FATIGUE OF MATERIALS & STRUCTURES

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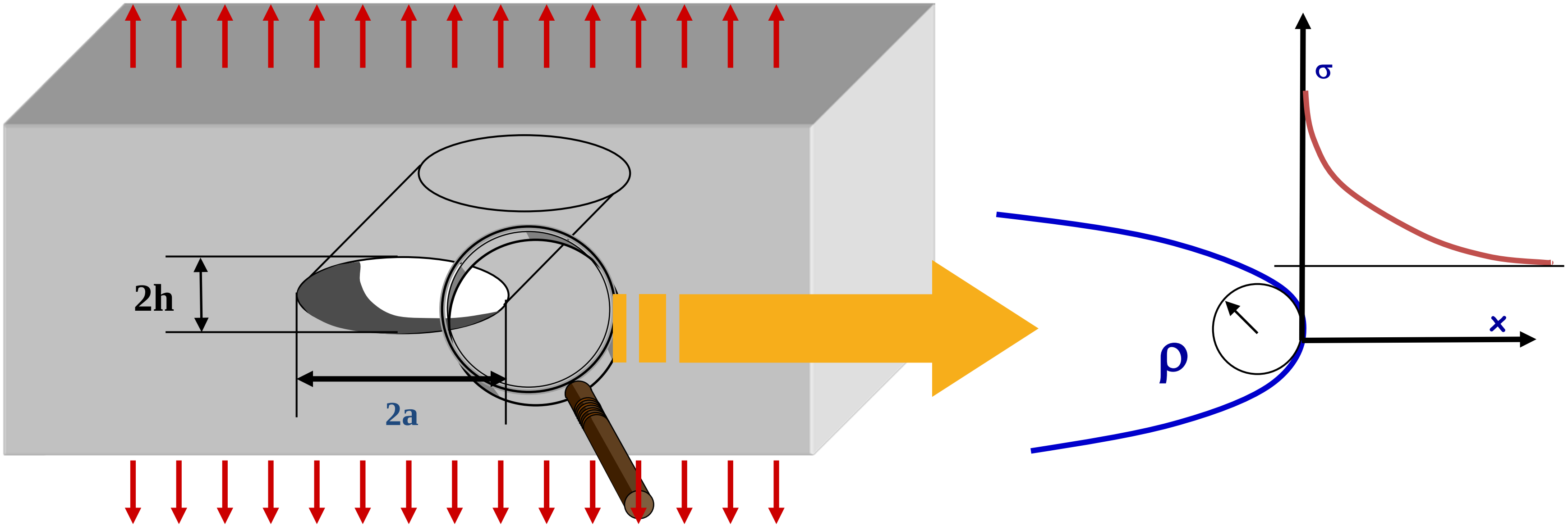
FATIGUE OF NOTCHED COMPONENTS

Notch effect



⇒ The local stress at the notch root is higher than the gross stress

Stress concentration factor

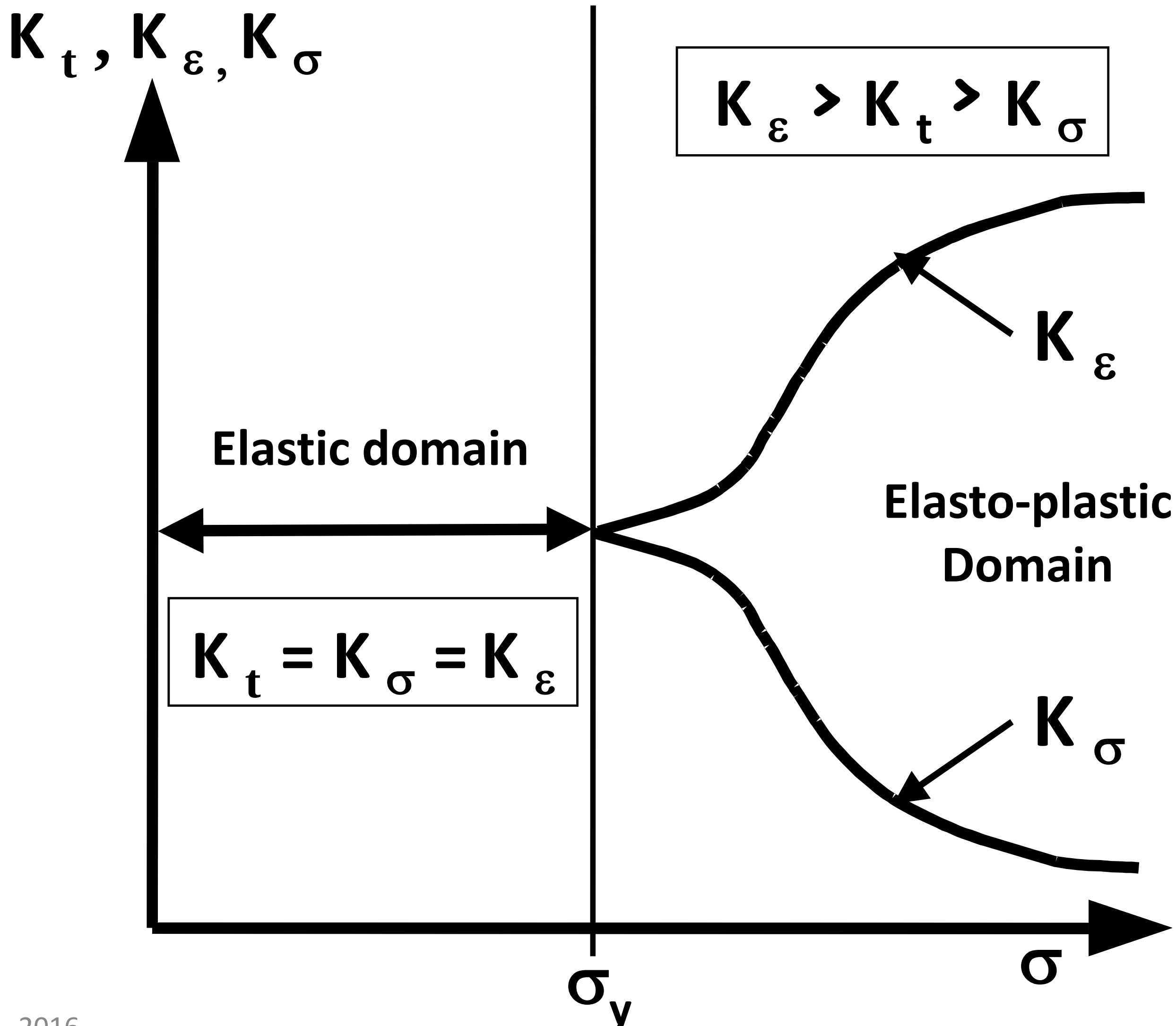
Definition : ratio "local stress/ gross stress on the net section"

$$K_t = \frac{\sigma_{local}}{\sigma_{net}}$$

K_t :

- is defined within the framework of elasticity;
- Only depends on geometry, in particular the notch tip radius! (typically not on the constitutive law of the considered material)

Relation between K_t, K_σ and K_ε



Reduction in fatigue life

Notch effect on fatigue limit quantified by the K_f cœfficient :

$$K_f = \frac{\sigma_D(\text{smooth})}{\sigma_D(\text{notched})}$$

Sensitivity to notch effect :

$$q = \frac{K_f - 1}{K_t - 1}$$

- q=0: insensitive to notch effect;
- q=1 : no adaptation (K_f=K_t)

Determination of the q coefficient

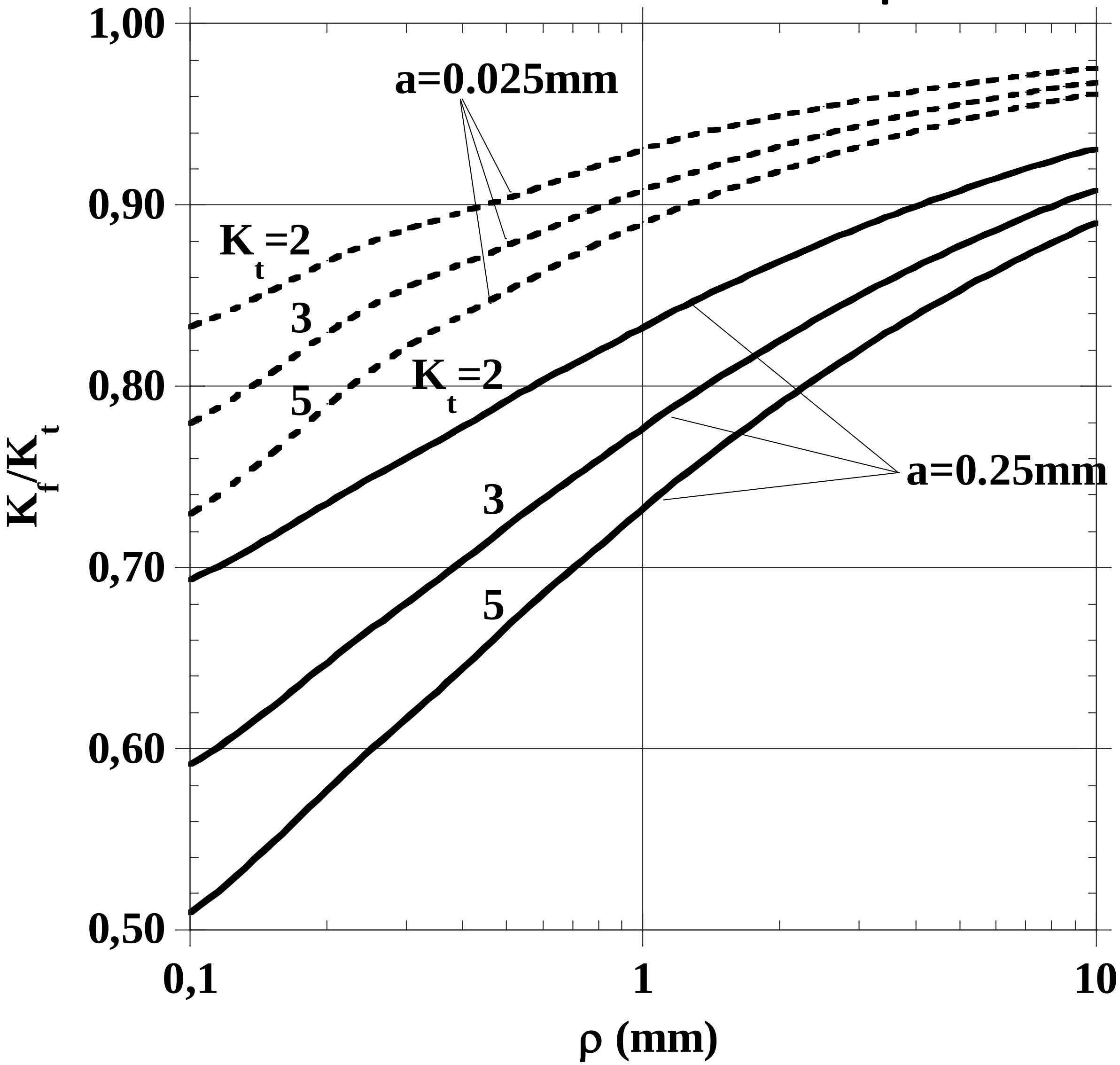
$$\mathbf{q} = \frac{1}{1+\left(\frac{\mathbf{a}}{\boldsymbol{\rho}}\right)^{\mathbf{m}}}$$

“Material” Parameter

Mild steels: a=0,25mm
High strength steels: a=0,025mm

Peterson: m=1
Neuber: m=1/2

Determination of K_f or q



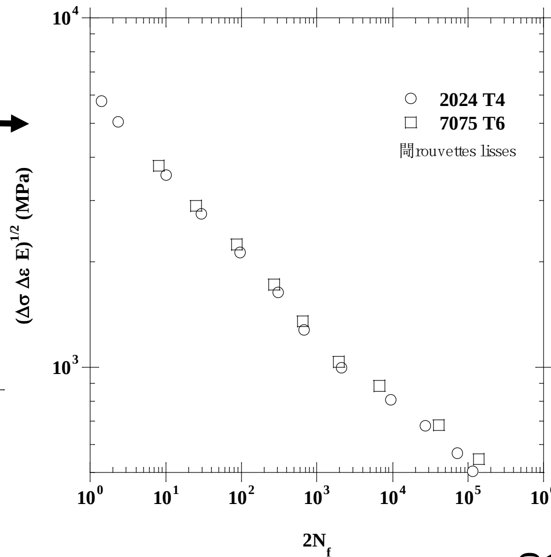
Application to Wöhler curves

$$K_f \sqrt{(\Delta S \times \Delta e)} = \sqrt{(\Delta\sigma \times \Delta\varepsilon)}$$

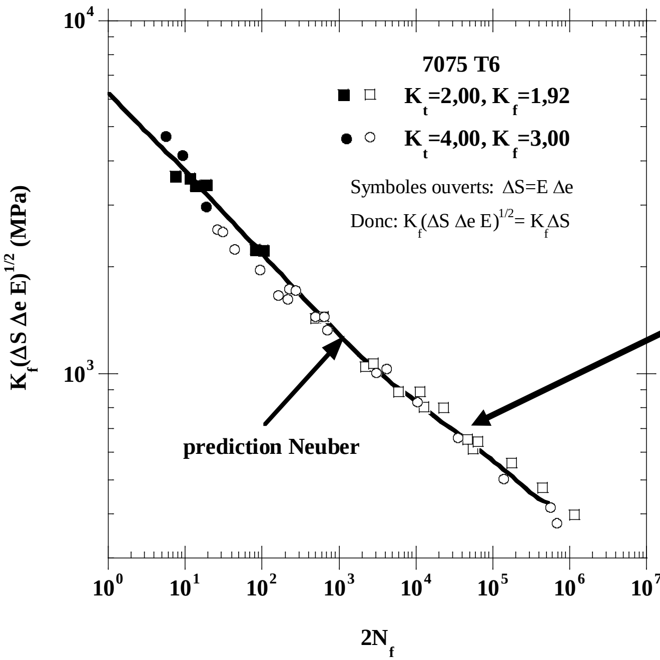
The determination of K_f permits the prediction of the fatigue life of notched components on the basis of the Wöhler curve established on smooth samples.

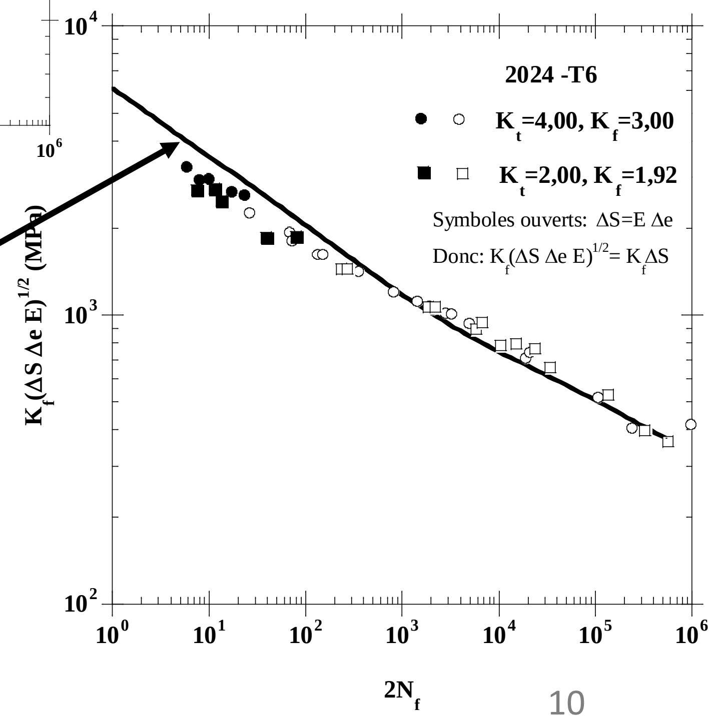
Application to Wöhler curves

Data on smooth samples



Master curve





Neuber's Rule

Problem: determine the local stress/strain amplitude at the notch root from the far field loading

→ simple solution in the framework of elasticity:

$$K_t^2 = K_\sigma \times K_\varepsilon$$

→ Idea: extrapolate the previous relation to the elasto-plastic domain

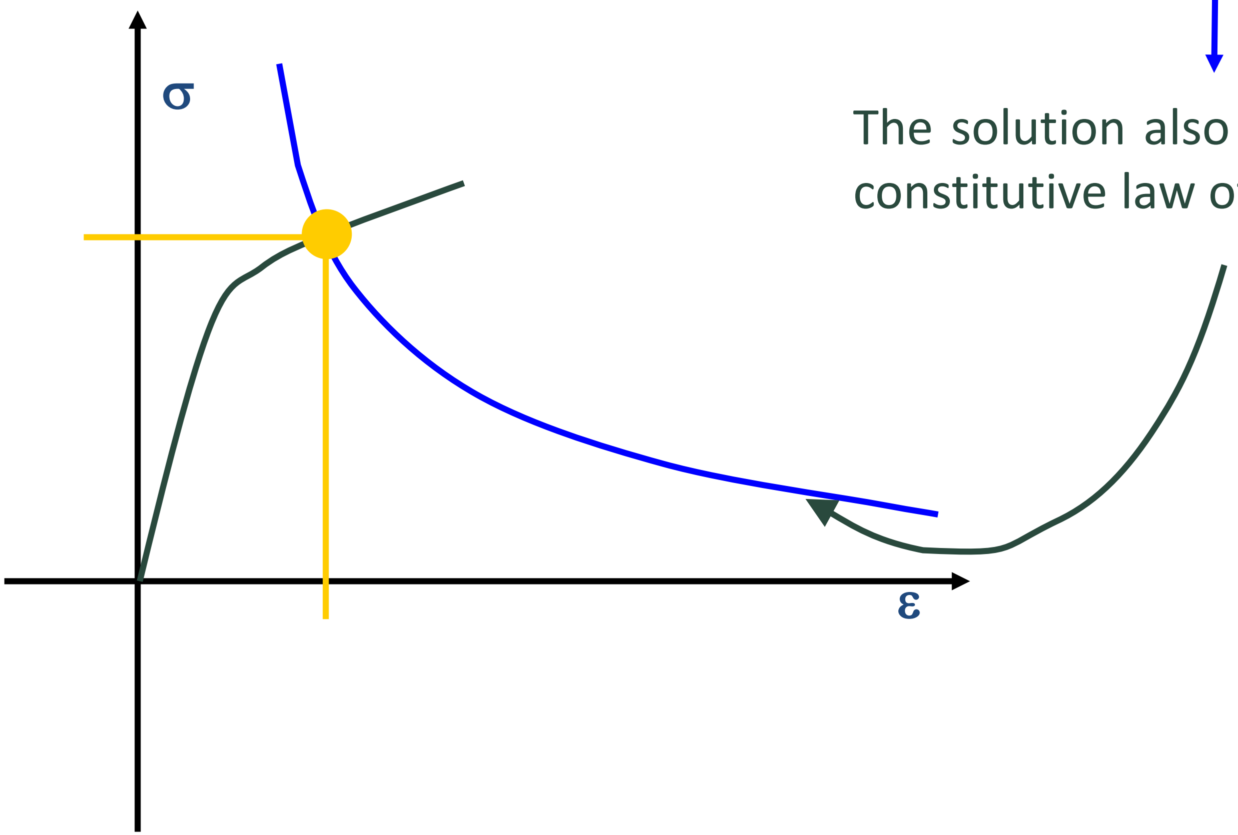
$$\mathbf{K_t^2 = K_\sigma \times K_\varepsilon}$$

Still valid in the easto-plastic domain

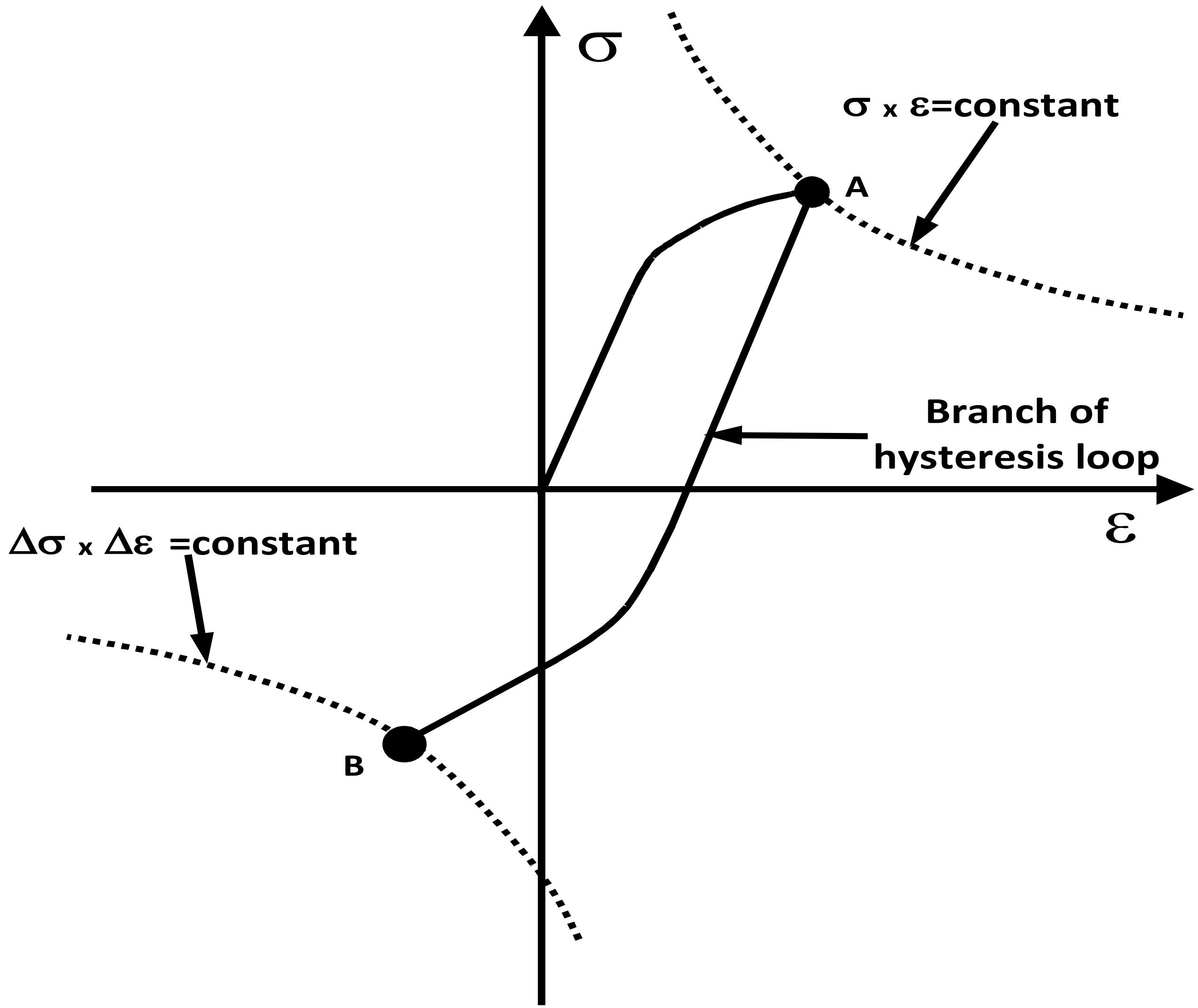
Neuber's Rule

Graphical solution of the equation: $K_t^2 = K_\sigma \times K_\varepsilon$

The solution also behaves to the constitutive law of the material



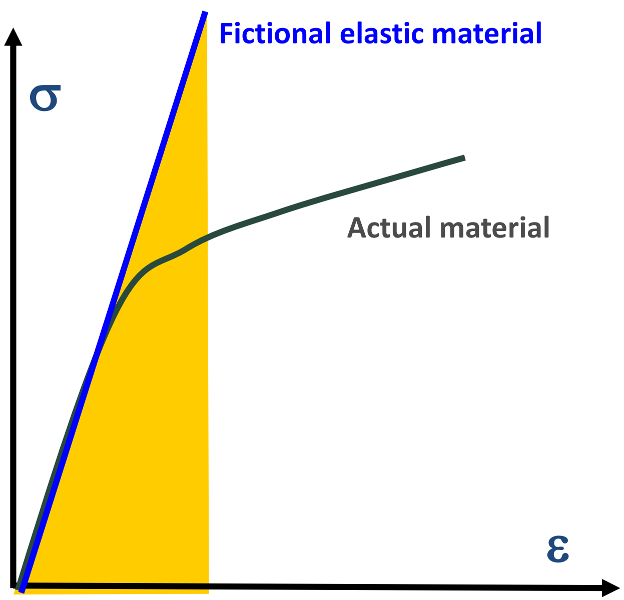
Extension to cyclic loading



Equivalent Deformation Energy (EDE) criterion

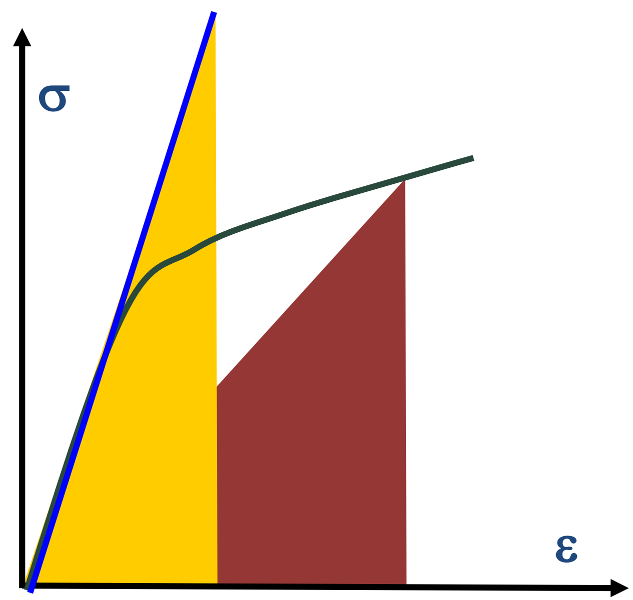
Strain energy density in elasticity: $$W_{local} = K_t^2 \times W_{global}$$

Hyp. : relation always satisfied in elasticity



EDE : the two areas are equal

$\neq$



Neuber's rule overestimates the local stress and strain

Comparison Neuber/EDE using Ramberg-Osgood constitutive law

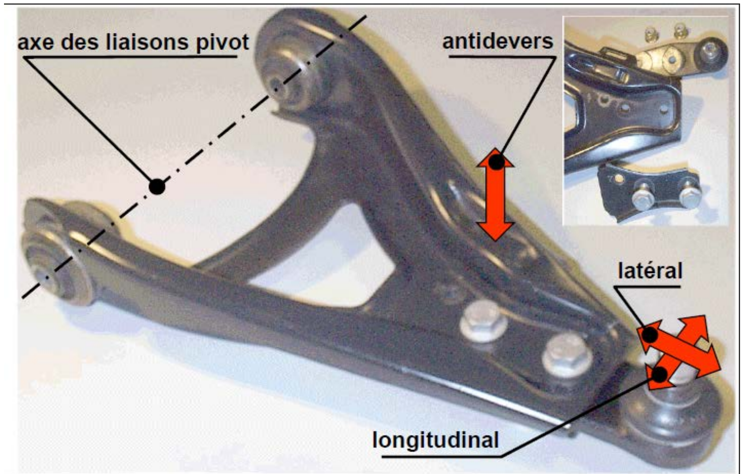
EDE: $$W_{locale} = \int_0^{\varepsilon} \sigma(\varepsilon)\,d\varepsilon = \left[\sigma \times \varepsilon\right]_0^{\varepsilon} - \int_0^{\varepsilon} \varepsilon\,d\sigma \longrightarrow W_{locale} = \frac{\sigma^2}{2E} + \frac{1}{1+n}\left(\frac{\sigma}{K}\right)^{1/n}$$

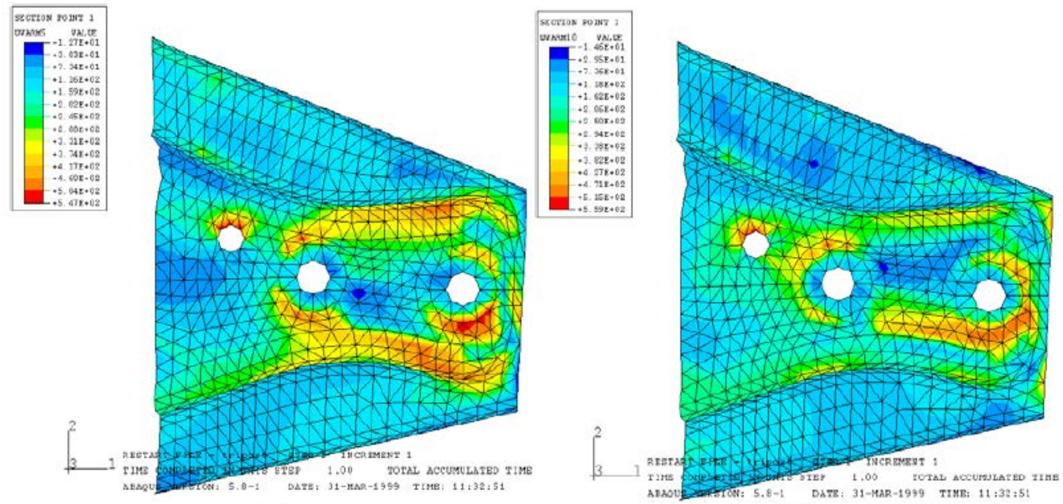
$$\frac{\sigma^2}{2E} + \frac{\sigma}{1+n}\left(\frac{\sigma}{K}\right)^{1/n} = \frac{(K_t \times S)^2}{2E}$$

Neuber

$$\frac{\sigma^2}{2E} + \frac{\sigma}{2}\left[\frac{\sigma}{K}\right]^{1/n} = \frac{(K_t \times S)^2}{2E}$$

Application of Neuber's rule : prediction of crack initiation in a suspension triangle





(a) (b)

initiation