# Aerodynamics
# Boundary Layer and Viscid, Incompressible Flow —TD3

## ■ N-S方程(流体力学控制方程)推导

讲义上：

$$\frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + \overline{\overline{\mathrm{grad}}}\,\vec{V}.\vec{V} = -\frac{1}{\rho}\overrightarrow{\mathrm{grad}}p + \frac{1}{\rho}\overrightarrow{\mathrm{div}}\left(\overline{\overline{\tau}}\right) + \vec{F}$$

其中： $\vec{V} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}$ $\vec{F} = \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix}$ （1）

$$\begin{cases} \tau_{xx} = \lambda \nabla \bullet \vec{V} + 2\mu \dfrac{\partial u}{\partial x} \\ \tau_{yy} = \lambda \nabla \bullet \vec{V} + 2\mu \dfrac{\partial v}{\partial y} \\ \tau_{zz} = \lambda \nabla \bullet \vec{V} + 2\mu \dfrac{\partial w}{\partial z} \end{cases}$$

$$\begin{cases} \tau_{xy} = \tau_{yx} = \mu\left(\dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y}\right) \\ \tau_{xz} = \tau_{zx} = \mu\left(\dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x}\right) \\ \tau_{yz} = \tau_{zy} = \mu\left(\dfrac{\partial w}{\partial y} + \dfrac{\partial v}{\partial z}\right) \end{cases}$$

$$\lambda = -\frac{2}{3}\mu$$

$$\begin{cases} div(\overline{\tau}_x) = \dfrac{\partial \tau_{xx}}{\partial x} + \dfrac{\partial \tau_{xy}}{\partial y} + \dfrac{\partial \tau_{xz}}{\partial z} = \mu\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2}\right) + (\mu + \lambda)\dfrac{\partial}{\partial x}(\nabla \bullet \vec{V}) \\ div(\overline{\tau}_y) = \dfrac{\partial \tau_{yx}}{\partial x} + \dfrac{\partial \tau_{yy}}{\partial y} + \dfrac{\partial \tau_{yz}}{\partial z} = \mu\left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2} + \dfrac{\partial^2 v}{\partial z^2}\right) + (\mu + \lambda)\dfrac{\partial}{\partial y}(\nabla \bullet \vec{V}) \\ div(\overline{\tau}_z) = \dfrac{\partial \tau_{zx}}{\partial x} + \dfrac{\partial \tau_{zy}}{\partial y} + \dfrac{\partial \tau_{zz}}{\partial z} = \mu\left(\dfrac{\partial^2 w}{\partial x^2} + \dfrac{\partial^2 w}{\partial y^2} + \dfrac{\partial^2 w}{\partial z^2}\right) + (\mu + \lambda)\dfrac{\partial}{\partial z}(\nabla \bullet \vec{V}) \end{cases}$$

（2）

# Aerodynamics
# Boundary Layer and Viscid, Incompressible Flow —TD3

## ■ N-S方程(流体力学控制方程)推导（继续）

不可压缩流体，因此 $\nabla \bullet \vec{V} = 0$ 将（1）（2）带入方程，得到：

$$\begin{cases} u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}+w\dfrac{\partial u}{\partial z}=X-\dfrac{1}{\rho}\dfrac{\partial P}{\partial x}+\dfrac{\mu}{\rho}(\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2}) \\ u\dfrac{\partial v}{\partial x}+v\dfrac{\partial v}{\partial y}+w\dfrac{\partial v}{\partial z}=Y-\dfrac{1}{\rho}\dfrac{\partial P}{\partial y}+\dfrac{\mu}{\rho}(\dfrac{\partial^2 v}{\partial x^2}+\dfrac{\partial^2 v}{\partial y^2}+\dfrac{\partial^2 v}{\partial z^2}) \\ u\dfrac{\partial w}{\partial x}+v\dfrac{\partial w}{\partial y}+w\dfrac{\partial w}{\partial z}=Z-\dfrac{1}{\rho}\dfrac{\partial P}{\partial z}+\dfrac{\mu}{\rho}(\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}+\dfrac{\partial^2 w}{\partial z^2}) \end{cases}$$

## ■ 库埃特流动

**Exercise 1**

由N-S方程得到流体控制方程：

$$u=\frac{U}{2}\left(1+\frac{y}{h}\right)-\frac{h^2}{2\mu}\frac{dp}{dx}\left[1-\left(\frac{y}{h}\right)^2\right]$$

$$\frac{u}{U}=\frac{1}{2}\left(1+\frac{y}{h}\right)-\frac{h^2}{2\mu U}\frac{dp}{dx}\left[1-\left(\frac{y}{h}\right)^2\right]$$

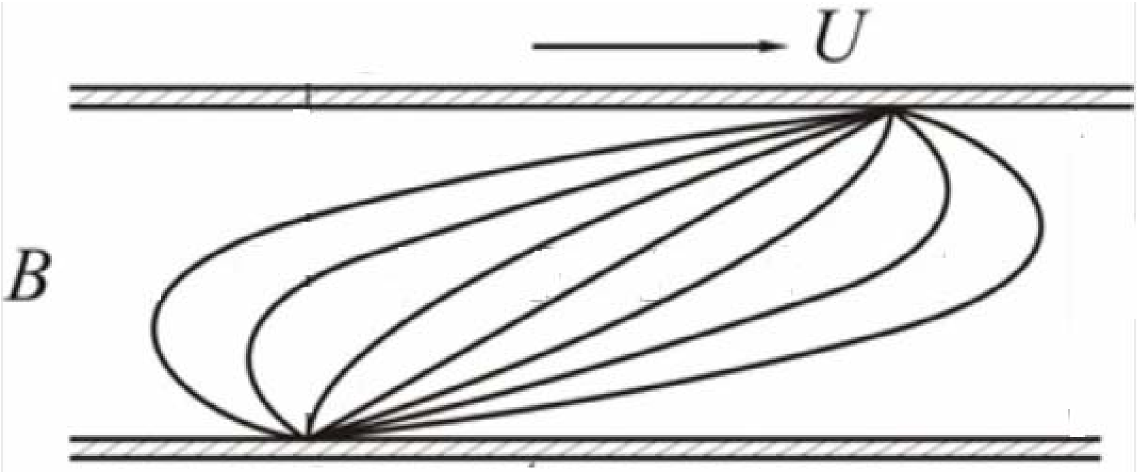
# Aerodynamics
# Boundary Layer and Viscid, Incompressible Flow —TD3

## ■ 库埃特流动 （续）

$$\frac{u}{U}=\frac{1}{2}\left(\frac{y}{h}+1\right)+B\left[\left(\frac{y}{h}\right)^2-1\right]$$

其中：$B=\dfrac{h^2}{2\mu U}\dfrac{dp}{dx}$

上式代表了上板运动和压强梯度共同作用下的平板间流场，称为库埃特流。 B取不同时，速度廓线不同。




(1) $B=0$，即压强梯度为零时，流体仅在上板带动下作纯剪切流动，速度廓线是斜直线；

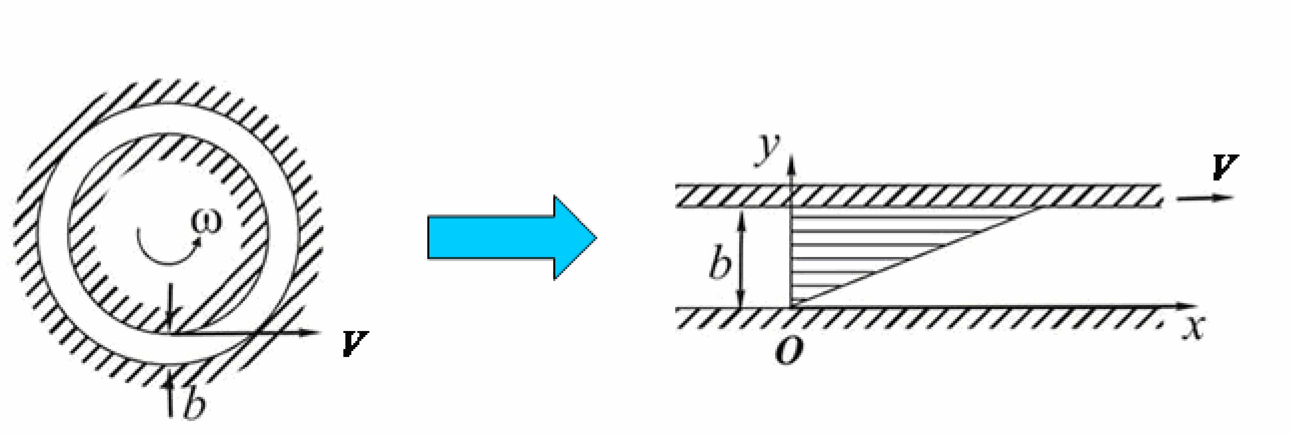
(2) $B>0$，在顺压梯度(压降方向与流动方向相同)作用下的库埃特流，速度廓线是斜直线与抛物线之相加；

(3) $B<0$，在逆压梯度（压降方向与流动方向相反）作用下的库埃特流，速度廓线是斜直线与抛物线之相减；

■ 无限大平行平板间的流动——库埃特流动

**Exercise 2**

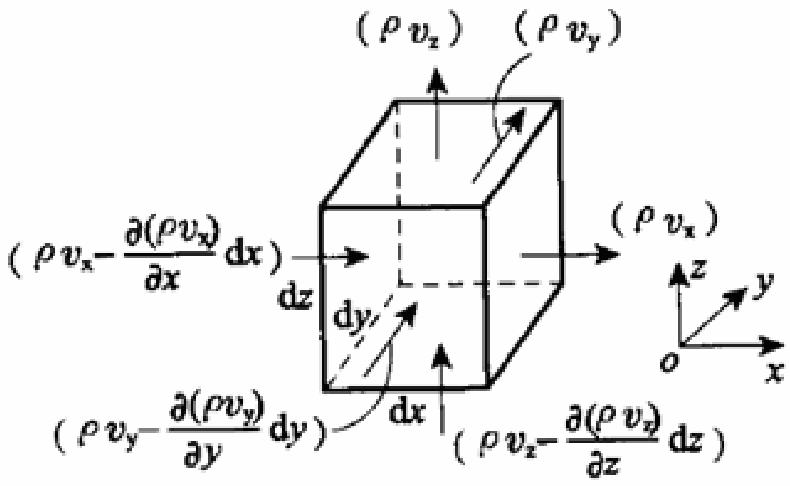
求解思路：



# Aerodynamics
# Viscid and Incompressible Flow —TD4

## ■ 平面应力

**Exercise 3**




微元控制体的流量平衡

切应力：

$$\begin{cases} \tau_{xy} = \tau_{yx} = \mu\left(\dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y}\right) \\ \tau_{xz} = \tau_{zx} = \mu\left(\dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial x}\right) \\ \tau_{yz} = \tau_{zy} = \mu\left(\dfrac{\partial w}{\partial y} + \dfrac{\partial v}{\partial z}\right) \end{cases}$$

正应力：

$$\begin{cases} p_{xx} = -p + 2\mu\dfrac{\partial u}{\partial x} - \dfrac{2}{3}\mu\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z}\right) \\ p_{yy} = -p + 2\mu\dfrac{\partial v}{\partial y} - \dfrac{2}{3}\mu\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z}\right) \\ p_{zz} = -p + 2\mu\dfrac{\partial w}{\partial z} - \dfrac{2}{3}\mu\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z}\right) \end{cases}$$