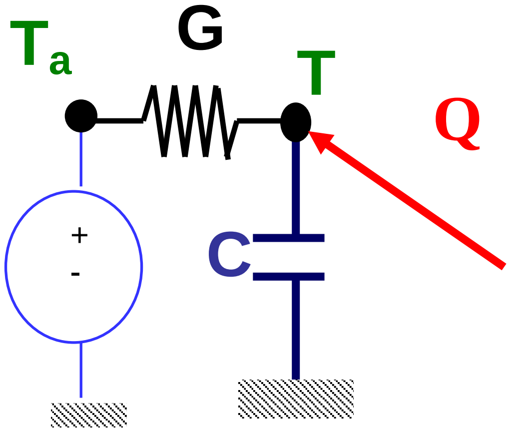
Partie 1-Solution

1)Biot number
Bi =h*D/λ

Bi =7.81 10^{-5}

2) When the Biot number is very small in front of 1, the object can be considered as isothermal

3) In a nodal representation, the fuse can be represented by a single node T.



C = ρ c Volume = ρ c s L with s = 3.14*(D/2)^2
C= 7.07 10^{-3} J/K

G = h S with S=3.14*D*L
G = 3.14 10^{-4}W/K

Q = R_e I^2 with R_e = ρ_e L/s
Q=1.22 W

4) Nodal equation

$$C\frac{dT}{dt} = G(T_a - T) + Q$$

5) $\frac{dT}{dt} = 0$, then $0 = G(T_a - T_{eq}) + Q$, and $T_{eq} = T_a + Q/G$
Teq = 3910 °C

6) τ = C/G
τ = 22.5 s

7) Solution

$$T = T_a + \frac{Q}{G}\left(1 - \exp\left(-t/\tau\right)\right)$$

8) $\frac{t_f}{\tau} = Ln\left(\frac{1}{1-\frac{G}{Q}\left(T_f - T_a\right)}\right)$

$t_f = 1.26$ s

The time to get the fusion is too long to protect efficiently!

9)
Radiation enhances the transfer, which means that the cooling of the fuse is more intense

10) As a consequence, the time to reach the steady state is shorter. In fact the fusion occurs at a time slightly shorter than without radiation

Solution de la Partie rayonnement

First exercice

1) Solar absorptivity

$\varepsilon_1 = \alpha_1 = 0{,}9$ $\varepsilon_2 = \alpha_2 = 0{,}1$

$\lambda_1 = 3\ \mu$ $T_S = 5800$ K $\lambda_1 T_S = 17400\ \mu$K $F_1 = 0{,}995$

$\alpha_S = \alpha_1 F_1 + \alpha_2 (1-F_1)$ $\alpha_S = 0{,}896$

IR emisivity

$\lambda_1 = 3\ \mu$ $T = 500$ K $\lambda_1 T = 1500\ \mu$K $F_1 = 0{,}0128$

$\varepsilon_{IR} = \varepsilon_1 F_1 + \varepsilon_2 (1-F_1)$ $\varepsilon_{IR} = 0{,}11$

$\lambda_1 = 3\ \mu$ $T = 600$ K $\lambda_1 T = 1800\ \mu$K $F_1 = 0{,}0393$

$\varepsilon_{IR} = \varepsilon_1 F_1 + \varepsilon_2 (1-F_1)$ $\varepsilon_{IR} = 0{,}13$

2) Radiative balance equation

$S\, \alpha_S\, \varphi_S = 2\, \varepsilon_{IR}\, S\, \sigma(T^4 - T_e^{\,4})$,
where T_e is the temperature of the environment. $T_e = 300$ K (usual ambient temperature in on earth conditions, which is the value given at the end of the text).

The equilibrium temperature is given by:

$$T = \left(T_e^4 + \frac{\alpha_S \varphi_S}{2\varepsilon_{IR}\sigma} \right)^{0,25}$$

T = 573,8 K

Comment:
The choice, for the calculation of ε_{IR} , of the equilibrium temperatures, T = 500 K or T = 600 K (question 1) has a small influence. For instance, T = 573 K for $\varepsilon_{IR} = 0{,}11$ or T= 551 K for $\varepsilon_{IR} = 0{,}13$. The exact value of T should be obtained with an iterative procedure.

Second exercice

1) View factor matrix
The single value which is given is $F_{23} = 0{,}059$

$F_{22} = F_{33} = 0$ (Plane surfaces)

$F_{21} = 1 - F_{22} - F_{23}$ (flux conservation) $F_{21} = 0{,}941$
$F_{12} = S_2 * F_{21} / S_1$ $F_{12} = 0{,}118$
$F_{13} = F_{12}$ (Symetry) $F_{13} = 0.118$
$F_{11} = 1 - F_{12} - F_{13}$ (flux conservation) $F_{11} = 0{,}765$
$F_{32} = F_{23}$ (Symetry) $F_{32} = 0.059$
$F_{31} = 1 - F_{32} - F_{33}$ (flux conservation) $F_{31} = 0{,}941$

2) The Gebhart factor B_{ij} represents the fraction of flux emitted by surface S_i and absorbed by surface S_j., when taking into account all the possible paths from S_i to S_j.

It has a first contribution corresponding to the direct transfer, that is the flux emitted by S_i which is absorbed by S_j, in this direct transfer:
$\alpha_j F_{ij}$.

The second contribution represents the flux emitted by S_i, reflected by all the S_k, which is finally absorbed by S_j, after all the possible reflections. For one such surface S_k it is given by :
$\rho_k F_{ik} B_{kj}$

Then for the complete cavities, we have:

$$B_{ij} = \alpha_j F_{ij} + \sum_k \rho_k F_{ik} B_{kj}$$

The flux conservation is written as :

$$\sum_j B_{ij} = 1$$

The reciprocity formula, between Gebhart factors can be written as :

$$\varepsilon_i S_i B_{ij} = \varepsilon_j S_j B_{ji}$$

3) To obtain B_{11}, B_{21}, B_{31}, the following system should be solved :

$$B_{11} = \alpha_1 F_{11} + \rho_1 F_{11} B_{11} + \rho_2 F_{12} B_{21} + \rho_3 F_{13} B_{31}$$
$$B_{21} = \alpha_1 F_{21} + \rho_1 F_{21} B_{11} + \rho_2 F_{22} B_{21} + \rho_3 F_{23} B_{31}$$
$$B_{31} = \alpha_1 F_{31} + \rho_1 F_{31} B_{11} + \rho_2 F_{32} B_{21} + \rho_3 F_{33} B_{31}$$

4) Matrix of Gebhart factors
The datas are the following coefficients :

$B_{11}=0{,}803$ $B_{21}=0{,}818$ $B_{31}=0{,}818$ $B_{22}=0{,}092$.
From the reciprocity formula we get:

$B_{12} = \varepsilon_2 S_2 B_{21} / \varepsilon_1 S_1$ $B_{12}=0{,}103$

The flux conservation gives then: $B_{13} = 1 - B_{11} - B_{12}$ $B_{13} = 0{,}094$
The flux conservation gives also : $B_{23} = 1 - B_{21} - B_{22}$ $B_{23} = 0{,}09$
$B_{32} = B_{23}$ (Symetry) $B_{32} = 0{,}09$

Finally, the flux conservation gives $B_{33}=1-B_{31}-B_{32}$ $B_{33}= 0,092$

5)

$$g_{12} = \varepsilon_1 S_1 B_{12}$$
$$g_{13} = \varepsilon_1 S_1 B_{13}$$
$$g_{23} = \varepsilon_2 S_2 B_{23}$$
$$g_{2e} = \varepsilon_{2e} S_2$$
$$g_{3e} = \varepsilon_{IR} S_3$$
$$Q_3 = \alpha_S \varphi_S S_3$$

6)

$S_1 = 3,53\ 10^{-2}$ $S_2 = 4,42\ 10^{-3}$ $S_3 = 4,42\ 10^{-3}$
$g_{12} = 3,62\ 10^{-4}$ $g_{13} = 3,62\ 10^{-4}$ $g_{23} = 4,31\ 10^{-5}$ $g_{2e} = 3,97\ 10^{-3}$
$g_{3e} = 4,87\ 10^{-4}$

$Q_3 = 5,54$ W

7)

$$\varepsilon_1 S_1 B_{13}\sigma\left(T_3^4 - T_1^4\right) + \varepsilon_1 S_1 B_{12}\sigma\left(T_2^4 - T_1^4\right) = 0$$
$$\varepsilon_1 S_1 B_{12}\sigma\left(T_1^4 - T_2^4\right) + \varepsilon_2 S_2 B_{23}\sigma\left(T_3^4 - T_2^4\right) + \varepsilon_{2e} S_2 \sigma\left(T_e^4 - T_2^4\right) = 0$$
$$\varepsilon_1 S_1 B_{13}\sigma\left(T_1^4 - T_3^4\right) + \varepsilon_2 S_2 B_{23}\sigma\left(T_2^4 - T_3^4\right) + \varepsilon_{IR} S_3 \sigma\left(T_e^4 - T_3^4\right) + Q_3 = 0$$

8) T_3 is the highest , T_2 the lowest and T_1 is in between.

$T_1 = 534$ K
$T_2 = 353$ K
$T_3 = 620$ K