Data Mining:

Concepts and Techniques

(3rd ed.)

— Chapter 6 —

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The slides are based on Jiawei Han's work

Chapter 5: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Frequent Itemset Mining Methods
- Which Patterns Are Interesting?—Pattern Evaluation Methods
- Summary

What Is Frequent Pattern Analysis?

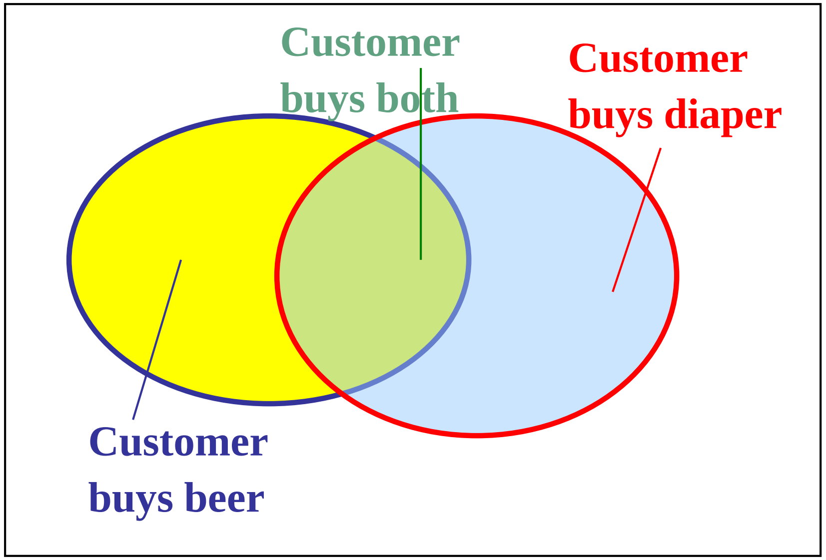
- Frequent pattern: a pattern (a set of items, subsequences, substructures, etc.) that occurs frequently in a data set
- First proposed by Agrawal, Imielinski, and Swami [AIS93] in the context of frequent itemsets and association rule mining
- Motivation: Finding inherent regularities in data
  - What products were often purchased together?— Beer and diapers?!
  - What are the subsequent purchases after buying a PC?
  - What kinds of DNA are sensitive to this new drug?
  - Can we automatically classify web documents?
- Applications
  - Basket data analysis, cross-marketing, catalog design, sale campaign analysis, Web log (click stream) analysis, and DNA sequence analysis.

Why Is Freq. Pattern Mining Important?

- Freq. pattern: An intrinsic and important property of datasets
- Foundation for many essential data mining tasks
  - Association, correlation, and causality analysis
  - Sequential, structural (e.g., sub-graph) patterns
  - Pattern analysis in spatiotemporal, multimedia, time-series, and stream data
  - Classification: discriminative, frequent pattern analysis
  - Cluster analysis: frequent pattern-based clustering
  - Data warehousing: iceberg cube and cube-gradient
  - Semantic data compression: fascicles
  - Broad applications

Basic Concepts: Frequent Patterns

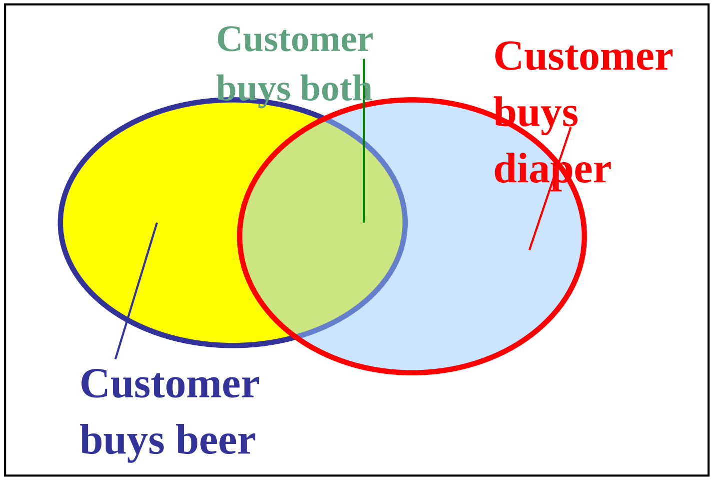
Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



- itemset: A set of one or more items
- k-itemset $X = \{x_1, \ldots, x_k\}$
- *(absolute) support*, or, *support count* of X: Frequency or occurrence of an itemset X
- *(relative) support*, *s*, is the fraction of transactions that contains X (i.e., the probability that a transaction contains X)
- An itemset X is *frequent* if X's support is no less than a *minsup* threshold

Basic Concepts: Association Rules

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



- Find all the rules $X \rightarrow Y$ with minimum support and confidence
  - support, *s*, probability that a transaction contains $X \cup Y$
  - confidence, *c*, conditional probability that a transaction having X also contains *Y*

Let minsup = 50%, minconf = 50%

Freq. Pat.: Beer:3, Nuts:3, Diaper:4, Eggs:3, {Beer, Diaper}:3

- Association rules: (many more!)
  - *Beer* → *Diaper* (60%, 100%)
  - *Diaper* → *Beer* (60%, 75%)

Closed Patterns and Max-Patterns

- A long pattern contains a combinatorial number of sub-patterns, e.g., $\{a_1, \ldots, a_{100}\}$ contains $\binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100} = 2^{100} - 1 = 1.27 \times 10^{30}$ sub-patterns!
- Solution: *Mine closed patterns and max-patterns instead*
- An itemset X is closed if X is *frequent* and there exists *no super-pattern* $Y \supset X$, *with the same support* as X (proposed by Pasquier, et al. @ ICDT'99)
- An itemset X is a max-pattern if X is frequent and there exists no frequent super-pattern $Y \supset X$ (proposed by Bayardo @ SIGMOD'98)
- Closed pattern is a lossless compression of freq. patterns
  - Reducing the # of patterns and rules

Closed Patterns and Max-Patterns

- Exercise. DB = {$<a_1, \ldots, a_{100}>$, $< a_1, \ldots, a_{50}>$}
  - Min_sup = 1.
- What is the set of closed itemset?
  - $<a_1, \ldots, a_{100}>$: 1
  - $< a_1, \ldots, a_{50}>$: 2
- What is the set of max-pattern?
  - $<a_1, \ldots, a_{100}>$: 1
- What is the set of all patterns?
  - !!

Computational Complexity of Frequent Itemset Mining

- How many itemsets are potentially to be generated in the worst case?
  - The number of frequent itemsets to be generated is sensitive to the minsup threshold
  - When minsup is low, there exist potentially an exponential number of frequent itemsets
  - The worst case: M^N where M: # distinct items, and N: max length of transactions

Chapter 5: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

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Scalable Frequent Itemset Mining Methods

- Apriori: A Candidate Generation-and-Test Approach
- Improving the Efficiency of Apriori
- FPGrowth: A Frequent Pattern-Growth Approach

The Downward Closure Property and Scalable Mining Methods

- The downward closure property of frequent patterns
  - <u>Any subset of a frequent itemset must be frequent</u>
  - If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
  - i.e., every transaction having {beer, diaper, nuts} also contains {beer, diaper}
- Scalable mining methods: Three major approaches
  - Apriori (Agrawal & Srikant@VLDB'94)
  - Freq. pattern growth (FPgrowth—Han, Pei & Yin @SIGMOD'00)
  - Vertical data format approach (Charm—Zaki & Hsiao @SDM'02)

Apriori: A Candidate Generation & Test Approach

- Apriori pruning principle: If there is any itemset which is infrequent, its superset should not be generated/tested! (Agrawal & Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
- Method:
  - Initially, scan DB once to get frequent 1-itemset
  - Generate length (k+1) candidate itemsets from length k frequent itemsets
  - Test the candidates against DB
  - Terminate when no frequent or candidate set can be generated

Tid	a	b	c	d	e	f
1	1	1	0	0	0	1
2	0	0	1	1	1	0
3	1	1	1	0	0	0
4	1	1	0	0	1	1
5	0	1	0	1	1	0
6	0	0	1	1	0	0
7	1	0	0	1	0	0
8	1	1	0	0	0	0
9	1	1	0	0	0	0
10	0	0	1	1	1	1
11	0	1	1	1	1	0
12	0	1	0	1	0	0
13	1	0	0	0	0	0
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15	0	1	0	1	1	0
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17	0	1	0	1	0	0
18	1	1	0	0	1	0
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21	0	1	1	1	1	0
22	0	1	1	1	0	0
23	0	0	0	0	1	0
24	1	0	0	0	0	0
25	1	1	1	1	0	0

{a}	{b}	{c}	{d}	{e}	{f}
0	0	0	0	0	0

Tid	a	b	c	d	e	f
1	1	1	0	0	0	1
2	0	0	1	1	1	0
3	1	1	1	0	0	0
4	1	1	0	0	1	1
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6	0	0	1	1	0	0
7	1	0	0	1	0	0
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9	1	1	0	0	0	0
10	0	0	1	1	1	1
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17	0	1	0	1	0	0
18	1	1	0	0	1	0
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20	1	1	1	0	1	0
21	0	1	1	1	1	0
22	0	1	1	1	0	0
23	0	0	0	0	1	0
24	1	0	0	0	0	0
25	1	1	1	1	0	0

{a}	{b}	{c}	{d}	{e}	{f}
1	1	0	0	0	1

Tid	a	b	c	d	e	f
1	1	1	0	0	0	1
2	0	0	1	1	1	0
3	1	1	1	0	0	0
4	1	1	0	0	1	1
5	0	1	0	1	1	0
6	0	0	1	1	0	0
7	1	0	0	1	0	0
8	1	1	0	0	0	0
9	1	1	0	0	0	0
10	0	0	1	1	1	1
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16	0	1	1	1	0	0
17	0	1	0	1	0	0
18	1	1	0	0	1	0
19	0	1	1	1	1	0
20	1	1	1	0	1	0
21	0	1	1	1	1	0
22	0	1	1	1	0	0
23	0	0	0	0	1	0
24	1	0	0	0	0	0
25	1	1	1	1	0	0

{a}	{b}	{c}	{d}	{e}	{f}
1	1	1	1	1	1

Tid	a	b	c	d	e	f
1	1	1	0	0	0	1
2	0	0	1	1	1	0
3	1	1	1	0	0	0
4	1	1	0	0	1	1
5	0	1	0	1	1	0
6	0	0	1	1	0	0
7	1	0	0	1	0	0
8	1	1	0	0	0	0
9	1	1	0	0	0	0
10	0	0	1	1	1	1
11	0	1	1	1	1	0
12	0	1	0	1	0	0
13	1	0	0	0	0	0
14	0	1	0	0	0	0
15	0	1	0	1	1	0
16	0	1	1	1	0	0
17	0	1	0	1	0	0
18	1	1	0	0	1	0
19	0	1	1	1	1	0
20	1	1	1	0	1	0
21	0	1	1	1	1	0
22	0	1	1	1	0	0
23	0	0	0	0	1	0
24	1	0	0	0	0	0
25	1	1	1	1	0	0

{a}	{b}	{c}	{d}	{e}	{f}
2	2	2	1	1	1

Tid	a	b	c	d	e	f
1	1	1	0	0	0	1
2	0	0	1	1	1	0
3	1	1	1	0	0	0
4	1	1	0	0	1	1
5	0	1	0	1	1	0
6	0	0	1	1	0	0
7	1	0	0	1	0	0
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21	0	1	1	1	1	0
22	0	1	1	1	0	0
23	0	0	0	0	1	0
24	1	0	0	0	0	0
25	1	1	1	1	0	0

{a}	{b}	{c}	{d}	{e}	{f}
11	18	11	14	11	3

Tid	a	b	c	d	e	f
1	1	1	0	0	0	1
2	0	0	1	1	1	0
3	1	1	1	0	0	0
4	1	1	0	0	1	1
5	0	1	0	1	1	0
6	0	0	1	1	0	0
7	1	0	0	1	0	0
8	1	1	0	0	0	0
9	1	1	0	0	0	0
10	0	0	1	1	1	1
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12	0	1	0	1	0	0
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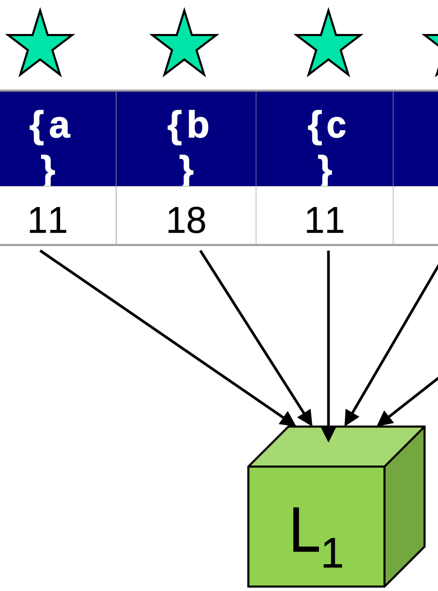
21	0	1	1	1	1	0
22	0	1	1	1	0	0
23	0	0	0	0	1	0
24	1	0	0	0	0	0
25	1	1	1	1	0	0

{a}	{b}	{c}	{d}	{e}	{f}
11	18	11	14	11	3



Tid	a	b	c	d	e	f
1	1	1	0	0	0	1
2	0	0	1	1	1	0
3	1	1	1	0	0	0
4	1	1	0	0	1	1
5	0	1	0	1	1	0
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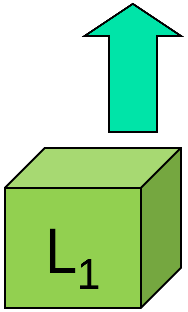
21	0	1	1	1	1	0
22	0	1	1	1	0	0
23	0	0	0	0	1	0
24	1	0	0	0	0	0
25	1	1	1	1	0	0

C_2

{a, b}	{a, c}	{a, d}	{a, e}	{b, c}
0	0	0	0	0
{b, d}	**{b, e}**	**{c, d}**	**{c, e}**	**{d, e}**
0	0	0	0	0



{a, b, c, d, e}

Tid	a	b	c	d	e	f
1	1	1	0	0	0	1
2	0	0	1	1	1	0
3	1	1	1	0	0	0
4	1	1	0	0	1	1
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6	0	0	1	1	0	0
7	1	0	0	1	0	0
8	1	1	0	0	0	0
9	1	1	0	0	0	0
10	0	0	1	1	1	1
11	0	1	1	1	1	0
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15	0	1	0	1	1	0
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21	0	1	1	1	1	0
22	0	1	1	1	0	0
23	0	0	0	0	1	0
24	1	0	0	0	0	0
25	1	1	1	1	0	0

{a, b}	{a, c}	{a, d}	{a, e}	{b, c}
1	0	0	0	0

{b, d}	{b, e}	{c, d}	{c, e}	{d, e}
0	0	0	0	0

Tid	a	b	c	d	e	f
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2	0	0	1	1	1	0
3	1	1	1	0	0	0
4	1	1	0	0	1	1
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18	1	1	0	0	1	0
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22	0	1	1	1	0	0
23	0	0	0	0	1	0
24	1	0	0	0	0	0
25	1	1	1	1	0	0

{a, b}	{a, c}	{a, d}	{a, e}	{b, c}
1	0	0	0	0
{b, d}	**{b, e}**	**{c, d}**	**{c, e}**	**{d, e}**
0	0	1	1	1

Tid	a	b	c	d	e	f
1	1	1	0	0	0	1
2	0	0	1	1	1	0
3	1	1	1	0	0	0
4	1	1	0	0	1	1
5	0	1	0	1	1	0
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23	0	0	0	0	1	0
24	1	0	0	0	0	0
25	1	1	1	1	0	0

{a, b}	{a, c}	{a, d}	{a, e}	{b, c}
2	1	0	0	1
{b, d}	**{b, e}**	**{c, d}**	**{c, e}**	**{d, e}**
0	0	1	1	1

Tid	a	b	c	d	e	f
1	1	1	0	0	0	1
2	0	0	1	1	1	0
3	1	1	1	0	0	0
4	1	1	0	0	1	1
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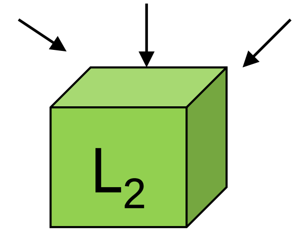
22	0	1	1	1	0	0
23	0	0	0	0	1	0
24	1	0	0	0	0	0
25	1	1	1	1	0	0

{a, b}	{a, c}	{a, d}	{a, e}	{b, c}
8	3	2	3	8
{b, d}	**{b, e}**	**{c, d}**	**{c, e}**	**{d, e}**
10	8	9	6	7



The Apriori Algorithm—An Example

TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

Scan *D* for count of each candidate →

C_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Compare candidate support count with minimum support count →

L_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Generate C_2 candidates from L_1 →

C_2

Itemset
{I1, I2}
{I1, I3}
{I1, I4}
{I1, I5}
{I2, I3}
{I2, I4}
{I2, I5}
{I3, I4}
{I3, I5}
{I4, I5}

Scan *D* for count of each candidate →

C_2

Itemset	Sup. count
{I1, I2}	4
{I1, I3}	4
{I1, I4}	1
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2
{I3, I4}	0
{I3, I5}	1
{I4, I5}	0

Compare candidate support count with minimum support count →

L_2

Itemset	Sup. count
{I1, I2}	4
{I1, I3}	4
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2

Generate C_3 candidates from L_2 →

C_3

Itemset
{I1, I2, I3}
{I1, I2, I5}

Scan *D* for count of each candidate →

C_3

Itemset	Sup. count
{I1, I2, I3}	2
{I1, I2, I5}	2

Compare candidate support count with minimum support count →

L_3

Itemset	Sup. count
{I1, I2, I3}	2
{I1, I2, I5}	2

The Apriori Algorithm (Pseudo-Code)

C_k: Candidate itemset of size k
L_k : frequent itemset of size k

L_1 = {frequent items};
for (k = 1; L_k !=$\emptyset$; k++) **do begin**
  C_{k+1} = candidates generated from L_k;
  for each transaction t in database do
    increment the count of all candidates in C_{k+1} that
     are contained in t
  L_{k+1} = candidates in C_{k+1} with min_support
  end
return $\cup_k L_k$;

Implementation of Apriori

- How to generate candidates?
  - Step 1: self-joining L_k
  - Step 2: pruning
- Example of Candidate-generation
  - $L_3=\{abc, abd, acd, ace, bcd\}$
  - Self-joining: L_3*L_3
    - *abcd* from *abc* and *abd*
    - *acde* from *acd* and *ace*
  - Pruning:
    - *acde* is removed because *ade* is not in L_3
  - $C_4 = \{abcd\}$

How to Count Supports of Candidates?

- Why counting supports of candidates a problem?
  - The total number of candidates can be very huge
  - One transaction may contain many candidates
- Method:
  - Candidate itemsets are stored in a *hash-tree*
  - *Leaf* node of hash-tree contains a list of itemsets and counts
  - *Interior* node contains a hash table
  - *Subset function*: finds all the candidates contained in a transaction

Scalable Frequent Itemset Mining Methods

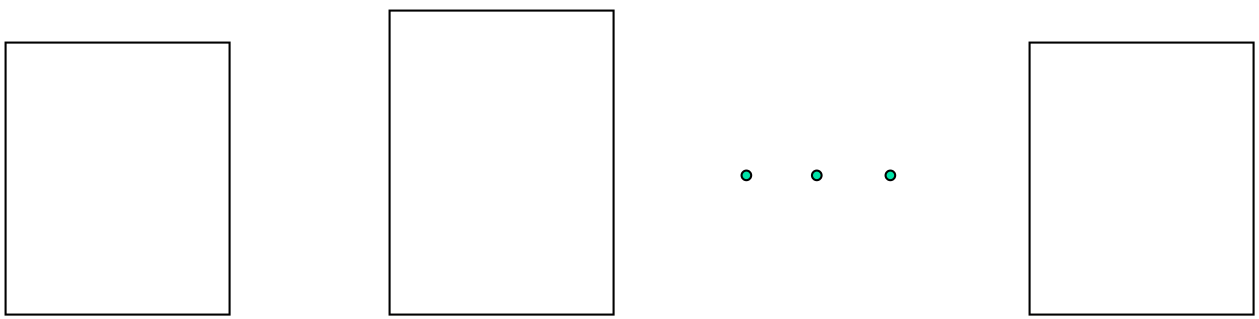
- Apriori: A Candidate Generation-and-Test Approach
- Improving the Efficiency of Apriori
- FPGrowth: A Frequent Pattern-Growth Approach
- Mining Close Frequent Patterns and Maxpatterns

Further Improvement of the Apriori Method

- Major computational challenges
  - Multiple scans of transaction database
  - Huge number of candidates
  - Tedious workload of support counting for candidates
- Improving Apriori: general ideas
  - Reduce passes of transaction database scans
  - Shrink number of candidates
  - Facilitate support counting of candidates

Partition: Scan Database Only Twice

- Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB
  - Scan 1: partition database and find local frequent patterns
  - Scan 2: consolidate global frequent patterns
- A. Savasere, E. Omiecinski and S. Navathe, *VLDB'95*



DB_1 + DB_2 + … + DB_k = DB

$sup_1(i) < \sigma DB_1$ $\quad$ $sup_2(i) < \sigma DB_2$ $\quad$ $sup_k(i) < \sigma DB_k$ $\quad$ $sup(i) < \sigma DB$

DHP: Reduce the Number of Candidates

- A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent

Create hash table H_2 using hash function $h(x, y) = ((\mathit{order\ of\ } x) \times 10 + (\mathit{order\ of\ } y)) \bmod 7$

H_2

bucket address	0	1	2	3	4	5	6
bucket count	2	2	4	2	2	4	4
bucket contents	{I1, I4} {I3, I5}	{I1, I5} {I1, I5}	{I2, I3} {I2, I3} {I2, I3} {I2, I3}	{I2, I4} {I2, I4}	{I2, I5} {I2, I5}	{I1, I2} {I1, I2} {I1, I2} {I1, I2}	{I1, I3} {I1, I3} {I1, I3} {I1, I3}

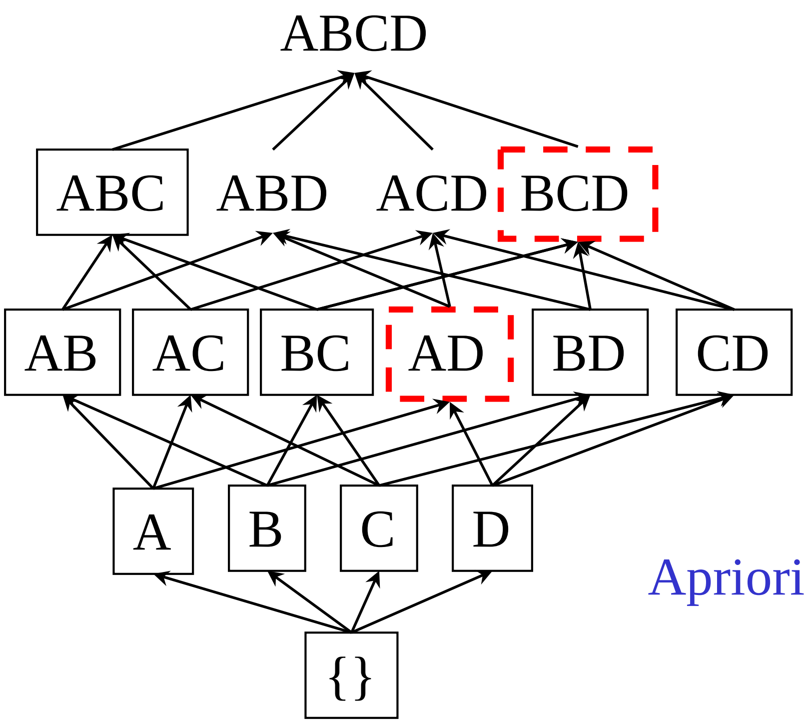
Hash table, H_2, for candidate 2-itemsets: This hash table was generated by scanning the transactions of Table 5.1 while determining L_1 from C_1. If the minimum support count is, say, 3, then the itemsets in buckets 0, 1, 3, and 4 cannot be frequent and so they should not be included in C_2.

- J. Park, M. Chen, and P. Yu. An effective hash-based algorithm for mining association rules. *SIGMOD'95*

Sampling for Frequent Patterns

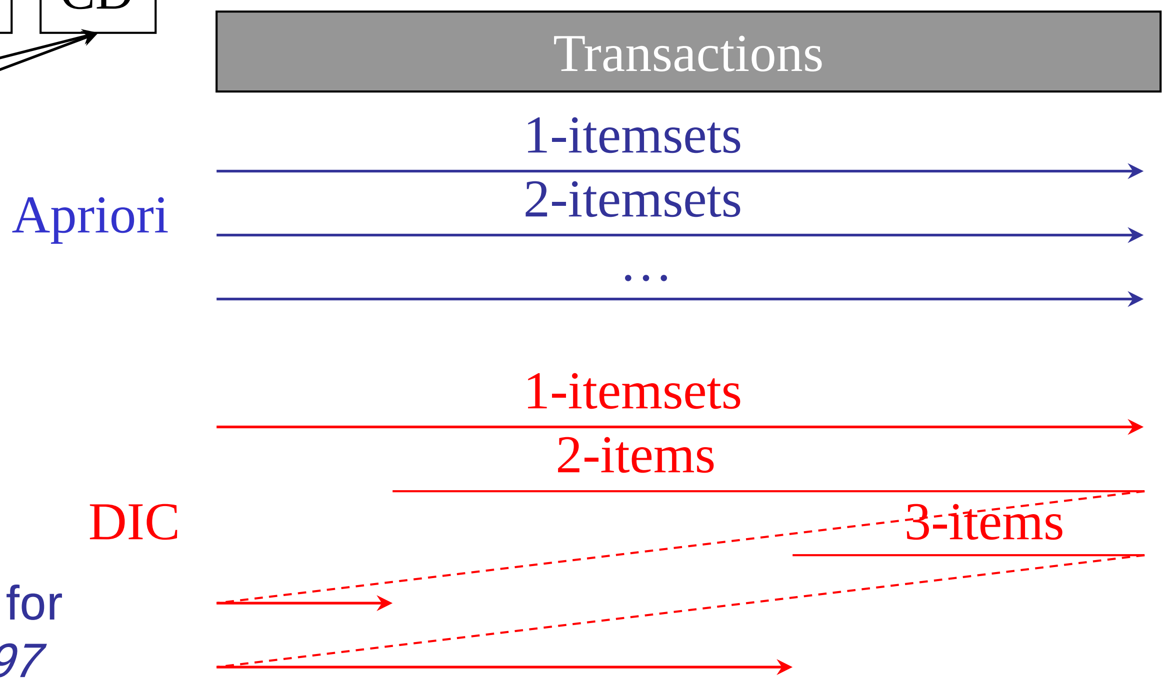
- Select a sample of original database, mine frequent patterns within sample using Apriori
- Scan database once to verify frequent itemsets found in sample, only *borders* of closure of frequent patterns are checked
  - Example: check *abcd* instead of *ab, ac, ..., etc.*
- Scan database again to find missed frequent patterns
- H. Toivonen. Sampling large databases for association rules. In *VLDB'96*

DIC: Reduce Number of Scans



Itemset lattice

- Once both A and D are determined frequent, the counting of AD begins
- Once all length-2 subsets of BCD are determined frequent, the counting of BCD begins



S. Brin R. Motwani, J. Ullman, and S. Tsur. Dynamic itemset counting and implication rules for market basket data. *SIGMOD'97*

Scalable Frequent Itemset Mining Methods

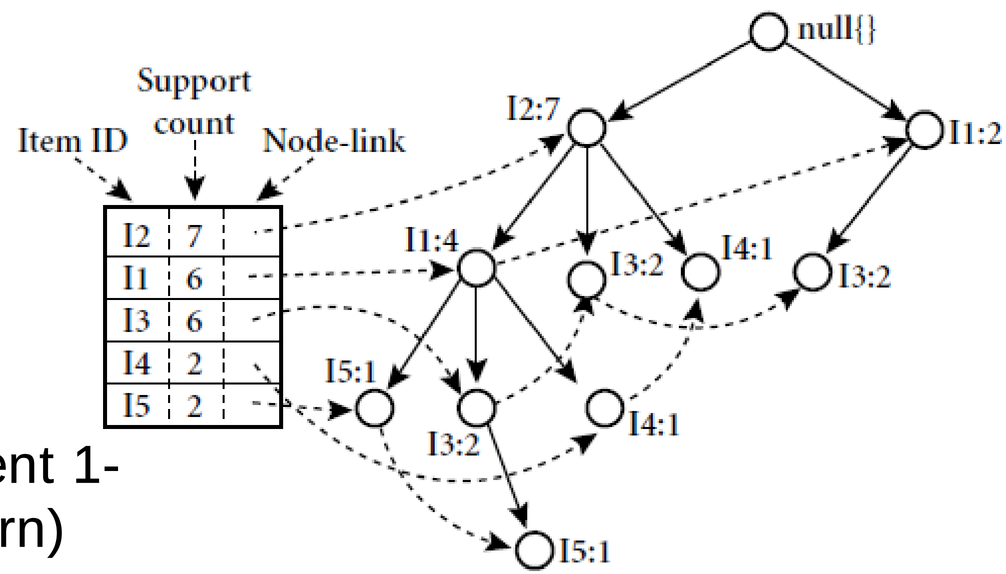
- Apriori: A Candidate Generation-and-Test Approach
- Improving the Efficiency of Apriori
- FPGrowth: A Frequent Pattern-Growth Approach
- ECLAT: Frequent Pattern Mining with Vertical Data Format
- Mining Close Frequent Patterns and Maxpatterns

Pattern-Growth Approach: Mining Frequent Patterns Without Candidate Generation

- Bottlenecks of the Apriori approach
  - Breadth-first (i.e., level-wise) search
  - Candidate generation and test
    - Often generates a huge number of candidates
- The FPGrowth Approach (J. Han, J. Pei, and Y. Yin, SIGMOD' 00)
  - Depth-first search
  - Avoid explicit candidate generation
- Major philosophy: Grow long patterns from short ones using local frequent items only
  - "abc" is a frequent pattern
  - Get all transactions having "abc", i.e., project DB on abc: DB|abc
  - "d" is a local frequent item in DB|abc → abcd is a frequent pattern

TID	List of item_IDs
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

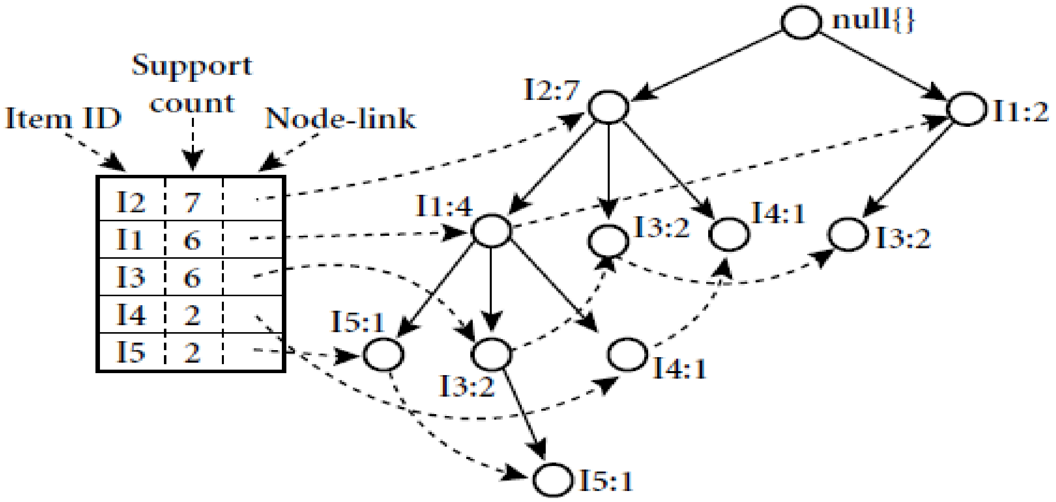
Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2



1. Scan DB once, find frequent 1-itemset (single item pattern)
2. Sort frequent items in frequency descending order, f-list
3. Scan DB again, construct FP-tree

Find Patterns Having P From P-conditional Database

- Starting at the frequent item header table in the FP-tree
- Traverse the FP-tree by following the link of each frequent item *p*
- Accumulate all of *transformed prefix paths* of item *p* to form *p*'s conditional pattern base



Item	*Conditional Pattern Base*	*Conditional FP-tree*	*Frequent Patterns Generated*
I5	{{I2, I1: 1}, {I2, I1, I3: 1}}	⟨I2: 2, I1: 2⟩	{I2, I5: 2}, {I1, I5: 2}, {I2, I1, I5: 2}
I4	{{I2, I1: 1}, {I2: 1}}	⟨I2: 2⟩	{I2, I4: 2}
I3	{{I2, I1: 2}, {I2: 2}, {I1: 2}}	⟨I2: 4, I1: 2⟩, ⟨I1: 2⟩	{I2, I3: 4}, {I1, I3: 4}, {I2, I1, I3: 2}
I1	{{I2: 4}}	⟨I2: 4⟩	{I2, I1: 4}

Benefits of the FP-tree Structure

- Completeness
  - Preserve complete information for frequent pattern mining
  - Never break a long pattern of any transaction
- Compactness
  - Reduce irrelevant info—infrequent items are gone
  - Items in frequency descending order: the more frequently occurring, the more likely to be shared
  - Never be larger than the original database (not count node-links and the *count* field)

The Frequent Pattern Growth Mining Method

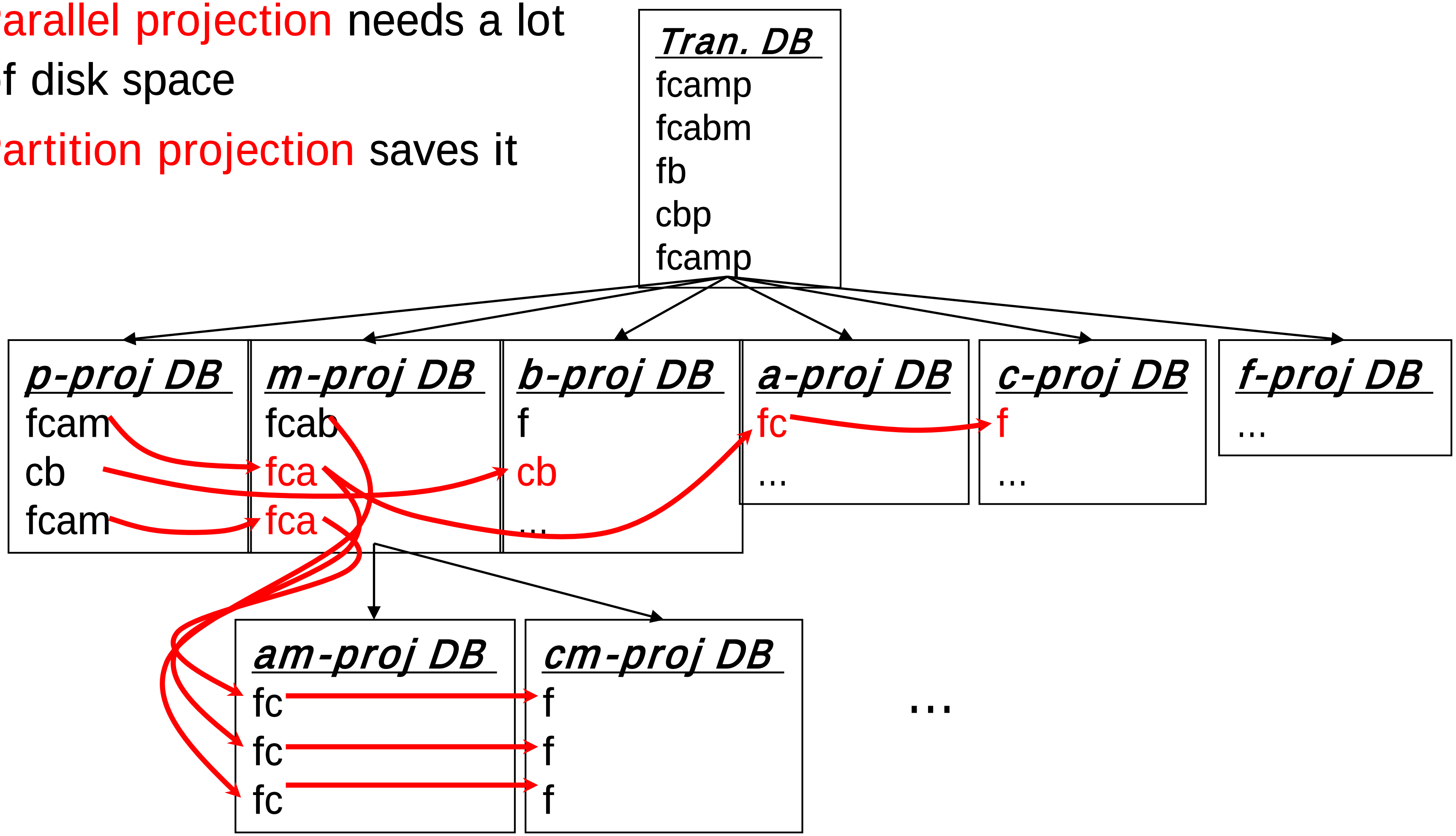
- Idea: Frequent pattern growth
  - Recursively grow frequent patterns by pattern and database partition
- Method
  - For each frequent item, construct its conditional pattern-base, and then its conditional FP-tree
  - Repeat the process on each newly created conditional FP-tree
  - Until the resulting FP-tree is empty, or it contains only one path—single path will generate all the combinations of its sub-paths, each of which is a frequent pattern

Scaling FP-growth by Database Projection

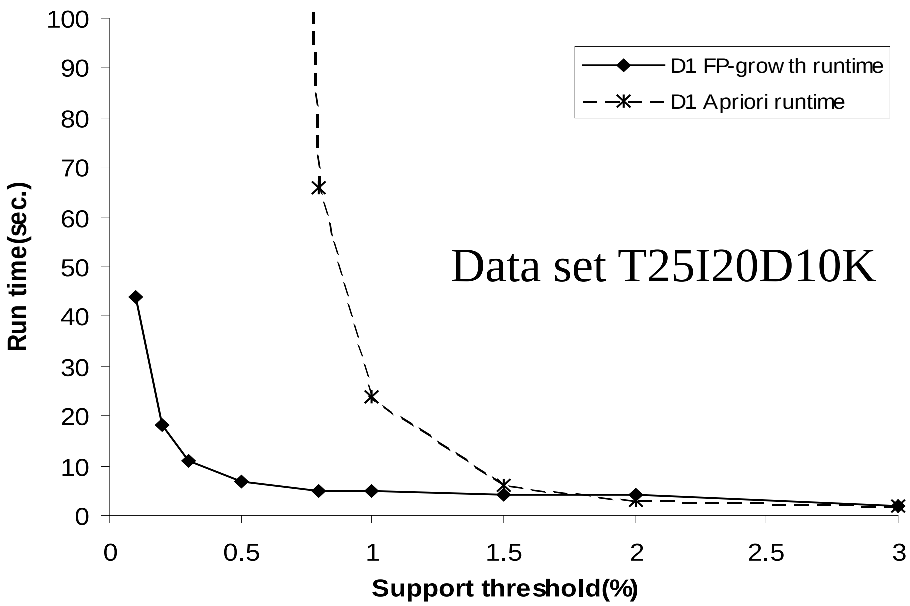
- What about if FP-tree cannot fit in memory?
  - DB projection
- First partition a database into a set of projected DBs
- Then construct and mine FP-tree for each projected DB
- Parallel projection vs. partition projection techniques
  - Parallel projection
    - Project the DB in parallel for each frequent item
    - Parallel projection is space costly
    - All the partitions can be processed in parallel
  - Partition projection
    - Partition the DB based on the ordered frequent items
    - Passing the unprocessed parts to the subsequent partitions

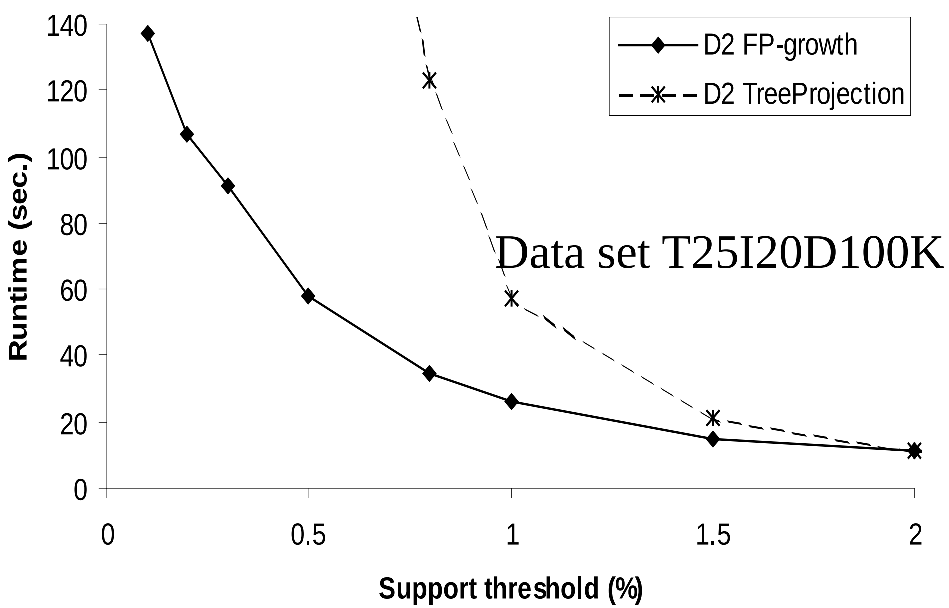
Partition-Based Projection

- Parallel projection needs a lot of disk space
- Partition projection saves it



Performance of FPGrowth in Large Datasets





FP-Growth vs. Apriori

FP-Growth vs. Tree-Projection

Advantages of the Pattern Growth Approach

- Divide-and-conquer:
  - Decompose both the mining task and DB according to the frequent patterns obtained so far
  - Lead to focused search of smaller databases
- Other factors
  - No candidate generation, no candidate test
  - Compressed database: FP-tree structure
  - No repeated scan of entire database
  - Basic ops: counting local freq items and building sub FP-tree, no pattern search and matching
- A good open-source implementation and refinement of FPGrowth
  - FPGrowth+ (Grahne and J. Zhu, FIMI'03)

Further Improvements of Mining Methods

- AFOPT (Liu, et al. @ KDD'03)
  - A "push-right" method for mining condensed frequent pattern (CFP) tree
- Carpenter (Pan, et al. @ KDD'03)
  - Mine data sets with small rows but numerous columns
  - Construct a row-enumeration tree for efficient mining
- FPgrowth+ (Grahne and Zhu, FIMI'03)
  - Efficiently Using Prefix-Trees in Mining Frequent Itemsets, Proc. ICDM'03 Int. Workshop on Frequent Itemset Mining Implementations (FIMI'03), Melbourne, FL, Nov. 2003
- TD-Close (Liu, et al, SDM'06)

Extension of Pattern Growth Mining Methodology

- Mining closed frequent itemsets and max-patterns
  - CLOSET (DMKD'00), FPclose, and FPMax (Grahne & Zhu, Fimi'03)
- Mining sequential patterns
  - PrefixSpan (ICDE'01), CloSpan (SDM'03), BIDE (ICDE'04)
- Mining graph patterns
  - gSpan (ICDM'02), CloseGraph (KDD'03)
- Constraint-based mining of frequent patterns
  - Convertible constraints (ICDE'01), gPrune (PAKDD'03)
- Computing iceberg data cubes with complex measures
  - H-tree, H-cubing, and Star-cubing (SIGMOD'01, VLDB'03)
- Pattern-growth-based Clustering
  - MaPle (Pei, et al., ICDM'03)
- Pattern-Growth-Based Classification
  - Mining frequent and discriminative patterns (Cheng, et al, ICDE'07)

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Mining Frequent Closed Patterns: CLOSET

- Flist: list of all frequent items in support ascending order
  - Flist: d-a-f-e-c
- Divide search space
  - Patterns having d
  - Patterns having d but no a, etc.
- Find frequent closed pattern recursively
  - Every transaction having d also has *cfa* → *cfad* is a frequent closed pattern
- J. Pei, J. Han & R. Mao. “CLOSET: An Efficient Algorithm for Mining Frequent Closed Itemsets", DMKD'00.

Min_sup=2

TID	Items
10	a, c, d, e, f
20	a, b, e
30	c, e, f
40	a, c, d, f
50	c, e, f

CLOSET+: Mining Closed Itemsets by Pattern-Growth

- Itemset merging: if Y appears in every occurrence of X, then Y is merged with X
- Sub-itemset pruning: if Y ɔ X, and sup(X) = sup(Y), X and all of X's descendants in the set enumeration tree can be pruned
- Hybrid tree projection
  - Bottom-up physical tree-projection
  - Top-down pseudo tree-projection
- Item skipping: if a local frequent item has the same support in several header tables at different levels, one can prune it from the header table at higher levels
- Efficient subset checking

MaxMiner: Mining Max-Patterns

Tid	Items
10	A, B, C, D, E
20	B, C, D, E,
30	A, C, D, F

- 1^{st} scan: find frequent items
  - A, B, C, D, E
- 2^{nd} scan: find support for
  - AB, AC, AD, AE, ABCDE
  - BC, BD, BE, BCDE
  - CD, CE, CDE, DE

Potential max-patterns

- Since BCDE is a max-pattern, no need to check BCD, BDE, CDE in later scan
- R. Bayardo. Efficiently mining long patterns from databases. *SIGMOD'98*

Chapter 5: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Frequent Itemset Mining Methods
- Which Patterns Are Interesting?—Pattern Evaluation Methods
- Summary

Interestingness Measure: Correlations (Lift)

- *play basketball* $\Rightarrow$ *eat cereal* [40%, 66.7%] is misleading
  - The overall % of students eating cereal is 75% > 66.7%.
- *play basketball* $\Rightarrow$ *not eat cereal* [20%, 33.3%] is more accurate, although with lower support and confidence
- Measure of dependent/correlated events: lift

$$lift = \frac{P(A \cup B)}{P(A)P(B)}$$

$$lift(B,C) = \frac{2000/5000}{3000/5000 * 3750/5000} = 0.89$$

$$lift(B,\neg C) = \frac{1000/5000}{3000/5000 * 1250/5000} = 1.33$$

	Basketball	Not basketball	Sum (row)
Cereal	2000	1750	3750
Not cereal	1000	250	1250
Sum(col.)	3000	2000	5000

Are *lift* and χ^2 Good Measures of Correlation?

- "*Buy walnuts* $\Rightarrow$ *buy milk* [1%, 80%]" is misleading if 85% of customers buy milk
- Support and confidence are not good to indicate correlations
- Over 20 interestingness measures have been proposed (see Tan, Kumar, Sritastava @KDD'02)
- Which are good ones?

symbol	measure	range	formula
ϕ	ϕ-coefficient	-1…1	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
Q	Yule's Q	-1 … 1	$\frac{P(A,B)P(\overline{A},\overline{B})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{A},\overline{B})+P(A,\overline{B})P(\overline{A},B)}$
Y	Yule's Y	-1 … 1	$\frac{\sqrt{P(A,B)P(\overline{A},\overline{B})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{A},\overline{B})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}}$
k	Cohen's	-1 … 1	$\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
PS	Piatetsky-Shapiro's	-0.25 … 0.25	$P(A,B)-P(A)P(B)$
F	Certainty factor	-1 … 1	$\max(\frac{P(B\|A)-P(B)}{1-P(B)}, \frac{P(A\|B)-P(A)}{1-P(A)})$
AV	added value	-0.5 … 1	$\max(P(B\|A)-P(B), P(A\|B)-P(A))$
K	Klosgen's Q	-0.33 … 0.38	$\sqrt{P(A,B)}\max(P(B\|A)-P(B), P(A\|B)-P(A))$
g	Goodman-kruskal's	0 … 1	$\frac{\Sigma_j \max_k P(A_j,B_k)+\Sigma_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-max_k P(B_k)}$
M	Mutual Information	0 … 1	$\frac{\Sigma_i\Sigma_j P(A_i,B_j)\log\frac{P(A_i,B_j)}{P(A_i)P(B_J)}}{\min(-\Sigma_i P(A_i)\log P(A_i)\log P(A_i), -\Sigma_i P(B_i)\log P(B_i)\log P(B_i))}$
J	J-Measure	0 … 1	$\max(P(A,B)\log(\frac{P(B\|A)}{P(B)})+P(A\overline{B})\log(\frac{P(\overline{B}\|A)}{P(\overline{B})}))$ $P(A,B)\log(\frac{P(A\|B)}{P(A)})+P(\overline{A}B)\log(\frac{P(\overline{A}\|B)}{P(\overline{A})})$
G	Gini index	0 … 1	$\max(P(A)[P(B\|A)^2+P(\overline{B}\|A)^2]+P(\overline{A}[P(B\|\overline{A})^2+P(\overline{B}\|\overline{A})^2]-P(B)^2-P(\overline{B})^2,$ $P(B)[P(A\|B)^2+P(\overline{A}\|B)^2]+P(\overline{B}[P(A\|\overline{B})^2+P(\overline{A}\|\overline{B})^2]-P(A)^2-P(\overline{A})^2)$
s	support	0 … 1	$P(A,B)$
c	confidence	0 … 1	$max(P(B\|A), P(A\|B))$
L	Laplace	0 … 1	$\max(\frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2})$
IS	Cosine	0 … 1	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
γ	coherence(Jaccard)	0 … 1	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
α	all_confidence	0 … 1	$\frac{P(A,B)}{\max(P(A),P(B))}$
o	odds ratio	0 … ∞	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(\overline{A},B)P(A,\overline{B})}$
V	Conviction	0.5 … ∞	$\max(\frac{P(A)P(\overline{B})}{P(A\overline{B})}, \frac{P(B)P(\overline{A})}{P(B\overline{A})})$
λ	lift	0 … ∞	$\frac{P(A,B)}{P(A)P(B)}$
S	Collective strength	0 … ∞	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})} \times \frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
χ^2	χ^2	0 … ∞	$\Sigma_i \frac{(P(A_i)-E_i)^2}{E_i}$

Null-Invariant Measures

Table 6: Properties of interestingness measures. Note that none of the measures satisfies all the properties.

Symbol	Measure	Range	P1	P2	P3	O1	O2	O3	O3'	O4
ϕ	ϕ-coefficient	$-1 \cdots 0 \cdots 1$	Yes	Yes	Yes	Yes	No	Yes	Yes	No
λ	Goodman-Kruskal's	$0 \cdots 1$	Yes	No	No	Yes	No	No*	Yes	No
α	odds ratio	$0 \cdots 1 \cdots \infty$	Yes*	Yes	Yes	Yes	Yes	Yes*	Yes	No
Q	Yule's Q	$-1 \cdots 0 \cdots 1$	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
Y	Yule's Y	$-1 \cdots 0 \cdots 1$	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
κ	Cohen's	$-1 \cdots 0 \cdots 1$	Yes	Yes	Yes	Yes	No	No	Yes	No
M	Mutual Information	$0 \cdots 1$	Yes	Yes	Yes	No**	No	No*	Yes	No
J	J-Measure	$0 \cdots 1$	Yes	No	No	No**	No	No	No	No
G	Gini index	$0 \cdots 1$	Yes	No	No	No**	No	No*	Yes	No
s	Support	$0 \cdots 1$	No	Yes	No	Yes	No	No	No	No
c	Confidence	$0 \cdots 1$	No	Yes	No	No**	No	No	No	Yes
L	Laplace	$0 \cdots 1$	No	Yes	No	No**	No	No	No	No
V	Conviction	$0.5 \cdots 1 \cdots \infty$	No	Yes	No	No**	No	No	Yes	No
I	Interest	$0 \cdots 1 \cdots \infty$	Yes*	Yes	Yes	Yes	No	No	No	No
IS	Cosine	$0 \cdots \sqrt{P(A,B)} \cdots 1$	No	Yes	Yes	Yes	No	No	No	Yes
PS	Piatetsky-Shapiro's	$-0.25 \cdots 0 \cdots 0.25$	Yes	Yes	Yes	Yes	No	Yes	Yes	No
F	Certainty factor	$-1 \cdots 0 \cdots 1$	Yes	Yes	Yes	No**	No	No	Yes	No
AV	Added value	$-0.5 \cdots 0 \cdots 1$	Yes	Yes	Yes	No**	No	No	No	No
S	Collective strength	$0 \cdots 1 \cdots \infty$	No	Yes	Yes	Yes	No	Yes*	Yes	No
ζ	Jaccard	$0 \cdots 1$	No	Yes	Yes	Yes	No	No	No	Yes
K	Klosgen's	$(\frac{2}{\sqrt{3}}-1)^{1/2}[2-\sqrt{3}-\frac{1}{\sqrt{3}}] \cdots 0 \cdots \frac{2}{3\sqrt{3}}$	Yes	Yes	Yes	No**	No	No	No	No

where: P1: $O(\mathbf{M}) = 0$ if $det(\mathbf{M}) = 0$, *i.e.*, whenever A and B are statistically independent.
P2: $O(\mathbf{M_2}) > O(\mathbf{M_1})$ if $\mathbf{M_2} = \mathbf{M_1} + [k \;\; -k; \;\; -k \;\; k]$.
P3: $O(\mathbf{M_2}) < O(\mathbf{M_1})$ if $\mathbf{M_2} = \mathbf{M_1} + [0 \;\; k; \;\; 0 \;\; -k]$ or $\mathbf{M_2} = \mathbf{M_1} + [0 \;\; 0; \;\; k \;\; -k]$.
O1: Property 1: Symmetry under variable permutation.
O2: Property 2: Row and Column scaling invariance.
O3: Property 3: Antisymmetry under row or column permutation.
O3': Property 4: Inversion invariance.
O4: Property 5: Null invariance.
Yes*: Yes if measure is normalized.
No*: Symmetry under row or column permutation.
No**: No unless the measure is symmetrized by taking $\max(M(A,B), M(B,A))$.

Comparison of Interestingness Measures

- Null-(transaction) invariance is crucial for correlation analysis
- Lift and χ^2 are not null-invariant
- 5 null-invariant measures

	Milk	No Milk	Sum (row)
Coffee	m, c	~m, c	c
No Coffee	m, ~c	~m, ~c	~c
Sum(col.)	m	~m	Σ

Measure	Definition	Range	Null-Invariant
$\chi^2(a,b)$	$\sum_{i,j=0,1} \frac{(e(a_i,b_j)-o(a_i,b_j))^2}{e(a_i,b_j)}$	$[0,\infty]$	No
$Lift(a,b)$	$\frac{P(ab)}{P(a)P(b)}$	$[0,\infty]$	No
$AllConf(a,b)$	$\frac{sup(ab)}{max\{sup(a),sup(b)\}}$	$[0,1]$	Yes
$Coherence(a,b)$	$\frac{sup(ab)}{sup(a)+sup(b)-sup(ab)}$	$[0,1]$	Yes
$Cosine(a,b)$	$\frac{sup(ab)}{\sqrt{sup(a)sup(b)}}$	$[0,1]$	Yes
$Kulc(a,b)$	$\frac{sup(ab)}{2}(\frac{1}{sup(a)}+\frac{1}{sup(b)})$	$[0,1]$	Yes
$MaxConf$(a,b)	$max\{\frac{sup(ab)}{sup(a)},\frac{sup(ab)}{sup(b)}\}$	$[0,1]$	Yes

Table 3. Interestingness measure definitions.

Null-transactions w.r.t. m and c

Kulczynski measure (1927)

Null-invariant

Data set	mc	$\overline{m}c$	$m\overline{c}$	$\overline{mc}$	χ^2	$Lift$	$AllConf$	$Coherence$	$Cosine$	$Kulc$	$MaxConf$
D_1	10,000	1,000	1,000	100,000	90557	9.26	0.91	0.83	0.91	0.91	0.91
D_2	10,000	1,000	1,000	100	0	1	0.91	0.83	0.91	0.91	0.91
D_3	100	1,000	1,000	100,000	670	8.44	0.09	0.05	0.09	0.09	0.09
D_4	1,000	1,000	1,000	100,000	24740	25.75	0.5	0.33	0.5	0.5	0.5
D_5	1,000	100	10,000	100,000	8173	9.18	0.09	0.09	0.29	0.5	0.91
D_6	1,000	10	100,000	100,000	965	1.97	0.01	0.01	0.10	0.5	0.99

Table 2. Example data sets.

Subtle: They disagree

Analysis of DBLP Coauthor Relationships

Recent DB conferences, removing balanced associations, low sup, etc.

ID	Author a	Author b	$sup(ab)$	$sup(a)$	$sup(b)$	$Coherence$	$Cosine$	$Kulc$
1	Hans-Peter Kriegel	Martin Ester	28	146	54	0.163 (2)	0.315 (7)	0.355 (9)
2	Michael Carey	Miron Livny	26	104	58	0.191 (1)	0.335 (4)	0.349 (10)
3	Hans-Peter Kriegel	Joerg Sander	24	146	36	0.152 (3)	0.331 (5)	0.416 (8)
4	Christos Faloutsos	Spiros Papadimitriou	20	162	26	0.119 (7)	0.308 (10)	0.446 (7)
5	Hans-Peter Kriegel	Martin Pfeifle	18	146	18	0.123 (6)	0.351 (2)	0.562 (2)
6	Hector Garcia-Molina	Wilburt Labio	16	144	18	0.110 (9)	0.314 (8)	0.500 (4)
7	Divyakant Agrawal	Wang Hsiung	16	120	16	0.133 (5)	0.365 (1)	0.567 (1)
8	Elke Rundensteiner	Murali Mani	16	104	20	0.148 (4)	0.351 (3)	0.477 (6)
9	Divyakant Agrawal	Oliver Po	12	120	12	0.100 (10)	0.316 (6)	0.550 (3)
10	Gerhard Weikum	Martin Theobald	12	106	14	0.111 (8)	0.312 (9)	0.485 (5)

Table 5. Experiment on DBLP data set.

Advisor-advisee relation: Kulc: high, coherence: low, cosine: middle

- Tianyi Wu, Yuguo Chen and Jiawei Han, "Association Mining in Large Databases: A Re-Examination of Its Measures", Proc. 2007 Int. Conf. Principles and Practice of Knowledge Discovery in Databases (PKDD'07), Sept. 2007

Which Null-Invariant Measure Is Better?

- IR (Imbalance Ratio): measure the imbalance of two itemsets A and B in rule implications

$$IR(A,B) = \frac{|sup(A) - sup(B)|}{sup(A) + sup(B) - sup(A \cup B)}$$

- Kulczynski and Imbalance Ratio (IR) together present a clear picture for all the three datasets D_4 through D_6
  - D_4 is balanced & neutral
  - D_5 is imbalanced & neutral
  - D_6 is very imbalanced & neutral

$Data$	mc	$\overline{m}c$	$m\overline{c}$	$\overline{mc}$	$all\_conf.$	$max\_conf.$	$Kulc.$	$cosine$	IR
D_1	10,000	1,000	1,000	100,000	0.91	0.91	0.91	0.91	0.0
D_2	10,000	1,000	1,000	100	0.91	0.91	0.91	0.91	0.0
D_3	100	1,000	1,000	100,000	0.09	0.09	0.09	0.09	0.0
D_4	1,000	1,000	1,000	100,000	0.5	0.5	0.5	0.5	0.0
D_5	1,000	100	10,000	100,000	0.09	0.91	0.5	0.29	0.89
D_6	1,000	10	100,000	100,000	0.01	0.99	0.5	0.10	0.99

Chapter 5: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Frequent Itemset Mining Methods
- Which Patterns Are Interesting?—Pattern Evaluation Methods
- Summary

Summary

- Basic concepts: association rules, support-confident framework, closed and max-patterns
- Scalable frequent pattern mining methods
  - Apriori (Candidate generation & test)
  - Projection-based (FPgrowth, CLOSET+, ...)
- Which patterns are interesting?
  - Pattern evaluation methods

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