is reduced for aircraft with engines mounted on the rear fuselage.



By analogy with the aerodynamic coefficients, we may define a yaw coefficient due to the thrust asymmetry.

$Cn_F = (F_G - F_D).\ d / (1/2\ \rho\ SlV^2)$ (>0 if right engine failed)

It also exists a rolling moment coefficient $\mathbf{Cl_F}$, above all for the aircraft with propellers (mainly due to the disappearance of the propeller blow generating a loss of lift on right wing here : i.e. $Cl_F>0$). For jets, it is often neglected.

When pushing the left rudder pedal, we have **balanced the airplane in yaw**.

But we need to **balance also the lateral forces** because the force $Ry(\delta n)$ generated by the **rudder deflection** is not neglected here.

There are then 3 solutions to balance this sideforce :

- *fly with sideslip (to the right here) without banking the airplane (in order that Ry(β) counteracts Ry(δn))*

- *fly with bank angle (to the left here) with null sideslip in order that the lateral weight counteracts Ry(δn)*

- *a combination of the 2 previous solutions (flight with sideslip and bank angle).*

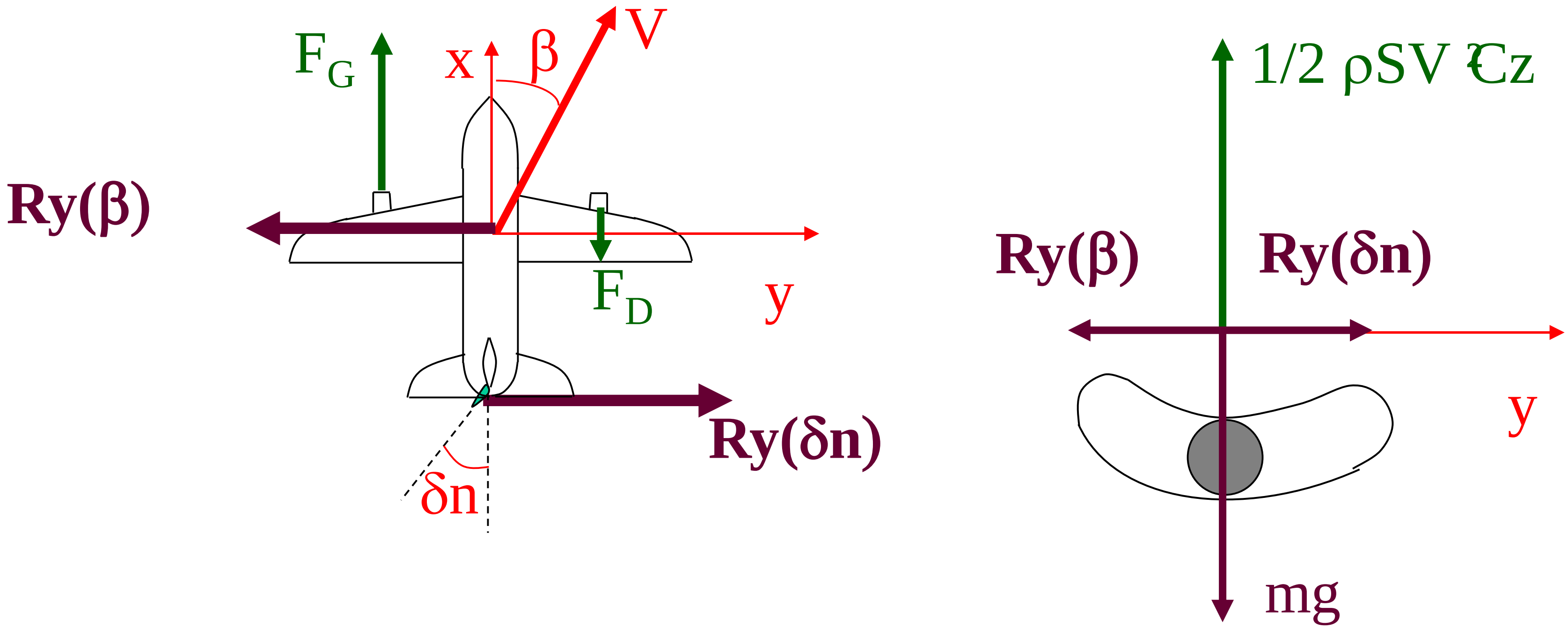
The equilibrium equations **in straight flight** (**p=r=0**) are written in en neglecting the cross coefficients Clδn, Cnδl, but no longer Cyδn:

$$\begin{cases} ny\, mg = mg \sin\phi \cos\theta = -1/2\, \rho S V^2 (Cy\beta .\, \beta + \mathbf{Cy\delta n .\, \delta n}) & \text{(1) Force Y} \\ Cl + Cl_F = Cl\beta.\beta + Cl\delta l .\, \delta l + \mathbf{Cl_F} = 0 & \text{(2) Roll axis X} \\ Cn + Cn_F = Cn\beta.\beta + Cn\delta n .\, \delta n + \mathbf{Cn_F} = 0 & \text{(3) Yaw axis Z} \end{cases}$$

7b) **Flight wings level** (horizontal wings) :

ϕ=0

In this case, as the weight cannot help to counteract the sideforce Ry(δn), it is the **sideslip** which is in charge of it.



We note that in this cas (ϕ=0), **the sideslip is on the side of the failed engine**.

We may also notice that, despite the sideslip, the ball is centered (it is not a slip indicator when Ry(δn) cannot be neglected)

This sideslip, on the failed engine side, may be embarrassing to balance the yawing moment equation: in fact, due to weathercock effect ($Cn\beta$), the aircraft nose will tend to move even more towards the failed engine side : $Cn(\beta)$ and Cn_F have the same sign. It will be necessary **to deflect more the rudder**.

The equation (1) provides a relationship between β and δn (because $\phi=0$):

$\beta = Cy\delta n / Cy\beta \, . \, \delta n$ (we find again $\beta > 0$ –i.e. to the right if $\delta n > 0$)

The equation 3 (yaw) coupled with the previous relationship allows to determine the necessary deflection δn to counteract $Cn_{F \text{ and}}$ $Cn(\beta)$.

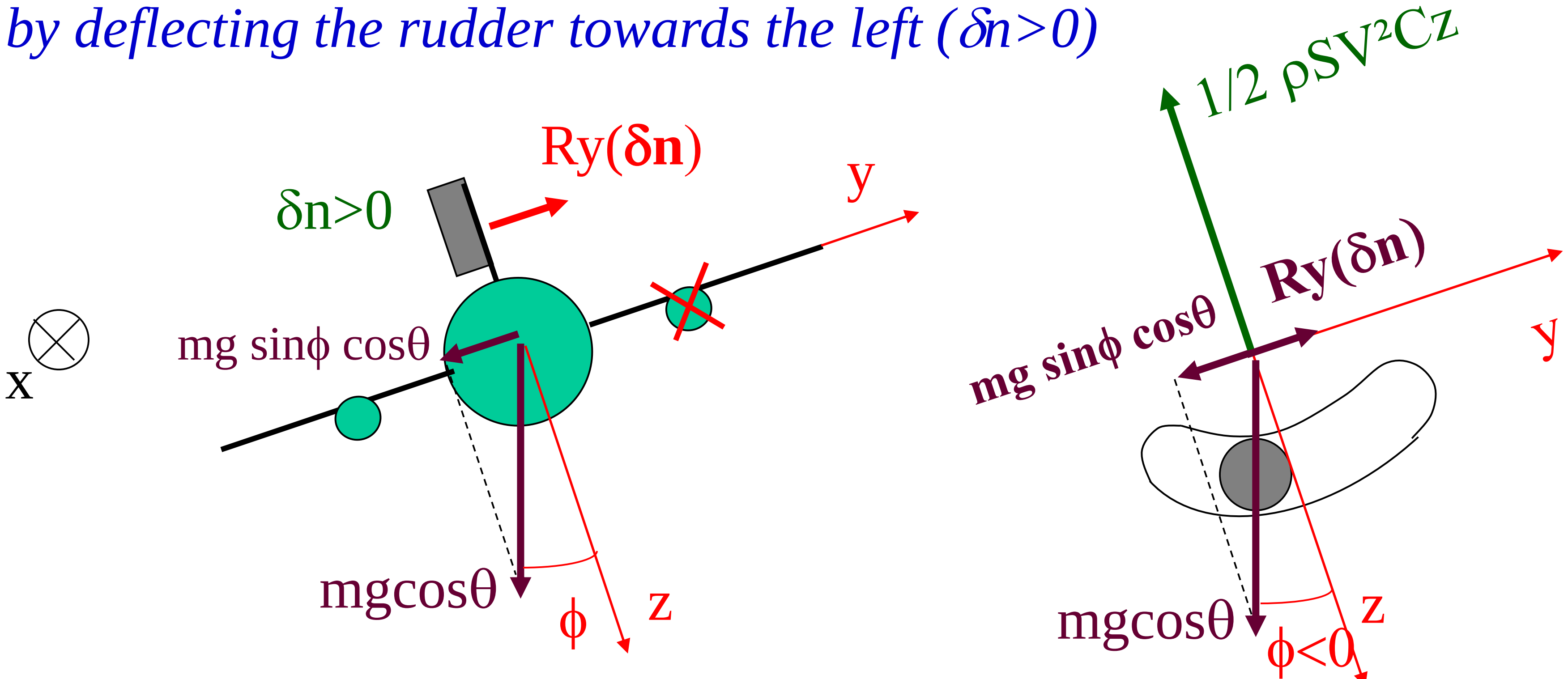
The equation 2 (roll) allows to compute the ailerons deflection δl necessary to counteract the sum of the dihedral effect $Cl(\beta)$, Cl_F and the adverse roll $Cl\delta n$ (if we do not neglect it).

7c) **Flight at zero sideslip :**

β=0

In this case, it is no longer the sideslip which counteracts the sideforce Ry(δn), but the lateral component of the **weight** which will be in charge of it, by **banking the airplane towards the sane engine side**

Reminder : *we have first to balance the thrust asymmetry Cn_F by deflecting the rudder towards the left (δn>0)*



In this case, the rudder δn only counteracts the yawing moment due to thrust asymmetry Cn_F *(see also equation (3))*.

According to (1), ny≠0 : **the ball is not centered despite the zero sideslip**. *It is caused by Ry(δn) which is not neglected here.*

We notice that the rudder deflection for the case β=0 is smaller than for the case ϕ=0 (no weathercock effect to counteract)

*Thus, **by banking on the sane engine side, we need less δn**.*

In other words , at iso-δn, we can control the thrust asymmetry at lower speed : see next § about the VMCA determination for which the regulation authorize to bank the aircraft towards the sane engine side (ϕ=5° as a maximum)

8°) Minimum Control speed (with failed engine): VMC.

8a) **VMCA (A stands for air) :**

Definition :

It is the **minimum speed** at which, in case of **engine failure**, it is possible to **keep a constant heading** with a bank angle $|\phi| \leq$ **5° ,** the other engines being at the maximum **take-off thrust**.

In order not to penalize the flight envelope and the performances of the aircraft, we try, during certification flight tests, to demonstrate the lowest possible VMCA.

Flight test method :

Test performed with a shut-down engine and with:

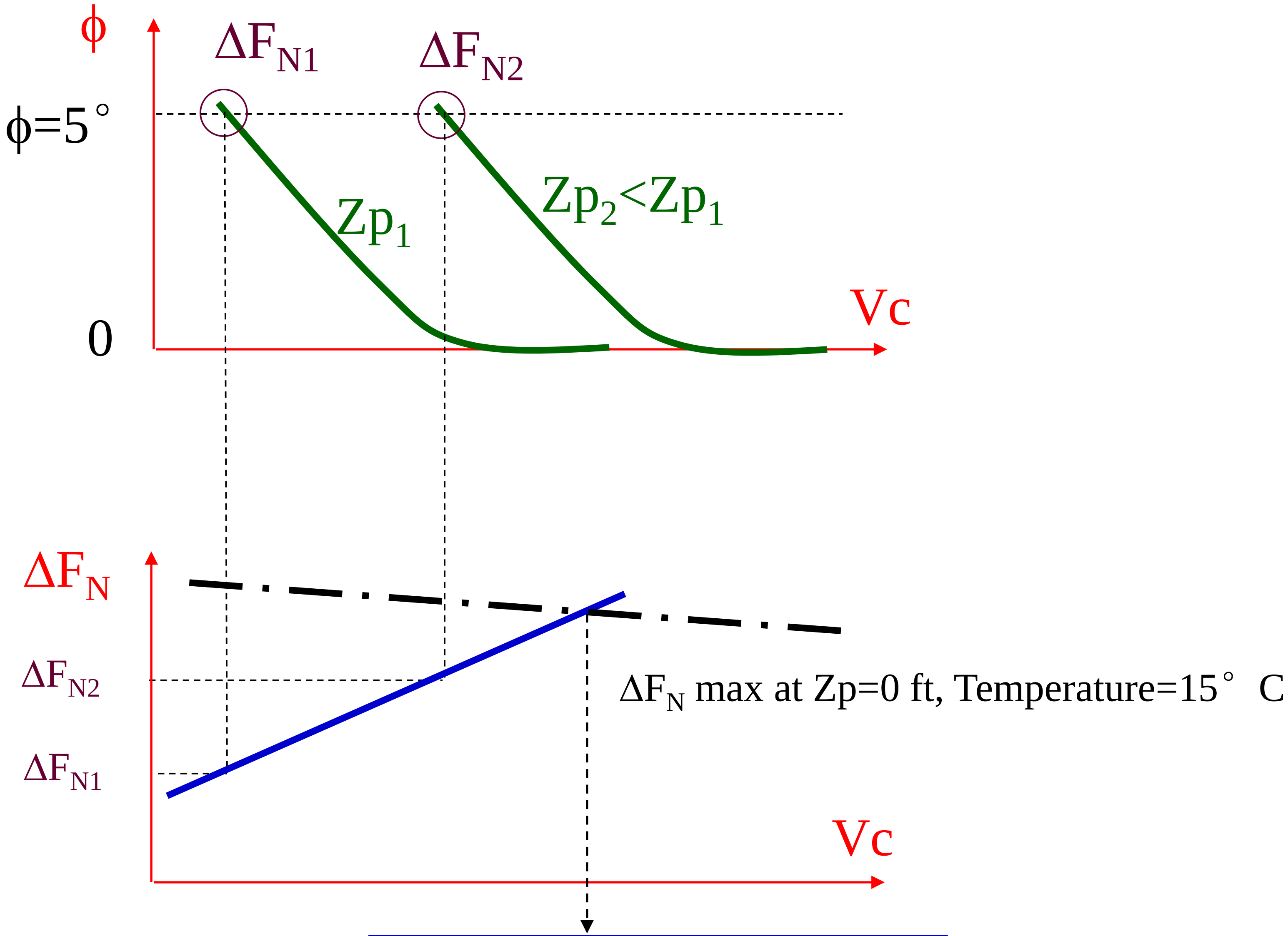
- *the max aft CG, most often (lever arm of the rudder is smaller)*
- *take-off configuration*
- *light weight (to get enough stall margin. In addition it is sizing because it allows to use smaller favorable sideslip)*

Static demonstration :

Slow deceleration while keeping constant heading. Below a certain speed, the rudder reaches its stop ; it is then necessary to keep to keep a constant heading by banking the airplane on the sane engine side up to $\phi=5°$.

This deceleration is performed at different altitudes Zp and so at different thrust asymmetries ΔF_N (if Zp increases, ΔF_N decreases).

Therefore, the VMCA is function of altitude and increases when altitude decreases (because thrust increases). We extrapolate at 0ft the results found in altitude.



Static VMCA at 0ft

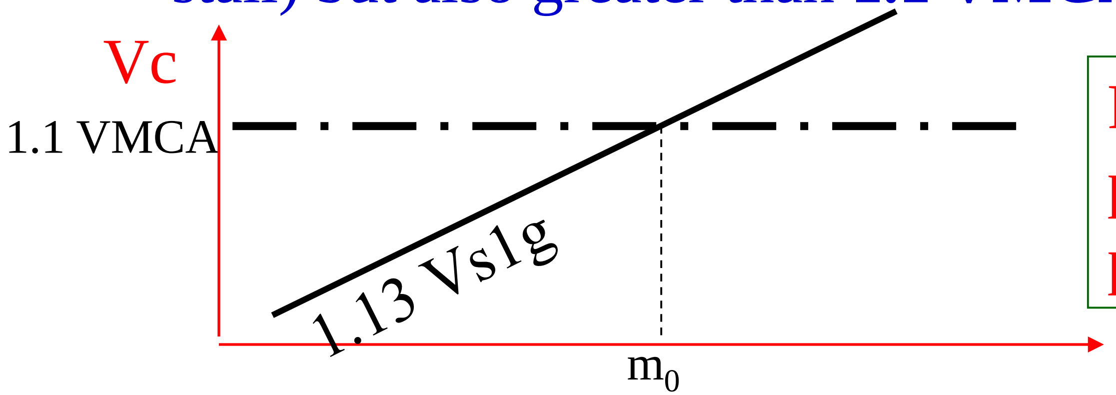
We also perform a **dynamic** **demonstration** :

the critical engine is suddenly made inoperative at the static VMCA previously demonstrated : the aircraft behavior has to be correct (heading change<20° and rudder pedal force <150 lbs).

If it is Ok, we keep VMCA= static VMCA.

Consequences for aircraft performances :

The regulation requires that minimum airspeed after take-off **V2min** is **greater than 1.13 Vs1g** (margin with respect to stall) but also greater than **1.1 VMCA**.



For mass<m0, VMCA penalizes take-off performances

8b) **VMCL (L stands for landing) :**

Definition :

It is the **minimum approach speed** at which, in case of **engine failure**, it is possible to **maintain a constant heading** with a lateral bank $|\phi| \leq \mathbf{5°}$ **,** the other engines being at the maximum take-off thrust or **go around thrust** (if different).

Flight test method :

Same kind of demonstration as for VMCA, the slat/flap configuration is here the **landing** configuration.

In addition, there is a **roll manoeuvrability** test on the sane engine side : it is more difficult to rapidly bank towards this side because the rudder is near its stop and it cannot prevent from a sideslip excursion when the bank angle is increased : the resulting dihedral effect can interfere with a quick roll rate.

8c) **VMCG (G stands for ground) :**

Definition :

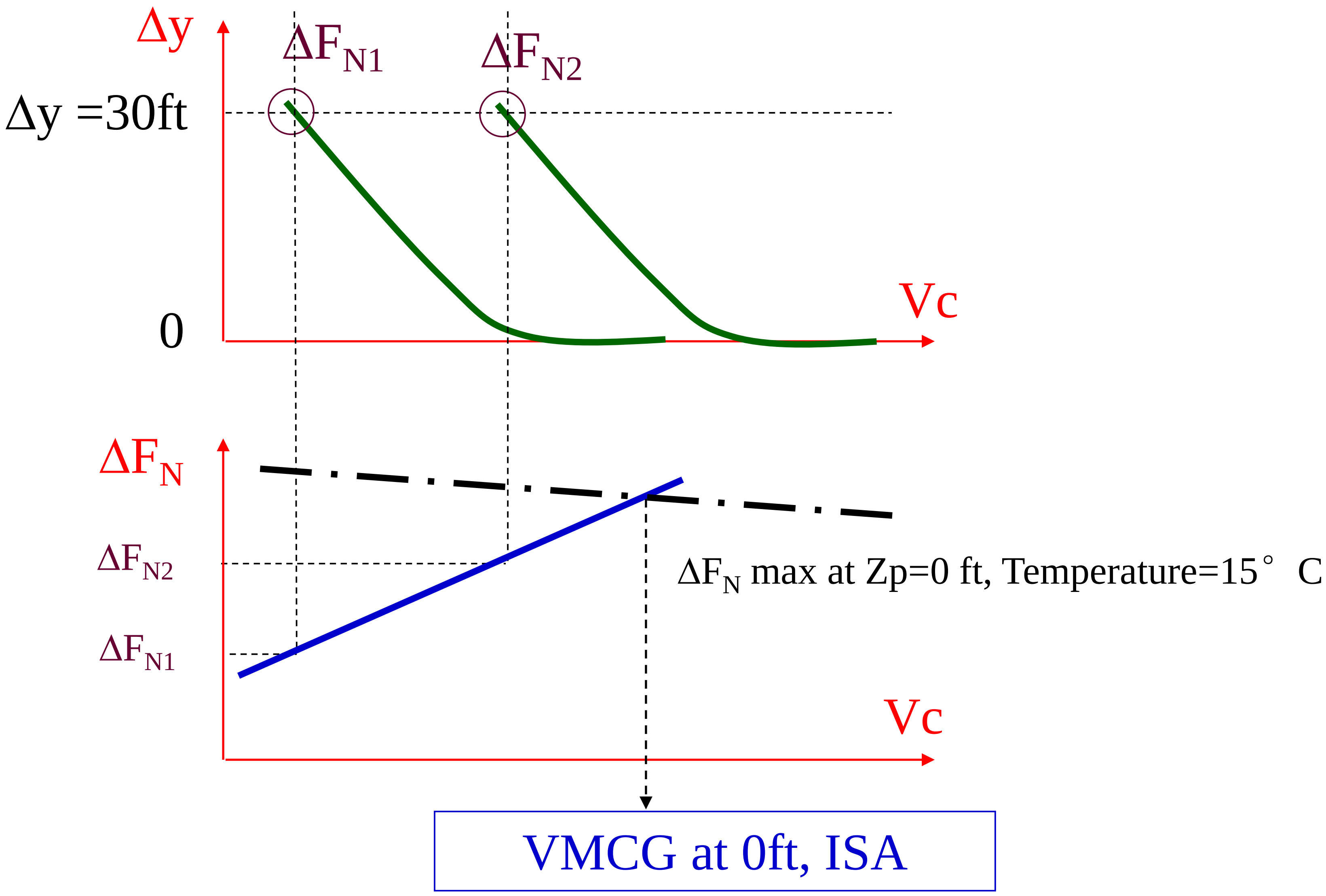
It is the **minimum speed on ground, at take-off,** at which, in case of **sudden engine failure**, it is possible to **maintain an almost rectilinear trajectory** (the maximum deviation from runway centerline, authorized by regulation, is 30 ft), when using only aerodynamic control surfaces; the other engines are set at max take-off thrust.

Flight test method :

Tests performed at aft CG with the nose wheel disconnected.

Engine failures before lift-off are triggered at lower and lower speeds . This is done at different thrust levels.

Thanks to inertial data or trajectory calculation, we compute the maximum lateral deviation Δy.



Consequences on aircraft performances :

The VMCG intervenes in the determination of the **decision speed V1**.

V1 is the speed beyond which, during take-off run, it is necessary to continue the take-off even in case of engine failure : the distance to stop has become longer than the available take-off distance.

We must have **V1>VMCG** :

indeed if V1<VMCG and if the engine failure occurs between V1 and VMCG, we should to carry on the take-off without being able to control the lateral trajectory of the aircraft.

1°) DIRECTIONAL STATIC STABILITY :

The **directional** static stability is a stability with respect to **sideslip** around the **yaw** axis.

The airplane is statically stable directionally if, for a sideslip variation, the yawing moment due to sideslip tends to decrease this sideslip :

$$\boxed{\text{Stable if } Cn\beta>0}$$

(*because a sideslip variation $\Delta\beta>0$ (on right) has to generate a yawing moment towards the right ($N(\beta)>0$) which tends therefore to decrease the sideslip*)

Reminder : Cnβ is also called « weathercock effect » ; the vertical tailplane has a preponderant effect.

Flight test method :

A steady sideslip is performed wings level and when the rudder pedals are released while maintaining constant the bank angle, we check that the aircraft tends to decrease its sideslip.

2°) ROLL STATIC STABILITY :

The **roll** static stability is a stability in **sideslip** around the **roll** axis**.**

The aircraft is statically stable in roll if, for a **sideslip** variation, the **rolling** moment due to sideslip tends to decrease this sideslip :

$\boxed{\text{Stable if } Cl\beta<0}$

(because if a sideslip variation $\Delta\beta>0$ (on right) generates a rolling moment (<0) , this will cause a skid towards the left thanks to the lateral component of the weight : this left skid will decrease the initial sideslip.)

Flight test method :

A steady sideslip is performed with constant heading and when releasing the stick, while maintaining constant the heading, we check that the low wing tends to go upwards (in a steady sideslip, the airplane is banked towards the sideslip : see next §).

3°) STEADY SIDESLIP :

3a) General points :

It is a flight with sideslip (!), **in straight line** and stabilised, i.e. :

$p=r=0$ whence $Cl=Cn=0$ and $\beta=cst$

(The cross coefficients are neglected)

$$\begin{cases} Cl = Cl\beta.\beta + Cl\delta l.\ \delta l = 0 & \text{roll axis X} \\ Cn = Cn\beta.\beta + Cn\delta n.\ \delta n = 0 & \text{yaw axis Z} \end{cases}$$

From the 2 previous equations, we may determine the ailerons deflection δl and rudder deflection δn necessary to fly with a stabilized sideslip in straight flight . We find that the deflections have an opposite sign.

In a simple way , we may say that for a given **sideslip, from right** for instance, it will be generated :

- a **rolling** moment on left thanks to the **dihedral effect** ($Cl\beta$) which needs to be counteracted with **stick on right**

- a **yawing** moment on right thanks to the **weathercock effect** ($Cn\beta$) which needs to be counteracted by pushing on **left rudder pedal**.

Thus we notice that in straight steady sideslip, the **controls are crossed** : **rudder pedal on the opposite side of the sideslip, stick towards the sideslip side**.

The lateral force equation is (if $Ry(\delta n)$ is negligible compared to $Ry(\beta)$) :

$$ny\ mg = mg \sin\phi \cos\theta = -1/2\ \rho S V^2 Cy\beta .\ \beta$$

The airplane has to **bank towards the sideslip side** in order that the lateral component of the weight balances the force $Ry(\beta)$.

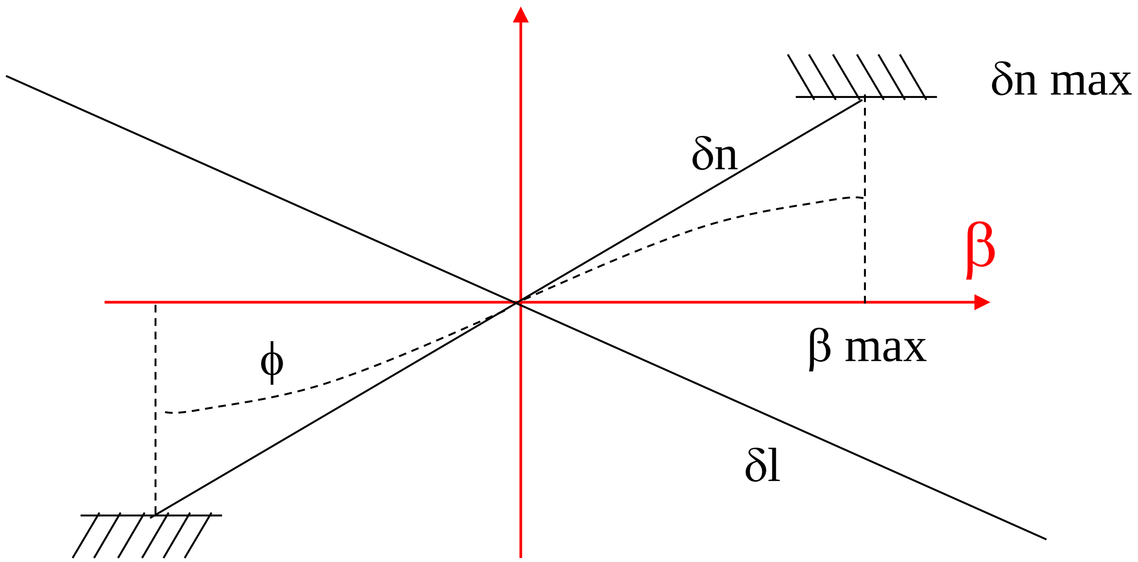
3b) Flight test method :

We stabilise the sideslip at increasing values while maintaining a constant heading and this, up to the first control stop (generally, the rudder stop , before the ailerons stop).

The regulation imposes that pilot control forces and controls deflection are proportional to the sideslip. It is also necessary that the steady sideslip is accompanied with enough bank angle (sideslip awareness for the pilot).

The flight tests of steady sideslip are notably performed at **operational speeds for take-off and landing**.

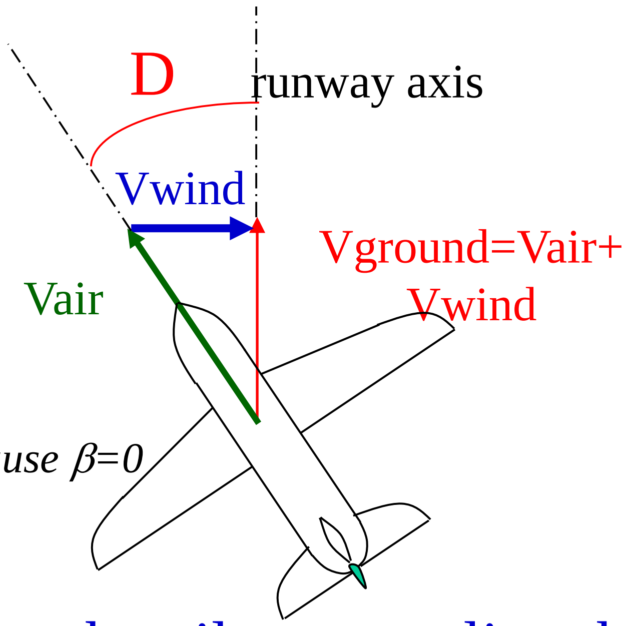
The **maxi sideslip values** noted down during these tests allow to estimate the **maximum crosswind allowable at take-off and landing**.



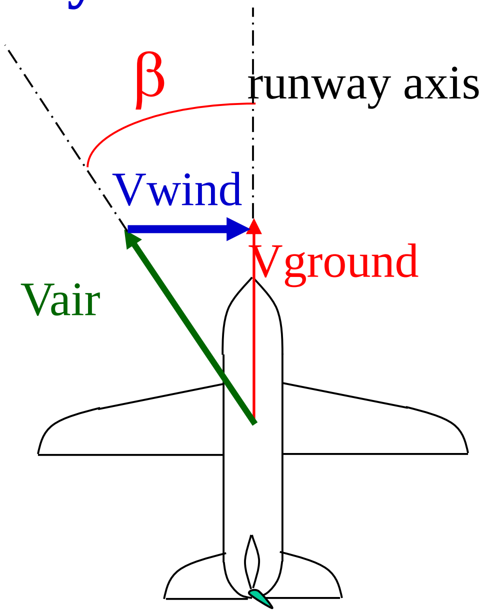
3c) Particular case: the decrab

During the final approach with crosswind, the aircraft fly like a «crab » although it flies without sideslip, in order that its **ground speed** is **aligned with the runway axis**.

$\boldsymbol{\beta=0}$



Here $\delta n=0$ because $\beta=0$



During flare, the pilot must align the X aircraft axis with the runway axis : it is the decrab.

He **transforms the angle of drift** D **into a sideslip** β. He reaches the runway with stick towards the wind and “foot” towards the opposite side.

4°) LATERAL DYNAMIC MANOEUVRABILITY :

It is mainly studied around the roll axis.

2 kinds of tests :

- square input solicitations with the stick in roll. We note down the achieved stabilised roll rate p. Tests performed with different amplitudes of the control input.

- Roll rate : It should be possible to roll the aeroplane from a steady 30° banked turn through an angle of 60° so as to reverse the direction of the turn in not more than 7 seconds for the large civil aircraft.

1°) GENERAL POINTS

We can **linearise** the **3 equations** of the lateral motion (with the assumption of small perturbations : in particular, we assume that ϕ is small enough) and we assume that the longitudinal parameters V, q, α and θ are constant.

The 4 unknown variables of the motion are then: **ϕ, β, p, r.**

We need a fourth equation, coming from an angular relationship:

$$\dot{\phi} = p + tg\theta(q\sin\phi) + r\cos\phi)$$

We may use the matrix form to describe the system of the 4 linearised equations, by defining :

the state vector $\mathbf{X} = \begin{bmatrix} \boldsymbol{\varphi} \\ \boldsymbol{\beta} \\ \mathbf{p} \\ \mathbf{r} \end{bmatrix}$ and the input vector $\mathbf{U} = \begin{bmatrix} \boldsymbol{\delta}\mathbf{l} \\ \boldsymbol{\delta}\mathbf{n} \end{bmatrix}$

The system comes down to : $\boxed{\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU}}$

STATE SPACE FORM

The characteristic equation of the matrix A has 4 roots , **2 real** and **2 imaginary** that define :

- 2 **aperiodic** motions:

the **pure roll (or roll subsidence)** very damped (quick mode)
the **spiral mode** little or not damped (slow mode)

- 1 **oscillatory** motion :

the **dutch roll** (generally rather well damped)

2°) THE PURE ROLL or ROLL SUBSIDENCE MODE

It is a **rapid mode**. For a roll perturbation, we get a pure roll motion which is **very damped**.

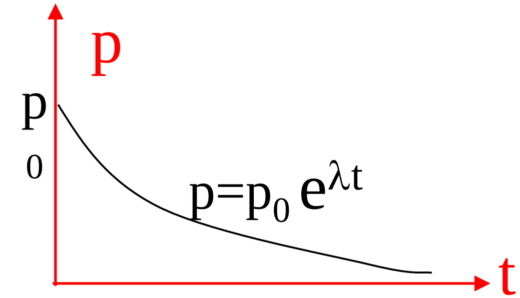
We can find the approximate time constant of this mode :

We assume cross inertia Ixz negligible, **β=0** and **r=0** and that control surfaces are fixed ($\delta l=\delta n=0$). The rolling moment equation may then be written :

$$I_x\, dp/dt = 1/2\, \rho S l V^2 C_{lp}\, pl/V$$

whence $\mathbf{p=p_0\, e^{\lambda t}}$ where $\lambda = 1/2\, \rho S l^2 V\, C_{lp}/I_x$ **<0** (because $C_{lp}<0$)

Very damped aperiodic motion.



Flight test method :

The roll perturbation is caused by a roll square input with the stick.

In the case where a yaw damper is used (implemented in the flight control system), the stick solicitation does not excite the dutch roll, and we see that the roll rate p is well damped

3°) The SPIRAL MODE

Whether the spiral mode is convergent or divergent, it is usually very **slow** to develop.

The spiral motion is an aperiodic one **very little** damped , characterized by slow variations in **roll and yaw** and a small sideslip.

We have already studied the **spiral stability** during the study of turns with stick only or rudder pedal only.

Simplified equation of the motion :

The motion is excite by a slow perturbation in roll or in yaw, for instance during the initiation of a turn. If we let the control surfaces at neutral position, the aerodynamic moments will tend to increase or decrease the bank angle : **spiral stability**.

We assume a small sideslip β and $\delta n = \delta l = 0$.

Thus no significant side-force : whence ny=0

$ny\, mg = mg \sin\phi \cos\theta - mV\Omega \cos\phi = 0$ implies $\phi = Vr/g$

et donc $\mathbf{p} =$ $d\phi/dt =$ $\mathbf{V/g\ dr/dt}$ *($r = \Omega \cos\phi \approx \Omega$; we assume ϕ to be small)*

As we assume slow variations of p and r, we may write Cl=Cn=0 , whence :

$Cl = Cl\beta.\beta + Clp\ pl/V + Clr.\ rl/V = 0$ Roll axis X

$Cn = Cn\beta.\beta + Cnp\ pl/V + Cnr.\ rl/V = 0$ Yaw axis Z

When eliminating β between these 2 equations, we get a differential equation of first order in r (by substituting p by V/g dr/dt) :

$$dr/r = -k\,(Cnr\ Cl\beta - Clr\ Cn\beta)\,dt$$ where $k>0$

whence $\mathbf{r=r_0\ e^{\lambda t}}$

The motion is **convergent aperiodic,** so **stable if $\lambda<0$.**

We find again the condition of the spiral stability :
$Cnr\ Cl\beta - Clr\ Cn\beta > 0$

For practical purposes, the aircraft can be stable or instable : the instability is not an issue because it is a slow mode.

<u>Flight test method :</u>

We stabilize the airplane in a 30° banked turn, then the stick is released. We check if the bank naturally increases or decreases.

4°) DUTCH ROLL

4a) Characteristics of the motion

• Motion **coupled in roll and yaw**.

• **Rapid mode** excited by a rapid perturbation in yaw, in roll or in **sideslip**.

• **Oscillatory motion** in **roll, yaw and sideslip,** that needs to be **convergent** and have a short period.

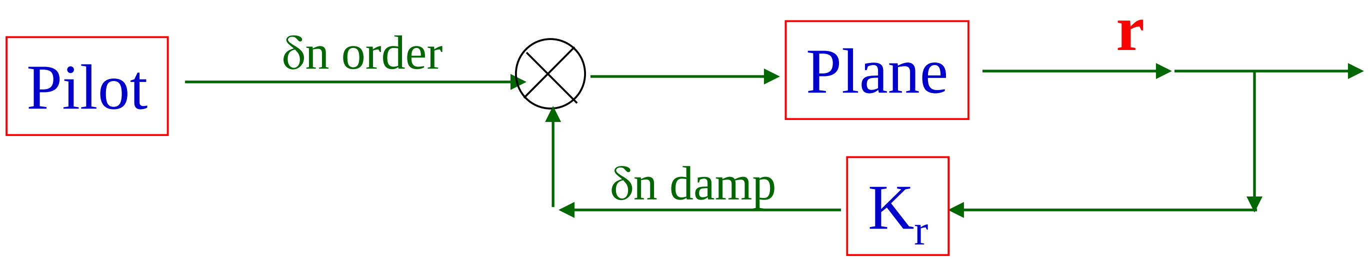
The oscillations in p, r and β are out of phase.

4b) Yaw damper

It is necessary that the dutch roll is well damped in order that the handling is not delicate and the flight remains comfortable for the passengers.

It is then sometimes necessary to fit some aircraft with a **yaw damper** in this aim.

It is a system which deflects the rudder with some phase-advance , using the yaw rate (measured by a gyroscope), thus damping the r oscillations.



On Airbus A310, the yaw damper has a kind of law as follow :

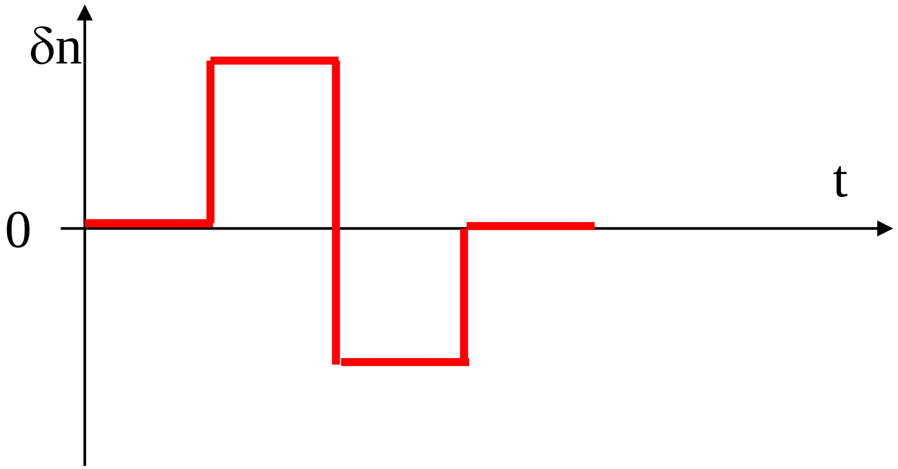
$$\delta n \text{ damp} = (r - g/V\ \phi).\ Kr\ H(p) \quad + \quad K\delta l\ .\ D\delta l$$

yaw damping function + *coordination turn term*

where Kr=Kr(Vc)>0 and H(p) is a phase advance term.

4c) Flight test method :

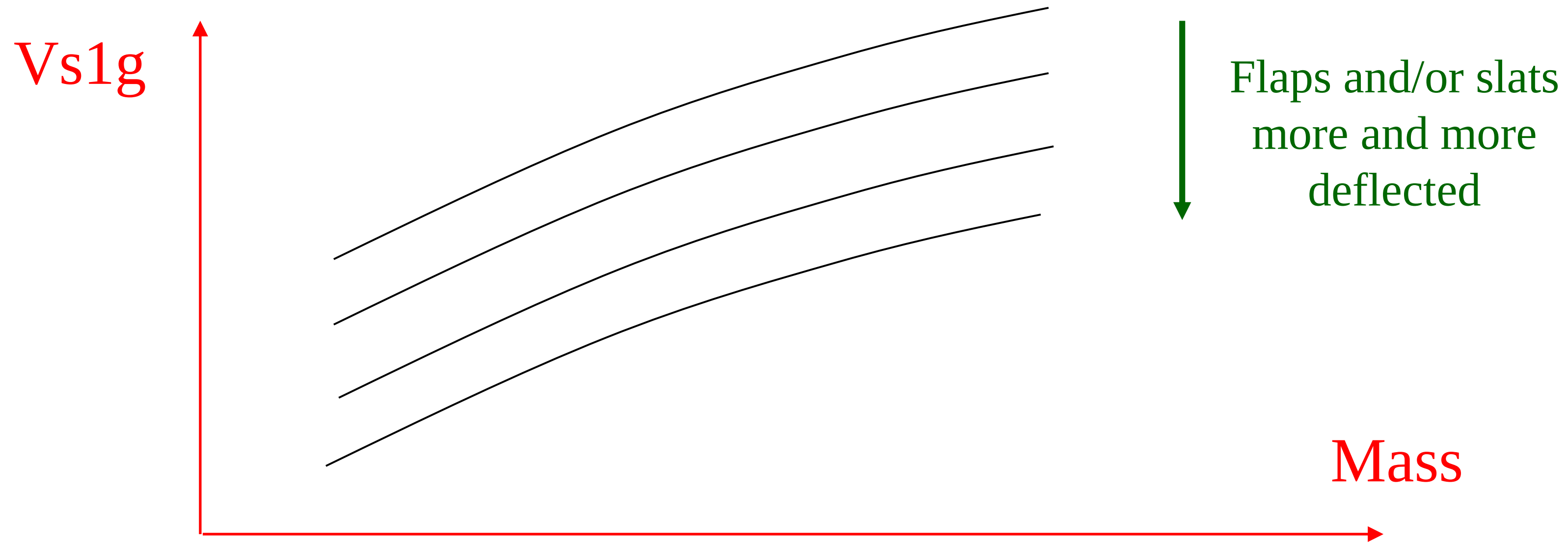
The excitation of the dutch roll is performed through a rudder input as follow :



LOW SPEEDS LIMITATIONS

The main limitation is the **stall speed** often noted **Vs1g** (stall at n=1g) or V_{SR}.

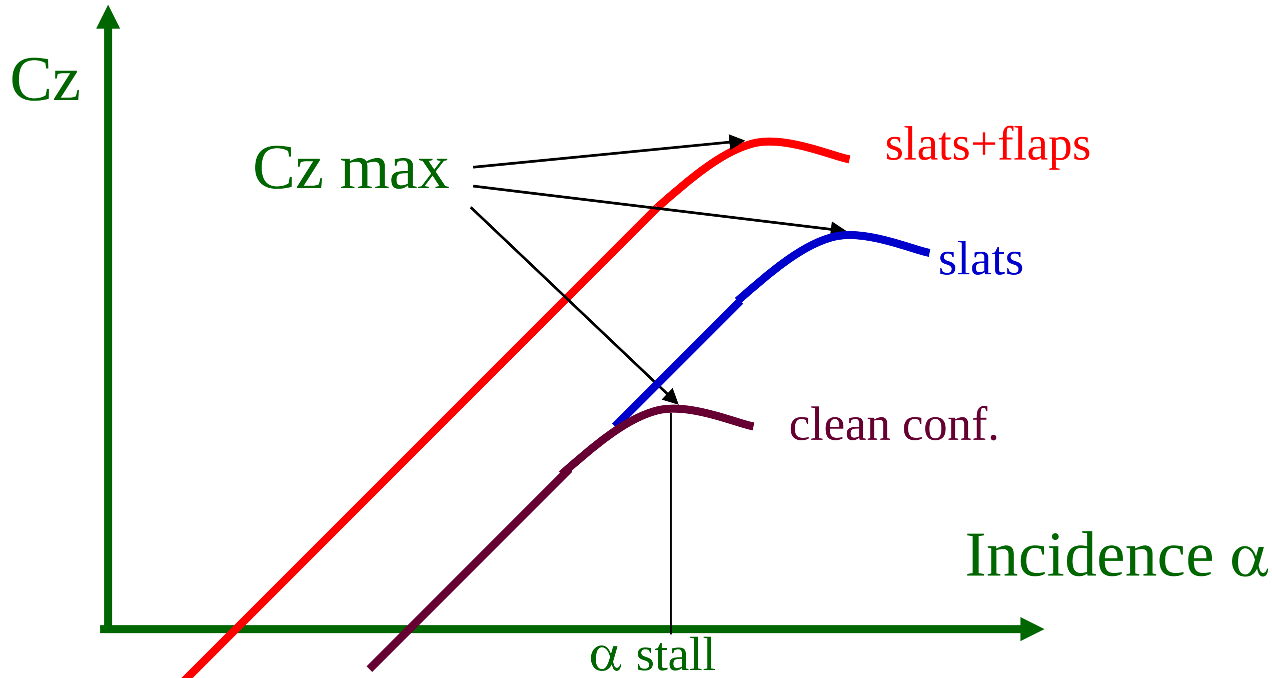
A Vs1g is defined for each slats/flaps configuration. This speed depends on the aircraft mass.



The Vs1g determination is very important because it is a significant determining factor of the **performances at take-off** (computed at V2≥1.13VS1g) and at **landing** (VREF≥ 1.23 VS1g)

STALL : aerodynamic phenomenon.

Beyond a given **angle of attack**, there is an **airflow separation** on the **wing upper surface** causing a **lift loss** : the aeroplane stalls.



Different kinds of stall :

We can perform stalls in straight flight or in turn, with engine at idle or not...

For the determination of the Vs1g, the stall are performed in straight flight, engine at idle at max forward CG, which is the most unfavorable case. Indeed, at this CG, the pitch rim is more nose up deflecting, thus decreasing the tailplane lift.

We also perform stalls at max aft CG to check the HQ behaviour of the aircraft : the longitudinal stability is in particular smaller.

Typical indications of the stall (but not systematic):

Pitch down
Loss of controls efficiency
Roll departure
Sudden change in the pilot control forces or deflection
Characteristic buffeting recognized by the pilot.

The recovery is performed by pushing the stick at neutral or nose down. Sometimes, thrust application is performed (not always the best solution during critical flight tests.)

Handling Qualities aspects of the stall

The regulation imposes :

- a **static longitudinal stability positive** up to the stall (assessed through the plots $\delta m(Cz)$)

- **no controls inversion** up to the stall

- **no abnormal nose-up pitching** at stall

- ability to **promptly prevent stalling** and to **recover from a stall by normal use of the controls**

- **warning** triggered at a speed greater by 5% than the stall speed. The buffeting may be a natural warning .It is most often a device set with angle of attack (aural gong, stick shaker,...)

Pilot helps :

• To fight against auto **pitch-up**, we can implement a pilot assistance : **automatic pitch trimming** (nose down) above a given angle of attack.

• The **pilot control forces** can also be increased before stall (through the springs stiffness) in order to get a better **longitudinal stability** and to prevent the pilot from going too far in incidence.

• **High Angle Of Attack protection** on aircraft fitted with **Electrical Flight Control Systems.**

Determination of the stall speeds Vs1g.

When slowly decelerating, wings level, up to the aerodynamic stall, the normal load factor **n_z is close to 1**.

We write the lift equation (neglecting the component of the thrust)

$$\mathbf{1/2\ \rho S V_{s1g}^{\,2}\ Czmax = n_z\ mg = mg}$$

where Czmax is the maximum lift coefficient for each slat/flap configuration.

Thus

$$V_{s1g} = \sqrt{(2mg / \rho_0 SCz\max)} = f(m)$$

Valid for Vc<200 Kt and Zp<15000 ft : Vs1g being the “Conventional AirSpeed or Computed AirSpeed: “CAS”)