Elastic linear isotropic Hook's law

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1.Definition of “classical elastic” schema - Hook's Law

-Relation of stress and strain:one dimensional Hook's law

-Generalized Hook's law

-Isotropy of materials

-**Physical equations**

2.Contants of materials

-Module Young and Poisson ratio

-Lame constant

-Relations of constants

1.Definition of "classical elastic" schema - Hook's Law

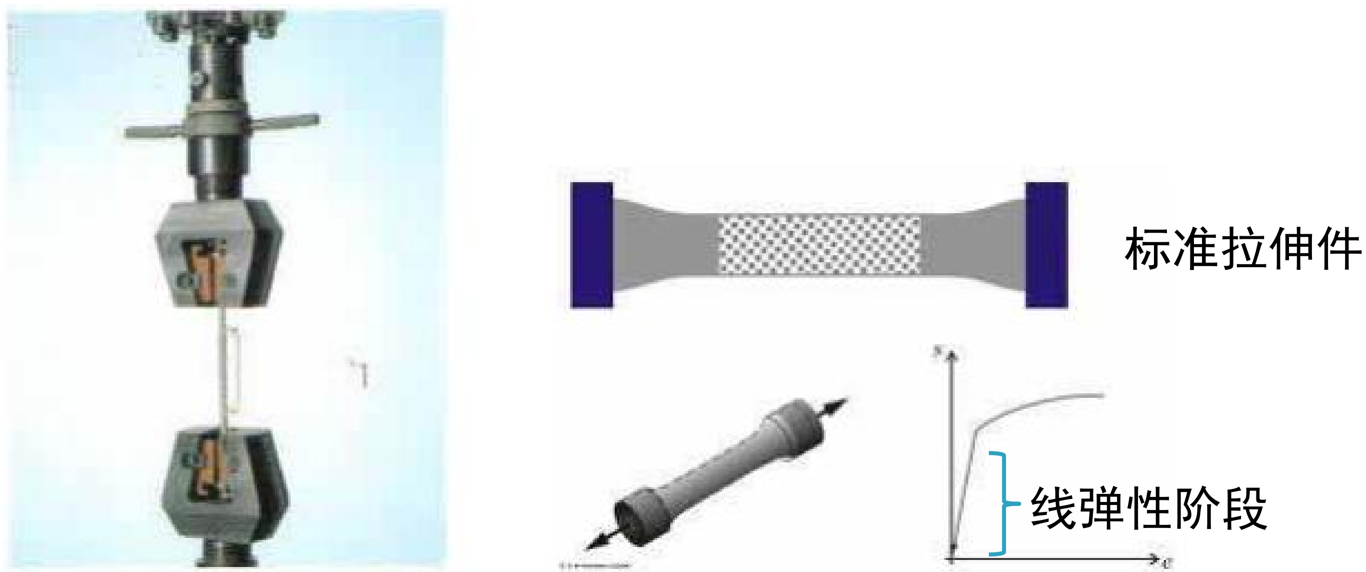

Figure 4.1a. Uniaxial tension

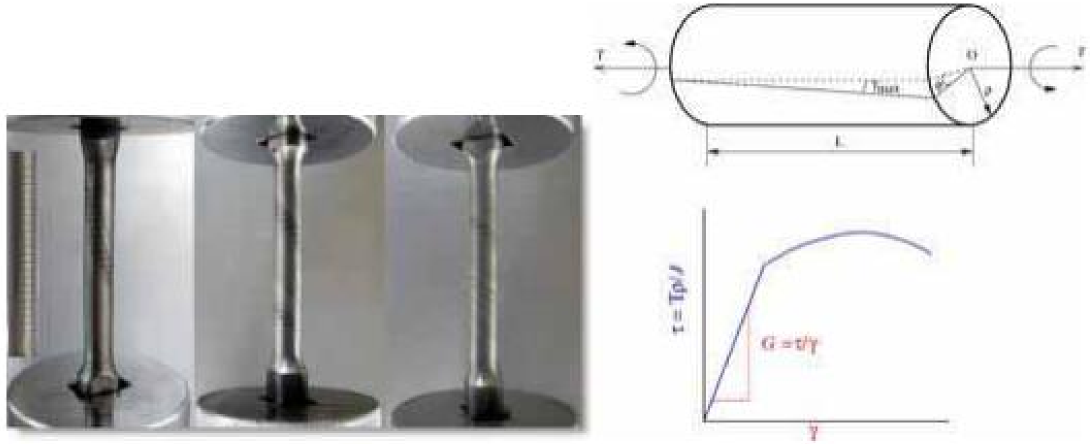

Figure 4.1b. Torsion

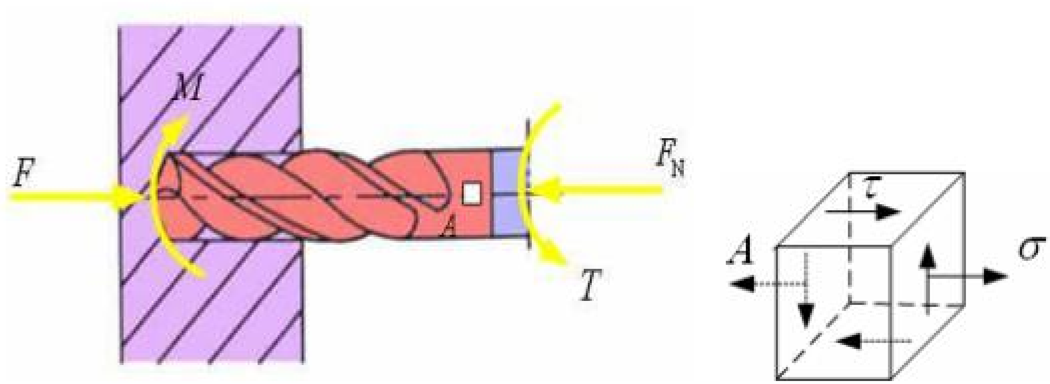

Figure 4.2. Complexe stress state

-Generalized Hook's law

$$\begin{cases} \sigma_x = C_{11}\varepsilon_x + C_{12}\varepsilon_y + C_{13}\varepsilon_z + C_{14}\gamma_{xy} + C_{15}\gamma_{yz} + C_{16}\gamma_{xz} \\ \sigma_y = C_{21}\varepsilon_x + C_{22}\varepsilon_y + C_{23}\varepsilon_z + C_{24}\gamma_{xy} + C_{25}\gamma_{yz} + C_{26}\gamma_{xz} \\ \sigma_z = C_{31}\varepsilon_x + C_{32}\varepsilon_y + C_{33}\varepsilon_z + C_{34}\gamma_{xy} + C_{35}\gamma_{yz} + C_{36}\gamma_{xz} \\ \tau_{xy} = C_{41}\varepsilon_x + C_{42}\varepsilon_y + C_{43}\varepsilon_z + C_{44}\gamma_{xy} + C_{45}\gamma_{yz} + C_{46}\gamma_{xz} \\ \tau_{yz} = C_{51}\varepsilon_x + C_{52}\varepsilon_y + C_{53}\varepsilon_z + C_{54}\gamma_{xy} + C_{55}\gamma_{yz} + C_{56}\gamma_{xz} \\ \tau_{xz} = C_{61}\varepsilon_x + C_{62}\varepsilon_y + C_{63}\varepsilon_z + C_{64}\gamma_{xy} + C_{65}\gamma_{yz} + C_{66}\gamma_{xz} \end{cases} \qquad (4.4)$$

-***Physical equations**

$$\begin{cases} \varepsilon_x = \frac{1}{E}[\sigma_x - \mu(\sigma_y + \sigma_z)] \\ \varepsilon_y = \frac{1}{E}[\sigma_y - \mu(\sigma_x + \sigma_z)] \\ \varepsilon_z = \frac{1}{E}[\sigma_z - \mu(\sigma_x + \sigma_y)] \\ \gamma_{xy} = \frac{1}{G}\tau_{xy} \\ \gamma_{yz} = \frac{1}{G}\tau_{yz} \\ \gamma_{xz} = \frac{1}{G}\tau_{xz} \end{cases} \qquad (4.5)$$

Volumetric strain and volumetric stress:

Adding the first three equations of equation (4.5),

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\mu}{E}(\sigma_x + \sigma_y + \sigma_z) \tag{4.6}$$

Volumetric stress is defined as

$$\theta = \frac{1-2\mu}{E}\Theta \tag{4.7}$$

-Constants of materials:

-Module Young

-Poisson ratio

-Lame constant

Transform physical equation (4.5), express stress components with strain components,

$$\begin{cases} \sigma_x = \lambda\theta + 2v\varepsilon_x \\ \sigma_y = \lambda\theta + 2v\varepsilon_y \\ \sigma_z = \lambda\theta + 2v\varepsilon_z \\ \tau_{xy} = v\gamma_{xy} \\ \tau_{yz} = v\gamma_{yz} \\ \tau_{xz} = v\gamma_{xz} \end{cases} \tag{4.8}$$

Where λ is Lamé parameter,

$$\lambda = \frac{\mu E}{(1+\mu)(1-2\mu)} \tag{4.9}$$

-Relation of constants:

There have relationships between engineering elastic constants and Lamé parameter,

$$E = \frac{\lambda + v}{v(2\lambda + 2v)}, \qquad \mu = \frac{\lambda}{2(\lambda + v)}, \qquad G = v$$

Two of three engineering elastic constants are actually independently, they have relation

$$G = \frac{E}{2(1+\mu)} \tag{4.10}$$

According to equation (4.10),

$$\lambda = \frac{2\mu G}{(1-2\mu)} \tag{4.11}$$

Now, we already obtained 15 basic formulations to solve elastostatics problems.