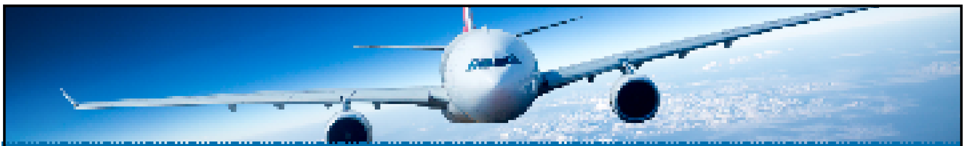


FATIGUE DESIGN OF STRUCTURES

G. Hénaff

1

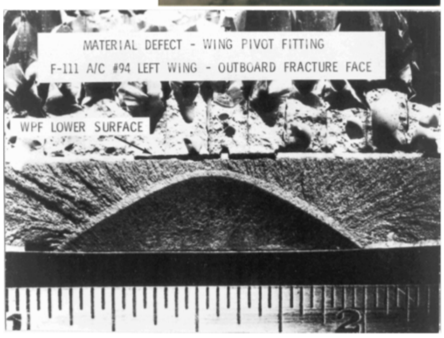
DAMAGE TOLERANCE

Damage Tolerance

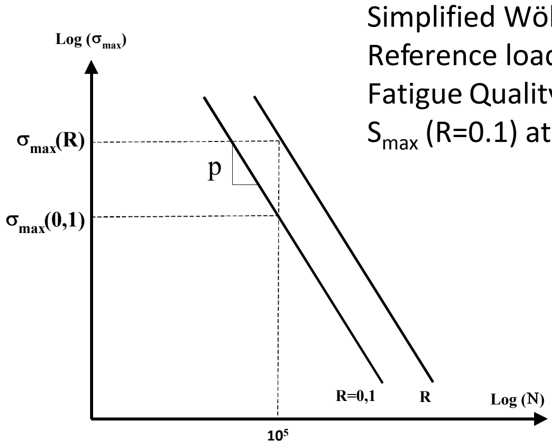
introduction

DT Motivations

- On Dec 22nd 1969, a F-111 fighter aircraft lost a wing during a training flight (2 pilots dead).
- Origin of the failure: forging flaw introduced during the first processing operations but not detected at the time of manufacture
- This accident triggered the largest survey ever conducted on a single material, in this case the D6AC steel but almost highlighte dthe need to take into account the presence of a flaw in the structure



Standard Design



Simplified Wöhler curve
Reference loading R=0.1
Fatigue Quality Index (IQF):
S_{max} (R=0.1) at 10^5 cycles

Standard Design (*cont'd*)

$$S_{max}(R) = \frac{S_{max}(0.1)}{f(R)} \quad \text{with} \quad f(R) = \left(\frac{1-R}{0.9}\right)^q$$

$$\Rightarrow N = 10^5 \times \left(\frac{IQF}{((1-R)/0.9)^q \times S_{max}(R)}\right)^p$$

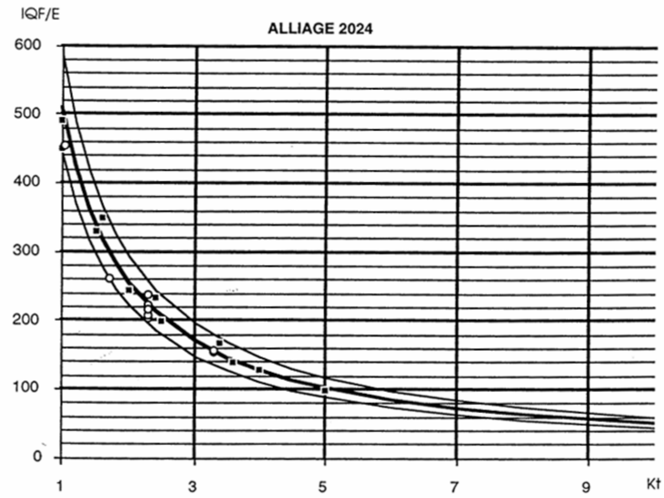
Standard Design (*cont'd*)

$$IQF = M \times E \times T_1 \times T_2 \times T_3 \times T_4 \times \frac{C}{K_t}$$

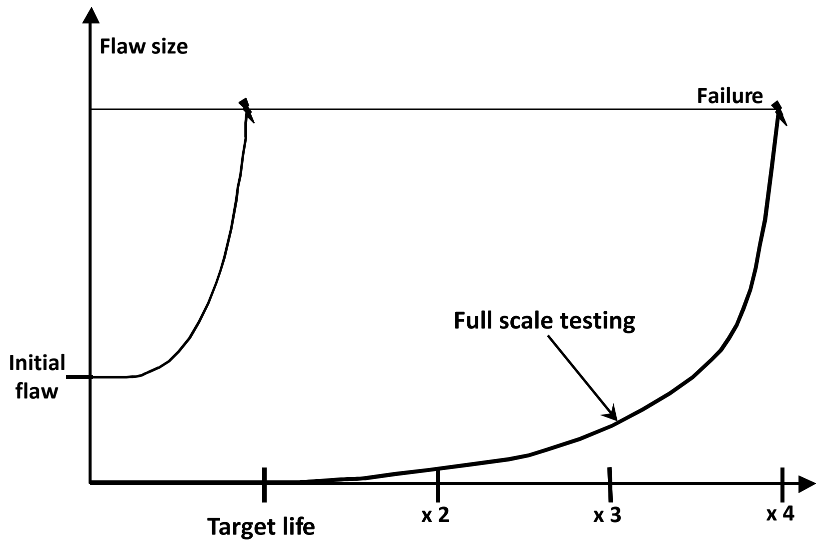
M : influence of material ;
E : scale effect ;
T : influence of technological processes
- T_1 : influence of surface treatment ;
- T_2 : influence of mechanical treatment ;
- T_3 : influence of the type of joining for bolted or riveted assemblies ;
- T_4 : Influence of adjustments for bolted joints .

K_t : stress concentration factor ;
C : Influence of the structural configuration (accidents of form, assemblies, lugs).

Standard Design (*end*)



Motivation



Damage Tolerance

Objectives

Objectives

- Any structure must be able to withstand, at any time during its use, a limit load fixed during the initial design. Thus, an exceptional effort with a low probability is considered, but this should not lead to the ruin of the structure. The limit load will therefore be defined as the product of the exceptional effort by a safety factor (typically 1.5).

- In the presence of cracks, the resistance is lower than the initial resistance estimated during design.

- Damage tolerance thus determines the ability of a structure to withstand the presence of damage in the form of cracks without catastrophic consequences, even for exceptional effort until the damaged part is detected and repaired or the structure reaches the end of its life.

Outputs

DT Analysis must provide:

- the evolution of residual strength as a function of crack size;

- the permissible crack size;

- the propagation time from the detection threshold;

- the permissible initial defect size in a new structure;

- the interval of inspection or replacement.

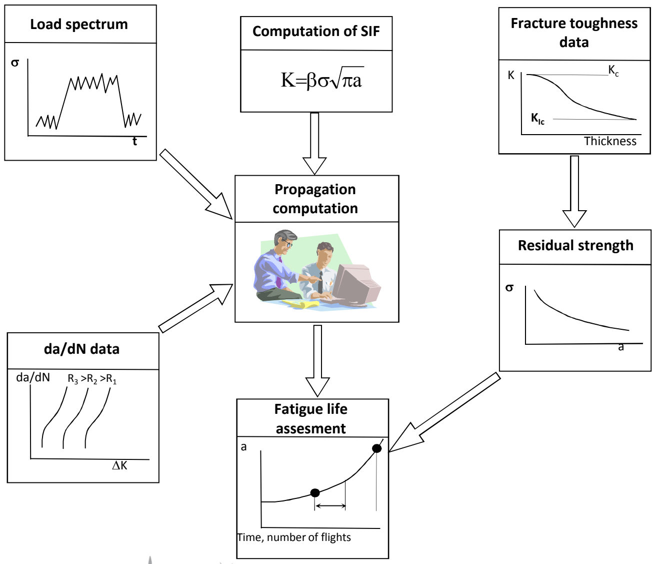
Tasks of the DT analysis

- Define the range of use of the structure
- Develop the spectrum of load factors
- Identify critical points
- Establishing the stress spectrum for each point
- Identify the environment for each point
- Establishing the cracking data

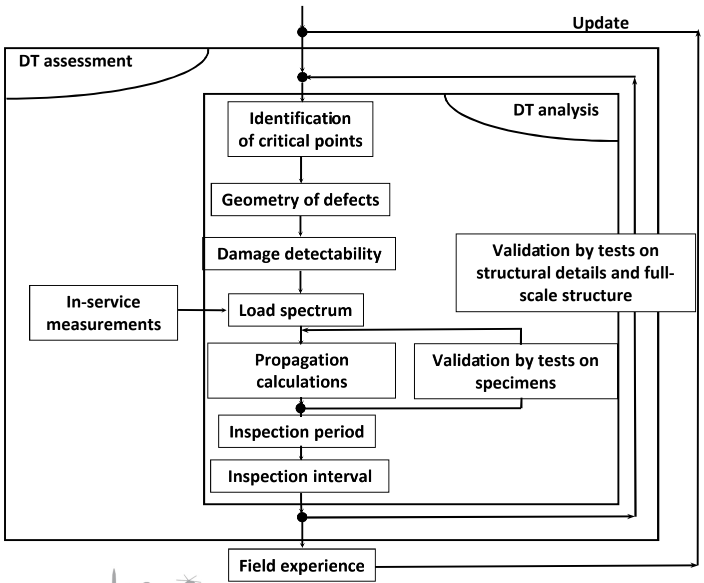
Tasks of the DT analysis (*cont'd*)

- Validate the crack growth analysis;
- Determine the fracture toughness for each material and geometry;
- Validate the residual resistance analysis methods;
- Bring together all the partners concerned and collectively define the inspection methods and their periodicity

DT Analysis



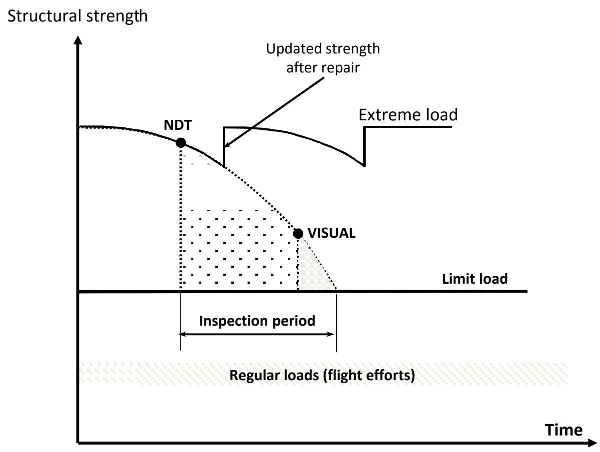
DT assessment



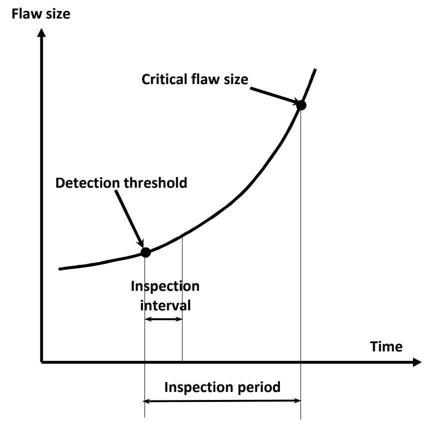
Damage Tolerance

Residual strength

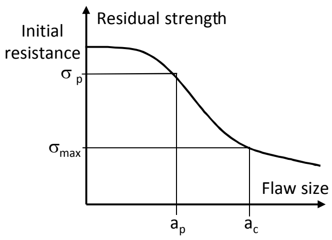
Residual strength

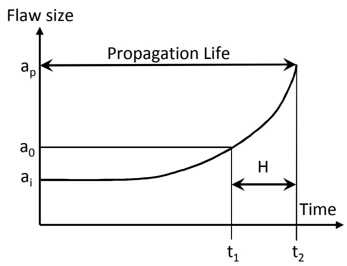


Residual strength



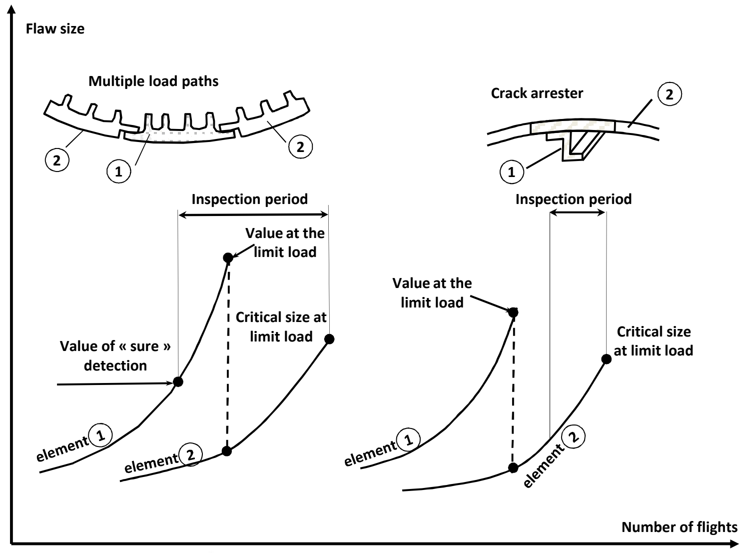
Residual strength





a_i: initial flaw size
a_0: NDT detection threshold

Application



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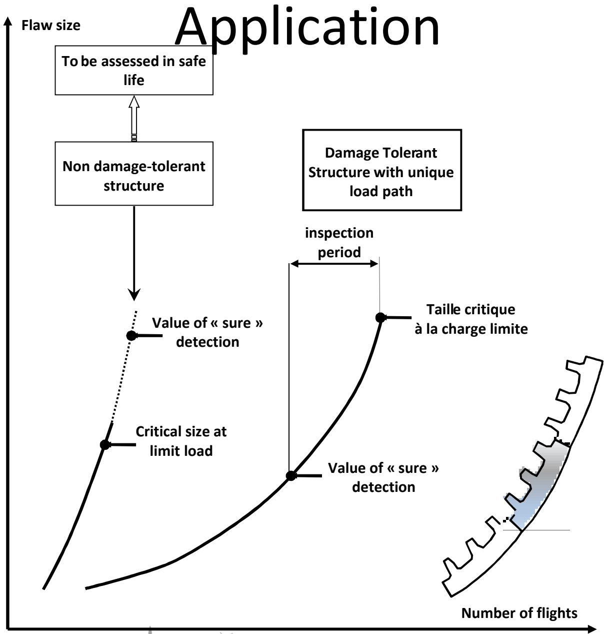
Application



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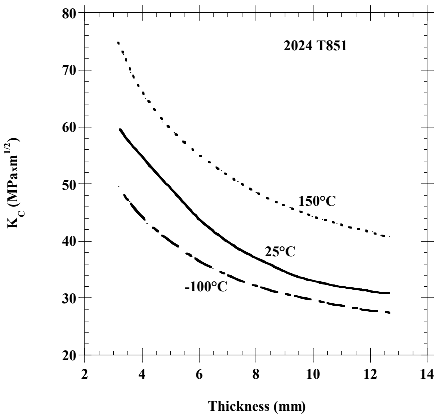
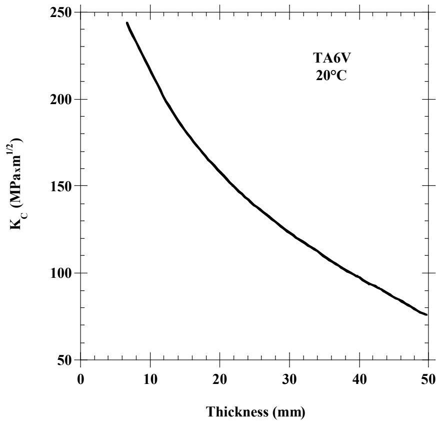
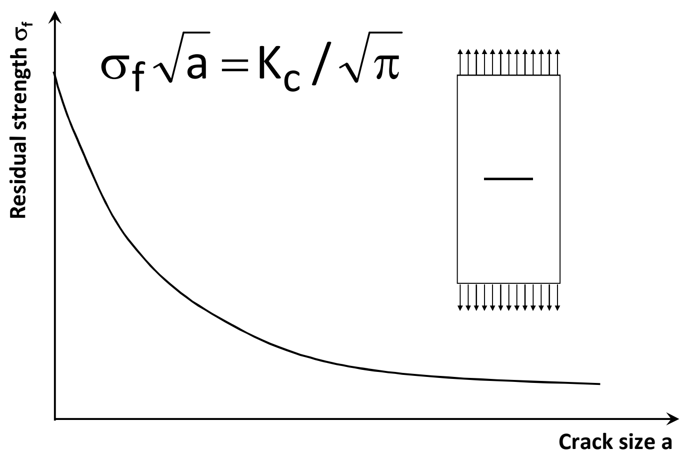
Toughness data as a function of thickness



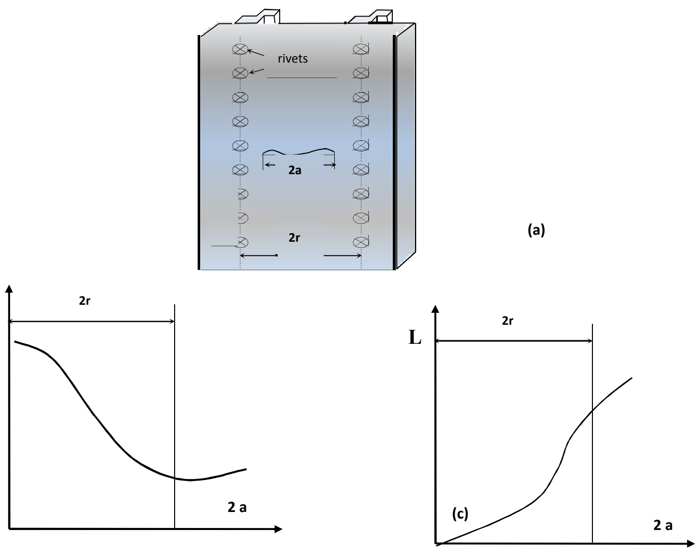


Residual strength of a panel

$$\sigma_f \sqrt{a} = K_C / \sqrt{\pi}$$

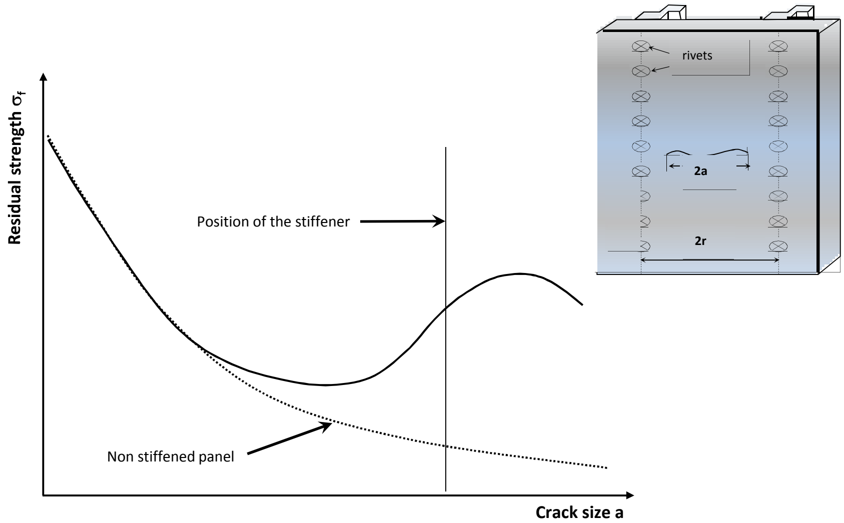


Residual strength of a stiffened panel

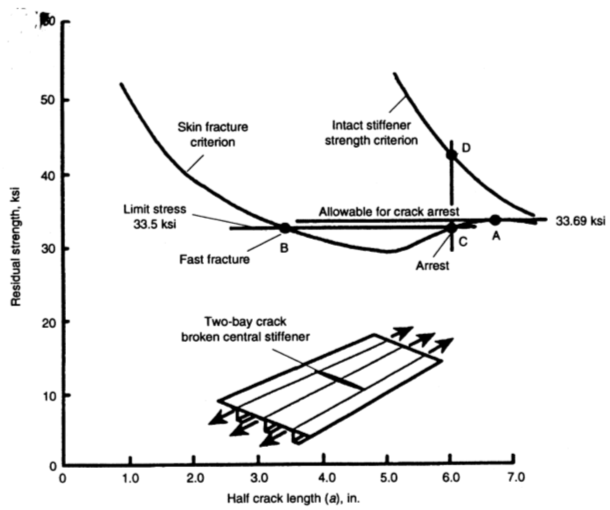


$$\beta = \frac{K}{\sigma\sqrt{\pi a}}$$

Residual strength of a stiffened panel



Example of residual strength diagram



Damage Tolerance

Different options

Options

A. periodic inspections: repair if flaw detection;

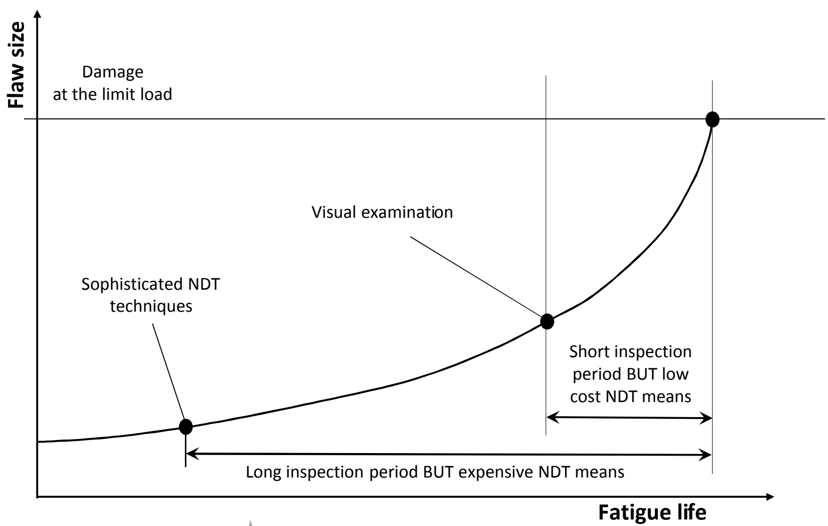
B. "Fail Safe" design: repair if partial failure;

C. Leak Before Breakage Design: Pressure Vessels

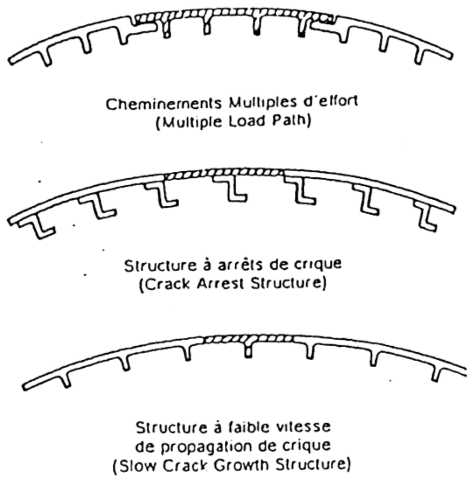
D. Durability design: replacement or withdrawal after time H;

E. Proof test (destructive test): repair if rupture during the test.

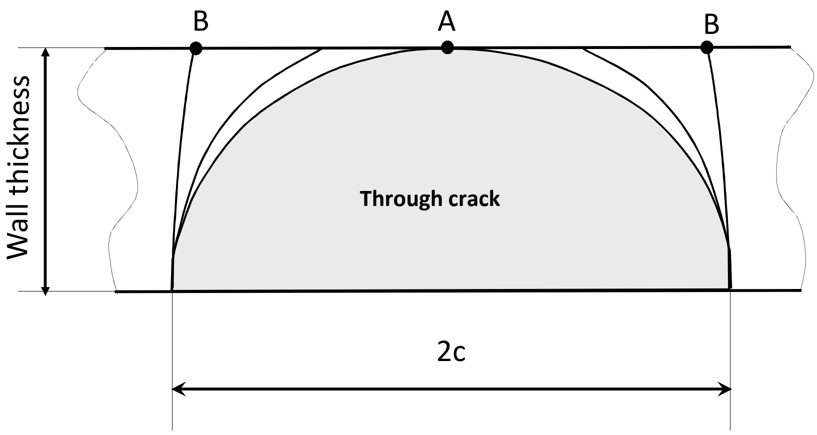
Periodic inspection



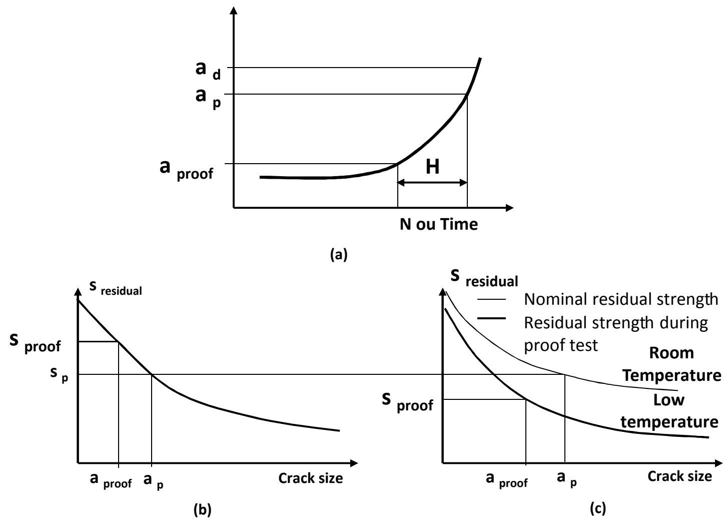
Fail Safe



Leak before breakage (LBB)



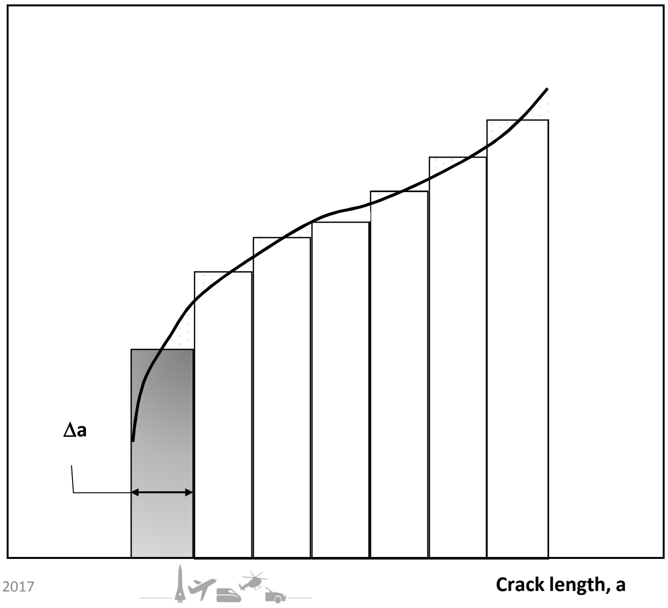
Proof test



Damage Tolerance

Calculation of propagation life

Integration



Iterative computation

Initialisation

$a_j = a_0; \quad N = 0;$

$$a_i = a_j$$

$$\beta_i = f\left(\frac{a_i}{W}\right)$$

$$\Delta K_i = \beta_i \times \sigma_i \times \sqrt{\pi a_i}$$

$$\left(\frac{da}{dN}\right)_i = F(\Delta K_i, R, \ldots)$$

$$\Delta a_i = \delta a_i \quad (0.01\text{mm for example})$$

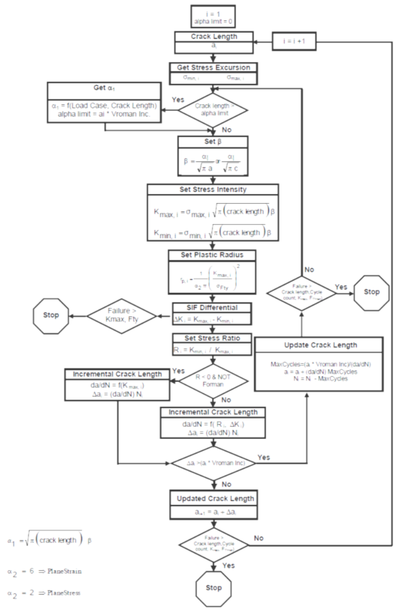
$$\Delta N_i = \frac{\Delta a_i}{\left(\frac{da}{dN}\right)_i}$$

$$N_j = N_i + \Delta N_i$$

$$a_j = a_i + \Delta a_i$$

$$\text{if } (a_j < a_c) \text{ continue}$$

AFGROW Flowchart

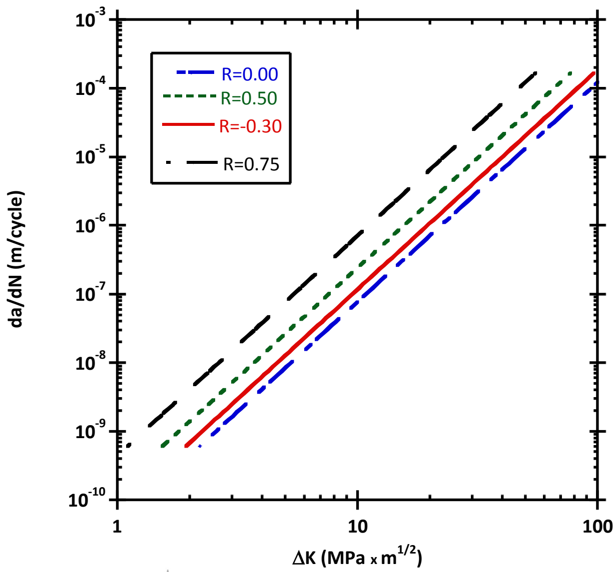


Walker equation

$$\frac{da}{dN} = C \times \left[K_{max} \times (1-R)^{1-m} \right]^{n} \qquad \text{for} \qquad R \leq 0$$

$$\frac{da}{dN} = C \times \left[\Delta K \times (1-R)^{m-1} \right]^{n} \qquad \text{for} \quad R \geq 0$$

Walker equation

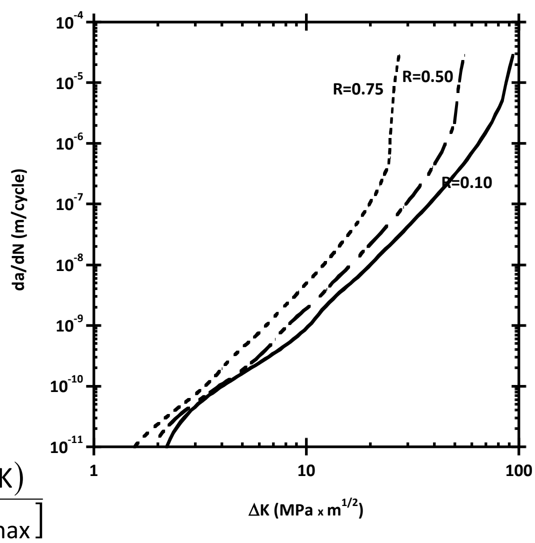


Forman equation

$$\frac{da}{dN} = C \times \frac{\Delta K^m}{[(1-R)K_c - \Delta K]}$$

McEvily - Forman

$$\frac{da}{dN} = C \times \left(\Delta K^2 - \Delta K_{seuil}^2\right) \frac{(1+\Delta K)}{[K_c - K_{max}]}$$



NASGRO equation

$$\frac{da}{dN} = C \times \left[\left(\frac{1-f}{1-R}\right) \times \Delta K\right]^n \times \frac{\left(1 - \frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1 - \frac{K_{max}}{K_{crit}}\right)^q}$$

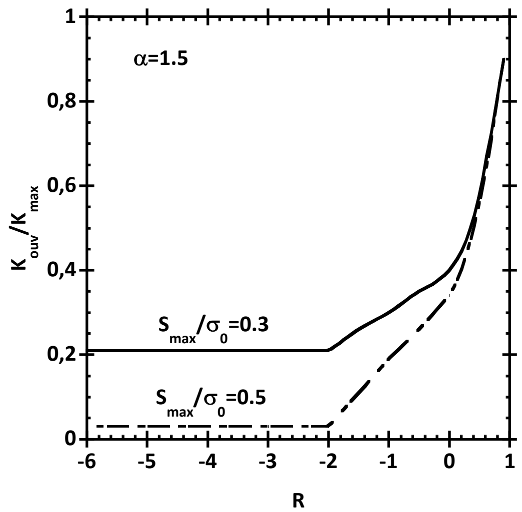
The *f* function accounts for closure effects $f = \frac{K_{ouv}}{K_{max}} = \max\left(R, A_0 + A_1 R + A_2 R^2 + A_3 R^3\right)$

$$A_0 = \left(0.825 - 0.34\alpha + 0.05\alpha^2\right) \times \left[\cos\left(\frac{\pi}{2} \times \frac{S_{max}}{\sigma_0}\right)\right] \qquad A_2 = 1 - A_0 - A_1 - A_3$$

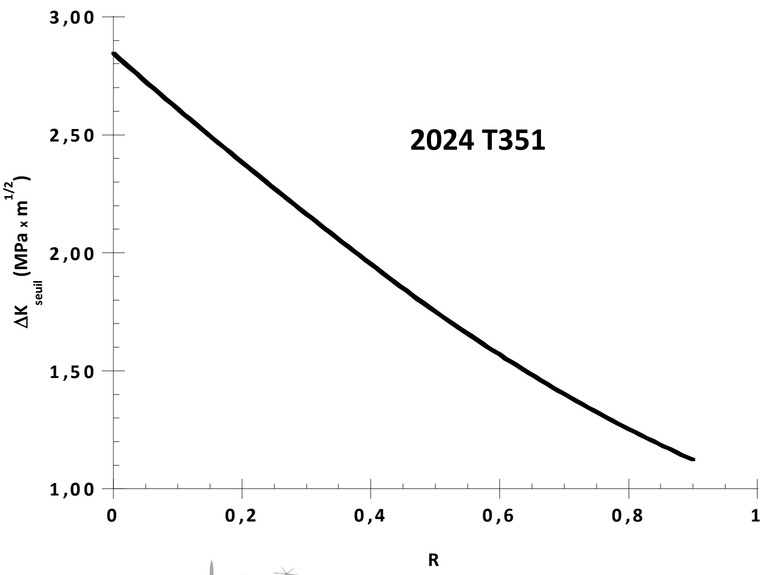
$$A_1 = (0.415 - 0.071\alpha) \times \frac{S_{max}}{\sigma_0} \qquad A_3 = 2A_0 + A_1 - 1$$

Threshold function : $\Delta K_{th} = \Delta K_0 \times \frac{\left(\frac{a}{a + a_0}\right)^{1/2}}{\left(\frac{1-f}{(1-A_0)(1-R)}\right)^{(1+C_{th}R)}}$

NASGRO: closure



NASGRO: Threshold



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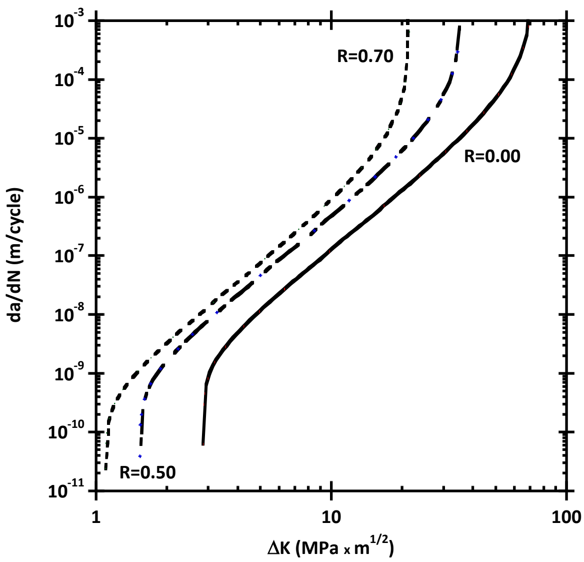
NASGRO



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Sources of error

Error	Source	Comment	Factor on predicted fatigue life
LEFM Intrinsic Limitations		Small error for small cracks or small components	1.1 - 2
Input data	K_{Ic} or K_c ou autre	Error on a_p	1.1-1.2
	da/dN	Natural scatter	1.1-1.5
	Environment	Weight average	1-2
	Crack growth equation	Adjustment	1.5
Assumptions	Crack growth direction (ex. LT vs SL)	Use of erroneous data	1-2
	Initial flaw size	Strong influence	1-3
	Shape	Surface flaws	1-2
Stress Intensity Factor	Loads/Stresses	Measurements	1-1.5
		Analysis	1-1.5
Load History	Sequence	Random or semi-random	1.5
	Truncation	Inadequate truncation	1.1-1.3
Retardation	Model	Small if well calibrated	1.1-1.3
	Stress state	If not taken into account	1.1-1.5
	Yield strength	For example 10% too low	1.5-2.0

Example: crack growth in a lug

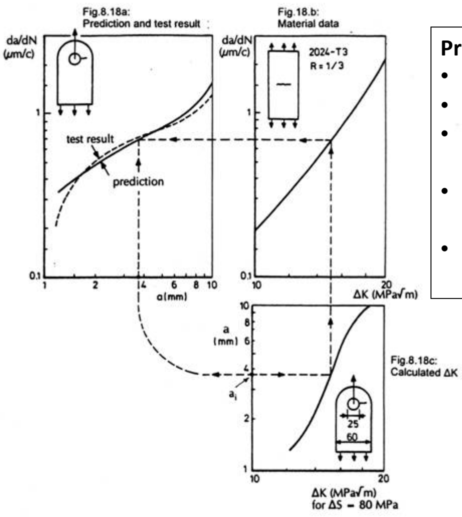
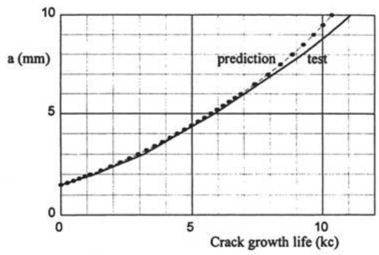
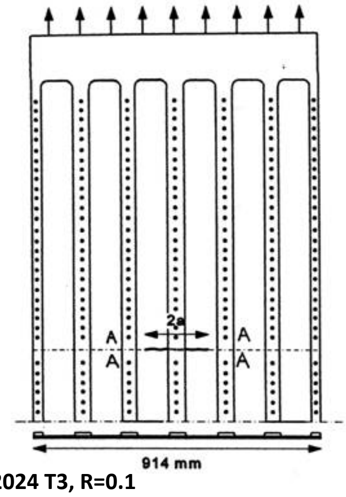


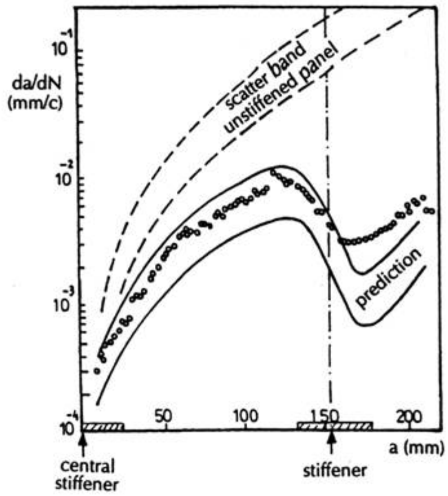
Fig.8.18: Prediction of the growth rate of a fatigue crack at the edge of a hole in a lug [33].

Procedure:
- Consider an initial default size a_i;
- Determine the ΔK in the screed for a = a_i;
- For this value read the corresponding crack growth rate;
- Report the growth rate on the da / dN = f (a) diagram
- The curve da / dN = f (a) must be integrated to obtain a = f (N)



Example 2: stiffened panel



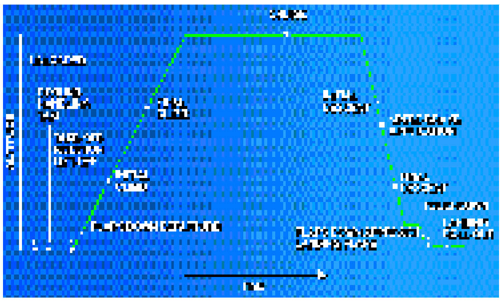


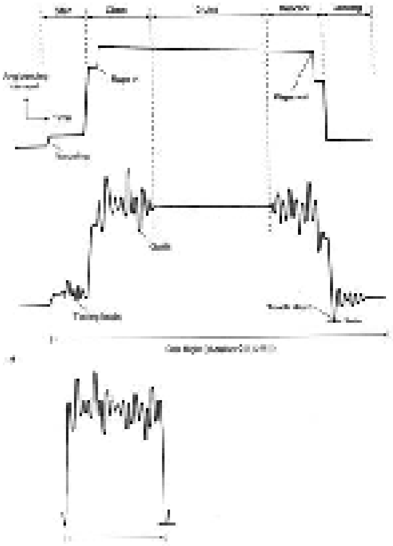
2024 T3, R=0.1
Crack initiated in the central bay

Damage Tolerance

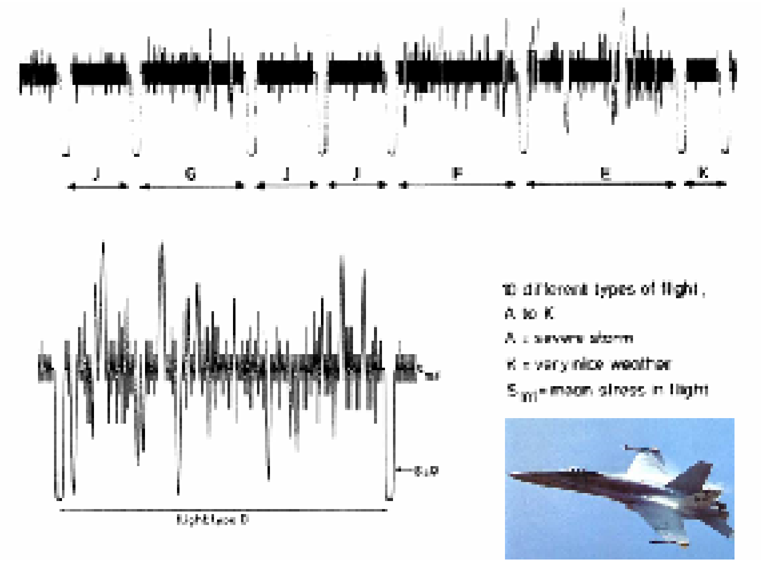
Variable amplitude loading -
Retardation

Load spectrum during a flight





Spectrum of a F-18 fighter aircraft wing

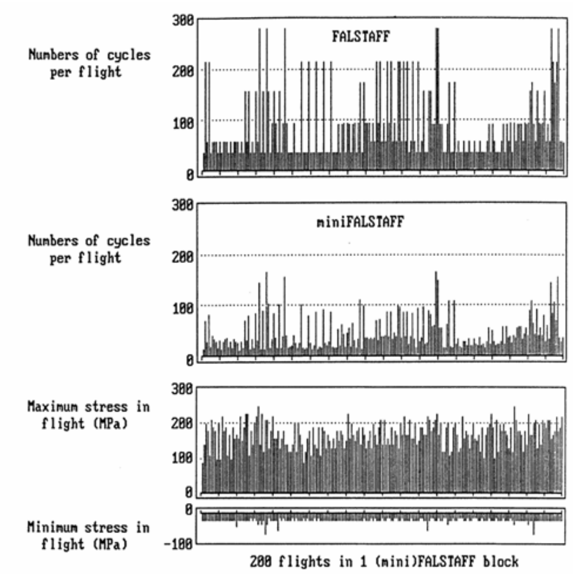


Standard load spectra

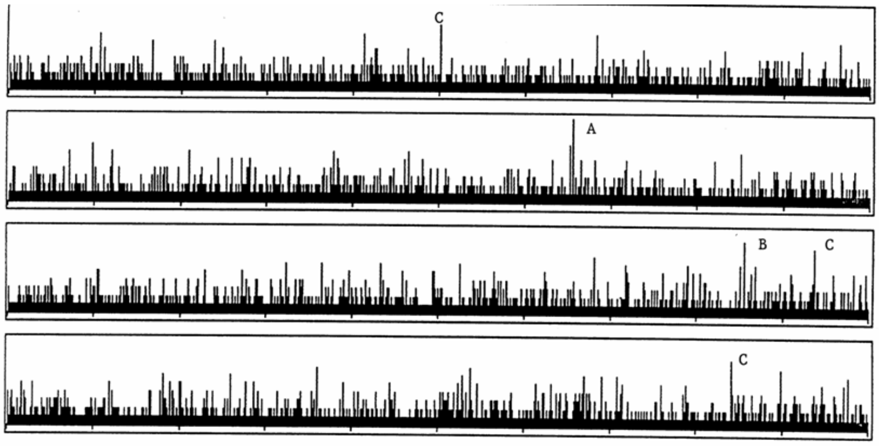
Year	Name	Load history for:
1973	TWIST	Transport aircraft lower wing skin
1976	FALSTAFF	Fighter aircraft lower wing skin
1977	GAUSSIAN	Random loading
1979	miniTWIST	Shortened TWIST
1983	HELIX/FELIX	Helicopter main rotor blades
1987	ENSTAFF	Tactical aircraft composite wing skin
1987	Cold TURBISTAN	Fighter aircraft engine, cold engine disks
1990	Hot TURBISTAN	Fighter aircraft engine, hot engine disks
1990	WASH	Offshore structures
1990	CARLOS	Car components
19xx	WALZ	Steel mill drive
1991	WISPER/WISPERX	Horizontal axis wind turbine blades

Source: Ref 35, 36

FALSTAFF



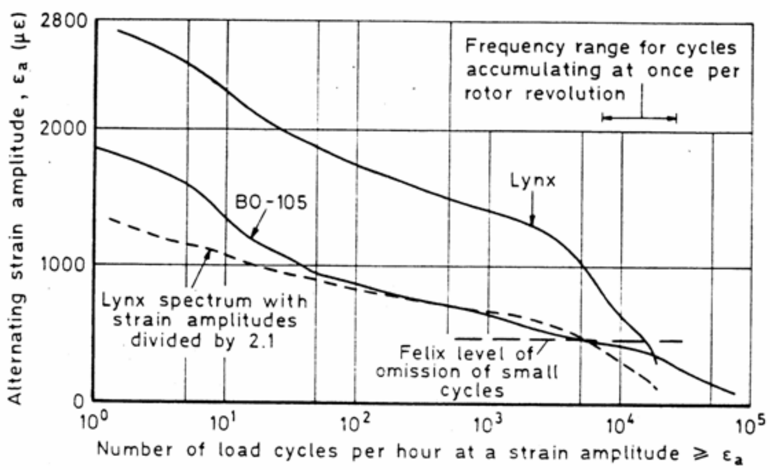
TWIST



TWIST and miniTWIST, flight sequence in one block of 4000 flights.

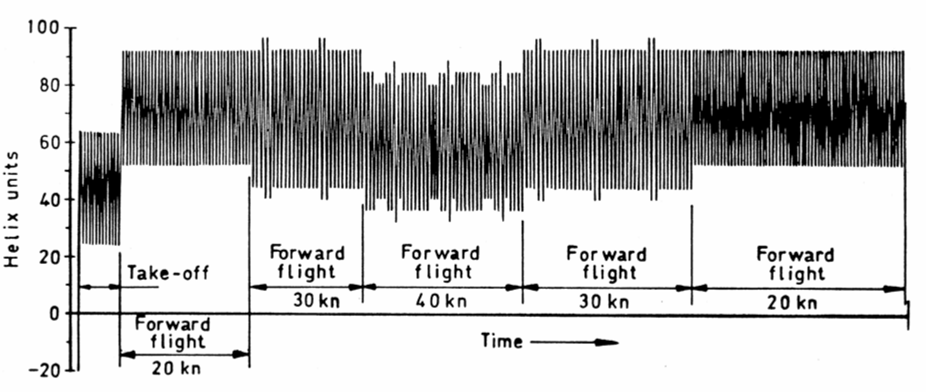
Numbers of most severe flights :
Type A: 1656, type B: 2856, type C: 501, 2936, 3841
Type D: 106, 412, 684, 1099, 1653, 2682, 3360, 3538, 3898

HELIX/FELIX



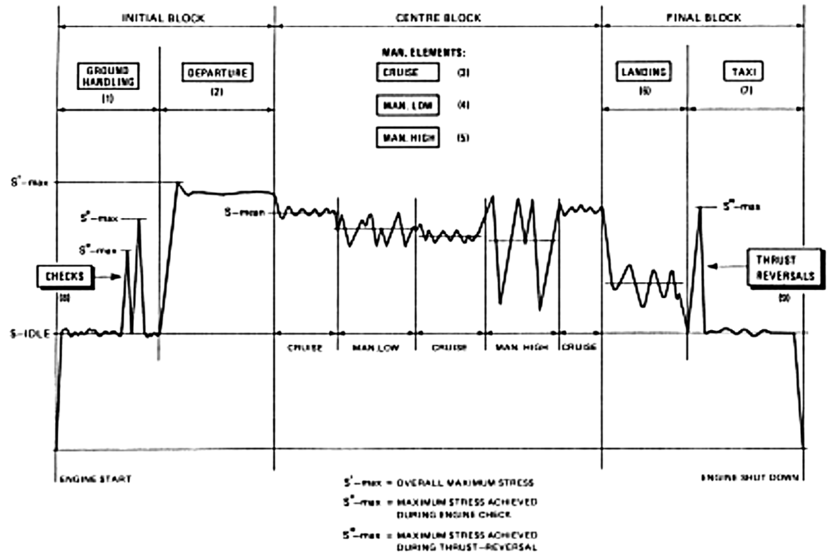
The distribution of cycles depends on the type of aircraft

HELIX



Example of the load time history for the first phase of a training flight in Helix

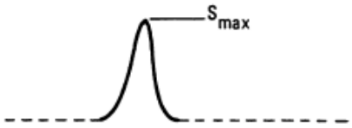
TURBISTAN



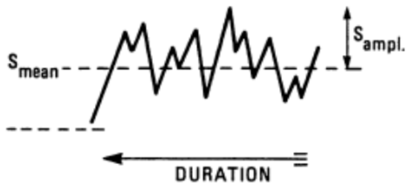
TURBISTAN

- **DETERMINISTIC**
  - ENGINE CHECKS
  - DEPARTURE *
  - THRUST REVERSAL

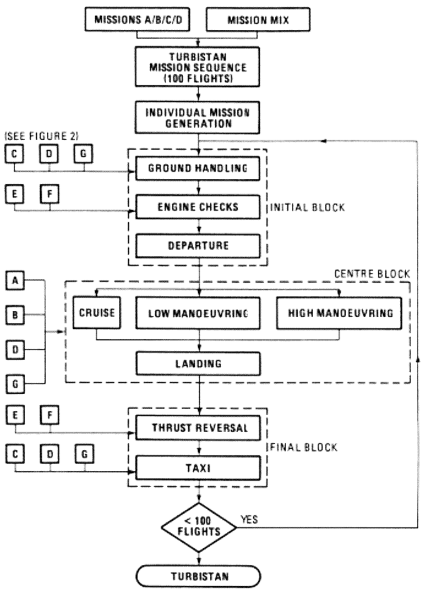
  * ONE EXCURSION TO 100 % LOAD



- **STOCHASTIC**
  - GROUND HANDLING
  - CRUISE
  - LOW MANOEUVRING
  - HIGH MANOEUVRING
  - LANDING
  - TAXI

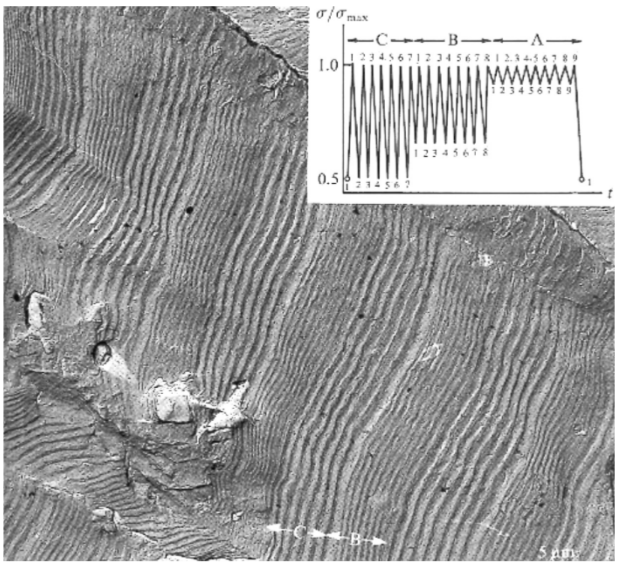


TURBISTAN

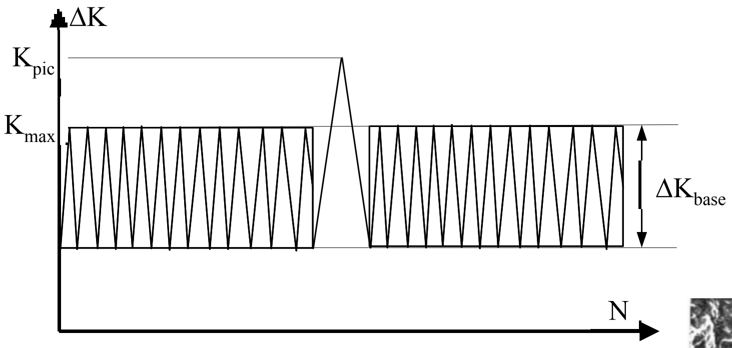


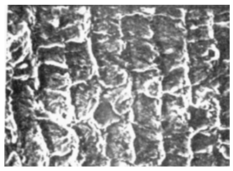
Load history effects

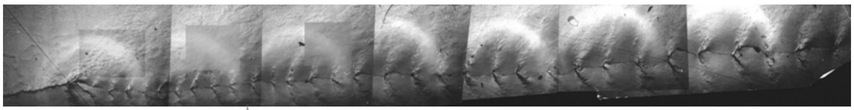
Markings on fracture surfaces



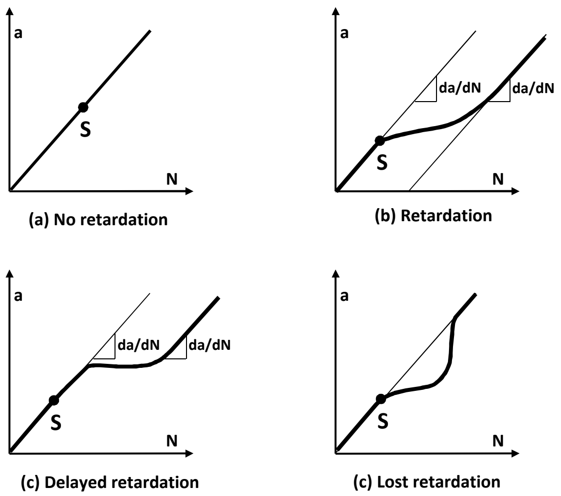
Overload



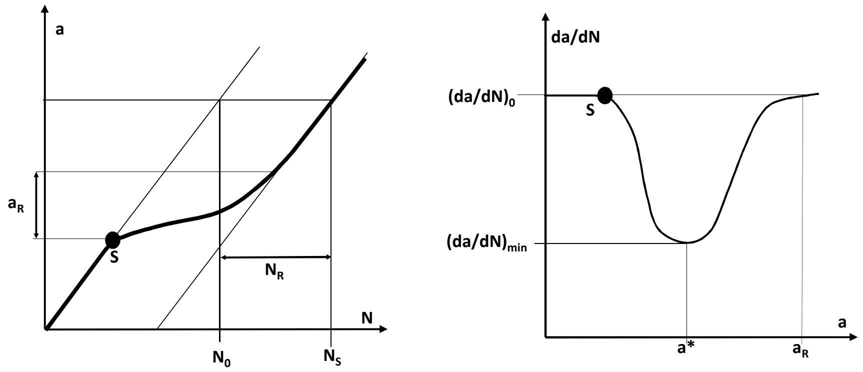




Retardation



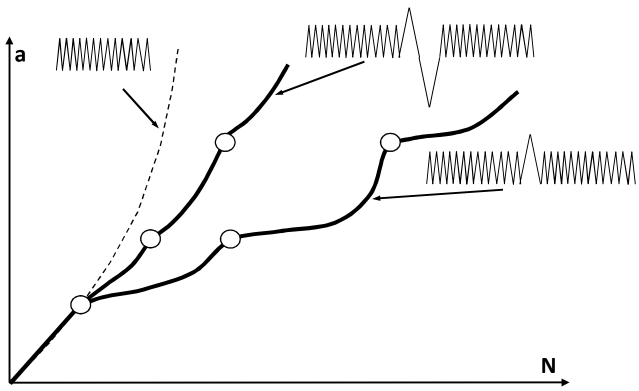
Characteristic retardation parameters



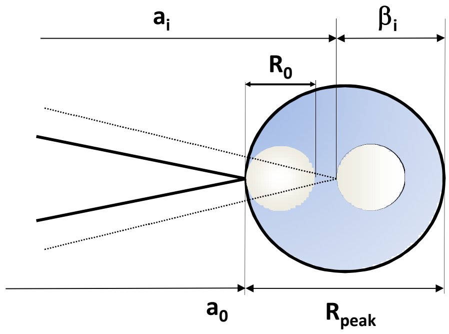
Retardation mechanisms

- The effect of residual stresses induced by the overload in front of the crack tip;
- An increase in the crack closure effects induced by the material highly stretched during the overload and subsequently passed in the crack wake;
- A crack tip blunting which would reduce the magnitude of stresses sustained at the crack tip assimilated to a micro-notch;
- Crack tip hardening of the material induced by the overload.

Overload/underload



Wheeler model



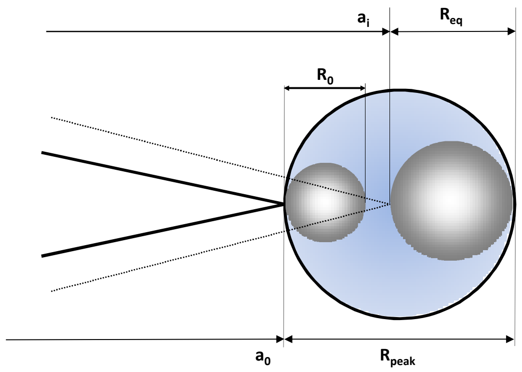
Plastic zone interactions

$$C_p = \left(\frac{R_0}{R_{pic}}\right)^{\lambda}$$

$$C_{p_i} = \left(\frac{R_0}{\beta_i}\right)^{\lambda}$$

$$\beta_i = a_0 + R_{peak} - a_i$$

Willenborg model



$$R_{eff} = \frac{K_{min_{eff}}}{K_{max_{eff}}}$$

$$K_{min_{eff}} = K_{min} - K_R$$

$$K_{max_{eff}} = K_{max} - K_R$$

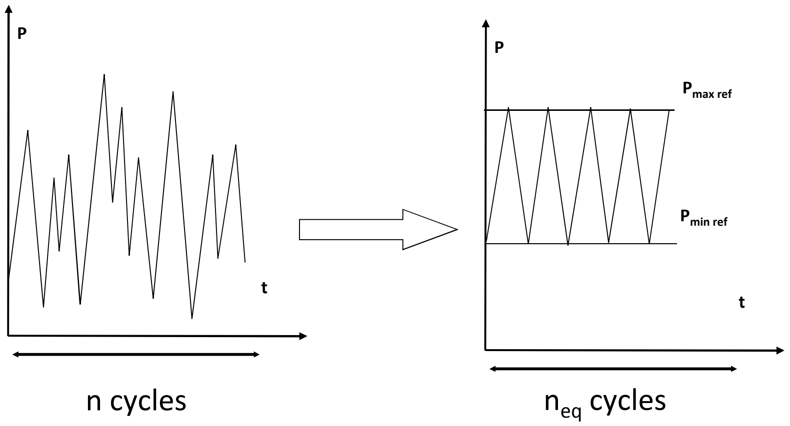
K_R: SIF required to create a PZ with a size R_{eq} defined by:

$$R_{eq} = a_0 + R_{pic} - a_i$$

$$K_R = K_{pic} \times \sqrt{1 - \frac{ai - a_0}{R_{pic}}}$$

Retardation predicted if a load ratio effect is noticed on the da/dN curves.

PREFFAS



CAL equivalent sequence

PREFFAS

- At cycle i, the crack growth is given by:

$\Delta a_i = C \times \Delta K_{eff}{}^m = C \times (K_{max,\,i} - K_{op,i})^m$

where C and m are the Paris law coefficients de la loi de Paris.

- The value of $K_{op,\,i}$ depends on the load history and is determined by a Elber relation :

$K_{max,\,i} - K_{op,i} = U \times (K_{max} - K_{min})$

With: $U(R) = A \times (K_{max,\,H} / K_{min,\,H}) + B$

Where $K_{min,\,H}$ et $K_{max,\,H}$ are values representative of the load history.

PREFFAS

$$K_{min,k} = \inf_{p=j,i-1}\left(K_{min,p}\right)$$

$$K_{op} = K_{max,i} - U(R) \times \left[K_{max,i} - K_{min,i}\right]$$

$$U(R) = A \times \frac{K_{min,k}}{K_{max,j}} + B$$

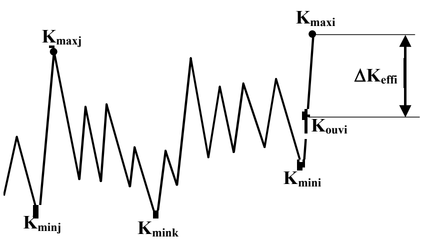
$$K_{op,i} = \sup_{j=1,i-1}\left(K_{op,j}\right)$$

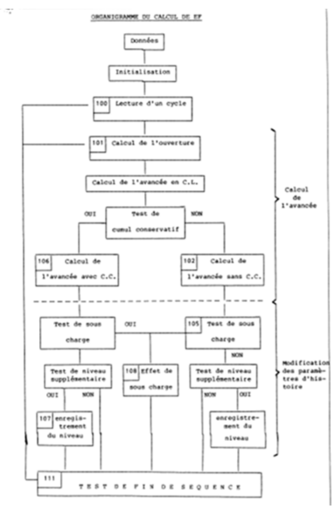
PREFFAS

- The prediction of the propagation under variable amplitude loading therefore only requires the determination of 4 parameters:
  - C and m, the coefficients of the PARIS law
  - A and B, the coefficients of the ELBER relation

- These 4 coefficients are derived from two tests
  - 1 under constant amplitude loading R=0.1
  - 1 under similar baseline loading with the introduction of an overload at τ = 1.7 every 1000 cycles.

PREFFAS





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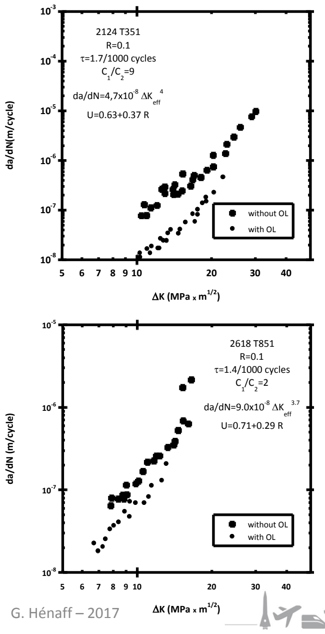

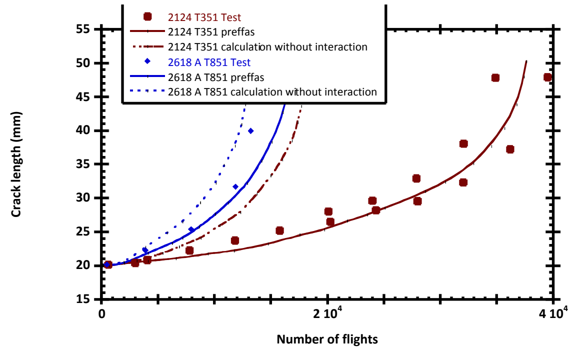

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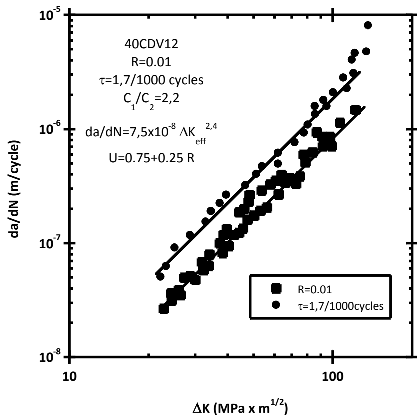
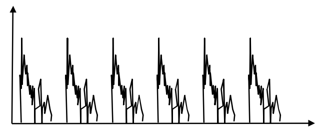
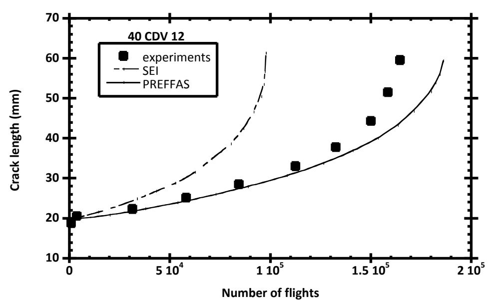
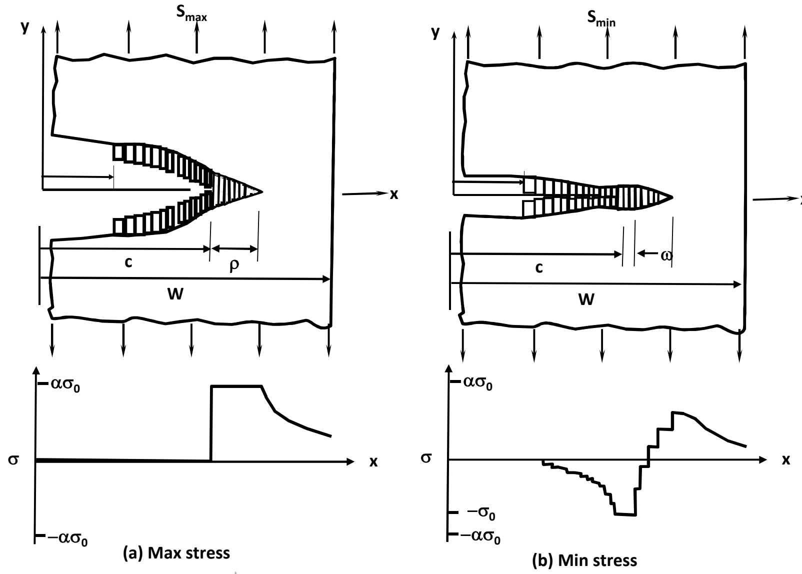




Pylon spectrum



FASTRAN



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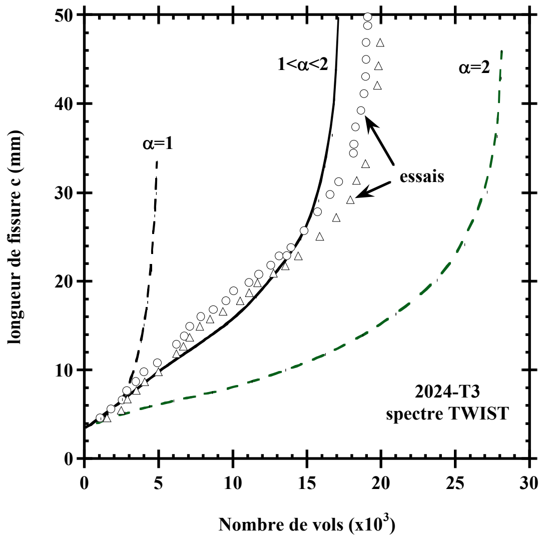

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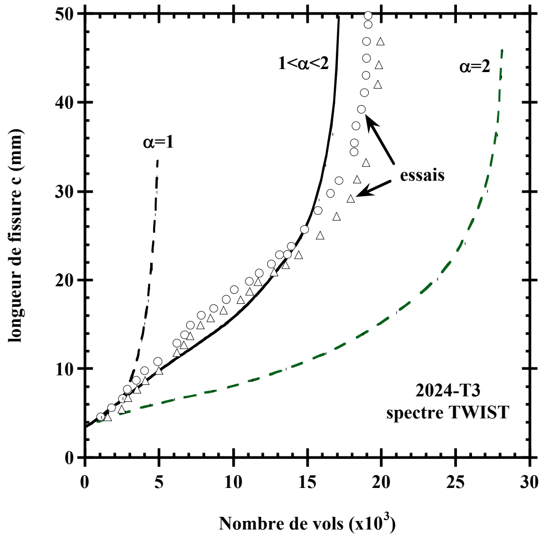
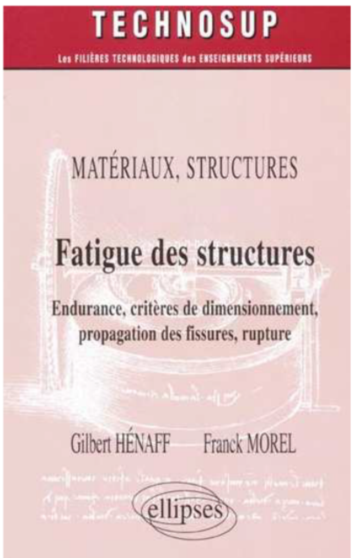

longueur de fissure c (mm)
Nombre de vols (x10³)
α=1
1<α<2
α=2
essais
2024-T3
spectre TWIST
0
5
10
15
20
25
30
0
10
20
30
40
50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