PS41 Propulsion – Chapter 3
Thrust and Nozzle

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Objectives

- Deals with the definitions and the basic relations of propulsive force, the exhaust velocity.
- Learn the thermodynamic relations of the processes inside a rocket nozzle and chamber.

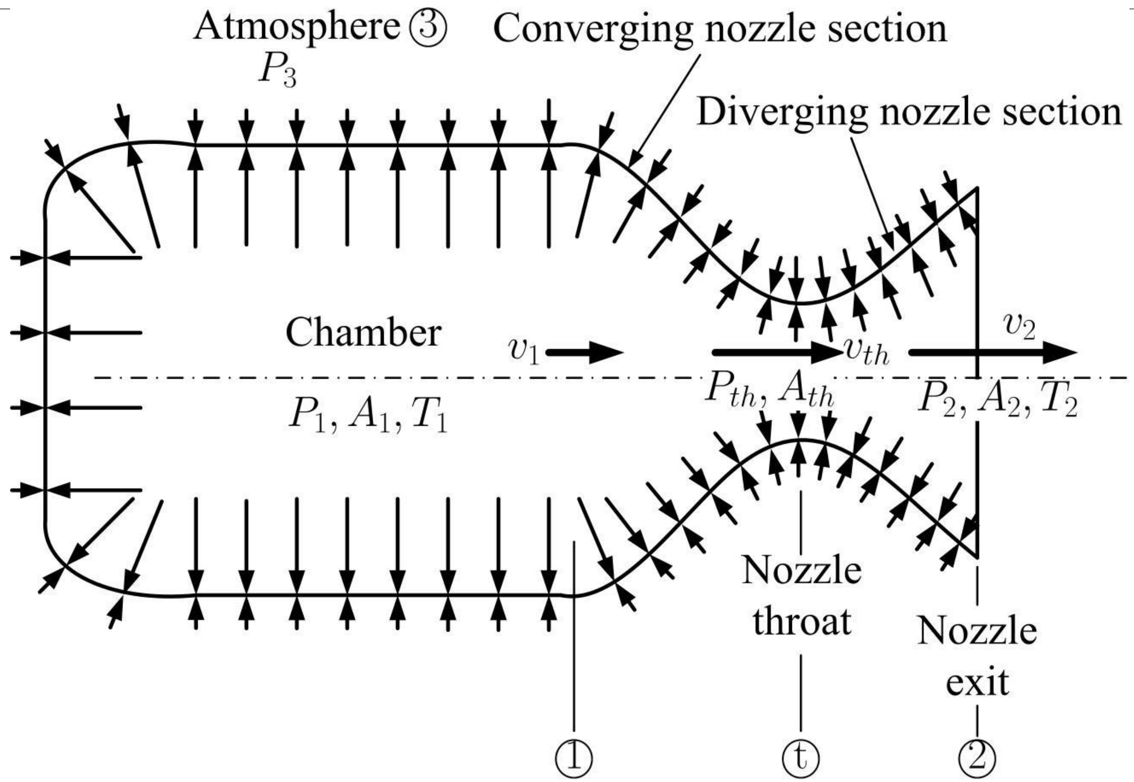
Introduction

- Propulsion: to drive away
- Jet propulsion
  - Rocket propulsion
  - Duct propulsion (air-breathing engines): turbojets, turbofans, ramjets, and pulsejets

Definitions and Fundamentals

- The total impulse: $I_t = \int_0^t F dt$
- The specific impulse: $I_s = \frac{\int_0^t F dt}{g \int \dot{m} dt}$ (s)
- The effective exhaust velocity: $c = I_s g = F / \dot{m}$
- The thrust-to-weight ratio: $F / (\dot{m} g)$
- The propellant mass fraction: $\zeta = \frac{m_p}{m_0} = \frac{m_0 - m_f}{m_0}$
- The impulse-to-weight ratio: $\frac{I_t}{w_0} = \frac{I_t}{(m_f + m_p) g}$

Thrust (1)



Pressure balance on chamber and nozzle interior walls is not uniform. The internal gas pressure (indicated by length of arrows) is highest in the chamber (P_1) and decreases steadily in the nozzle until it reaches the nozzle exit pressure P_2. The external or atmospheric pressure P_3 is uniform. At the throat the pressure is P_{th}.

Thrust (2)

- $F = \dot{m}v_2 + (P_2 - P_3)A_2$
- The optimum nozzle: $P_2 = P_3, F = \dot{m}v_2$
- The effective exhaust velocity:

$$c = F/\dot{m} = v_2 + (P_2 - P_3)A_2/\dot{m}$$

- The characteristic velocity: $c^* = P_1 A_{th}/\dot{m}$

Example 1

Ideal rocket (1)

- An ideal rocket unit is one for which the following assumptions are valid:
  - The working substance (or chemical reaction products) is *homogeneous*.
  - All the species of the working fluid are *gaseous*. Any condensed phases (liquid or solid) add a negligible amount to the total mass.
  - The working substance obeys the *perfect gas law*.
  - There is *no heat transfer* across the rocket walls; therefore, the flow is adiabatic.
  - There is no appreciable *friction* and all *boundary layer* effects are neglected.
  - There is no *shock waves* or *discontinuities* in the nozzle flow.

Isentropic expansion

Ideal rocket (2)

- The *propellant flow* is *steady* and *constant*. The expansion of the working fluid is uniform and steady, without vibration. Transient effects (i.e., start-up and shutdown) are of very short duration and may be neglected.
- All exhaust gases leaving the rocket have an *axially directed velocity*.
- The gas velocity, pressure, temperature, and density are all uniform across any section normal to the nozzle axis.
- *Chemical equilibrium* is established within the rocket chamber and the gas composition does not change in the nozzle (frozen flow).
- Stored propellants are at room temperature. Cryogenic propellants are at their boiling points.

Summary of thermodynamic relations (1)

- $h_t = h + v^2/2$
- $h_x - h_y = \frac{1}{2}\left(v_y^2 - v_x^2\right) = c_p\left(T_x - T_y\right)$
- $\dot{m}_x = \dot{m}_y = \dot{m} = Av/\nu$ (ν-the specific volume)
- $P_x \nu_x = RT_x, \gamma = c_p/c_v, c_p - c_v = R$
- $\boxed{\frac{T_x}{T_y} = \left(\frac{P_x}{P_y}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{\nu_y}{\nu_x}\right)^{\gamma-1}}$

Summary of thermodynamic relations (2)

- $T_t = T + \frac{v^2}{2c_p}$
- $\frac{P_t}{P} = \left[1 + \frac{v^2}{2c_pT}\right]^{\frac{\gamma}{\gamma-1}} = \left(\frac{\nu}{\nu_t}\right)^{\gamma}$
- $a = \sqrt{\gamma RT}, M = \frac{v}{a} = \frac{v}{\sqrt{\gamma RT}}$
- $T_t = T\left[1 + \frac{1}{2}(\gamma - 1)M^2\right]$
- $P_t = P\left[1 + \frac{1}{2}(\gamma - 1)M^2\right]^{\frac{\gamma}{\gamma-1}}$

Isentropic flow through nozzles (1)

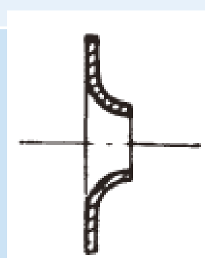
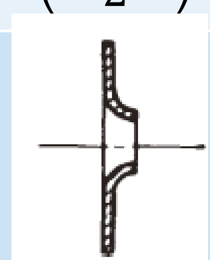
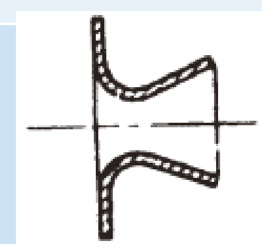
- $v_2 = \sqrt{2(h_1 - h_2) + v_1^2}$
- $v_2 = \sqrt{\frac{2\gamma}{\gamma - 1} RT_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right] + \cancel{v_1^2}}$
- For optimum expansion $P_2 = P_3$, $v_2 = c_{opt}$
- $\frac{A_y}{A_x} = \frac{M_x}{M_y} \sqrt{\left\{\frac{1 + \frac{\gamma-1}{2} M_y^2}{1 + \frac{\gamma-1}{2} M_x^2}\right\}^{\frac{\gamma+1}{\gamma-1}}}$
- The nozzle expansion area ratio:

$\epsilon = A_2/A_{th}$ (A_{th}- the throat area)

Isentropic flow through nozzles (2)

- At throat: $M = 1$
  - $\frac{P_{th}}{P_1} = \left[\frac{2}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}}$ (P_{th}-the critical pressure)
  - $T_{th} = \frac{2T_1}{\gamma+1}$
  - $\nu_{th} = \nu \left[\frac{\gamma+1}{2}\right]^{\frac{1}{\gamma-1}}$
  - $v_{th} = \sqrt{\frac{2\gamma}{\gamma+1} RT_1} = a_{th} = \sqrt{\gamma RT_{th}}$

Isentropic flow through nozzles (3)

	Subsonic	Sonic	Supersonic
Throat velocity	$v_{th} < a_{th}$	$v_{th} = a_{th}$	$v_{th} = a_{th}$
Exit velocity	$v_2 < a_2$	$v_2 = v_{th}$	$v_2 > v_{th}$
Mach number	$M_2 < 1$	$M_2 = M_1 = 1$	$M_2 > 1$
Pressure	$\frac{P_1}{P_2} < \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}$	$\frac{P_1}{P_2} = \frac{P_1}{P_{th}} = \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}$	$\frac{P_1}{P_2} > \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}$
Shape			

Example 1

The following measurements were made in a sea-level test of a solid propellant rocket motor (all cross sections are circular and unchanging):

Burn duration	40 sec
Initial propulsion system mass	1210 kg
Mass of rocket motor after test	215 kg
Sea-level thrust	62250 N
Chamber pressure	7.0 Mpa
Nozzle exit pressure	70.0 kPa
Nozzle throat diameter	8.55 cm
Nozzle exit diameter	27.03 cm

Determine $\dot{m}$, v_2, c^* and c at seal level. Also, determine the specific impulse at sea level, 1000 m and 25000 m altitude. Assume the momentum thrust is invariant during the ascent, and the start and stop transients are short.