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Final Examination on
Linear Vibrations
Clmaped_clamped beam with
mass and spring
CAUC 2011/2012 Alain Berlioz
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```
> restart;with(plots):with(LinearAlgebra):unprotect(D):
#
#  Validation of the choice of the function of displacement.
#
> dep_R:=Cte*f(t)*(1-cos(2*Pi*x/L));
  displacement_at_x_0=eval(subs(x=0,dep_R));
> displacement_at_x_L=eval(subs(x=L,1-cos(2*Pi*x/L)));
> slope_at_x_0=eval(subs(x=0,diff(1-cos(2*Pi*x/L),x)));
  slope_at_x_L=eval(subs(x=L,diff(1-cos(2*Pi*x/L),x)));
```

$$dep\_R := Cte\, \mathrm{f}(t) \left(1 - \cos\left(\frac{2\,\pi\,x}{L}\right)\right)$$

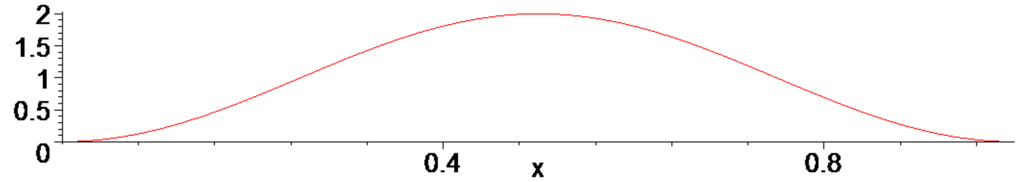
$$displacement\_at\_x\_0 = 0$$

$$displacement\_at\_x\_L = 0$$

$$slope\_at\_x\_0 = 0$$

$$slope\_at\_x\_L = 0$$

```
> unassign('L');
  tr3:=plot({subs(Cte=1,L=1,f(t)=1,dep_R)},x=0..1):
  display(tr3);
```




```
#
#  Stress Energy of the beam
#
> dep_R:=1*f(t)*(1-cos(2*Pi*x/L));
  temp:=diff(dep_R,x,x)^2:
> U_beam:=E*II*int(temp,x=0..L)/2;
```

$$dep\_R := \mathrm{f}(t) \left(1 - \cos\left(\frac{2\,\pi\,x}{L}\right)\right)$$

$$U\_beam := \frac{4\,E\,II\,\pi^4\,\mathrm{f}(t)^2}{L^3}$$

```
#
#  Kinetic Energy of the beam
#
> temp:=diff(dep_R,t)^2:
> T_beam:=rho*S*int(temp,x=0..L)/2;
```

$$T\_beam := \frac{3}{4}\rho\, S\, L \left(\frac{d}{dt} f(t)\right)^2$$

```
#
# Lagrange's equation for beam
#
> KK:=diff(subs(f(t)=f_t,U_beam),f_t);
  k:=coeff(subs(C=1,KK),f_t);
> MM:=diff(subs(d_f_t=diff(f(t),t),diff(subs(diff(f(t),t)=d_f_t,T_beam),d_f_t)),t);
> m:=coeff(subs(C=1,MM),diff(f(t),`$`(t,2)));
```

$$KK := \frac{8\, E\, II\, \pi^4\, f\_t}{L^3}$$

$$k := \frac{8\, E\, II\, \pi^4}{L^3}$$

$$MM := \frac{3}{2}\rho\, S\, L \left(\frac{d^2}{dt^2} f(t)\right)$$

$$m := \frac{3\,\rho\, S\, L}{2}$$

```
> EOM:=MM+KK;
```

$$EOM := \frac{3}{2}\rho\, S\, L \left(\frac{d^2}{dt^2} f(t)\right) + \frac{8\, E\, II\, \pi^4\, f\_t}{L^3}$$

```
#
# Angular frequency and compare
#
> temp:=(sqrt(k/m)):
  omega_Beam:=evalf(%);
  omega_Th:=22.37*(E*II/L^4/rho/S)^(1/2);
  Delta_percent_Th:=(omega_Beam-omega_Th)/omega_Th;
```

$$omega\_Beam := 22.79287503 \sqrt{\frac{E\, II}{L^4\, \rho\, S}}$$

$$omega\_Th := 22.37 \sqrt{\frac{E\, II}{L^4\, \rho\, S}}$$

$$Delta\_percent\_Th := 0.01890366696$$

```
#
#  Add. mass at x = 2L/3
#  New Kinetic Energy
#
> temp1:=subs(x=2*L/3,dep_R);
  temp2:=diff(temp1,t);
  Delta_T_mass:=(rho*S*L/20)*temp2^2/2;
```

$$temp1 := f(t)\left(1-\cos\left(\frac{4\pi}{3}\right)\right)$$

$$temp2 := \frac{3}{2}\left(\frac{d}{dt}f(t)\right)$$

$$Delta\_T\_mass := \frac{9}{160}\rho S L \left(\frac{d}{dt}f(t)\right)^2$$

```
#
#  New Lagrange's equation for beam + mass
#
```

```
> temp3:=subs(diff(f(t),t)=f_t,Delta_T_mass):
  temp4:=diff(temp3,f_t):
  temp5:=subs(f_t=diff(f(t),t),temp4):
  temp6:=diff(temp5,t):
  Delta_m1:=coeff(subs(C=1,temp6),diff(f(t),`$`(t,2)));
```

$$Delta\_m1 := \frac{9\rho S L}{80}$$

```
> temp:=(sqrt(k/(m+Delta_m1)));
  omega_Beam_Mass:=evalf(%);
```

$$temp := \frac{8\sqrt{1290}\,\pi^2\sqrt{\frac{E\,II}{L^4\rho S}}}{129}$$

$$omega\_Beam\_Mass := 21.98340076\sqrt{\frac{E\,II}{L^4\rho S}}$$

```
> Delta_percent_Mass:=evalf(omega_Beam_Mass-omega_Th)/omega_Th;
```

$$Delta\_percent\_Mass := -0.01728204023$$

```
#
#  Add. spring at x = 2L/3
#  New Sress Energy
#
```

```
> temp1:=subs(x=2*L/3,dep_R);
  temp2:=eval((temp1)^2):
  Ke:=E*II/(L^3);
```

$$temp1 := f(t)\left(1-\cos\left(\frac{4\pi}{3}\right)\right)$$

$$Ke := \frac{E\,II}{L^3}$$

```
> delta_T_spring:=Ke*temp2/2:
  temp3:=subs(f(t)=f_t,delta_T_spring):
  temp4:=diff(temp3,f_t):
  delta_k1:=subs(C=1,coeff(temp4,f_t));
```

$$delta\_k1 := \frac{9\,E\,II}{4\,L^3}$$

```
> temp:=(sqrt(((k+delta_k1))/m));
  omega_Beam_Spring:=evalf(%);
  omega_Th:=22.37*(E*II/L^4/rho/S)^(1/2);
  delta_percent:=evalf(omega_Beam_Spring-omega_Th)/omega_Th;
```

$$temp := \frac{\sqrt{6}\sqrt{\frac{\frac{8\,E\,II\,\pi^4}{L^3}+\frac{9\,E\,II}{4\,L^3}}{\rho\,S\,L}}}{3}$$

$$omega\_Beam\_Spring := 22.82575634\sqrt{\frac{E\,II}{L^4\,\rho\,S}}$$

$$omega\_Th := 22.37\sqrt{\frac{E\,II}{L^4\,\rho\,S}}$$

$$delta\_percent := 0.02037355118$$

```
#
#  Summary
#
> omega_Th;omega_Beam;Delta_percent_Th;
```

$$22.37\sqrt{\frac{E\,II}{L^4\,\rho\,S}}$$

$$22.79287503\sqrt{\frac{E\,II}{L^4\,\rho\,S}}$$

$$0.01890366696$$

```
> omega_Beam_Mass;omega_Beam_Spring;
```

$$21.98340076\sqrt{\frac{E\,II}{L^4\,\rho\,S}}$$

$$22.82575634\sqrt{\frac{E\,II}{L^4\,\rho\,S}}$$

```
#
# Theoretical Part (not required)
#
> Mat:=Matrix(4,4):Vect:=Vector(4,[C ,D, E, F]):
> PHI:=C*sin(beta*x)+D*cos(beta*x)+E*sinh(beta*x)+F*cosh(beta*x);
> Pente:=diff(PHI,x);
> Moment_F:=diff(PHI,x,x):
> Effort_T:=diff(PHI,x,x,x):
```

$$PHI := C\sin(\beta\,x) + \mathrm{D}\cos(\beta\,x) + E\sinh(\beta\,x) + F\cosh(\beta\,x)$$

$$Pente := C\cos(\beta\,x)\,\beta - \mathrm{D}\sin(\beta\,x)\,\beta + E\cosh(\beta\,x)\,\beta + F\sinh(\beta\,x)\,\beta$$

```
> temp:=subs(x=0,PHI):
> Mat[1,1]:=coeff(temp,C):Mat[1,2]:=coeff(temp,D):Mat[1,3]:=coeff(
  temp,E):Mat[1,4]:=coeff(temp,F):
> temp:=subs(x=0,Pente):
```

```
> Mat[2,1]:=coeff(temp,C):Mat[2,2]:=coeff(temp,D):Mat[2,3]:=coeff(
  temp,E):Mat[2,4]:=coeff(temp,F):
> temp:=subs(x=L,PHI):
> Mat[3,1]:=coeff(temp,C):Mat[3,2]:=coeff(temp,D):Mat[3,3]:=coeff(
  temp,E):Mat[3,4]:=coeff(temp,F):
> temp:=subs(x=L,Pente):
> Mat[4,1]:=coeff(temp,C):Mat[4,2]:=coeff(temp,D):Mat[4,3]:=coeff(
  temp,E):Mat[4,4]:=coeff(temp,F):
> #print(Mat):
> Prod:=(Mat.Vect):
> simplify(Determinant(Mat)/beta^2/2=0):
> sol1:=solve({Prod[1],Prod[2]},{E,F}):
> PHI:=collect(collect(collect(collect(subs(sol1,PHI),F),E),D),C):
> Pente:=collect(collect(collect(collect(subs(sol1,Pente),F),E),D)
  ,C):
> sol1:=solve({subs(x=L,PHI)},C):
> Phi1:=subs(sol1,PHI):
> Phi1:=collect(Phi1,D);
```

$$\Phi 1 := \left(-\frac{(\sin(\beta\, x) - \sinh(\beta\, x))\,(-\cos(\beta\, L) + \cosh(\beta\, L))}{-\sin(\beta\, L) + \sinh(\beta\, L)} + \cos(\beta\, x) - \cosh(\beta\, x) \right) \mathrm{D}$$

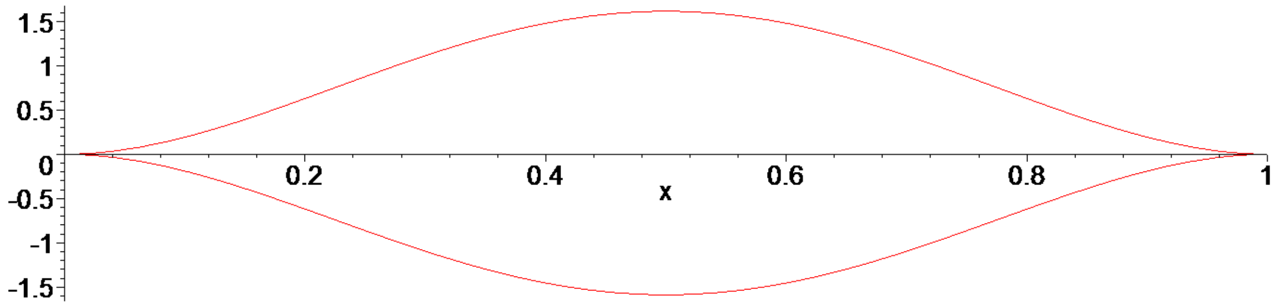
```
> sol2:=solve({subs(x=L,Pente)},D):
> Phi2:=subs(sol2,PHI):
> Phi2:=collect(Phi2,C);
```

$$\Phi 2 := \left(\sin(\beta\, x) - \sinh(\beta\, x) - \frac{(\cos(\beta\, x) - \cosh(\beta\, x))\,(-\cos(\beta\, L) + \cosh(\beta\, L))}{\sin(\beta\, L) + \sinh(\beta\, L)} \right) C$$

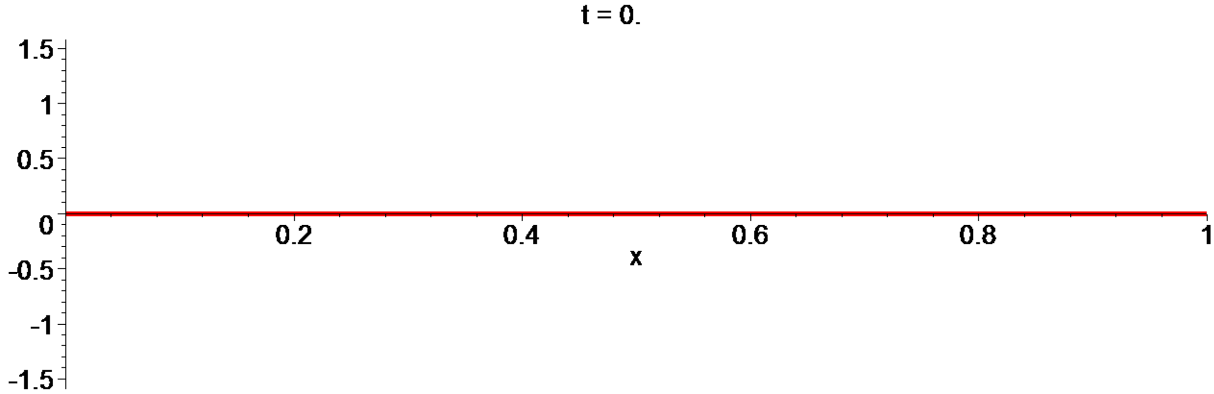
```
> beta:=sqrt(22.37);
  L:=1;
> tr1:=plot({subs(D=1,Phi1)},x=0..1):
  tr2:=plot({subs(C=1,Phi2)},x=0..1):
> display({tr1,tr2});
```

$$\beta := 4.729693436$$

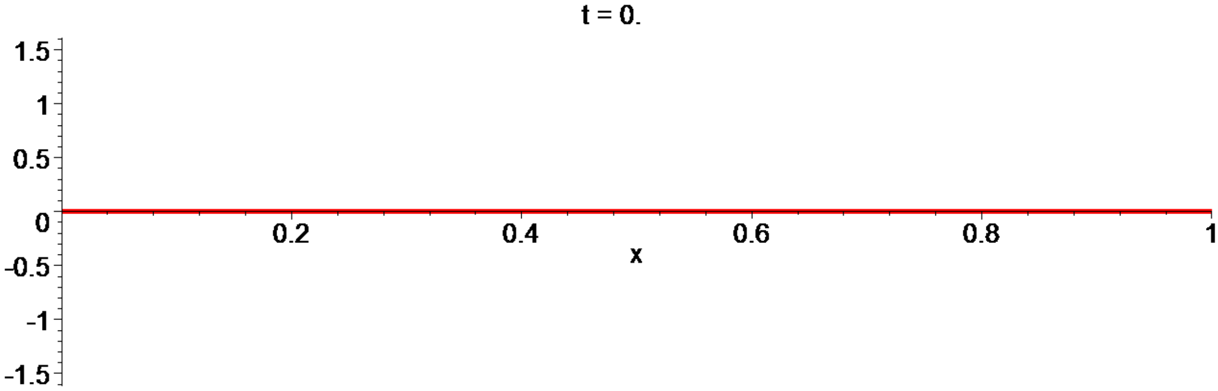
$$L := 1$$




```
> animate(plot,[subs(D=1,Phi1)*sin(t),x=0..1],t=0..2*Pi*23/24,fram
  es=24,thickness=5);
```




```
> animate(plot,[subs(C=1,Phi2)*sin(t),x=0..1],t=0..2*Pi*23/24,frames=24,thickness=5);
```

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>
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